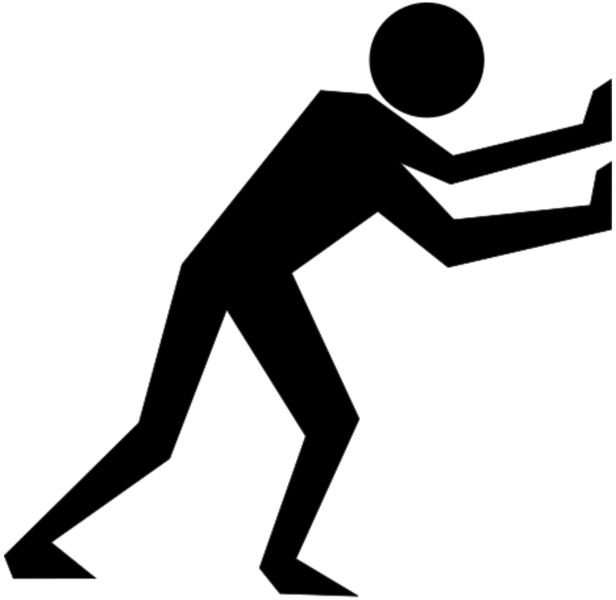


Pushing the Limits in Dark Matter Production

Julia Harz

October 15th 2024
17th International Conference on Interconnections between Particle Physics and Cosmology
Hyderabad

PRiSMA+
JOHANNES GUTENBERG UNIVERSITÄT MAINZ
JG|U

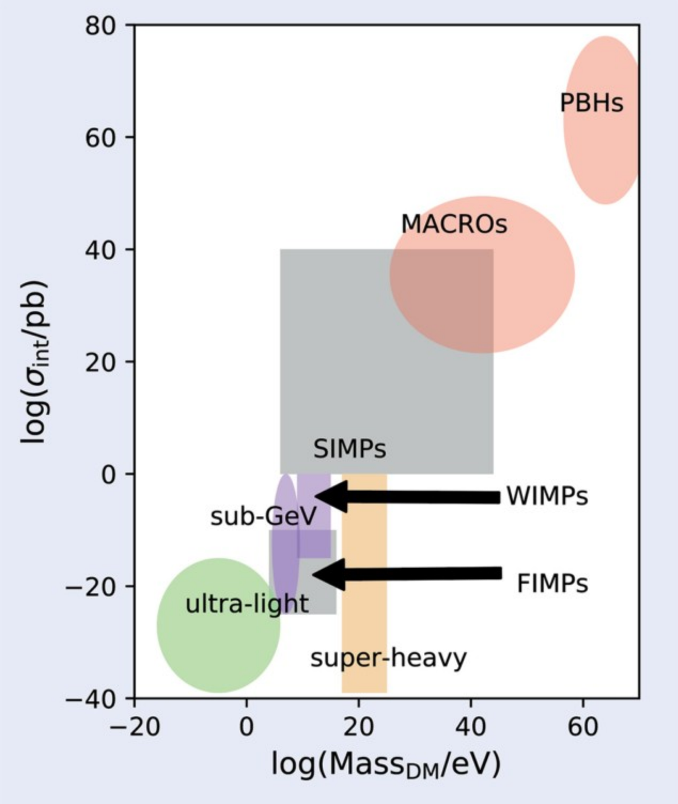
Cornering the dark matter landscape

Of utmost importance:

- reliable theory predictions for
- accurate interpretation of experimental data

➡ methodological advancements

"pushing the limits"

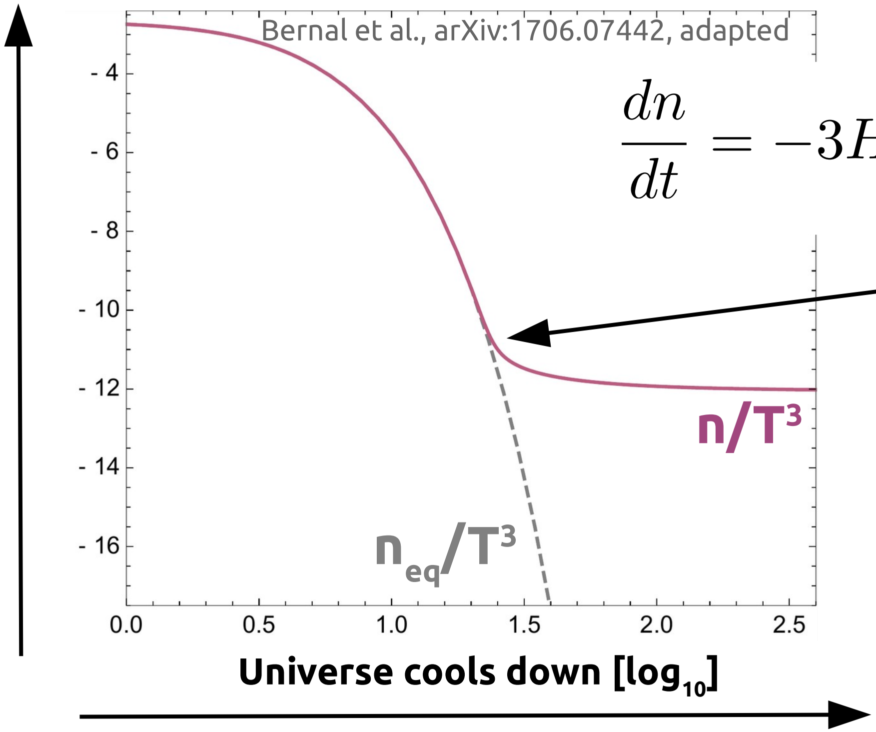


C. Arina, CERN Courier, 4 March 2021

Freeze-out of Dark Matter

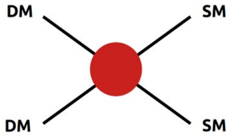
- **Assumption**: WIMP as DM candidate is due to its interaction rate in thermal equilibrium with the standard model bath

Normalized number density of Dark Matter [$\log_{10}$]



$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2)$$

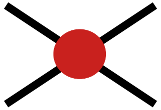
Expansion faster than interaction rate of dark matter ("freeze-out")



Towards new standards for the DM abundance prediction

- Improving precision of cross sections crucial for DM abundance calculation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\rm eff} v \rangle (n^2 - n_{\rm eq}^2)$$



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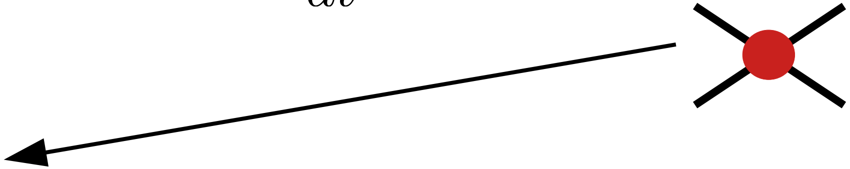
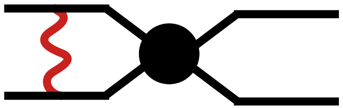
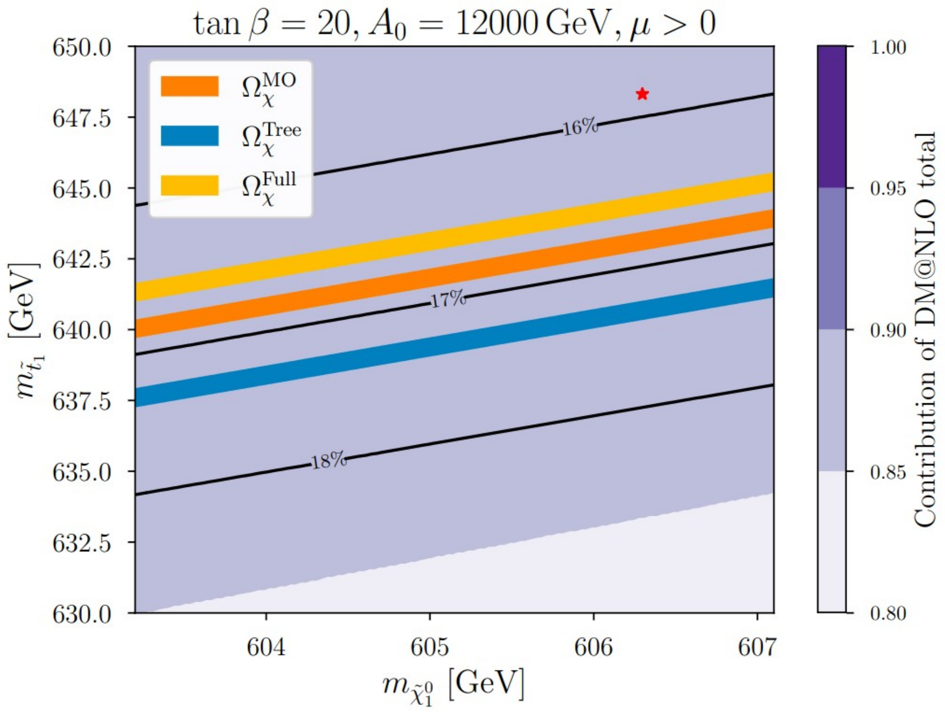


Higher order corrections

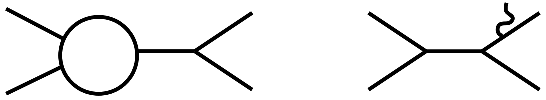


DM@NLO

- **DM abundance calculation with QCD@NLO within the MSSM**



Finite =



Solved bottleneck of Dipole subtraction for massive initial state particles needed for early Universe!

→ **Code is publicly available**

JH, Klasen, Younes Sassi, Wiggering (2022)
JH, Herrmann, Klasen, Kovarik, Wiggering (2023)

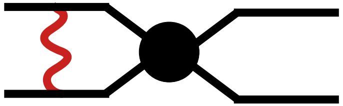
Towards new standards for the DM abundance prediction

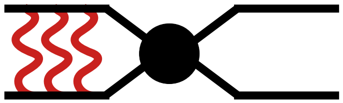
- Improving precision of cross sections crucial for DM abundance calculation

$$\frac{dn}{dt} = -3Hn - \langle\sigma_{\text{eff}} v\rangle (n^2 - n_{\text{eq}}^2)$$

Higher order corrections

Sommerfeld effect





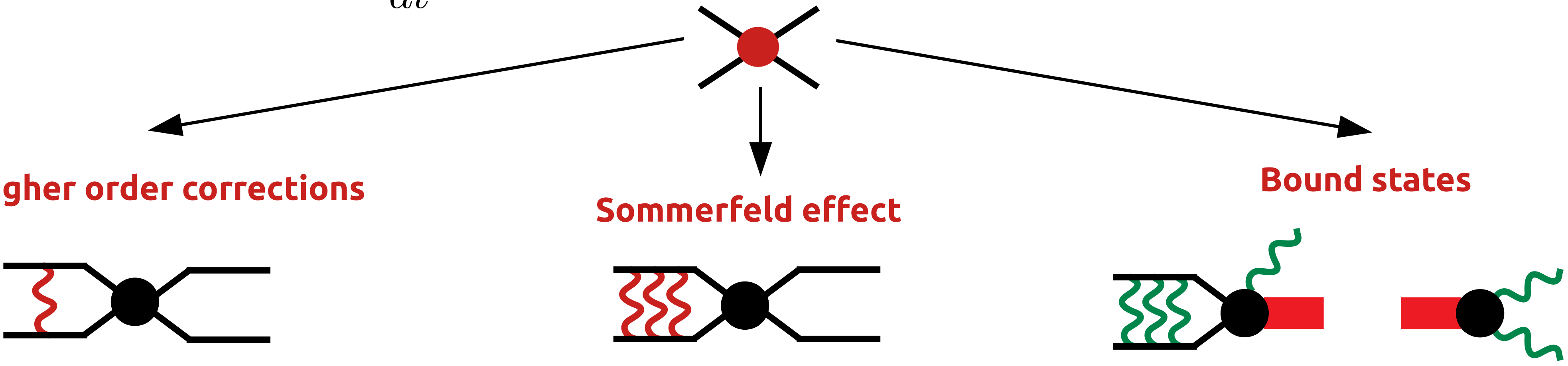
If $\alpha \sim v_{\text{rel}}$, exchange of *n* particles lead to

$\left(\frac{\alpha}{v_{\text{rel}}}\right)^n \sim 1$ → resummation

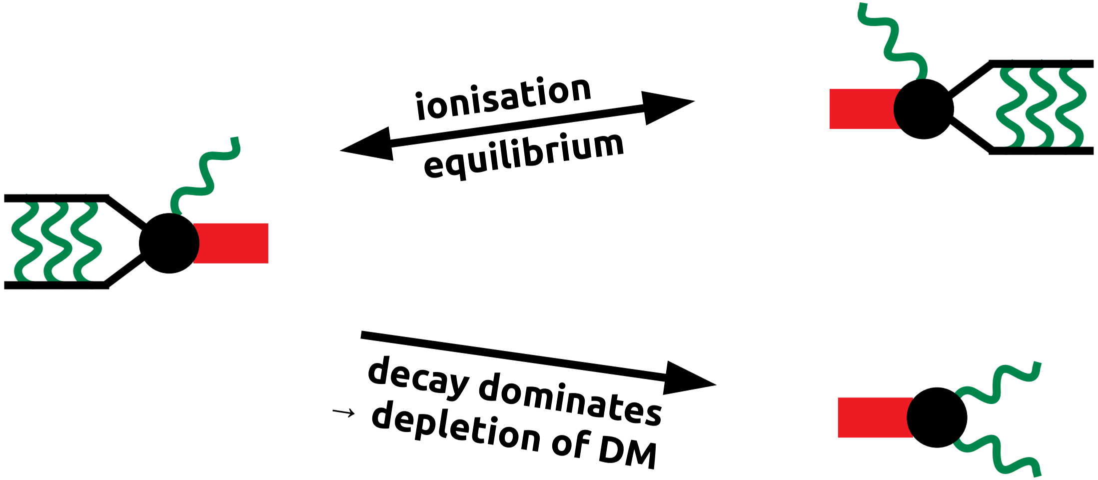
Towards new standards for the DM abundance prediction

- Improving precision of cross sections crucial for DM abundance calculation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\mathrm{eff}} v \rangle (n^2 - n_{\mathrm{eq}}^2)$$



Bound state formation and decay



$T > E_\mathrm{B}$

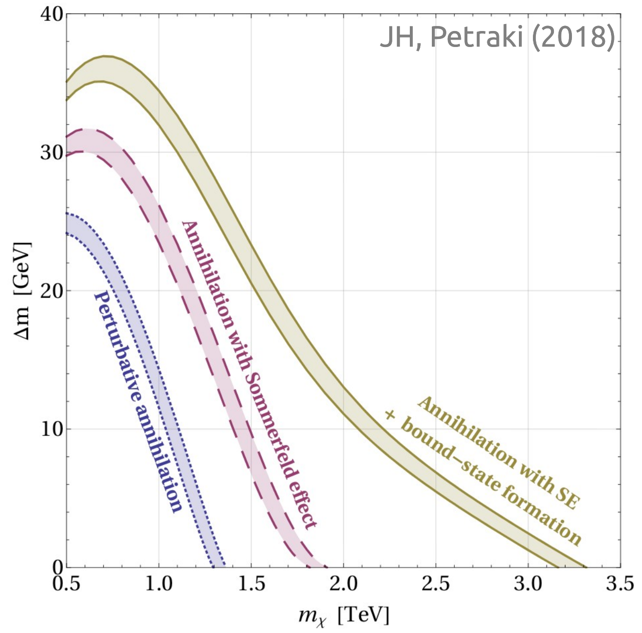
$T < E_\mathrm{B}$

$$\langle \sigma_\mathrm{BSF} v_\mathrm{rel} \rangle_\mathrm{eff} = \langle \sigma_\mathrm{BSF} v_\mathrm{rel} \rangle \times \left(\frac{\Gamma_\mathrm{dec}}{\Gamma_\mathrm{dec} + \Gamma_\mathrm{ion}} \right)$$

What has been done so far?

- Development of **formalism** to describe boundstate formation (and Sommerfeld effect) for dark matter abundance at zero-T incl. non-equilibrium dynamics
  Von Harling, Petraki (2014), Petraki, Postma, Wiechers (2015), Petraki, Postma, de Vries (2016)

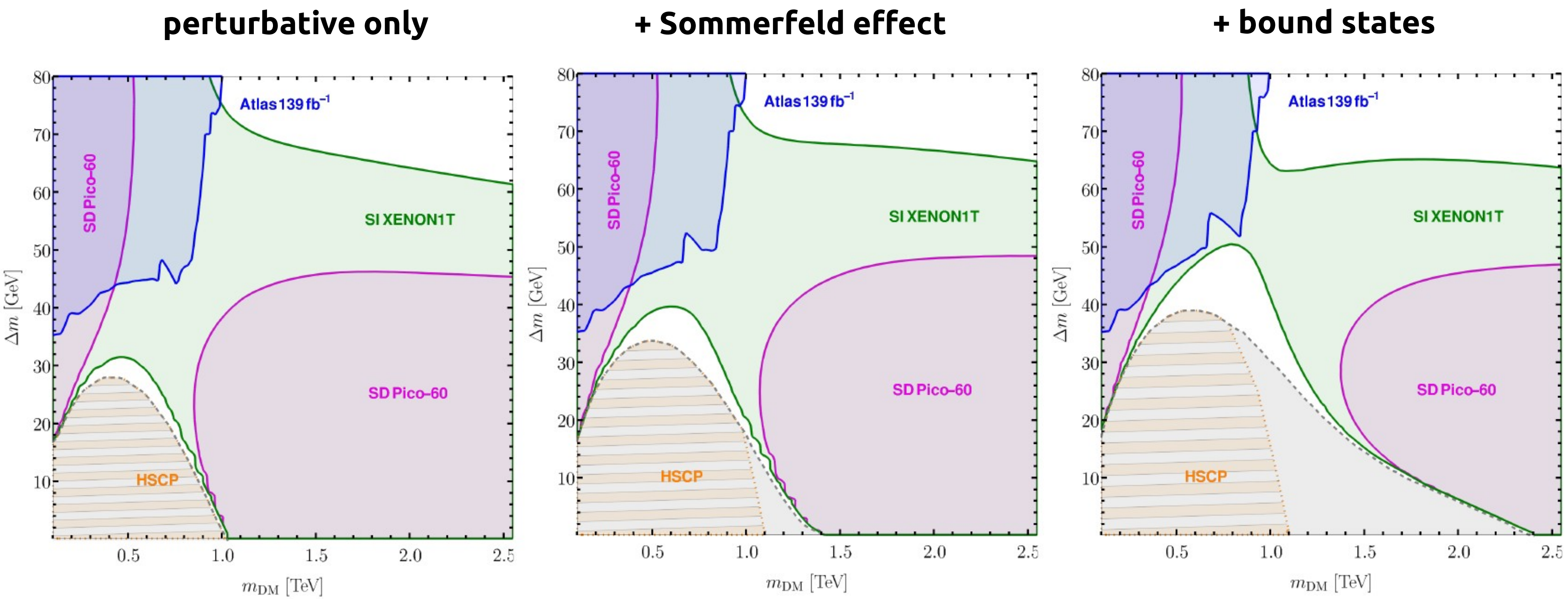
- Demonstrating **phenomenological impact** on EW and strongly coupled WIMP scenarios
  e.g. Asadi, Baumgart, Fitzpatrick, Krupczak, Slatyer (2016), JH, Petraki (2018), JH, Petraki (2018), JH, Petraki (2019)



Common conclusion:
Bound state formation and Sommerfeld effect can significantly affect relic abundance prediction

What is the impact on the interpretation of experimental data?

