Oct 15, 2024

LARGE BLUE-TILTED ISOCURVATURE FROM ROTATING COMPLEX SCALAR

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AXIONIC THEORY

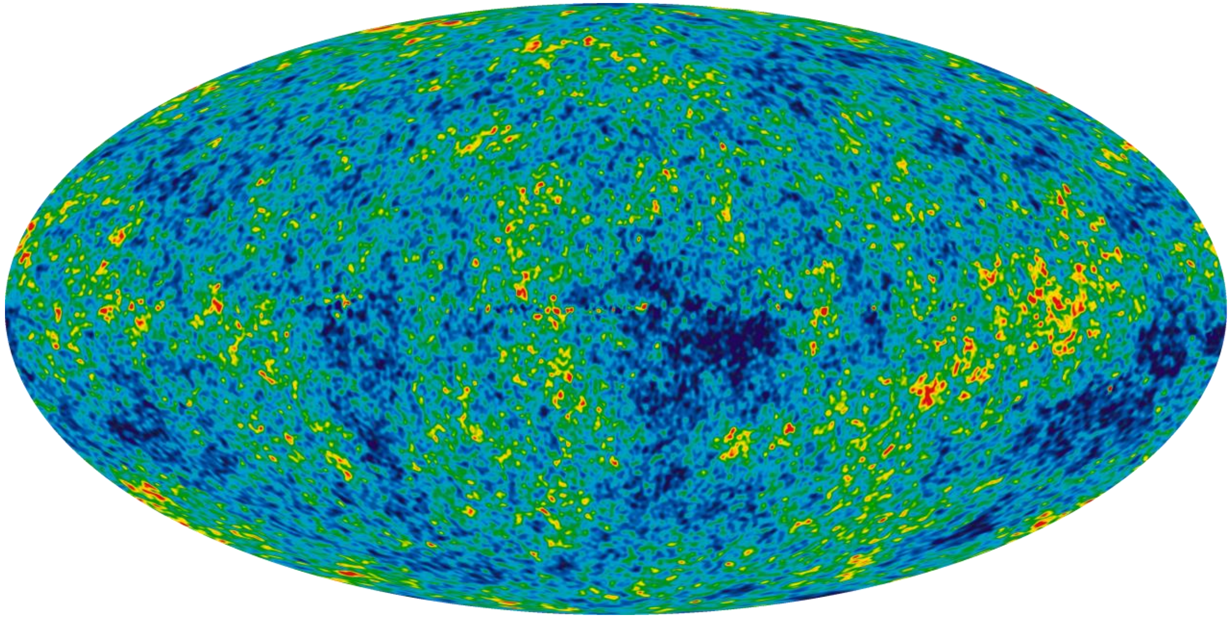
during inflation

A *dynamical* axionic theory in a strongly interacting background

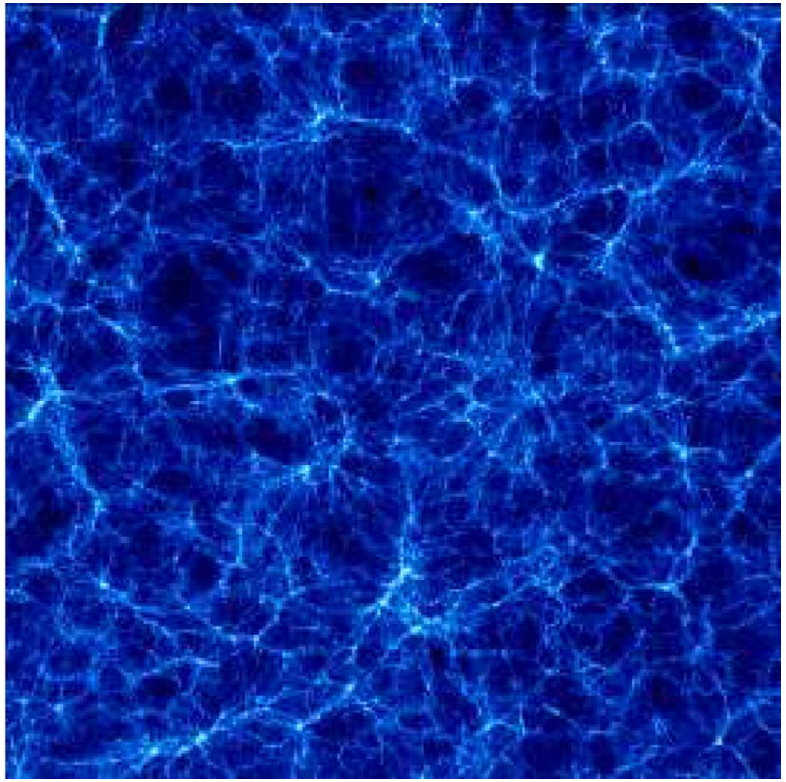
Has a $\mathbf{k}^2$ *power spectrum* for the fluctuations

Light eigen mode resembles a relativistic perfect fluid

Cosmic Inhomogeneities



Temperature inhomogeneity in CMB



Inhomogeneity in matter density in the form of clustering

These cosmic inhomogeneities can be explained as being sourced by some primordial fluctuations.

These fluctuations are usually categorized in two distinct and orthogonal forms:
Adiabatic and **Isocurvature**

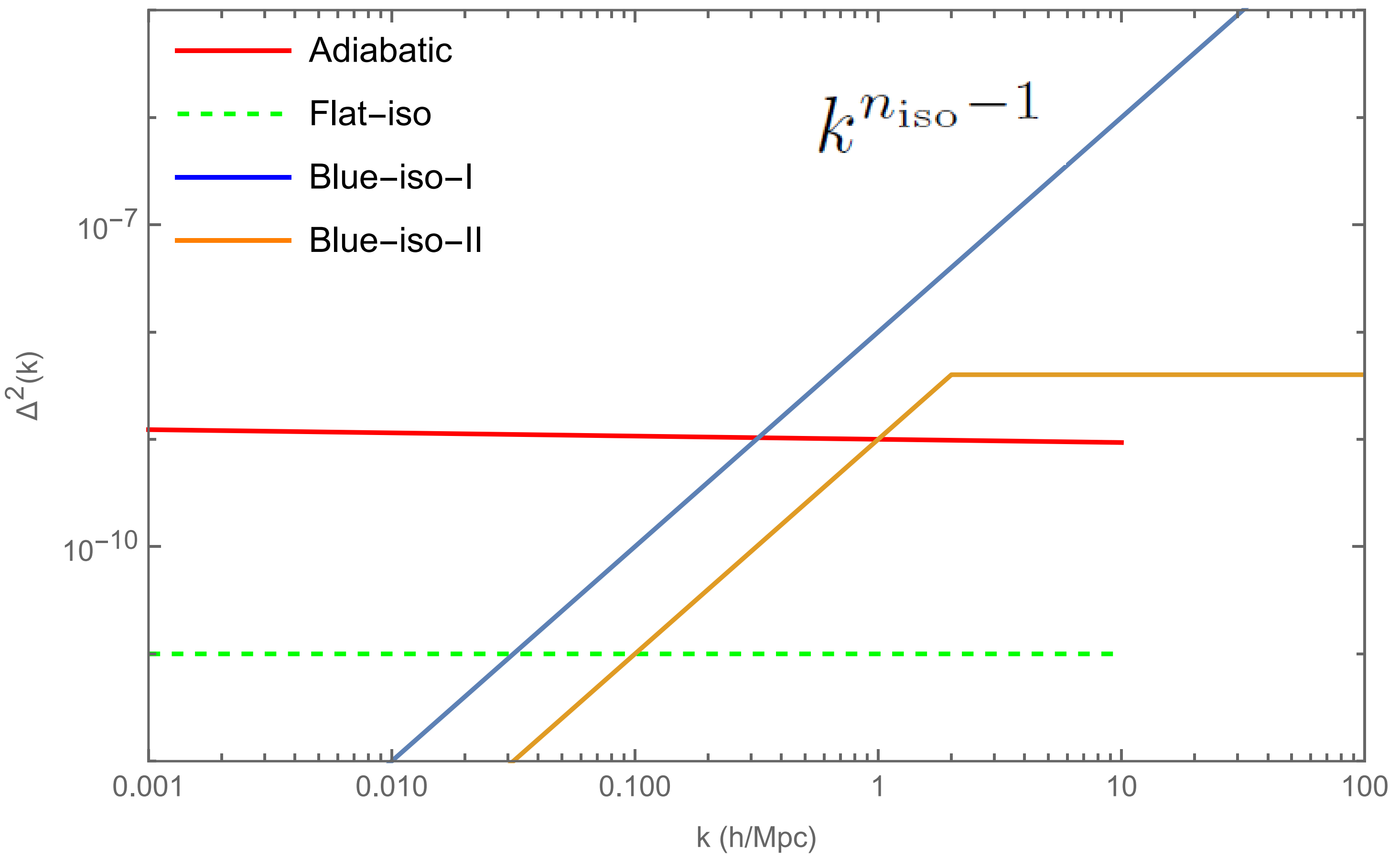
$$\delta = \frac{\delta\rho}{\rho}$$

Adiabatic	Isocurvature
single dynamical growing mode	Multiple independent dynamical growing modes

$$\delta_c^{\rm ad} = \delta_b^{\rm ad} = \frac{3}{4}\delta_\gamma^{\rm ad} = \frac{3}{4}\delta_\nu^{\rm ad}$$

$$\delta_c \neq \frac{3}{4}\delta_\gamma$$

Primordial blue-tilted CDM isocurvature (generated during inflation)



$$\alpha = \left.\frac{A_{\rm iso}}{A_{\rm ad}}\right|_{k=0.05/{\rm Mpc}}$$ **< 0.03**

