FLAVOR (MODELS) IN FINITE UNIFIED THEORIES

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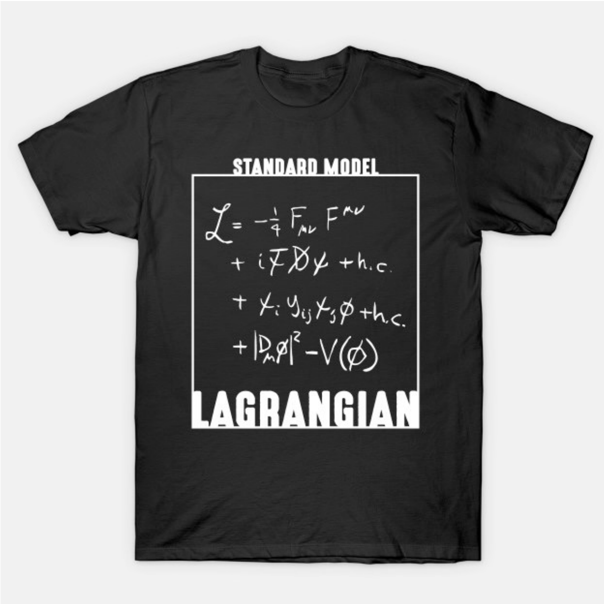
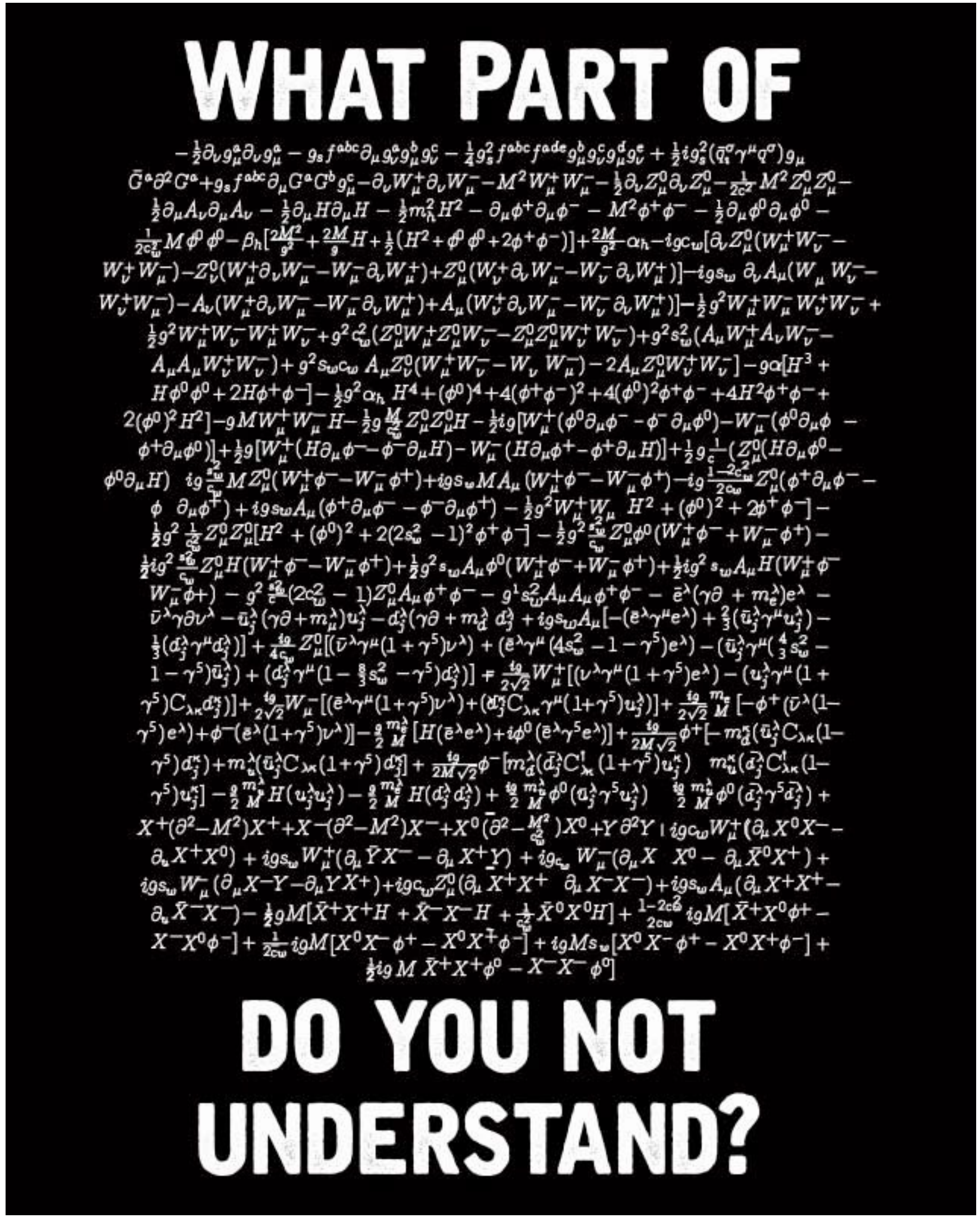
CONAHCYT
CONSEJO NACIONAL DE HUMANIDADES CIENCIAS Y TECNOLOGÍAS