Impact of SE and BSF on experimental interpretation

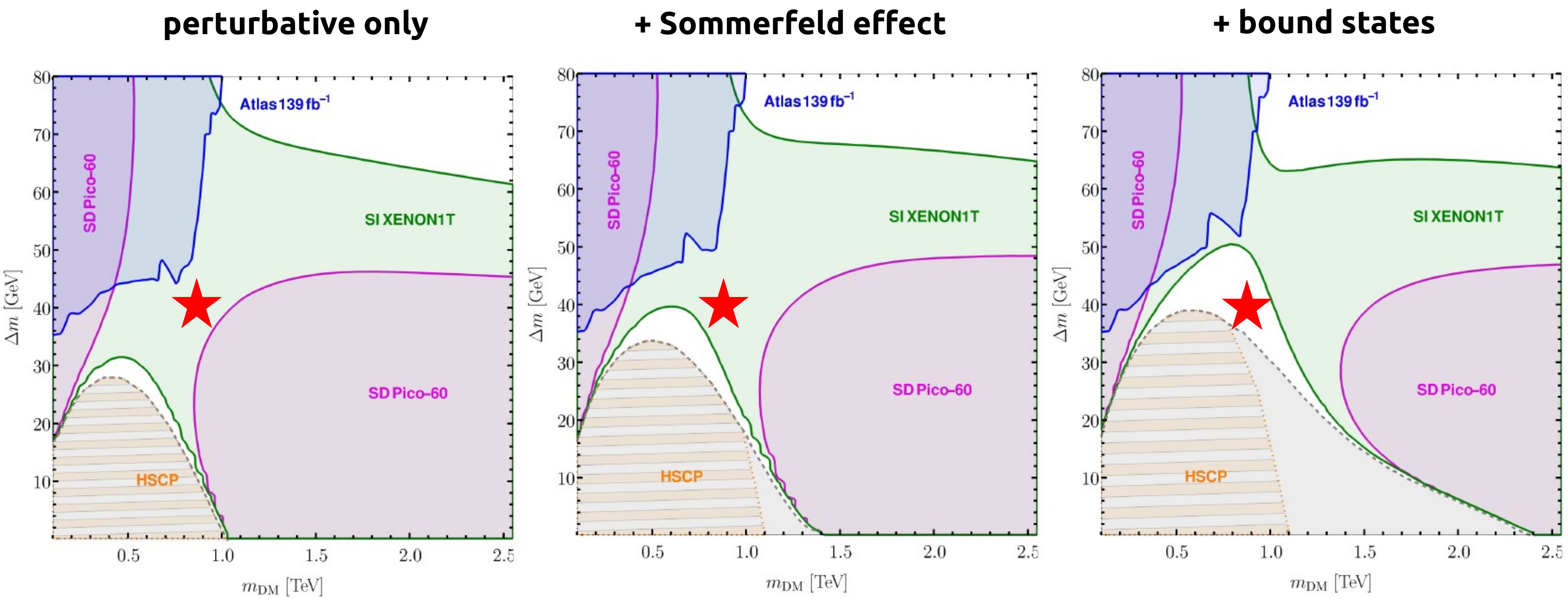


For dedicated study of white / grey area, see
Bollig, Vogl (2021), Decant, Heisig, Hooper, Lopez-Honorez (2021), Garny, Heisig (2021)

$$\mathcal{L} \supset g_{\mathrm{DM},ij} X_i^\dagger \bar{\chi} P_R q_j + h.c.$$

Becker, Copello, JH, Mohan, Sengupta (2022)

Impact of SE and BSF on experimental interpretation



For dedicated study of white / grey area, see
Bollig, Vogl (2021), Decant, Heisig, Hooper, Lopez-Honorez (2021), Garny, Heisig (2021)

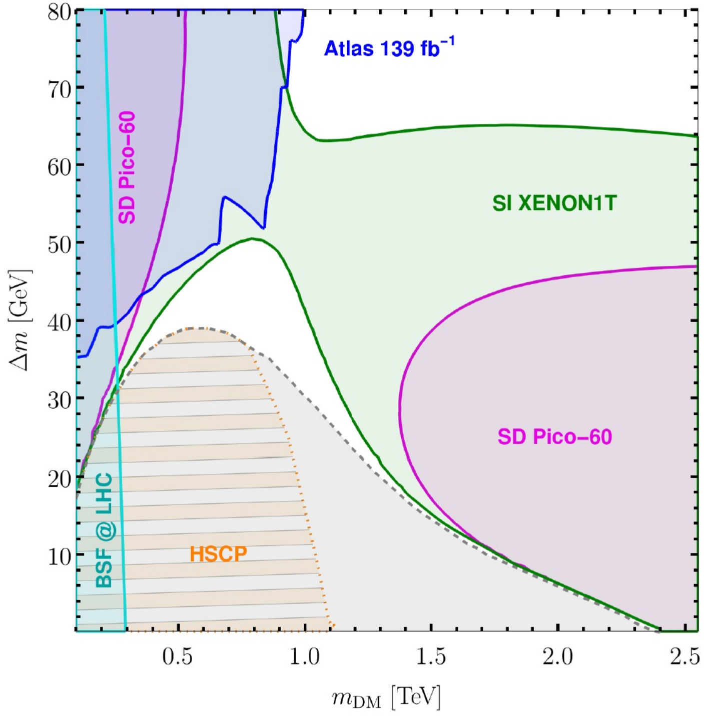
→ previously excluded parameter space is NOT yet excluded!

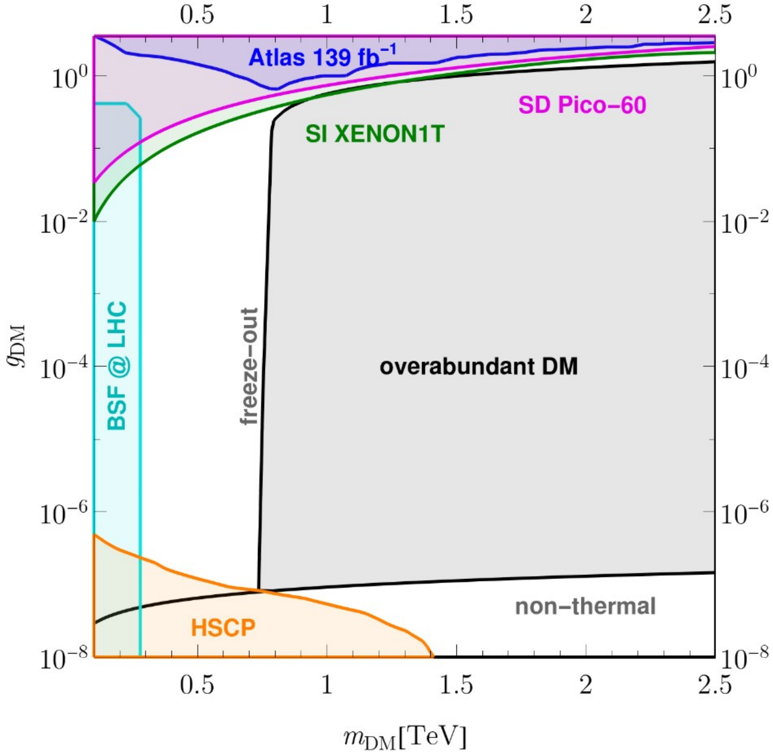
Becker, Copello, JH, Mohan, Sengupta (2022)

Potential of bound state formation at colliders

$$\mathcal{L} \supset g_{\mathrm{DM},ij} X_i^{\dagger} \bar{\chi} P_R q_j + h.c.$$

$$\Delta = 0.05 m_{\mathrm{DM}}$$

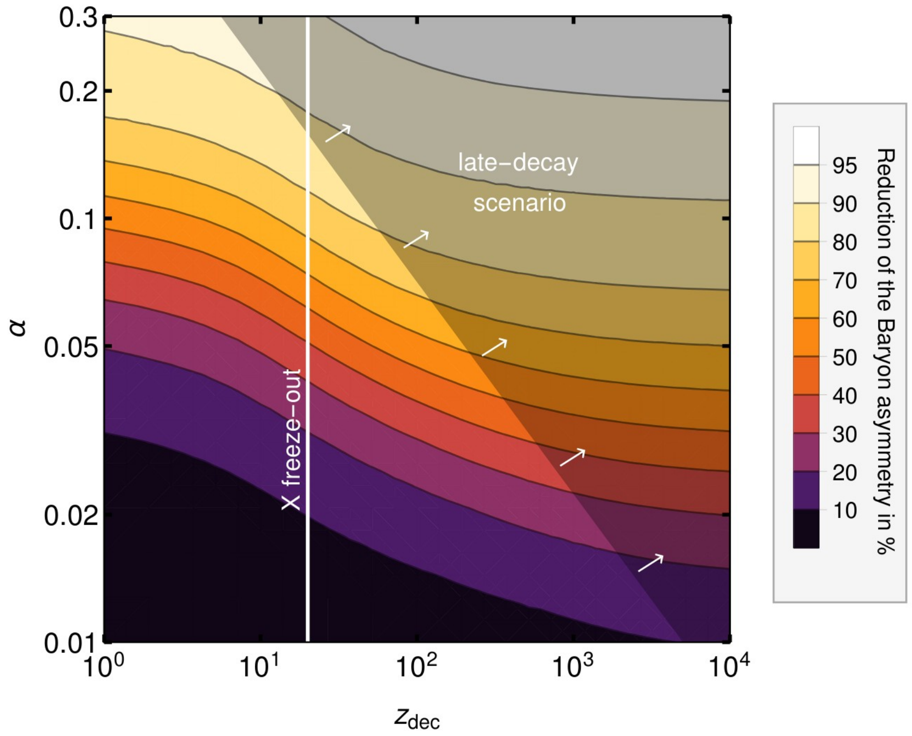




→ **BSF@LHC closes gap between prompt and LLP searches**

Becker, Copello, JH, Mohan, Sengupta (2022)

What is the impact of BSF on baryogenesis?



→ check out our latest paper (2408.08361) !

Becker, Fridell, JH, Hati (2024)

Thermal effects in DM freeze-out

- Thermal effects **negligible** in comparison to dark matter calculations at **NLO**

  Beneke, Dighera, Hryczuk (2014)
  Beneke, Dighera, Hryczuk (2016)

- thermal effects **subdominant** for **Sommerfeld** effect Kim, Laine (2016)

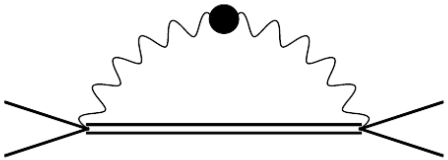
- temperature effects **relevant** for **bound state formation** for large temperature (melting)

- development of **finite-T treatment of bound state formation** while still in ionization equilibrium

  Kim, Laine (2017), Biondini, Laine (2018), Biondini (2018), Covi, Binder, Mukaida (2018)

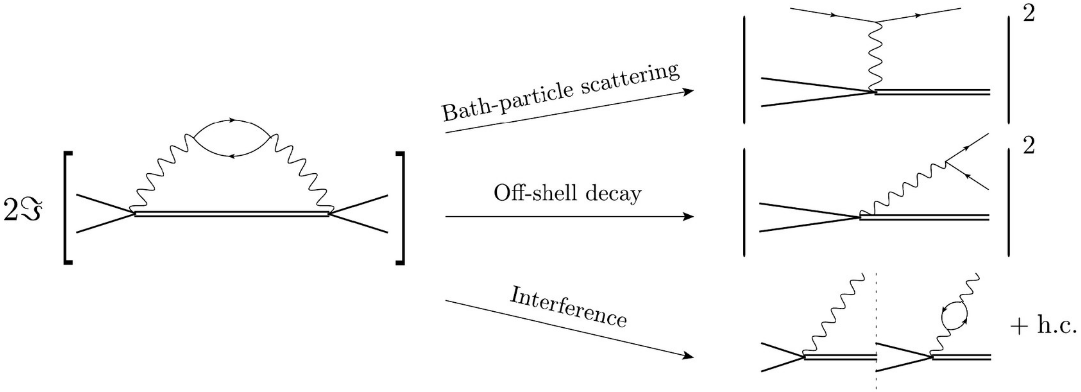
- **phenomenology** of finite-T treatment of bound state formation while still in ionization equilibrium

  Biondini, Vogl (2018), Biondini, Vogl (2019)



Bound state formation at NLO: a non-equilibrium approach

- **Out-of-equilibrium description *from first principles* at NLO (including bath particle scattering and emission) for a *massless* mediator via density matrix formalism**



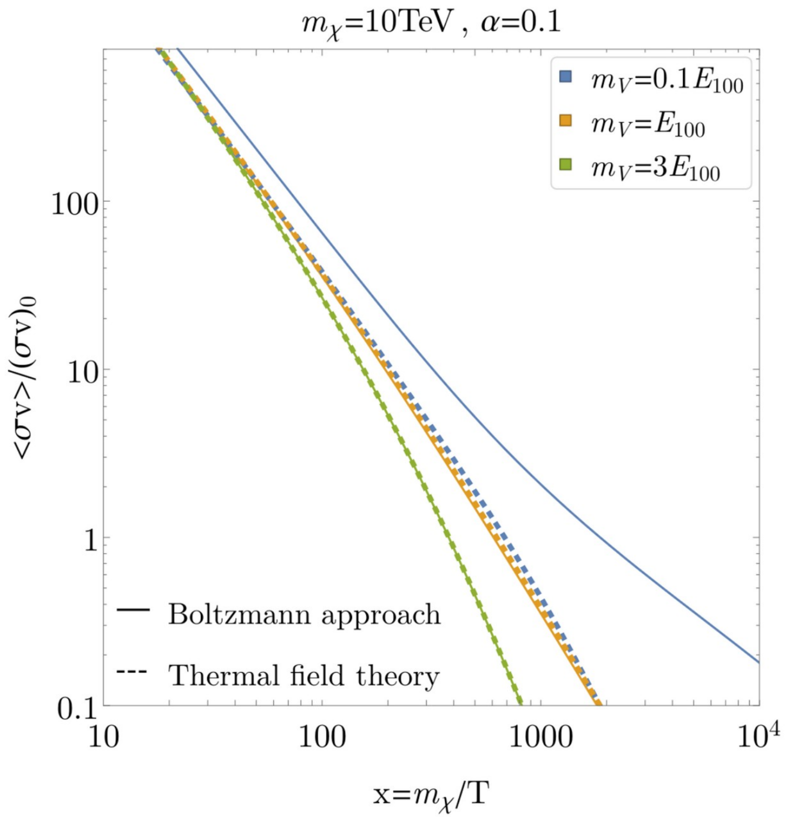
→ first consistent thermal description of bound state formation at NLO beyond ionisation equilibrium

→ proof of consistent cancellation of all IR (and UV) divergencies

Binder, Blobel, JH, Mukaida (2020)

For impact of bath particle scattering with massive mediator, see Binder, Mukaida, Petraki (2019)

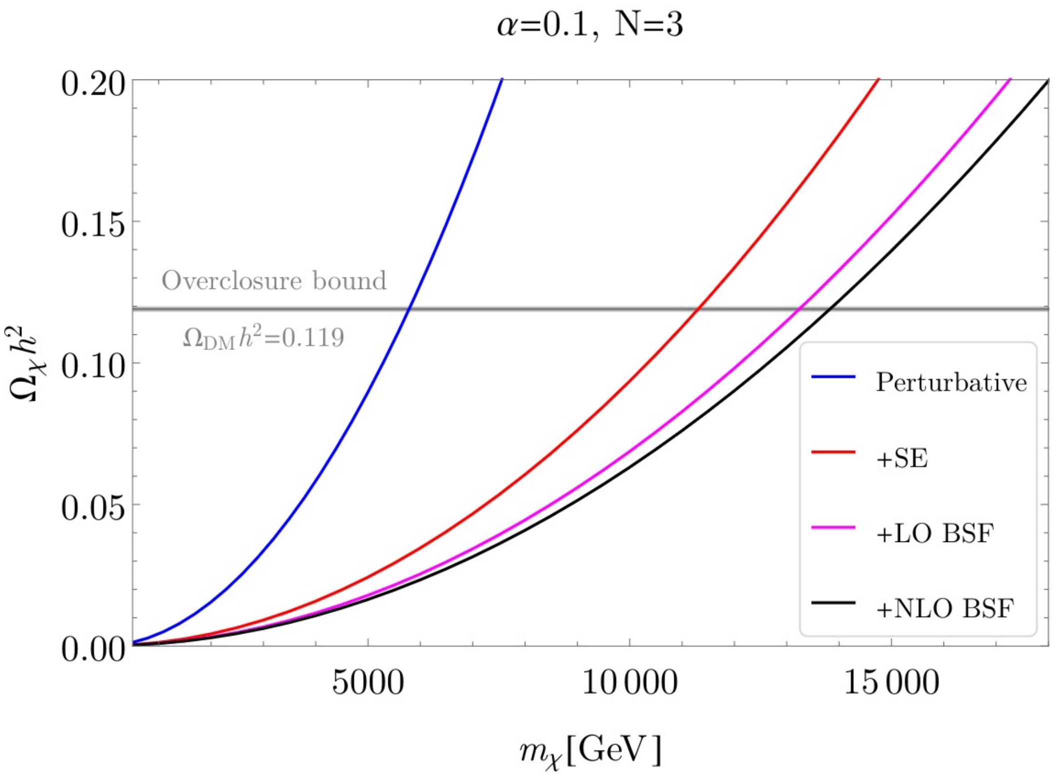
Bound state formation at NLO: a non-equilibrium approach



semi-classical Boltzmann approach with thermal mass regularisation insufficient for $m_V << \Delta E$

Binder, Blobel, JH, Mukaida (2020)

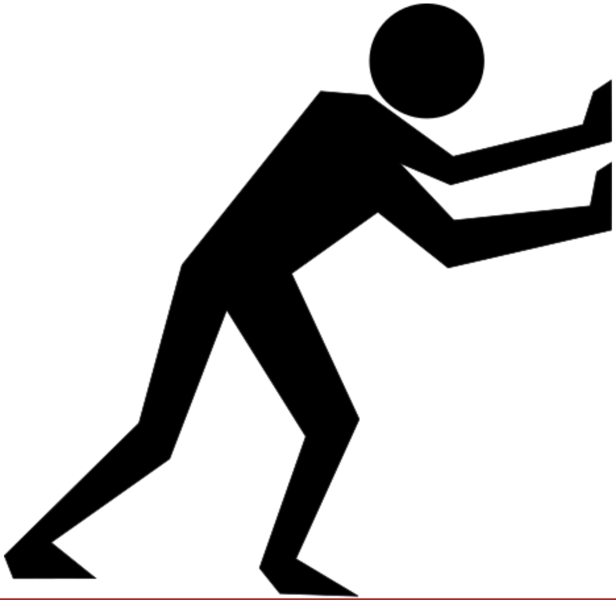
Bound state formation at NLO: a non-equilibrium approach



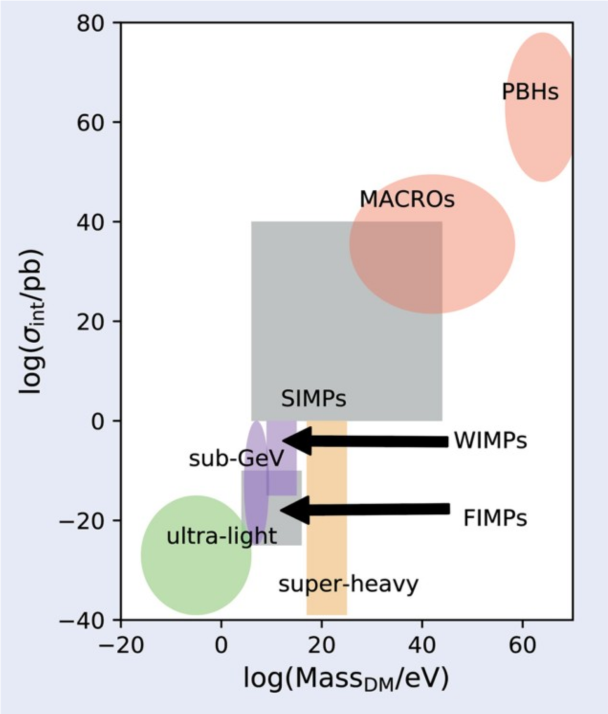
Impact on relic abundance moderate with respect to Sommerfeld effect or LO BSF