2-sigma hint found in **1711.06736**, **1707.09354** and **1807.06211** from combined *Planck+BOSS* analysis for uncorrelated CDM isocurvature with **1.5<niso<3.5**

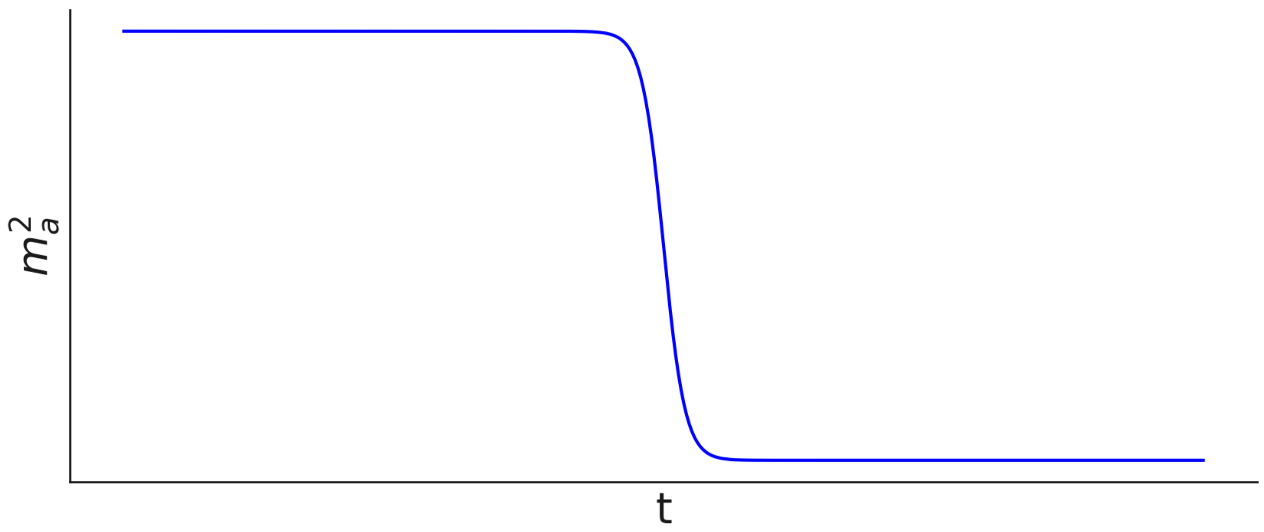
Large blue-tilted isocurvature can be 3 times Adiabatic at 0.1/Mpc !!!

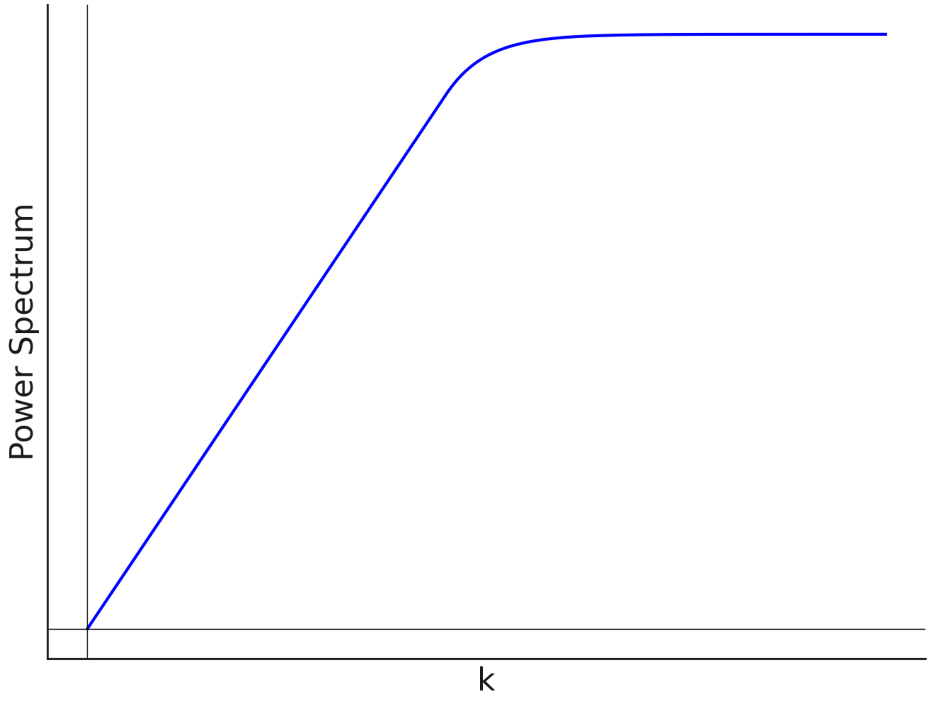
Why study blue isocurvature

$$n_{iso} \equiv n_{iso}(m_\chi)$$

- Similar to non-gaussianity and primordial GWs, isocurvature can offer valuable insights into inflation and the presence of spectator fields and their mass scales.
- The theorem in **1509.0585** states that blue isocurvatures with niso > 2.4, uniquely hint towards spectator fields with time-dependent mass during inflation.
- Blue isocurvature can relax the constraint on H-f parametric region $O\left(\frac{H_{\rm inf}}{f_{\rm PQ}\theta_i}\right)$
- They modify small scale physics. May explain early galaxies seen by JWST.

How do we generate primordial blue isocurvature for axions?





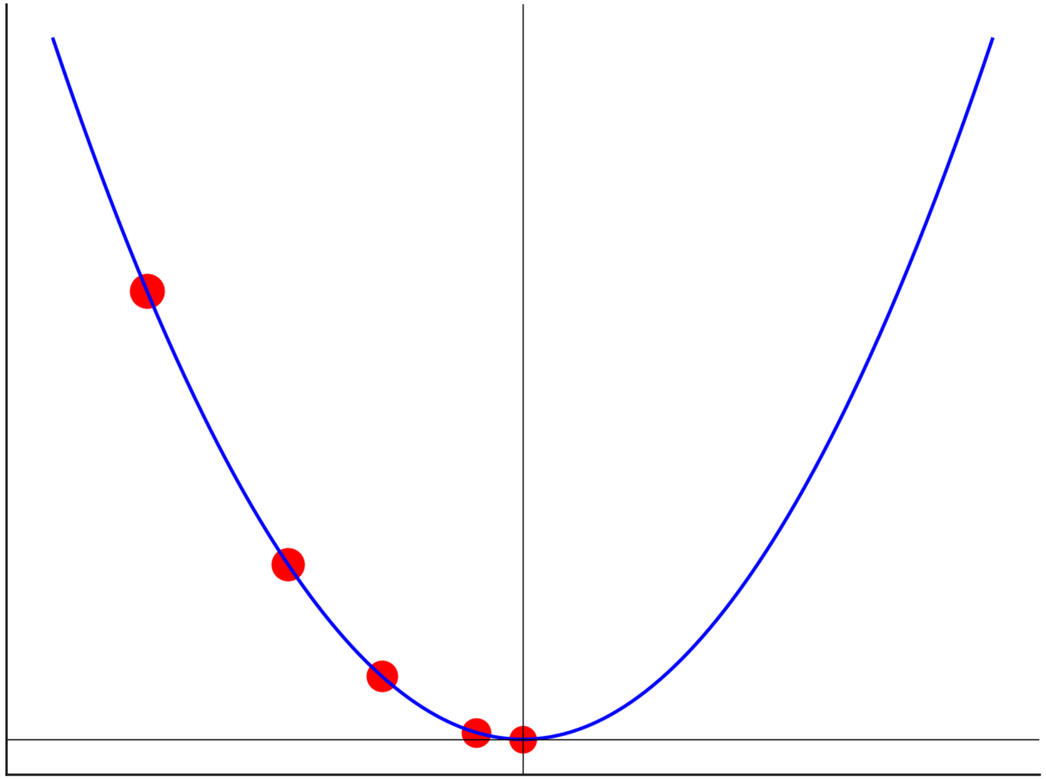
Generate blue-tilted spectra via **non-equilibrium** radial field