UNIVERSIDAD NACIONAL AUTONOMA DE MEXICO
POR MI RAZA HABLARA EL ESPIRITU


STANDARD MODEL
LAGRANGIAN


WHAT PART OF
DO YOU NOT UNDERSTAND?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+ ΛCDM…

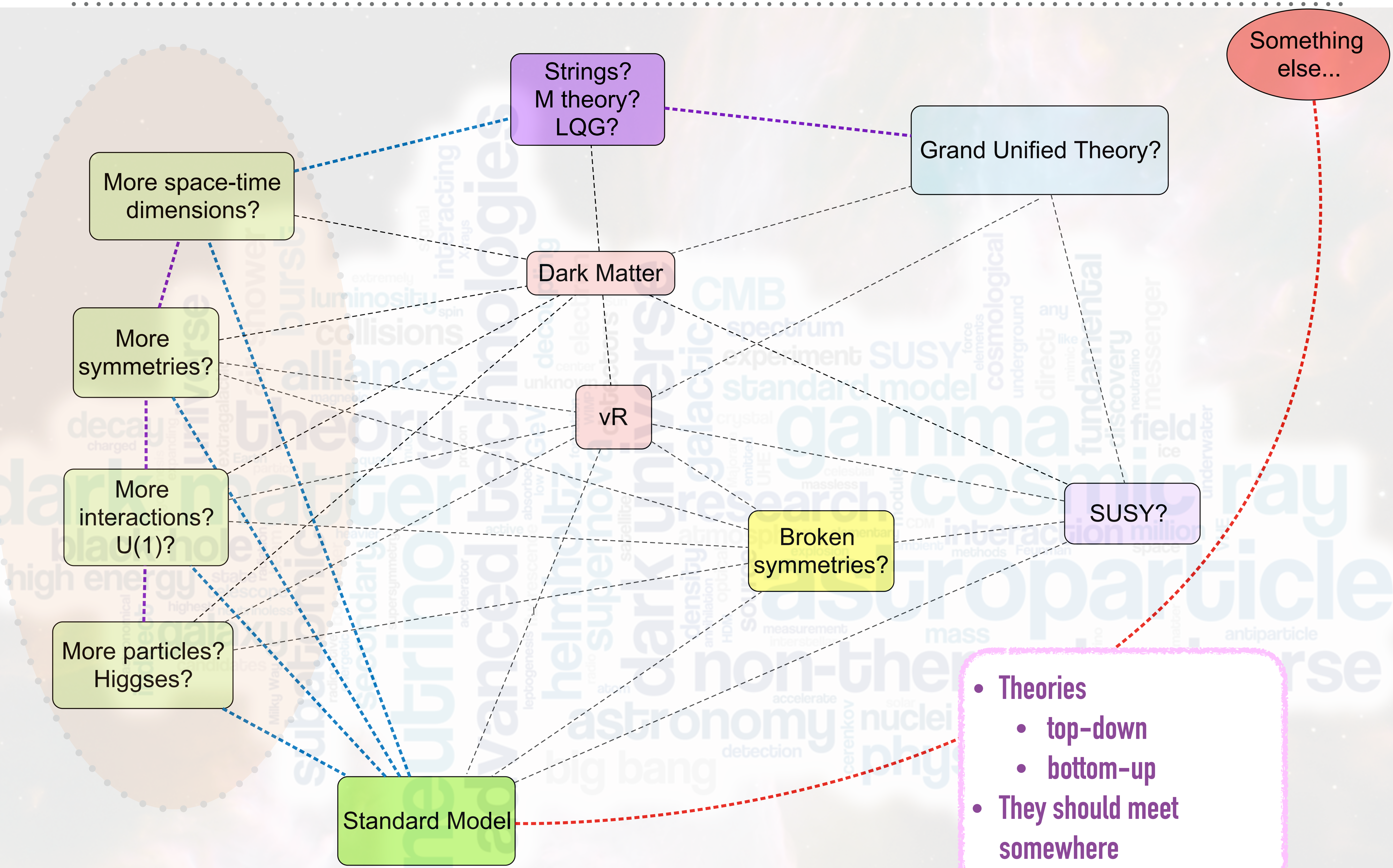
WHAT'S GOING ON?

- What happens as we approach the Planck scale?
- What happened at the early Universe?
- How do we go from an effective theory like the SM to a more fundamental one?
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?

- Why/how are the elementary particle masses so different?
- Is there more than one Higgs, more scalars?
- What about flavor?

- **Where is the new physics?**

HOW DO WE GO BEYOND THE SM?