Binder, Blobel, JH, Mukaida (2020)

Take home messages for the WIMP



- **Sommerfeld effect and BSF can have significant impact on DM production**
- **Thermal effects subdominant wrt NLO corrections and Sommerfeld effect**
- **Thermal effects for BSF moderately relevant for light mediators**



C. Arina, CERN Courier, 4 March 2021

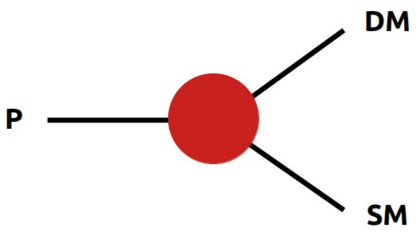
The FIMP – feebly interacting massive particle

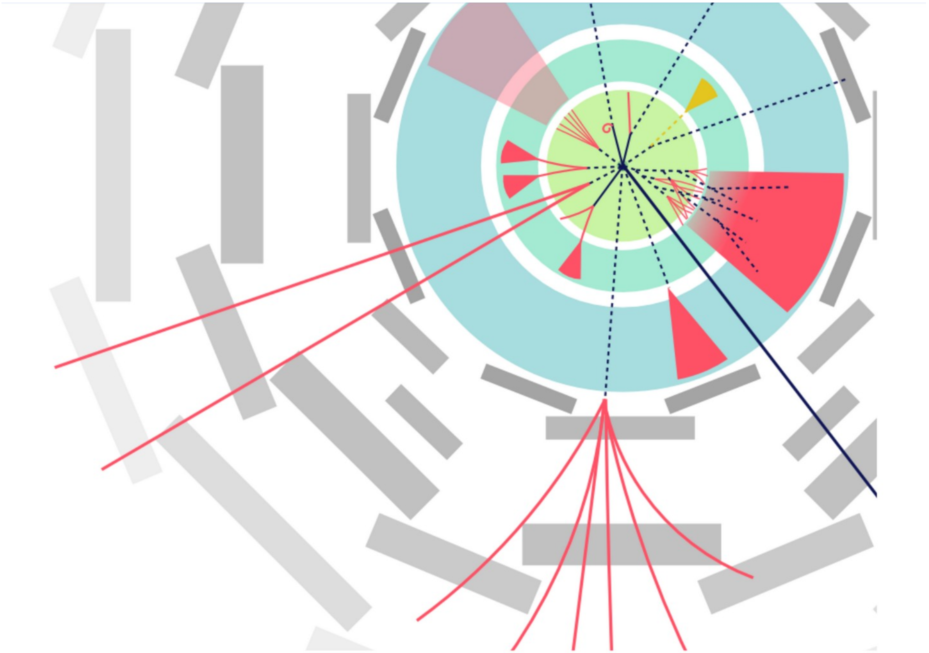
The FIMP

- **Assumption**: FIMPs feature very small interaction rates with SM particles and are hence not in thermal equilibrium with the standard model bath

- FIMPs as **DM** produced via decay of parent particle

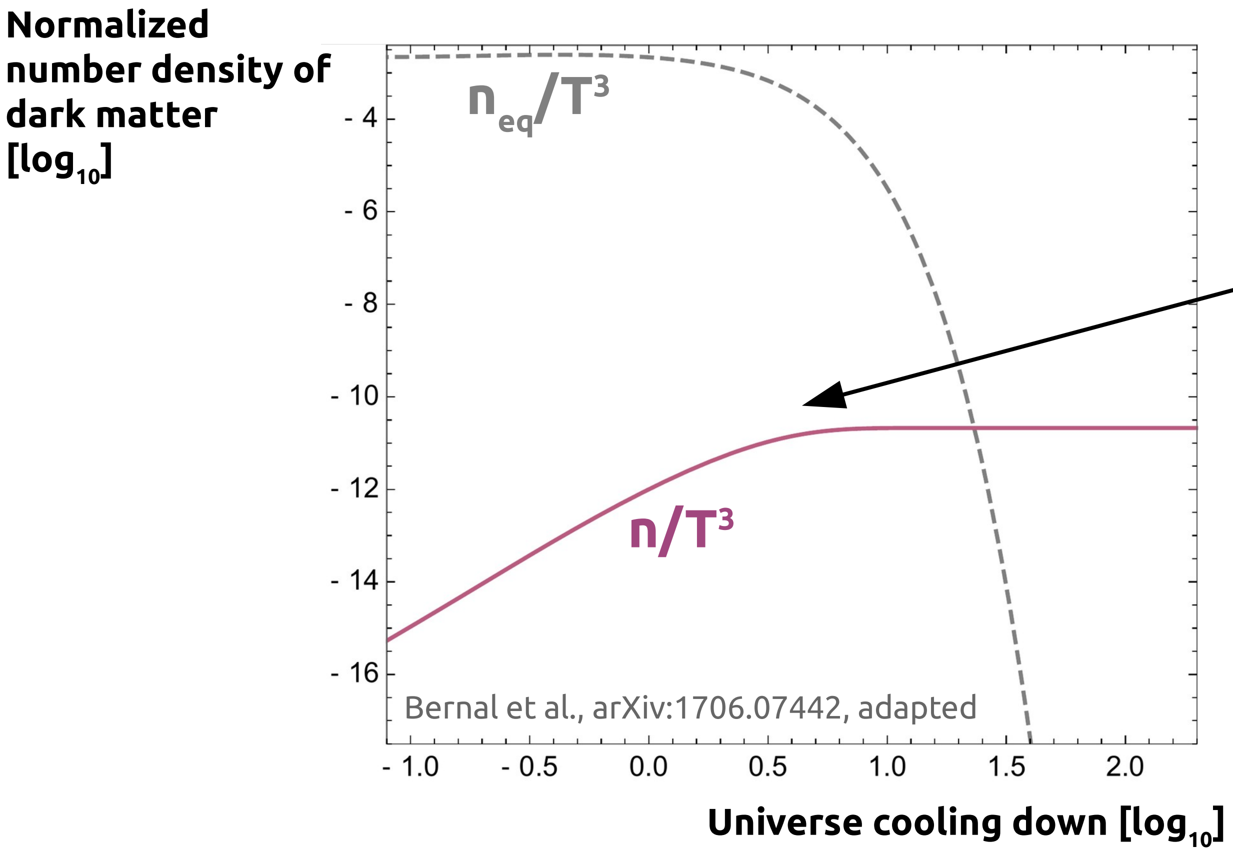
$$y_{\rm DM}\, P\, \chi\, f_{\rm SM}$$




credit: Heather Russell, McGill University, 2017

Feeble interaction leads to interesting long lived particle (LLP) signatures at colliders!

The FIMP – freeze-in of dark matter



Expansion faster than production rate of DM ("Freeze-in")

$$z_{\mathrm{FI}} = \frac{m_F}{T} \approx 5$$

Freeze-in: Linking the early Universe with physics in the lab

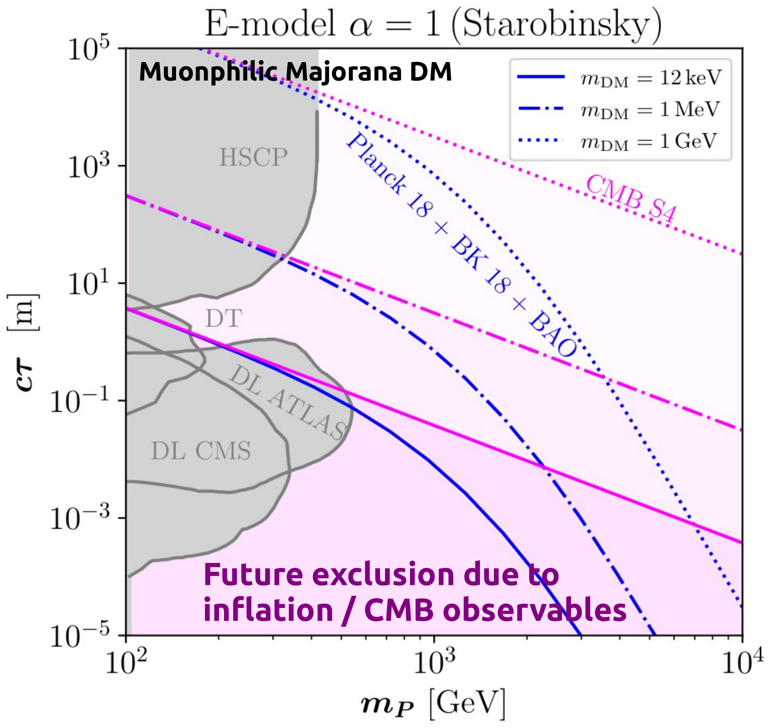
FIMPs / freeze-in feature interesting phenomenology:

- Testable via LLP searches at colliders
- Consistent cosmological history (inflationary model, reheating, DM production via freeze-in) leads to constraints from cosmological data

→ what are **uncertainties** in the prediction?

→ how does the **thermal plasma** affect freeze-in mechanism?

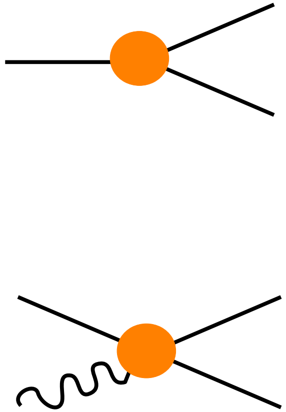
$$z_{\rm FI} = \frac{m_F}{T} \approx 5$$

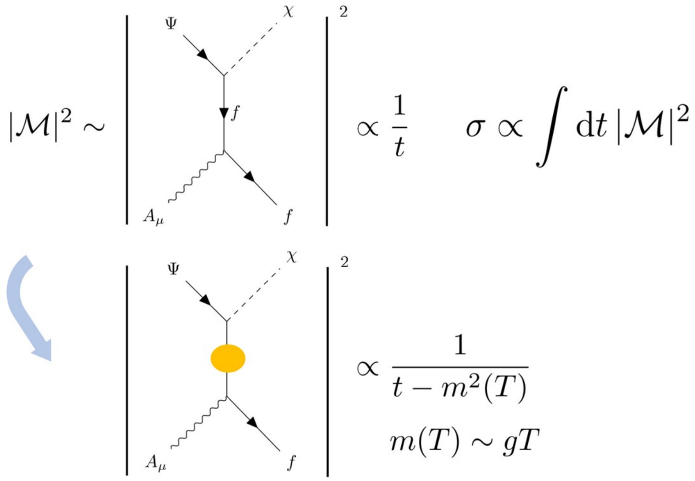


Becker, Copello, JH, Lang, Xu (2023)

Relevance of thermal masses for freeze-in via scattering

- **Scattering with the thermal plasma**





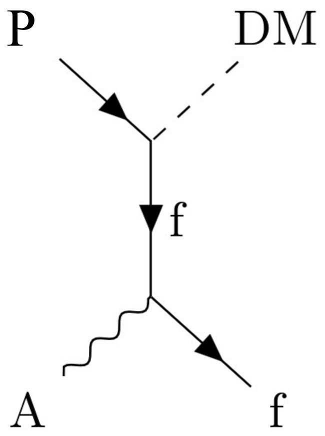
Credits: E. Copello

→ **divergence for massless particle in the t-channel propagator usually regularized with thermal mass**

Thermal corrections to freeze-in in the literature

Different treatments can be found

- Boltzmann approach with decays in vacuum only
- Boltzmann approach with decays only including thermal masses
- Boltzmann approach with decays and scattering including thermal masses
- Non-equilibrium approach with tree-level propagators
- Non-equilibrium approach with HTL approximated propagators



→ **How do different treatments in the literature compare?**

→ **What is phenomenologically the most recommended method?**

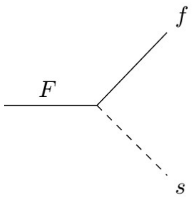
For comparing **thermal masses and quantum statistics** see Bringmann, Heeba, Kahlhoefer, Vangsnes (2021)
For production rate of scalar DM with real time formalism and **HTL approximation**, see Drewes, Kang (2015)
For fermionic DM with imaginary time formalism including **scattering** and **partially resummed propagators for decays** incl. **LPM effect**, see Biondini, Ghiglieri (2020)

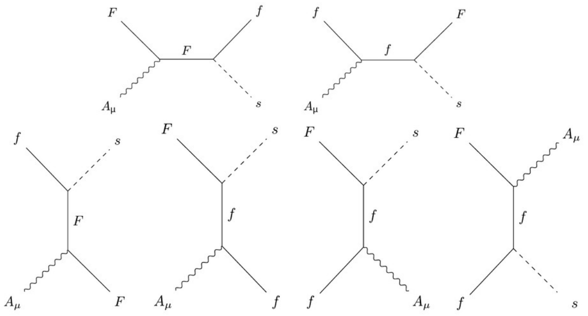
DM freeze-in in the Closed Time Path (CTP) formalism

GOAL: Calculate freeze-in within non-equilibrium framework (closed time path formalism) with 1PI-resummed propagators at LO in the loop expansion of the 2PI effective action and compare with other approaches

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu s)^2 - \frac{1}{2}m_s^2 s^2 - V(s,H) + \bar{F}\left(i\not{D} - m_F\right)F - \left[y_{\rm DM}\bar{F}fs + h.c.\right]$$

Free parameters: $\delta = \frac{m_F - m_{\rm DM}}{m_{\rm DM}}$ $\quad y_{\rm DM}$ $\quad z = \frac{m_F}{T}$ $\quad G = Y^2 g_1^2 + C_2(\mathcal{R}_2)\, g_2^2 + C_2(\mathcal{R}_3)\, g_3^2$





	Y	$SU(2)$	$SU(3)$	G	$\mu = M_Z$	10^4 GeV	10^7 GeV	10^{10} GeV
e_L	$-1/2$	**2**	**1**	$\frac{g_1^2}{4} + \frac{3g_2^2}{4}$	0.38	0.4	0.46	0.52
q_L	$+1/6$	**2**	**3**	$\frac{g_1^2}{36} + \frac{3g_2^2}{4} + \frac{4g_3^2}{3}$	2.3	1.6	1.2	1.0
e_R	-1	**1**	**1**	g_1^2	0.21	0.22	0.24	0.26
u_R	$+2/3$	**1**	**3**	$\frac{4g_1^2}{9} + \frac{4g_3^2}{3}$	2.1	1.3	0.9	0.7
d_R	$-1/3$	**1**	**3**	$\frac{g_1^2}{9} + \frac{4g_3^2}{3}$	2.0	1.2	0.8	0.6

Becker, Copello, JH, Tamarit (2023)

DM freeze-in in the Closed Time Path (CTP) formalism

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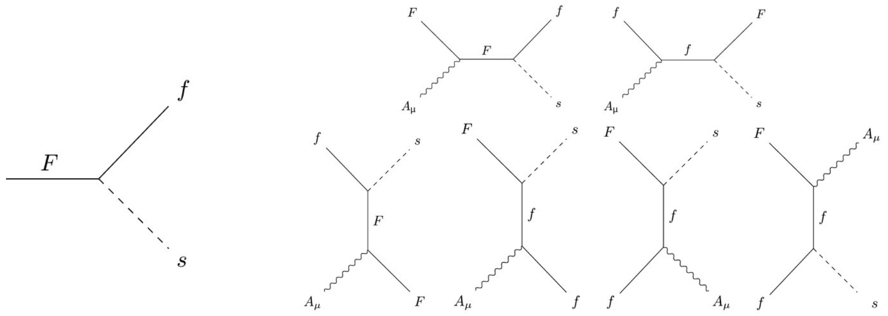
Free parameters: $\delta = \frac{m_F - m_{\text{DM}}}{m_{\text{DM}}}$ y_{DM} $z = \frac{m_F}{T}$ $G = Y^2 g_1^2 + C_2(\mathcal{R}_2)\, g_2^2 + C_2(\mathcal{R}_3)\, g_3^2$



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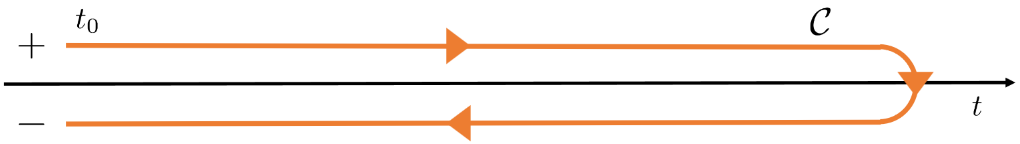
Becker, Copello, JH, Tamarit (2023)

The CTP formalism I

- **S-matrix formalism (in-out) in vacuum for transition amplitudes between well-defined initial and final states**
- **CTP formalism (in-in) for time-dependent expectation values, e.g. the evolution of the statistical ensemble of a primordial plasma with continuously interacting fields**



$$iG^{ab}(x,y) = \langle T_C\, \phi(x^a)\bar{\phi}(y^b)\rangle$$

$$iG^{<} \equiv iG^{+-}$$

$$iG^{>} \equiv iG^{-+}$$

- **Number density**

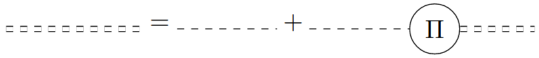
$$n_s = \int \frac{d^3p}{(2\pi)^3} f_s(\vec{p}) = \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{p^0}{\pi} p^0 i\Delta_s^<(p)$$

The CTP formalism II

Derive evolution equation for the scalar self energy based on Schwinger-Dyson equations

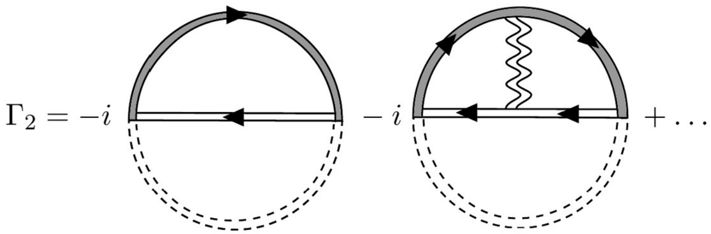
$$\partial_t f_s(t, |\vec{p}|) = \int_0^\infty \frac{p^0}{\pi} \frac{1}{2} \left[\left(i\Pi_s^<\right) \left(i\Delta_s^>\right) - \left(i\Pi_s^>\right) \left(i\Delta_s^<\right) \right]$$

$$i\Delta_0^{ab^{-1}}(x, y) = i\Delta^{ab^{-1}}(x, y) + i\Pi^{ab}(x, y)$$



We perform our calculation at LO in the loop expansion of the 2PI effective action

$$\Pi^{ab}(x, y) = i\, ab\, \frac{\delta \Gamma_2[\Delta, S]}{i\delta \Delta^{ba}(y, x)}$$



and include the fully 1PI-resummed propagators for fermions

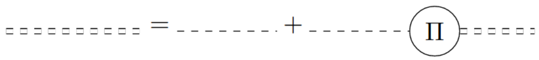
Becker, Copello, JH, Tamarit (2023)

The CTP formalism II

Derive evolution equation for the scalar self energy based on Schwinger-Dyson equations

$$\partial_t f_s(t,|\vec{p}|) = \int_0^\infty \frac{p^0}{\pi}\,\frac{1}{2}\left[\left(i\Pi_s^<\right)\left(i\Delta_s^>\right) - \left(i\Pi_s^>\right)\left(i\Delta_s^<\right)\right]$$

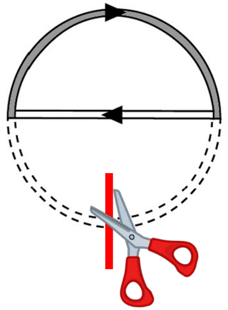
$$i\Delta_0^{ab^{-1}}(x,y) = i\Delta^{ab^{-1}}(x,y) + i\Pi^{ab}(x,y)$$



