Polarisation computations in the STRIPPER framework

Rene Poncelet

THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

COMETA

NNLO QCD

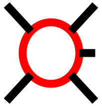
$$\sigma_{h_1h_2\to X} = \sum_{ij}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\phi_{i,h_1}(x_1,\mu_F^2)\phi_{j/h_2}(x_2,\mu_F^2)\hat\sigma_{ij\to X}(\alpha_s(\mu_R^2),\mu_R^2,\mu_F^2)$$

Perturbative expansion: $\hat\sigma_{ab\to X} = \hat\sigma^{(0)}_{ab\to X} + \hat\sigma^{(1)}_{ab\to X} + \hat\sigma^{(2)}_{ab\to X} + \mathcal{O}(\alpha_s^3)$

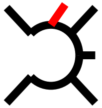
Next-to-leading order

$$\hat\sigma^{(1)}_{ab} = \hat\sigma^{\mathrm{R}}_{ab} + \hat\sigma^{\mathrm{V}}_{ab} + \hat\sigma^{\mathrm{C}}_{ab}$$

$$\hat\sigma^{\mathrm{V}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_n\, 2\mathrm{Re}\left\langle \mathcal{M}_n^{(0)}\middle|\mathcal{M}_n^{(1)}\right\rangle \mathrm{F}_n$$



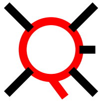
$$\hat\sigma^{\mathrm{R}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_{n+1}\left\langle \mathcal{M}_{n+1}^{(0)}\middle|\mathcal{M}_{n+1}^{(0)}\right\rangle \mathrm{F}_{n+1}$$



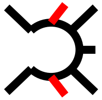
Next-to-next-to-leading order

$$\hat\sigma^{(2)}_{ab} = \hat\sigma^{\mathrm{VV}}_{ab} + \hat\sigma^{\mathrm{RV}}_{ab} + \hat\sigma^{\mathrm{RR}}_{ab} + \hat\sigma^{\mathrm{C2}}_{ab} + \hat\sigma^{\mathrm{C1}}_{ab}$$

$$\hat\sigma^{\mathrm{VV}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_n\left(2\mathrm{Re}\left\langle \mathcal{M}_n^{(0)}\middle|\mathcal{M}_n^{(2)}\right\rangle + \left\langle \mathcal{M}_n^{(1)}\middle|\mathcal{M}_n^{(1)}\right\rangle\right)\mathrm{F}_n$$



$$\hat\sigma^{\mathrm{RV}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_{n+1}\, 2\mathrm{Re}\left\langle \mathcal{M}_{n+1}^{(0)}\middle|\mathcal{M}_{n+1}^{(1)}\right\rangle \mathrm{F}_{n+1}$$



$$\hat\sigma^{\mathrm{RR}}_{ab} = \frac{1}{2\hat s}\int \mathrm{d}\Phi_{n+2}\left\langle \mathcal{M}_{n+2}^{(0)}\middle|\mathcal{M}_{n+2}^{(0)}\right\rangle \mathrm{F}_{n+2}$$

NNLO QCD

$$\sigma_{h_1 h_2 \to X} = \sum_{ij} \int_0^1 \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \phi_{i,h_1}(x_1, \mu_F^2) \phi_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}_{ij \to X}(\alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

Perturbative expansion: $\hat{\sigma}_{ab \to X} = \hat{\sigma}^{(0)}_{ab \to X} + \hat{\sigma}^{(1)}_{ab \to X} + \hat{\sigma}^{(2)}_{ab \to X} + \mathcal{O}(\alpha_s^3)$

NNLO QCD schemes

Slicing

qT-slicing [Catain'07],
N-jettiness slicing [Gaunt'15/Boughezal'15]

Subtraction

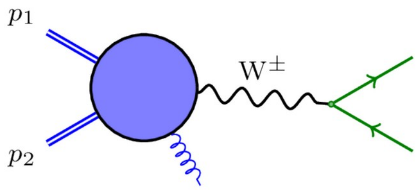
Antenna [Gehrmann'05-'08]
Colorful [DelDuca'05-'15]
Projection [Cacciari'15]
Geometric [Herzog'18]
Unsubtraction [Aguilera-Verdugo'19]
Nested collinear [Caola'17]
Sector-improved residue subtraction [Czakon'10-'14'19]

Sector-improved residue subtraction C++ framework

- Formulation allows efficient algorithmic implementation → STRIPPER
- **High degree of automation**:
  - Partonic processes (taking into account all symmetries)
  - Sectors and subtraction terms
  - Interfaces to Matrix-element providers + O(100) hardcoded: AvH, OpenLoops, Recola, NJET, HardCoded

