Magnetic field amplification and decay in cosmic string wakes

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References

1. D.B., D. Kumar, S. Nayak, S. Sanyal, Magnetic field amplification and decay in cosmic string wakes [arXiv:2408.01778]
2. S. Nayak, S. Sau, S. Sanyal, Evolution of magnetic fields in cosmic string wakes, *Astroparticle Physics*, Volume 146, 2023, 102805 [arXiv:2204.13303v2]
3. D. Kumar and S. Sanyal, Magnetic Reconnection in the Wakes of Cosmic Strings *Astrophysical Journal* 944 183 (2023) [arXiv:2210.17164v1]
4. S. Sau and S. Sanyal, Neutrino currents in wakes of cosmic strings. *Eur. Phys. J. C* 80, 152 (2020) [arXiv:1906.09733v2]

Introduction

Cosmic String as a 1D topological defect

- Toy example: $\mathcal{L} = |\partial_\mu \Phi|^2 - \frac{\lambda}{2}(|\Phi|^2 - \eta^2)^2$ after global $U(1)$ symmetry breaking
- VEV: $\langle \Phi \rangle = \eta e^{i\alpha}$ (new stable minima)
- Which stable vacua (α) is picked depends on fluctuations $\Rightarrow$ domain formation
- $\Delta\alpha = 2\pi n$ over spatial loop. Shrinking loop continuously can't make it zero
- Discontinuity is encountered. Trapped old vacua: cosmic string solution

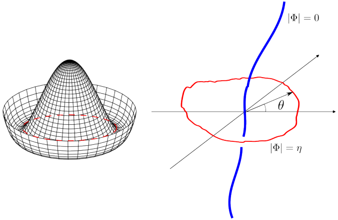

Figure 1: Going over a loop in physical space

[Cosmic strings 1506.04039]

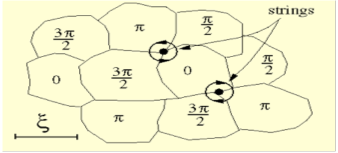

Figure 2: Domains of different vacua

[CTC Cambridge]

Motivation

Cosmological importance of topological defects and cosmic strings

- Topological defects: directly connected to very early universe
- Likely in several phase transition scenarios
- In particular, cosmic strings have trapped energy density; don't contradict basic cosmological observations

Cosmic string characteristics:

- Abelian strings have finite mass per unit length: $G\mu \sim 10^{-6}$
- Signatures like gravitational waves, lensing, synchrotron radiation etc.
- role in large-scale structure formation, primordial magnetic field origin, evolution

Cosmic string wakes play a critical role

Cosmic String Wake formation

Sheet-like structures behind cosmic strings

- Cosmic string metric is locally flat but **globally conical** (deficit angle $\theta = 8\pi G\mu$)
- Gives background matter/plasma a **kinematic boost** $\Delta v = 4\pi G\mu v_s \gamma_v$

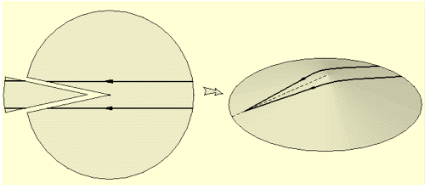

Figure 3: Globally conical, locally flat metric

[*"Cosmic Strings and Large Scale Structures"*, CTC Cambridge]

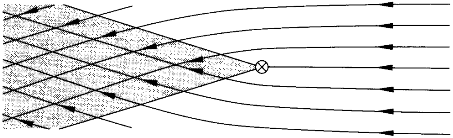

Figure 4: Overdense wake structure in string's rest frame

[*Cosmic Strings and Other Topological Defects*, A. Vilenkin]

Magnetized Wakes

Wakes with magnetic field in the plasma

- Magnetic fields can be generated in cosmic string wakes
- The background plasma and magnetic field interact and evolve with the wake
- Possibility of magnetic reconnections $\Rightarrow$ new signatures

[S. Sau and S. Sanyal, Eur. Phys. J. C, 80(2), February 2020]
[D. Kumar and S. Sanyal, ApJ 944(2):183, feb 2023.]

These motivate us to explore

- Magnetohydrodynamic simulations of wakes in magnetized plasma
- Identifying the core features of the field evolution
- Characteristic scales, relevance to magnetic reconnections?