We perform our calculation at LO in the loop expansion of the 2PI effective action

$$\Pi^{ab}(x,y) = i\,ab\,\frac{\delta\Gamma_2[\Delta,S]}{i\delta\Delta^{ba}(y,x)}$$

$\Gamma_2 = -i$

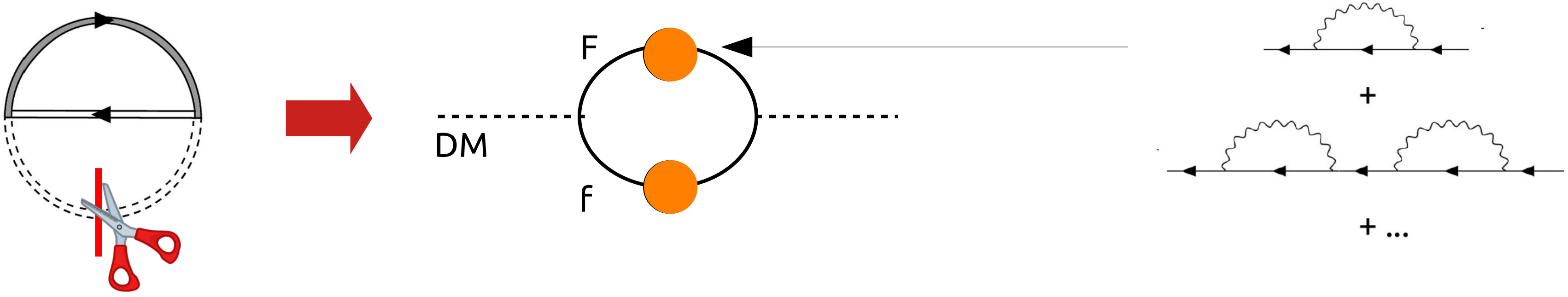


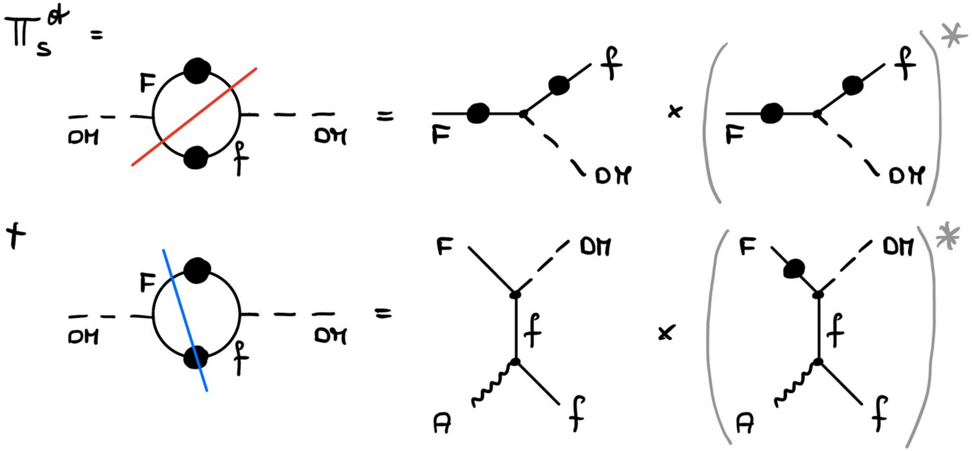
and include the fully 1PI-resummed propagators for fermions

Becker, Copello, JH, Tamarit (2023)

The CTP formalism III

The phenomenological interpretation





+ 3-body decays etc.

Becker, Copello, JH, Tamarit (2023)

The CTP formalism IV

Finally, we obtain an evolution equation for the number density of the scalar particle

$$\dot{n}_s + 3Hn_s = \gamma_{\rm DM} \equiv \frac{1}{2\pi^2}\int d|\vec{p}|\frac{|\vec{p}|^2}{\omega_p}\Pi_s^{\mathcal{A}}(\omega_p, |\vec{p}|)f_-(\omega_p)$$

With the reaction density

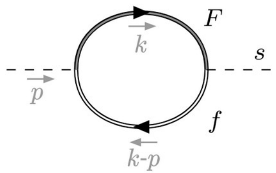
$$\gamma_{\rm DM} = \frac{y_{\rm DM}^2}{4\pi^5}\int d|\vec{p}|\, dk^0\, d|\vec{k}|\, d\cos\theta\, \frac{|\vec{k}|^2|\vec{p}|^2}{\omega_p} tr\left\{P_L \not{S}_F^{\mathcal{A}}(k) P_R \not{S}_f^{\mathcal{A}}(k-p)\right\} f_-(\omega_p)\left[1 - f_+(k^0) - f_+(\omega_p - k^0)\right]$$

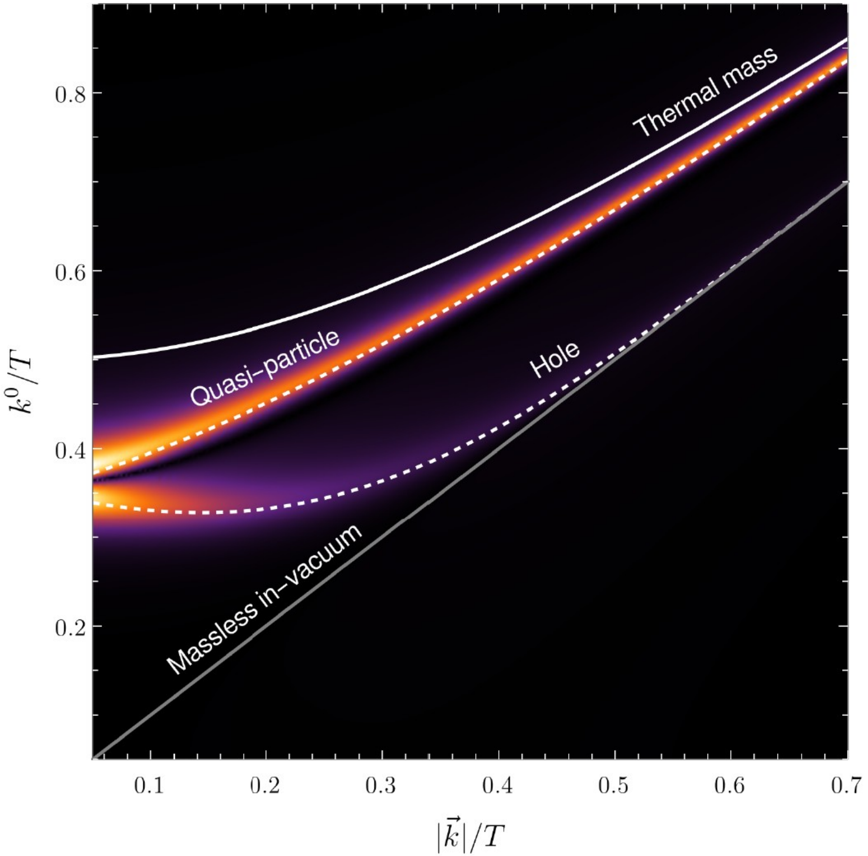
And the spectral propagators

$$\not{S}_F^{\mathcal{A}}(k) = \left(\not{k} - \not{\Sigma}_F^{\mathcal{H}}(k) + m_F\right)\frac{\Gamma_F(k)}{\Omega_F^2(k) + \Gamma_F^2(k)} - \not{\Sigma}_F^{\mathcal{A}}(k)\frac{\Omega_F(k)}{\Omega_F^2(k) + \Gamma_F^2(k)}$$

Becker, Copello, JH, Tamarit (2023)

Comparing methods – spectral propagator





- **In vacuum** $\not{S}^{\mathcal{A}} \sim \delta(k^2 - m_0^2)$
- **With thermal masses** $\not{S}^{\mathcal{A}} \sim \delta(k^2 - m_{\text{th}}^2)$
- **1PI resummed**

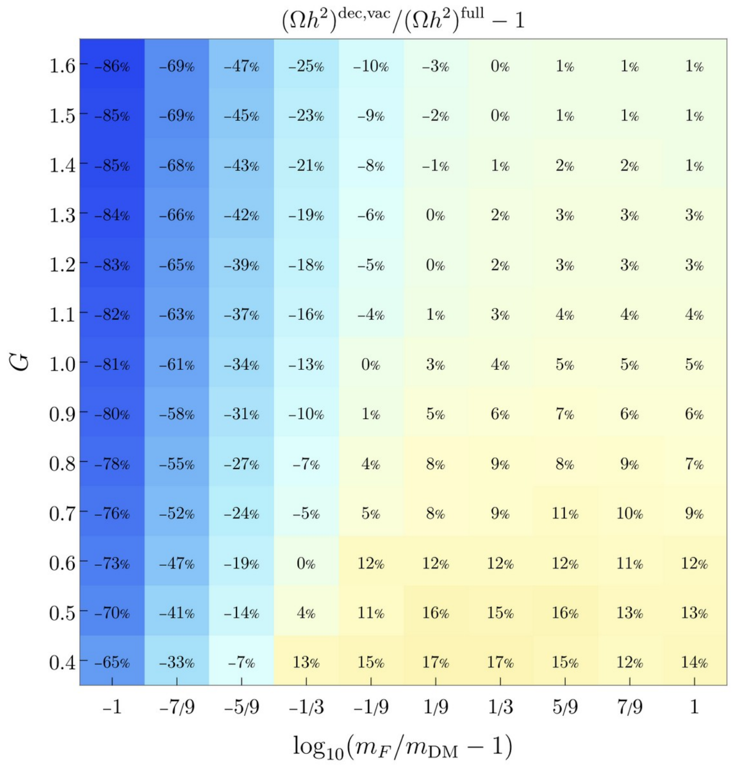
$$\not{S}_F^{\mathcal{A}}(k) = \left(\not{k} - \not{\Sigma}_F^{\mathcal{H}}(k) + m_F\right) \frac{\Gamma_F(k)}{\Omega_F^2(k) + \Gamma_F^2(k)} - \not{\Sigma}_F^{\mathcal{A}}(k) \frac{\Omega_F(k)}{\Omega_F^2(k) + \Gamma_F^2(k)}$$

→ **thermal width broadens the spectrum**

Becker, Copello, JH, Tamarit (2023)

Comparing methods – relic abundance

Compared to vacuum decay only



Decays start to dominate for large z

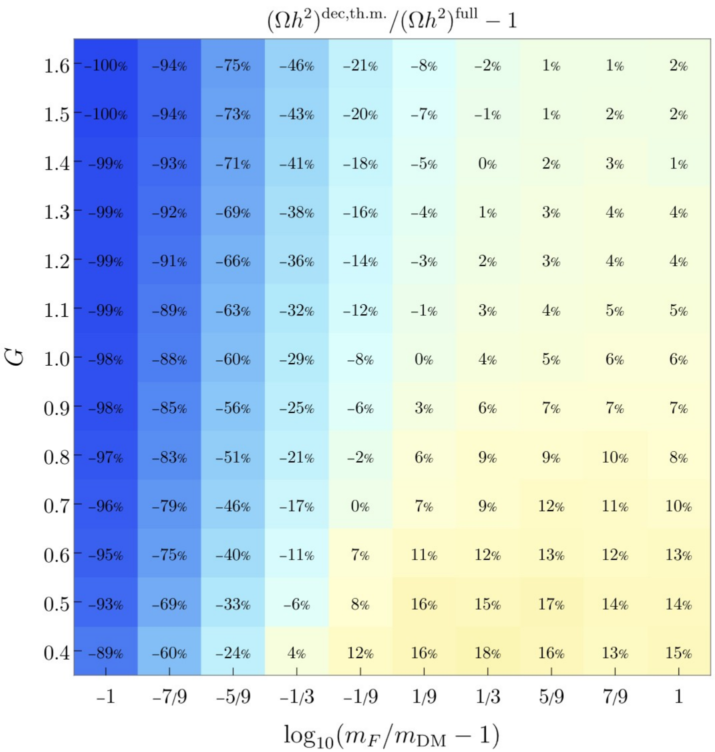
$$z > \sqrt{G}\,\frac{1-\delta}{2\delta-\delta^2}$$

- Ω_{DM} strongly **underestimated** for small mass splittings (missing scatterings)
- Ω_{DM} partially **overestimated** for larger mass splittings (quantum statistics)

Becker, Copello, JH, Tamarit (2023)

Comparing methods – relic abundance

Compared to decays with thermal masses



Decays start to dominate for large z

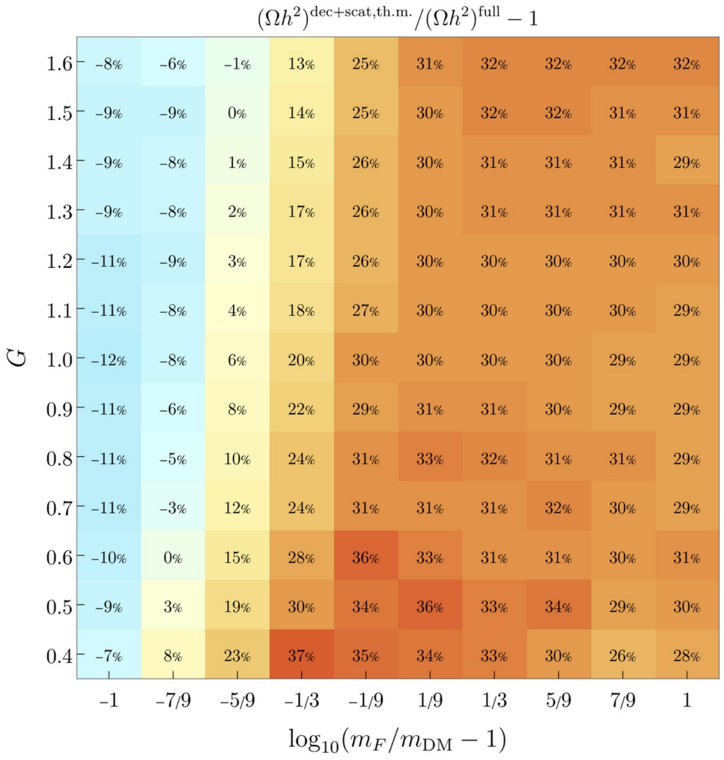
$$z > \sqrt{G}\frac{1-\delta}{2\delta - \delta^2}$$

- Ω_{DM} strongly **underestimated** for small mass splittings (missing scatterings)
- Ω_{DM} partially **overestimated** for larger mass splittings (quantum statistics)
- **Thermal masses increase the deviation**

Becker, Copello, JH, Tamarit (2023)

Comparing methods – relic abundance

Compared to decays and scattering with thermal masses

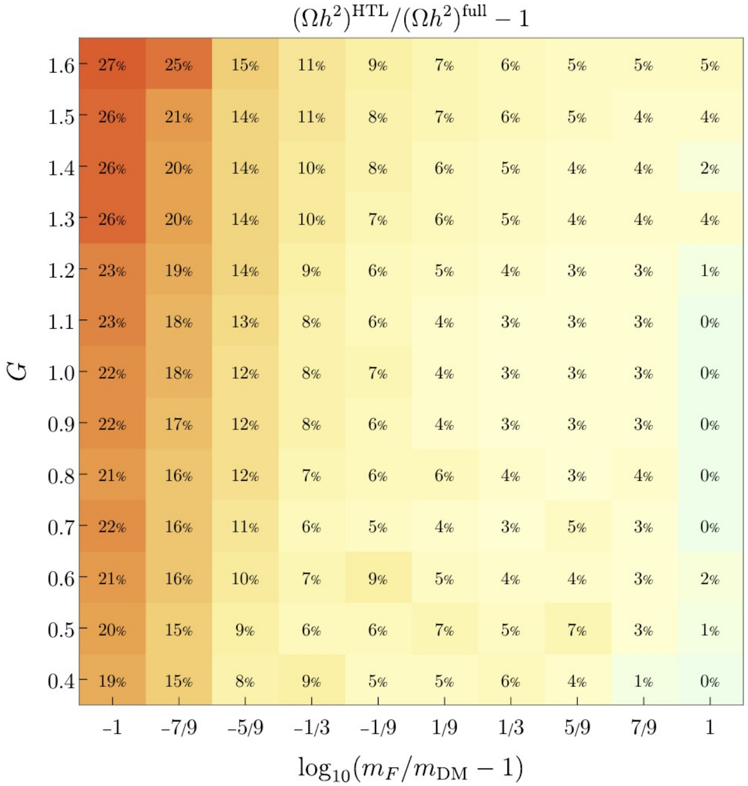


- Ω_{DM} still slightly **underestimated** for small mass splittings
- Ω_{DM} strongly **overestimated** for large mass splittings (e.g. larger masses, …)
- **When including Fermi-Dirac / Bose-Einstein statistics in semi-classical BEQ, deviation reduced by approx. 50%**

Becker, Copello, JH, Tamarit (2023)

Comparing methods – relic abundance

Compared to HTL approximation

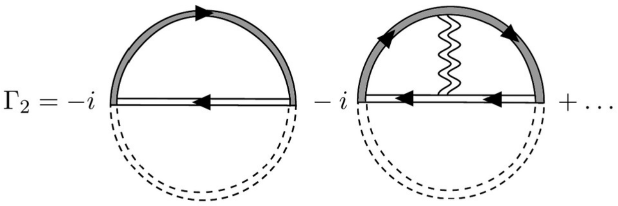


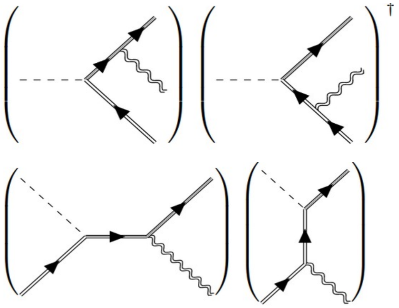
- Ω_{DM} **strongly overestimated** for small mass splittings (HTL overestimates scattering)
- Ω_{DM} **slightly overestimated** for large mass splittings (vanishing thermal width)
- Larger deviations for larger G
- **significant corrections on Ω_{DM} dependent on mass splitting and gauge coupling G**

Becker, Copello, JH, Tamarit (2023)

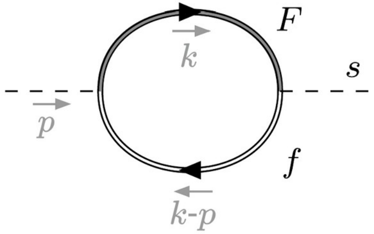
Going beyond LO in expansion of 2PI effective action

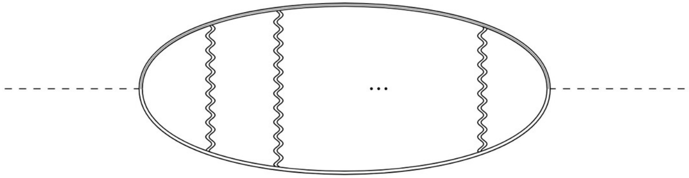
- **How important can NLO contributions to 2PI effective action be?**





→ **Power counting** for soft gauge bosons for scalar self energy (for method see e.g. Arnold, Moore, Yaffe, 2001)





Hard collinear fermion momenta:	g^2T^2	g^2T^2	**LPM effect → resummation**
Hard non-collinear fermion momenta:	g^2T^2	$g^{4n+2}T^2$	

JH, Fernandez Lozano, in preparation (2024)