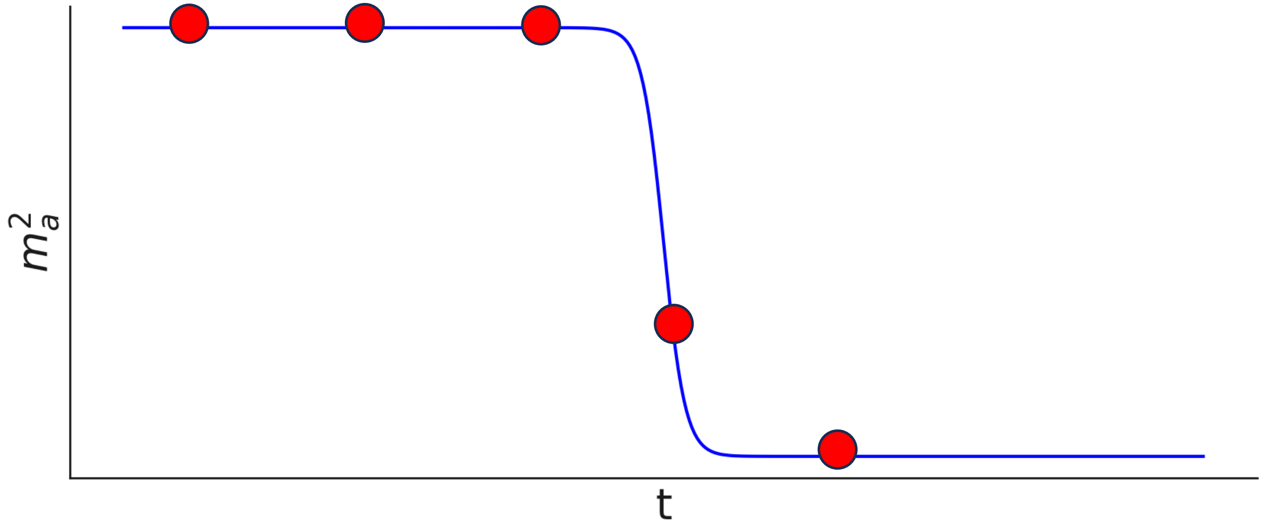
S. Kasuya, M. Kawasaki [0904.3800]

$$R : O\left(M_{\rm pl}\right) \to O\left(f_{\rm PQ}\right)$$

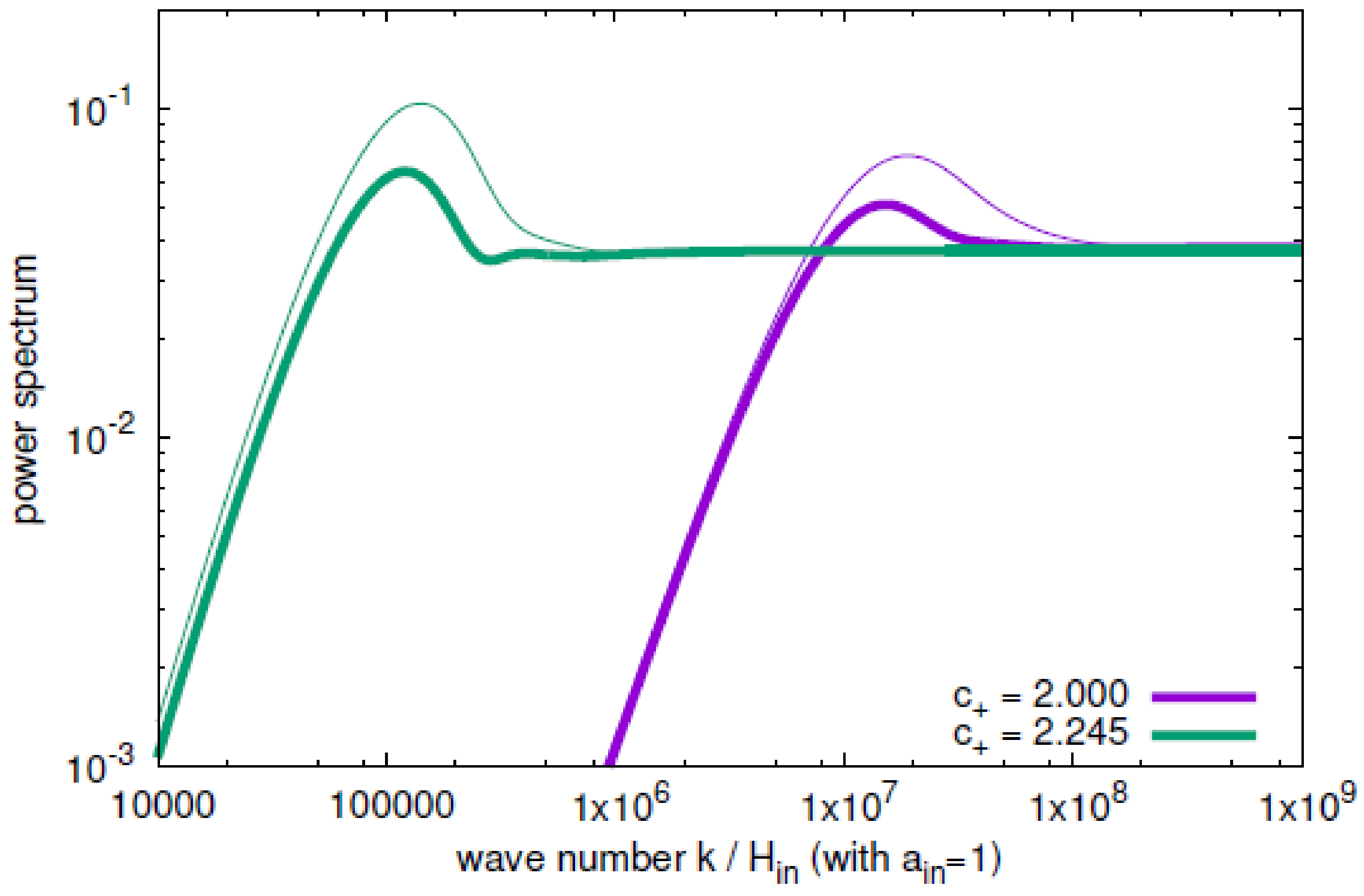
$$\Phi = Re^{i\frac{a}{R}}$$

$$\Box a - \left(\frac{\Box R}{R}\right) a \approx 0$$





We require displacement away from minimum vev

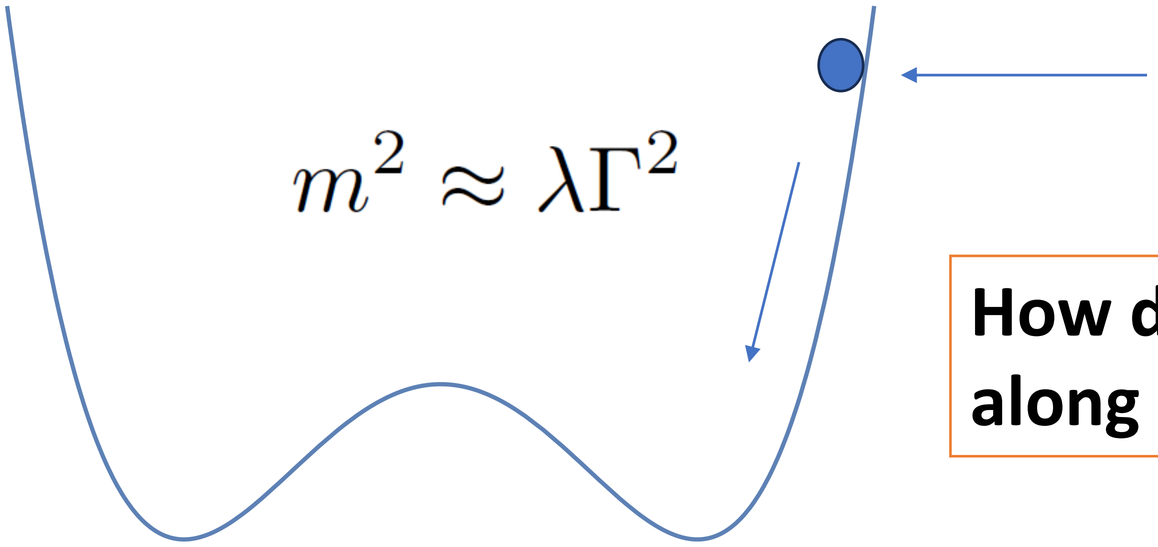


Blue-tilted power spectrum generated during the rolling of the field down the potential

Generic PQ model

U(1) PQ field $$\Phi = \frac{1}{\sqrt{2}}\Gamma e^{i\theta} \equiv \frac{1}{\sqrt{2}}\Gamma e^{i\frac{A}{\Gamma}}$$

PQ symmetry breaking potential $$V = -M^2\Gamma^2 + \frac{\lambda}{4}\Gamma^4$$



$$m^2 \approx \lambda\Gamma^2$$

Displaced away
from minimum vev

How do we slow down this roll along the quartic potential ?

