भौतिक अनुसंधान प्रयोगशाला Physical Research Laboratory

The Neutrino Landscape

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17th International Conference on interconnections between particle physics and cosmology, 14-18 October, Hyderabad, India

Three fundamental questions

- **Do neutrinos have mass ? If so how small ? (Pauli, Fermi, 1930s)**

  Planck data => $\sum m_i \leq 0.26eV$

- **Do neutrinos of different flavour oscillate amongst one another ? (Pontecorvo, Maki, Nakagawa, Sakata,1960s)**

  Neutrino oscillations observed => neutrino mass and flavour mixing

- **Are neutrinos their own antiparticles ? (Majorana 1930s)**

  Can be tested in neutrino less double beta decay

- **The three issues are interconnected and may throw light on the physics beyond the Standard Model**

The neutrino mass matrix

- The neutrino mass matrix at low energy

$$m_\nu = U^*_{PMNS} Diag(m_1, m_2, m_3) U^\dagger_{PMNS}$$

- Mixing matrix relating the flavour and mass states

$$U_{PMNS} = R_{23}(\theta_{23}) R_{13}(\theta_{13},\delta) R_{12}(\theta_{12}) P(\sigma, \rho)$$

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P(\sigma, \rho)$$

Parameters of the neutrino mass matrix

- **9 unknown parameters for three neutrino flavours**

  3 masses $m_1,\ m_2,\ m_3$

  3 mixing angles $\theta_{12},\ \theta_{13},\ \theta_{23}$

  3 phases $\delta,\ \alpha,\ \beta$

- **Oscillation experiments sensitive to**

  2 mass squared differences $\Delta m_{21}^2 = m_2^2 - m_1^2,\ \Delta m_{31}^2 = m_3^2 - m_1^2$

  3 mixing angles $\theta_{12},\ \theta_{13},\ \theta_{23}$

  1 Dirac phase δ

- **Absolute neutrino mass comes from**

  Tritium Beta-decay

  Neutrino-less double beta decay

  Cosmology

- **Majorana CP phases from lepton number violating processes**

Oscillation Probability (vacuum)

If neutrinos have Mass: :

Flavour states $\nu_\alpha = \Sigma_i U_{\alpha i} \nu_i$ Mass states

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} U_{\beta i}\right) \sin^2 \Delta_{ij} + 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} U_{\beta i}\right) \sin 2\Delta_{ij}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\Delta_{ij} = \Delta m^2_{ij} L / 4E$$

$$\Delta m^2_{ij} = m^2_i - m^2_j$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2(\Delta m^2 L / 4E)$$

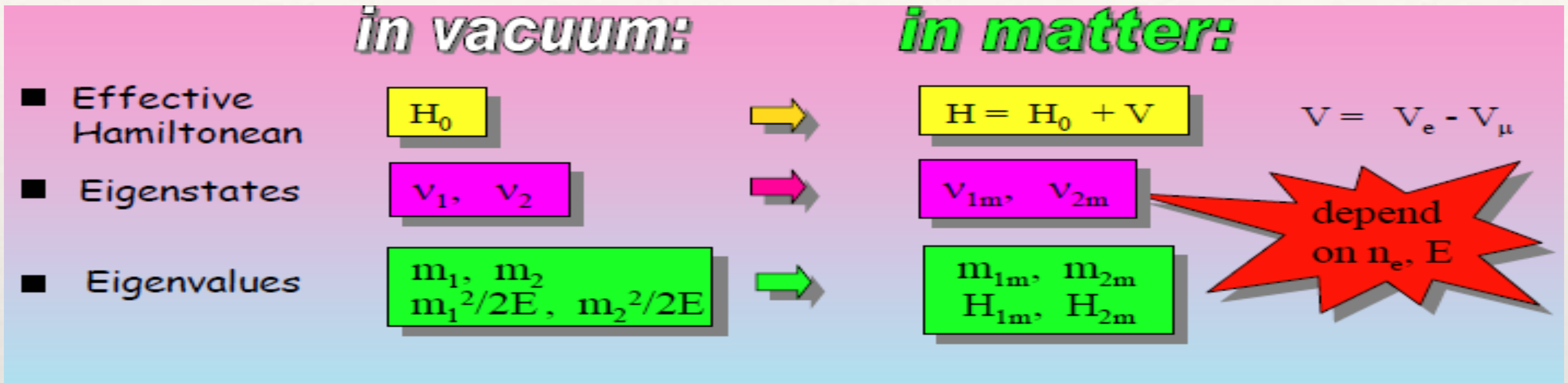
- Neutrino Oscillation requires
  - Non-zero neurino mass
  - Non-zero mixing angles
  - Oscillation effect $\Delta m^2 \sim E/L$

$$\bar{\nu} : U \to U^*$$

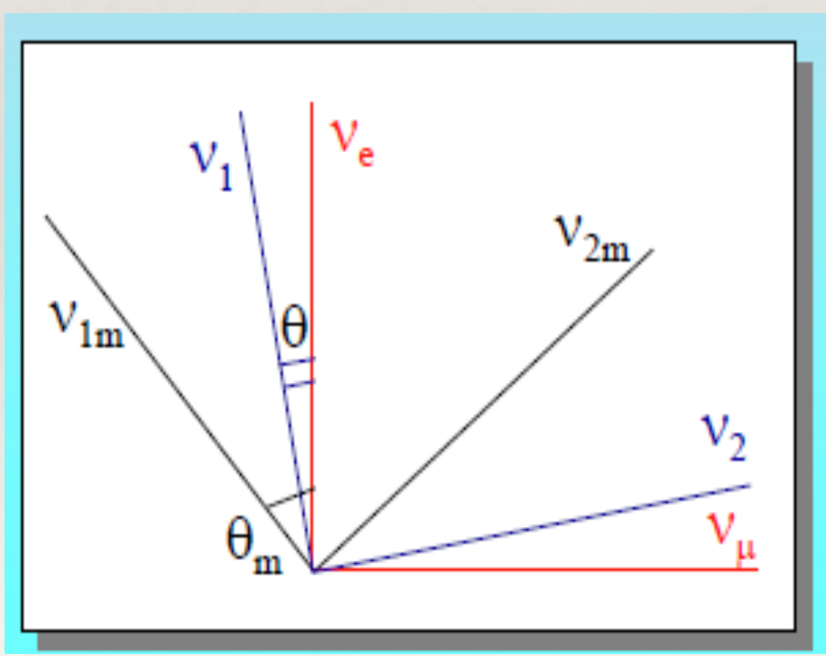
Not sensitive to the sign of Δm^2
Not sensitive to the octant of θ

Not sensitive to absolute masses

Matter Effect



Courtsey: A. Yu. Smirnov



Mixing angle in matter is defined with respect to the matter eigenstates

The eigenvalues and the mixing angle are determined by diagonalizing the effective Hamiltonian in matter

Matter Effect

- In matter, only ν_e's undergoes Charged current interaction $\rightarrow$ an effective potential of $\sqrt{2}G_F N_e$

Effective mixing angle θ_M in matter

$$\tan 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F n_e E}$$

$$\Delta m^2 \cos 2\theta = 2\sqrt{2}G_F n_e E, \ \theta_M \rightarrow \pi/4$$ MSW Resonance

L. Wolfenstein, PRD **17**, 1978 S.P. Mikhyev, A.Yu. Smirnov, SJNP **42**, 1985

$$\Delta m_m^2 = [(\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F n_e E)^2 + \Delta m^2 \sin^2 2\theta]^{1/2}$$

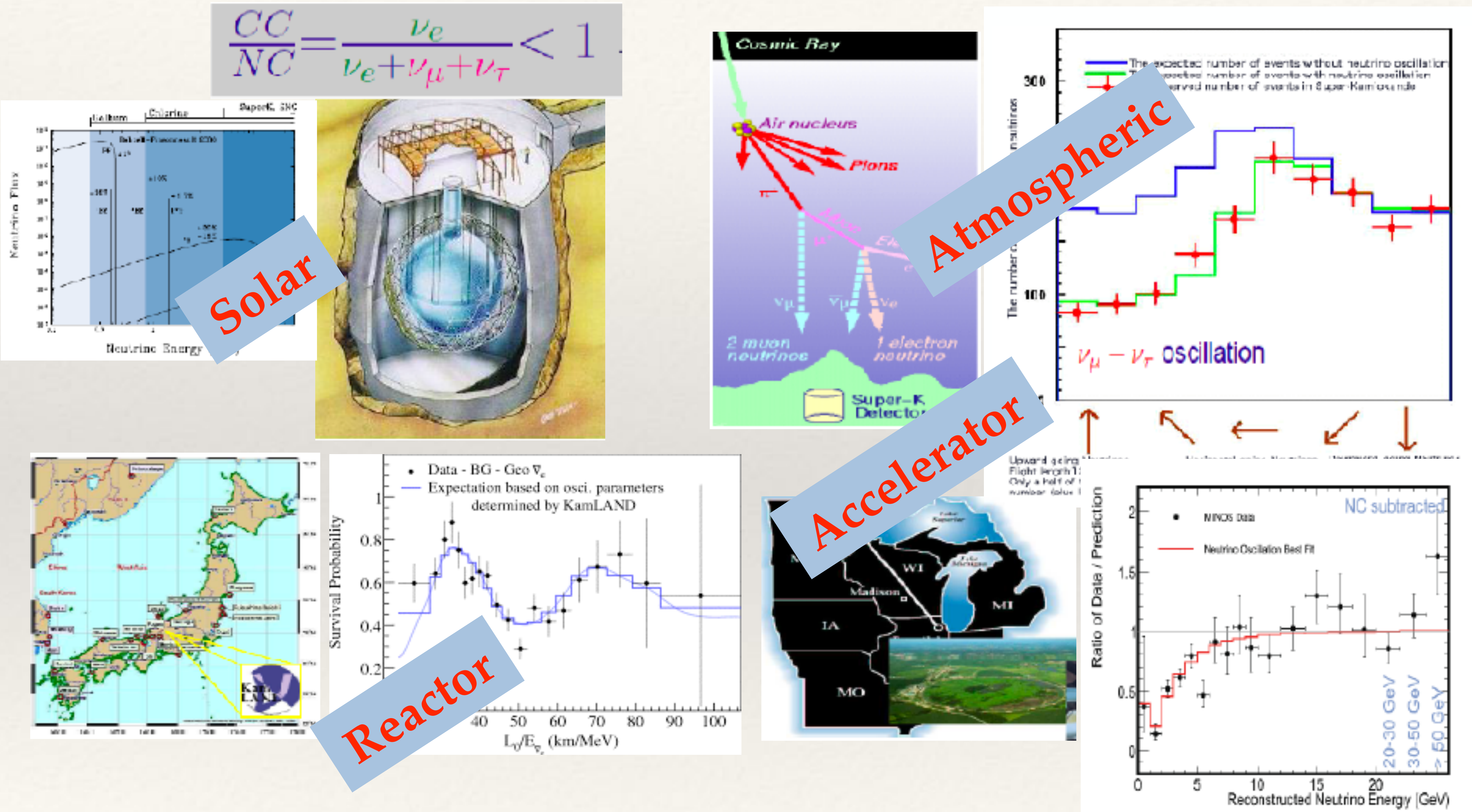
For antineutrinos potential changes sign

Resonance for $\Delta m^2 < 0$

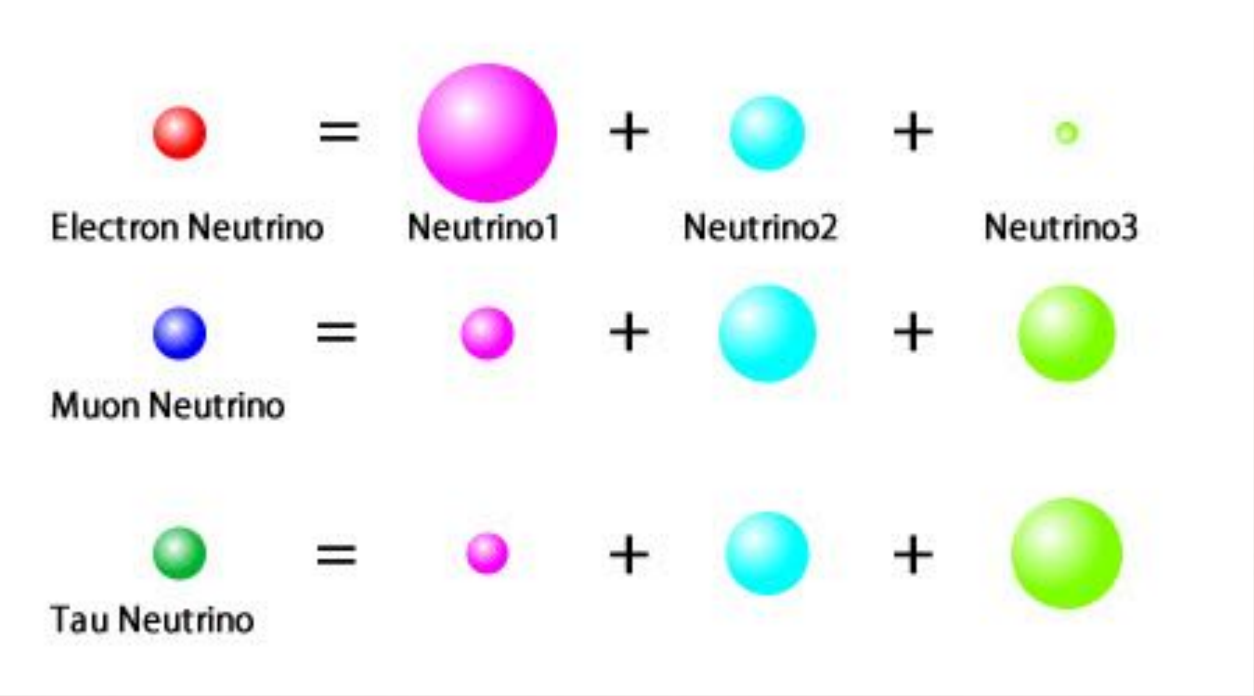
Matter effect is sensitive to the ordering of the mass states

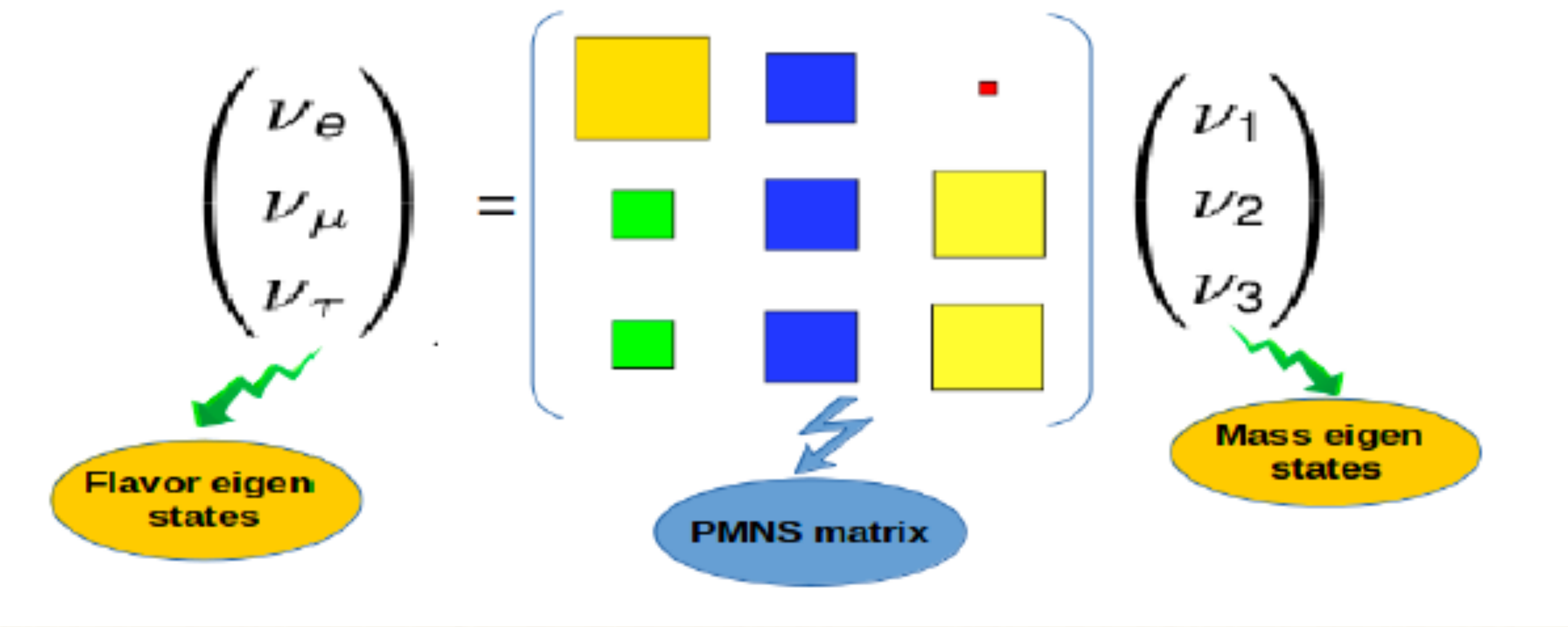
- Probabilities are obtained by solving propagation equation in matter
- Depends on density profile of matter

Neutrino Oscillation: Evidences



Three Neutrino Mixing





Three Neutrino Paradigm

- Measurement of non-zero θ_{13} in reactor experiments → three neutrino picture

Atm +LBL | Sol+KL

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & e^{-i\delta} s_{13} \\ & 1 & \\ -e^{i\delta} s_{13} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$c_{12} = \cos\theta_{12}$ etc., δ CP-violating phase