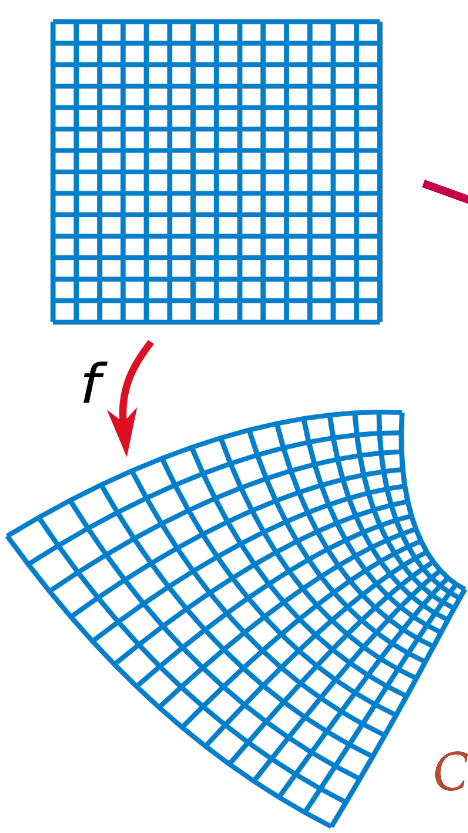


- Theories
  - top-down
  - bottom-up
- They should meet somewhere

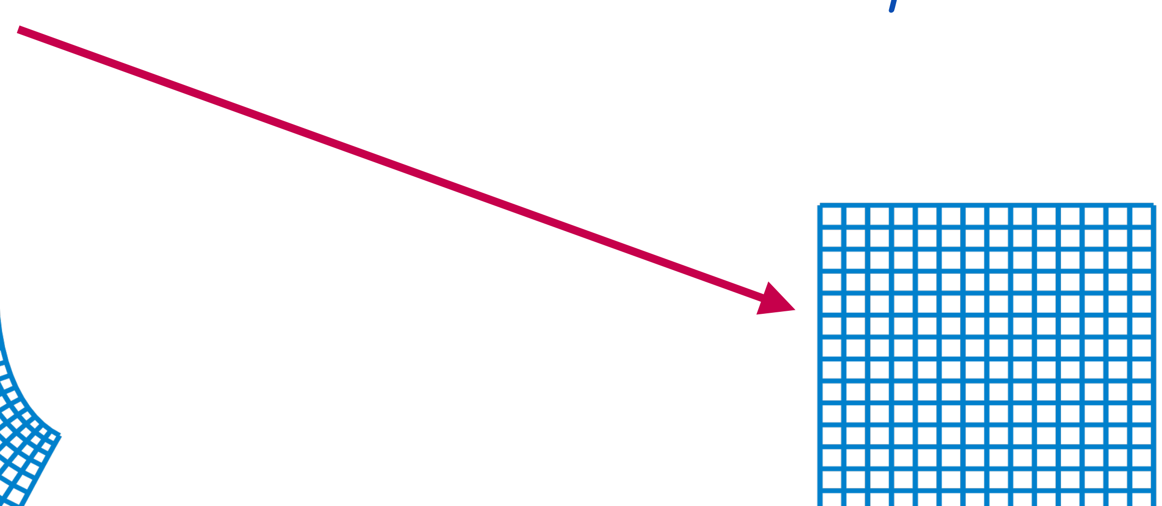
HOW DO WE MOVE UP (OR DOWN) IN ENERGY?

- We know how a QFT behaves at different scales through the renormalization group RG
- The theory has the same structure at different energy scales, but the parameters — couplings and masses — change with energy
- Related to scale invariance and conformal invariance

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} \qquad \gamma(\phi) = \mu \frac{\partial \ln Z}{\partial \mu}$$



Conformally invariant



Scale invariant

Scale invariance appears in many physical systems

HOW TO GO BEYOND THE STANDARD MODEL (BSM)?

- Traditional way $\Rightarrow$ addition of symmetries

  N=1 SUSY

- Very effective, but too many free parameters

  Can get messy…

- Complementary approach
  Look for renormalization group invariant relations at high energies

  GUT $\Rightarrow$ Planck

- Resulting theory has few free parameters $\therefore$ very predictive

Relates gauge and Yukawa sector
Predictions for 3rd generation masses

RENORMALIZATION GROUP INVARIANTS RGI

- Search for more fundamental theory ⇒ less parameters
  Renormalization Group Invariants (RGI)

$$\Phi(g_1, \ldots, g_N) = 0$$

$$\mu \, d\Phi/d\mu = \sum_{i=1}^{N} \beta_i \, \partial\Phi/\partial g_i = 0$$

- Equivalent to solve reduction equations

$$\beta_g \left(dg_i/dg \right) = \beta_i$$

$$i = 1, \ldots, N$$

- Reduced theory has only one coupling and its beta function
- Reduction $\rightarrow$ power series solution
- Uniqueness of solution can be studied at one-loop

Zimmermann (1985); Zimmermann, Oehme, Sibold (1984-1985)

REDUCTION OF COUPLINGS

- Couplings related to a primary coupling
  totally reduced — all couplings depend on one
  partially reduced — some couplings depend on one
- Can be applied to SUSY and non-SUSY models
- SM analyzed — results now ruled out, still impressive