Generic PQ model

U(1) PQ field $$\Phi = \frac{1}{\sqrt{2}}\Gamma e^{i\theta} \equiv \frac{1}{\sqrt{2}}\Gamma e^{i\frac{A}{\Gamma}}$$

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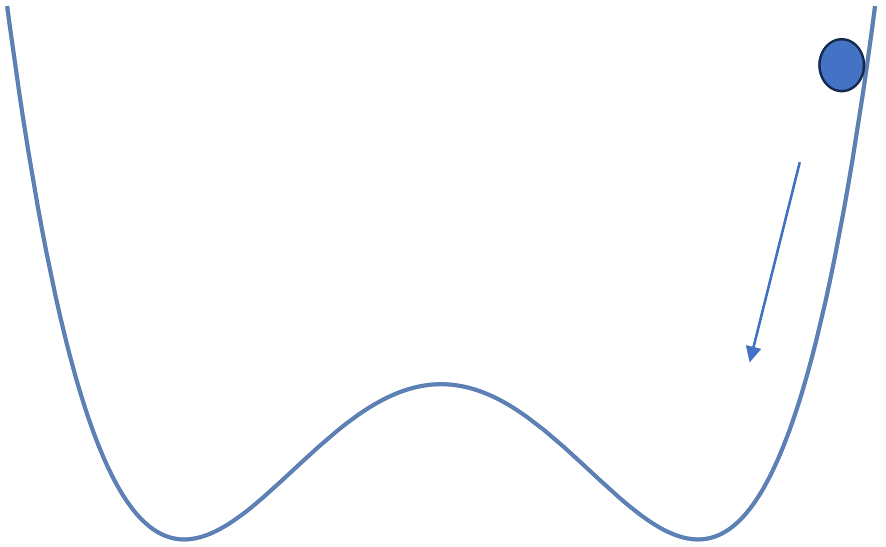
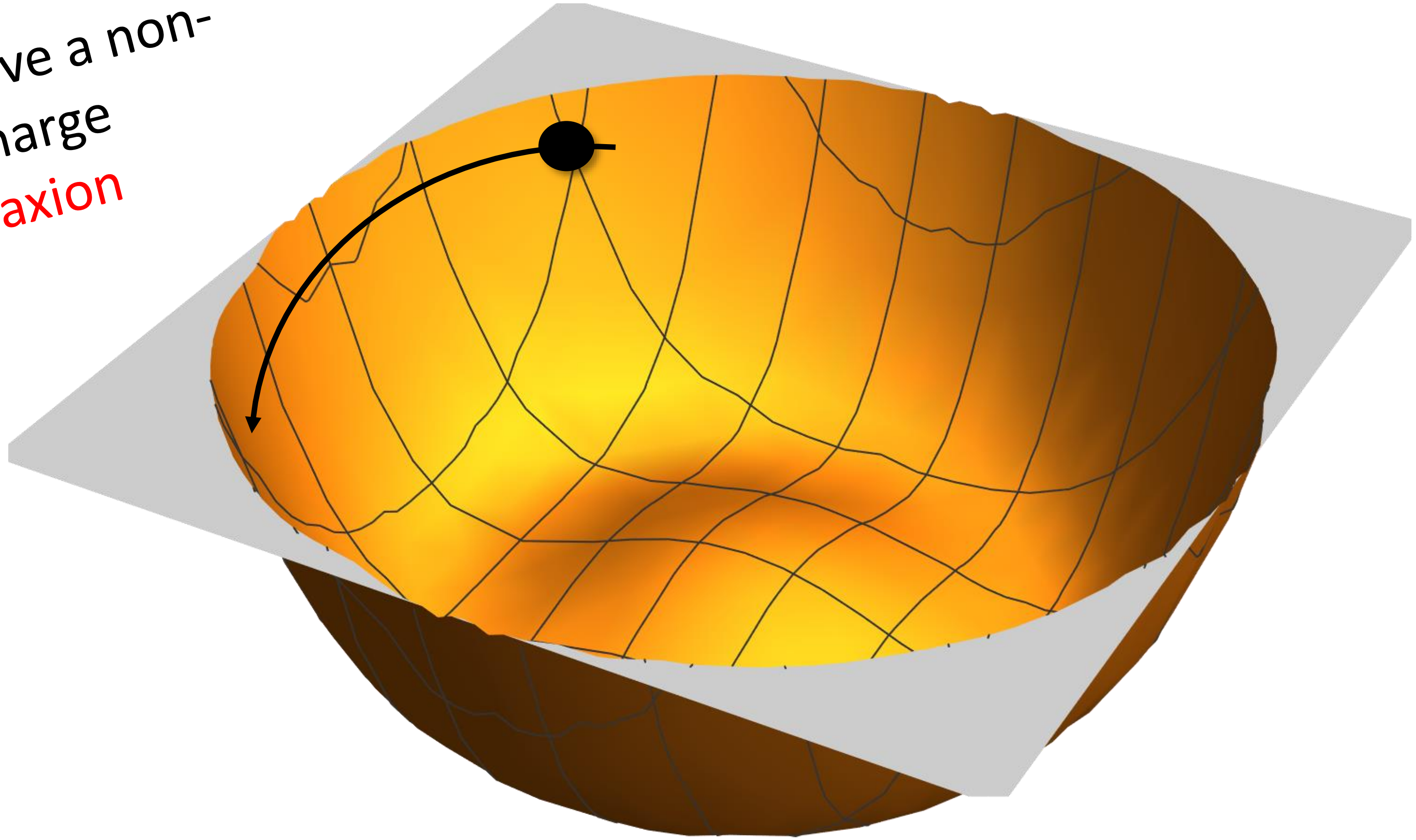


Displaced away from minimum vev

How do we slow down this roll to last a large amount of time?

Impart initial angular momentum, or, equivalently have a non-zero U(1)PQ charge
a.k.a rotating axion