- Δm^2_{21}, θ_{12}, θ_{13} Solar + KamLND
- Δm^2_{31}, θ_{13} Reactor
- Δm^2_{31}, θ_{23}, θ_{13}, δ_{CP} Atm + LBL

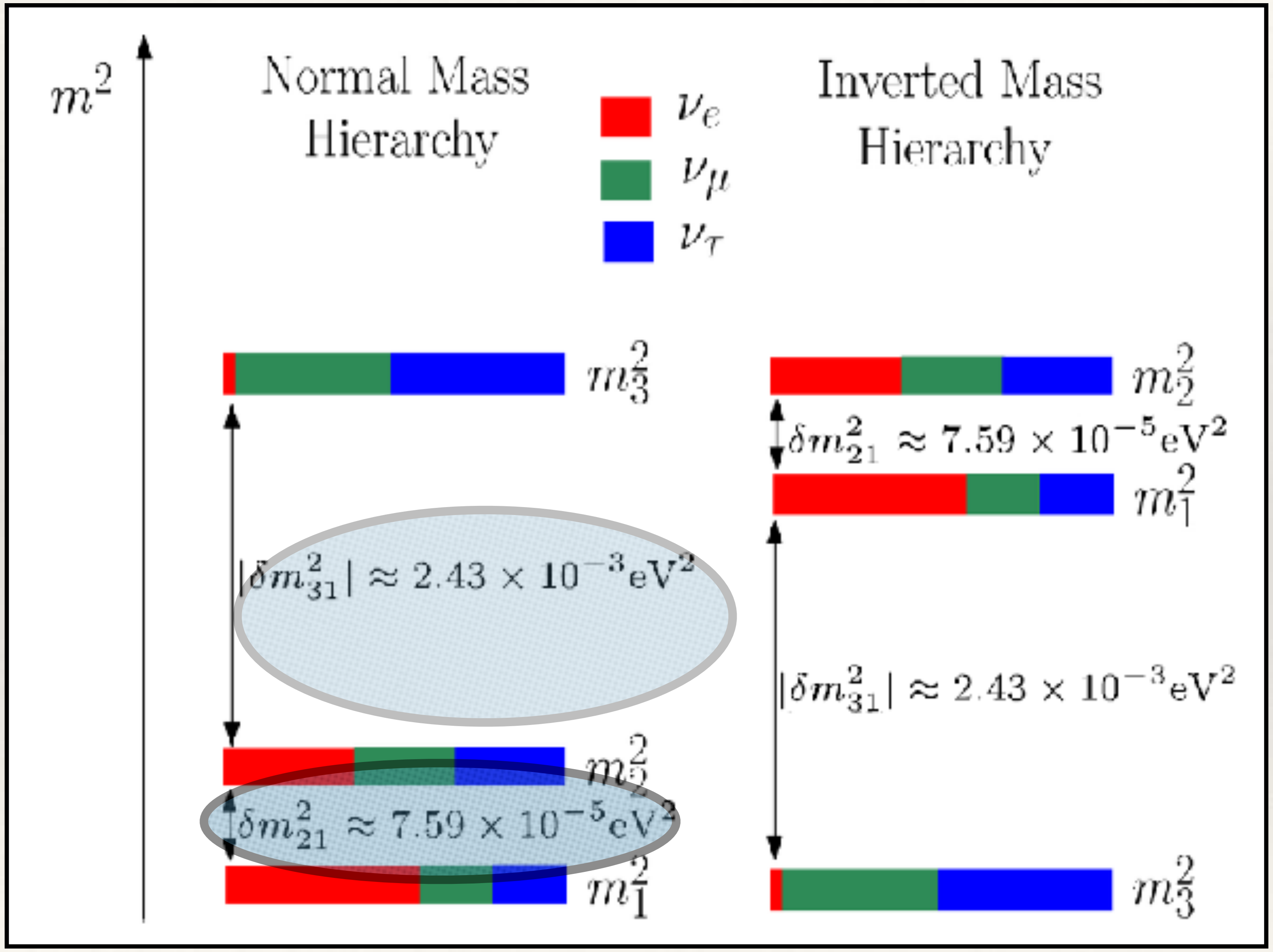
→ Global analysis (combines statistics)

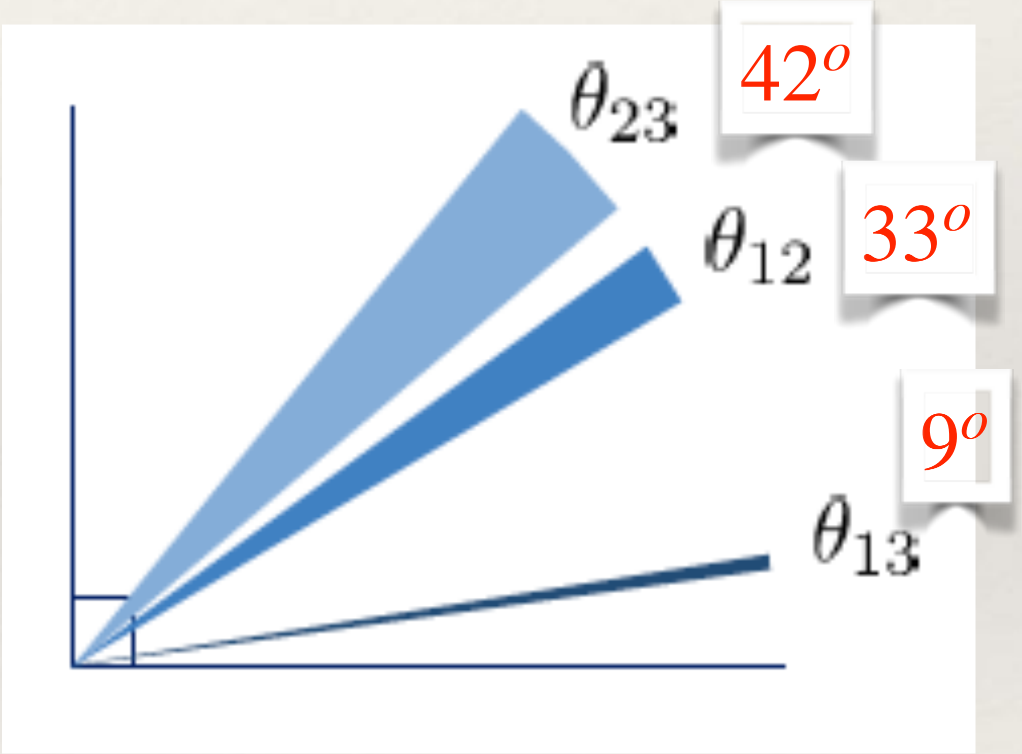
→ Tensions in the data

→ Complementarity and Synergy

Interplay among different sectors because of θ_{13}

Neutrino Oscillation Parameters





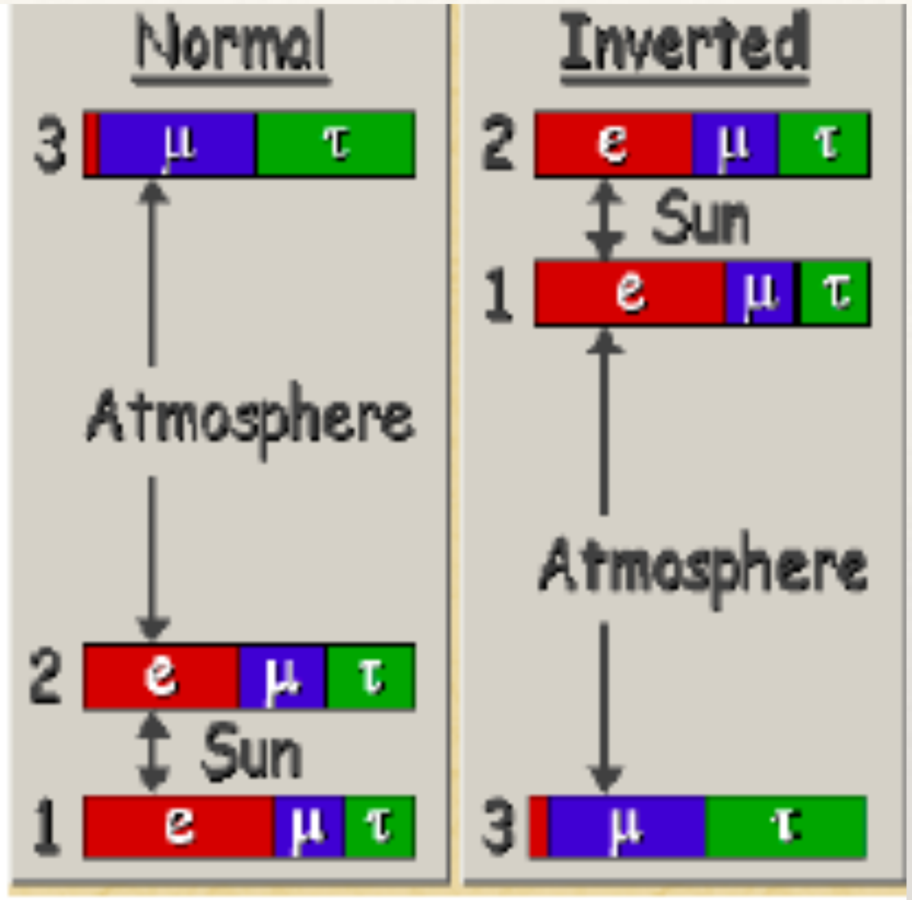
Neutrino Oscillation : from discovery to precision

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

NuFIT 5.1 (2021)

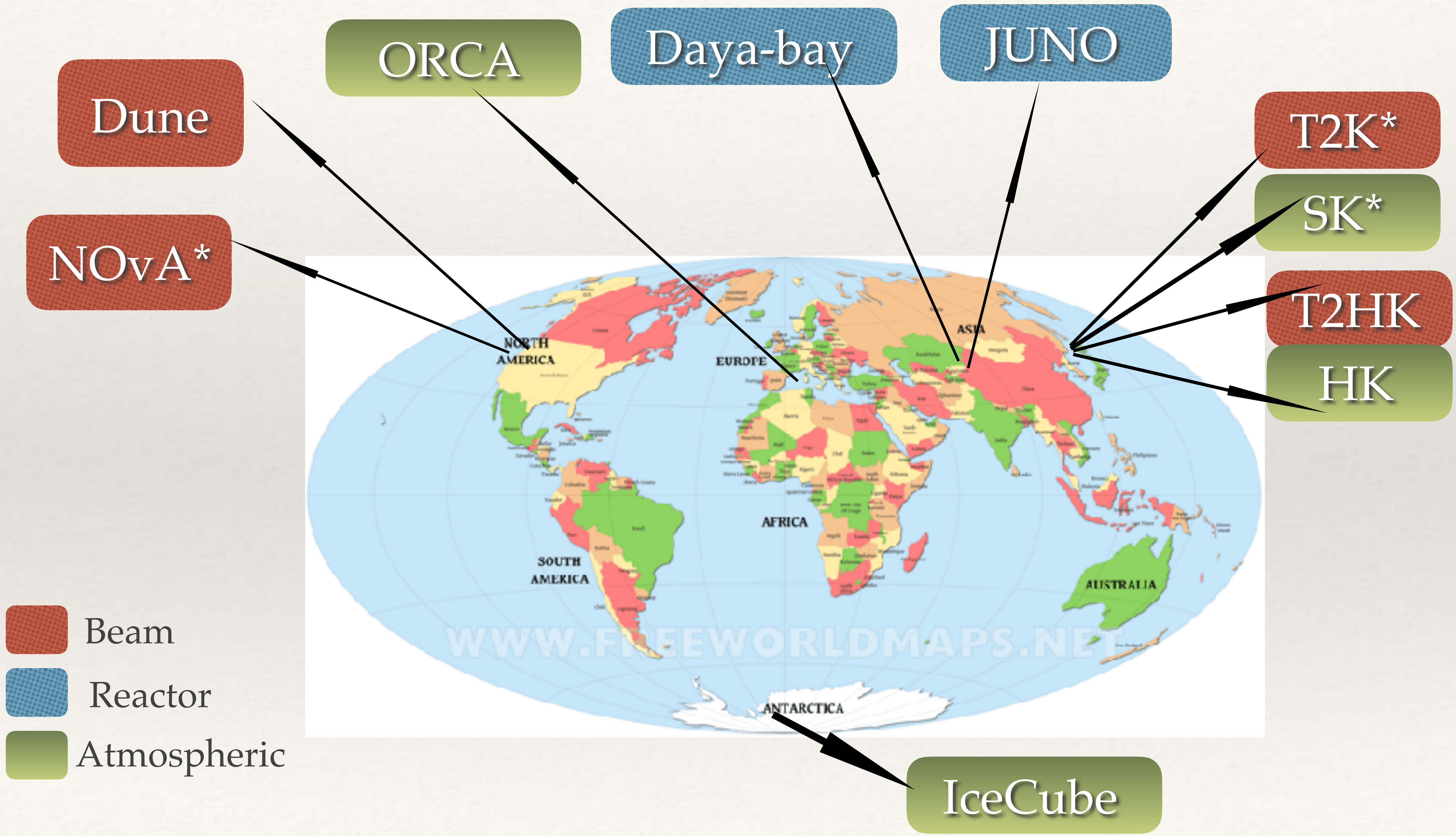
Oscillation parameters we do not know

- The neutrino mass ordering i.e. if the third state is above or below the first two states
- The octant of the 2-3 mixing angle i.e. if $\theta_{23} > 45^o$ or $< 45^o$
- The value of the CP phase



Why these parameters are particularly difficult to determine ?

Current and future oscillation experiments



Long baseline experiments : salient features

Expt	Baseline	E (GeV)		Details
T2K	295 km, Tokai to Kamioka	0.6		0.76 MW Super Kamiokande Water Cerenkov
NOVA	810 km, FNAL to ASH River	1.7		0.7 MW 14 kt TASD
DUNE	1300 km FNAL to South Dakota	0.5-8		1.2 MW Liquid Argon 10kt/40 kt
T2HK	295 km JPARC to Kamioka	0.6		1.3 MW , 187 kt X2 Hyper Kamiokande
T2HKK	295km, 1100 km	0.6		HK, Water Cherenkov in Korea
ESSnuSB	540 km , Lund to Gapenberg	0.3		5 MW 500 kt Water Cerenkov,

High Intensity Beams, bigger detectors

Atmospheric neutrino detectors: salient features

	Prototype	Salient features
Magnetized IRON	ICAL@INO	50 kt, muon energy and direction measurement, charge id, neutrino energy reconstruction
Water Cherenkov	Hyper Kamiokande	Megaton, no charge id, both electron and muon energy and direction
Water Cherenkov (Mediteranian)	ORCA	Multi- Megaton, tracks and showers, no charge id
ICE Cherenkov (Southpole)	PINGU IceCube	Multi megaton, tracks and showers , no charge id
Liquid Argon	DUNE	Liquid Argon, both muon and electron events Charge id for both ??

