Self-interactions of ultralight spinless dark matter to the rescue?

Gaurav Goswami

Ahmedabad University

XVII International Conference on Interconnections between Particle Physics and Cosmology
Indian Institute of Technology, Hyderabad

- Bihag Dave, **Gaurav Goswami**, "*ULDM self-interactions, tidal effects and tunnelling out of satellite galaxies*," J. Cosmol. Astropart. Phys., 02 (**2024**) 044. E-Print: 2310.19664 [astro-ph.CO]
- Bihag Dave, **Gaurav Goswami**, "*Self-interactions of ULDM to the rescue?*," J. Cosmol. Astropart. Phys., 07 (**2023**) 015. E-Print: 2304.04463 [astro-ph.CO]
- Sayan Chakrabarti, Bihag Dave, Koushik Dutta, **Gaurav Goswami**, "*Constraints on the mass and self-coupling of Ultra-Light Scalar Field Dark Matter using observational limits on galactic central mass*," J. Cosmol. Astropart. Phys., 09 (**2022**) 074. E-print: 2202.11081 [astro-ph.CO]

The Punchline...

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - U(\varphi) \right)$$

$$U(\varphi) = m_a^2 f^2 \left[1 - \cos\left(\frac{\varphi}{f}\right) \right]$$

$$U(\varphi) = \frac{m^2 \varphi^2}{2} + \frac{\lambda \varphi^4}{4!}$$

$$\lambda_a = -\left(\frac{m_a}{f}\right)^2$$

$$\lambda > 0$$

$m \sim 10^{-22}$ eV and $f \sim 10^{17}$ GeV, then, $\lambda \sim 10^{-96}$

Some observations can probe self couplings of e.g. $\mathcal{O}(10^{-90})$

This non-negligible coupling could often be useful

Could be helpful in uncovering the identity of DM

The Plan

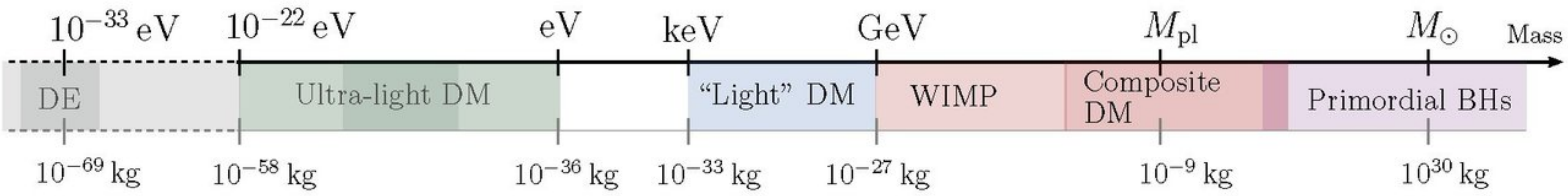
- Wave Dark Matter
- Self Interactions of Ultra Light Dark Matter
- Observable effects of self coupling - I
- Observable effects of self coupling - II
- Conclusions

Wave Dark Matter

Dark Matter: particle physics

Microscopic origin?

- Stable / long lived (lifetime cosmological)
- Electric charge very small (or zero)
- Mass unknown
- Spin unknown
- Non-gravitational interactions / couplings unknown (but constrained)



Ultra Light Dark Matter

Dark Matter particles

- Stable
- Zero electric charge
- Small mass (how small?)
  - **Note**: non-thermal
- Zero intrinsic spin (i.e. is scalar or pseudo scalar)
- Self interactions (inevitable for scalar)
  - Singlet under SM gauge group

If DM particle mass is too small, it can't be fermion

Wave Dark Matter

- Very small particle mass implies very large number density
- Bosonic quantum fields → Particles and waves in classical limits
  - Gamma ray photons vs radio waves
  - Particle DM vs wave DM

- Ultra light Bosonic Dark matter can be described by classical field equations

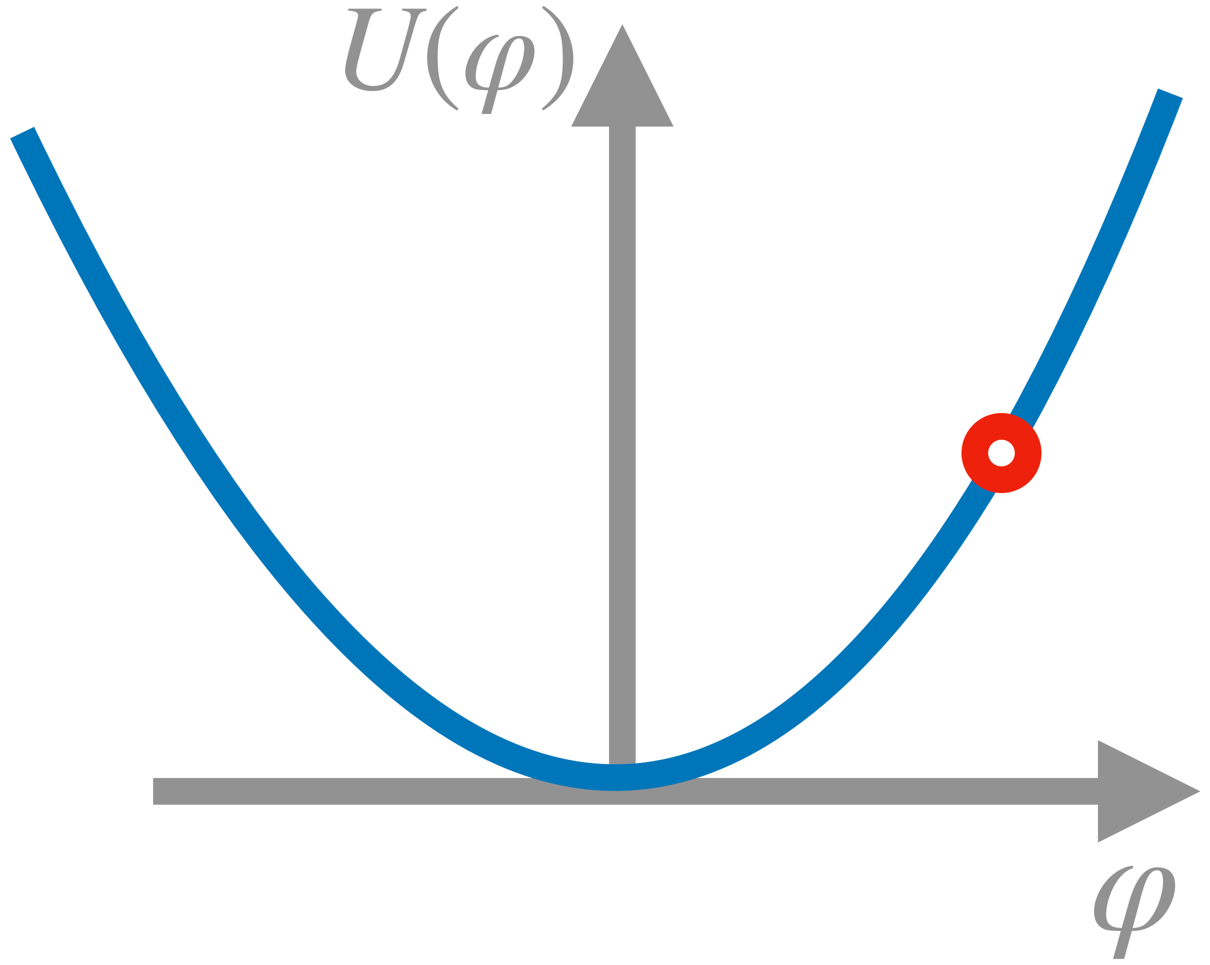
Cosmology with UL(SF)DM

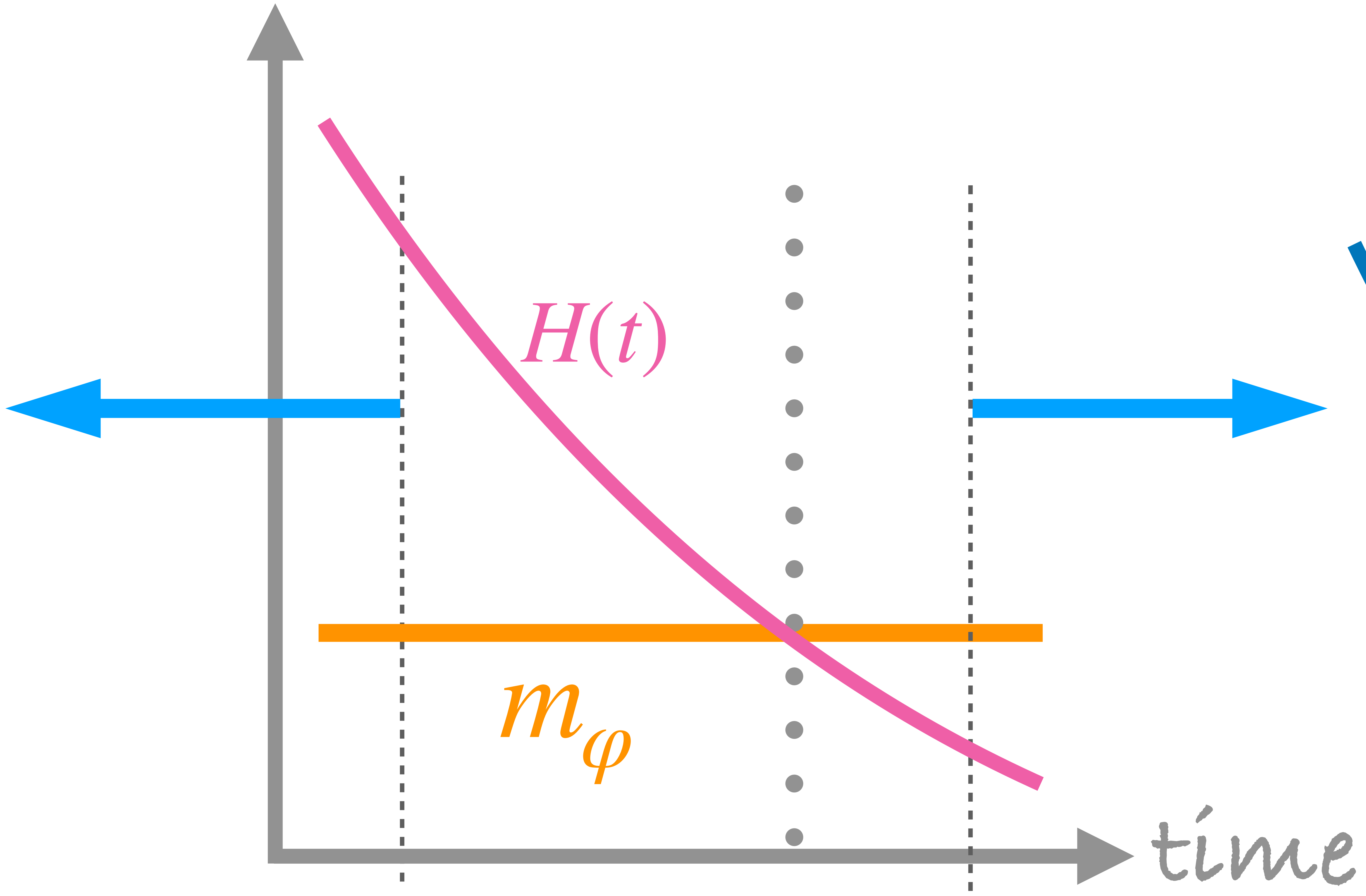
$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - U(\varphi)\right)$$