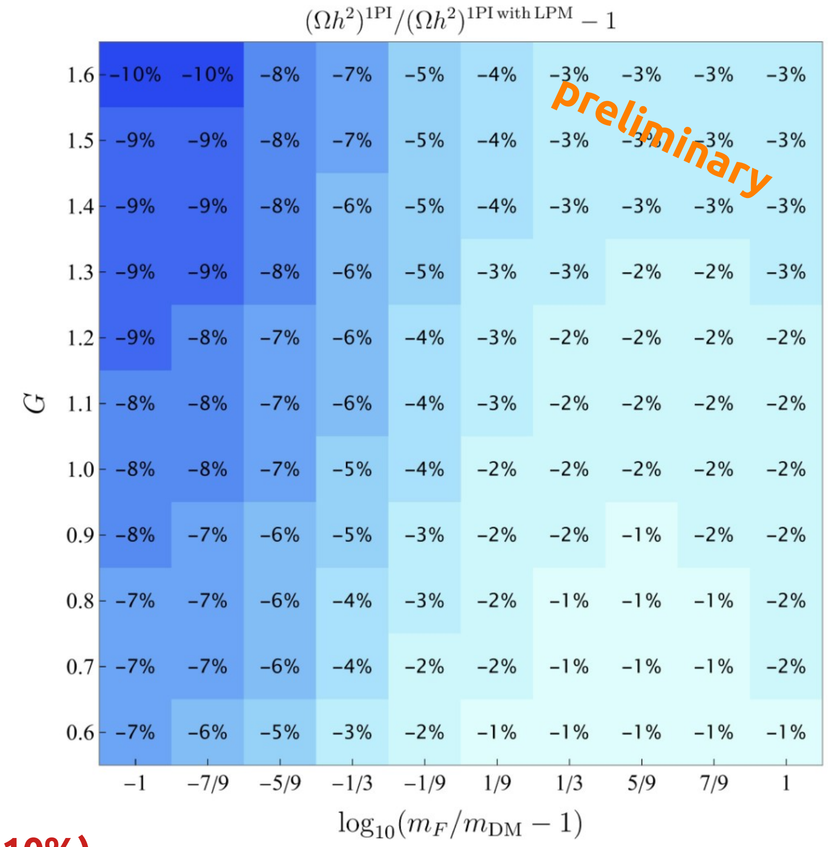
Going beyond LO in expansion of 2PI effective action

- **LPM resummation expected to be relevant also for scalar self energy**

  Previously studied only for fermion self energies in the context of
  → leptogenesis (e.g. Besak, Bödeker)
  → Fermionic FIMPs (Biondini, Ghiglieri)

- **Use of recursion relation and integrating out of soft gauge bosons** (Besak 2010)

$$\gamma_{\rm DM} = \left(\gamma_{\rm DM}^{\rm LPM} - \gamma_{\rm DM}^{\rm LPM\,Born}\right)\kappa(m_F) + \gamma_{\rm DM}^{\rm 1PI\text{-}resummed}$$



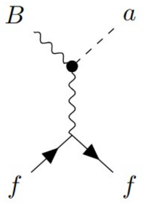
→ **moderate corrections to fully 1PI-resummed result (max. 10%)**

JH, Fernandez Lozano, in preparation (2024)

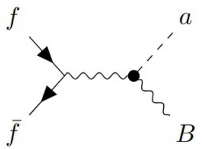
Another application: Ly-α limits on freeze-in of ALP DM

- **Photophilic ALP DM freezes-in via scattering processes**

$f\,V \to f\,a$



$f\,\bar{f} \to V\,a$



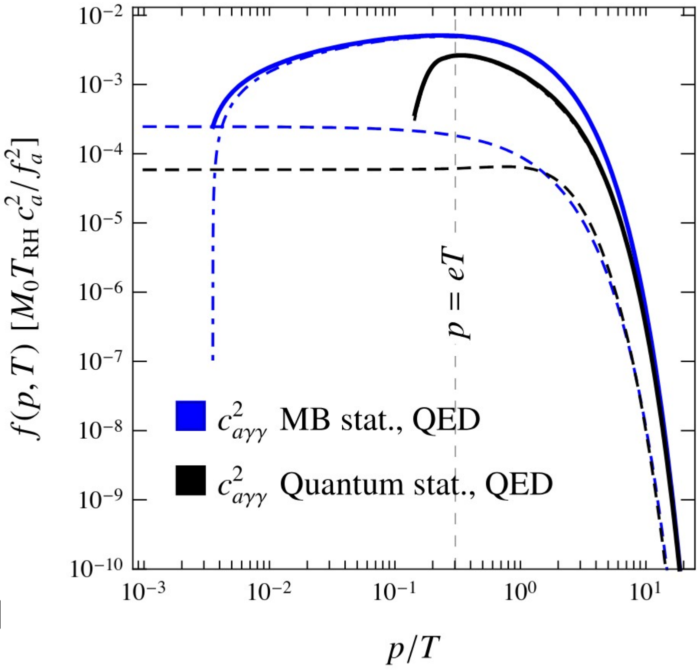
- **Limit from Lyman-α on free-streaming length as warm DM**

$$\lambda_{\rm fs} \propto \frac{\langle p \rangle / T}{m_a} \qquad \langle p \rangle = \frac{\int dp\, p^3 f_a(p)}{\int dp\, p^2 f_a(p)} \qquad \langle p \rangle_{\rm cut} = 3.24$$

- **T-channel divergence in f V → f a diagram**

Introduce cut-of scale k_* below which HTL resummed propagator is used

$gT \ll k_\star \ll T$, only valid for $p_a > gT$ Bolz, Brandenburg, Buchmuller (2001)

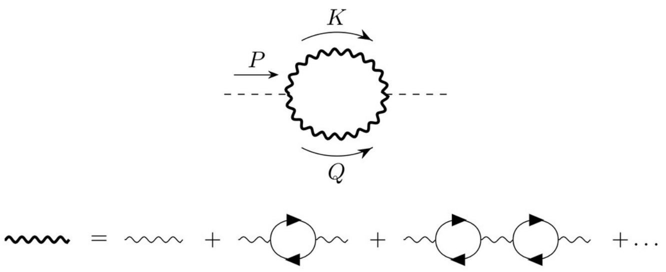


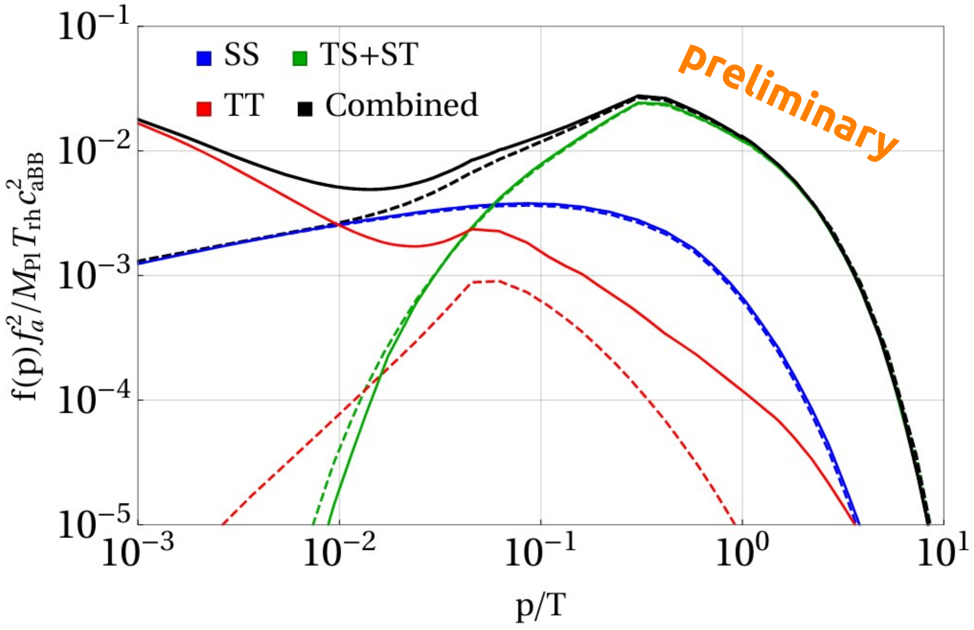
→ Momentum distribution functions turn negative for soft axion momenta!

Baumholzer, Brdar, Morgante (2021)

Another application: Ly-α limits on freeze-in of ALP DM

- **Calculate distribution functions using CTP including fully 1PI-resummed propagators**





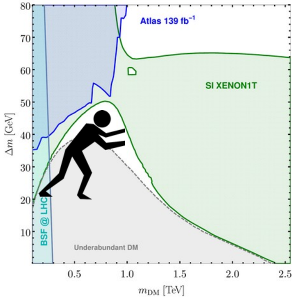
- **HTL solves already negative rates**
- **1PI-resummed propagators show previously neglected TT contribution for low momenta**
- **Impacts directly Lyman-α constraints**

$$\lambda_{fs} \propto \frac{\langle p \rangle / T}{m_a} \qquad \langle p \rangle_{cut} = 3.24 \qquad \langle p \rangle_{HTL} = 3.19 \qquad \langle p \rangle_{Full} = 3.08$$

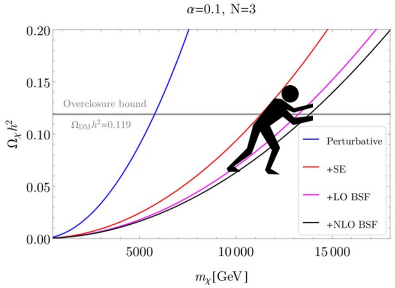
Becker, JH, Morgante, Puchades-Ibanez, Schwaller (in preparation)

For different approach for non-abelian theories, see Bouzoud and Ghiglieri (2024)

Conclusions

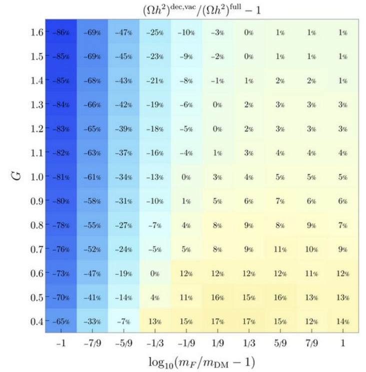


- Dark matter still one of the biggest puzzles of modern (astro)particle physics
- Cutting-edge methods needed for accurate theory predictions and correct experimental interpretation



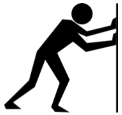
WIMP freeze-out

- Sommerfeld effect and BSF can have significant impact on DM production
- Thermal effects subdominant wrt NLO corrections and Sommerfeld effect
- Thermal effects for BSF moderately relevant for light mediators



FIMP freeze-in

- Thermal effects can have sizeable effect depending on gauge interactions with the thermal plasma and mass difference of DM with parent particle



Let's push the limits in the search of dark matter!

Thank you for your attention!

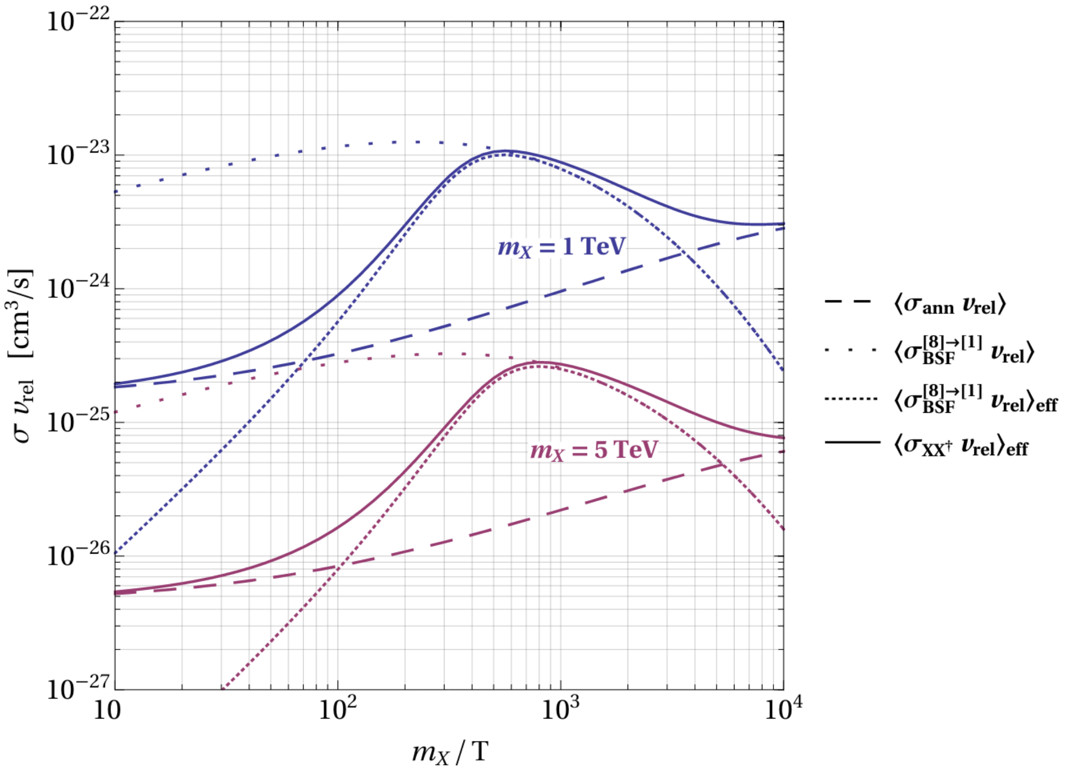
NASA, ESA, and J. Lotz, M. Mountain, A. Koekemoer, and the HFF Team (STScI)

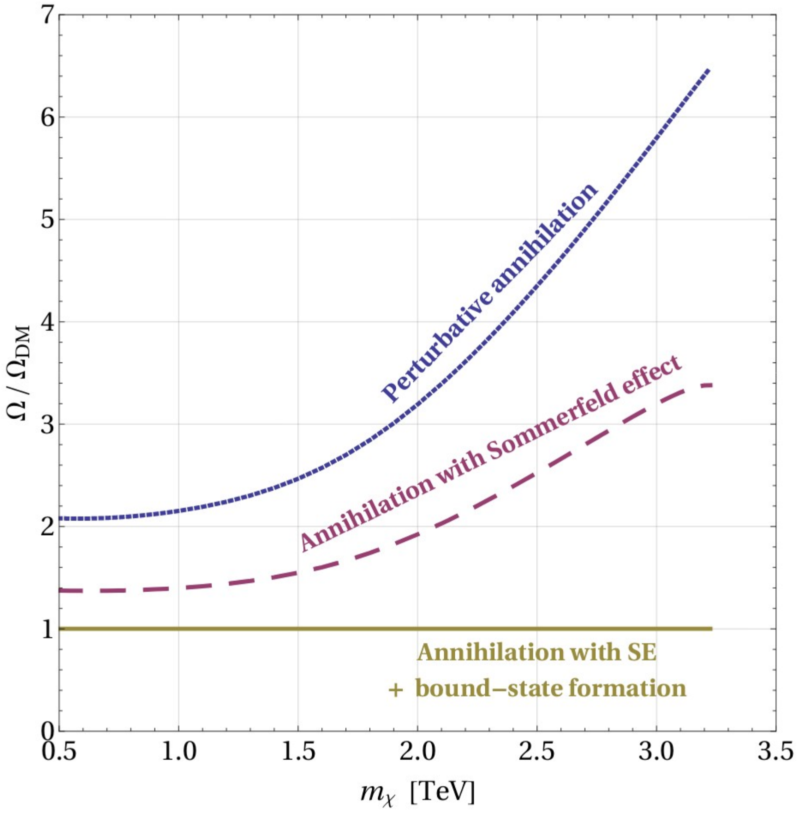
NLO corrections to the Relic Abundance

Full NLO calculation historically mainly done for the MSSM

- **SloopS – NLO EW for MSSM** Boudjema et al. (2006), Baro et al. (2008, 2010), Boudjema (2011), Chatterjee et al. (2012), Boudjema et al. (2014)
- **DM@NLO – NLO QCD for MSSM** Herrmann et al. (2009), JH et al. (2013, Herrmann et al. (2014), JH et al. (2015, 2016, 2019, 2023)
- **Sommerfeld effect in MSSM** Drees et al. (2013), Beneke et al. (2013, 2015), JH et al. (2015), Beneke et al. (2016), Schmiemann et al. (2019), Branahl et al. (2019)
- **Sommerfeld effect in other models** Chowdhury Nasri (2017), Baldes, Petraki (2017), El Hedri, Kaminska, Vries (2017), JH, Petraki (2018), Biondini (2018), and by now by many, many more...

Impact of bound state formation on the relic abundance



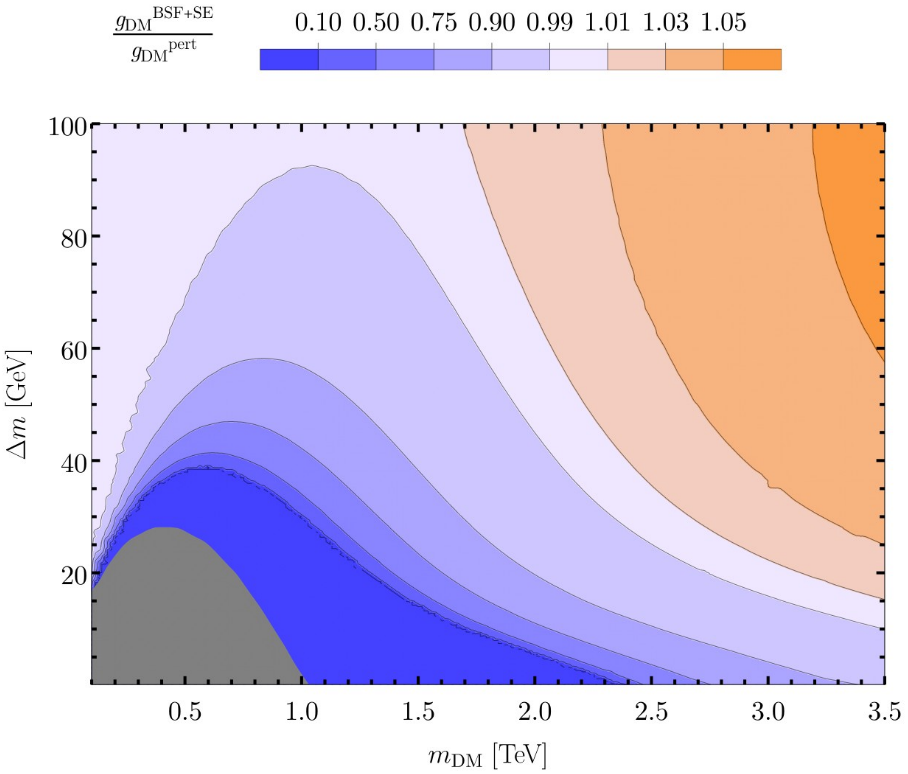


→ **How correct without thermal effects and bath particle scattering?**

JH, Petraki (2018)

Impact on minimal dark matter coupling strength

Identify lower bound on g_{DM} in order not to overproduce DM



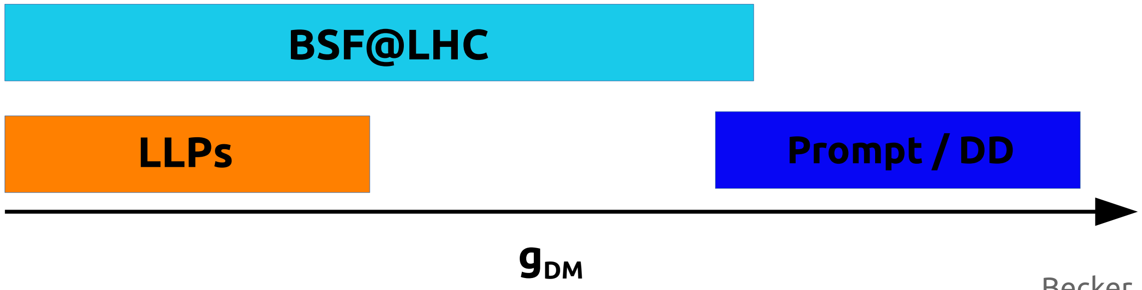
- **Non-perturbative effects result in corrections on minimal g_{DM}**
- **Depending on parameter space: positive or negative correction**

Becker, Copello, JH, Mohan, Sengupta (2022)