Path length — 10- 10000 km,
matter effects important

Degeneracy problem

- The main problem in determination of hierarchy, octant and δ_{CP} in LBL experiments is due to presence of degeneracies
- Degeneracy → different set of parameters giving the same probability → equally good fit to the data
-

Hierarchy - δ_{CP} degeneracy

$$P_{\mu e}(\Delta, \delta_{CP}) = P_{\mu e}(-\Delta, \delta'_{CP})$$

Minakata, NunoKawa, 2001

Intrinsic octant degeneracy

$$P_{\mu\mu}(\theta_{23}) = P_{\mu\mu}(\theta_{23} - \pi/2 - \theta_{23})$$

Fogli and Lisi, 1996

Octant - δ_{CP} degeneracy

$$P_{\mu e}(\theta_{23}, \delta_{CP}) = P_{\mu e}(\theta'_{23}, \delta'_{CP})$$

Gandhi, Ghosal, Goswami, Shankar 2005

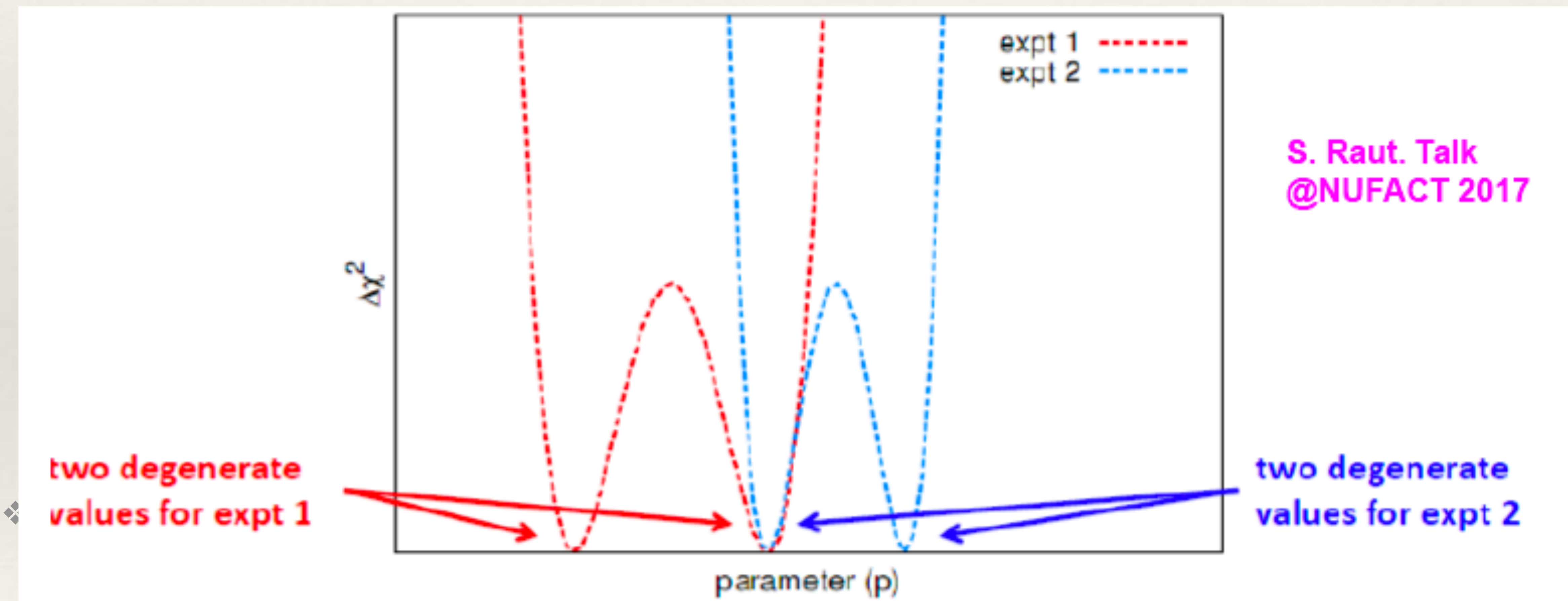
Comprehensive Approach

$$P_{\mu e}(\theta_{23}, \Delta, \delta_{CP}) = P_{\mu e}(\theta'_{23}, -\Delta', \delta'_{CP}) \Rightarrow \text{generalized}\,(\text{hierarchy} - \theta_{23} - \delta_{CP})\,\text{degeneracy}.$$

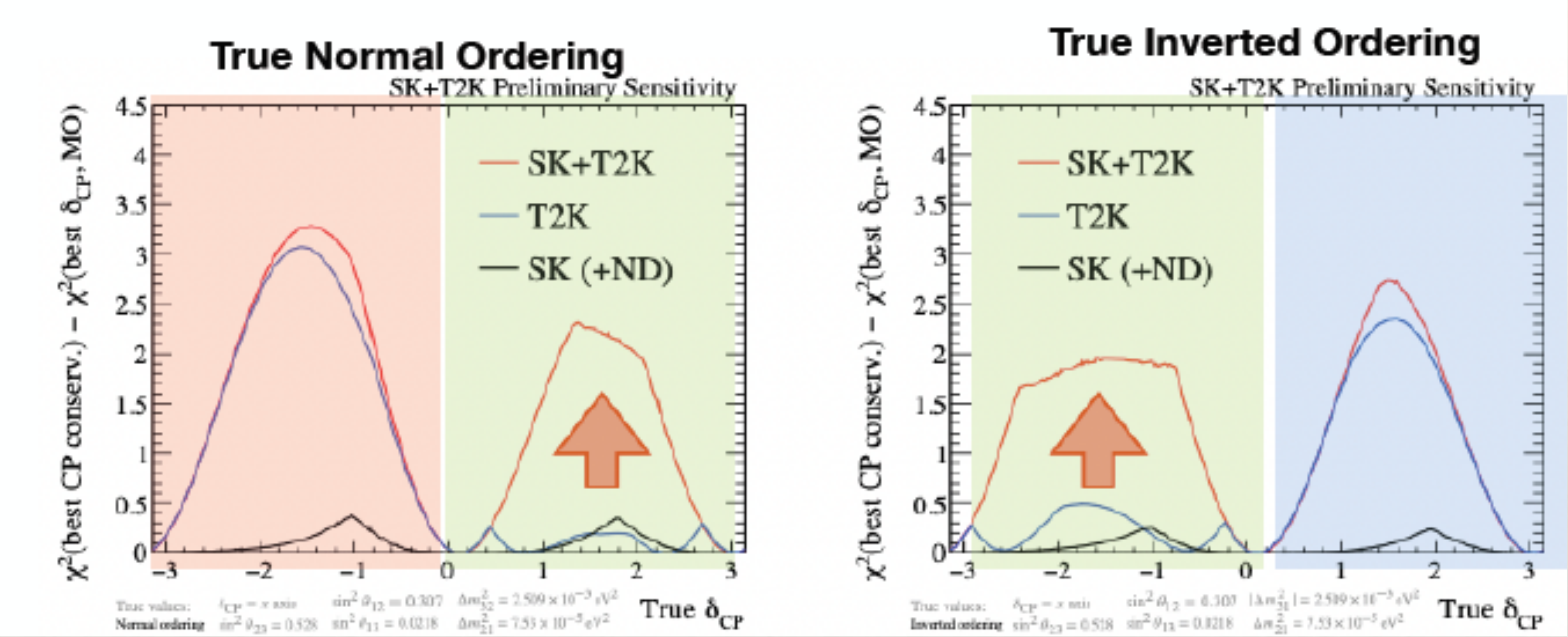
Ghosh,Ghoshal,Goswami,Nath,Raut, 2015

Synergy between experiments

- Different experiments have different L, E dependence and therefore the dependence on the oscillation probability on a given parameter can be different



