A MINIMAL EXTENSION OF INERT 2HDM :

THE DISAPPEARING AND PREVAILING ANOMALIES

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Outline

Model : Minimal Extension of I2HDM

- i2HDM (Φ_1, Φ_2) + A neutral complex gauge singlet scalar field Φ_3

$$\Phi_1 = \begin{bmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}\left(v_{\rm SM} + \phi_1^0 + i\,\eta_1^0\right) \end{bmatrix};\ \Phi_2 = \begin{bmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}\left(\phi_2^0 + i\,\eta_2^0\right) \end{bmatrix};\ \Phi_3 = \frac{1}{\sqrt{2}}\left(v_s + \phi_3^0 + i\,\eta_3^0\right)$$

- Invoke a Z_2 symmetry : All SM Fields, Φ_1 : Even ; Φ_2, Φ_3 : Odd

$$\begin{aligned}
\mathscr{L}_{\rm scalar} &= \left(D_\mu\Phi_1\right)^\dagger \left(D^\mu\Phi_1\right) + \left(D_\mu\Phi_2\right)^\dagger \left(D_\mu\Phi_2\right) + \left(D_\mu\Phi_3\right)^* \left(D_\mu\Phi_3\right) - V_{\rm scalar} \\
V_{\rm scalar} &= -\frac{1}{2}m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) - \frac{1}{2}m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\
&+\lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{1}{2}\left[\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right] \\
&-\frac{1}{2}m_{33}^2\,\Phi_3^*\Phi_3 + \frac{\lambda_8}{2}\left(\Phi_3^*\Phi_3\right)^2 + \lambda_{11}\left|\Phi_1\right|^2\Phi_3^*\Phi_3 + \lambda_{13}\left|\Phi_2\right|^2\Phi_3^*\Phi_3 \\
&-i\,\kappa\left[\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right)\left(\Phi_3 - \Phi_3^\star\right)\right]
\end{aligned}$$

Model is further constrained by imposing additional global $U(1)$ symmetry :

$U(1)$: $\Phi_1 \to \Phi_1$, $\Phi_2 \to \Phi_2$, $\Phi_3 \to e^{i\alpha}\Phi_3$ – explicitly broken by the κ term.

Model

$$-\mathscr{L}_{\text{Yukawa}} = y_u\, \overline{Q_L}\, \widetilde{\Phi_1}\, u_R + y_d\, \overline{Q_L}\, \Phi_1\, d_R + y_\ell\, \overline{l_L}\, \Phi_1\, e_R + y_1\, \overline{l_L}\, \Phi_2\, e_R + \text{h.c.}$$

- All couplings real in order to preserve the CP invariance.
- m Explicit breaking of Z_2 in $\mathscr{L}_{\text{Yukawa}}$: to facilitate coupling of SM leptons with scalars pseudo-scalars.
- $\Phi_1 - \Phi_3$ mixing $\rightarrow$ CP even Scalars

$$m^2_{h_1} = \cos^2\theta_{13}\,\lambda_1\, v^2_{\text{SM}} + \sin(2\theta_{13})\, v_s\,\lambda_{11}\, v_{\text{SM}} + \sin^2\theta_{13}\, v_s^2\,\lambda_8$$
$$m^2_{h_3} = \sin^2\theta_{13}\,\lambda_1\, v^2_{\text{SM}} - \sin(2\theta_{13})\, v_s\,\lambda_{11}\, v_{\text{SM}} + \cos^2\theta_{13}\, v_s^2\,\lambda_8$$

$$\tan 2\theta_{13} = \frac{\lambda_{11}\, v_{\text{SM}}\, v_s}{\lambda_1\, v^2_{\text{SM}} - \lambda_8\, v_s^2}$$

- $\Phi_2 - \Phi_3$ mixing $\rightarrow$ CP odd Scalars

$$m^2_{A^0} = \frac{1}{2}\left(\overline{\lambda}_{345}\, v^2_{\text{SM}} - m^2_{22} + \lambda_{13} v_s^2\right)\cos^2\theta_{23} - \sqrt{2}\kappa v_{\text{SM}}\sin 2\theta_{23}$$
$$m^2_{P^0} = \frac{1}{2}\left(\overline{\lambda}_{345}\, v^2_{\text{SM}} - m^2_{22} + \lambda_{13} v_s^2\right)\sin^2\theta_{23} + \sqrt{2}\kappa v_{\text{SM}}\sin 2\theta_{23}$$

$$\tan(2\,\theta_{23}) = -\frac{2\sqrt{2}\, v_{\text{SM}}\,\kappa}{m^2_{P^0} + m^2_{A^0}}$$

Model

- **Other Scalars**

Massless Nambu-Goldstone Bosons : $\eta_1^0 \to G^0$; $\phi_1^\pm \to G^\pm$

Massive Components of Φ_2 : $\phi_2^0 \to h_2$; $\phi_2^\pm \to H^\pm$

with masses

$$m^2_{h_2} = \frac{1}{2}\left[-m^2_{22} + (\lambda_3+\lambda_4+\lambda_5)\, v^2_{\rm SM} + \lambda_{13} v_s^2\right]$$
$$m^2_{H^\pm} = -m^2_{22} + \lambda_3\, v^2_{\rm SM} + \lambda_{13} v_s^2$$