$$m^2 \approx 2H^2$$

$$n_I = 3$$

$$\Delta_s^2 \propto k^2$$

Generates blue isocurvature spectrum

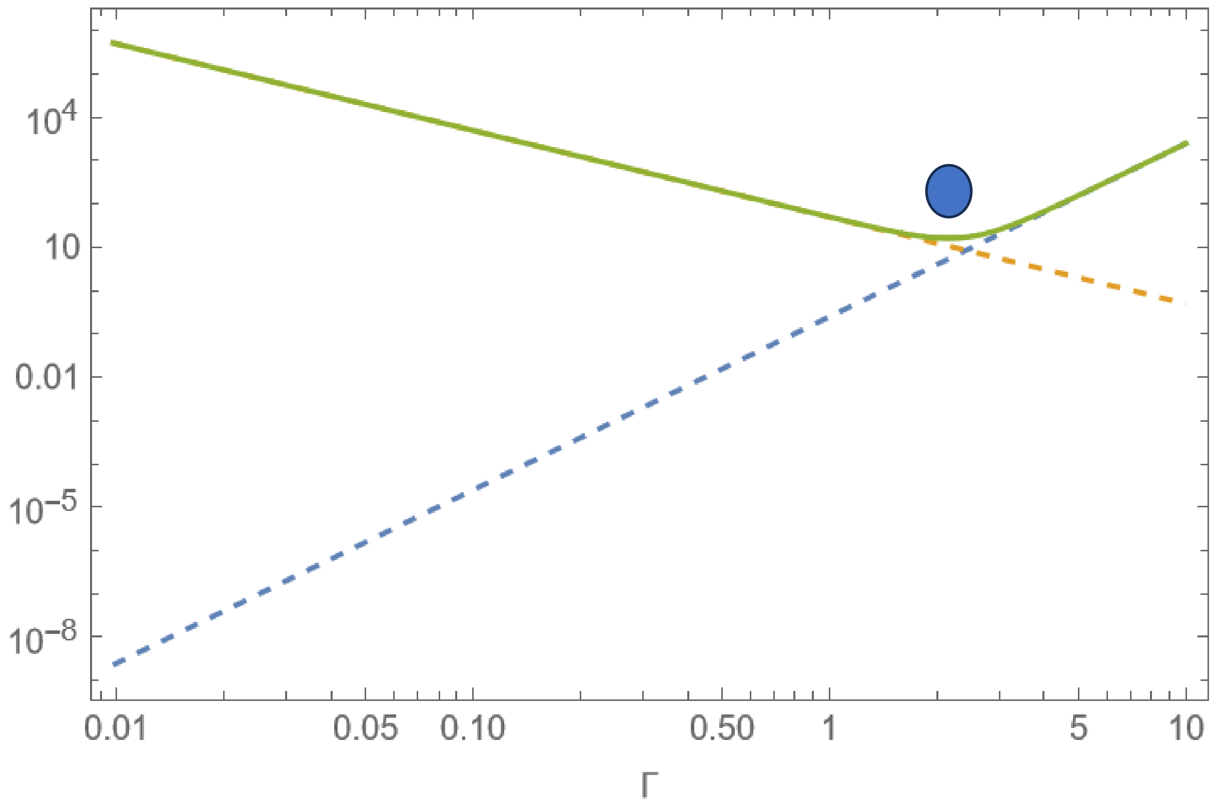
How rotations stabilize the vev

Conserved angular momentum

$$L = a^3 \Gamma^2 \dot{\theta}$$

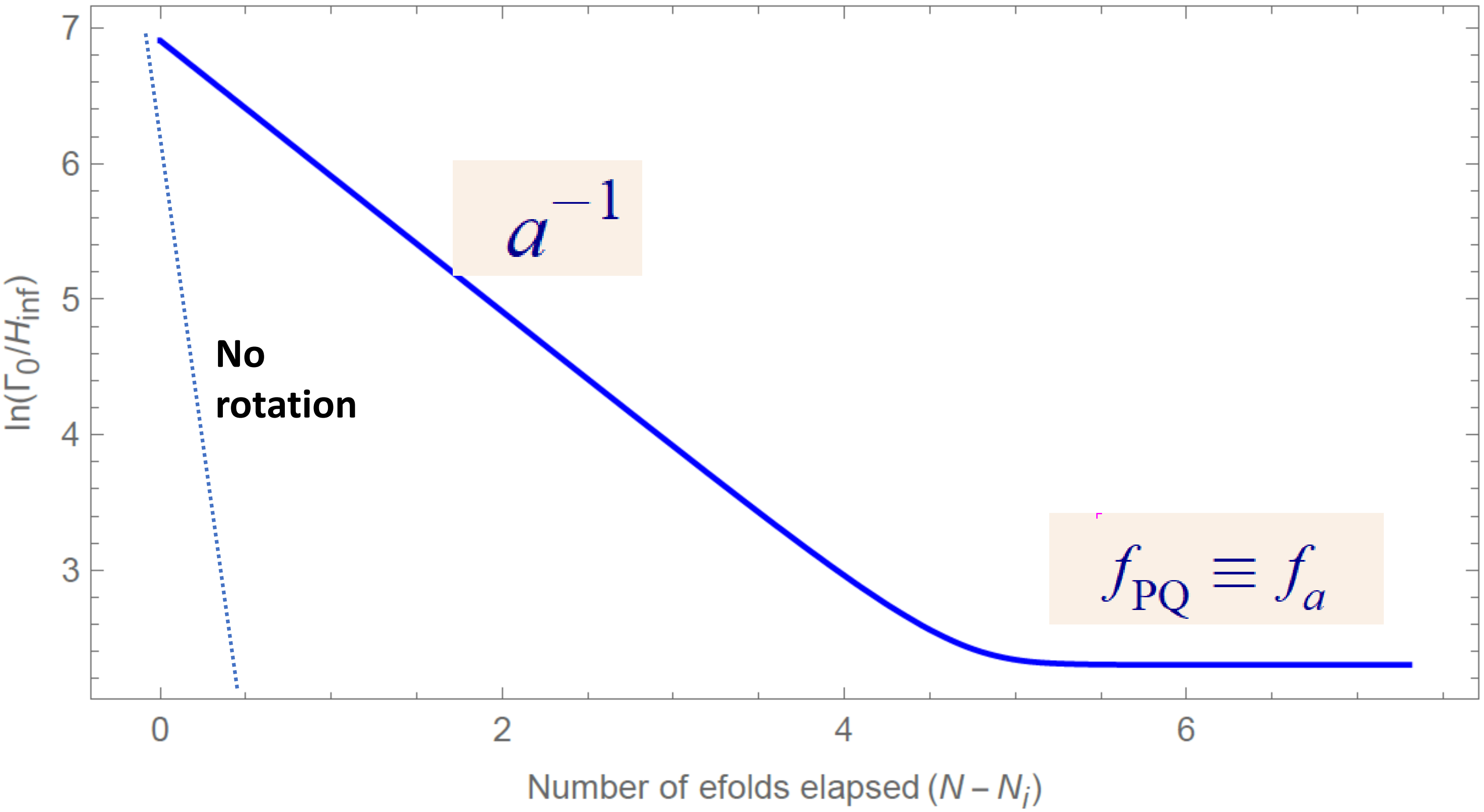
$$V_{\mathrm{E}} = \frac{\lambda}{4}\Gamma^4 + \frac{1}{2}\Gamma^2 \left(\partial_t \theta\right)^2$$

$$V_E(\Gamma, a) = \frac{\lambda}{4}\Gamma^4 + \frac{1}{2}\frac{L^2}{a^6\Gamma^2}$$



This delicate balance requires that the radial field $\Gamma \to a^{-1}\Gamma_0$

Indicates a classical conformal theory



$$Y \equiv a\Gamma$$

$$Y_0 = \frac{\partial_\eta \theta_0}{\sqrt{\lambda}} = \text{constant}$$

An axionic theory in a coupled background

$$\Phi = \Gamma e^{i\theta} \equiv \Gamma e^{i\frac{A}{\Gamma}}$$

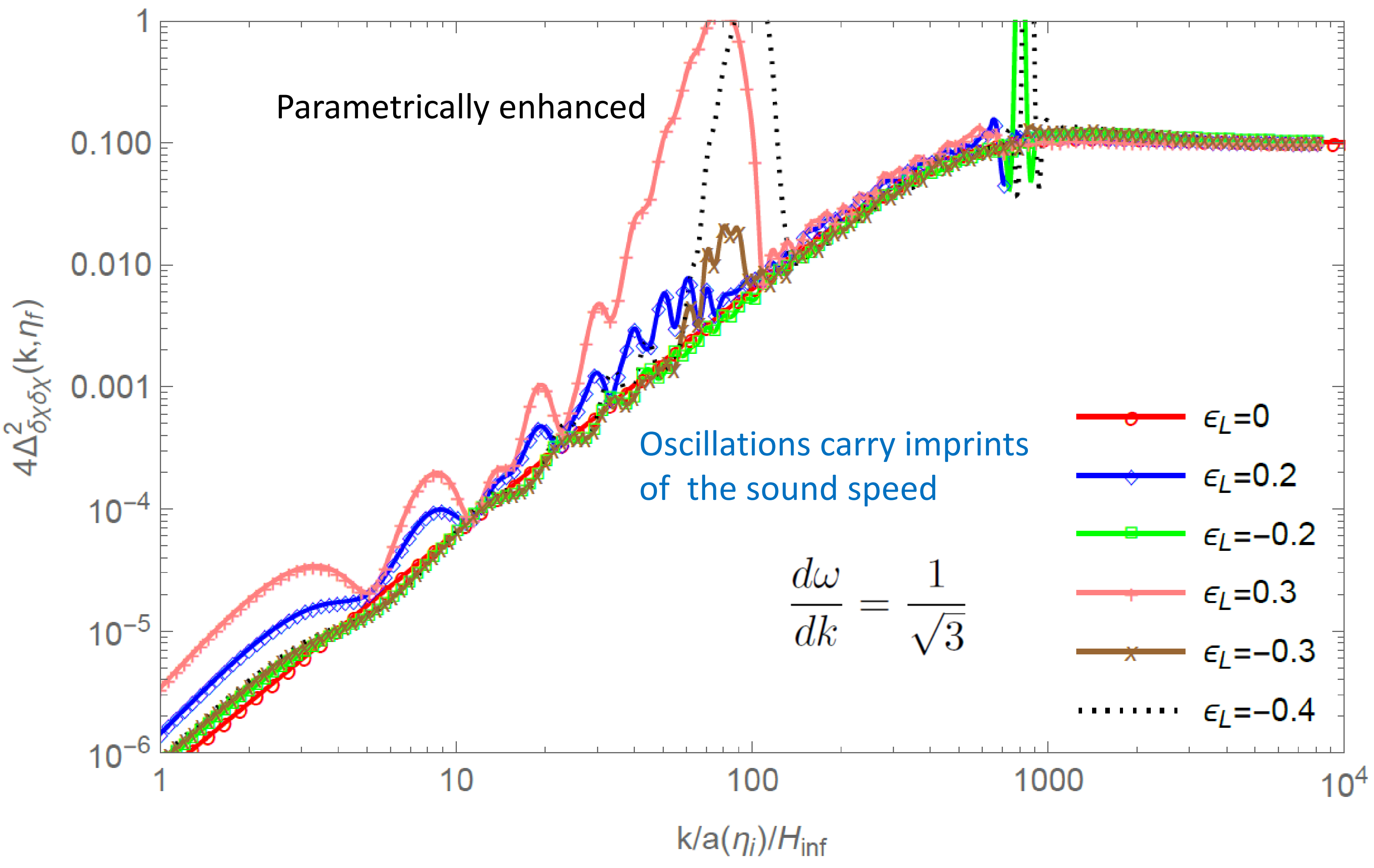
We need consistent quantization due to strong coupling !