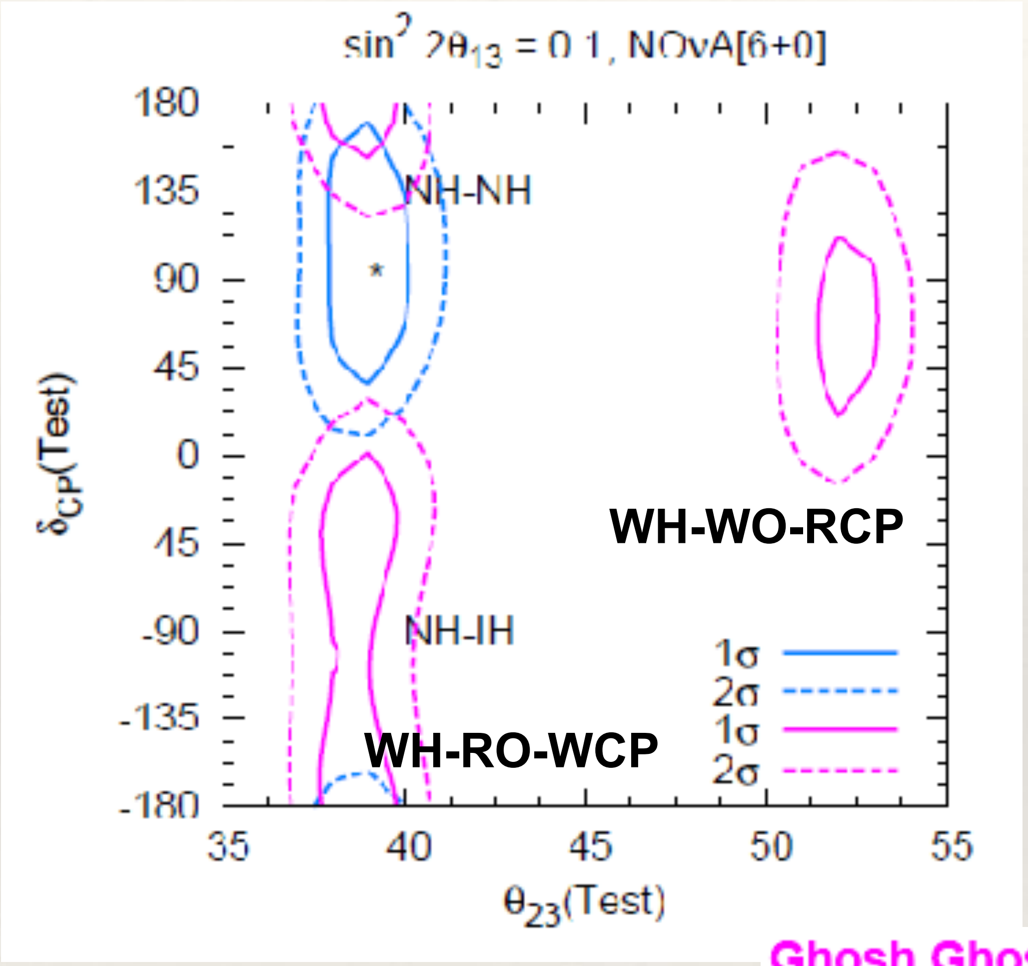
T2K+SK

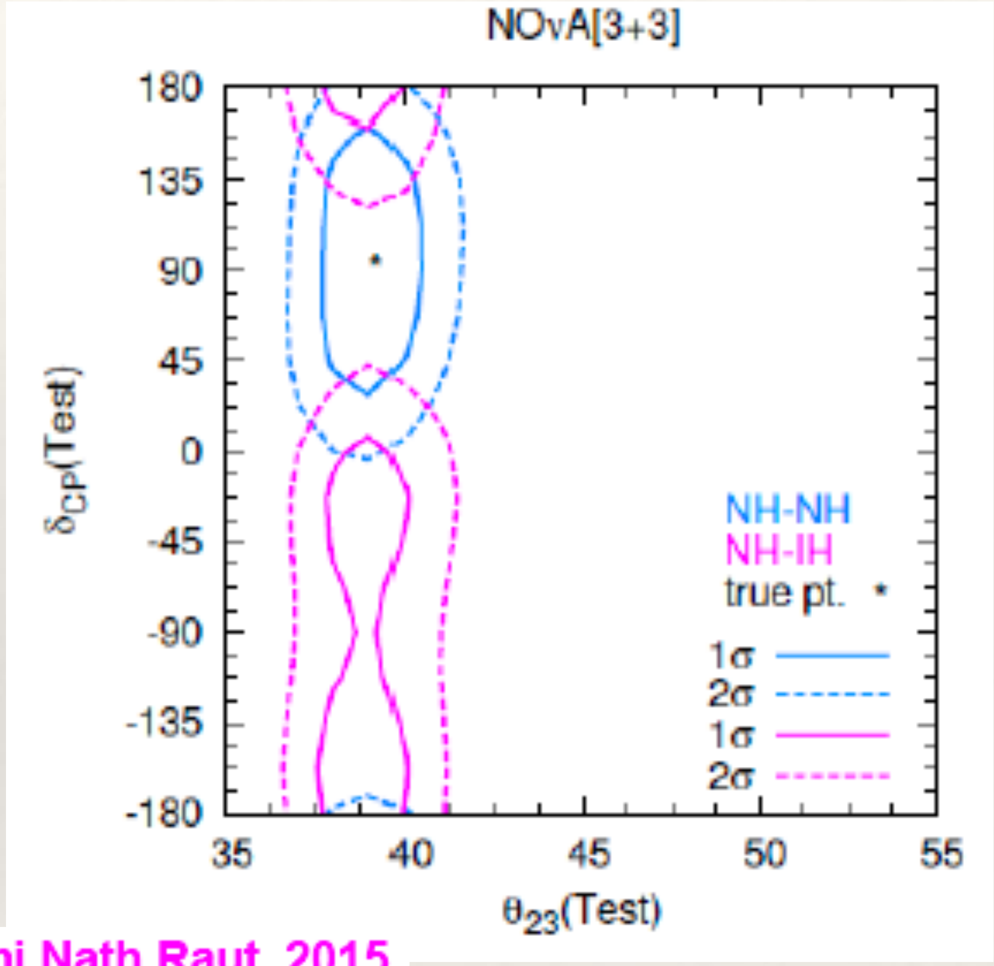


Claudio Giganti, Neutrino 2024

Can resolve
hierarchy -CP degeneracy

Synergy between neutrinos and antineutrinos

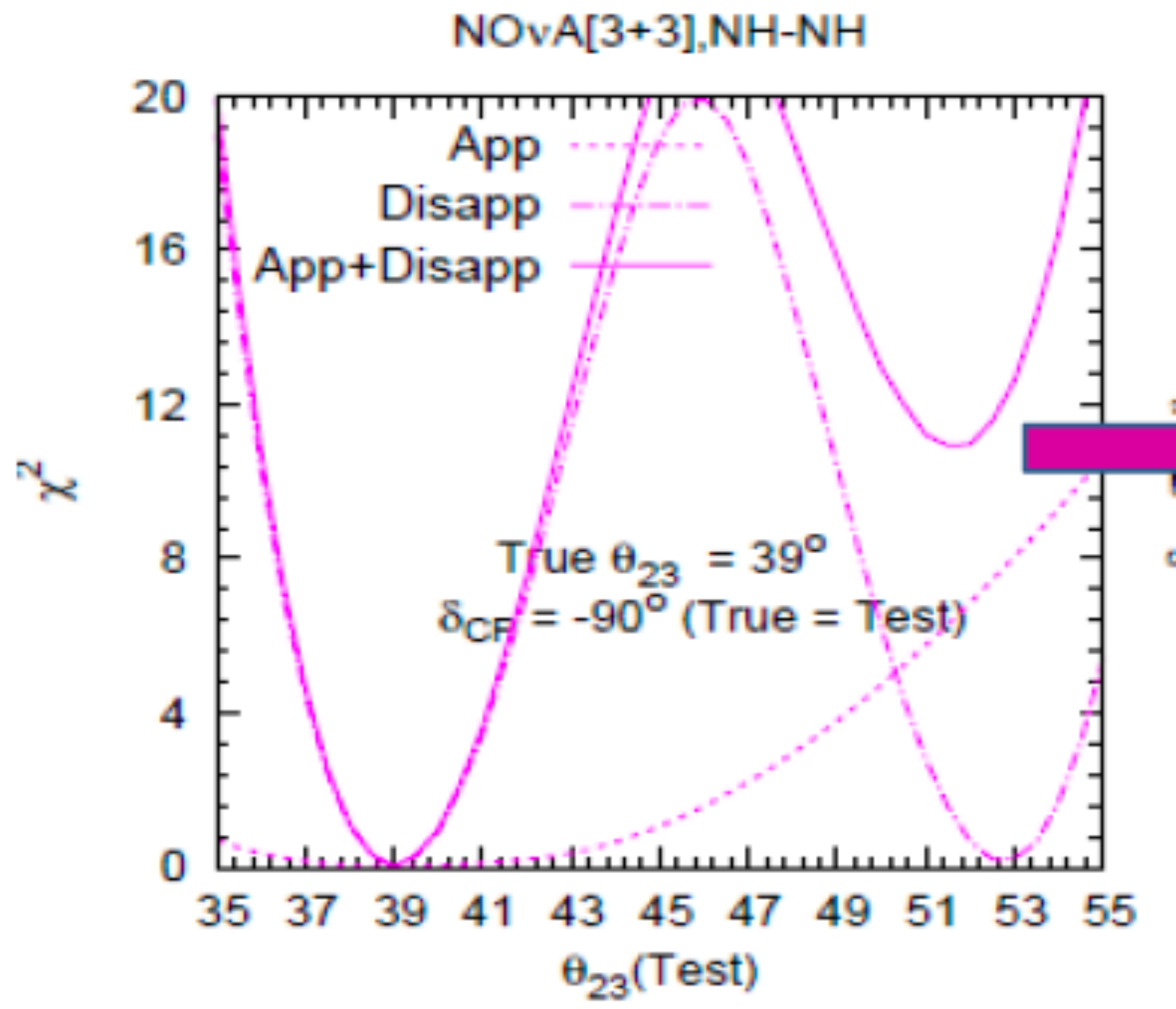




Ghosh,Ghoshal,Goswami,Nath,Raut, 2015

Combination of neutrino and antineutrino run removes wrong octant solutions

Synergy between channels

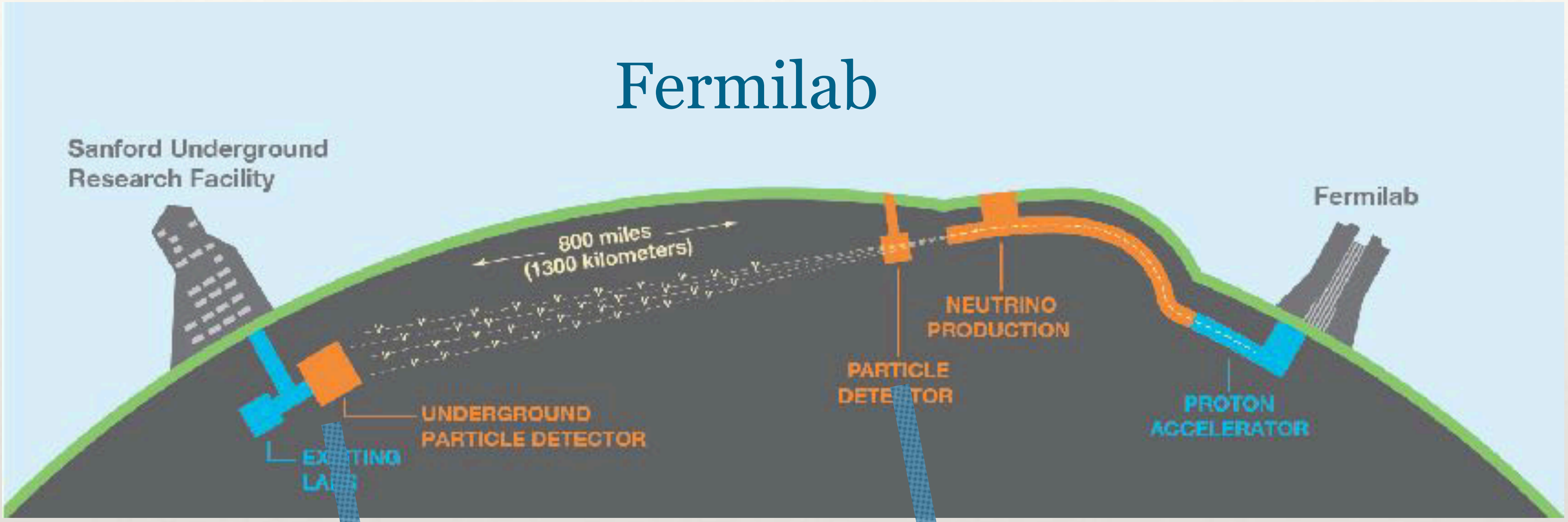


Total χ^2 greater than individual Contribution

Additional challenges

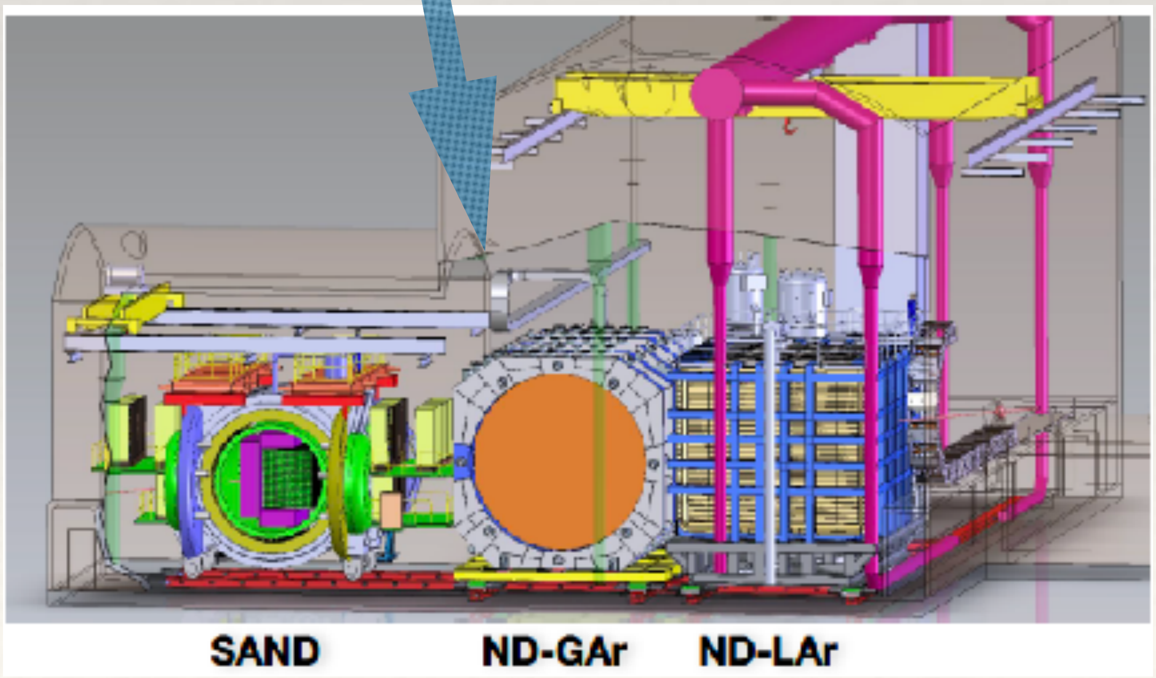
- CP violation is manifest in differences between neutrino and antineutrino probabilities
- The difference is at the level of few percents
- Needs large data samples
- Systematic uncertainties play important role for precision measurements
- Near detectors are helpful in reducing these

DUNE



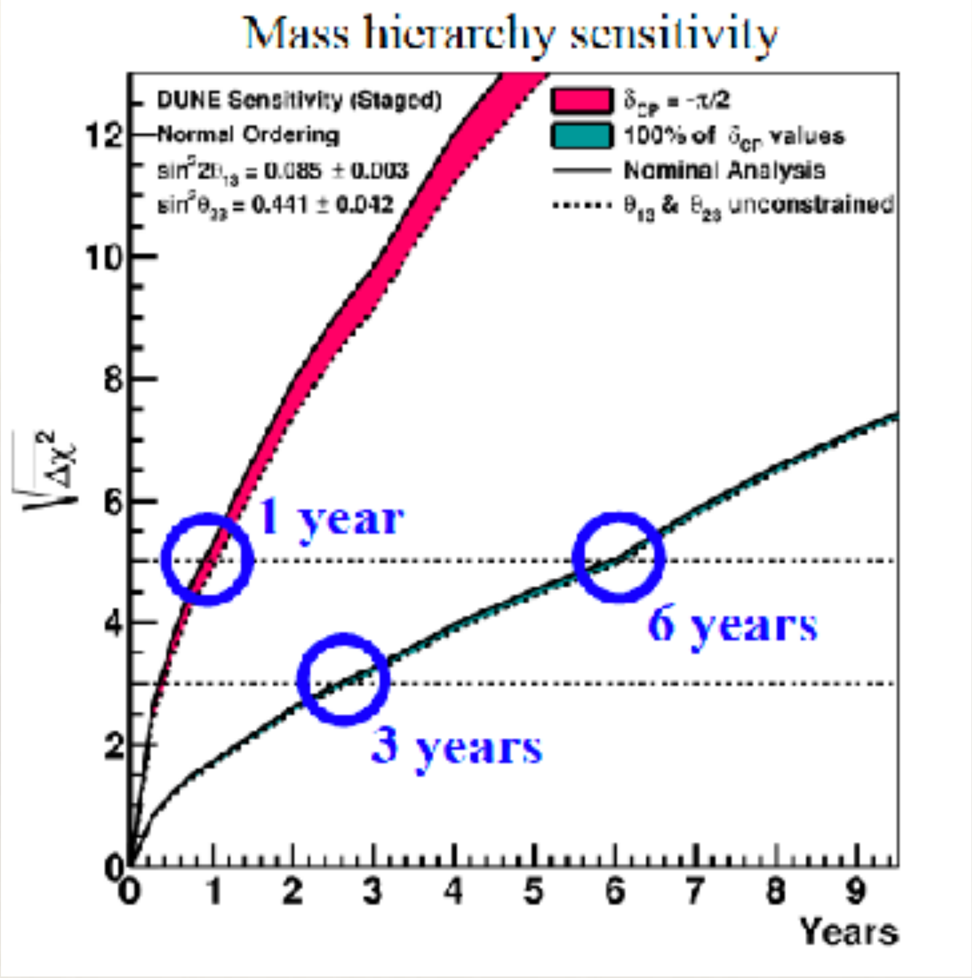
Far Detector

Liquid Argon Time Projection Chamber

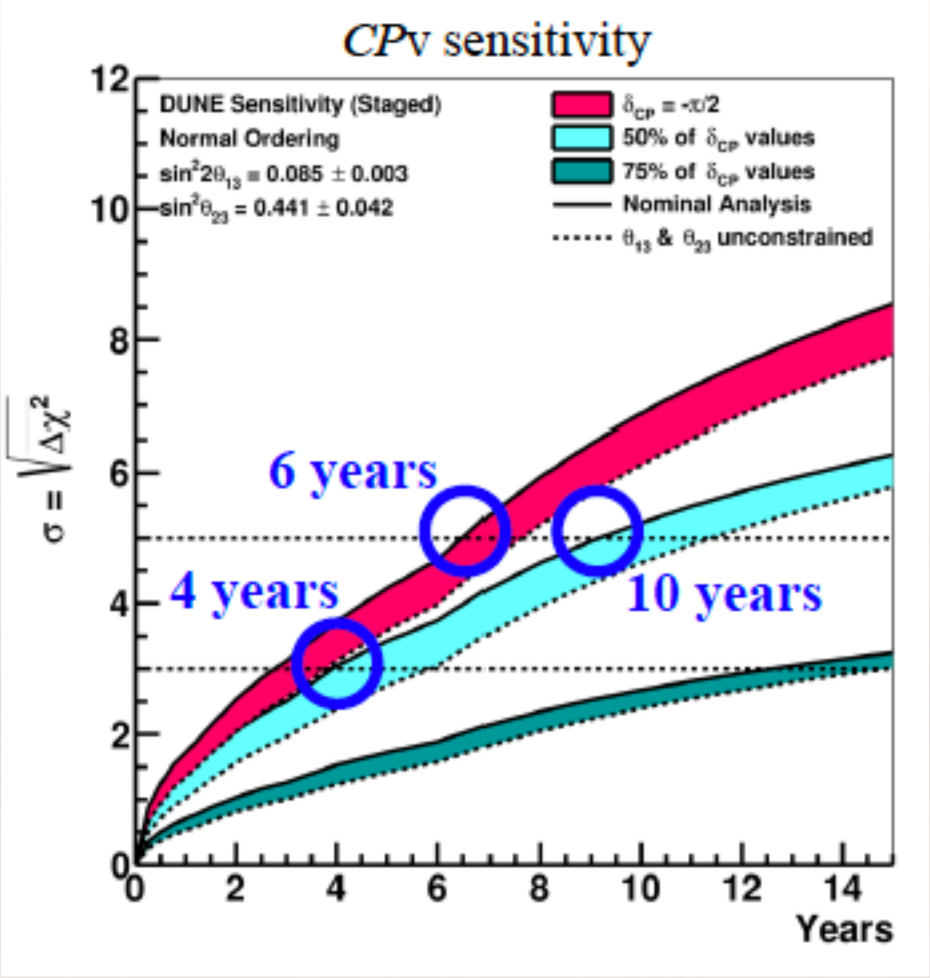


DUNE PRISM

Mass hierarchy and CP with DUNE



Hierarchy sensitivity due to enhanced matter effects

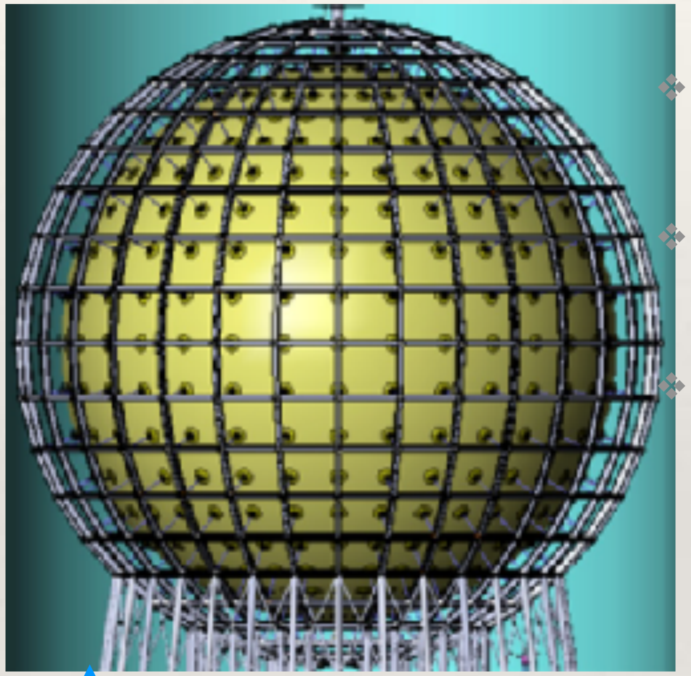


Matter effects help in removing wrong hierarchy-wrong CP solutions

From: R. Patterson's slides

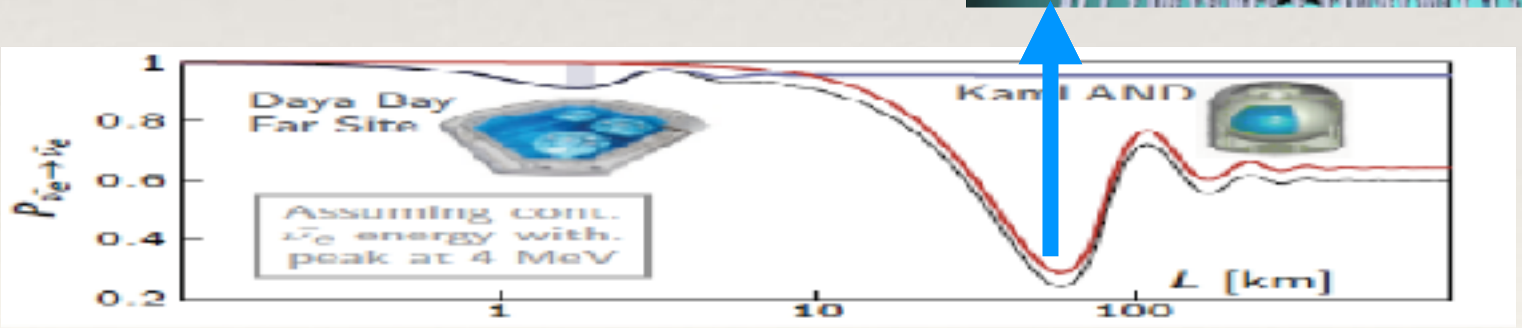
JUNO: Jianmen Underground Neutrino Observatory

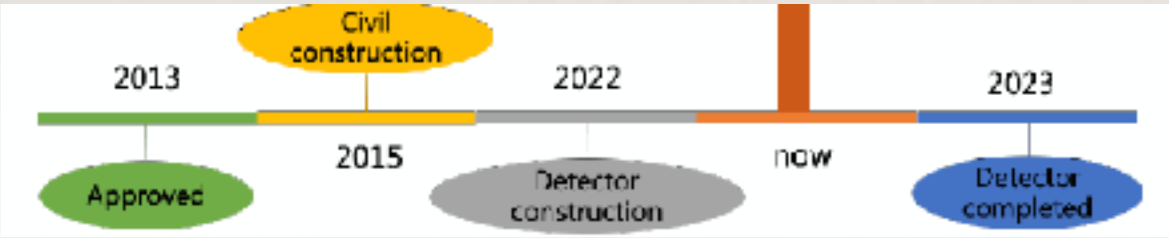
- Reactor Antineutrino Experiment
- 20 kt Liquid Scintillator detector



- Determination of mass Hierarchy
- Precision of osc. parameters
- Solar, supernova, atmospheric and geo-neutrinos

- Detector at 53 km from the two reactors





Expected to complete detector construction in 2023

JUNO: Hierarchy and Precision

$$P_R(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \left\{ \begin{array}{l} \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ +\sin^2 2\theta_{13}\sin^2\theta_{12}\left(\cos 2\Delta_{31}\sin^2\Delta_{21} - \frac{1}{2}\sin 2\Delta_{31}\sin 2\Delta_{21}\right) \end{array} \right\}$$

Precision of θ_{12}

Bandyopadhay,Choubey, S.G. 2003

Petcov, Piai, 2001,
Choubey,Petcov,Piai, 2003

Hierarchy sensitivity

Distortions in the energy spectrum

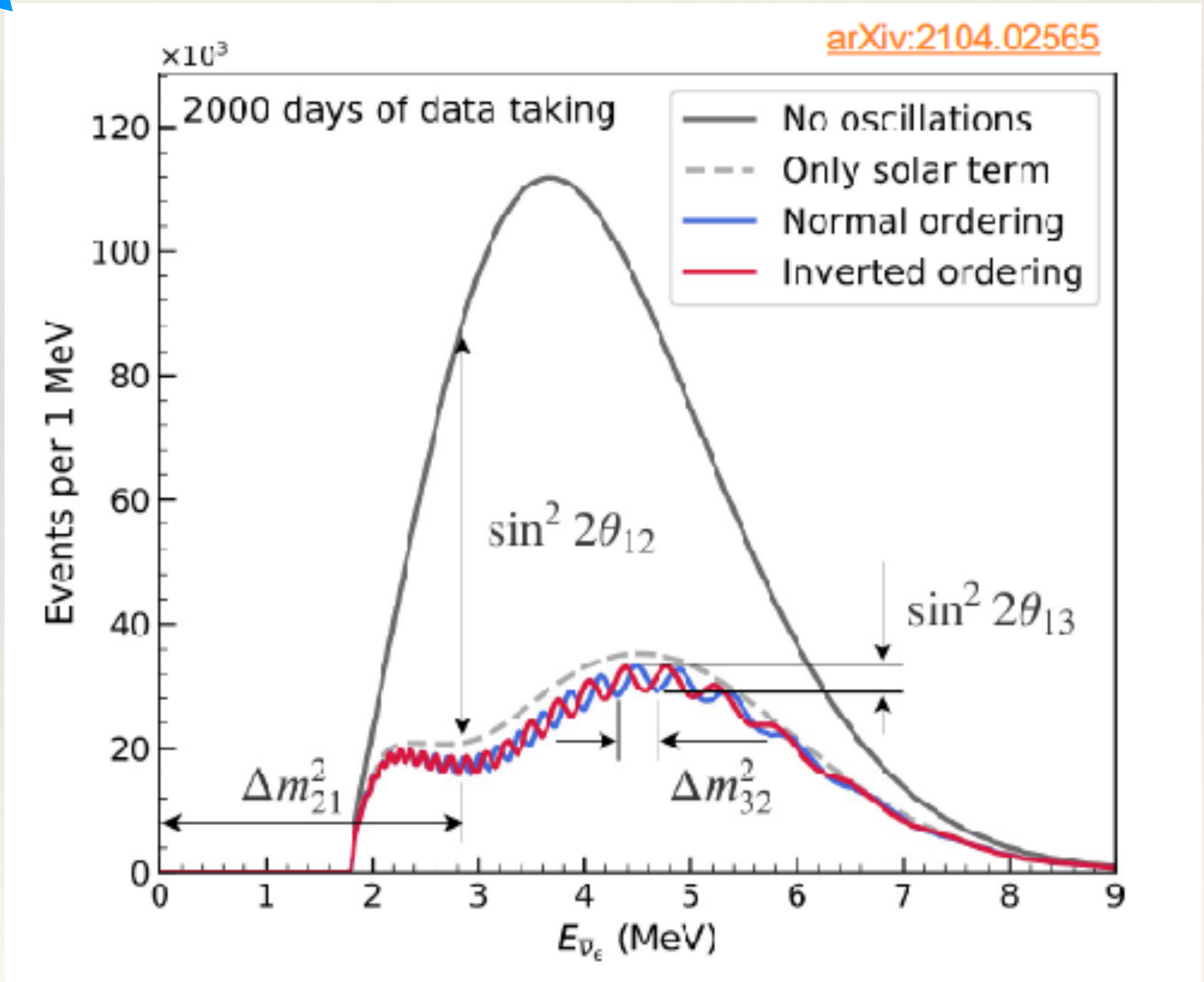
Better than 3% energy resolution needed

3σ hierarchy sensitivity in 6 years

Precision of oscillation parameters

	Δm^2_{31}	Δm^2_{21}	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$
JUNO 6 years	~0.2%	~0.3%	~0.5%	~12%
PDG2020	1.4%	2.4%	4.2%	3.2%