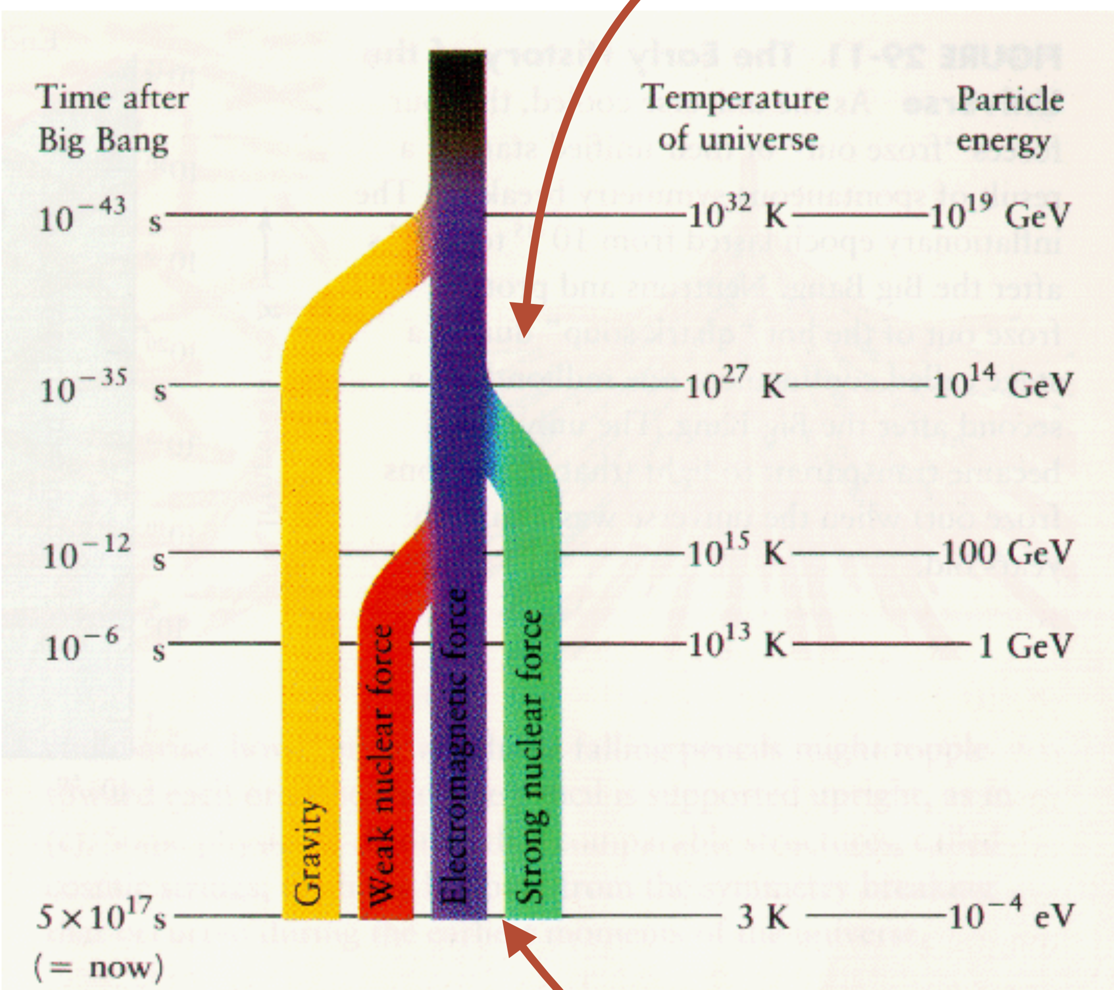
  Kubo, Sibold, Zimmermann (1984-1987)

- 2HDM analyzed Denner (1990) — now re-analysed:
  possible to have one-loop reduced equations in type II 2HDM at a high-scale boundary

  May Pech, MM, Patellis, Zoupanos (2023)

- Under some conditions SUSY unification models might be **finite**

FINITENESS = SCALE/CONFORMAL INVARIANCE



- All-loop finiteness ⇒ $\beta = 0$
  to all orders in perturbation theory
- Scale/conformal invariance
  Conformal and scale invariant = Yukawa couplings
  Scale invariant = Soft breaking terms
  Do not depend on energy scale
  Based on RGI and reduction of couplings
- Gives UV completion of the QFT
- Reduces greatly the number of free parameters
  ⇒ new symmetries
- Partial reduction ⇒ predictions for 3rd generation masses

FINITE SU(5) THEORIES — THIRD GENERATION

- Prediction for top mass — very clean

$M_{top}^{th} \sim 178$ GeV 1993 Kapetanakis, M.M., Zoupanos

m_bot also predicted, large tan beta

$M_{top}^{exp} = 176 \pm 18$ GeV 1995

$M_{top}^{th} \sim 172.5$ GeV 2007 Heinemeyer, M.M.,Zoupanos

$M_{top}^{exp} = 173.1 \pm .09$ GeV 2013

- Prediction for Higgs mass — depends on soft breaking terms, also very restricted

$M_{Higgs}^{th} \sim 121 - 126$ GeV 2008, 2013 Heinemeyer, M.M., Zoupanos

$M_{Higgs}^{exp} = 126 \pm 1$ GeV 2013

FINITESS $\Longrightarrow$ GAUGE YUKAWA UNIFICATION

Grand Unified SUSY N=1, no gauge anomalies:

$$W = \frac{1}{2} m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6} C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k$$

$$\beta_g^{(1)} = 0 = \gamma_i^{j\,(1)}$$

$$\sum_i T(R_i) = 3C_2(G)\,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i)$$

T Dynkin index of irrep, C_2 Casimir invariant of group *C_{ijk} Yukawa couplings, g gauge coupling*

- Restricts the gauge group
- Relates gauge and Yukawa couplings
- If finite to all orders $\Rightarrow$ Conformal invariance
- May imply extra symmetries, in this case discrete

- Just analyze one-loop solution
- One-loop finite $\Rightarrow$ two-loop finite
- Isolated and non-degenerate solution $\Rightarrow$ all-loop finite Lucchesi, Piguet, Sibold

β= 0 non-renormalization of coupling constants, not complete UV finiteness where field renormalization is absent

SUSY BREAKING SSB

- Explicit/soft breaking >100 new free parameters 😫

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i \phi_j \phi_k + \frac{1}{2}\, b^{ij}\, \phi_i \phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i} \phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

- SSB can also be restricted through RGI ⇒ β = 0
- Leads to a sum rule among scalars and gauging masses

$$(\, m_i^2 + m_j^2 + m_k^2\,)/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(2)} + O(g^4)$$

- Breaks conformal invariance BUT remains scale invariant!

