Fermions Mass and Mixing Hierarchies through $U(1)_X$ and Z_N Symmetries

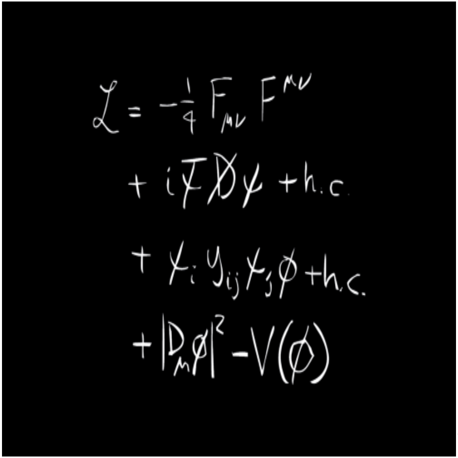
Abdul Rahaman Shaikh

Centre for Theoretical Physics
Jamia Millia Islamia
New Delhi - 110025

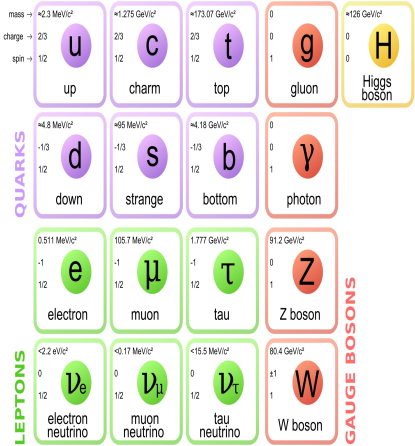
Based On - arXiv:2404.11570

Outline

Standard Model of Particle Physics



[1]



[2]

[1] https://visit.cern/node/612

[2] https://en.wikipedia.org/wiki/Standard_Model

A Possible Solution of Hierarchy of Fermion Masses and Mixing Angles

- For $0.02 < \epsilon < 0.03$ the masses of quarks and charged lepton along with quarks mixing angles can be written as :

$$\begin{array}{cccccc} m_t \approx 1 & m_b \approx \epsilon & m_s \approx \epsilon & m_s \approx \epsilon^2 & m_u \approx \epsilon^3 & m_d \approx \epsilon^3 \\ m_\tau \approx \epsilon & m_\mu \approx \epsilon^2 & m_e \approx \epsilon^3 & s^q_{12} \approx \epsilon & s^q_{23} \approx \epsilon & s^q_{13} \approx \epsilon^2 \end{array}$$

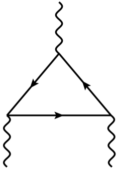
- These suppression in masses charged fermions and mixing angles of quarks can be generated by [3]:

$$m_u \approx \begin{pmatrix} \epsilon^3 & \epsilon & 1 \\ \epsilon^3 & \epsilon & 1 \\ \epsilon^3 & \epsilon & 1 \end{pmatrix} \quad m_d \approx \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \end{pmatrix} \quad m_l \approx \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \end{pmatrix}$$

[3] Nucl.Phys.B 147 (1979), 277-298

Anomaly Cancellation in Chiral Gauge theory

- Axial current $j^{\mu 5} = \overline{\psi}\gamma^{\mu}\gamma^{5}\psi$ which is conserved at the classical level, becomes divergent through one-loop triangle diagrams.



$$\propto \qquad \mathrm{tr}\left(\gamma^5 t^a \{t^b, t^c\}\right)$$

- If we want to gauge a $U(1)_X$ symmetry into the Standard Model gauge group, we must consider following anomaly cancellation

$$[U(1)_X]^3 \qquad SU(2)[U(1)_X]^2 \qquad [SU(2)]^2 U(1)_X$$
$$U(1)_X G^2 \qquad SU(3)SU(2)U(1)_X \qquad [SU(3)]^2 U(1)_X$$

- The $U(1)_X$ charges of the fermions are given by

$$Q_{iL} \to n_1^i, \quad u_{iR} \to n_2^i, \quad d_{iR} \to n_3^i, \quad L_{iL} \to n_4^i, \quad e_{iR} \to n_5^i, \quad N_{iR} \to n_6^i$$