    → In practice: Only two-loop matrix elements required
- **Broad range of applications** through additional facilities:
  - Narrow-Width & Double-Pole Approximation
  - Fragmentation
  - Polarised intermediate massive bosons
  - (Partial) Unweighting → Event generation for **HighTEA**
  - Interfaces: FastNLO, FastJet

Ingredients for polarised predictions



$$M_\lambda = \mathbf{P}_\mu \cdot \frac{-g_{\mu\nu} + \frac{k^\mu k^\nu}{k^2}}{k^2 - M_V^2 + iM_V\Gamma_V} \cdot \mathbf{D}_\nu$$

on-shell polarization sum:

$$\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right) \to \sum_\lambda \epsilon_\lambda^{*\mu} \epsilon_\lambda^\nu$$

On-shell mapping:

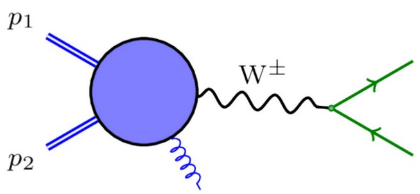
- Narrow-width-approximation (NWA)
- (double) Pole-approximation (DPA)

Polarisation vectors:

- frame dependent

Subtraction scheme modifications (?)

Ingredients for polarised predictions



$$M_\lambda = \mathbf{P}_\mu \cdot \frac{-g_{\mu\nu} + \frac{k^\mu k^\nu}{k^2}}{k^2 - M_V^2 + iM_V\Gamma_V} \cdot \mathbf{D}_\nu$$

on-shell polarization sum:

$$\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right) \to \sum_\lambda \epsilon_\lambda^{*\mu}\epsilon_\lambda^\nu$$

'arbitrary' choices → impact on pol. observables

On-shell mapping:

- Narrow-width-approximation (NWA)
- (double) Pole-approximation (DPA)

Polarisation vectors:

- frame dependent

Subtraction scheme modifications (?)

On-shell mappings in STRIPPER

1. Narrow-width-approximation (NWA) [hep-ph/9904472]

- Uniquely defined limit (at cross sections level)
- Full factorization of production and decay (while keeping spin dependence)
- No off-shell effects, restricted phase space
- Corrections of $\mathcal{O}\left(\frac{\Gamma}{m}\right)$ expected

For example NWA for top-quark pairs

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}p_t^2}{2\pi} T_D(p_t^2) \xrightarrow{\frac{\Gamma_t}{m_t}\to 0} \int_{-\infty}^{\infty} \mathrm{d}p_t^2 \frac{\delta(p_t^2 - m_t^2)}{2m_t\Gamma_t} \qquad T(p_t) = \frac{i}{\not{p}_t - \mu_t} \quad \text{with} \quad \mu_t^2 = m_t^2 - i m_t \Gamma_t$$

$$\hat{\sigma}^{(0)} \xrightarrow{\frac{\Gamma_t}{m_t}\to 0} \hat{\sigma}^{(0)}_{\mathrm{NWA}} \equiv \frac{1}{\hat{s}}\frac{1}{\mathcal{N}} \int \mathrm{d}\Phi_{t\bar{t}}\,\mathrm{d}\Phi_{\Gamma_t}\,\mathrm{d}\Phi_{\Gamma_{\bar{t}}} \frac{\langle \mathcal{M}_{\mathrm{res}} | \mathcal{M}_{\mathrm{res}} \rangle}{(2m_t\Gamma_t)^2}\bigg|_{p_t^2 = m_t^2,\, p_{\bar{t}}^2 = m_t^2}$$