- one- and two-loop finiteness conditions known
- all-loop finiteness possible

Kazakov, Jack, Jones, Pickering…

- Depends on the gaugino mass scale M
- Scale invariant but not conformal

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman; Kobayashi, Kubo, Zoupanos

SU(5) FINITE UNIFIED MODELS

The one- and two-loop finiteness conditions imply following matter content:

$$3\ \overline{\mathbf{5}} + 3\ \overline{\mathbf{10}} + 4\ (\mathbf{5} + \overline{\mathbf{5}}) + \overline{\mathbf{24}}$$

3 generations, 4 pairs of Higgs doublets one field in the adjoint

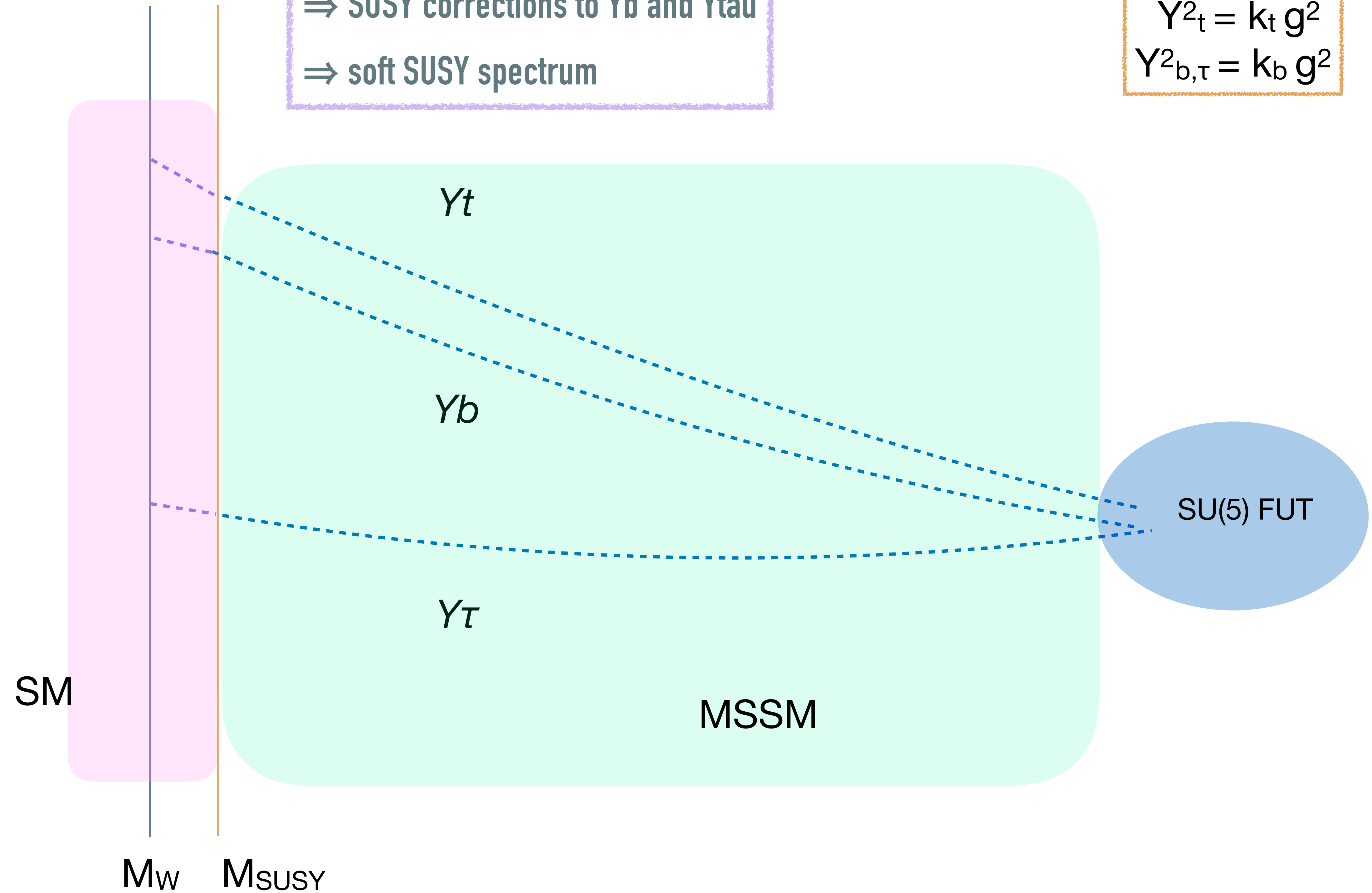
- Soft scalar masses obey sum rule
- No proton decay
- At GUT scale finiteness is broken $\Rightarrow$ MSSM finiteness broken
- Rotation of FUT Higgs sector $\Rightarrow$ 2 Higgs doublets of MSSM maximally coupled to third generations