- **Production**: e.g. misalignment mechanism
- **Background dynamics** same as that of N.R. particles as long as scalar field oscillates at the bottom of a quadratic potential $m \gtrsim 10^{-28}$ eV
  - Unlike inflation, dark energy
- **Linear** perturbations
  - Matter power spectrum: small scale power suppression: classical wave can't be squashed into too small a region $m \gtrsim 10^{-23}$ eV
- **Nonlinear** scales

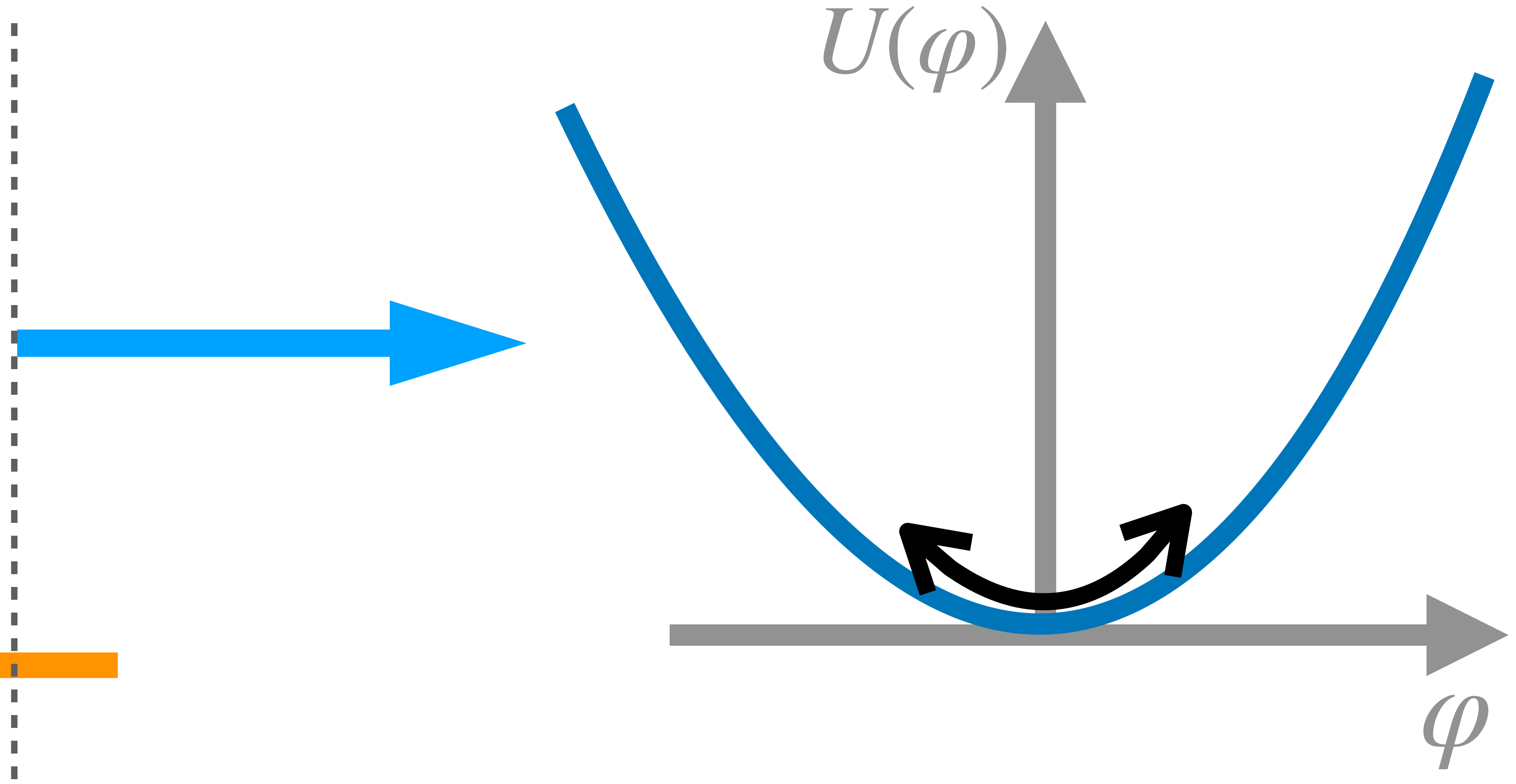
Production mechanism

Stuck





Oscillates



$P \ll \rho$

$\rho \sim a^{-3}$

$m \gtrsim H_{eq} \approx 10^{-28}$ eV

Klein-Gordon-Einstein to Schrodinger-Poisson

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - U(\varphi) \right)$$

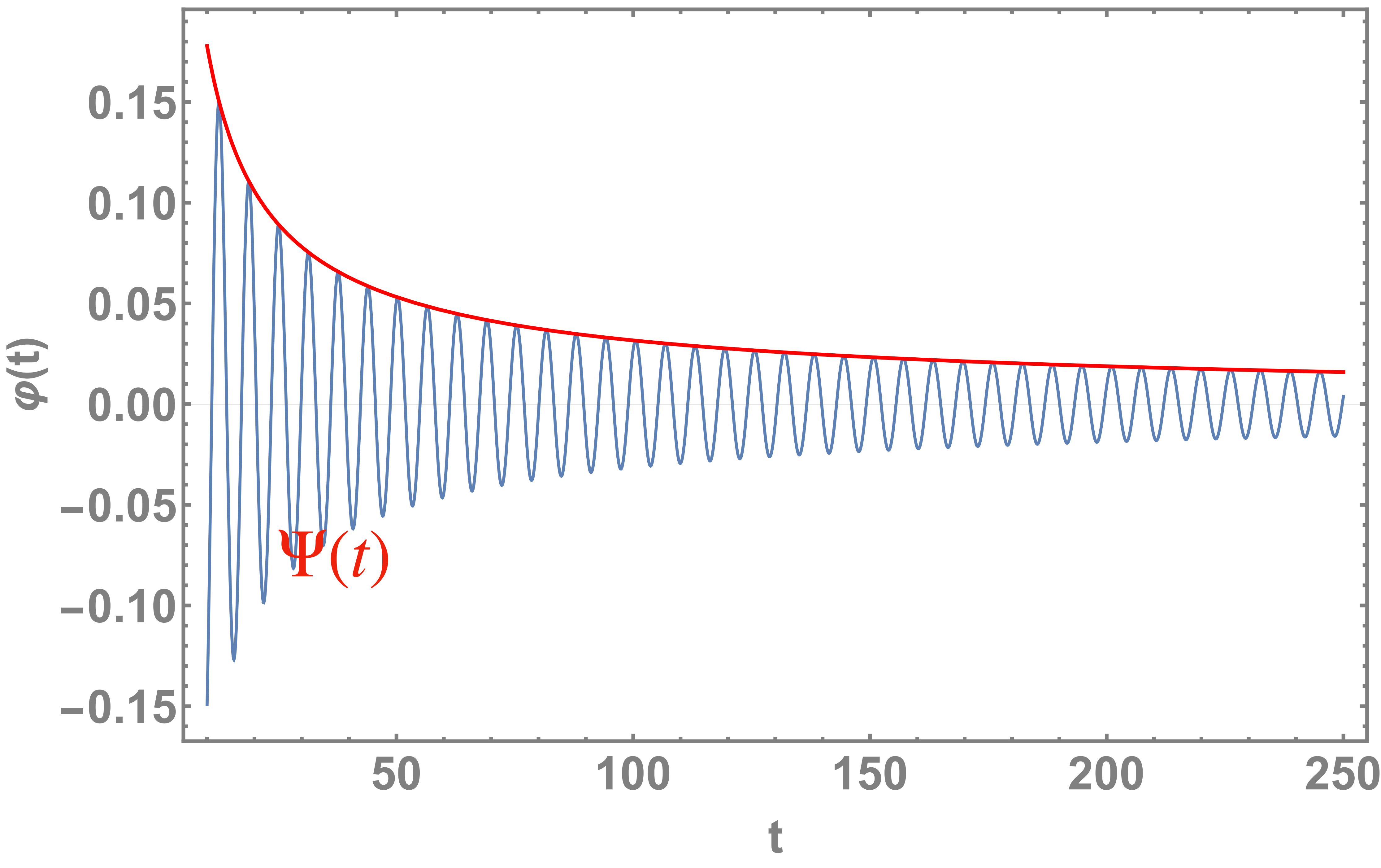
$$U(\varphi) = \frac{m^2\varphi^2}{2} + \frac{\lambda\varphi^4}{4!}$$

$$U(\varphi) = m_a^2 f^2 \left[1 - \cos\left(\frac{\varphi}{f}\right) \right]$$

Slowly varying, non-relativistic limit…

$$\varphi(t, \vec{x}) = \frac{1}{\sqrt{2m}} \left[e^{-imt} \Psi(t, \vec{x}) + \text{c.c.} \right]$$