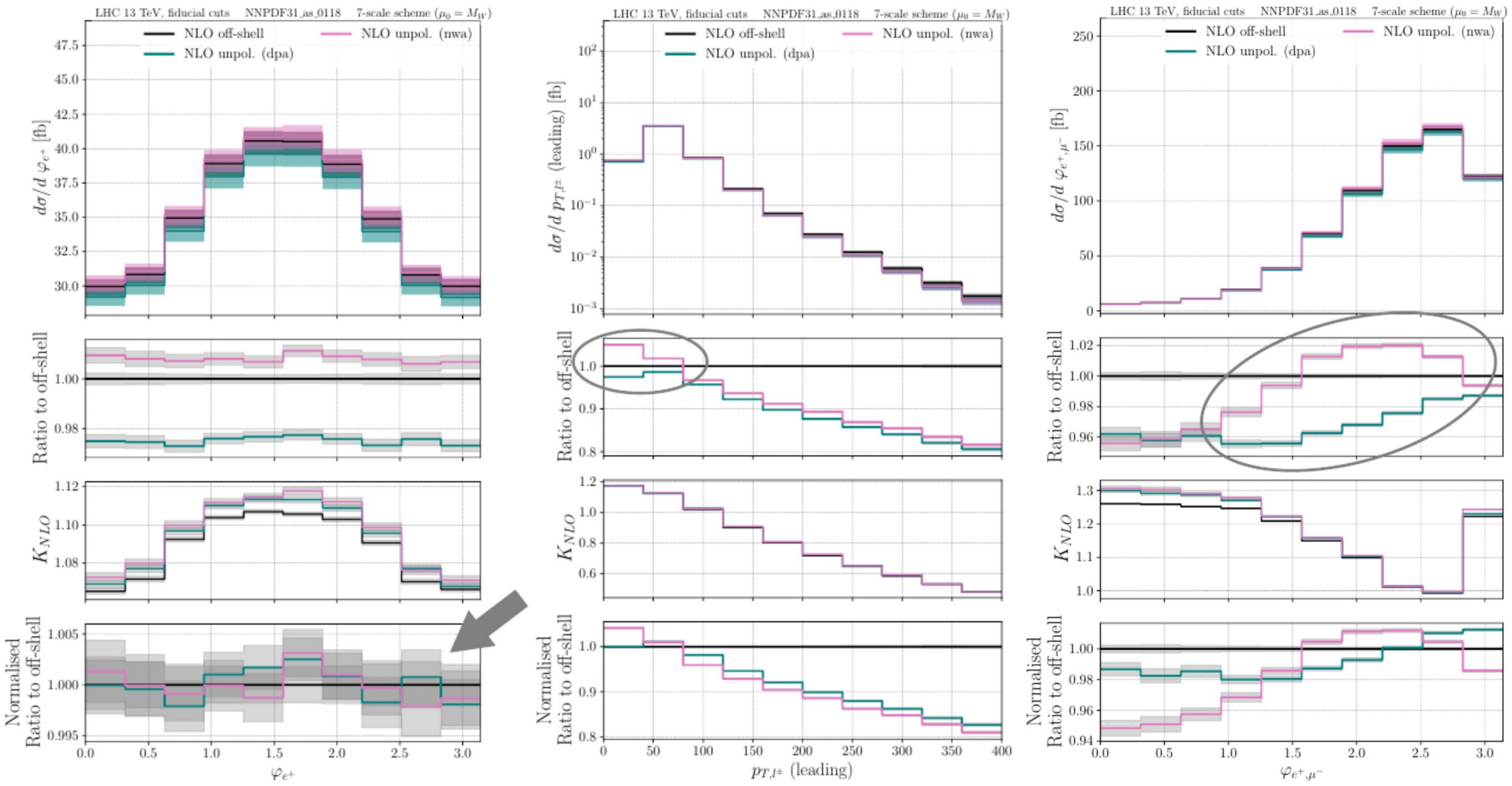
$$\langle \mathcal{M}_{\mathrm{res}} | \mathcal{M}_{\mathrm{res}} \rangle = \sum_{h,h',\bar{h},\bar{h}'} \big\langle M_{\mathrm{prod}}(h',\bar{h}') \big| M_{\mathrm{prod}}(h,\bar{h}) \big\rangle \big\langle \Gamma_t(h') \big| \Gamma_t(h) \big\rangle \big\langle \Gamma_{\bar{t}}(\bar{h}') \big| \Gamma_{\bar{t}}(\bar{h}) \big\rangle$$

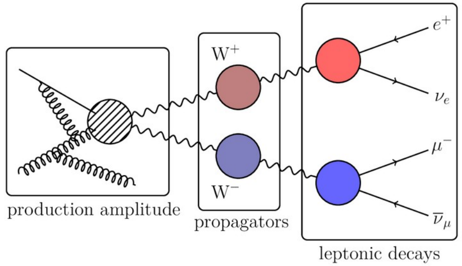
On-shell mappings in STRIPPER

2. (double) Pole-approximation (DPA) [1310.1564]

- Full phase space
- On-shell matrix elements (exactly the same that go into NWA) defined through on-shell projection
- Off-shell Breit-Wigner propagators → capture kinematic off-shell effects
- Choices to be made about what quantities to preserve (we follow [2107.06579]):
    - Invariant mass of boson system
    - Certain angles in specific frames