Potential of bound state formation at colliders

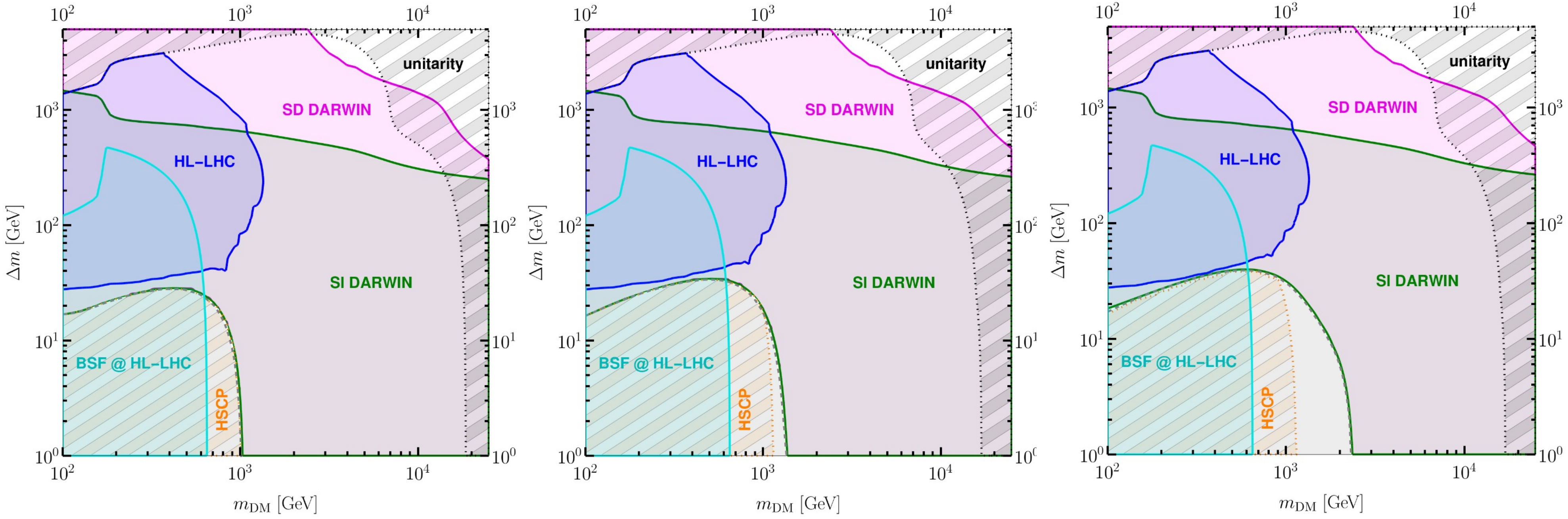
$$\sigma(pp \to \mathcal{B}(XX^\dagger)) = \frac{\pi^2}{8m_\mathcal{B}^3}\mathcal{P}_{gg}\left(\frac{m_\mathcal{B}}{13\ \mathrm{TeV}}\right)\Gamma(\mathcal{B}(XX^\dagger) \to gg)$$

- Resonant production of bound state and subsequent decay (e.g. into photons)
- Dedicated searches, see e.g. *ATLAS coll. Phys. Lett. B 775 (2017) 105*
- Efficient for large range of g_{DM}, as long as $\Gamma_X < E_B$ ($g_{DM} < g_s$, when bound states are efficiently produced)



Becker, Copello, JH, Mohan, Sengupta (2022)

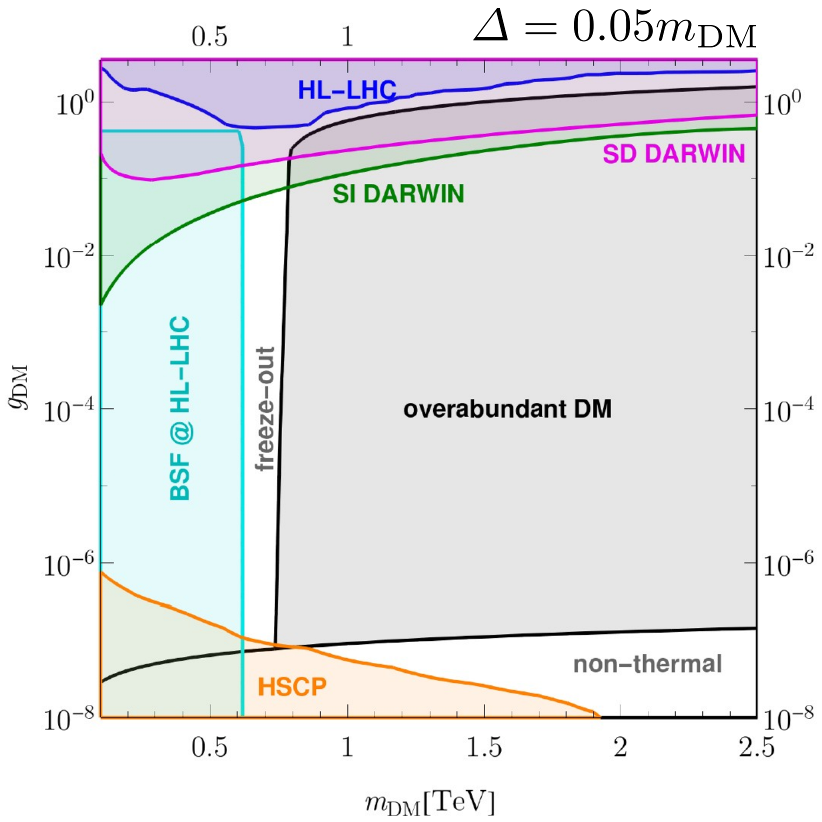
Future prospects



- **HSPC not strict exclusion limit (BSF@LHC is!)**
- **Highly testable: parameter space can be almost entirely probed**
- **BSF effects enlarge parameter range that still needs to be tested**

Becker, Copello, JH, Mohan, Sengupta (2022)

Future prospects for bound states at colliders



BSF@LHC has potential to unambiguously close parameter space for small DM masses and mass splittings

Becker, Copello, JH, Mohan, Sengupta (2022)

Impact of thermal corrections to dark matter annihilation

- Cancellation of soft and collinear divergences at individual CTP self energy diagram
- Helicity suppression for Majorana fermion s-wave annihilation not lifted by thermal corrections
- Leading correction of order $\mathcal{O}\left(\frac{T^4}{m_\chi^4}\right)$ with T << m_χ around freeze-out

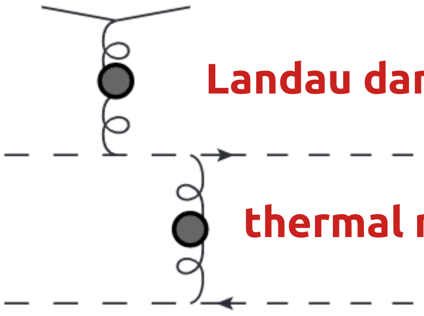
  smaller than zero-temperature NLO corrections → can therefore be neglected
- For co-annihilation expected at $\mathcal{O}\left(\frac{T^2}{m_\chi^2}\right)$

→ **Zero-temperature calculation sufficient for dark matter (co)-annihilation**

Beneke, Dighera, Hryczuk (2014)
Beneke, Dighera, Hryczuk (2016)

Sommerfeld effect in the thermal plasma

Different thermal effects



- Thermal width
  - $\Gamma \sim \alpha^2 T^2/M$ scattering states
  - $\Gamma \sim T^3/M^2$ bound states
- Thermal masses
  - $m_{\rm th} \sim \alpha^{1/2} T$ Debye mass
  - $M_{\rm th} \sim -\alpha^{3/2} T$ Debye screening / Salpeter correction

Sommerfeld effect with $v \sim (T/M)^{1/2}$ and $v \sim \alpha$ follows $T \sim \alpha^2 M$

momenta large compared to Debye mass $Mv \sim (MT)^{1/2} \gg m_{\rm th} \sim \alpha^{1/2} T$

Kin. energy large compared to thermal width $T \gg \Gamma \sim \alpha^2 T^2/M$

→ **thermal effects subdominant for Sommerfeld effect**

Kim, Laine (2016)

Bound state formation in the thermal plasma

Different thermal effects

- Thermal width

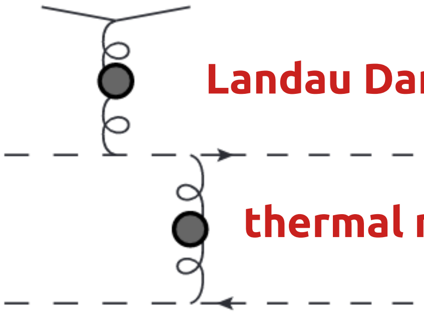
$\Gamma \sim \alpha^2 T^2/M$ scattering states

$\Gamma \sim T^3/M^2$ bound states

- Thermal masses

$m_{\rm th} \sim \alpha^{1/2} T$ Debye mass

$M_{\rm th} \sim -\alpha^{3/2} T$ Salpeter correction



Bound states melt when

$$\Gamma \sim T^3/M^2 \geq \Delta E \sim \alpha^2 M$$

$$T \geq \alpha M$$

→ **BUT: bound states relevant when out of ionisation equlibrium and low(er) T**

Kim, Laine (2016)

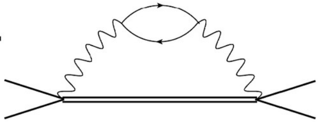
Bound state formation at NLO: a non-equilibrium approach

- **Out-of-equilibrium description *from first principles* at NLO (including bath particle scattering and emission) for a *massless* mediator via density matrix formalism**

$$\dot{n}_\chi + 3Hn_\chi = -\sum_{\mathcal{B}} \langle \sigma_{\mathcal{B}}^{\mathrm{bsf}} v_{\mathrm{rel}} \rangle \left[n_\chi n_{\bar{\chi}} - n_{\mathcal{B}} \frac{n_\chi^{\mathrm{eq}} n_{\bar{\chi}}^{\mathrm{eq}}}{n_{\mathcal{B}}^{\mathrm{eq}}} \right] - \langle \sigma^{\mathrm{an}} v_{\mathrm{rel}} \rangle \left[n_\chi n_{\bar{\chi}} - n_\chi^{\mathrm{eq}} n_{\bar{\chi}}^{\mathrm{eq}} \right]$$

$$\sigma_{\mathcal{B}}^{\mathrm{bsf}} v_{\mathrm{rel}} \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[1 + f_\gamma^{\mathrm{eq}}(\Delta E) \right] D^\rho_{\mu\nu}(\Delta E, \mathbf{p}) \sum_{\mathrm{Spin}} \mathcal{T}^\mu_{\mathbf{k},\mathcal{B}}(\Delta E, \mathbf{p}) \mathcal{T}^{\nu\star}_{\mathbf{k},\mathcal{B}}(\Delta E, \mathbf{p})$$

$$D^\rho_{\mu\nu} = 2\Im \left[i D^R_{\mu\nu} \right] = 2\Im \left[D^{R,0}_{\mu\nu} + D^{R,0}_{\mu\alpha} \Pi_R^{\alpha\beta} D^{R,0}_{\beta\nu} + \ldots \right]$$

$2\Im\Big[$  $\Big]$

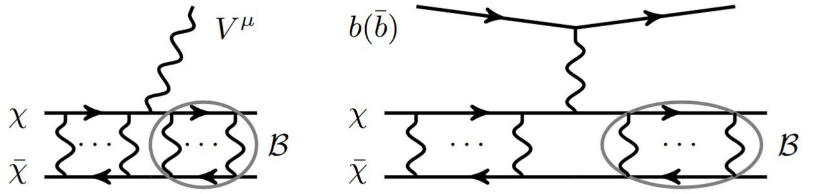
→ **first consistent thermal description of bound state formation at NLO beyond ionisation equilibrium for DM freeze-out**

→ **proof of consistent cancellation of all IR (and UV) divergencies**

Binder, Blobel, JH, Mukaida (2020)

Comparison bath particle scattering & emission of mediator

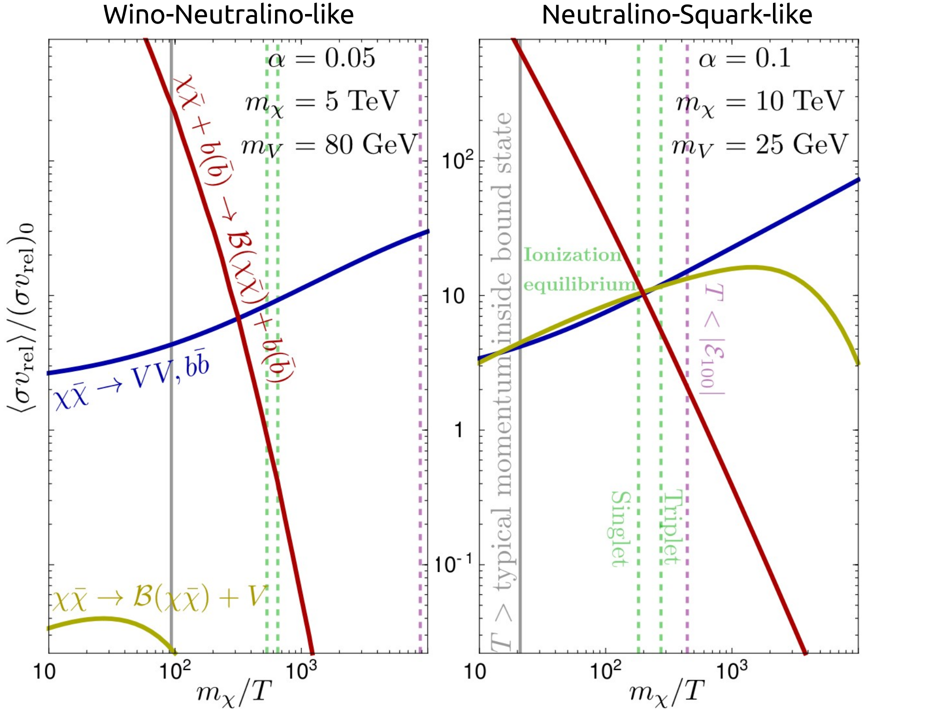
- **Effective out-of-equilibrium description including bath particle scattering and emission for *massive* mediator**



$$\dot{n}_\chi + 3Hn_\chi = -\left[\langle\sigma^{\rm an} v_{\rm rel}\rangle + W(T)\right]\left[n_\chi^2 - (n_\chi^{\rm eq})^2\right]$$

$$W \equiv \langle\sigma_{100}^{\rm BSF} v_{\rm rel}\rangle \left[\frac{(1/4)\,\Gamma^{\rm dec}_{100,\rm S}}{\Gamma^{\rm dec}_{100,\rm S} + \Gamma^{\rm dis}_{100}} + \frac{(3/4)\,\Gamma^{\rm dec}_{100,\rm T}}{\Gamma^{\rm dec}_{100,\rm T} + \Gamma^{\rm dis}_{100}}\right]$$

$$\sigma_{nlm}^{\rm BSF} v_{\rm rel} = \int \frac{d^3p}{(2\pi)^3} D^{-+}_{\mu\nu}(P) \sum_{\rm spins} \mathcal{T}^{\mu}_{\mathbf{k},nlm}(P)\,\mathcal{T}^{\nu\star}_{\mathbf{k},nlm}(P)$$



→ bath particle scattering can play relevant role

Binder, Mukaida, Petraki (2019)

Implications on the decay length

$$\frac{\Omega_{\rm DM}h^2}{0.12} \simeq \left(\frac{1.5\,\rm m}{c\tau}\right)\left(\frac{106.75}{g_s}\right)^{3/2}\left(\frac{m_{\rm DM}}{100\,\rm keV}\right)\left(\frac{200\,\rm GeV}{m_P}\right)^2 \times \begin{cases} \frac{2k+4}{3}\left(\frac{T_{\rm rh}}{m_P}\right)^{4k-1} \mathcal{I}_{\rm rh,b} + \mathcal{I}^0_{\rm RD} & \text{in BR} \\ \frac{2k+4}{3k-3}\left(\frac{T_{\rm rh}}{m_P}\right)^{\frac{9-k}{k-1}} \mathcal{I}_{\rm rh,f} + \mathcal{I}^0_{\rm RD} & \text{in FR} \end{cases},$$

$$V(\Phi) = \lambda\frac{|\Phi|^k}{M^{k-4}}$$

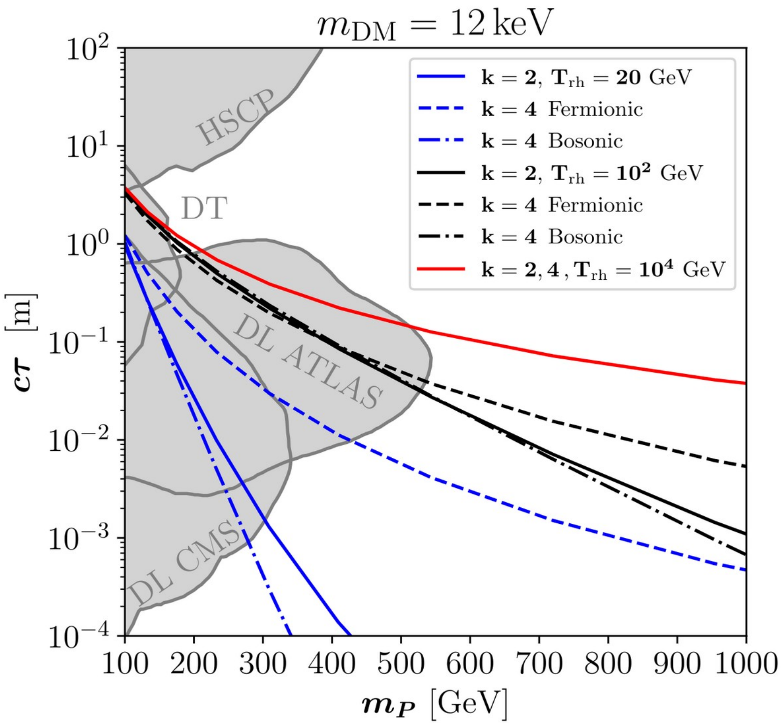
Type	$T_{\rm rh}$ [GeV]	$c\tau$ [m]
$k=2$	10	2.2×10^{-7}
$k=4$ BR	10	2.2×10^{-11}
$k=4$ FR	10	2.0×10^{-3}
$k=2$	20	2.6×10^{-5}
$k=4$ BR	20	4.3×10^{-7}
$k=4$ FR	20	5.6×10^{-3}
$k=2$	100	3.9×10^{-2}
$k=4$ BR	100	4.1×10^{-2}
$k=4$ FR	100	4.9×10^{-2}
$k=2$	10^4	0.15
$k=4$ BR	10^4	0.15
$k=4$ FR	10^4	0.15

m_{DM} = 12keV, m_P= 500 GeV

Becker, Copello, JH, Lang, Xu (2023)

Constraints from LLP searches at the LHC

Muonphilic Majorana DM model



→ **Interpretation of exclusion limits dependent on cosmological history, e.g. reheating temperature!**

Becker, Copello, JH, Lang, Xu (2023)

Linking to inflationary models

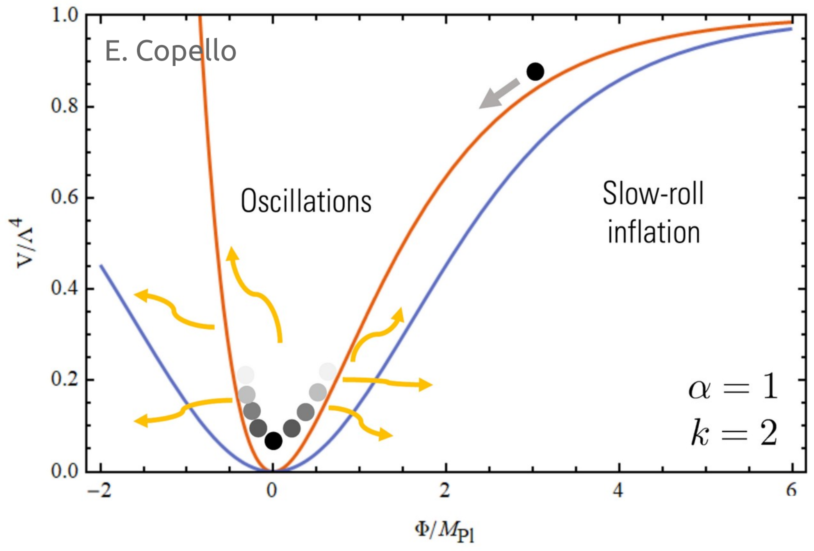
$$V(\Phi) = \lambda \frac{|\Phi|^k}{M^{k-4}}$$