Slowly varying field



$$\Psi(t + m^{-1}) = \Psi(t) + \frac{\dot{\Psi}(t)}{m} + \frac{\ddot{\Psi}(t)}{2m^2} + \cdots$$

$$\Psi(t) \gg \frac{\dot{\Psi}(t)}{m} \gg \frac{\ddot{\Psi}(t)}{m^2} \text{ etc}$$

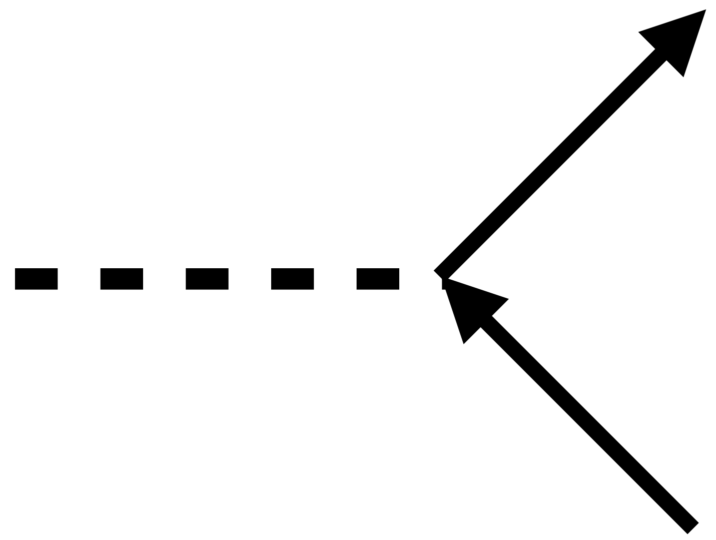
$$i\frac{\partial \Psi}{\partial t} = -\frac{\nabla^2}{2m}\Psi + m\Phi\Psi$$

$$\nabla^2 \Phi = \frac{|\Psi|^2}{2M_{\rm pl}^2} + \cdots .$$

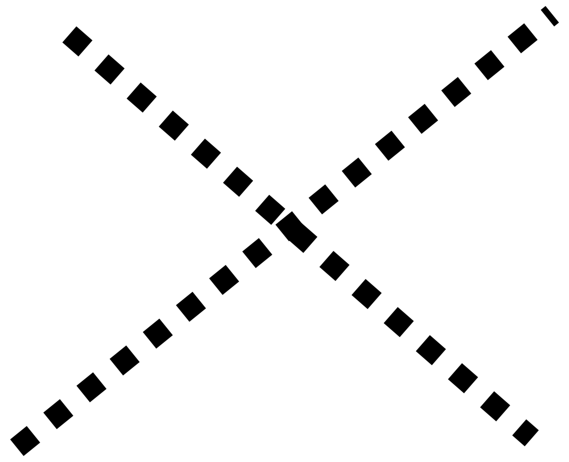
Self interactions of ULDM?

Ultra-light scalar field DM self coupling

- Ultra light scalar fields, mass of O (10^{-22} eV), could act as DM,



- Does this scalar couple to other particles?



- What is the self coupling, λ, of this scalar?
    - It must exist, the question is, is it small enough be ignored?
    - This must be established by observations
    - **Even a very small value of self coupling, λ, can have dramatic implications**

Benchmark value of self coupling

$$U(\varphi) = m_a^2 f^2 \left[1 - \cos\left(\frac{\varphi}{f}\right)\right]$$

$$\lambda_a = -\left(\frac{m_a}{f}\right)^2$$

- **Benchmark value**
  - Misalignment mechanism: correct relic abundance
    - if $m \sim 10^{-22}$ eV and $f \sim 10^{17}$ GeV, then $\lambda \sim 10^{-96}$

$$\Omega_a \sim 0.1 \left(\frac{f_a}{10^{17}\ \text{GeV}}\right)^2 \left(\frac{m_a}{10^{-22}\ \text{eV}}\right)^{1/2} \implies \lambda_a \sim 10^{-96}$$

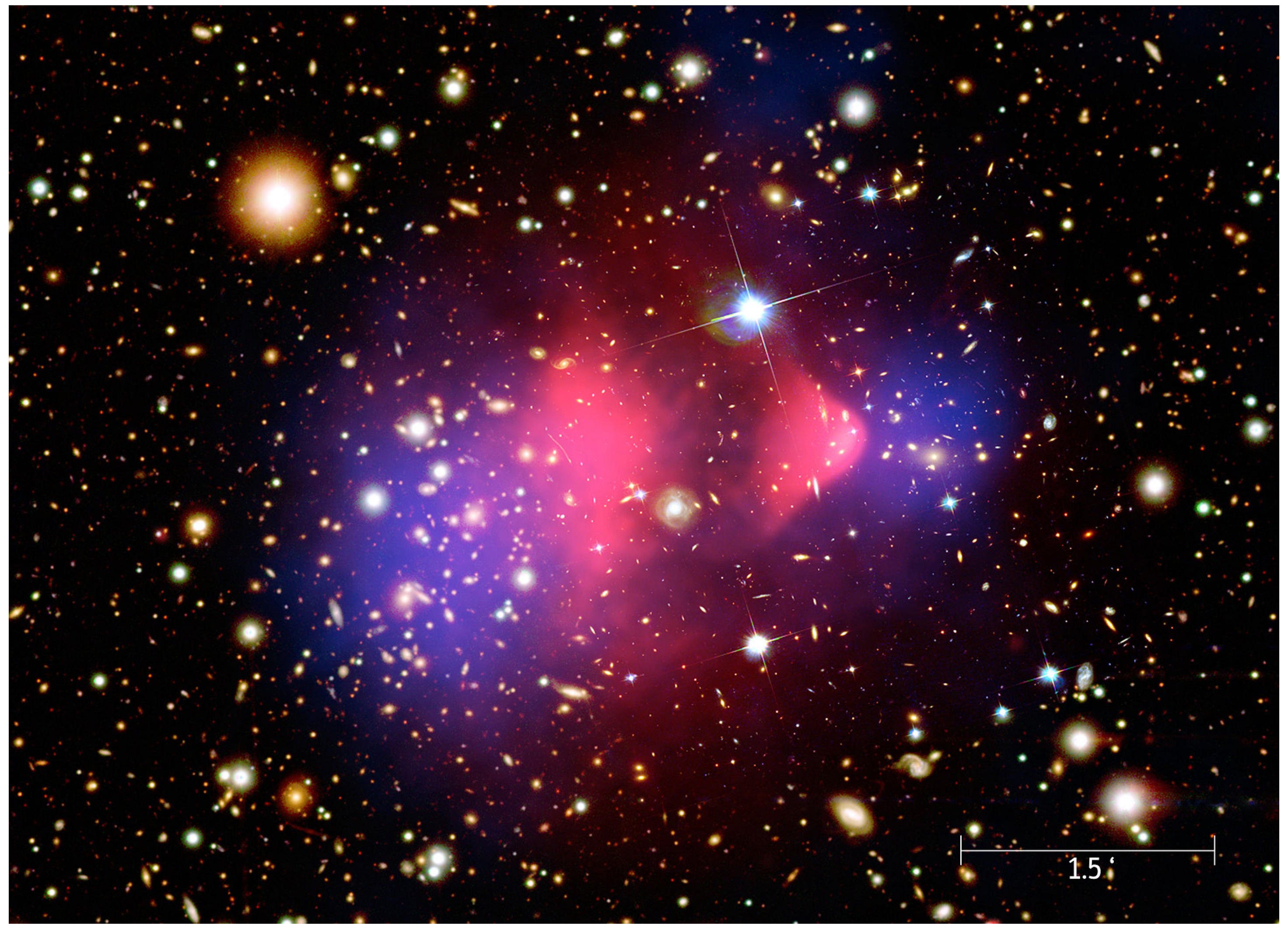
Small self-interactions

$$i\,\dot{\Psi} = -\frac{1}{2m}\nabla^2\Psi + m\Phi\Psi + \frac{\lambda}{8m^3}|\Psi|^2\Psi$$

$$\nabla^2\Phi = 4\pi G|\Psi|^2$$

$$\frac{\hbar^2}{2m}\frac{1}{L^2}\Psi \sim \lambda\frac{M}{L^3}\Psi \implies \lambda \sim 4\left(\frac{m}{10^8\ \mathrm{M}_\odot}\right)\left(\frac{\mathrm{kpc}}{\hbar/mc}\right) \sim 10^{-92}$$

$\rho = |\Psi|^2$ (Mass density)



chandra.harvard.edu/photo/2006/1e0657/more.html

Bullet cluster constraints on self-interactions

$$\sigma/m \lesssim 0.5\mathrm{cm}^2 g^{-1}$$

$$\sigma = \frac{\lambda^2}{32\pi s} \sim \frac{\lambda^2}{32\pi m^2}$$

$$\implies \lambda \lesssim 10^{-44} \qquad m = 10^{-22}\ \mathrm{eV}$$

Sign of self coupling

$$U(\varphi) = \frac{m^2\varphi^2}{2} + \frac{\lambda\varphi^4}{4!}$$

$$U(\varphi) = m_a^2 f^2 \left[1 - \cos\left(\frac{\varphi}{f}\right)\right]$$

$$\lambda_a = -\left(\frac{m_a}{f}\right)^2$$

- Thus,
  - What is the **sign** of the self-coupling? I.e. attractive or repulsive?
  - What is the **strength** of the self-coupling?