Due to the *strong radial-axial coupling*: $\mathcal{L} \supset -a^2 \Gamma_0 \partial_\eta \theta_0 \, \delta\Gamma \partial_\eta \delta\theta$

Dispersion relation of the Goldstone mode resembles that of a *coupled radiation fluid*

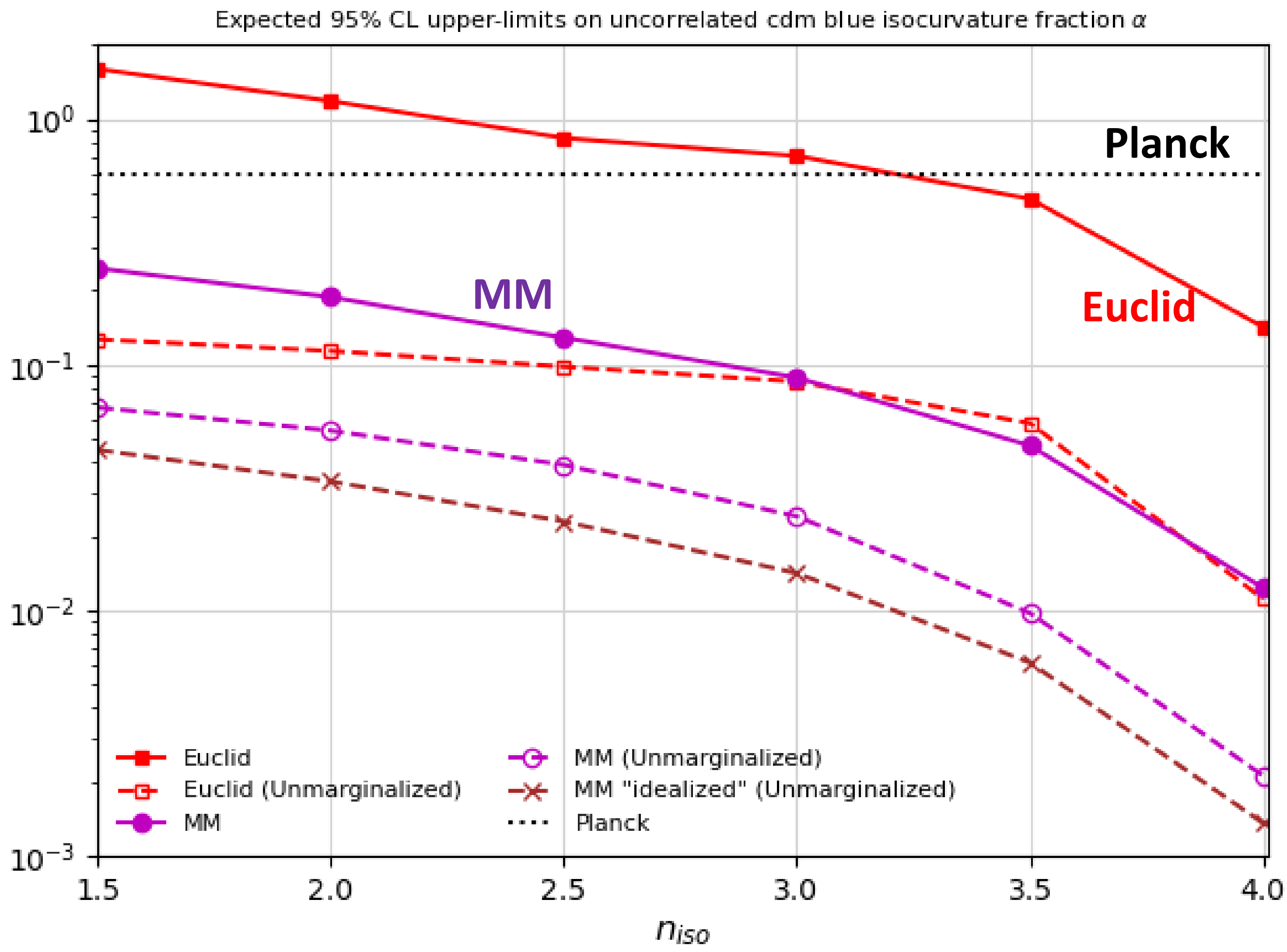
$$\boxed{\frac{d\omega}{dk} = \frac{1}{\sqrt{3}}}$$

A $\partial_\eta \theta_0$-induced non-trivial aspect of this theory unlike usual Goldstone modes



Parametrically enhanced
Oscillations carry imprints
of the sound speed
$$\frac{d\omega}{dk} = \frac{1}{\sqrt{3}}$$
ϵ_L=0
ϵ_L=0.2
ϵ_L=−0.2
ϵ_L=0.3
ϵ_L=−0.3
ϵ_L=−0.4
$4\Delta^2_{\delta\chi\delta\chi}(k,\eta_f)$
$k/a(\eta_i)/H_{\rm inf}$

Estimates TSC, D.Chung, M.Muenchmeyer [2306.09456]



$$\alpha = \frac{A_{\rm iso}}{A_{\rm ad}}\Big|_{k=0.05/{\rm Mpc}}$$

1) Additional factor of 2-3 improvement from RSD

2) Further improvement from strong priors on bias parameters

Factor of ~10 improvement from Stage5 surveys

Rotating axion during inflation has cool properties

- Resembles a CT that "near-spontaneously" flows from *time-independent* to *time-dependent* phase.
- Kinetic correlation $\langle \partial_\eta \delta\Gamma \partial_\eta \delta\chi \rangle \neq 0$ even though $\langle \delta\Gamma \delta\chi \rangle = 0$
- Sizeable non-gaussianity from the strong kinetic coupling
- Goldston mode behaves like a radiation-matter fluid during rotation ($k^2/3$)
- The conformality results in a fixed ~k^2 isocurvature spectrum.

Thanks…

LIVE

Science driver 1: Inflation and Early Universe Physics

Spec-S5 will extensively explore inflation by measuring its non-gaussianity and primordial features, complementary to CMB-S4's measurement of B-modes from gravity waves

1. **Tensor-to-scalar ratio**: energy scale of inflation → best measured by CMB-S4 in B-modes
   - insight into rate of expansion during inflation
   - detection provides evidence that gravity is quantized

2. **Non-gaussianity**: physics of the inflaton field → best measured by Spec-S5 and CMB-S4
   - number of fields responsible for inflation
   - excited state of the inflaton field
   - interactions between particles and the inflaton field

3. **Primordial features:** deviations in scale invariance from well-motivated particle physics
   - early universe particle production
   - periodic corrections to the inflationary potential
   - changes to the power-law power spectrum on small scales
   - ***best measured by Spec-S5***

David Schlegel's screen

Other sources of small-scale blue-power ?

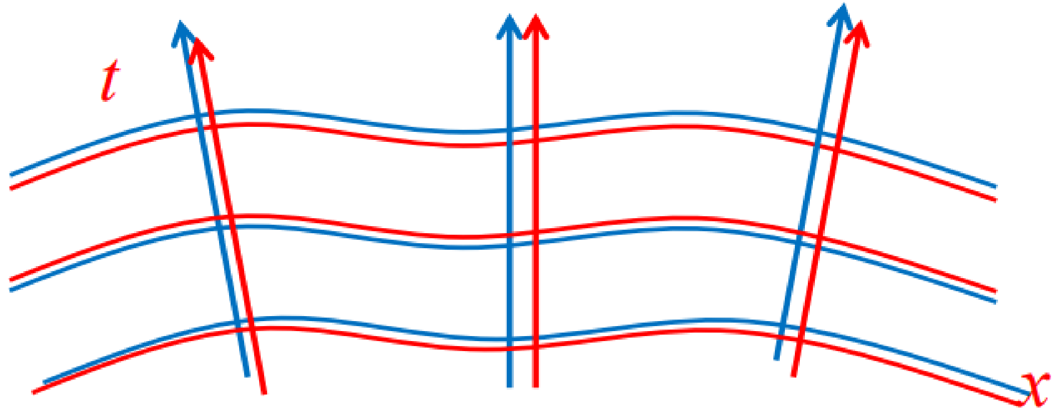
A rise in the power on small-scales is generically predicted in several phenomenological models during early-universe:

- PBH isocurvature (induces an almost k^3 spectrum on short scales) [**2012.03698**]

- Phase transition isocurvature (~ k^3) [**2311.16222**]