Reheating potential can be obtained ($\Phi < M_{Pl}$), e.g. from

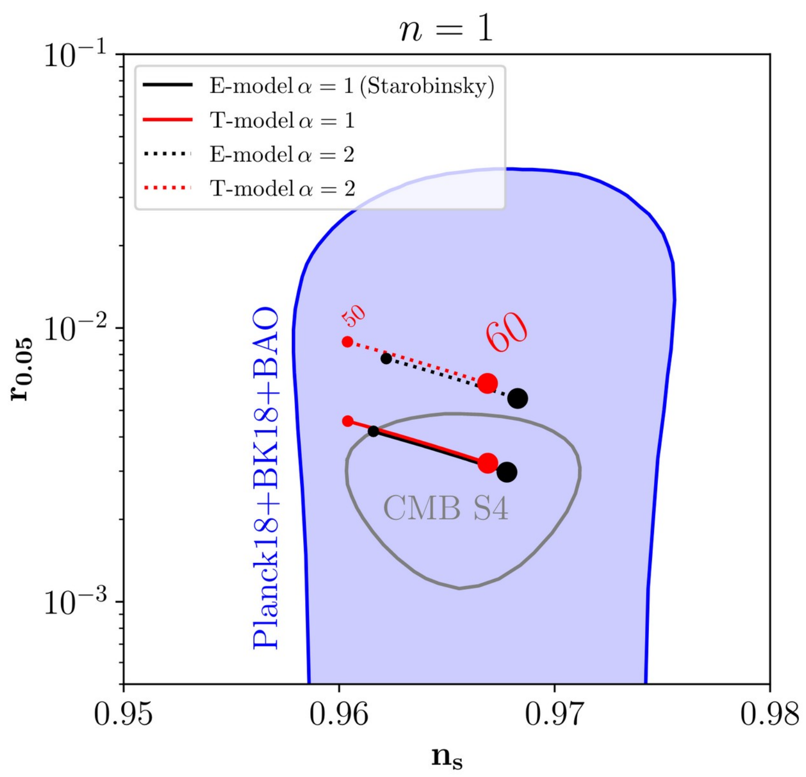
E-model (Starobinsky inflation for α=1):

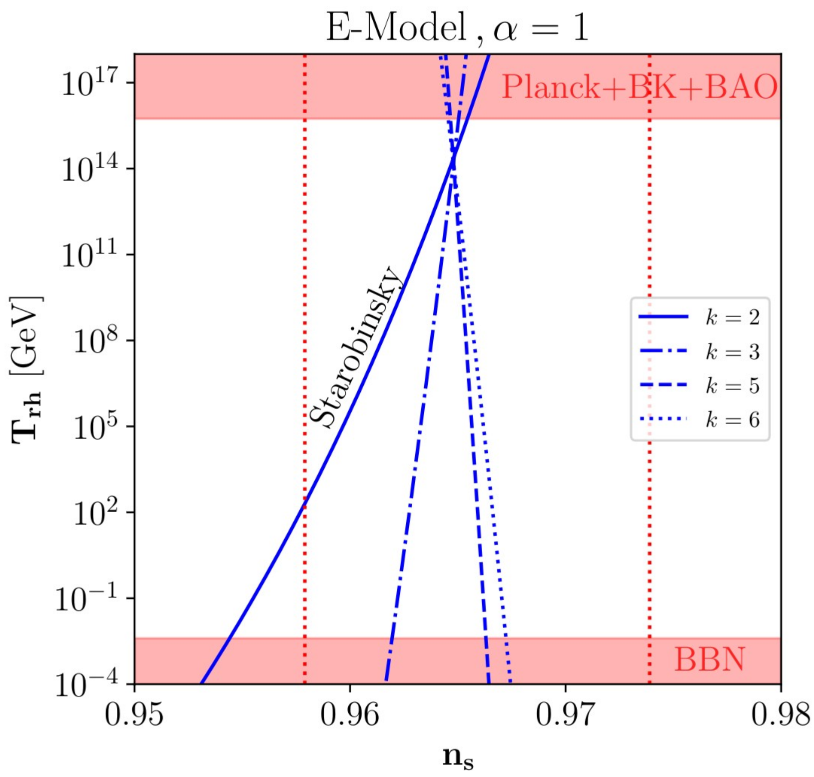
$$V(\Phi) = \Lambda^4 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\frac{\Phi}{M_{\rm Pl}}}\right)^{2n}$$

$$V(\Phi) \simeq \Lambda^4 \left(\frac{2}{3\alpha}\right)^n \left(\frac{\Phi}{M_{\rm Pl}}\right)^{2n} \equiv \frac{\lambda}{M_{\rm Pl}^{k-4}}\Phi^k$$



Constraints on the reheating temperature from inflation



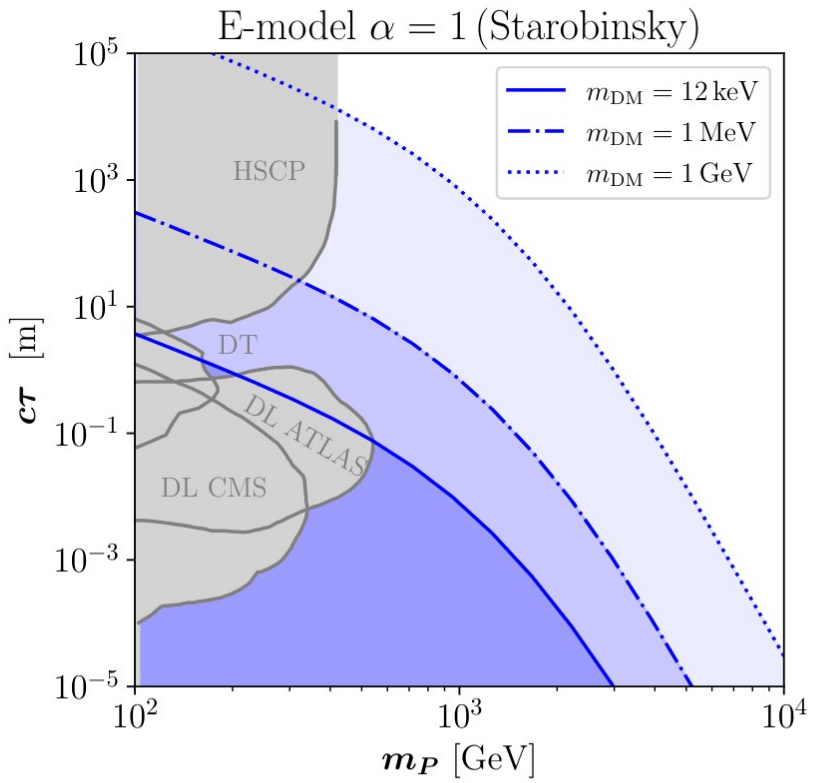


→ spectral index sets lower limit on T_{rh}

Becker, Copello, JH, Lang, Xu (2023)

Linking the early Universe with physics in the lab

Muonphilic Majorana DM model



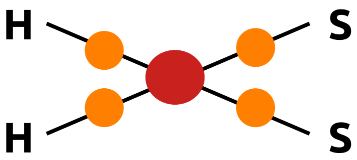
→ **bound on spectral index**
→ **lower bound on T_{rh}**
→ **lower bound on cτ**

$$\Omega_{DM} h^2 \sim m_{DM} \, (c\tau)^{-1}$$

→ **higher m_{DM}**
→ **more stringent constraints**

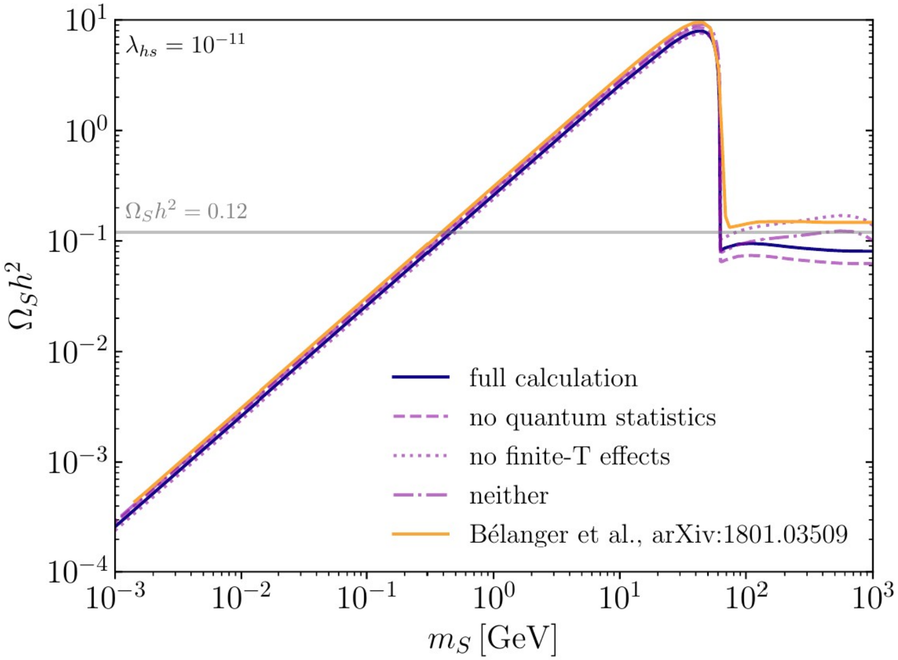
Semi-classical Boltzmann approach with thermal masses

- **Freeze-in via scattering**



$$\mathcal{L} = \frac{1}{2}\partial_\mu S \partial^\mu S + \frac{1}{2}\mu_S^2 S^2 + \frac{1}{2}\lambda_{hs} S^2 |H|^2 + \frac{1}{4}\lambda_s S^4$$

- Including thermal masses
- Including thermal effects on Higgs potential
- Full quantum statistics



Bringmann, Heeba, Kahlhoefer, Vangsnes (2021)

Impact of thermal plasma in ultrarelativistic regime

- **Interpolation with susceptibility needed to bridge between ultrarelativistic and (non)relativistic regime** Biondini, Ghiglieri (2020)

$$\mathrm{Im}\Pi_R^{1\leftrightarrow 2} = (\mathrm{Im}\Pi_R^{\mathrm{LPM}} - \mathrm{Im}\Pi_R^{\mathrm{LPM\ Born}})\kappa(M_\eta) + \mathrm{Im}\Pi_R^{\mathrm{Born}}$$

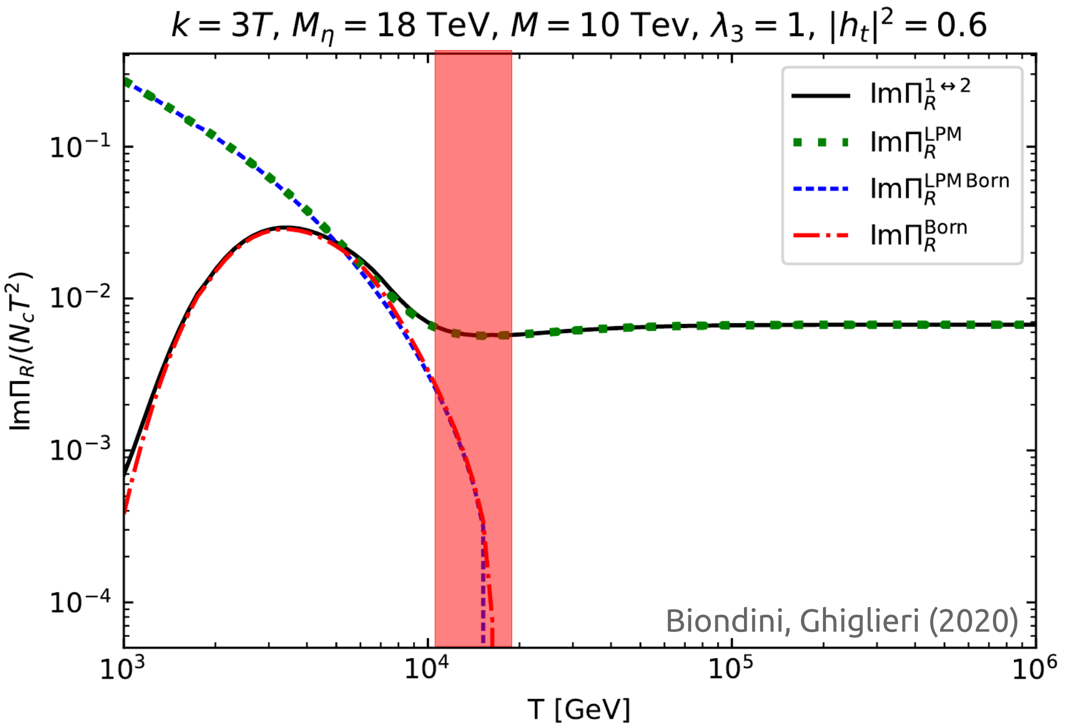
- **pure HTL approximation**

  For production rate of scalar DM, see Drewes, Kang (2015)

$$z_{\mathrm{FI}} = \frac{m_F}{T} \approx 5 \qquad m_{F,0}^2 + G\frac{T^2}{4} \approx 25T^2$$

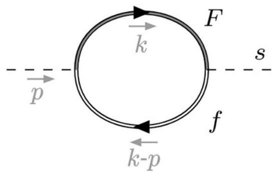
$$m_{F,0} \approx \mathcal{O}(T)$$

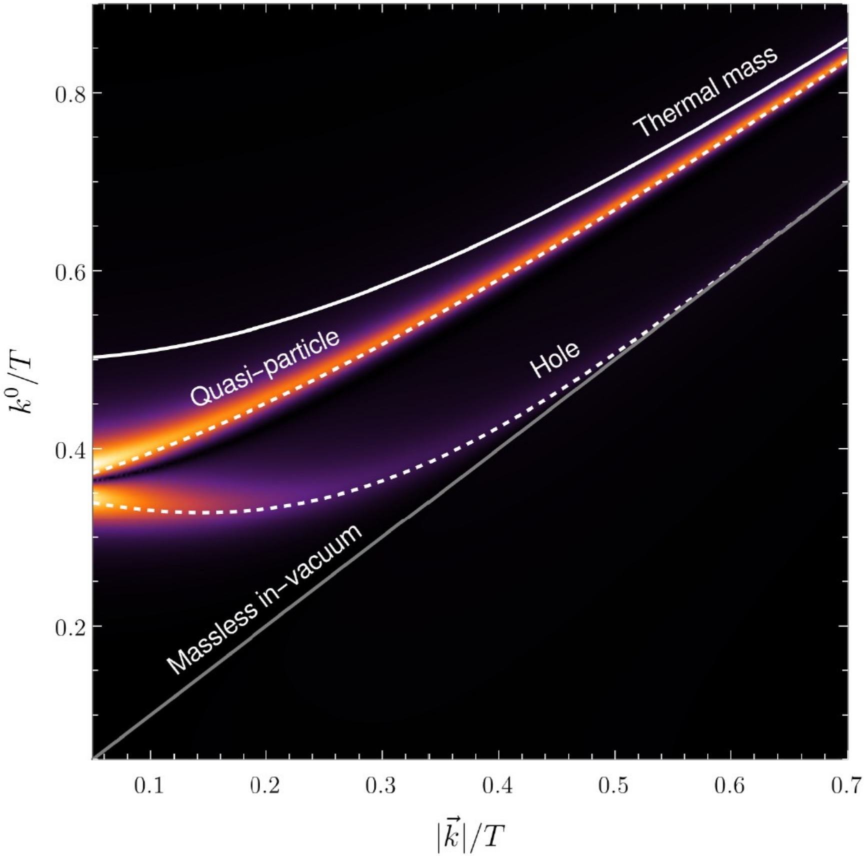
→ **breaks down**



→ freeze-in highly dependent on regime when parent particle becomes non-relativistic

Comparing methods – spectral propagator III





- **HTL approximation**

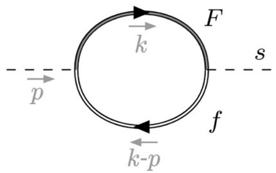
$$\Gamma_F^{\mathrm{HTL}} \propto \theta\left(-k^2\right) \frac{G}{\pi} 8|\vec{k}|T^2$$

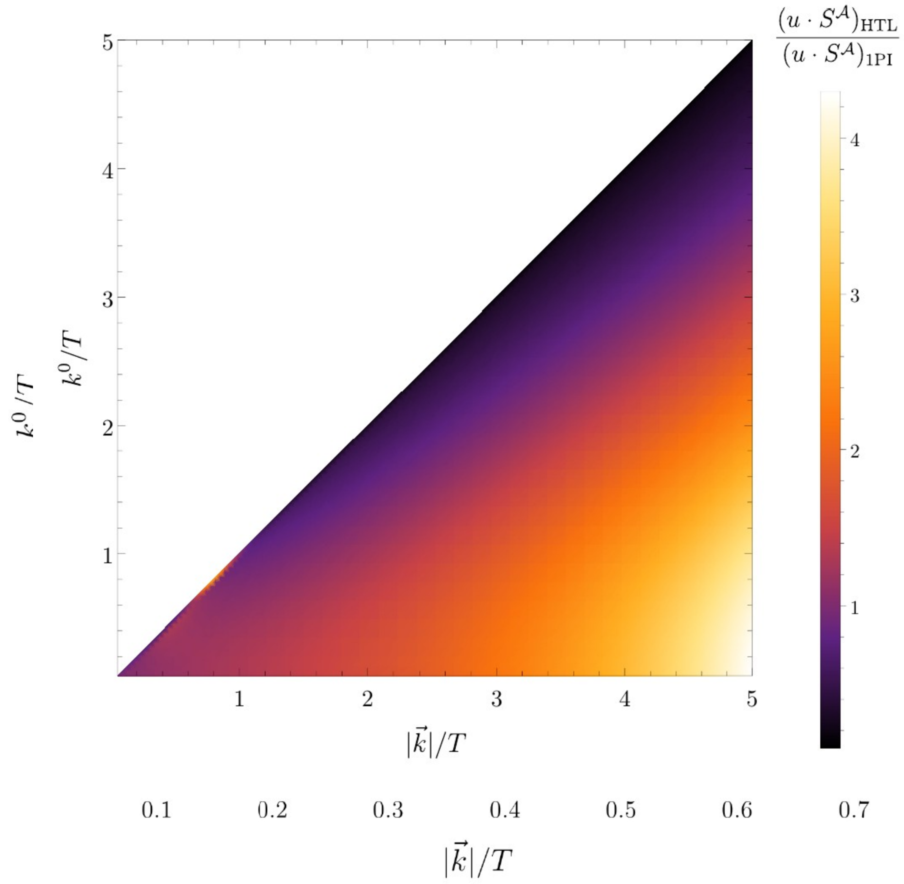
  - Thermal width non-zero only for space-like momenta k → continuum ("Landau damping")
  - For time-like momenta k, vanishing thermal width and recovery of particle-like dispersion relation

$$\$^{\mathcal{A}}_{F/f}(k) = \pi\,\mathrm{sign}(k^0)\left(\not{k} - m_{F/f} - \not{\Sigma}^{\mathcal{H},\mathrm{HTL}}_{F/f}(k)\right)\delta\left(\left[k - \Sigma^{\mathcal{H},\mathrm{HTL}}_{F/f}(k)\right]^2 - m^2_{F/f}\right)$$

Becker, Copello, JH, Tamarit (2023)

Comparing methods – spectral propagator III





- **HTL approximation**

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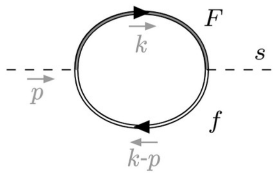
- Thermal width non-zero only for space-like momenta k → continuum ("Landau damping")