- λ_2 does not contribute to the mass spectrum

- The remaining eleven parameters in scalar potential

$$m^2_{11},\ m^2_{22},\ m^2_{33},\ \lambda_{i=1,3,4,5,8,11,13},\ \text{and}\ \kappa$$

are expressed in terms of

$$v_{\rm SM},\, v_s,\, m^2_{22},\, m^2_{h_1},\, m^2_{h_1},\, m^2_{h_2},\, m^2_{H^\pm},\ m^2_{A^0},\, m^2_{P^0},\, \theta_{13},\, \theta_{23}$$

Constraints on Parameter Space (Theoretical)

- **Positivity Conditions**

$$\lambda_1, \lambda_2, \lambda_8 > 0,$$

$$\bar{\lambda}_{12} \equiv \lambda_3 + \Theta\left[|\lambda_5| - \lambda_4\right]\ (\lambda_4 - |\lambda_5|) + \sqrt{\lambda_1\lambda_2} > 0,$$

$$\bar{\lambda}_{13} \equiv \lambda_{11} + \sqrt{\lambda_1\lambda_8} > 0,\ \bar{\lambda}_{23} \equiv \lambda_{13} + \sqrt{\lambda_2\lambda_8} > 0 \text{ and}$$

$$\sqrt{\lambda_1\lambda_2\lambda_8} + [\lambda_3 + \Theta[|\lambda_5| - \lambda_4](\lambda_4 - |\lambda_5|)]\sqrt{\lambda_8} + \lambda_{11}\sqrt{\lambda_2} + \sqrt{2\ \bar{\lambda}_{12}\bar{\lambda}_{13}\bar{\lambda}_{23}} > 0$$

- **Tree level Perturbative Unitarity:**

- **Stability and Positivity Conditions :**
  With $\lambda_5 = \left[m^2_{h_2} - m^2_{A^0} - m^2_{P^0}\right] / v^2_{\rm SM}$, get two mutually exclusive allowed regions of parameter space:

For Region I	:	$\lambda_5 > 0$, $m^2_{h_2} > m^2_{A^0} + m^2_{P^0}$,	and	$m^2_{H^\pm} > m^2_{A^0} + m^2_{P^0}$
For Region II	:	$\lambda_5 < 0$, $m^2_{h_2} < m^2_{A^0} + m^2_{P^0}$,	and	$m^2_{H^\pm} < m^2_{h_2}$

We Explore Region I in this work

Constraints on Parameter Space(Experimental)

Observables

- Higgs Decay
- LEP Data
- Anomalous Muon magnetic Dipole Moment Δa_μ
- W-mass measurement at CDF and CMS

Parameters

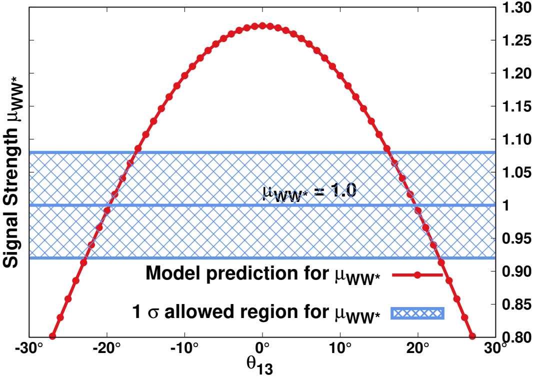
$$
\begin{aligned}
\text{Masses} &: \; m_{h_1}, m_{h_2}, m_{h_3}, m_{H^\pm}, \; m_{A^0}, m_{P^0} \\
\text{Mixing Angles} &: \; \theta_{13}, \theta_{23} \\
\text{Couplings} &: \; y_1, \; \lambda_{h_1 H^+ H^-}, \; \lambda_{h_3 H^+ H^-}
\end{aligned}
$$

Constraints From Higgs Decay

- Identify CP even lightest neutral scalar h_1 with the observed scalar H and take $m_{h_1} \simeq 125\,\text{GeV}$.
- Compare total Higgs decay width in SM $\Gamma(h^{\rm SM} \to \text{all}) \sim 4.07\,\text{MeV}$ with the total Higgs decay width at LHC $\Gamma(H \to \text{all})_{\rm LHC} = 3.2^{+2.4}_{-1.7}\,\text{MeV}$ *[PDG2022]*
- Determine the constrained parameter space by demanding that, in our model, h_1 decays can account for the measured value of the total Higgs decay width.
- Use signal strength μ_{XY} *w.r.t.* h_1 production *via* dominant gluon fusion in $p-p$ collision, followed by its decay to $X\,Y$ pairs in the narrow width approximation

$$\mu_{XY} \quad = \quad \frac{\sigma(pp \to h_1 \to XY)}{\sigma(pp \to h \to XY)^{\rm SM}} = \cos^2\theta_{13}\,\frac{{\rm BR}\,(h_1 \to X\,Y)}{{\rm BR}\,\left(h^{\rm SM} \to X\,Y\right)}$$

Constraints From Higgs Decay



$$\mu_{WW^\star} = \cos^4\theta_{13}\,\frac{\Gamma(h^{\rm SM}\to {\rm all})}{\Gamma(H\to {\rm all})_{\rm LHC}}$$