- Recall: quartic self coupling implies contact interactions i.e. in N.R. limit, interaction PE is $V(\mathbf{r}_i, \mathbf{r}_j) = \#\ \delta^3(\mathbf{r}_i - \mathbf{r}_j)$

- Could eventually help in **identifying** the scalar field i.e. Dark Matter

Gross-Pitaevskii-Poisson equations

Spreading

Self gravity

Scalar self interactions

$$i\frac{\partial\Psi}{\partial t} = -\frac{\nabla^2}{2m}\Psi + m\Phi\Psi + \frac{\lambda}{8m^3}|\Psi|^2\Psi \;+\; \cdots$$

$$\nabla^2\Phi = \frac{|\Psi|^2}{2M_{\rm pl}^2} \;+\; \cdots\,.$$

Observable effects of self coupling - I

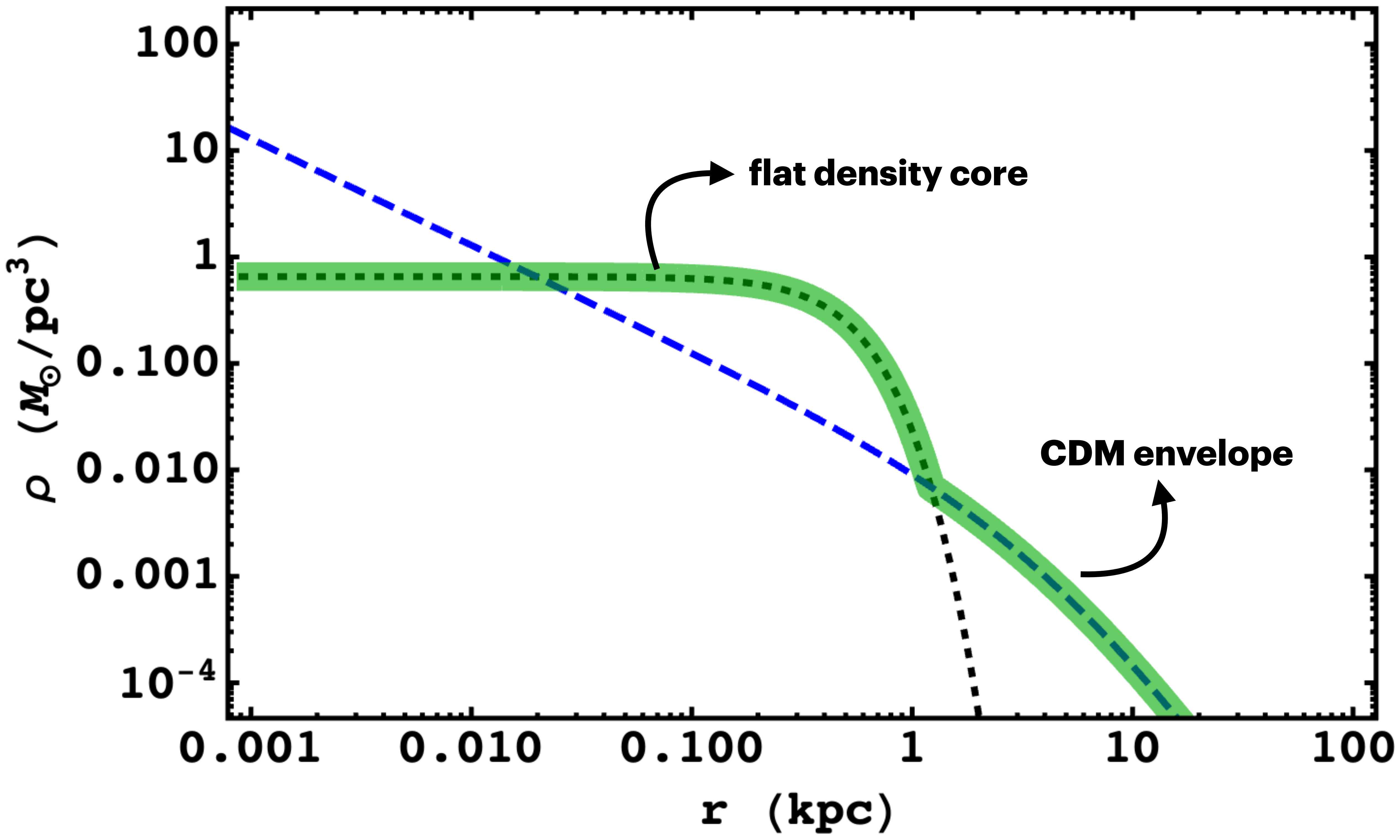
Parameters and observables

- Cores of DM halos formed from wave DM
- Solve GPP equations
- Tunable parameters:
  - Particle mass “m”,
  - Self coupling “λ”,
  - Number of particles (parameterised by a scaling parameter “s”)
- These parameters affect
  - density profile,
  - core mass
  - rotation curve

Velocity from density

Simulations suggest a Core-Halo structure:

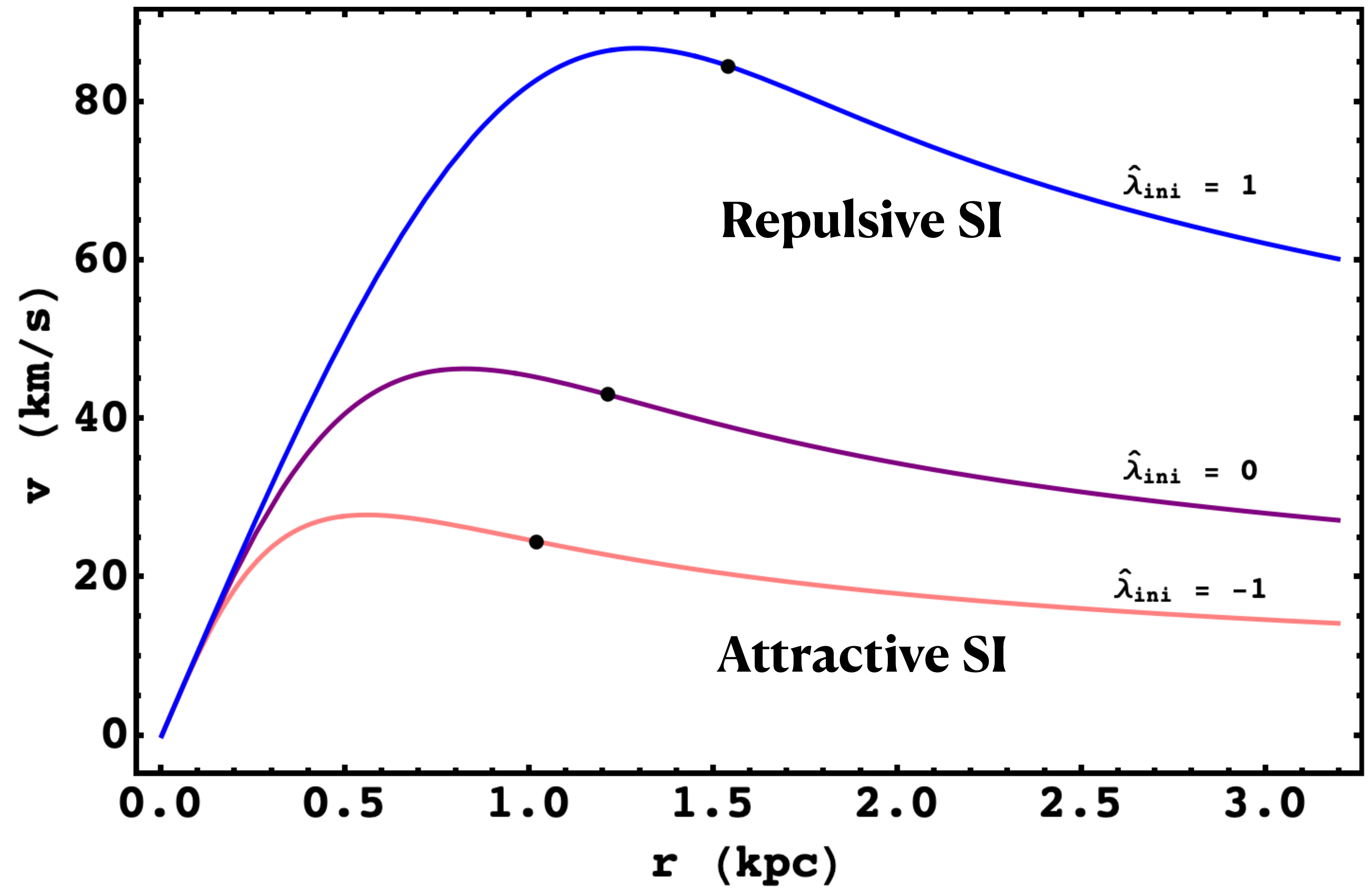
$$\rho_{DM} = \Theta(r_t - r)\rho_{ULDM}(r) + \Theta(r - r_t)\rho_{CDM}(r)$$