Ideal Magnetohydrodynamics (MHD)

Framework for numerical simulation

Large-scale matter flow around string is collisional; astrophysical/early universe plasma is highly conductive

[A. Beresnyak, Astrophys. J., 804(2):121, 2015]

- So, we use Ideal MHD: inviscid and infinitely conductive fluid

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0 \,, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \,,$$
$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} \right) = -\nabla \left(P + \frac{B^2}{8\pi} \right) + \frac{(\mathbf{B}.\nabla)\mathbf{B}}{4\pi} \,. \tag{1}$$

- Plus an equation of state (polytropic: $P = K\rho^{\gamma}$, $\gamma = 5/3$)
- Vorticity $\omega = \nabla \times \mathbf{v}$ depends on velocity and magnetic fields

Numerical Simulation

Package and Initial setup

- Software package: `OpenMHD`, a finite-volume Fortran code for MHD
- Simulations confined to **2D discretized xy plane**; Stepsize/lengthscale $\Delta l = 1$ unit.
- Open boundary conditions. Turbulence not generated due to boundary conditions or by vorticity at different lengthscales

Scales

Typical/initial values of field variables:
ρ_0, B_0, v_0, P_0
Sound speed $c_s^2 = \gamma P_0 / \rho_0$

Parameters

plasma-$\beta = 2P/B^2$,
Mach number $\mathcal{M} = v_0 / c_s$,
Cosmic string deficit angle 2θ

$$\text{time scale} \propto \frac{\text{length scale}}{\mathcal{M}}$$

Simulating the Wake

- Simulation happens in **string's rest frame**.
- To the string's right plasma has constant flow $\mathbf{v} = -v_0 \hat{x}$
- To the string's left, **perturbation** is given such that total velocity is unchanged
- If wake angle $= 2\theta$, we have $v'_y = \delta v_y = v_0 \sin\theta, v'_x = v_0 \cos\theta$
- This velocity discontinuity implies **local vorticity**

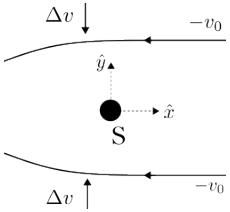

Figure 5: Wake simulation geometry and setup

[P. P. Avelino and E. P. S. Shellard, Phys. Rev. D, 51:5946–5949, May 1995.]

Hydrodynamic wake simulation

Visualizing hydrodynamic wake using streamlines

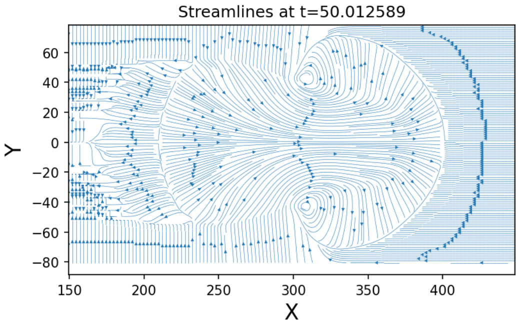

Figure 6: String at $(x_s, 0)$. Vortices are formed due to evolution of the initial velocity perturbations and vorticity flow. No magnetic field is taken. Total vorticity is conserved. $\mathcal{M} = 10$, $\theta = 15^\circ$

Wakes with magnetic field

- MHD wake simulations with magnetic field turned on in xy plane
- Magnitude fixed by β, taken 10. Higher βs give similar results. Flow dynamics **not dominated by magnetic field**
- Total vorticity is conserved.
- Non-uniform, sheared fields; more likely to occur in wakes
- Different field configurations spatially varying over a **range of lengthscales** are used for simulations
  (As fields in wake can be generated at varied lengthscales)

Amplification of B field

Sheared perturbation along the flow

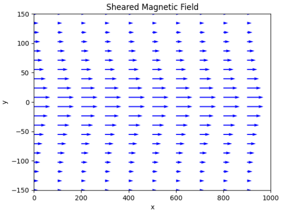

Figure 7: $\mathbf{B} = B_0 \hat{x}\, e^{-\alpha_1 |y|}$ sheared magnetic field with lengthscale $l_0 \sim 1/\alpha_1$

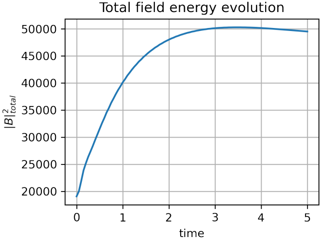

Figure 8: Total magnetic field energy increasing with time

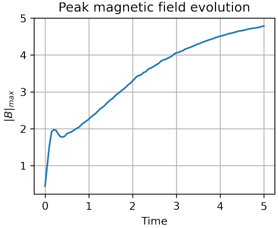

Figure 9: Peak value of magnetic field increasing with time

Magnetic field amplifies for all α_1 and geometries sheared along the flow. Results shown for $\alpha_1 = 0.01$

Reasons for amplification

- **Alfven's theorem**, valid in ideal MHD:
  Magnetic field lines are frozen in the fluid
- So, fluid overdensity and deformation in the wake causes B field amplification
- Skew deformations of fluid affect both B field direction and strength
- Analysis by deformation tensor of fluid shows magnetic field increases with density

$$B \propto \rho^a$$

- So final field strength is more than initial field strength in the wake

[Emma J. King and Peter Coles, MNRAS 365(4):1288–1294, February 2006.]