Table: A. Paoloni, 2021

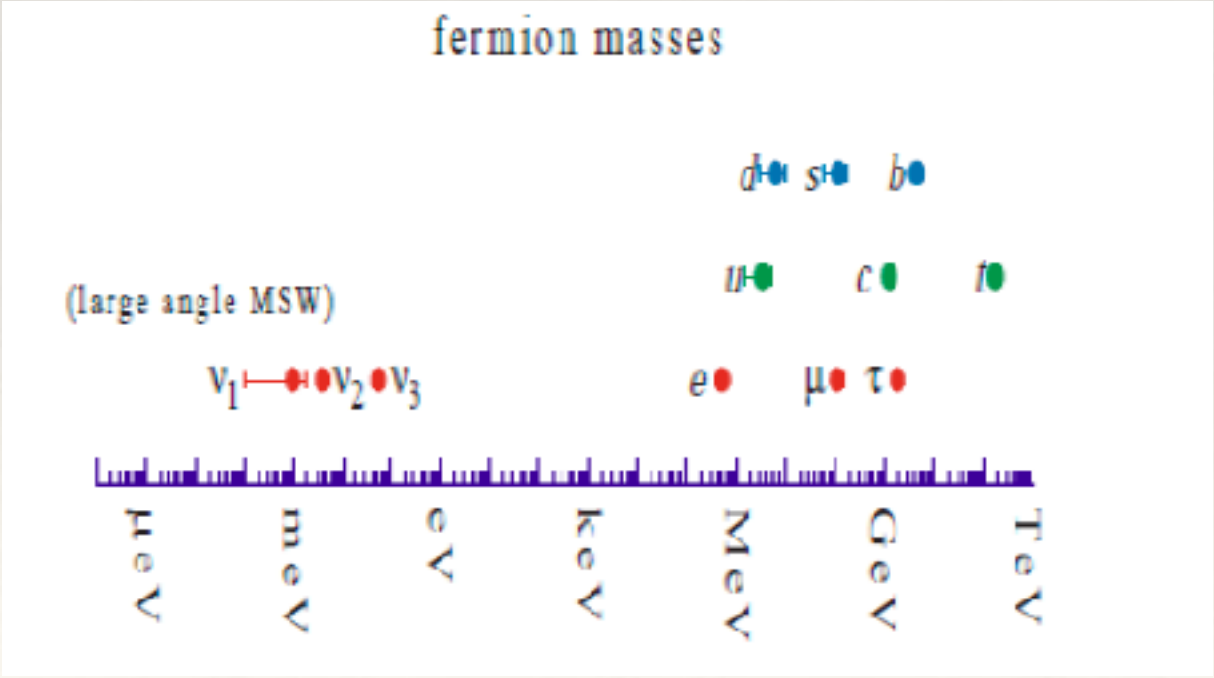


Beyond vanilla oscillations

- Possibility of probing new physics beyond the SM in neutrino oscillation experiments
- Sterile neutrinos, non-standard interactions, non-unitarity of neutrino mixing matrices, CPT violation, long range force, unstable neutrinos ….
- Sub-leading effect
- Changes the oscillation probability
- Impact on the 3-neutrino picture — extra parameters and degeneracies
- Constraining new physics parameters
- Unique signatures of BSM physics in neutrino experiments ?

Neutrinos have mass

- Neutrino Oscillation Experiments => neutrinos have mass and mixing
- Cosmological bound on neutrino mass $\Sigma m_\nu < 0.26\ eV\ (90\%\ C.L.)$
- => **Neutrino masses are small**
- Neutrino mass and mixing => **physics beyond the Standard Model**



=> Much smaller than the other fermion masses

Simplest way to give mass to neutrinos

- Add a right handed neutrino to the Standard Model
- Mass generation by Higgs Mechanism
- $m_\nu = Y_\nu v$ → $y_\nu \sim 10^{-12} - 10^{-13}$
- In comparison $m_e = Y_e v$ → $Y_e \sim 10^{-6}$
- Too small to be interesting

The effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{M}\mathcal{L}^{d=5} + \frac{1}{M^2}\mathcal{L}^{d=6} + \ldots$$

$$\mathscr{L}^{d=5} = c_5 \bar{L}^c L H H$$

Neutrino Mass
Lepton Number Violation

Weinberg Operator

$$\mathscr{L}^{d=6} = c_6 \bar{L}^c L \bar{L}^c L$$

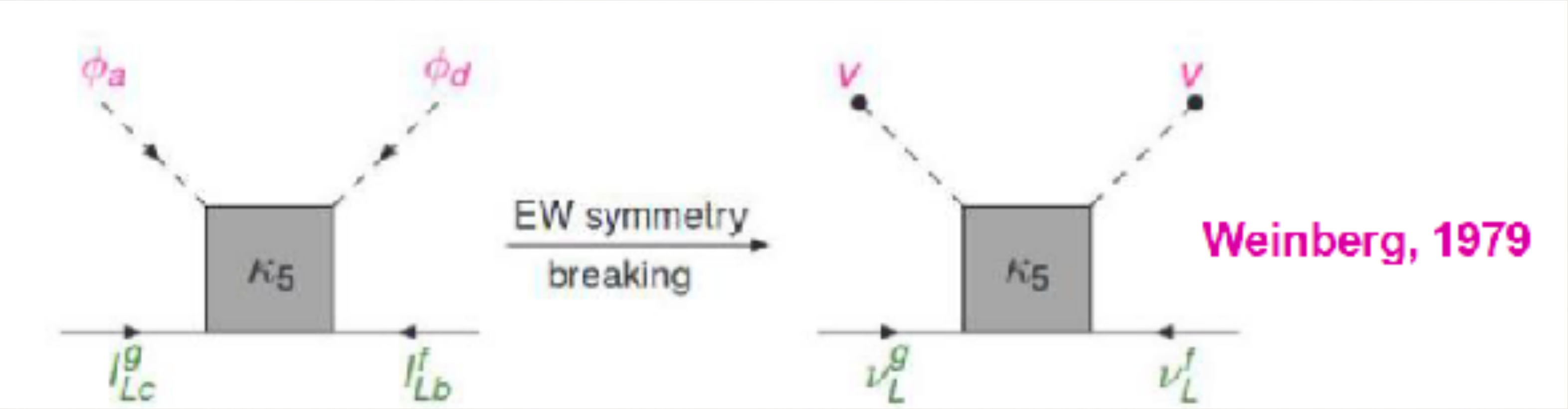
Lepton Flavour Violation,
NSI

Non- Renormalizable Operators —> what is the UV completion ?

New Physics at a high scale

Seesaw Mechanism

- A natural way to explain small neutrino masses is via **seesaw mechanism**
- **Relates smallness of neutrino masses with new physics at a high scale**
- Tree level exchange of some heavy particle gives rise than effective dimension 5 operator at the low scale
-



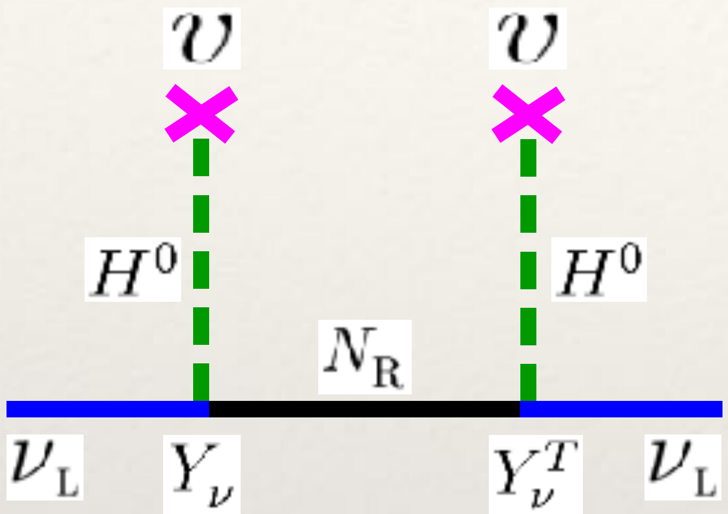
$$\mathscr{L}^{d=5} = c_5 \bar{L}^c L H H$$

$$m_\nu \sim c_5 v^2 / M$$

- Violation of lepton number → **Majorana nature of neutrinos**
- LNV decays of heavy mediators → **leptogenesis**

Three types of seesaw

Type-I seesaw



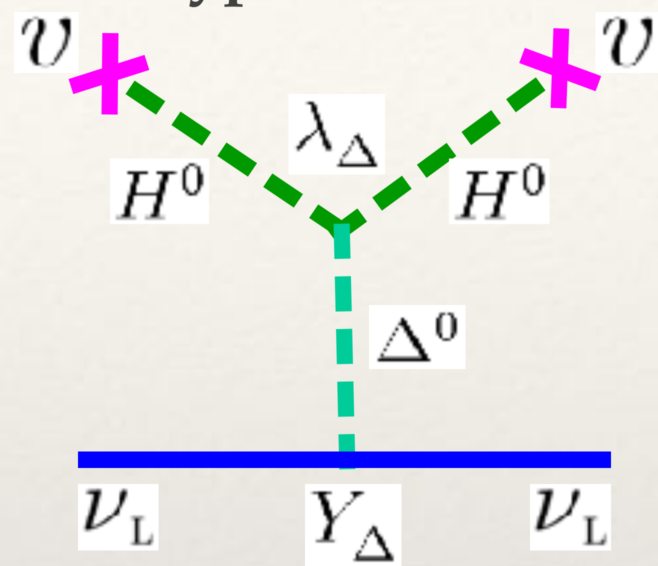
$$M_\nu \approx -v^2 Y_\nu \frac{1}{M_R} Y_\nu^T$$

Right handed neutrinos

SO(10) GUTS

Left Right Models

Type-II seesaw



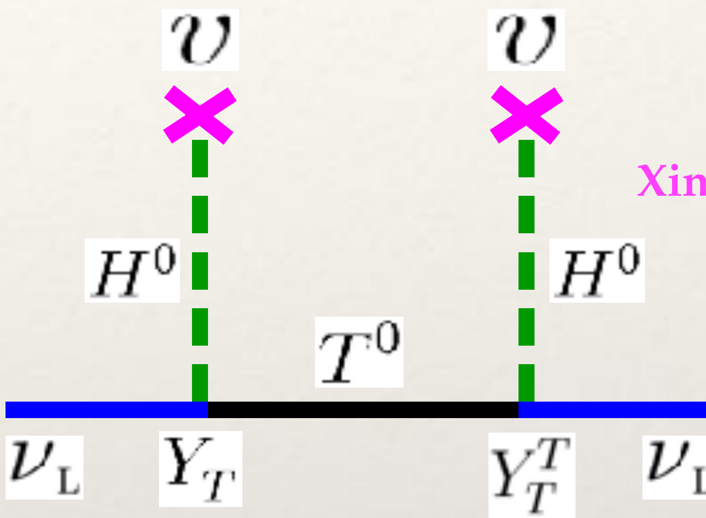
$$M_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

Triplet Higgs

Left-Right Models

Type-III seesaw

Xing



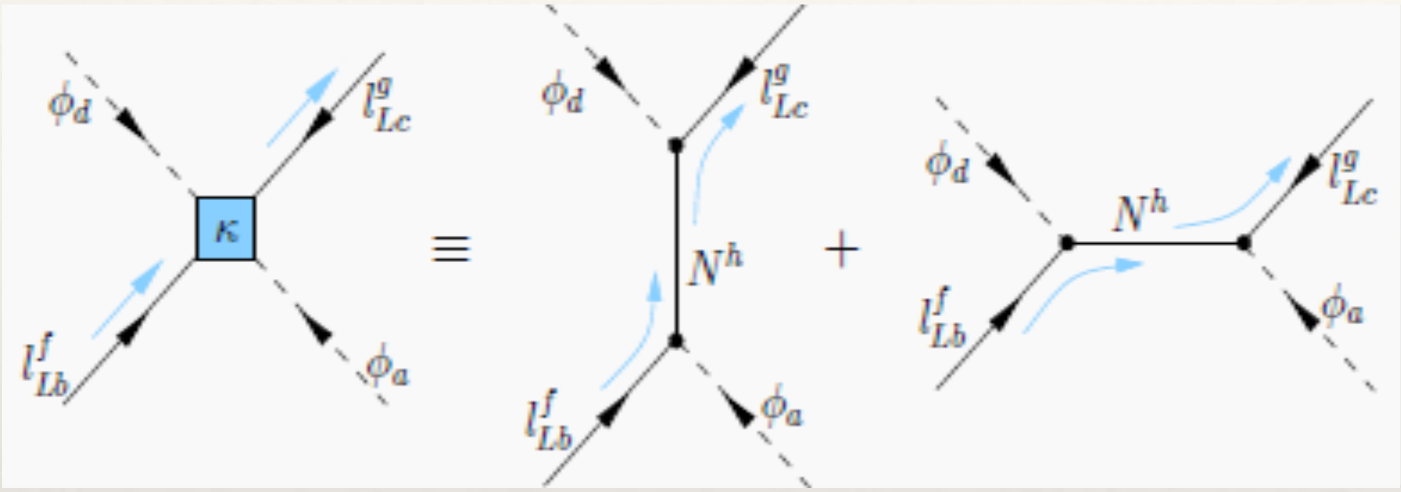
$$M_\nu \approx -v^2 Y_T \frac{1}{M_T} Y_T^T$$

Triplet Fermions

SU(5) GUTS

Type-I seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + (Y_\nu)\overline{N_R}\,\tilde{\phi}^\dagger l_L + \frac{1}{2}\overline{N_R{}^c}(M_R)_{ij}N_R + \text{h.c}$$



$\kappa = Y_\nu^T M_R^{-1} Y_\nu$ (at M_R) $\quad m_\nu = \kappa v^2$

$M_\nu = m_D^T M_R^{-1} m_D$, $m_D = v Y_\nu$

$m_\nu \sim 0.01$ eV for $M_R \sim 10^{16}$ GeV, $m_D \sim 100$ GeV, $Y_\nu \sim 1$

Cannot be tested directly.