- Velocity of a test particle in the gravitational potential of the halo:

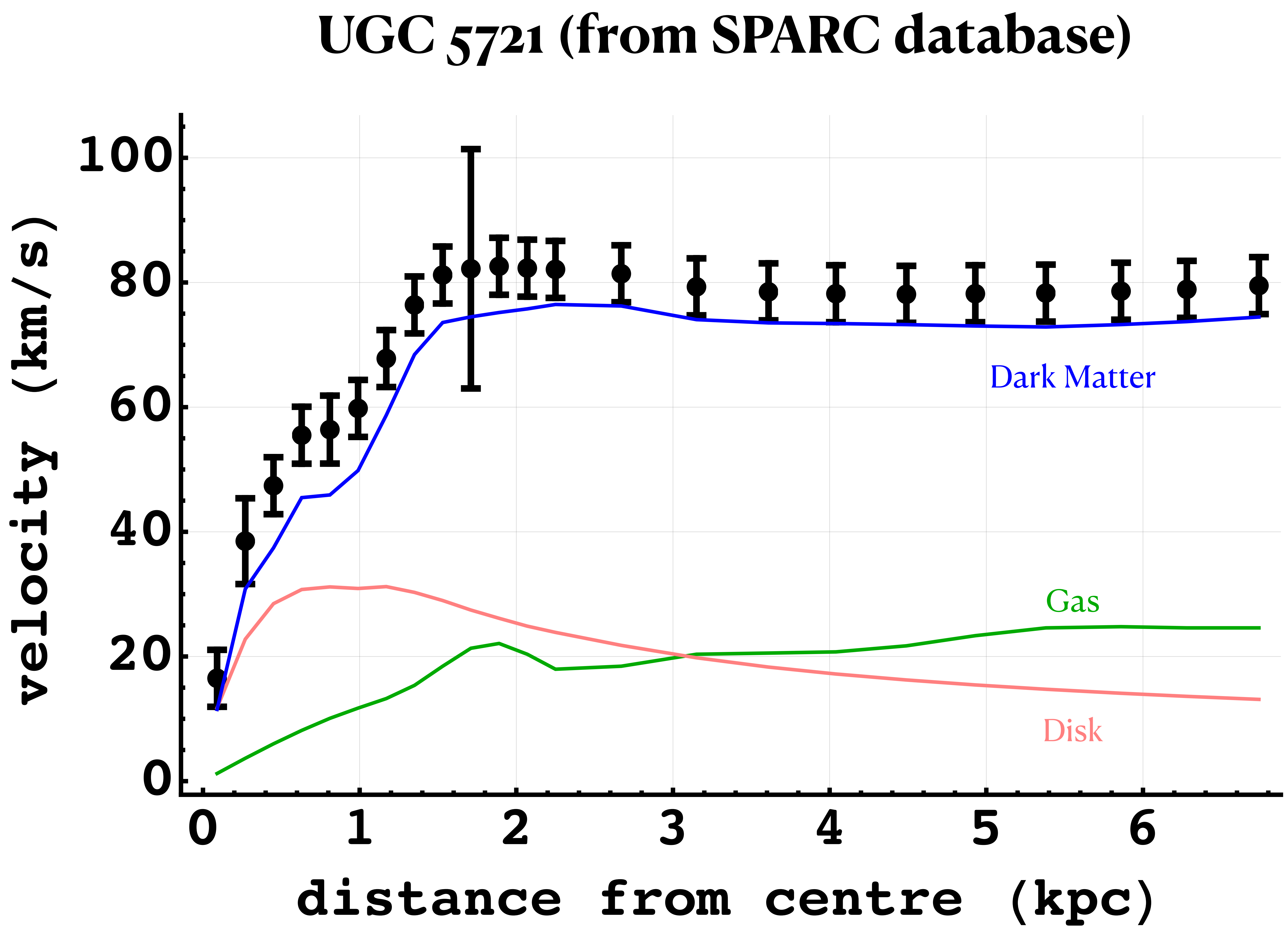
$$v(r) = \sqrt{\frac{GM(r)}{r}} = \sqrt{\frac{4\pi G \int_0^r r'^2 dr' \rho}{r}}$$

- $\rho_{ULDM}(r)$ is parameterised by $\left\{ \boldsymbol{m}, \boldsymbol{\hat{\lambda}_{ini}}, \boldsymbol{s} \right\}$



Self-interactions alter velocity curves as well

Observed rotation curves



Observed velocity can be separated into contributions from different components

$$V_{obs} = \sqrt{V_{DM}^2 + \Upsilon_d |V_d| V_d + \Upsilon_b |V_b| V_b + |V_g| V_g}$$

Dark Matter — Disk — Bulge — Gas

Baryonic contribution can be tuned using Υ_d and Υ_b

↓

Even if we assume no information about the Baryonic contribution

$$V_{DM} \leq V_{obs}$$

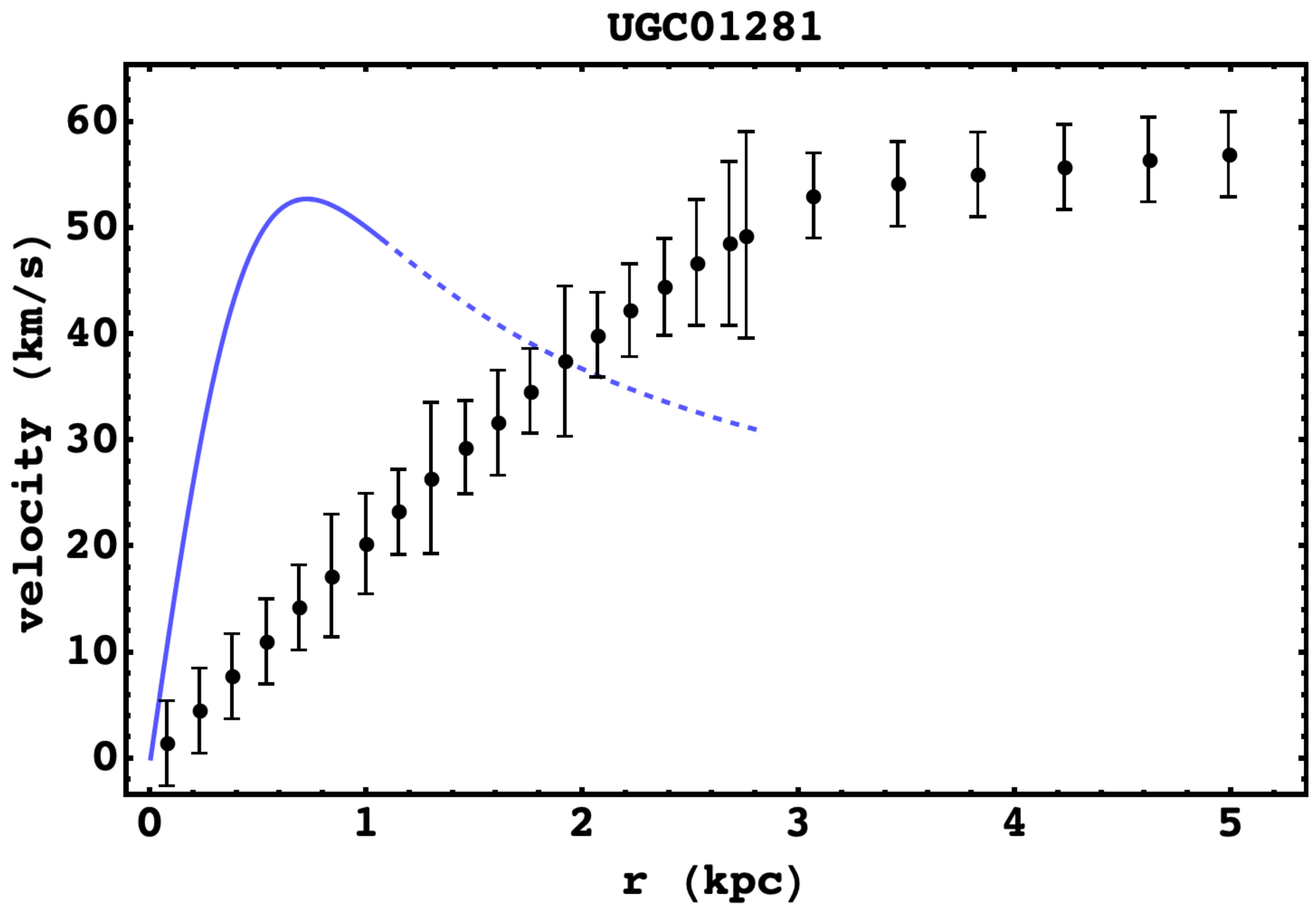
Must always hold

Ruling out FDM

Power-law relation (*Schive et al., 2014*) between mass of soliton and mass of halo:

$$\left(\frac{M_{SH}}{10^9\ M_\odot}\right) = 1.4\left(\frac{M_h}{10^{12}\ M_\odot}\right)^{1/3}\left(\frac{m}{10^{-22}\ \text{eV}}\right)^{-1}.$$