Comparisons between DPA and NWA





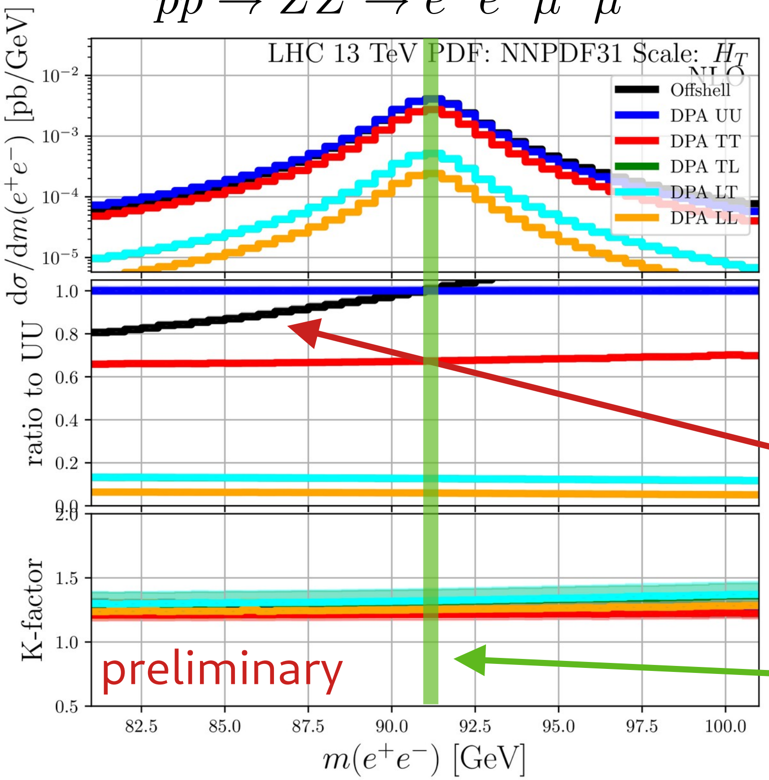
Systematic shape differences of few percent

NNLO QCD study of polarised W+W- production at the LHC,
Poncelet, Popescu 2102.13583

DPA off-shell effects

DPA captures certain off-shell effects

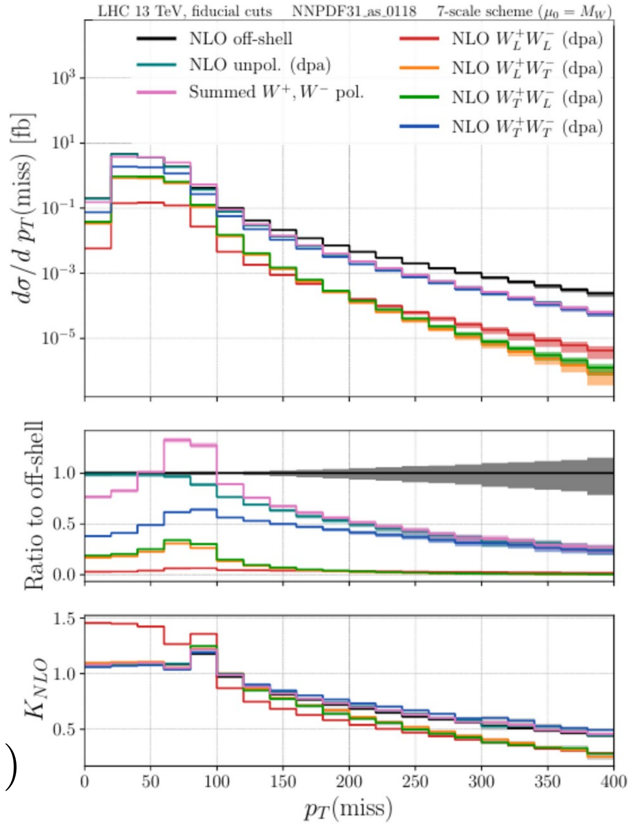
$pp \to ZZ \to e^+e^-\mu^+\mu^-$



DPA
vs
Off-shell

NWA
→ $\delta(m(e^+e^-) - m_Z)$

But not all…
e.g. single-resonant contributions



Polarization frames

Two options implemented:

- Laboratory frame
- Respective vector-boson rest-frame (directions by simple boost from lap-frame)

$$\varepsilon^\mu_- = \frac{1}{\sqrt{2}}\left(0, \cos\theta_V \cos\phi_V + \mathrm{i}\sin\phi_V, \cos\theta_V \sin\phi_V - \mathrm{i}\cos\phi_V, -\sin\theta_V\right),$$
$$\varepsilon^\mu_+ = \frac{1}{\sqrt{2}}\left(0, -\cos\theta_V \cos\phi_V + \mathrm{i}\sin\phi_V, -\cos\theta_V \sin\phi_V - \mathrm{i}\cos\phi_V, \sin\theta_V\right),$$
$$\varepsilon^\mu_{\mathrm{L}} = \frac{1}{M}\left(p, E\sin\theta_V \cos\phi_V, E\sin\theta_V \sin\phi_V, E\cos\theta_V\right),$$

Interplay between polarization frame and higher-order QCD corrections?
→ recoil **might** introduce 'spurious' effects in polarization fractions
→ enhancement of NNLO QCD effects on longitudinal fractions ?