Using the observed value,

$\mu_{WW^\star} = 1.00 \pm 0.08$,

θ_{13} can be strongly constrained

Throughout this work, we take $\theta_{13} = 20^\circ$

Constraints From Higgs Decay

$$\frac{\mu_{\gamma\gamma}}{\mu_{WW^\star}} = \frac{\Gamma(h_1 \to \gamma\gamma)}{\Gamma(h_1 \to WW^\star)} \times \frac{\Gamma(h^{SM} \to WW^\star)}{\Gamma(h^{SM} \to \gamma\gamma)} = |1+\zeta_{\gamma\gamma}|^2$$

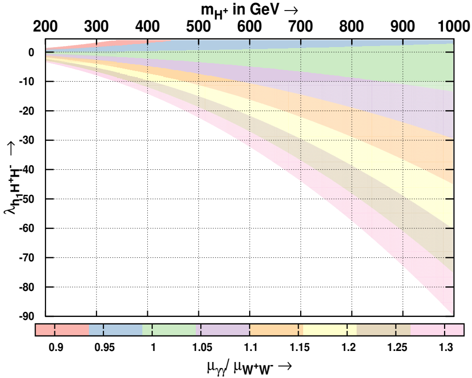
- We calculate the partial decay width $\Gamma(h_1 \to \gamma\gamma)$ at one loop in our model

.

- Using the average experimental values of $\mu_{\gamma\gamma}$ and $\mu_{WW^\star}$, the ratio $\mu_{\gamma\gamma}/\mu_{WW^\star} = 1.1 \pm 0.11$
- The coupling $g_{h_1 H^+ H^-}$ is bounded from below and above for a given value of $m_{H^\pm}$. For example, with $m_{H^\pm} = 1\,\text{TeV}$, the range allowed by $\mu_{\gamma\gamma}/\mu_{WW^\star}$ is

$$-90 < \lambda_{h_1 H^+ H^-} < 4 \quad \text{at} \quad 2\sigma$$

$$-60 < \lambda_{h_1 H^+ H^-} < 3 \quad \text{at} \quad 1\sigma$$



Constraints from LEP II Data

- LEP Limit from direct production of scalars *[1302.3415(Phys. Re.(2013)), 1301.6065(EPJC (2013))]* :

$$m_{H^\pm} \gtrsim (80-100)\text{GeV} \quad \text{and} \quad \sum m_{h_i} \gtrsim 200\, GeV$$

We take all scalar and pseudoscalar mases above 210 GeV

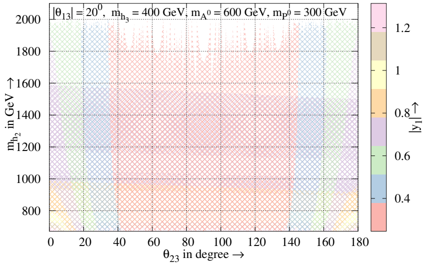
- The excess contribution to $\sigma(e^+ \, e^- \to \mu^+ \, \mu^-)$ in our model over SM value

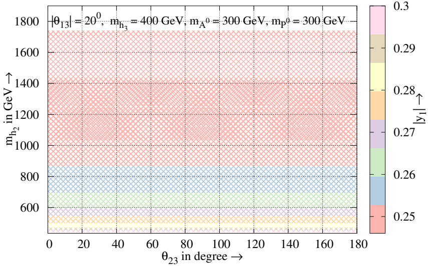
$$\sigma^{\rm Excess}_{\mu^+\mu^-} = \frac{s}{64\pi}\sqrt{\frac{s-4m_\mu^2}{s-4m_e^2}} \times \left[y_1^2 \left(-\frac{\cos^2\theta_{23}}{s-m^2_{A^0}} - \frac{\sin^2\theta_{23}}{s-m^2_{P^0}} + \frac{1}{s-m^2_{h_2}} \right) + \frac{2m_e m_\mu}{v^2_{\rm SM}} \left(\frac{\cos^2\theta_{13}}{s-m^2_{h_1}} + \frac{\sin^2\theta_{13}}{s-m^2_{h_3}} \right) \right]^2 - \left[\frac{2m_e m_\mu}{v^2_{\rm SM}} \left(\frac{1}{s-m^2_{h_{SM}}} \right) \right]^2$$

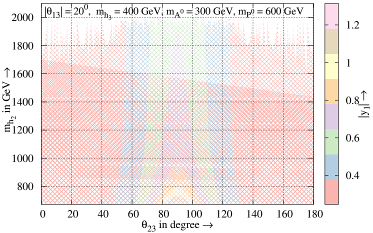
is used to constrain the model parameters by accommodating it within the 1σ uncertainty in the measured value $\sigma(e^+ \, e^- \to \mu^+ \, \mu^-) = 3.072 \pm 0.108 \pm 0.018\,\text{pb}$ by combined analysis of DELPHI and L3 at LEP II

[Schael et al, Phys. Rep. (2013) 1302.3415]

Constraints from LEP II Data







- LEP data tightly constrains the magnitude of $|y_1|$ and $|\theta_{23}|$
- The allowed range of $|y_1|$ governed by the choice of θ_{23} and $R_P = m_{P^0}/m_{A^0}$
- Except for $R_P = 1$, the allowed values of $|y_1|$ are not very sensitive to m_{h_2}