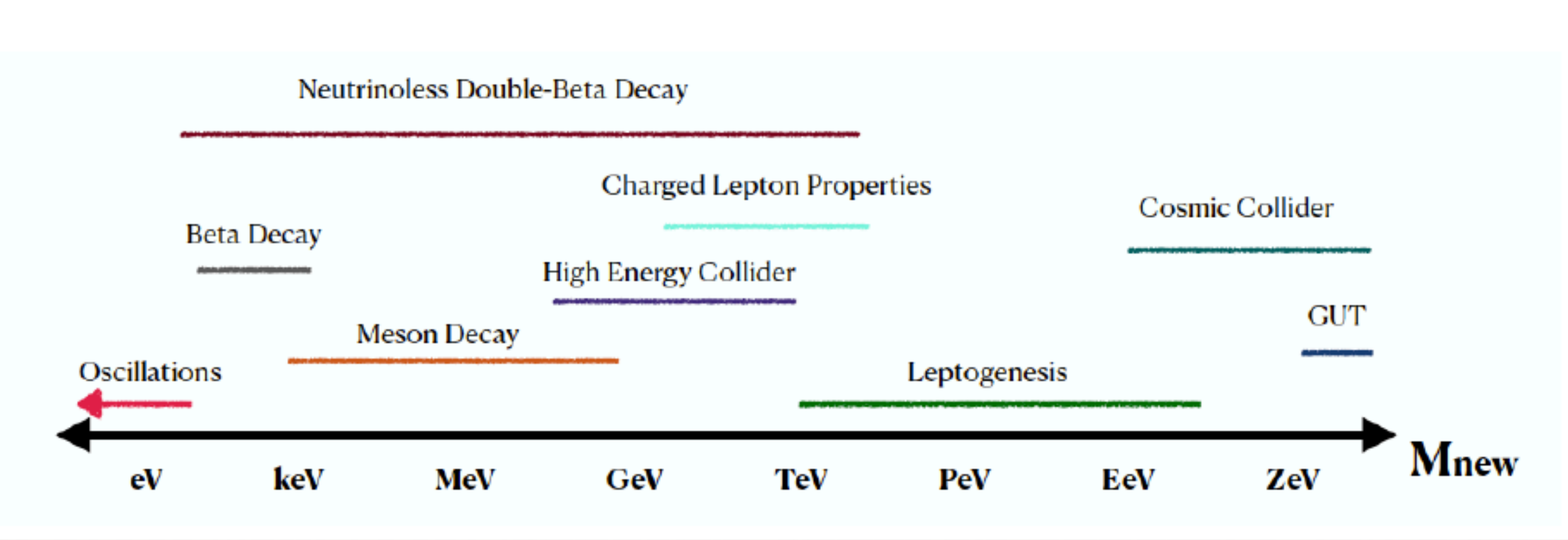
Minkowski, 1977
Yanagida, 1979
Glashow, 1979
Gelman, Ramond, Slansky, 1979
Mohapatra, Senjanovic, 1980

TeV scale seesaw

- GUT scale seesaw no testability at colliders $\rightarrow$ TeV Scale seesaw
- Extra singlets — inverse / linear seesaw
- Scale of LNV different than scale of mass generation
- Large light-heavy mixing $\Rightarrow$ testability at colliders
- Large light-heavy mixing $\Rightarrow$ Lepton flavour violation
- Stability of electroweak vacuum
- **Combined constraints** on model parameters

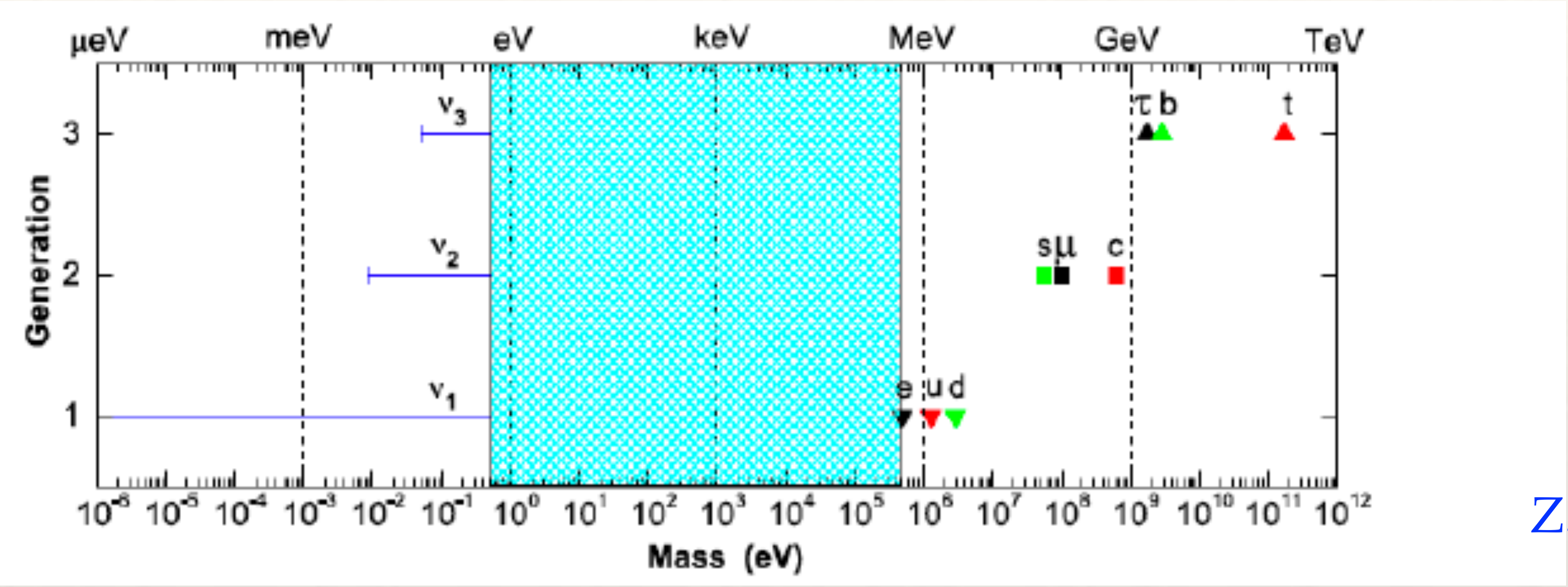
Other possibilities : Radiative mass models, higher dimensional models

What is the scale of new physics ?



Ack: Andre De Gouvea , Snowmass 2022

Fermion mass spectrum



Z.Z. Xing

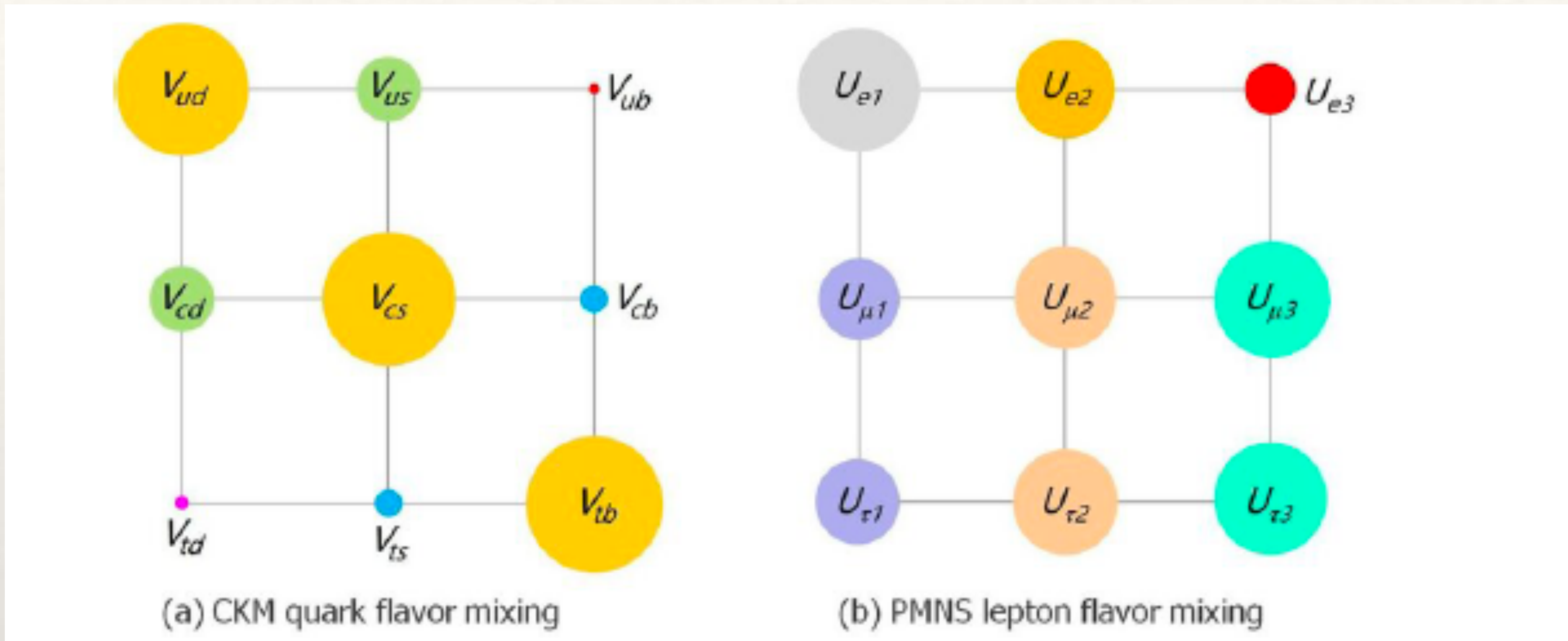
Neutrino masses << quarks and charged lepton masses

Hierarchy of neutrino masses not strong $m_3/m_2 \leq 6$

Inverted hierarchy, quasi-degeneracy

⟶ No analogue in the quark sector

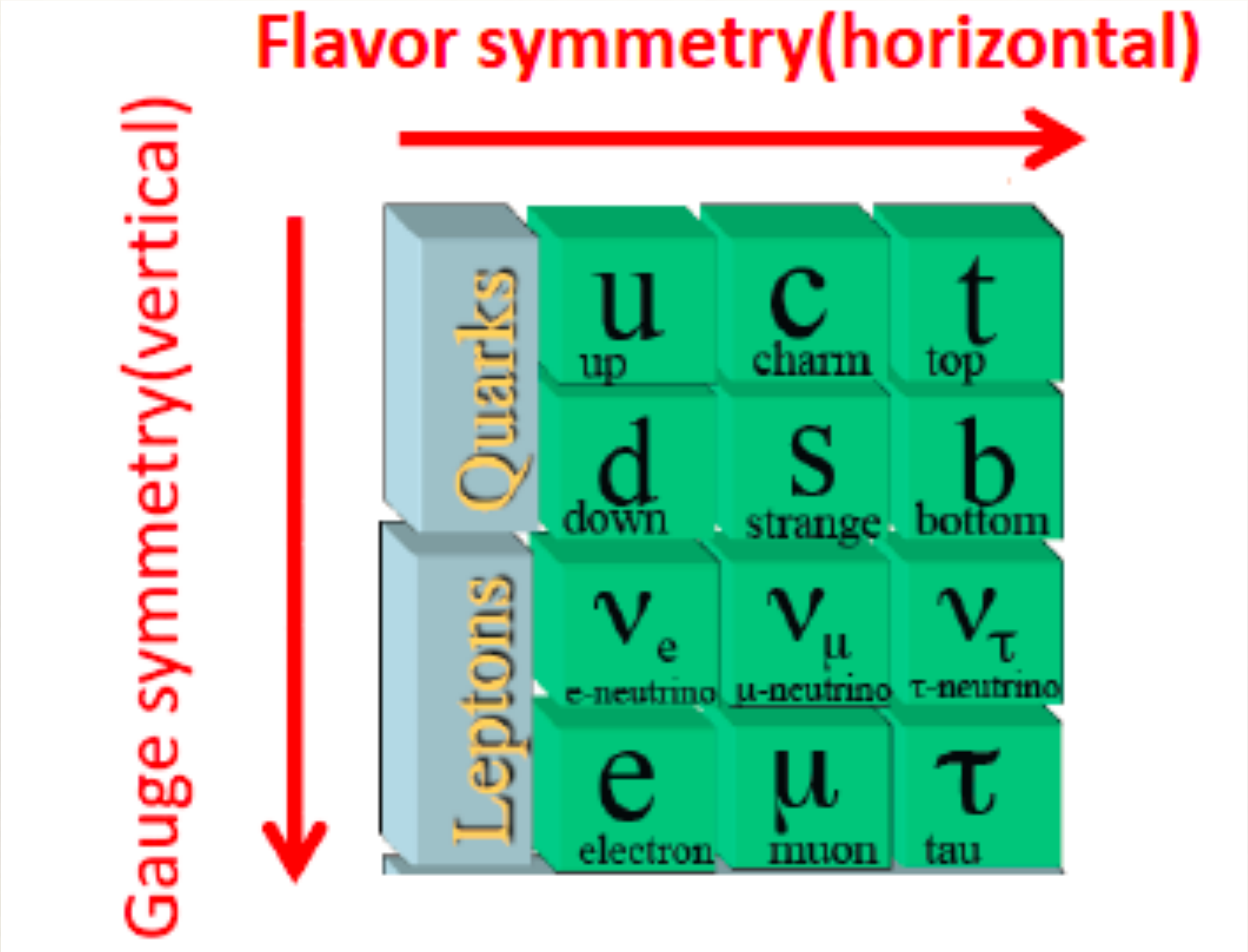
The flavour puzzle



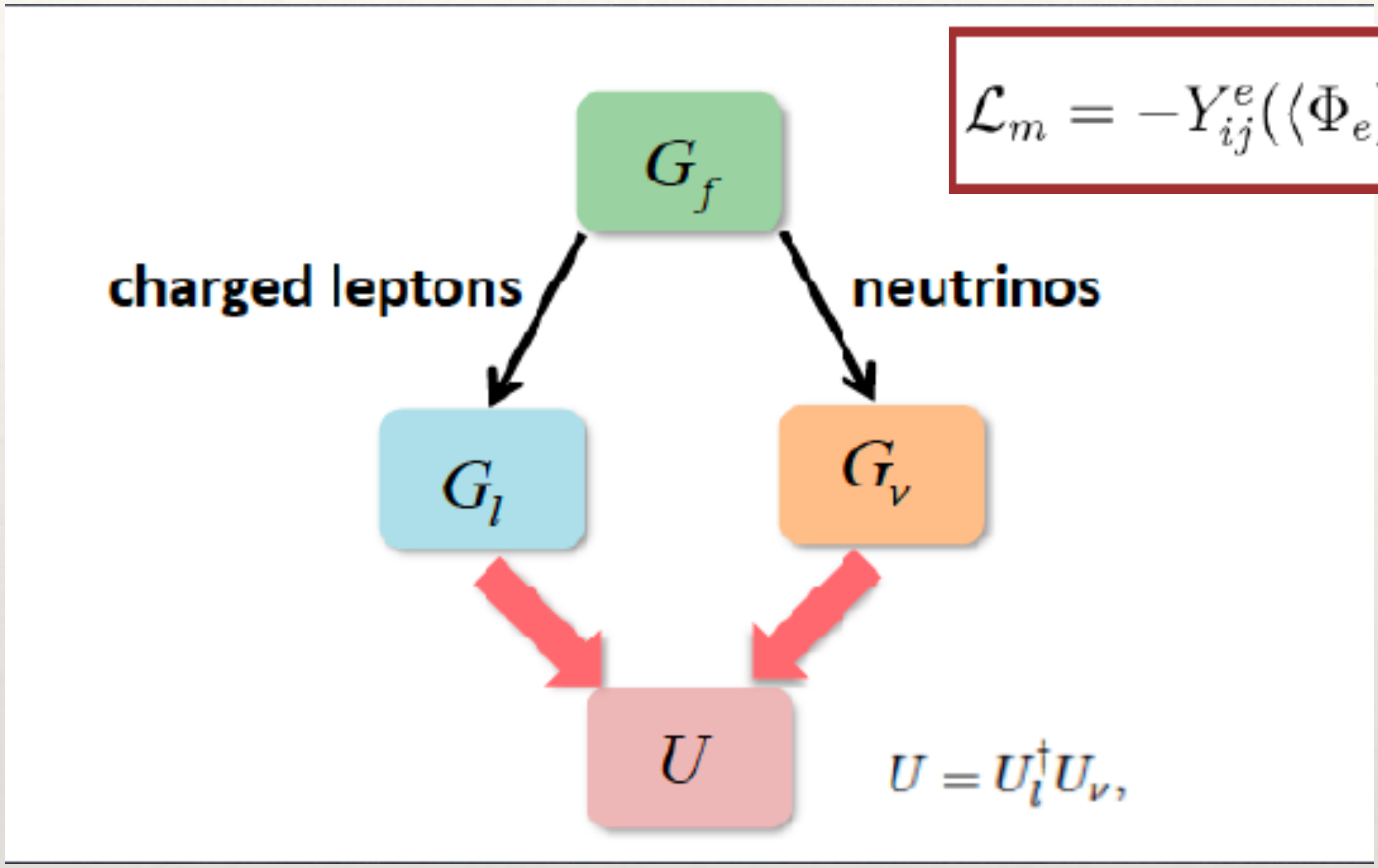
Quark Mixing angles	Neutrino mixing angles
$\theta_{12} = 13^0$	$\theta_{12} = 33.5^0$
$\theta_{23} = 2.3^0$	$\theta_{23} = 49^0$
$\theta_{13} = 0.2^0$	$\theta_{13} = 8.5^0$
$\delta_{CP} = 68.5^0$	$\delta_{CP} = 250^0$

$$|U^{\text{Current}}|^2_{3\sigma} = \begin{pmatrix} [0.606, 0.742] & [0.265, 0.337] & [0.020, 0.024] \\ [0.051, 0.270] & [0.198, 0.484] & [0.392, 0.620] \\ [0.028, 0.469] & [0.098, 0.685] & [0.140, 0.929] \end{pmatrix},$$

Flavour symmetry approach



Residual symmetries



$$\mathcal{L}_m = -Y^e_{ij}(\langle\Phi_e\rangle)\overline{L_i}He_{Rj} - \frac{1}{2}Y^\nu_{ij}(\langle\Phi_\nu\rangle)\overline{L^c_i}HH^TL_j + \ldots$$