Charge Assignment of the Fields

- $U(1)_X$ charges can be written in term of n_1 and n_2

$$n_3^i = 2n_1^i - n_2^i \qquad n_4^i = -3n_1^i$$
$$n_5^i = -(2n_1^i + n_2^i) \qquad n_6^i = (n_2^i - 4n_1^i)$$

- To create the matrix structure discuss above we need to assign a sequential $U(1)_X$ charges

$$(n_1^1, n_1^2, n_1^3) = (0, 0, 0) \qquad (n_2^1, n_2^2, n_2^3) = (3, 1, 0)$$

- Then charges of other fermions are

$$(n_3^1, n_3^2, n_3^3) = (-3, -1, 0) \qquad (n_4^1, n_4^2, n_4^3) = (0, 0, 0)$$
$$(n_5^1, n_5^2, n_5^3) = (-3, -1, 0) \qquad (n_6^1, n_6^2, n_6^3) = (3, 1, 0)$$

Higgs Sector:-

$$\phi \sim (1, 2, 1/2, 0) \qquad \chi_1 \sim (1, 1, 0, -1)$$

The Discrete symmetry

- According the this above $U(1)_X$ charge assignment the mass matrix of all the fermions take the form

$$m = \begin{pmatrix} h_{11}\epsilon^3 & h_{12}\epsilon & h_{13} \\ h_{21}\epsilon^3 & h_{22}\epsilon & h_{23} \\ h_{31}\epsilon^3 & h_{32}\epsilon & h_{33} \end{pmatrix} \frac{v}{\sqrt{2}} \; ;$$

  Where, $\epsilon = \frac{v_{\chi_1}}{\Lambda}$, $v = <\phi>$, $v_{\chi_1} =<\chi_1>$ and Λ is the cut-off scale of the theory.
- SM fermions have a strong hierarchies in masses of the up, down sector of quark as well as charged lepton sector.
- Intra-generational hierarchies could be obtained by introducing another scalar singlet χ_2 with a Z_2 symmetry[4].

[4] Eur.Phys.J.C 83 (2023), 305

Charges of the particles

Table: Charges of Scalars and Fermions

Particles	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_X$	Z_2	$U(1)_L$
Q_{iL}	3	2	1/6	(0, 0, 0)	(+, +, +)	0
u_{iR}	3	1	2/3	(3, 1, 0)	(+, +, +)	0
d_{iR}	3	1	- 1/3	(-3, -1, 0)	(+, -, -)	0
L_{iL}	1	2	-1/2	(0,0,0)	(+, +, +)	1
e_{iR}	1	1	-1	(-3, -1, 0)	(+, -, -)	1
N_{iR}	1	1	0	(3, 1, 0)	(+, +, +)	0
ϕ	1	2	1/2	0	+	0
χ_1	1	1	0	-1	+	0
χ_2	1	1	0	0	-	0
η	1	2	1/2	0	+	-1

Mass and Mixing of Charged Fermions

- The Yukawa Lagrangian of the quarks is given by :

$$\mathcal{L}_Q^Y = \left(\frac{\chi_1}{\Lambda}\right)^3 h^u_{i1}\,\bar{Q}_{iL}\tilde{\phi}\,u_R + \left(\frac{\chi_1}{\Lambda}\right) h^u_{i2}\,\bar{Q}_{iL}\tilde{\phi}\,c_R + h^u_{i3}\,\bar{Q}_{iL}\tilde{\phi}\,t_R + \left(\frac{\chi_1^*}{\Lambda}\right)^3 h^d_{i1}\,\bar{Q}_{iL}\phi\,d_R + \left(\frac{\chi_1^*\chi_2}{\Lambda^2}\right) h^d_{i2}\,\bar{Q}_{iL}\phi s_R + \left(\frac{\chi_2}{\Lambda}\right) h^d_{i3}\,\bar{Q}_{iL}\phi\,b_R + h.c$$

- After χ_1 & χ_2 get *vev* the mass matrix of the quarks are :

$$m_u = \begin{pmatrix} h^u_{11}\epsilon^3 & h^u_{12}\epsilon & h^u_{13} \\ h^u_{21}\epsilon^3 & h^u_{22}\epsilon & h^u_{23} \\ h^u_{31}\epsilon^3 & h^u_{32}\epsilon & h^u_{33} \end{pmatrix} \frac{v}{\sqrt{2}}; \quad m_d = \begin{pmatrix} h^d_{11}\epsilon^3 & h^d_{12}\epsilon\epsilon' & h^d_{13}\epsilon' \\ h^d_{21}\epsilon^3 & h^d_{22}\epsilon\epsilon' & h^d_{23}\epsilon' \\ h^d_{31}\epsilon^3 & h^d_{32}\epsilon\epsilon' & h^d_{33}\epsilon' \end{pmatrix} \frac{v}{\sqrt{2}}$$

- We will get the charged leptons mass matrix by $h^d_{ij} \to h^l_{ij}$.