Muon Anomalous Magnetic Moment

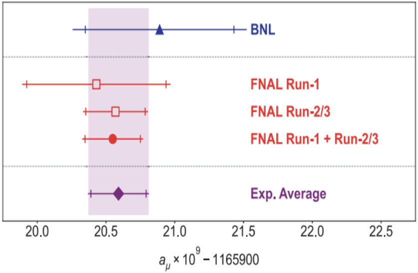
➤ indirect clues for physics beyond the SM

The Prevailing Anomaly in a_μ Anomalous magnetic moment : $a = \frac{g-2}{2}$

[T. Aoyama et al., Phys. Rep. (2020)]

$$a_\mu^{\rm SM} = 116591810(43) \times 10^{-11}$$

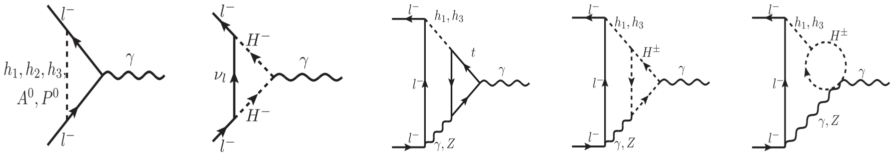
$$\Delta a_\mu = a_\mu^{\rm exp.} - a_\mu^{\rm SM} = (2.49 \pm 0.48) \times 10^{-9}$$



[Muon g - 2 Collaboration, PRL (2023)]

Improvements in uncertainties of SM prediction may reduce the discrepancy

Muon Anomalous Magnetic Moment



$$\delta a_l^{1\,\text{loop}} = \frac{1}{16\,\pi^2}\left[2\,\frac{m_l^4}{v_{\text{SM}}^2}\left(\frac{\cos^2\theta_{13}}{m_{h_1}^2}+\frac{\sin^2\theta_{13}}{m_{h_3}^2}-\frac{1}{m_{h^{\text{SM}}}^2}\right)\mathscr{I}_1+m_l^2\left(\frac{\cos^2\theta_{23}}{m_{A^0}^2}+\frac{\sin^2\theta_{23}}{m_{P^0}^2}\right)y_1^2\mathscr{I}_2 + \frac{m_l^2}{m_{h_2}^2}\,y_1^2\mathscr{I}_1+|y_1|^2\,\frac{m_l^2}{m_{H^\pm}^2}\mathscr{I}_3\right]$$

$$\delta a_l{}^{2\ loop} = \frac{\alpha_{\text{em}}}{4\ \pi^3}\left[\frac{m_l}{v_{\text{SM}}}\,\frac{m_t}{v_{\text{SM}}}\,\sin^2\theta_{13}\left\{f\left(\frac{m_t^2}{m_{h_3}^2}\right)-f\left(\frac{m_t^2}{m_{h_1}^2}\right)\right\} - \frac{1}{4}\,\frac{m_l}{v_{\text{SM}}}\left\{(\lambda_{h_1H^+H^-})\,\frac{m_l^2}{m_{h_1}^2}\,\cos\theta_{13}\,\tilde{f}\left(\frac{m_{H^\pm}^2}{m_{h_1}^2}\right)-\lambda_{h_3H^+H^-}\,\frac{m_l^2}{m_{h_3}^2}\,\sin\theta_{13}\,\tilde{f}\left(\frac{m_{H^\pm}^2}{m_{h_3}^2}\right)\right\}\right]$$

The Contribution of Charged Higgs negligibly small

The Disappearing W-Mass Anomaly

$$m_W^{\rm CDF} = (80.4335 \pm 0.0094)\,{\rm GeV}$$

[T. Aaltonen et al. (CDF), Science (2022)]

$$m_W^{\rm PDG} = (80.3692 \pm 0.0133)\,{\rm GeV}.$$

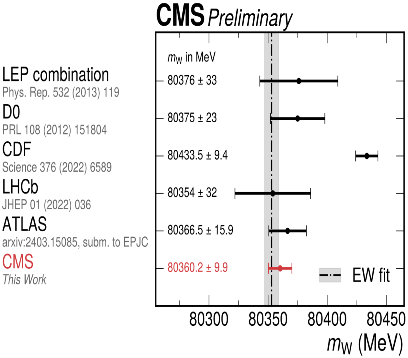
Global fit to electroweak data :

$$m_W^{\rm SM} = (80.353 \pm 0.006)\,{\rm GeV}$$

[PDG (2024)]

Recent measurement by CMS Collaboration

$$m_W^{\rm CMS} = (80.3602 \pm 0.0099)\,{\rm GeV}$$



[CMS-PAS-SMP-23-002 (2024)]

Aim : To explain the observed upward pull for m_W and Δa_μ

Computation of W- Mass

- In any BSM Model Discrepancy between the SM prediction and experimental value of m_W
$$\Delta m_W = \frac{\alpha\, m_W^{\mathrm{SM}}}{2(c_w^2 - s_w^2)} \left(-\frac{1}{2}\Delta S + c_w^2 \Delta T + \frac{c_w^2 - s_w^2}{4 s_w^2} \Delta U \right)$$
ΔS, ΔT, and ΔU are the deviations from their corresponding SM values in the estimation of the oblique parameters in any new physics models
- The deviations from SM are computed in our model at one loop level
- Quantum corrections to the relation is a function of the scalar and pseudoscalar masses and the gauge couplings.
- This relationship is employed for predicting the W-boson mass m_W by an iterative procedure