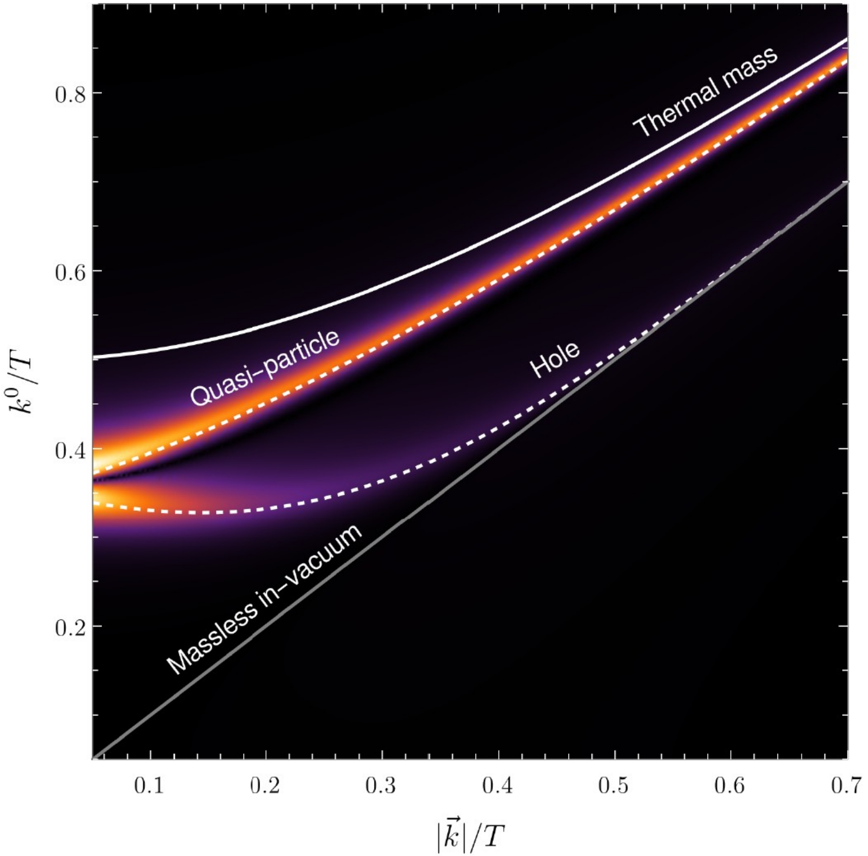
- For time-like momenta k, vanishing thermal width and recovery of particle-like dispersion relation

$$\not{S}^{\mathcal{A}}_{F/f}(k) = \pi\,\mathrm{sign}\left(k^0\right)\left(\not{k} - m_{F/f} - \not{\Sigma}^{\mathcal{H},\mathrm{HTL}}_{F/f}(k)\right)\delta\left(\left[k - \Sigma^{\mathcal{H},\mathrm{HTL}}_{F/f}(k)\right]^2 - m^2_{F/f}\right)$$

Becker, Copello, JH, Tamarit (2023)

Comparing methods – spectral propagator





- **Tree-level CTP approximation**
  - Thermal widths identically zero
  - Dispersion relation with momentum-independent thermal masses and vacuum mass

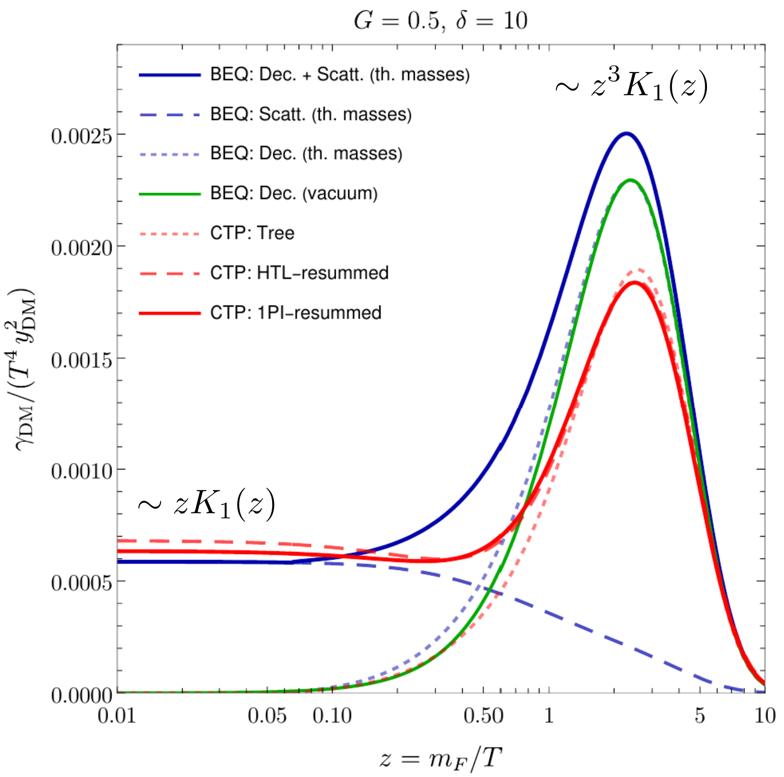
$$\not{S}_F^{\mathcal{A}}(k) = \pi\delta(k^2 - m_F^2)(\not{k} + m_F)\,\mathrm{sign}(k^0)$$

  - Corresponds to DM decay from on-shell F including in-vacuum masses and thermal masses by accounting for the proper quantum statistics

$$\Pi_s^{\mathcal{A}}(p) = \frac{y_{\mathrm{DM}}^2}{16\pi\,|\vec{p}|}\left|p^2 - m_F^2 - m_f^2\right| \int_{\mathcal{B}} k_0\left[1 - f_+(k_0) - f_+(p_0 - k_0)\right]$$

Becker, Copello, JH, Tamarit (2023)

Comparing methods – interaction rate density



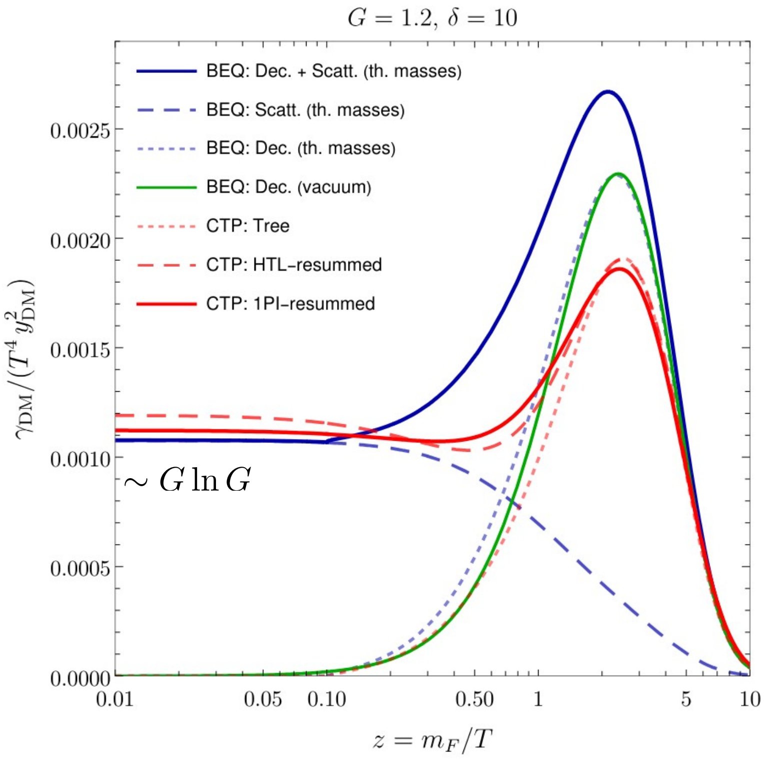
- **Scatterings** dominate for small z (plateau)
- **Decays** start to dominate for large z

$$z > \sqrt{G}\,\frac{m_{\rm DM} m_F}{m_F^2 - m_{\rm DM}^2}$$

- Relative mass difference δ sets **height** of the **decay peak**

Becker, Copello, JH, Tamarit (2023)

Comparing methods – interaction rate density



- **Scatterings** dominate for small z (plateau)
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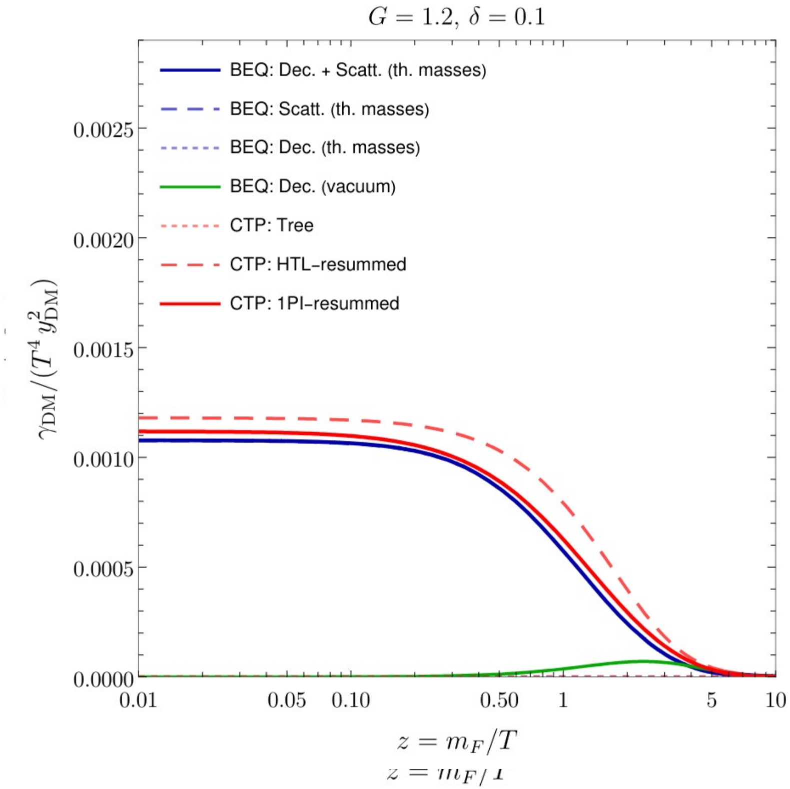
$$z > \sqrt{G}\frac{m_{\rm DM} m_F}{m_F^2 - m_{\rm DM}^2}$$

- Relative mass difference δ sets **height** of the **decay peak**
- Gauge coupling G sets the **height** of the **plateau**

Becker, Copello, JH, Tamarit (2023)

Comparing methods – interaction rate density



- **Scatterings** dominate for small z (plateau)
- **Decays** start to dominate for large z

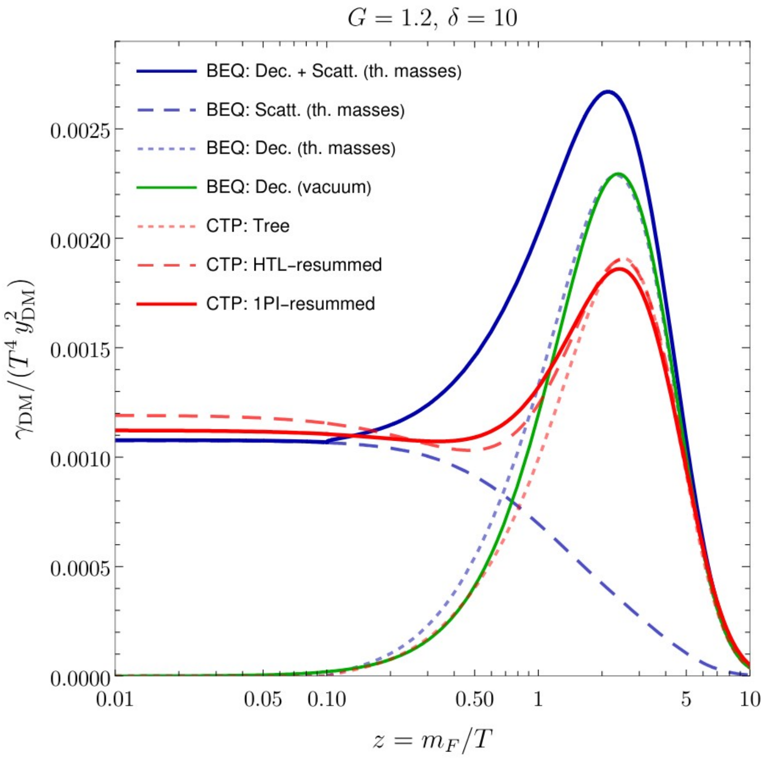
$$z > \sqrt{G}\frac{m_{\rm DM} m_F}{m_F^2 - m_{\rm DM}^2}$$

- Relative mass difference δ sets **height** of the **decay peak**
- Gauge coupling G sets the **height** of the **plateau**
- For smaller δ , decays become less relevant due to **phase space suppression**

$$z > \sqrt{G}\frac{1-\delta}{2\delta - \delta^2}$$

Becker, Copello, JH, Tamarit (2023)

Comparing methods – interaction rate density



BEQ with decay only (vacuum masses)

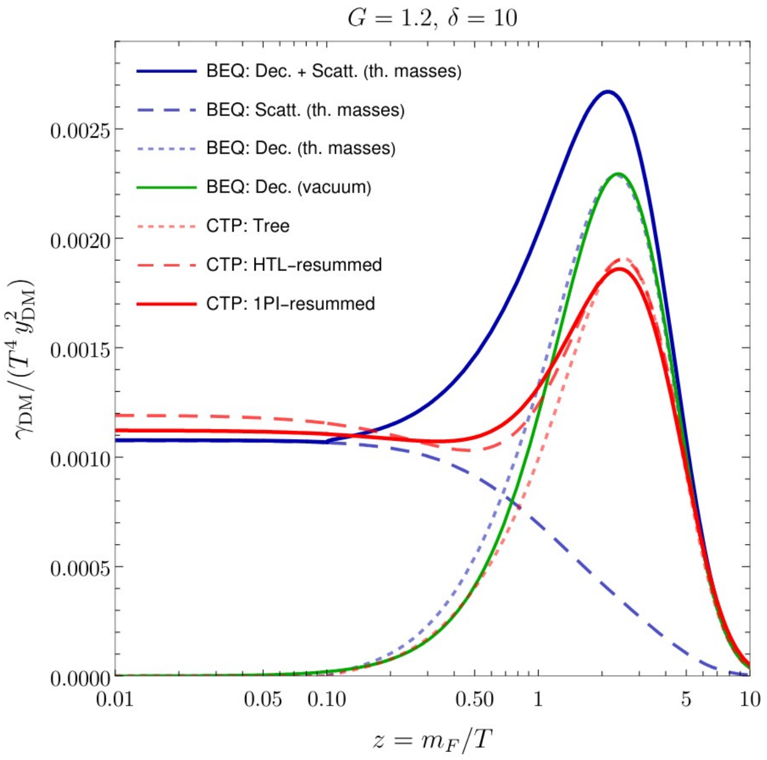
- Heavily **underestimates** production for **small z** (missing scatterings)
- **Overestimates** production for **large z** due to missing quantum statistics

BEQ with decay and scatterings (incl. thermal masses)

- General differences due to missing quantum statistics
- Decay **overestimated** due to higher thermal mass, earlier closure of the decay window, larger and longer contribution from BEQ than for 1PI resummed

Becker, Copello, JH, Tamarit (2023)

Comparing methods – interaction rate density



CTP with tree-level propagators

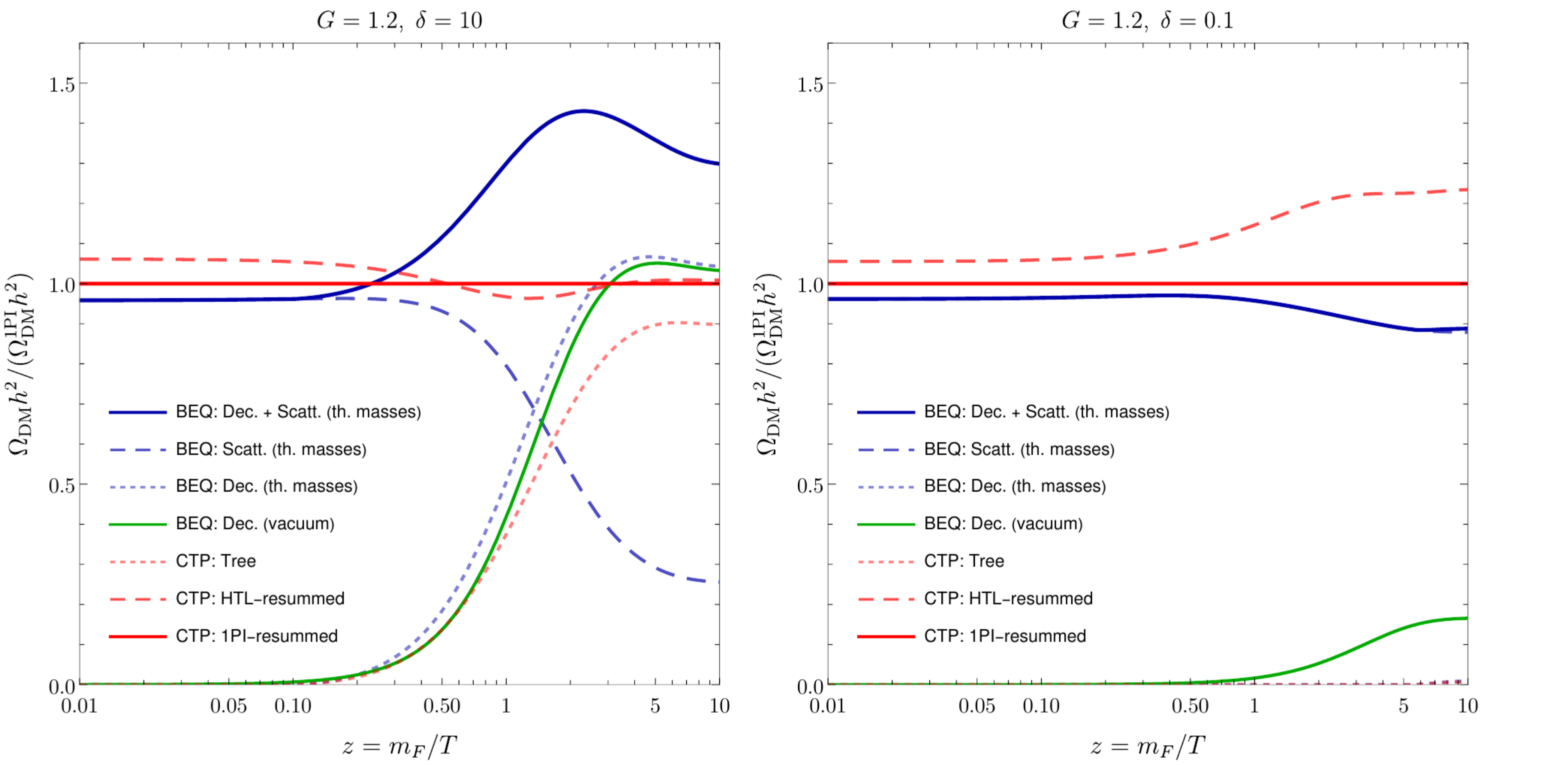
- Accounts for **decays with thermal masses** and proper **quantum statistics** while neglecting scatterings
- Comparable with **HTL for decays**

CTP with HTL approximation

- For **small z**, HTL overestimates, as dominated by **ST**-contributions that lack suppression by vacuum mass for large space-like momenta
- Decay contribution kicks in later, as HTL-propagator is delta function for time-like momenta in contrast to 1PI-resummed one
- For **large z**, **HTL overestimates** decays as finite width in 1PI-resummed propagators smear out quasi-particle solution

Becker, Copello, JH, Tamarit (2023)

Comparing methods – relic abundance



Becker, Copello, JH, Tamarit (2023)