Finite soft breaking terms included

⇒ SUSY corrections to Yb and Ytau

⇒ soft SUSY spectrum

$$Y^2_t = k_t \, g^2$$
$$Y^2_{b,\tau} = k_b \, g^2$$



Results confronted to experimental constraints ⇒

gives available parameter space

$m_t = Y_t \, v_u$ $\qquad v_u / v_d = \tan \beta$

$m_{b,\tau} = Y_{b,\tau} \, v_d$ $\qquad v_d = m_\tau^{exp} / Y_\tau$

INTERPLAY HIGH-LOW ENERGIES: SEARCHES AT FUTURE COLLIDERS

GUT scale, Finiteness gives:

- Relations between gauge-Yukawa couplings
- Sum rule for soft breaking terms
- $\Rightarrow$ Very few free parameters

Require:

- Absence of proton decay
- Proper unification of gauge couplings
- MSSM

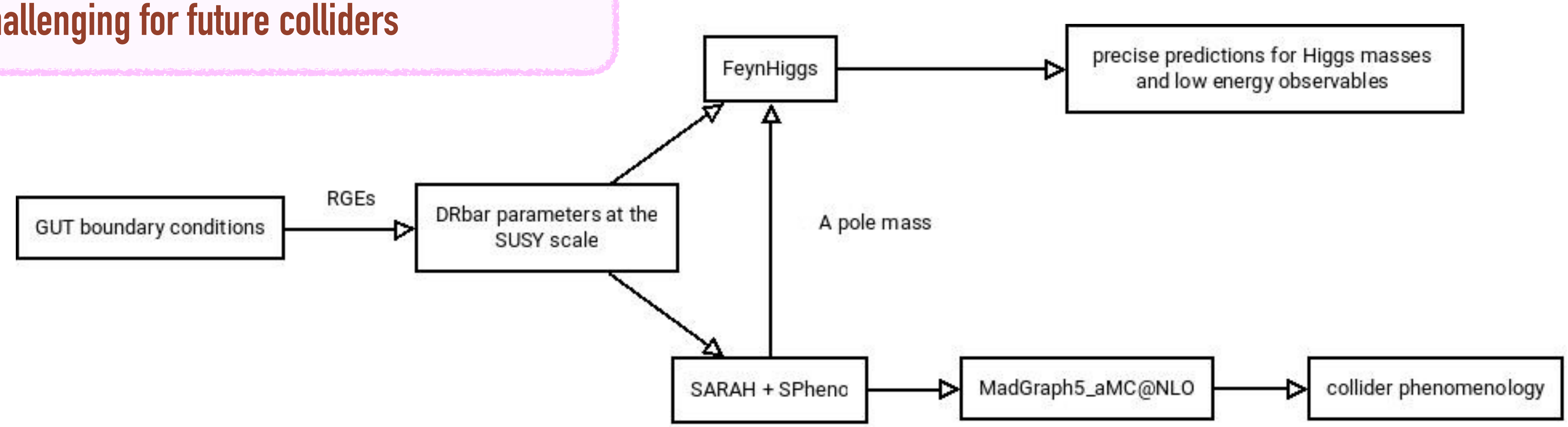
Low energies:

- Radiative eW symmetry breaking
- Include SUSY radiative corrections
- Quark and Higgs masses in experimental range
- Compliance with B physics (not trivial)

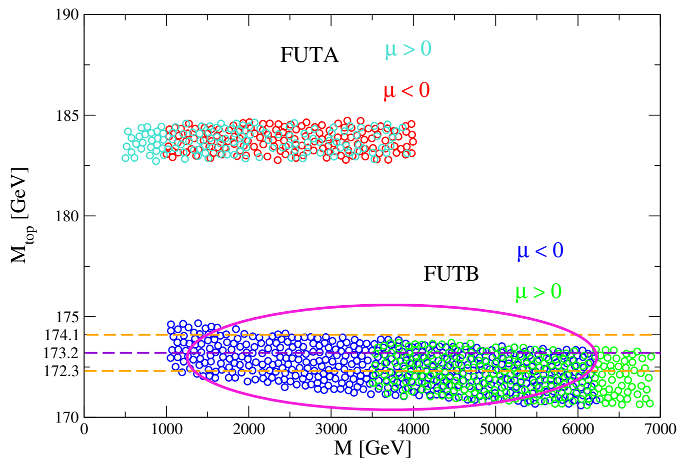
Large tan β
High SUSY spectrum > 1 TeV
Challenging for future colliders

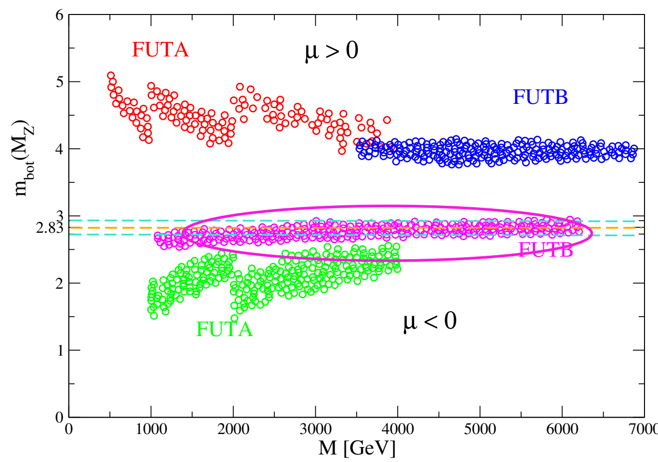
B constraints:

- BR ($b \rightarrow s\gamma$)
- BR ($B_s \rightarrow \mu^+\mu^-$)
- BR ($B_u \rightarrow \tau\upsilon$) B_s
- $\Delta M_{B_s\ SM/MSSM}$



Heinemeyer, Kalinowski, Kotlarski , Mondragon, Patellis, Tracas, Zoupanos (2021)

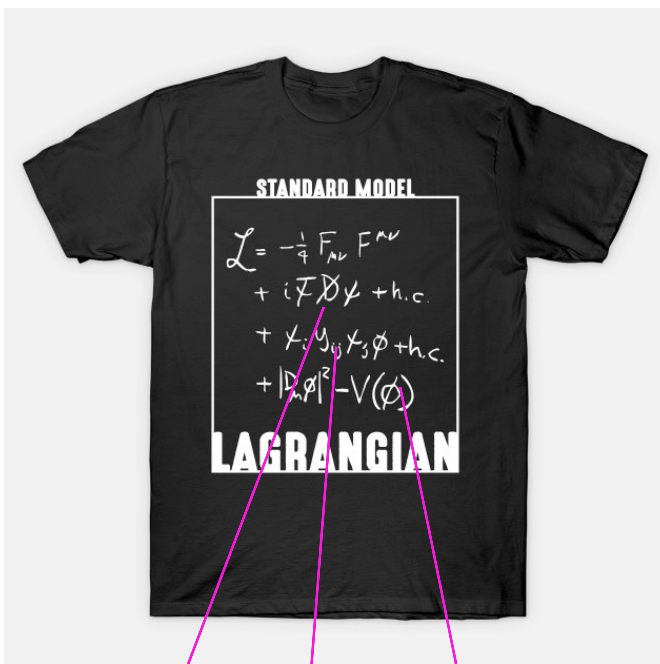




FUTB — 3rd generation

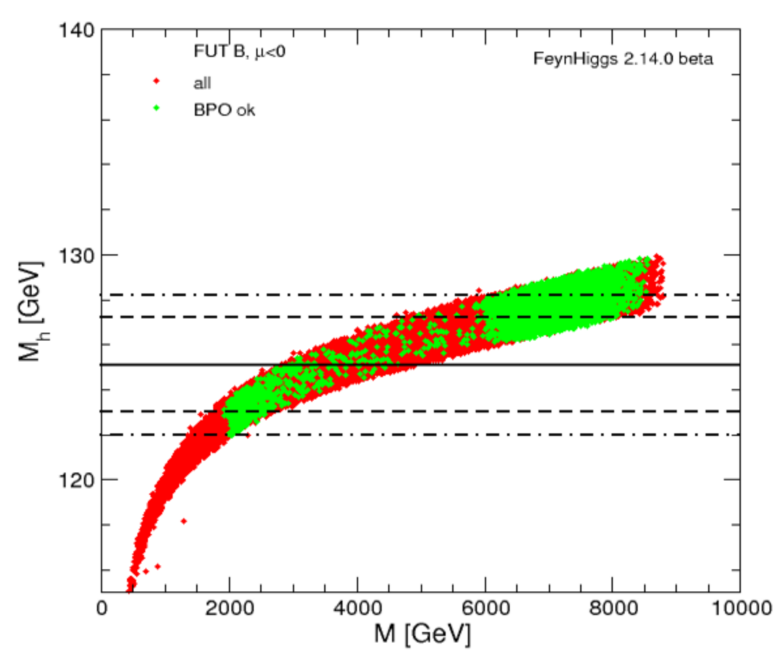
1 free parameter in gauge-Yukawa sector

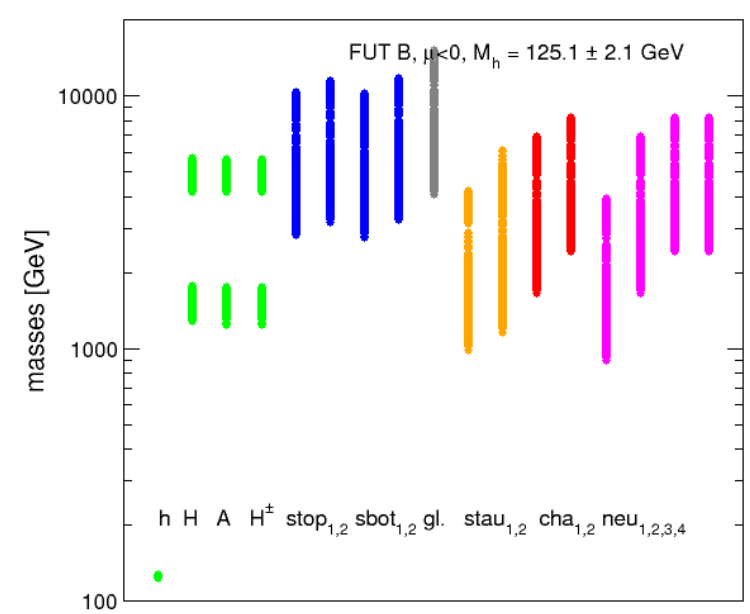
2 free parameters in soft SUSY breaking



Higgs mass range determined by finiteness, sum rule, B physics constraints and radiative top contributions to Higgs mass ⇒ heavy spectrum

These are now related!