Viable Parameter Space with All Constraints

- From Higgs Decay:

  $|\theta_{13}| = 20^\circ, \ -\lambda_{\min} < \lambda_{h_1 H^+ H^-} < \lambda_{\max}$

  $\lambda_{\min}, \lambda_{\max}$ being bounds determined by $\mu_{\gamma\gamma}/\mu_{WW^\star}$

- Representative Values

  $$R_P = \frac{m_{P^0}}{m_{A^0}} = 0.5, \ 1, \ 2$$

  $(m_{A^0}, m_{P^0}) =$

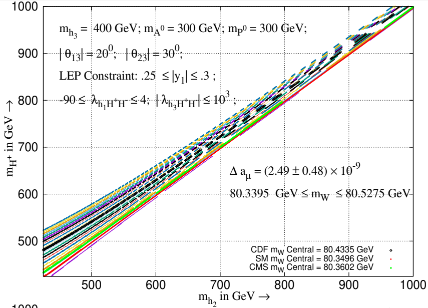
  $[(600, 300), (300, 300), (300, 600)]\, GeV$

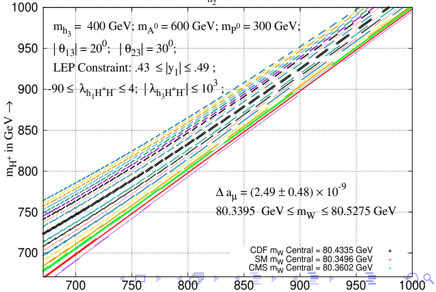
  $\theta_{23} = 30^\circ, 45^\circ, 60^\circ$

- Vary masses in range

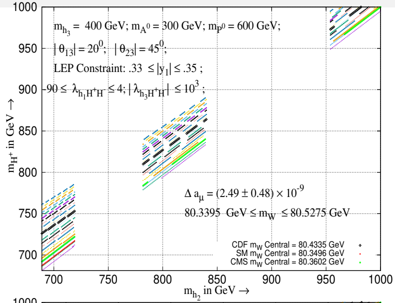
  $$\sqrt{m_{A^0}^2 + m_{P^0}^2} < m_{h_2}, m_{H^\pm} \leq 1000\, GeV$$

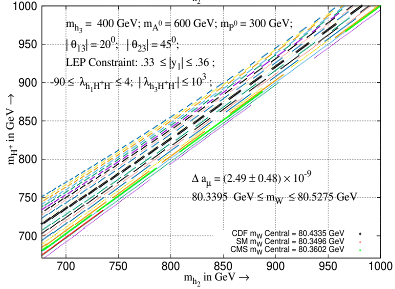
  and $\quad 200 \text{ GeV} < m_{h_3} < 1000 \text{ GeV}$





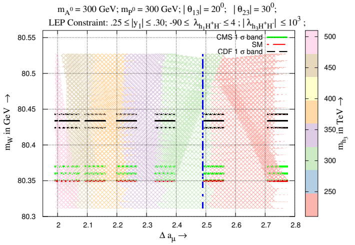
Viable Parameter Space

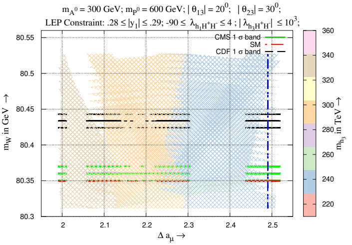


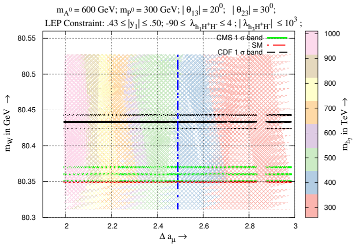


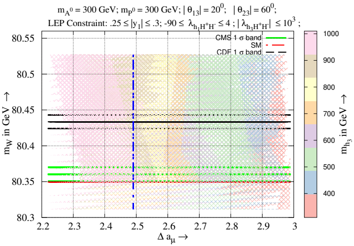
- $\theta_{23} = 30^\circ$, $R_P = 0.5$ case is similar to $\theta_{23} = 60^\circ$, $R_P = 2$
- For $R_P = 1$, similar patterns are obtained in the contour plots for all θ_{23}
- No viable solution for m_W in the required range is found for $R_P = 2(0.5)$, with $\theta_{23} = 30^\circ(60^\circ)$ at fixed $m_{h_3} = 400$ GeV
- The long discontinuities of loci of points indicate the noncompliance of the model parameters to simultaneously accommodate measured values of both observables