Lepton mixing from mismatch of

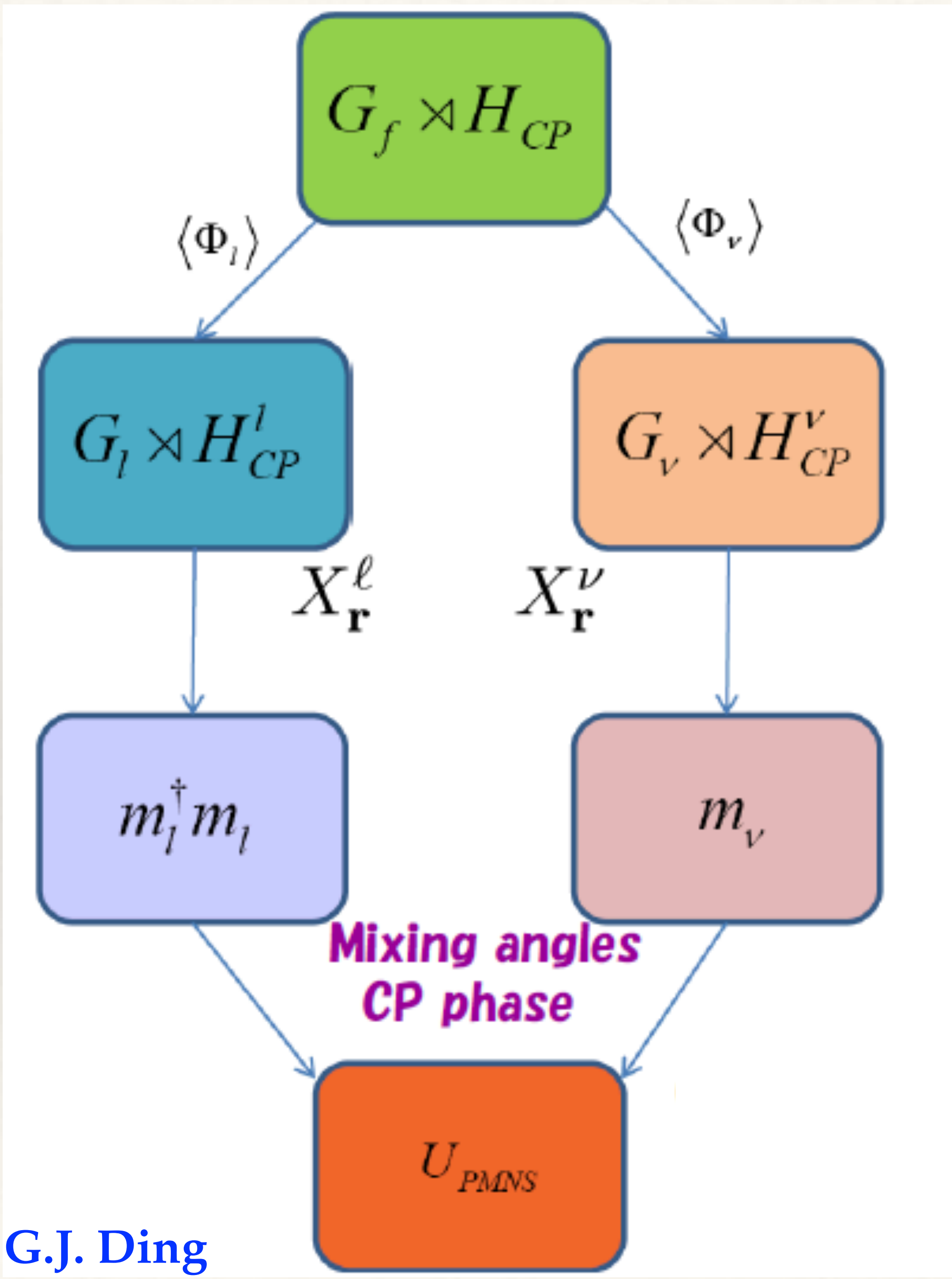
G_l and G_ν

Invariance of the Lagrangian under G_f requires flavons

Vacuum alignment of flavons

G_f	Continuous	Discrete
Abelian	U(1)	Z_n
Non-Abelian	U(2), SU(3),SO(3) …	A_4, S_4, A_5, $\Delta(6n^2)$, …

Generalized CP invariance and flavour symmetry



Courtsey: G.J. Ding

CP invariance →

$$X^{\nu T}_{\mathbf{r}} m_{\nu LL} X^{\nu}_{\mathbf{r}} = m^*_{\nu LL}$$

$$X^{\ell\dagger}_{\mathbf{r}} (m^\dagger_\ell m_\ell) X^{\ell}_{\mathbf{r}} = (m^\dagger_\ell m_\ell)^*$$

Predictions for mixing angle and CP phases

Generalized $\mu - \tau$ symmetry

$$\nu_e \to \nu_e^c\,, \quad \nu_\mu \to \nu_\tau^c\,, \quad \nu_\tau \to \nu_\mu^c$$

$$\nu_e \to \nu_e\,, \quad \nu_\mu \to \nu_\tau\,, \quad \nu_\tau \to \nu_\mu$$

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \mapsto \begin{pmatrix}\nu_e^c\\ \nu_\tau^c\\ \nu_\mu^c\end{pmatrix} = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}\nu_e^c\\ \nu_\mu^c\\ \nu_\tau^c\end{pmatrix}.$$

$$\theta_{23} = \pi/4,\ \theta_{13} = 0$$

Generalized CP invariance → $$\mathcal{S}^T M_0 \mathcal{S} = M_0^*,$$

→ $$M_0 = \begin{pmatrix} a & d & d^* \\ d & c & b \\ d^* & b & c^* \end{pmatrix},$$ → $$U_0 = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ v_1^* & v_2^* & v_3^* \end{pmatrix},$$

→ $$|U_{\mu i}| = |U_{\tau i}| \quad \text{where} \quad i = 1,2,3.$$

→ $$\theta_{23} = \frac{\pi}{4}, \quad s_{13}\cos\delta_{\rm CP} = 0,$$

$$|U_{\mu i}| = |U_{\tau i}| \iff \begin{cases} \theta_{23} = \frac{\pi}{4},\ \theta_{13} = 0\,; \\ \text{or} \\ \theta_{23} = \frac{\pi}{4},\ \delta = \pm\frac{\pi}{2}. \end{cases}$$

Modular invariance approach

arXiv > hep-ph > arXiv:1706.08749

High Energy Physics – Phenomenology

[Submitted on 27 Jun 2017 (v1), last revised 29 Sep 2017 (this version, v2)]

Are neutrino masses modular forms?

Ferruccio Feruglio

The Yukawa couplings are functions of modular forms

Flavour symmetry broken by the VEV of a single scalar field — the modulus τ

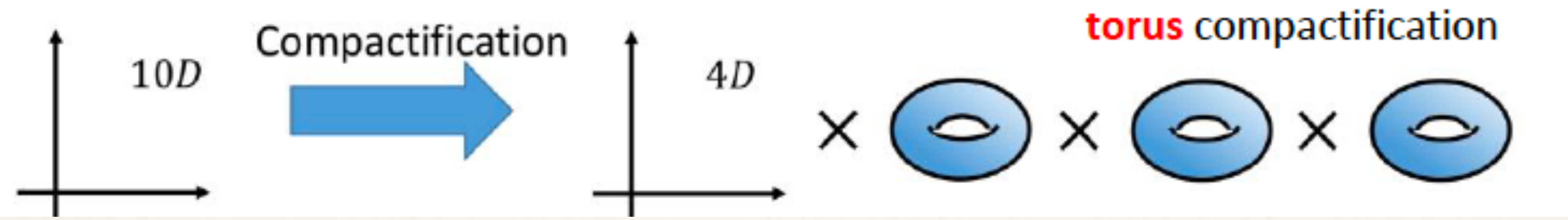
No other flavons or complexity of vacuum alignment

Close to 200 papers …

Minimal, predictive

Ding and King 2311.09282 (review)

Modular Symmetries



$$S = \int d^4x\, d^6y\, \mathcal{L}_{10D} \to \int d^4x\, \mathcal{L}_{\text{eff}}$$

$\mathcal{L}_{\text{eff}}$ depends on the structure of 

The shape of torus is represented by a modulus $\tau \in \mathbb{C}$.

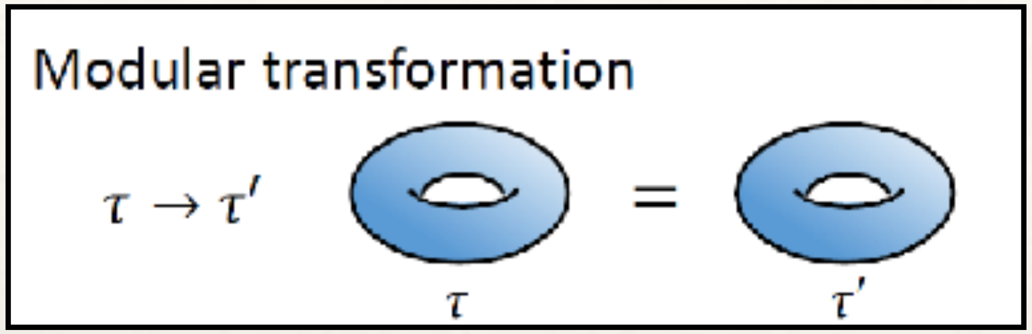
$\tau = \tau_1$ $\quad$ $\tau = \tau_2$

The different value of τ realize the different shape of T^2

$$\mathcal{L}_{\text{eff}} \supset Y(\tau)_{ij}\, \phi \overline{\psi_i} \psi_j + \cdots$$

Tanimoto
Penedo

Modular transformation



Specific transformations which does not change T^2

$$\tau' = \frac{a\tau + b}{c\tau + d}$$

$$SL(2,\mathbb{Z}) = \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a, b, c, d \in \mathbb{Z}, \ \det\gamma = 1 \right\}$$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z), \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod N \right\}$$

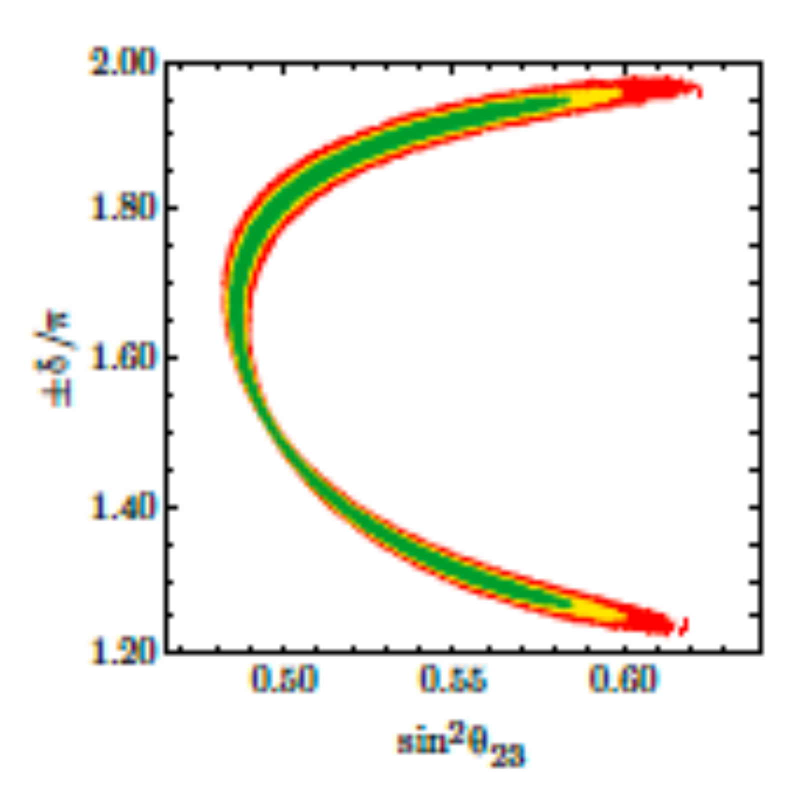
$\Gamma_N \equiv \Gamma / \Gamma(N)$ quotient group finite group of level N

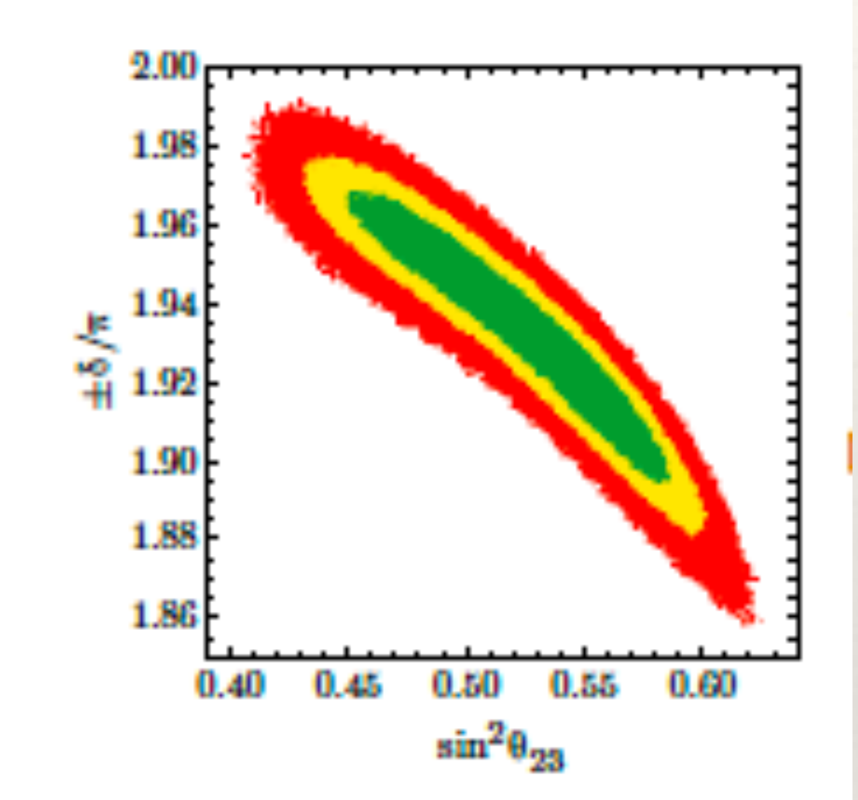
$\Gamma_2 \simeq S_3$ $\Gamma_3 \simeq A_4$ $\Gamma_4 \simeq S_4$ $\Gamma_5 \simeq A_5$

Penedo

Results from a model based on modular S_4

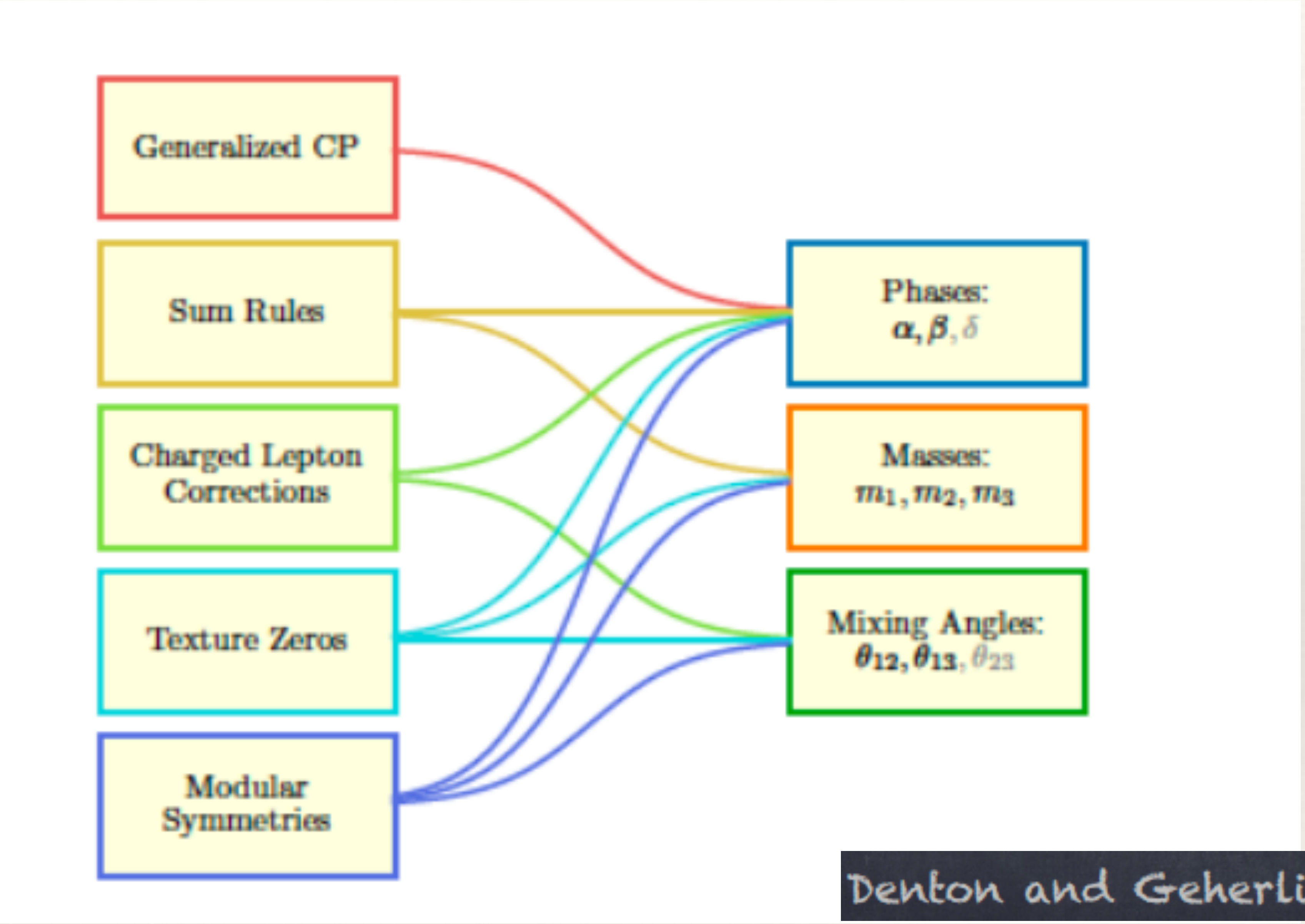
P. P. Novichkov , J. T. Penedo , S. T. Petcov. , A. V. Titov, JHEP 04 (2019) 005