NNLO K-factor
UU: 1.02
LL : 1.06

QCD Order	$pp \to W^+W^-$		$pp \to ZZ$	
Pol. frame	Lab		V rest	
	UU	LL	UU	LL
NLO	214.55(7) fb	9.064(6) fb	15.159(1) fb	0.889(1) fb
NNLO	219.4(4) fb	9.88(3) fb	16.06(2) fb	0.975(1) fb
NNLO+LI	232.7(4) fb	10.57(3) fb	17.39(2) fb	1.073(1) fb

NNLO K-factor
UU: 1.06
LL : 1.10

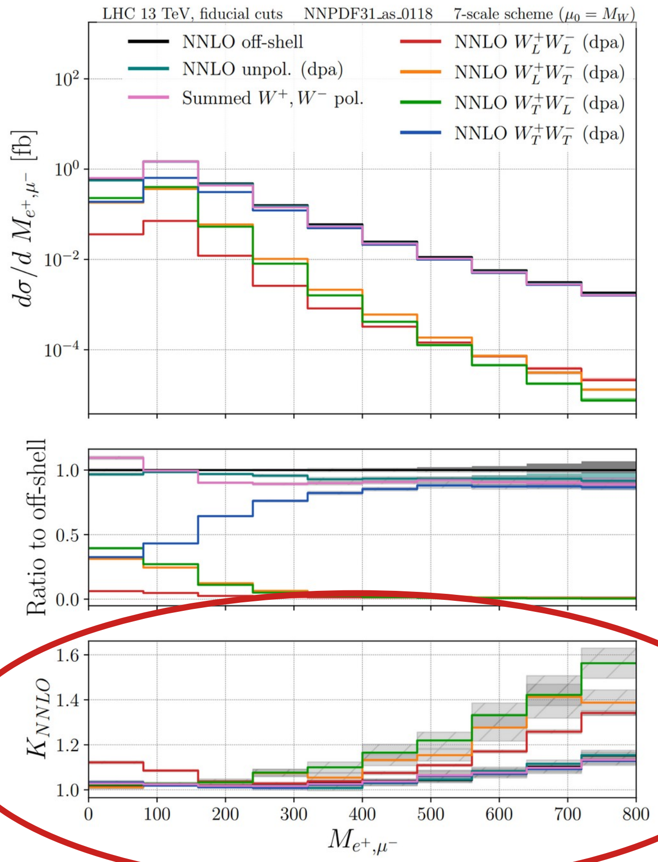
Impact on LL component from NNLO QCD in both frames!

Polarization frames – higher QCD

(Sorry for the confusing colour coding ;-))