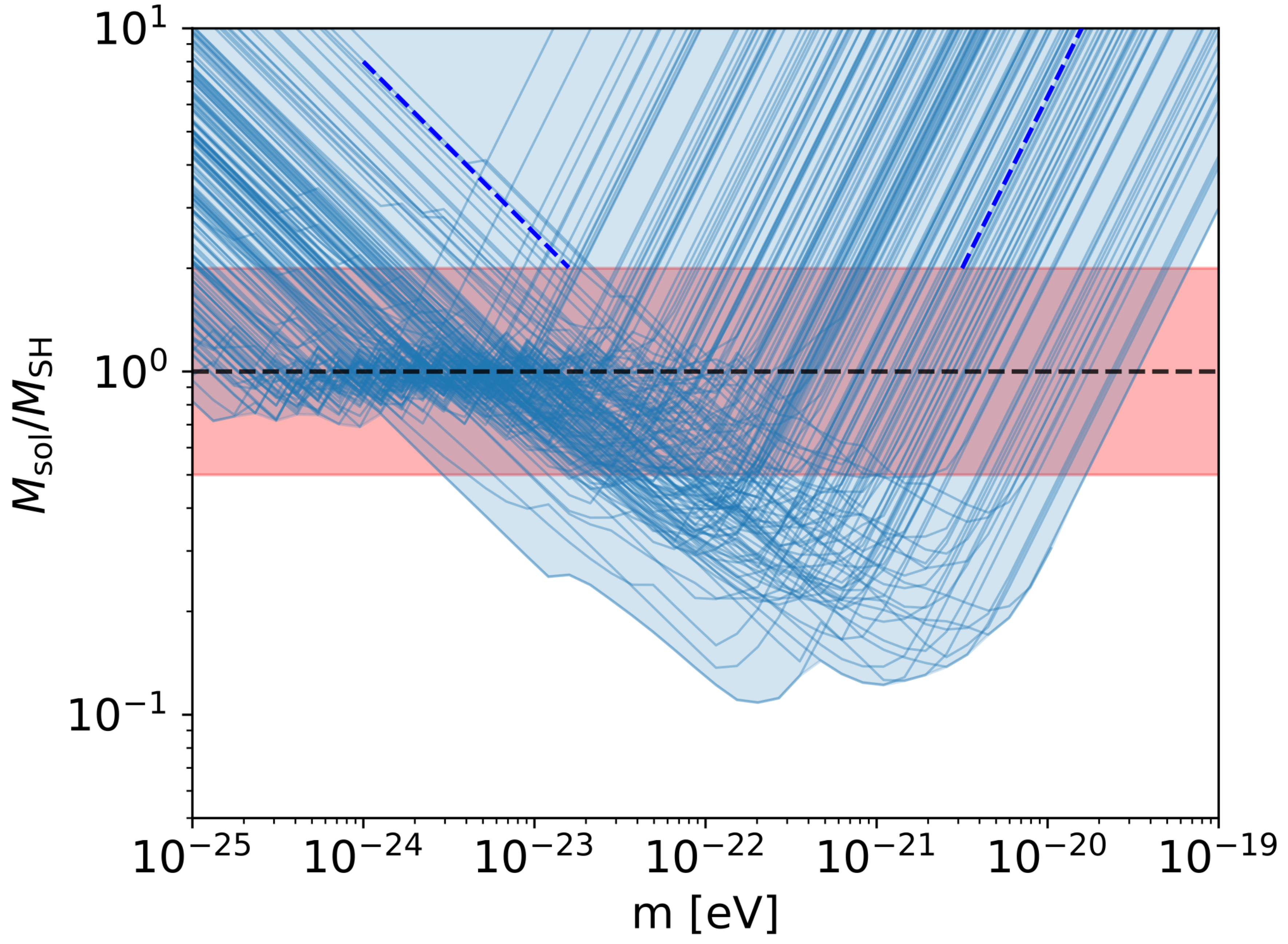
- Soliton masses that satisfy the SH relation are not allowed by observed rotation curves.



What soliton masses are allowed?

N. Bar et al. (2018)

N. Bar et al. (2022)



Self-interactions to the rescue?

SH relation is expected to change in the presence of self-interactions[1,2]

$$\left(\frac{M_{SH}}{10^9\ M_\odot}\right) = 1.4\left(\frac{M_h}{10^{12}\ M_\odot}\right)^{1/3}\left(\frac{m}{10^{-22}\ \mathrm{eV}}\right)^{-1}\sqrt{1 + (1.16\times10^{-7})2\hat{\lambda}\left(\frac{M_h}{10^{12}\ M_\odot}\right)^{2/3}}$$

We can then ask...

For a fixed m, (in this case 10^{-22} eV)

Can ULDM with SI fit observed rotation curves
AND
satisfy an expected soliton-halo relation?

1. L. E. Padilla, et al. *Phys. Rev. D* **103,** no. 6, 063012 (2021)
2. P. H. Chavanis, *Phys. Rev. D* **100,** no. 12, 123506 (2019)

Numerical Procedure

Fix m, (e.g. $m = 10^{-22}$ eV)

↓

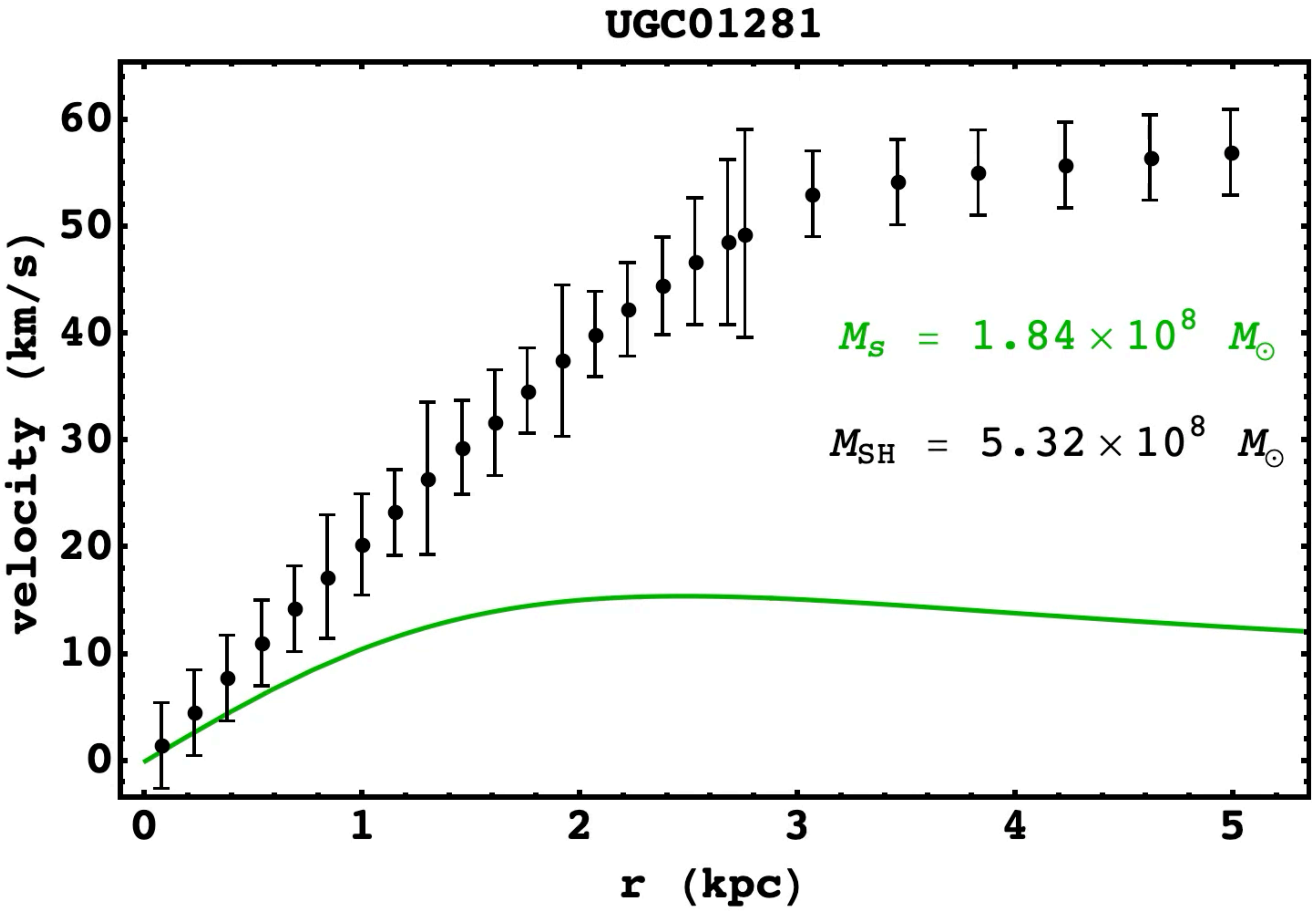
Choose a large s (i.e. a small M_s) such that $V_{DM} < V_{obs}$

↓

Decrease s until $V_{DM} > V_{obs}$ and $\frac{(V_{DM} - V_{Obs})^2}{\sigma^2} \geq 1$ for even one data-point