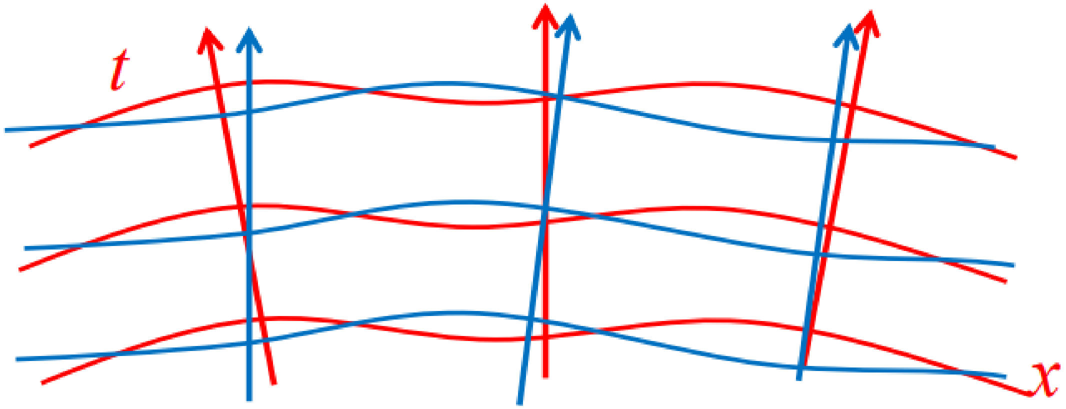
- Post-inflationary PQ SSB (~ k^3) [**2004.02926**]

- Lumpy DM (~ k^n) [**2306.04674**]



Adiabatic: Uniform-matter and uniform-radiation hypersurfaces coincide

$$\zeta_i = \zeta_j$$



Isocurvature: Hypersurfaces do not coincide

$$\delta\ddot{\chi}^{(N)} + 3H\delta\dot{\chi}^{(N)} + V''_{\chi}(\chi_0)\delta\chi^{(N)} - 4\dot{\chi}_0\dot{\Psi}^{(N)} + 2V'_{\chi}(\chi_0)\Psi^{(N)} = 0.$$

$$\mathcal{H}^{-1}\mathcal{R}' = \frac{2}{3(1+w)}\left(\frac{k}{\mathcal{H}}\right)^2\left[c_s^2\Psi + \tfrac{1}{3}(\Psi - \Phi)\right] + 3c_s^2\mathcal{S}\,.$$

$$\mathcal{R}_{\vec{k}} \;=\; \mathcal{R}_{\vec{k}}(\mathrm{rad}) + \frac{y}{4+3y}S_{\vec{k}}(\mathrm{rad}) \qquad \text{for} \qquad k \ll \mathcal{H}\,.$$

Axion energy density

$$\rho_a = \frac{1}{2} m_a^2 a^2 = m_a^2 \theta_i^2 f_a^2$$

θ_i
initial misalignment angle

$m_{a,\mathrm{QCD}} \approx O\left(\Lambda_{\mathrm{QCD}}^2 / f_a\right)$ when $T \lesssim \Lambda_{\mathrm{QCD}}$

$$\Omega_a h^2 \approx .24 \times \theta_i^2 \left[\ln\left(\frac{e}{1-\theta_i^2/\pi^2}\right)\right]^{7/6} \left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{7/6}$$

Large blue-tilted spectra

Blue nI > 1
Red nI < 1

Isocurvature spectrum $$\Delta^2_{S_\chi}(k) \sim k^3 \int d^3k' \left\langle S_{\chi,\gamma,\vec{k}} S_{\chi,\gamma,\vec{p}} \right\rangle \sim k^{n_I - 1}$$

$$S \approx \frac{\delta\chi}{\chi}$$

spectral index $$n_I - 1 = 3 - 2\sqrt{9/4 - m_\chi^2/H^2}$$

$$n_I - 1 \sim O(1) \implies m_\chi^2/H^2 \sim O(1)$$

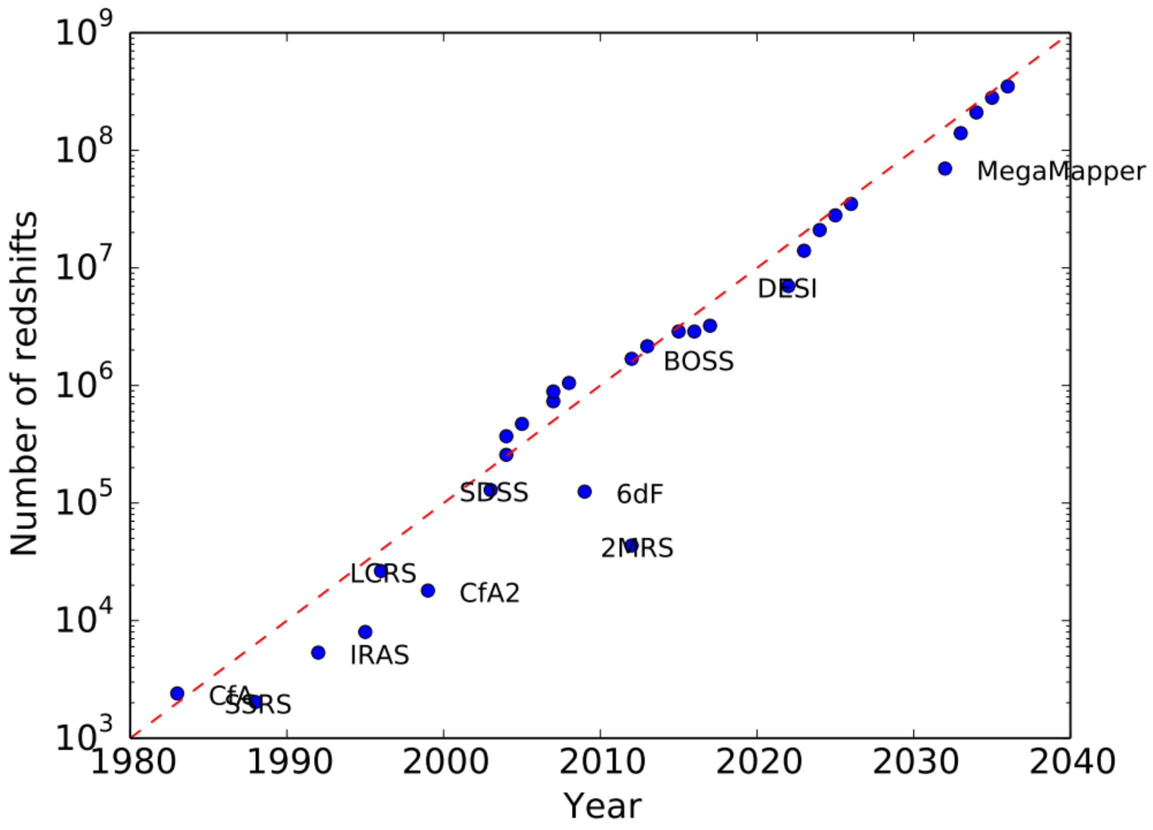
$$\Delta^2_S(k) = \omega_\chi^2 \Delta^2_{S_\chi}(k)$$

$$\omega_\chi^2 \sim \chi^2$$

$$\chi \sim \chi_0 \exp\left(-t\left(3/2 - \sqrt{9/4 - m_\chi^2/H^2}\right)\right)$$

Table 1: Survey speeds for multi-fiber spectrographs as measured by the product of the telescope clear aperture, number of fibers and losses from mirror reflections. This speed assumes a dedicated facility, which would not be possible in all cases. Keck/FOBOS[17], MSE[18], SpecTel[19] and MegaMapper[20] are proposed experiments. LSSTspec[21, 22] is a notional number using MegaMapper positioners on the LSST focal plane, if optical design limitations could be overcome injecting f/1.2 light into fibers.