Amplification with perpendicular shear

Sheared magnetic perturbation perpendicular to the flow

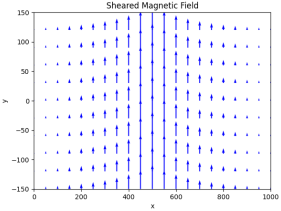

Figure 10: $\mathbf{B} = B_0 \hat{y} e^{-\alpha_2 |x - x_s|}$ perpendicular sheared field with lengthscale $l_0 \sim 1/\alpha_2$

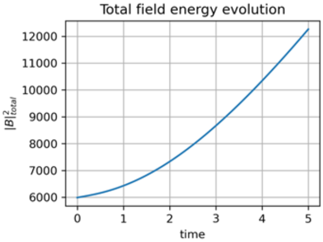

Figure 11: Total magnetic field energy increasing with time

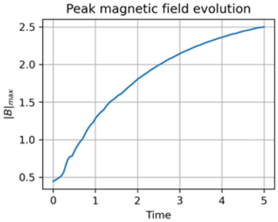

Figure 12: Peak value of magnetic field increasing with time

Amplification for $\alpha_2 \lesssim 1$ (large lengthscales). Results shown for $\alpha_2 = 0.01$.

Decay with perpendicular shear

Sheared magnetic perturbation perpendicular to the flow

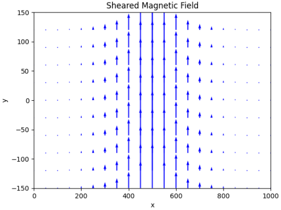

Figure 13: $\mathbf{B} = B_0\hat{y}e^{-\alpha_2|x-x_s|}$ perpendicular sheared field with smaller l_0

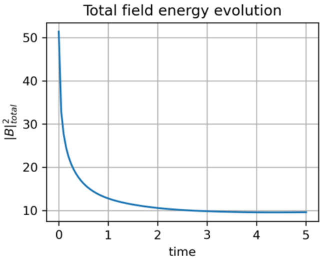

Figure 14: Total magnetic field energy decreasing with time

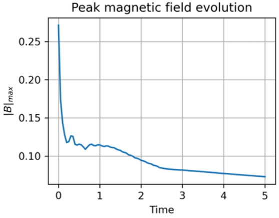

Figure 15: Peak value of magnetic field decreasing with time

Decay of magnetic field for $\alpha_2 \gtrsim 1$ (smaller lengthscale). Results shown for $\alpha_2 = 1$.

Reasons for decay

Breakdown of Alfven's theorem

- Alfven theorem implies magnetic field lines flow with the velocity of the fluid
- At individual charged particle level: guiding center approximation is used
- Valid as long as the gradient drift is small

$$v_g \propto \frac{r_g}{l_0} \Rightarrow r_g << l_0 \ , \quad \left(r_g = \frac{mcv}{qB} \ , \quad l_0 : B\text{-field lengthscale} \right) \tag{2}$$

- Guiding center approx. breaks down when $r_g \sim l_0$. Magnetic field lines don't flow with the plasma.
- In the simulation this is verified by the values of r_g and l_0
- Magnetic reconnection requires similar breakdown of Alfven's theorem

[Gregory L. Eyink, Hussein Aluie, Physica D, Volume 223, Issue 1, 2006]
[Bhimsen K. Shivamoggi, Physics Reports, Volume 127, Issue 2, 1985]

Summary

- Amplification and decay of magnetic fields in cosmic string wakes is studied in absence of dynamo mechanism
- Found strong dependence on magnetic field geometry and lengthscales in the field's evolution.
- Decay of magnetic field occurs for lengthscales below gyroradius r_g associated with the field and plasma particle
- Alfven's theorem breakdown near r_g suggests a scale for magnetic reconnections
- Resistive MHD, $3D$ simulations etc. are possible future directions

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Thank You!