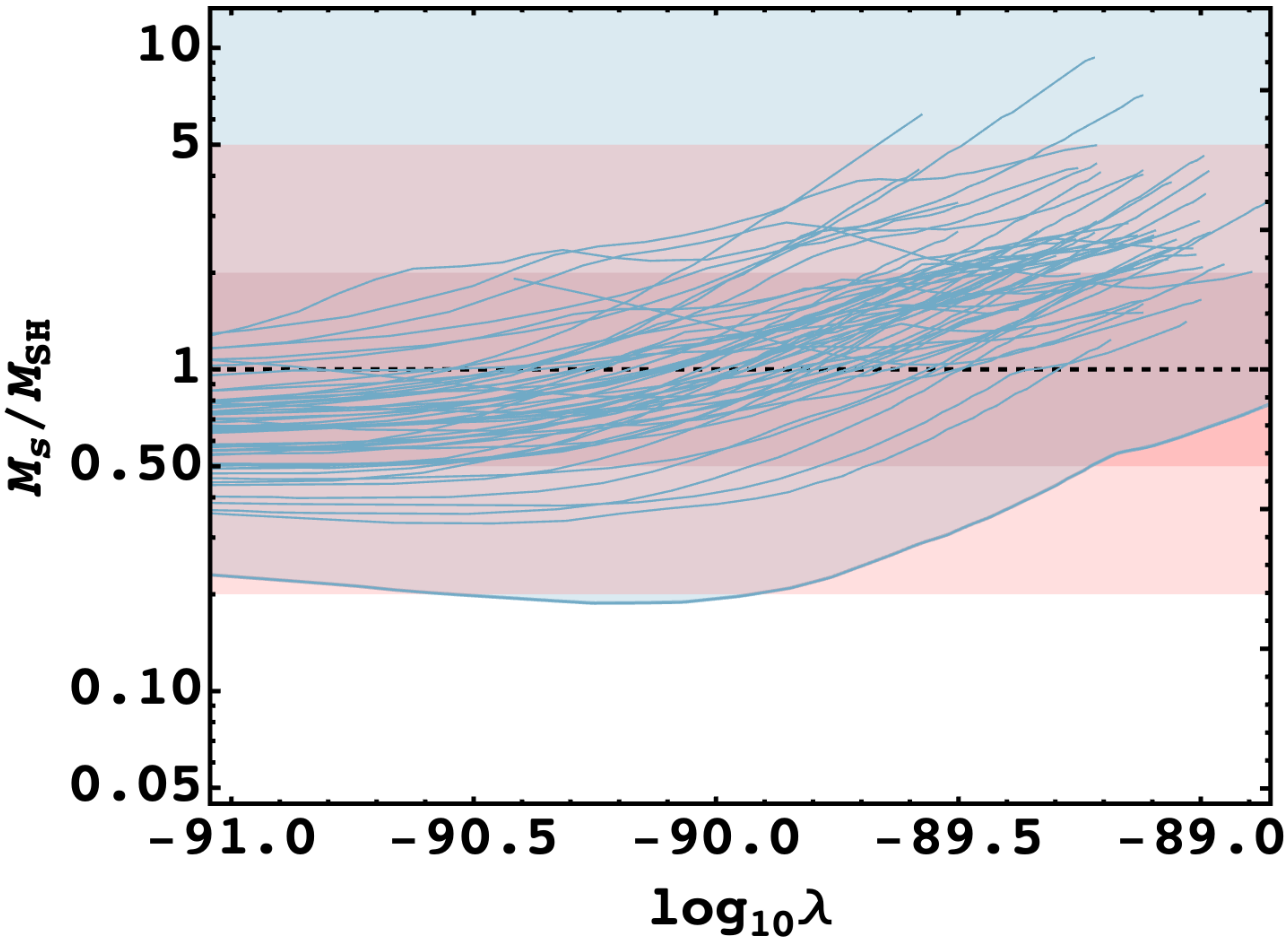
↓

Last value of s (M_s) allowed by the data forms the boundary of the excluded region



$M_h = 5.49 \times 10^{10}\ M_\odot$

Saving ULDM



Bihag Dave, **Gaurav Goswami**, "*Self-interactions of ULDM to the rescue?*," J. Cosmol. Astropart. Phys., 07 (**2023**) 015. E-Print: 2304.04463 [astro-ph.CO]

Observable effects of self coupling - II

Tidal effects for satellite galaxy

Host Galaxy

- A satellite galaxy in a circular orbit around the centre of a larger host DM halo

- Two points in satellite freely falling under the gravity of the host halo

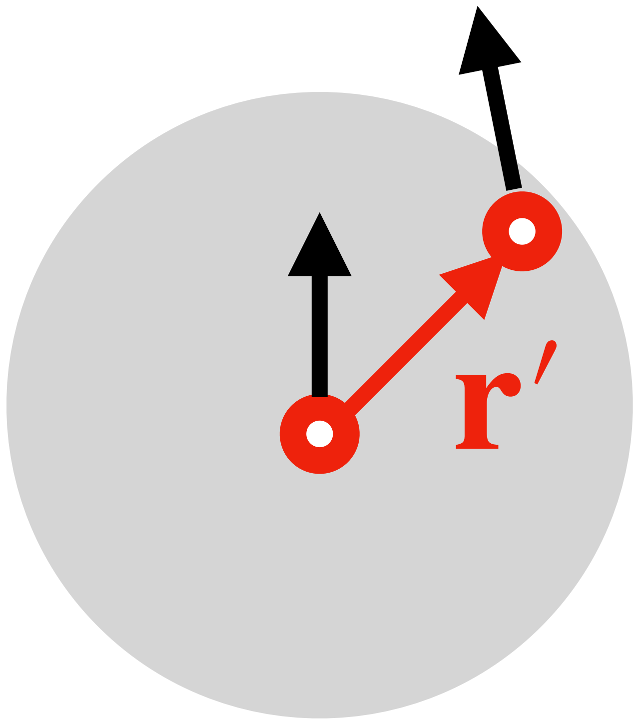
- Acceleration of the relative position vector ($\mathbf{r}'$) of the second point w.r.t first point is $\mathbf{a}(\mathbf{r}')$

- Tidal potential

$$\Phi_{\mathrm{tidal}}(\mathbf{r}) - \Phi_{\mathrm{tidal}}(\mathbf{r}_0) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{a}(\mathbf{r}') \cdot d\mathbf{r}'$$

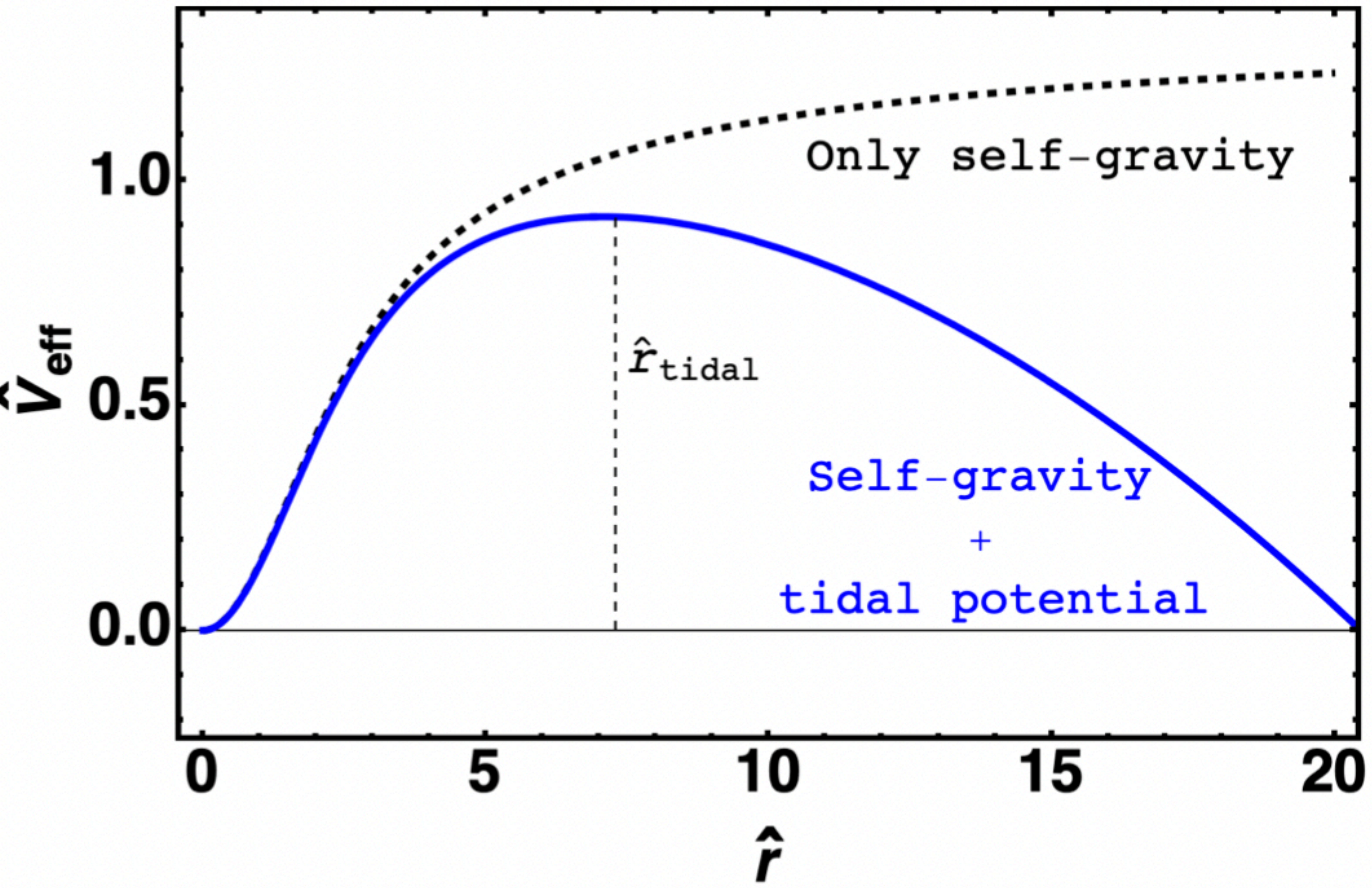
- In addition to self gravity, tidal disruption effects also important

- For particle like CDM (self gravity and tidal effects), for wave dark matter (self gravity, quantum pressure and tidal effects)



Satellite Galaxy

Trouble for wave Dark Matter?



For **particle-like** Cold Dark Matter (CDM), matter contained within the tidal radius is safe from tidal disruption indefinitely.

For **wave dark matter**, tunnelling can cause the DM within tidal radius to penetrate the potential barrier

Can all satellite galaxies exist over cosmological time scale?