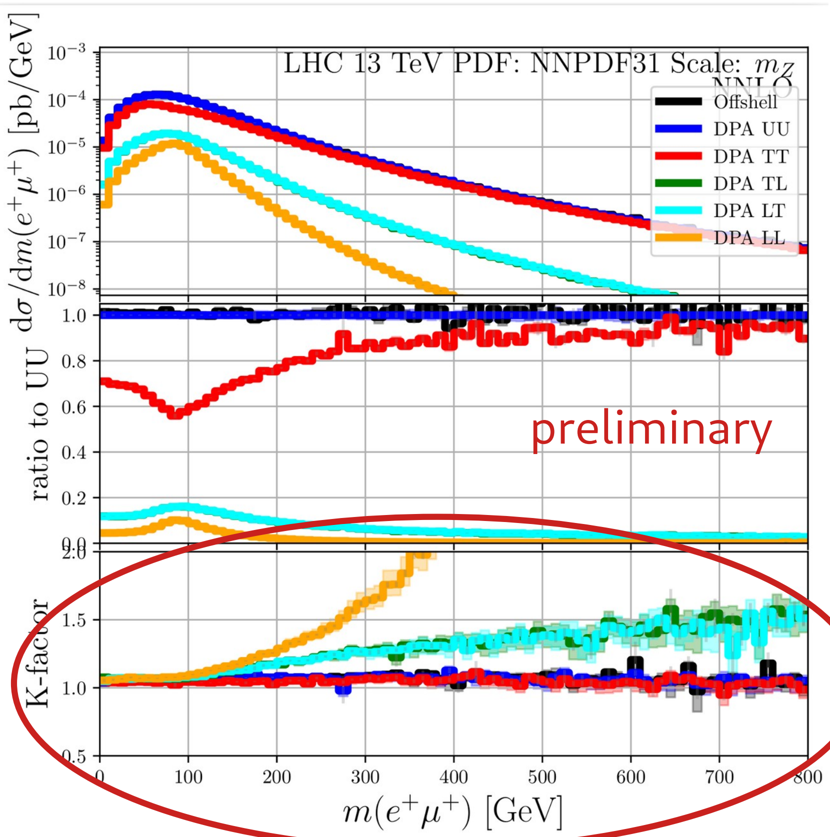
$pp \to W^+W^-$

Lab frame



$pp \to ZZ$

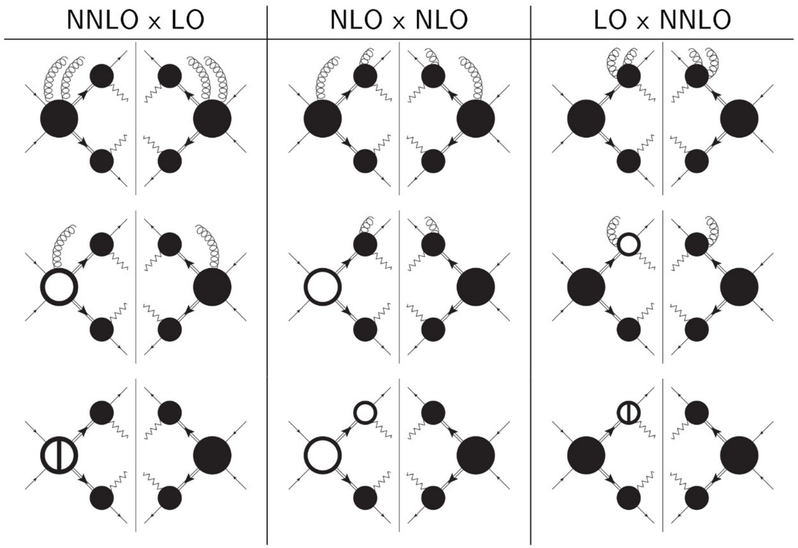
V rest frame



Subtraction scheme and polarization

- So far only QCD corrections considered
  → no need to modifications
- Works also for loop-induced processes example: pp → H with b+t mass effects [2312.09896 & 2407.12413]
- What about non-leptonic decays?
  - Higher-order QCD corrections to NWA decays
  - used for the much more complicated top-pair case already
  - DPA would require to keep track of resonances

$$pp \to t\bar{t} \to \ell\bar{\ell}v\bar{v}b\bar{b}$$



[2008.11133 & 1901.05407]

HighTEA

How to make NNLO QCD calculations easy?

high tea
for your freshly brewed analysis

Try it, it's fun :)

https://www.precision.hep.phy.cam.ac.uk/hightea

<u>What is HighTEA in a nutshell?</u>

- Database of pre-calculated fixed-order events (think Ntuples [BlackHat '08'13])
  → Equivalent to a fully fledged calculation
- Equipped with an easy-to-use interface: bash, python, webform

HighTEA: **High energy Theory Event Analyser**

[2304.05993]

Michał Czakon,[a] Zahari Kassabov,[b] Alexander Mitov,[c] Rene Poncelet,[c] Andrei Popescu[c]

[a] *Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany*
[b] *DAMTP, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom*
[c] *Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom*

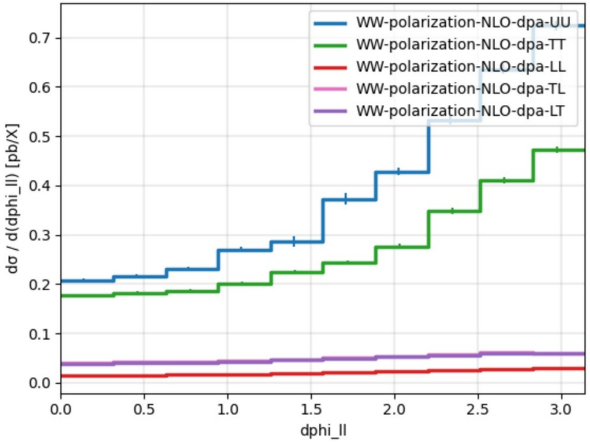
E-mail: mczakon@physik.rwth-aachen.de, zk261@cam.ac.uk, adm74@cam.ac.uk, poncelet@hep.phy.cam.ac.uk, andrei.popescu@cantab.net

Polarized VV in HighTEA

Work-in-progress, but first example online

pp→ WW @ NLO QCD (takes only seconds to run)