Viable Parameter Space



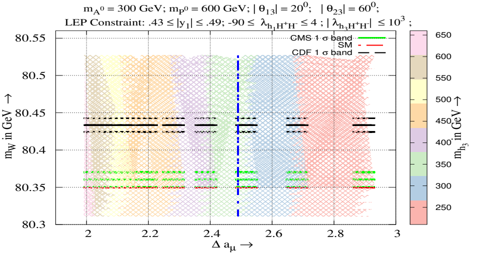


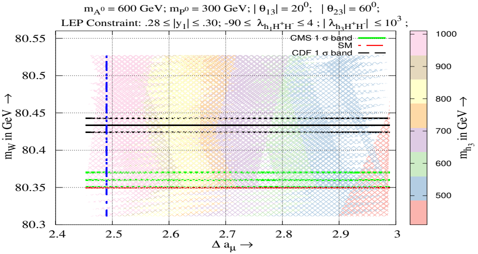




Viable Parameter Space

- For a given R_P, lower values of m_{h_3} are favored for lower values of mixing angle θ_{23}. Similarly, for a given value of θ_{23}, lower values of m_{h_3} are favored for higher R_P.
- For $\theta_{23} = 30^\circ$ and $R_P = 2, m_{A^0} = 300\,\text{GeV}(\theta_{23} = 60^\circ$ and $R_P = 0.5, m_{A^0} = 600\,\text{GeV})$, the common parameter space allowed by m_W value favors Δa_μ in the lower(upper) half of 1σ band.





Summary

- The I2HDM minimally extended by a complex singlet scalar is explored for the possibility to explain Δa_μ and m_W observed values simultaneously
- The parameter space is constrained by
  - Theoretical Considerations
  - Higgs decay and signal strength at LHC
  - LEP II data
- The constrained parameter space is scanned systematically to search for simultaneous solution for W mass lying in the range $m_W \in [80.3395 : 80.5275]$ which includes $m_W^{\rm SM}$, $m_W^{\rm CMS}$ as well as $m_W^{\rm CDF}$ and the anomalous magnetic moment of muon lying within one sigma band $\Delta a_\mu \in [2.01 : \ 2.97] \times 10^{-9}$
- Work on Constraints from DM relic Density is under progress

Thank You

For Your Patience

Back up Slides

Scalar Couplings in terms of Mass Parameters

$$
\begin{aligned}
\lambda_3 &= \frac{1}{v_{\rm SM}^2}\left[2\, m_{H^\pm}^2 + m_{22}^2 - \lambda_{13}\, v_s^2\right] \\
\lambda_4 &= \frac{1}{v_{\rm SM}^2}\left[m_{h_2}^2 + m_{A^0}^2 + m_{P^0}^2 - 2\, m_{H^\pm}^2\right]. \\
\lambda_5 &= \frac{1}{v_{\rm SM}^2}\left[m_{h_2}^2 - m_{A^0}^2 - m_{P^0}^2\right] \\
\lambda_8 &= \frac{1}{v_s^2}\left[m_{h_1}^2 + m_{h_3}^2 - \lambda_1\, v_{\rm SM}^2\right] \\
\lambda_{11} &= \frac{1}{v_{\rm SM}\, v_s}\left(\lambda_1\, v_{\rm SM}^2 - \lambda_8\, v_s^2\right)\tan(2\,\theta_{13}) \\
\kappa &= -\frac{1}{2\sqrt{2}\, v_{\rm SM}}\left(m_{P^0}^2 + m_{A^0}^2\right)\tan(2\,\theta_{23})
\end{aligned}
$$

Mixing of Scalars

$$M^2_{\phi_1^0\,\phi_3^0} = \frac{1}{2}\begin{pmatrix}\phi_1^0 & \phi_3^0\end{pmatrix}\begin{pmatrix}\lambda_1\, v_{\rm SM}^2 & \lambda_{11}\, v_{\rm SM}\, v_s \\ \lambda_{11}\, v_{\rm SM}\, v_s & \lambda_8\, v_s^2\end{pmatrix}\begin{pmatrix}\phi_1^0 \\ \phi_3^0\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}\eta_2^0 & \eta_3^0\end{pmatrix}\begin{pmatrix}-\frac{1}{2}m_{22}^2 + \frac{1}{2}\overline{\lambda}_{345}\, v_{\rm SM}^2 + \frac{1}{2}v_s^2\lambda_{13} & -\sqrt{2}\kappa v_{\rm SM} \\ -\sqrt{2}\kappa v_{\rm SM} & 0\end{pmatrix}\begin{pmatrix}\eta_2^0 \\ \eta_3^0\end{pmatrix}$$

Model: Relevant Couplings

Yukawa couplings with scalar/ pseudoscalar mass eigenstates

y_{ffh_1}	$\left(\sqrt{2}m_f/v_{\rm SM}\right)\cos\theta_{13}$	y_{llh_2}	y_1
y_{ffh_3}	$-\left(\sqrt{2}m_f/v_{\rm SM}\right)\sin\theta_{13}$	y_{llP^0}	$-i\ y_1\sin\theta_{23}$
$y_{l\nu H^-}$	y_1	y_{llA^0}	$i\ y_1\cos\theta_{23}$

Scalar Triple Couplings of Charged Higgs

$$\lambda_{h_1H^+H^-} = \lambda_3\cos\theta_{13} + \frac{v_s}{v_{\rm SM}}\lambda_{13}\sin\theta_{13}$$

$$\lambda_{h_3H^+H^-} = \frac{v_s}{v_{\rm SM}}\lambda_{13}\cos\theta_{13} - \lambda_3\ \sin\theta_{13}$$