M.P. Hertzberg and A. Loeb, *Quantum tunneling of ultralight dark matter out of satellite galaxies*, *JCAP* **02** (2023) 059 [arXiv:2212.07386]

Could self interactions help?

Spreading

Self gravity

Scalar self interactions

$$\gamma\phi = -\frac{\hbar^2}{2m}\nabla_r^2\phi + \left(m\Phi_{\rm SG} - \frac{3}{2}m\omega^2 r^2 + \frac{\lambda\hbar^3}{8m^3c}|\phi|^2\right)\phi\,,$$

$$\nabla_r^2\Phi_{\rm SG} = 4\pi G|\phi|^2\,,$$

Tidal effect

B. Dave and **G. Goswami**, "*ULDM self-interactions, tidal effects and tunnelling out of satellite galaxies,*", J. Cosmol. Astropart. Phys., 02 (2024) 044. arXiv:2310.19664 [astro-ph.CO].

Could self interactions help?

$$\gamma\phi = -\frac{\hbar^2}{2m}\nabla_r^2\phi + \left(m\Phi_{\rm SG} - \frac{3}{2}m\omega^2 r^2 + \frac{\lambda\hbar^3}{8m^3c}|\phi|^2\right)\phi\,,$$

$$\nabla_r^2\Phi_{\rm SG} = 4\pi G|\phi|^2\,,$$

- ☑ Regular everywhere
- ☑ Spherically symmetric
- ☐ Nodeless
- ☐ Spatially localised
- ☐ Stationary

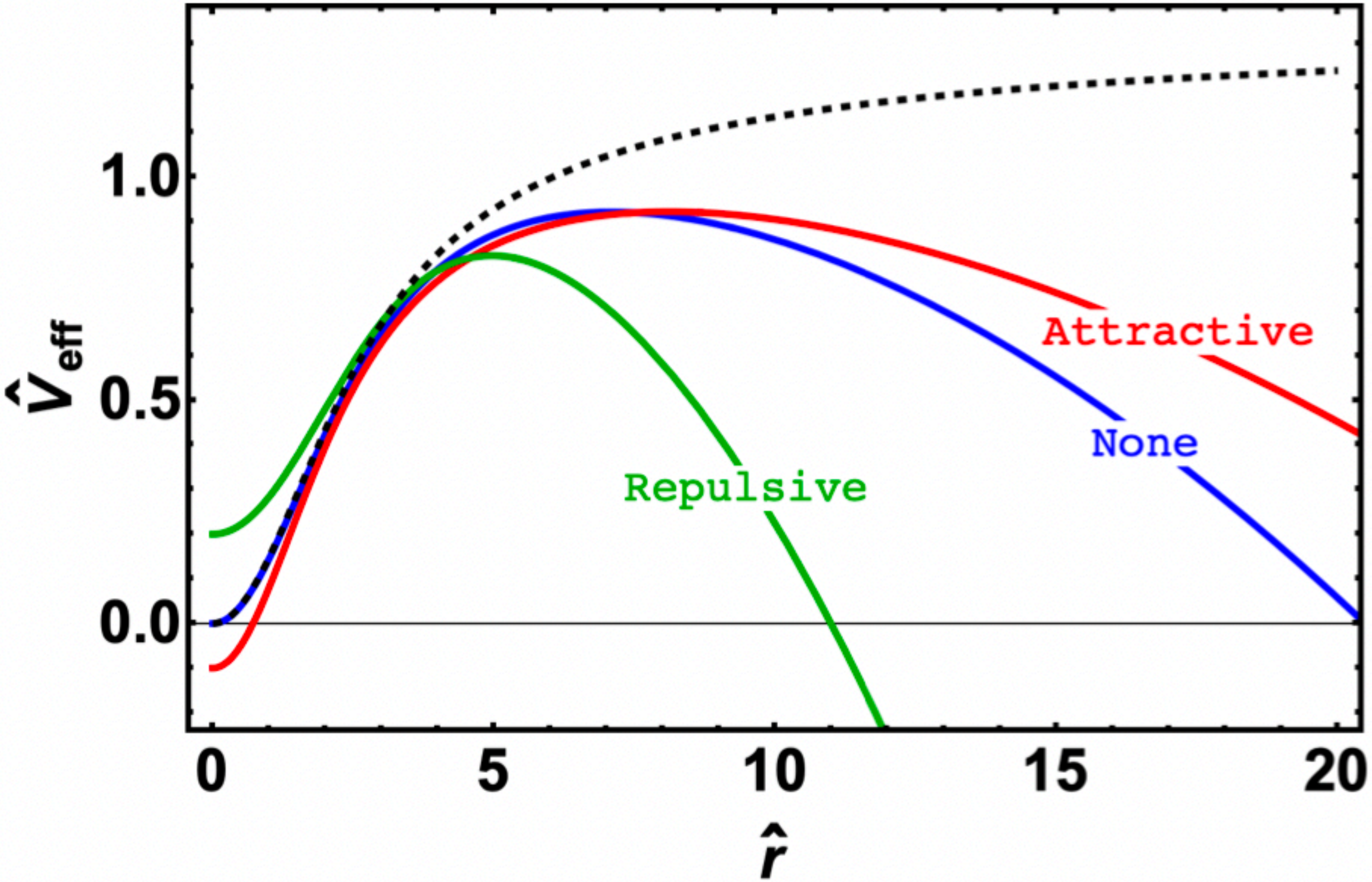
Look for solutions with outgoing wave boundary conditions

Allow the "energy" to be complex

Quasi stationary states

B. Dave and **G. Goswami**, "*ULDM self-interactions, tidal effects and tunnelling out of satellite galaxies,*", J. Cosmol. Astropart. Phys., 02 (2024) 044. arXiv:2310.19664 [astro-ph.CO].

Saving wave Dark Matter!



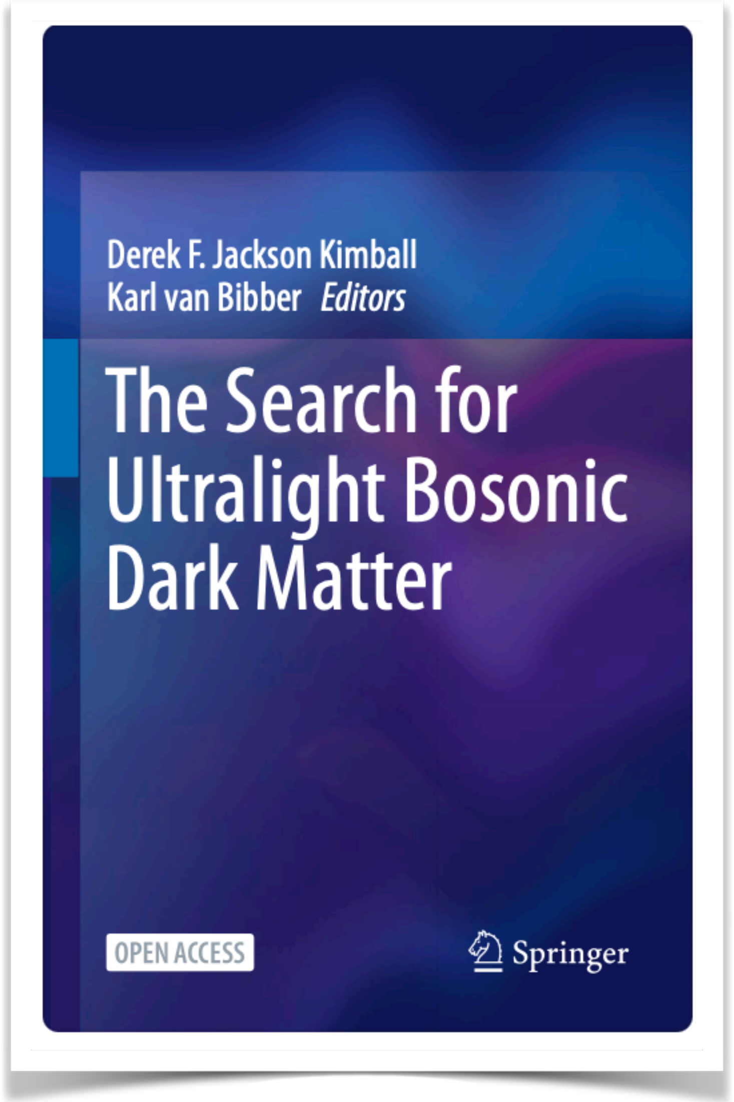
The potential barrier shrinks in the presence of repulsive self-interactions (green), while it stretches when self-interactions are attractive (red)

Can all satellite galaxies exist over cosmological time scale?

B. Dave and **G. Goswami**, "*ULDM self-interactions, tidal effects and tunnelling out of satellite galaxies,*", J. Cosmol. Astropart. Phys., 02 (2024) 044. arXiv:2310.19664 [astro-ph.CO].

Remarks

- Often, when it is claimed that FDM is ruled out, it is assumed that the self interactions are negligibly small,
  - Where, negligibly small means much smaller than even 10^{-90}
- Even other celebrated constraints e.g. those based on Lyman α can be evaded by self interactions
  - See e.g. 1709.07946, 2301.10266, chapter 3 of this book



- Could other (all?) cases in which FDM is ruled out be saved by self interactions?
  - Work in progress!
- Attractive or repulsive?

Further Remarks

- Benchmark scenario: axions with a cosine potential:
  - Self coupling negative and (if m is of the order of 10^{-22} eV) of magnitude 10^{-96}
- How do I get enhancement of λ (and still get the right relic abundance)?
- Single axion with multiple instantons (note that f could be very close to Planck scale) could give correct relic abundance (misalignment mechanism) and a coupling which is a few order of magnitude larger (in progress).
- Coupling to SM particles, fifth forces, modified gravity etc?

Thank You