Instrument (year)	Primary/m^2	Nfiber	Reflections	Product	Speed vs. SDSS
SDSS (1999)	3.68	640	0.9^2	1908	1.00
BOSS (2009)	3.68	1000	0.9^2	2980	1.56
DESI (2020)	9.5	5000	0.9^1	42,750	22.4
PFS (2023)	50	2400	0.9^1	108,000	56.6
4MOST (2023)	12	1624	0.9^2	15,800	8.3
DESI-Upgrade (2027)	9.5	11,250	0.9^1	96,200	50.4
MegaMapper	**28**	**26,100**	$\mathbf{0.9^2}$	**590,000**	**309**.
Keck/FOBOS	77.9	1800	0.9^3	102,000	53.6
MSE	78	3249	0.9^1	228,000	119.
LSSTspec	35.3	8640	0.9^3	222,000	116.
SpecTel	87.9	15,000	0.9^2	1,070,000	560.



$$\int d^3k\, e^{i\vec{k}\cdot\vec{r}}\frac{1}{k} \sim \frac{1}{r^2}$$

$$\lim_{\eta\to-\infty} S_Y^{(2)} = \int d\eta d^3x\left[\frac{1}{2}\left(\partial_\eta \delta Y\right)^2 - \frac{1}{2}\left(\partial_i \delta Y\right)^2\right]$$
$$+ \int d\eta d^3x\left[\frac{1}{2}\delta Y^2\left(\partial_\eta \theta_0\right)^2\right] + \int d\eta d^3x\left[-\left(\frac{3\lambda}{2}Y_0^2\right)\delta Y^2\right]$$

$$\lim_{k\ll\partial_\eta\theta_0} \Delta^2_{\frac{\delta\Gamma}{\Gamma_0}\frac{\delta\Gamma}{\Gamma_0}}(\eta<\eta_{\rm tr}) \approx \left(\frac{1}{3^{1/2}2^{3/2}}\right)\frac{1}{\Gamma_0^2(\eta)a^2(\eta)}\frac{k^3}{2\pi^2\partial_\eta\theta_0},$$

$$\lim_{\eta\to-\infty} S_\theta^{(2)} = \int d\eta d^3x\left[\frac{1}{2}\left(\partial_\eta \delta A\right)^2 - \frac{1}{2}\left(\partial_i \delta A\right)^2\right]$$

Questions:

1. Mass of spectator field to obtain large blue-tilt (niso > 2)?
   - > H

2. How do you switch off the mass at the end of the rolling to ensure that the axion density isn’t diluted away by inflation?

3. How do we get a large initial displacement and not worry about quartic interaction term?

All conditions satisfied by Kasuya-Kwasaki’s SUSY axion model.

Q1 via Hubble-induced mass terms

and Q2&3 via “SUSY flat-directions”.

At early time

$$S=-\int d^4x\frac{1}{2}\left\{\eta^{\mu\nu}\partial_\mu(a\phi)\,\partial_\nu(a\phi)-\cancel{\left(\frac{a'}{a}\right)^2(a\phi)^2}\right\}$$

$$\langle\delta(a\phi)(t,\vec{x})\delta(a\phi)(t,\vec{y})\rangle\sim\frac{1}{|\vec{x}-\vec{y}|^2}\quad\Rightarrow\quad\langle\delta\phi(t,\vec{x})\delta\phi(t,\vec{y})\rangle\sim\frac{1}{a(t)^2|\vec{x}-\vec{y}|^2}\equiv\frac{1}{|\vec{x}_{phy}-\vec{y}_{phy}|^2}.$$

Breaks time-translation

Later

$$S=-\int d^4x\frac{1}{2}\left\{\eta^{\mu\nu}\partial_\mu(a\phi)\,\partial_\nu(a\phi)-\left(\frac{a'}{a}\right)^2(a\phi)^2\right\}$$

$$\langle\delta\phi(t,\vec{x})\delta\phi(t,\vec{y})\rangle\sim H^2,\qquad a(t)|\vec{x}-\vec{y}|\gg H^{-1}$$

Scale-invariant spectrum

Massless scalar field

$$S = -\int d^4x a^2 \eta^{\mu\nu} \frac{1}{2} \partial_\mu \phi \partial_\nu \phi$$

$$g_{\mu\nu} = a^2(t)\eta_{\mu\nu}$$

$$S = -\int d^4x \frac{1}{2} \left\{ \eta^{\mu\nu} \partial_\mu (a\phi) \partial_\nu (a\phi) - \left(\frac{a'}{a}\right)^2 (a\phi)^2 \right\}$$

Note that this theory has a conformal symmetry restricted to "dilatations" defined as

$$a \to a u^{-1}$$

$$\phi \to \phi u$$

We will call this a "time-dependent conformal theory"

$$\zeta = \Phi^{\mathrm{N}} - H\frac{\delta\rho^{\mathrm{N}}}{\dot{\rho}}$$

Guage-invariant curvature perturbation

Adiabatic		Isocurvature
$\zeta_i = \zeta_j$	Gauge-invariant definition	$S_{ij} = 3\left(\zeta_i - \zeta_j\right)$
$\Phi \neq 0, S = 0$	Initial conditions for solving fluid equations	$\Phi = 0, S \neq 0$

SUSY axion model

S. Kasuya, M. Kawasaki [0904.3800]

Renormalizable superpotential:

$$W_{\rm PQ} = h\left(\Phi_+\Phi_- - F_a^2\right)\Phi_0$$

subscripts on Φ indicate U(1) PQ charges.

$$\Phi_+\Phi_- - F_a^2 = 0 \qquad \Phi_0 = 0 \qquad \text{flat-direction}$$

$$V = \frac{1}{2}c_+H^2\left|\Phi_+\right|^2 + \frac{1}{2}c_-H^2\left|\Phi_-\right|^2 + \frac{1}{2}\left|\Phi_+\Phi_- - F_a^2\right|^2$$

Kaehler induced mass terms

F-term

Kasuya-Kawasaki model relies on having a SUSY flat-direction (and two dynamical chiral PQ fields) and no quartic self interaction.

Is there a generic way to generate large blue isocurvature without flat-direction, for single dynamical PQ field and with quartic self-interaction term and with a plateau $< O(1)/\mathrm{Mpc}$?

The axial rotations cause a strong mixing between the radial and axial fluctuations.

$$a^2\left[\partial_\eta \delta a, \partial_\eta \delta\Gamma\right] \approx -2i\partial_\eta \theta_0 \delta^{(3)}(\vec{x}-\vec{y})$$

$$\mathcal{L}_{\rm int} \supset Y_0 \delta Y \eta^{\mu\nu} \partial_\mu \theta_0 \partial_\nu \delta\theta$$

In such a scenario, it is not obvious how to consistently quantize the two <u>strongly coupled</u> fields, define proper vacuum, identify the Goldstone mode and get correct correlation functions.

We need to consistently quantize the theory in a rotating background scenario

Explicit quantization

$$\phi = \Gamma e^{i\theta} \to a\phi = Y e^{i\theta}$$

$$\delta\psi^n = \quad (a\delta\Gamma, a\Gamma_0\delta\theta)^n$$

$$\begin{aligned}
[\delta\psi^n(\eta,\vec{x}), \delta\psi^m(\eta,\vec{x})] &= 0, \\
[\pi^n(\eta,\vec{x}), \pi^m(\eta,\vec{x})] &= 0, \\
[\delta\psi^n(\eta,\vec{x}), \pi^m(\eta,\vec{x})] &= i\delta^{nm}\delta^{(3)}(\vec{x}-\vec{y})
\end{aligned}
\longrightarrow
\begin{aligned}
[\delta\psi^n, \delta\psi^m] &= 0, \\
[\delta\psi^n, \partial_\eta\delta\psi^m] &= i\delta^{nm}\delta^{(3)}(\vec{x}-\vec{y}) \\
[\partial_\eta\delta\psi^n, \partial_\eta\delta\psi^m] &= i\delta^{(3)}(\vec{x}-\vec{y})\begin{bmatrix} 0 & 2\partial_\eta\theta_0 \\ -2\partial_\eta\theta_0 & 0 \end{bmatrix}.
\end{aligned}$$

Coupled EoM (cannot be decoupled due to anti-symm derivative coupling)

$$\partial_\eta^2\delta\psi^n - \partial_i^2\delta\psi^n + \kappa^{nm}\partial_\eta\psi^m + \left(\mathcal{M}^2\right)^{nm}\delta\psi^m = 0$$

Explicit quantization

2 annihilating ladder operators with ++ and +- belonging to different eigen-solutions

$$\delta\psi^n = \int \frac{d^3p}{(2\pi)^{3/2}} \left[a_{\vec{p}}^{++} c_{++} V_{++}^n e^{-i\omega_{++}\eta} + a_{\vec{p}}^{+-} c_{+-} V_{+-}^n e^{-i\omega_{+-}\eta} + h.c. \right] e^{i\vec{p}\cdot\vec{x}}$$

Eigenvectors not orthogonal

$$V_{++}^n = \begin{pmatrix} 1 \\ \mathcal{R}_{++} \end{pmatrix}, \quad V_{+-}^n = \begin{pmatrix} 1 \\ \mathcal{R}_{+-} \end{pmatrix}$$

Goldstone mode

$$V_{+-}^n \sim \begin{pmatrix} \delta\Gamma \\ \delta a \end{pmatrix} \sim \begin{pmatrix} k/\partial_\eta \theta_0 \\ 1 \end{pmatrix}$$

Lighter mode

$$\omega_{\pm-}^2 \approx \frac{k^2}{3} + O\left(\frac{k^4}{\lambda Y_c^2}\right),$$

Heavier mode

$$\omega_{\pm+}^2 \approx 6\lambda Y_c^2 + \frac{5k^2}{3} + O\left(\frac{k^4}{\lambda Y_c^2}\right)$$