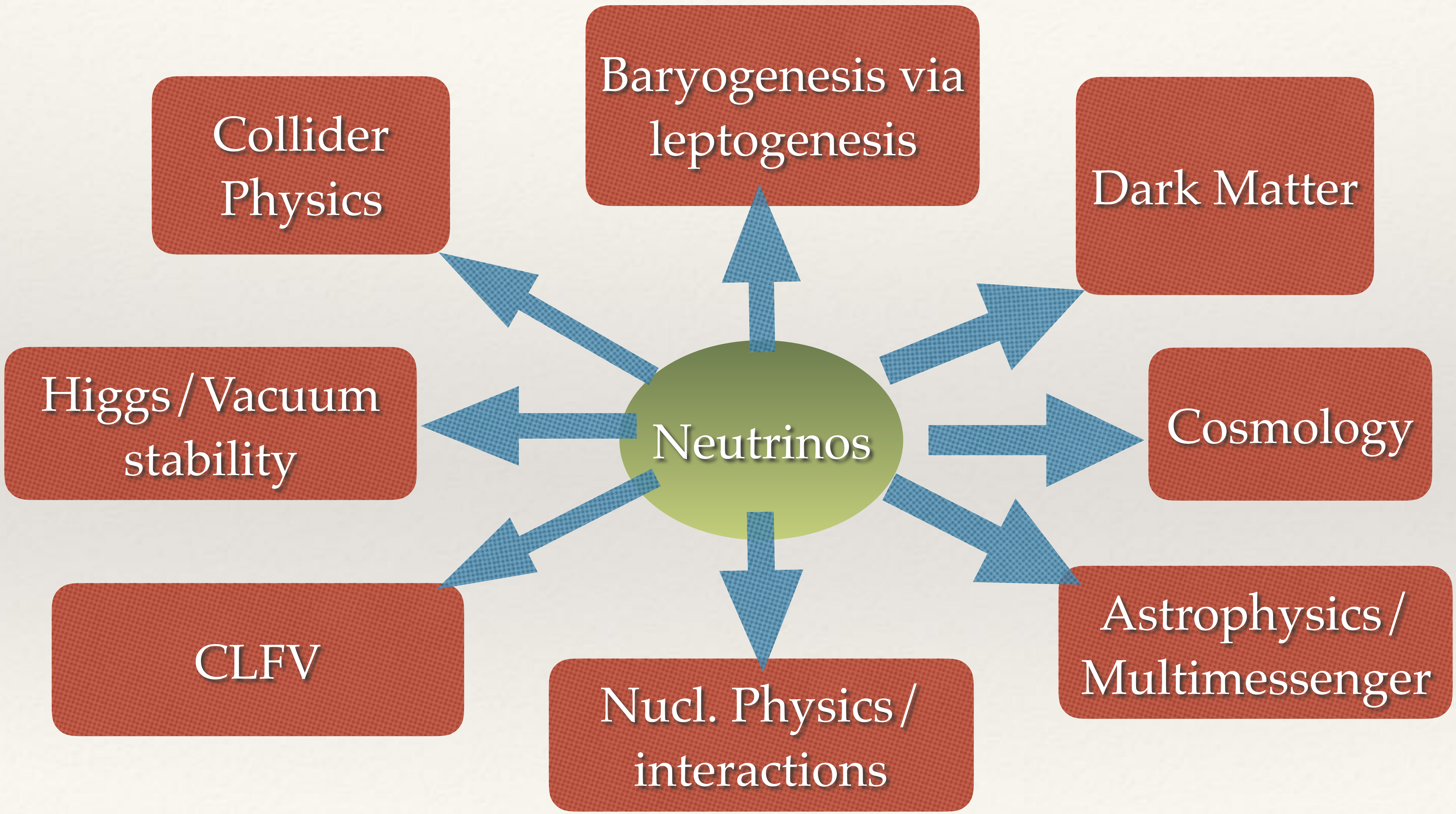
Correlation between θ_{23} and δ

A snapshot of different methods



Denton and Geherlin, 2308.09737

Neutrino Connections



Future Goals in Neutrino Physics

- Determination of hierarchy, octant and CP phase
- Synergy between different experiments
- Signatures of sterile neutrinos

} Immediate Goals

- Precise Measurement of neutrino cross-sections
- Nature of neutrinos , absolute neutrino mass

} Ongoing

- Probing new physics in oscillation experiments
- Search for Dark Matter in neutrino facilities
- Probing BSM physics through new interactions

} Emerging Goals

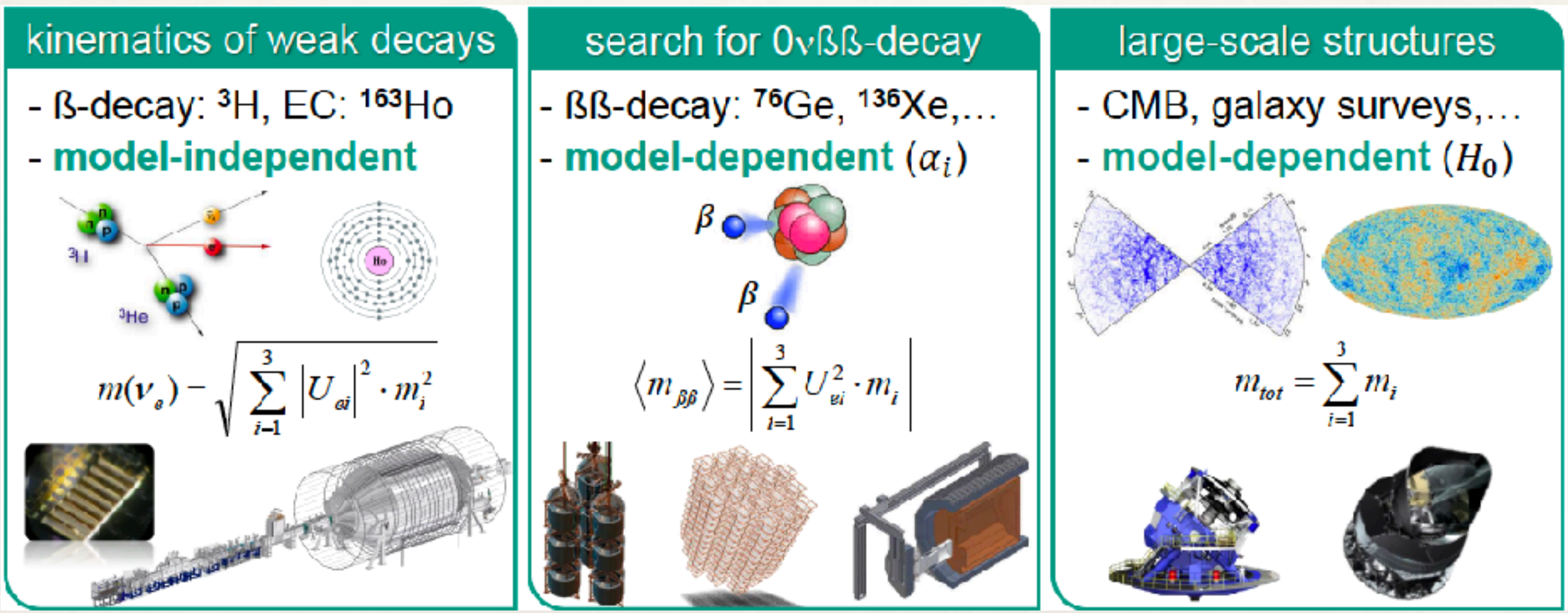
- Neutrinos and Multimessenger Astronomy
- Origin of neutrino masses and mixing

} Continuing


Neutrino physics — still a vast landscape
Thank you!!

Absolute neutrino mass

Information can come from three different sectors



Slide : G. Drexlin, NOW 2022

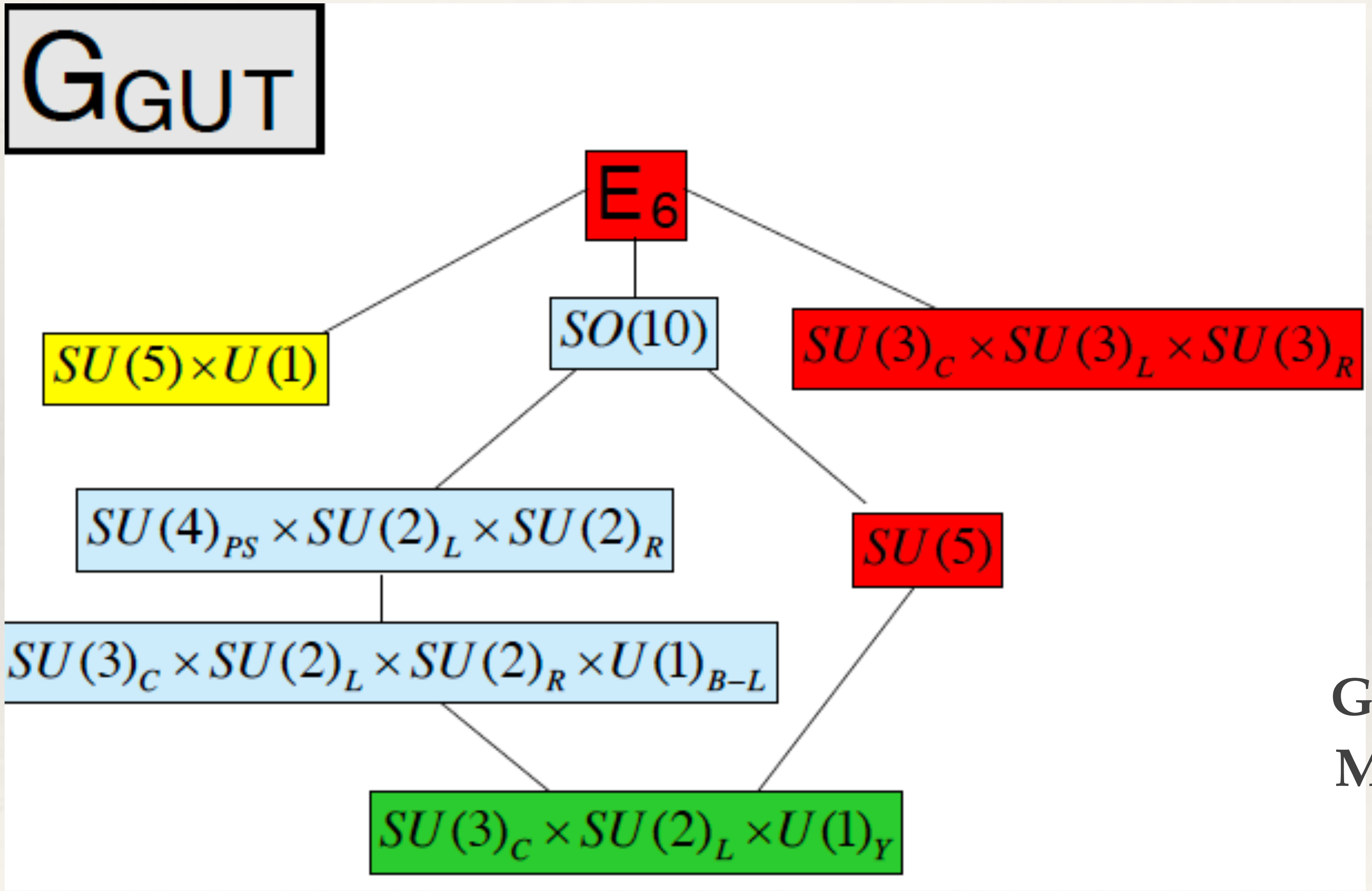
Grand Unified Theories

$$G_{\rm SM} \subset G$$

$$G = SU(5),\ SU(6),\ SO(10),\ ...$$

Quark-lepton unification

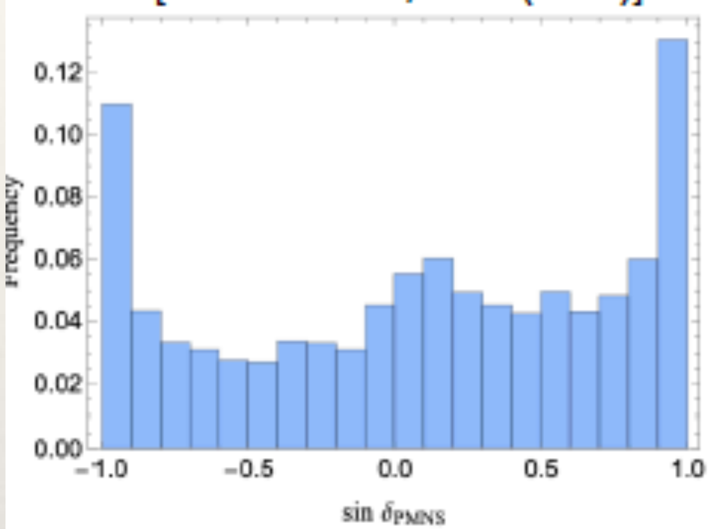
$$\psi_i \ni (Q_i, u_i^c, d_i^c, L_i, e_i^c, ...)$$



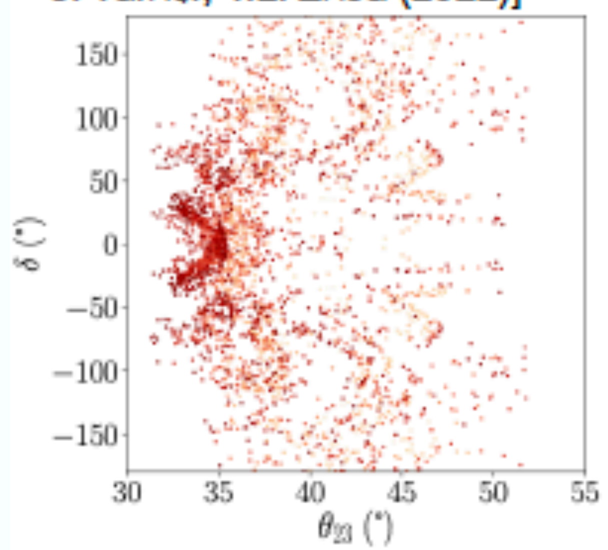
GUT X flavour
Modular GUT

Prediction of CP phase from SO(10)

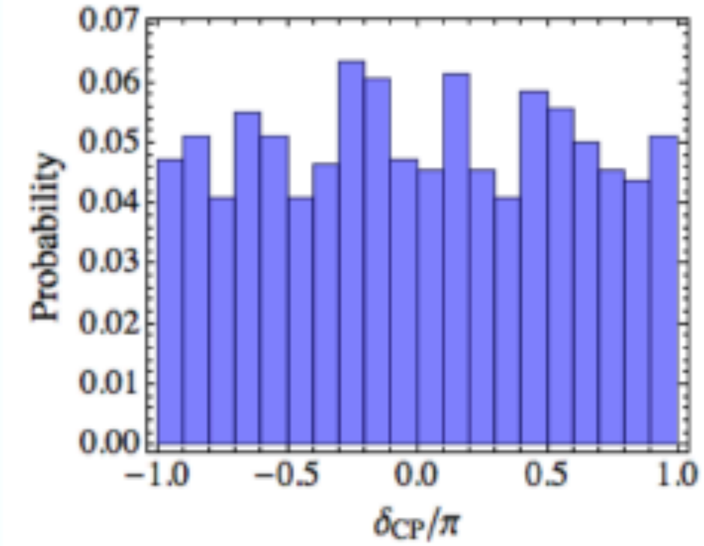
[V.S. Mummidi, KMP (2021)]



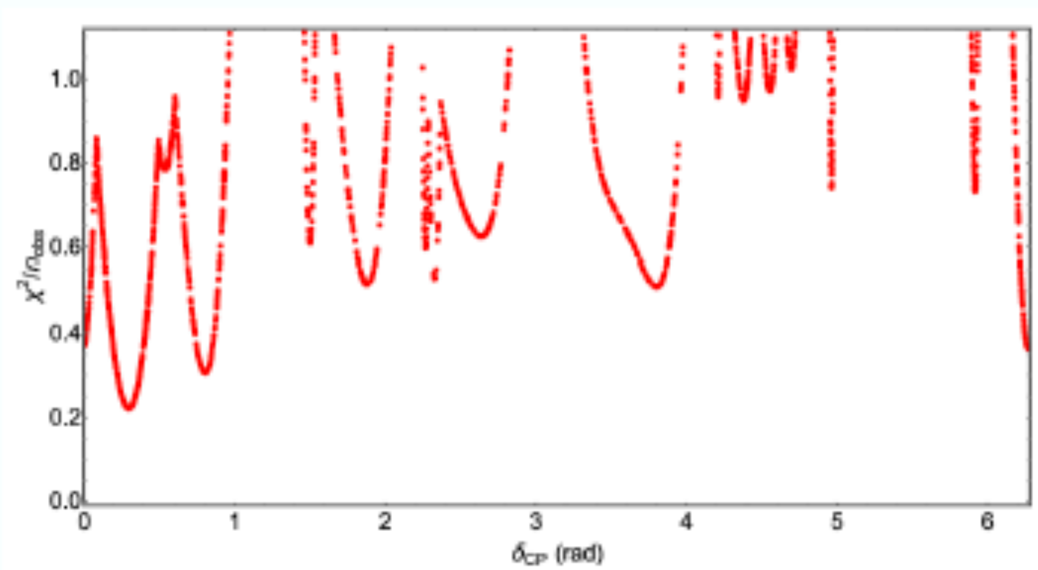
[B. Fu, S.F. King, L. Marsili, S. Pascoli, J. Turner, Y.L. Zhou (2022)]



[F. Feruglio, D. Vicino, KMP (2014)]



[K.S. Babu, B. Bajc, S. Saad (2018)]



Courtsey: K.M. Patel

Prediction of CP phase from SO(10)