Loop Form Factors in Higgs Decay Width

$$\Gamma(h^{\rm SM}\to\gamma\gamma) = \frac{G_F\alpha^2\ m_h^3}{128\sqrt{2}\pi^3}\left|\frac{4}{3}\mathscr{M}^{\gamma\gamma}_{1/2}\left(\frac{4m_t^2}{m_h^2}\right)+\mathscr{M}^{\gamma\gamma}_{1}\left(\frac{4m_{\rm W}^2}{m_h^2}\right)\right|^2$$

$$\zeta_{\gamma\gamma} = \frac{v_{\rm SM}}{\cos\theta_{13}}\left[\frac{\frac{g_{h_1H^+H^-}}{2\,m^2_{H^\pm}}\mathscr{M}^{\gamma\gamma}_0\left(\frac{4m^2_{H^\pm}}{m^2_{h_1}}\right)}{\mathscr{M}^{\gamma\gamma}_1\left(\frac{4\,m^2_{\rm W}}{m^2_{h_1}}\right)+\frac{4}{3}\mathscr{M}^{\gamma\gamma}_{1/2}\left(\frac{4\,m^2_t}{m^2_{h_1}}\right)}\right]$$

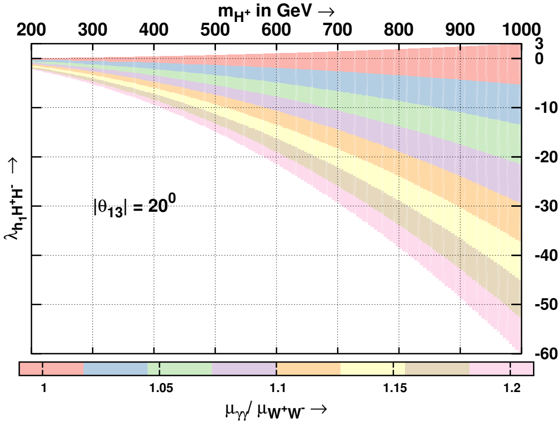
$$\mathscr{M}^{\gamma\gamma}_0(\tau) = -\tau[1-\tau f(\tau)]$$
$$\mathscr{M}^{\gamma\gamma}_{1/2}(\tau) = 2\tau[1+(1-\tau)f(\tau)],$$
$$\mathscr{M}^{\gamma\gamma}_1(\tau) = -[2+3\tau+3\tau(2-\tau)f(\tau)]$$

$$f(\tau)=\begin{cases}\arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) & \text{for } \tau\geqslant 1,\\ -\frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right)-i\pi\right]^2 & \text{for } \tau<1\end{cases}$$

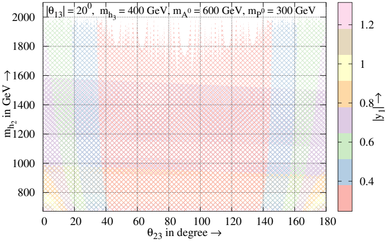
The dimensionless parameter τ is essentially function of the ratios of mass squared of physical scalars, pseudo-scalars, gauge bosons and fermions.

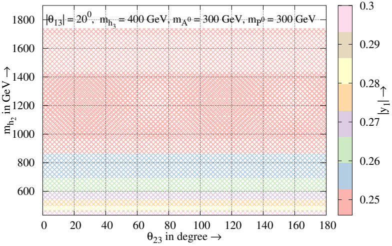
Higgs Decay

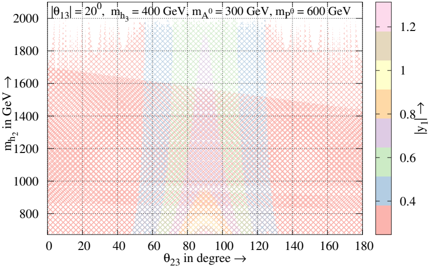
Allowed value of $\lambda_{h_1 H^+ H^-}$ at 1σ value of signal strength



LEP Constraints







One loop and two loop functions for MDM

The integrals in one loop contribution to the muon magnetic moment of leptons

$$
\begin{aligned}
\mathscr{I}_1(r^2) &= \int_0^1 dx\, \frac{(1+x)(1-x)^2}{(1-x)^2\; r^2 + x} \\
\mathscr{I}_2(r^2) &= \int_0^1 dx\, \frac{-(1-x)^3}{(1-x)^2\; r^2 + x}, \\
\mathscr{I}_3(r^2) &= \int_0^1 dx\, \frac{-x(1-x)}{1-(1-x)\; r^2}
\end{aligned}
$$

with $r = \frac{m_l}{m_{s_i}}$, and $s_i = h_1,\ h_2,\ h_3,\ A^0,\ P^0$.

The integrals contributing to the muon magnetic moment of leptons at two loop level

$$
\begin{aligned}
f(r^2) &= \frac{r^2}{2}\int_0^1 dx\, \frac{1\ -\ 2x(1-x)}{x(1-x)\ -\ r^2}\, \ln\left[\frac{x(1-x)}{r^2}\right] \\
\tilde{f}(r^2) &= \int_0^1 dx\, \frac{x(1-x)}{r^2 -\ x(1-x)}\, \ln\left[\frac{x(1-x)}{r^2}\right]
\end{aligned}
$$

The Oblique Parameters

The precision observables derived from the radiative corrections of the gauge Boson propagator are essentially the two point vacuum polarization tensor functions of $\Pi_{ij}^{\mu\nu}(q^2)$, q^2 is the four-momentum of the vector boson $(V = W, Z or \gamma)$.