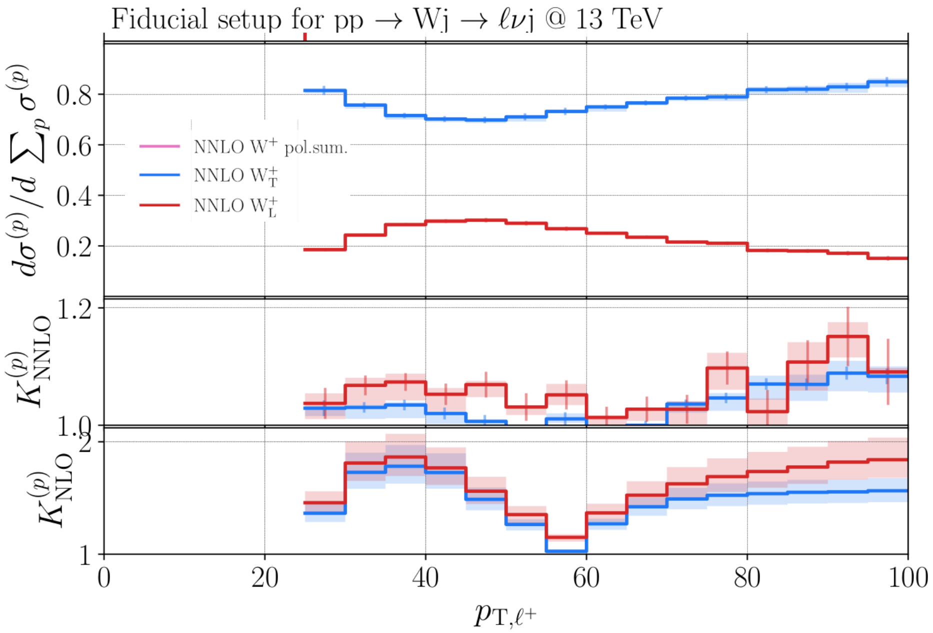
```
jobs = {}

printit = True

for mode in ["LO-dpa-UU","LO-dpa-TT","LO-dpa-LL","LO-dpa-TL","LO-dpa-LT",
             "NLO-dpa-UU","NLO-dpa-TT","NLO-dpa-LL","NLO-dpa-TL","NLO-dpa-LT"]:

    print('Current mode: ',mode)

    jobs[mode] = hightea('WW-polarization-'+mode,directory=USERDIR,overwrite=False)
    jobs[mode].process('pp_ww_pol_13TeV',printit)
    printit=False

    jobs[mode].contribution(mode)

    jobs[mode].define_new_variable('a_lp','arctan2(p_lp_1,p_lp_2)')
    jobs[mode].define_new_variable('a_lm','arctan2(p_lm_1,p_lm_2)')
    jobs[mode].define_new_variable('dphi_ll','fabs(a_lp-a_lm)*(pi > fabs(a_lp-a_lm))+(2*pi-fabs(a_lp-a_lm))*(pi < fabs(a_lp-a_lm))')

    jobs[mode].observable('dphi_ll',list(np.linspace(0,np.pi,11)))

    jobs[mode].request()
```



NNLO QCD on the way…

Do we need NNLO QCD corrections?

Example: $pp \to W^{\pm}(\to l\nu)j$



Important

Non-trivial differential NNLO K-factors!

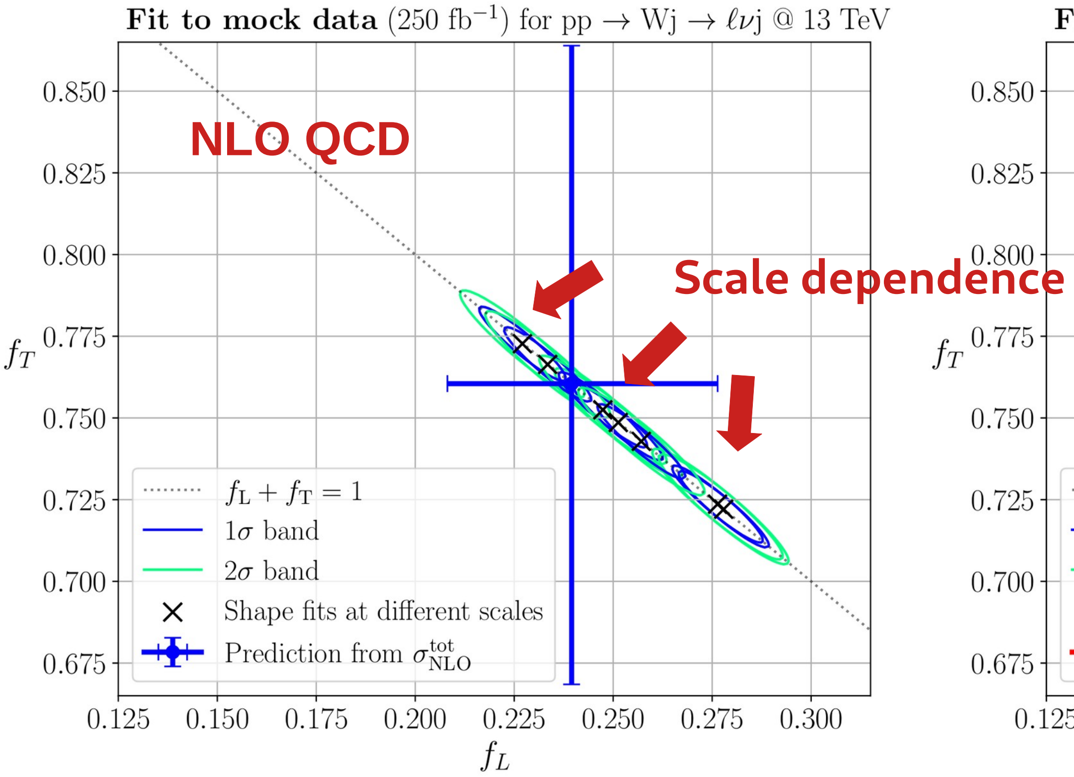
1) Differential polarization fraction have shapes
2) Higher-order corrections dependent on polarization! Just using unpolarized K-factor would lead to distortion of spectrum.
3) NNLO QCD needed to reach percent-level scale-dependence → MHO