Masses and Mixing Of the Fermions

- The masses of quarks and CKM matrix elements are given by [5] :

$$(m_t, m_c, m_u) \approx (1, \epsilon, \epsilon^3); \quad (m_b, m_s, m_d) \approx (\epsilon', \epsilon\epsilon', \epsilon^3);$$

$$V_{CKM}^{th} \approx \begin{pmatrix} 1 & \epsilon^2/\epsilon' & \epsilon^3/\epsilon' \\ \epsilon^2/\epsilon' & 1 & \epsilon \\ \epsilon^3/\epsilon' & \epsilon & 1 \end{pmatrix}$$

- The Dirac and Majorana mass matrix for neutrino are given by

$$M_D = \begin{pmatrix} h_{11}^{\nu}\epsilon^3 & h_{12}^{\nu}\epsilon & h_{13}^{\nu} \\ h_{21}^{\nu}\epsilon^3 & h_{22}^{\nu}\epsilon & h_{23}^{\nu} \\ h_{31}^{\nu}\epsilon^3 & h_{32}^{\nu}\epsilon & h_{33}^{\nu} \end{pmatrix} \frac{v_\eta}{\sqrt{2}}; \quad M_R = \begin{pmatrix} h_{11}^{m}\epsilon^5 & h_{12}^{m}\epsilon^3 & h_{13}^{m}\epsilon^2 \\ h_{21}^{m}\epsilon^3 & h_{22}^{m}\epsilon & h_{23}^{m} \\ h_{31}^{m}\epsilon^2 & h_{32}^{m} & h_{33}^{m}\epsilon' \end{pmatrix} \frac{v_1}{\sqrt{2}}$$

$$M_\nu = M_D M_R^{-1} M_D^T$$

- Rotational matrix for the charged leptons is approximately identity

$$V_{PMNS} = R^{\nu\dagger} R^l \approx R^{\nu\dagger}$$

[5] Phys.Rev.D 58 (1998), 096012

A Benchmark Values of Yukawa Couplings

- For fitting the quark masses and mixing, we minimize

$$\chi^2 = \sum_{i,j=1}^{3} \frac{(m_{ui} - m_{ui}^{model})^2}{\sigma^2_{m_{ui}}} + \frac{(m_{di} - m_{di}^{model})^2}{\sigma^2_{m_{di}}} + \frac{(sin\theta_{ij} - sin\theta_{ij}^{model})^2}{\sigma^2_{sin\theta_{ij}}}$$

- We take $\epsilon \approx \epsilon' \approx 0.02$ and get the values of Yukawa couplings

$$h^u = \begin{pmatrix} 0.1 & 0.3 & 0.5 \\ 3.3 & 0.1 & 0.3 \\ 0.1 & 0.4 & 0.6 \end{pmatrix} \quad h^d = \begin{pmatrix} 4 & 0.2 & 0.2 \\ 1.1 & 1.4 & 0.5 \\ 4 & 0.1 & 0.4 \end{pmatrix} \quad h^l = \begin{pmatrix} 0.2 & 0.4 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 1.9 & 0.2 \end{pmatrix}$$

- For $v_\eta = 1$ KeV and we find the Yukawa couplings

$$(h^\nu_{11}, h^\nu_{22}, h^\nu_{33}, h^m_{11}) = (0.52, 1.27, 2.85, 0.92)$$

All other Yukawa couplings are taken equal to 1

Phenomenology of the Model

- Gauge interaction of the new gauge boson with SM fermions are

$$\mathcal{L}_{int} = g_F^{\prime} Z_F^{\prime} \Big[3\bar{u}_R^{\prime}\gamma^{\mu}u_R^{\prime} + \bar{c}_R^{\prime}\gamma^{\mu}c_R^{\prime} - 3\bar{d}_R^{\prime}\gamma^{\mu}d_R^{\prime} - \bar{s}_R^{\prime}\gamma^{\mu}s_R^{\prime} - 3\bar{e}_R^{\prime}\gamma^{\mu}e_R^{\prime} - \bar{\mu}_R^{\prime}\gamma^{\mu}\mu_R^{\prime} \Big] + h.c$$