The vacuum polarization tensor functions corresponding to pair of gauge Bosons V_i, V_j

$$i\Pi_{ij}^{\mu\nu}(q) = ig^{\mu\nu}A_{ij}(q^2) + iq^{\mu}q^{\nu}B_{ij}(q^2) \quad ; \qquad A_{ij}(q^2) = A_{ij}(0) + q^2 F_{ij}(q^2)$$

The oblique parameters are defined as:

$$S \equiv \frac{1}{g^2}\left(16\pi\cos\theta_W^2\right)\left[F_{ZZ}(m_Z^2) - F_{\gamma\gamma}(m_Z^2) + \left(\frac{2\sin\theta_W^2 - 1}{\sin\theta_W\cos\theta_W}\right)F_{Z\gamma}(m_Z^2)\right] \tag{1}$$

$$T \equiv \frac{1}{\alpha_{em}}\left[\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}\right] \tag{2}$$

$$U \equiv \frac{1}{g^2}(16\pi)\left[F_{WW}(m_W^2) - F_{\gamma\gamma}(m_W^2) - \frac{\cos\theta_W}{\sin\theta_W}F_{Z\gamma}(m_W^2)\right] - S\,. \tag{3}$$

The Oblique Parameters

The additional contribution to the oblique parameters (apart from SM) in our model can be computed to give

$$\Delta S = \frac{G_F\,\alpha_{em}^{-1}}{2\sqrt{2}\pi^2}\sin^2(2\theta_W)\Bigg[\sin^2\theta_{13}\Big\{m_Z^2\Big(\mathscr{B}_0(m_Z^2;m_Z^2,m_{h_1}^2)-\mathscr{B}_0(m_Z^2;m_Z^2,m_{h_3}^2)\Big)$$
$$+\mathscr{B}_{22}(m_Z^2;m_Z^2,m_{h_3}^2)-\mathscr{B}_{22}(m_Z^2;m_Z^2,m_{h_1}^2)\Big\}$$
$$+\cos^2\theta_{23}\mathscr{B}_{22}(m_Z^2;m_{h_2}^2,m_{A^0}^2)+\sin^2\theta_{23}\mathscr{B}_{22}(m_Z^2;m_{h_2}^2,m_{P^0}^2)-\mathscr{B}_{22}(m_Z^2;m_{H^\pm}^2,m_{H^\pm}^2)\Bigg]$$

where

$$\mathscr{B}_{22}(q^2;m_1^2,m_2^2) = B_{22}(q^2;m_1^2,m_2^2)-B_{22}(0;m_1^2,m_2^2)$$
$$\mathscr{B}_0(q^2;m_1^2,m_2^2) = B_0(q^2;m_1^2,m_2^2)-B_0(0;m_1^2,m_2^2)$$

THE OBLIQUE PARAMETERS

$$\begin{aligned}
\Delta T \quad = \quad & \frac{G_F\, \alpha_{em}^{-1}}{2\sqrt{2}\pi^2}\Bigg[\sin^2\theta_{13}\Bigg\{ m_W^2\Big(B_0(0; m_W^2, m_{h_1}^2) - B_0(0; m_W^2, m_{h_3}^2)\Big) \\
& - m_Z^2\Big(B_0(0; m_Z^2, m_{h_1}^2) - B_0(0; m_Z^2, m_{h_3}^2)\Big) + B_{22}(0; m_W^2, m_{h_3}^2) - B_{22}(0; m_W^2, m_{h_1}^2) \\
& + B_{22}(0; m_Z^2, m_{h_1}^2) - B_{22}(0; m_Z^2, m_{h_3}^2)\Bigg\} - \frac{1}{2} A_0(m_{H^\pm}^2) + B_{22}(0; m_{H^\pm}{}^2, m_{h_2}^2) \\
& + \cos^2\theta_{23}\Big(B_{22}(0; m_{H^\pm}{}^2, m_{A^0}^2) - B_{22}(0; m_{h_2}^2, m_{A^0}^2)\Big) \\
& + \sin^2\theta_{23}\Big(B_{22}(0; m_{H^\pm}{}^2, m_{P^0}^2) - B_{22}(0; m_{h_2}^2, m_{P^0}^2)\Big)\Bigg]
\end{aligned}$$

$$\begin{aligned}
A_0(m^2) \quad &= \quad m^2\ (\Delta + 1 - \ln m^2) \\
B_0(q^2; m_1^2, m_2^2) \quad &= \quad \Delta - \int_0^1 dx \ln(X - i\varepsilon) \\
B_{22}(q^2; m_1^2, m_2^2) \quad &= \quad \frac{1}{4}(\Delta + 1)\left[m_1^2 + m_2^2 - \frac{1}{3}q^2\right] - \frac{1}{2}\int_0^1 dx\, X \ln(X - i\varepsilon)
\end{aligned}$$

where $X \equiv m_1^2 x + m_2^2(1-x) - q^2 x(1-x)$ and $\Delta \equiv \frac{2}{4-d} + \ln(4\pi) + \gamma_E$ in d space-time dimensions