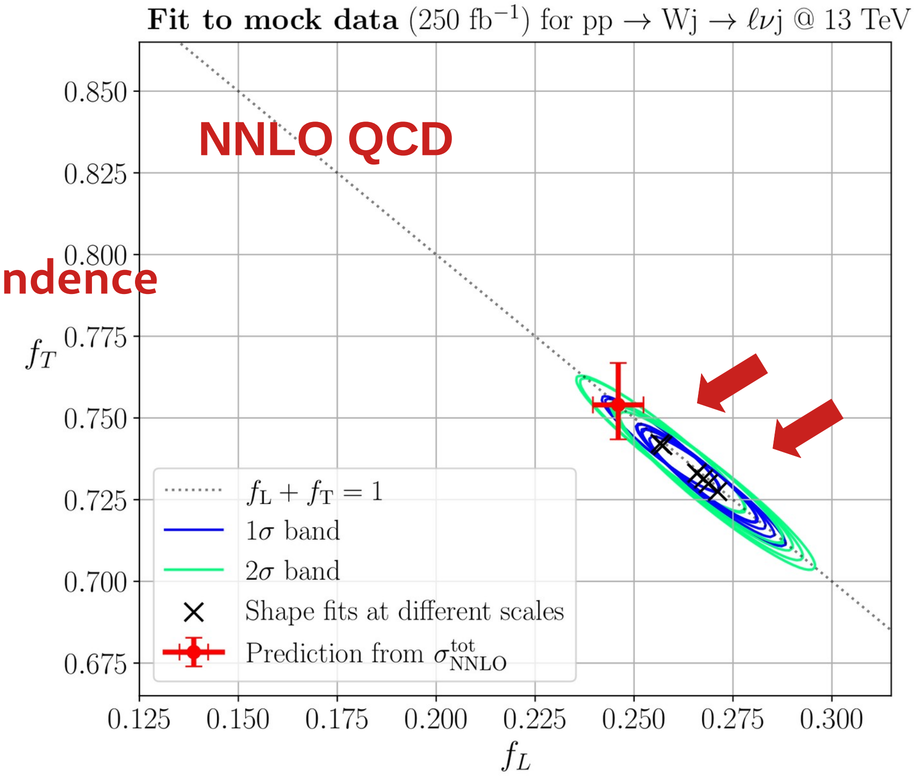
Polarised W+j production at the LHC: a study at NNLO QCD accuracy, Pellen, Poncelet, Popescu 2109.14336

Do we need NNLO QCD corrections?

Fit to mock-data (based on NNLO QCD and 250 fb^{-1} stats):
→ extreme case to see effect of scale dependence reduction

Observable: $\cos(\ell, j_1)$





Summary and Outlook

Summary

- STRIPPER: automated NNLO QCD subtraction scheme
- Polarization features: DPA & NWA, LAB & V-boson folarization frames
- Polarized processes @ NNLO QCD: pp → Vj, pp → WW, NEW: pp → ZZ
- NEW: polarized cross sections in HighTEA (wip)

Outlook

- The other pp → VV underway
- Extending HighTEA setup
- Polarization in top-quark production and decay
- Non-leptonic decays in NWA including NNLO QCD corrections