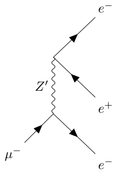
- The ration of branching fraction of $Z' \to e^- e^+$ and $Z' \to \mu^-\mu^+$ in our model

$$\frac{\Gamma(Z' \to e^- e^+)}{\Gamma(Z' \to \mu^-\mu^+)} = \frac{(n_5^1)^2}{(n_5^2)^2} = 9$$

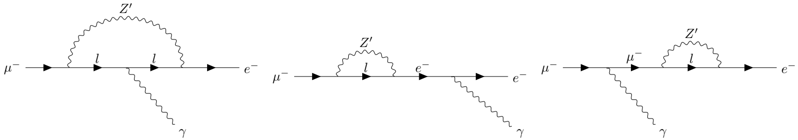
- Due to the sequential nature of the new gauge symmetry there is a possibility of FCNC through Z'.

Phenomenology of the Models (cont.)

- The $\mu^- \to e^- e^- e^+$ decay is possible in our model in tree level.



- The $\mu^- \to e^- \gamma$ decay is possible in our model at one loop level [6]



[6] https://doi.org/10.22323/1.350.0023

Phenomenology of the Model (cont.)

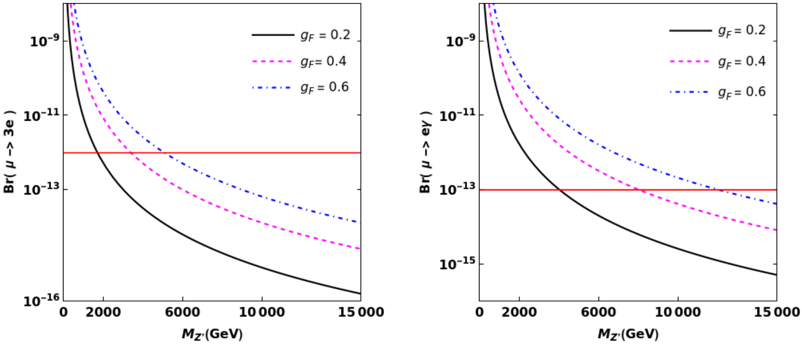


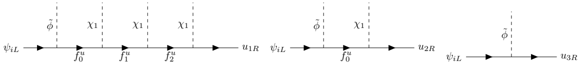
Fig: Variation of Br($\mu \to 3e$) and Br($\mu \to e\,\gamma$) with Model Parameters.

A Possible UV Completion of the Model

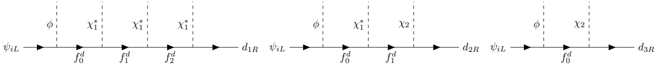
- We have added some vector like fermions for UV completion

Particles	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	Z_2
f_i^u	3	1	2/3	(0,1,2)	(+,+,+)
f_i^d	3	1	-1/3	(0,-1,-2)	(+,+,+)
f_i^l	3	1	-1	(0, -1,-2)	(+,+,+)

- Mass generation of up type quarks through dim-4 operators



- Mass generation of down type quarks through dim-4 operators



Conclusion

- We successfully explain the fermion mass and mixing hierarchies, including neutrino masses and mixing, by assuming all Yukawa couplings to be of $\mathcal{O}(1)$ in a anomaly free $U(1)_X$ and Z_2 extended symmetry of SM.
- Due to sequential nature of $U(1)_X$ symmetry it produces FCNC through new gauge bosons Z'.
- We have studied various FCNC processes, particularly $\mu \to 3e$ and $\mu \to e\,\gamma$ and we have found the bound on new gauge bsons mass for a particular values of g_F .
- We found that $\mu \to e\,\gamma$ give stronger bound on the Z' mass for a particular g_F.
- We also provide a possible UV completion of the theory by introducing some extra vector-like fermions.


Thank
you