MANY ASPECTS OF FINITENESS STUDIED

- SU(5) models extensively studied — Rabi et al; Kazakov et al; Quirós et al; MM, Zoupanos et a
- One coincides with a non-standard Calabi-Yau — MM, Zoupanos
- Finite string theories and criteria for branes — Ibáñez
- Models with three generations — Babu, Enkhbat, Gogoladze; MM & Jiménez; Estrada, MM, Patellis, Zoupanos
- $SU(N)^k$ models finite $\Longleftrightarrow$ 3 generations
  only $SU(3)^3$ compatible with phenomenology — MM, Ma, Zoupanos
- Relations non-commutative theories and finiteness — Jack, Jones
- Proof of conformal invariance (dimensionless part) — Kazakov, Bork; MM & Reyes
- Relation between finiteness and QFT in curved space-time & inflation
  Elizalde, Odintsov, et al
- Recent reviews — Heinemeyer, M.M, Tracas, Zoupanos, Phys.Rept. 814 (2019); Fortsch.Phys. 68 (2020)

SUPERPOTENTIAL

- The SU(5) superpotential of possible finite models is
  $\bar{\mathscr{H}}_{ai} = \bar{\mathbf{5}}\ , \quad \mathscr{H}^{i}_{\ a} = \mathbf{5}\ , \quad \bar{\Psi}_{a'i} = \bar{\mathbf{5}}\ , \quad X^{ij}_{a'} = \mathbf{10}\ , \quad \Sigma^{i}_{\ j} = \mathbf{24}$

  3 generations, 4 pairs of Higgs doublets and one field in the adjoint

$$3\ \bar{\mathbf{5}} + 3\ \overline{\mathbf{10}} + 4\ (\mathbf{5} + \bar{\mathbf{5}}) + \overline{\mathbf{24}}$$

$$\mathcal{W}_{SU(5)-R} = \bar{g}_{a'b'a}\bar{\Psi}_{b'i}X^{ij}_{a'}\bar{\mathcal{H}}_{aj} + \frac{1}{2}g_{a'b'a}\epsilon_{ijklm}X^{ij}_{a'}X^{kl}_{b'}\mathcal{H}^{m}_{a} + f_{ab}\bar{\mathcal{H}}_{ai}\Sigma^{i}{}_{j}\mathcal{H}^{j}_{b} + \frac{1}{3!}p\Sigma^{i}{}_{j}\ \Sigma^{j}{}_{k}\ \Sigma^{k}{}_{i} + \frac{1}{2}\lambda^{(\Sigma)}\Sigma^{i}{}_{j}\Sigma^{j}{}_{i} + m_{ab}\bar{\mathcal{H}}_{ai}\mathcal{H}^{i}_{b}\ .$$

$\bar{g}_{ijk}$ = down Yukawa couplings, g_{ijk} = up Yukawa couplings

WHAT ABOUT FLAVOR? 3 GENERATIONS

Classification of SU(5) FUT with off-diagonal γ done already

Coupled to 3 Higgs doublets

$$V_3^{(1)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & g_{123}\langle\mathcal{H}_3^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{213}\langle\mathcal{H}_3^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{231}\langle\mathcal{H}_1^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{321}\langle\mathcal{H}_1^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_3^{(2)} = \begin{pmatrix} g_{112}\langle\mathcal{H}_2^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

$$V_3^{(3)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_3^{(4)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & 0 & 0 \\ 0 & g_{223}\langle\mathcal{H}_3^5\rangle & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

Coupled to 4 Higgs doublets

$$V_4^{(1)} = \begin{pmatrix} g_{111}\langle\mathcal{H}_1^5\rangle & g_{124}\langle\mathcal{H}_4^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{214}\langle\mathcal{H}_4^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{231}\langle\mathcal{H}_1^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{321}\langle\mathcal{H}_1^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_4^{(2)} = \begin{pmatrix} g_{112}\langle\mathcal{H}_2^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ 0 & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

$$V_4^{(3)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{222}\langle\mathcal{H}_2^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} \quad , \quad V_4^{(4)} = \begin{pmatrix} g_{113}\langle\mathcal{H}_3^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & g_{132}\langle\mathcal{H}_2^5\rangle \\ g_{211}\langle\mathcal{H}_1^5\rangle & g_{223}\langle\mathcal{H}_3^5\rangle & g_{234}\langle\mathcal{H}_4^5\rangle \\ g_{312}\langle\mathcal{H}_2^5\rangle & g_{324}\langle\mathcal{H}_4^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix}$$

Top and down mass matrices with same structure

Babu, Enkhbat, Gogoladze (2003)

2-LOOP FINITE MODEL — $V_4{}^1$

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

Z_n	$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	X_1	X_2	X_3	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	$\bar{\mathcal{H}}_1$	$\bar{\mathcal{H}}_2$	$\bar{\mathcal{H}}_3$	$\bar{\mathcal{H}}_4$	Σ
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_8	4	3	5	0	7	1	0	2	6	1	4	6	2	5	0

We find the following symmetries ⇒
parametric relations among couplings ⇒ 2-loop solution

up-type Yukawa

$$|g_{124}|^2 = |g_{214}|^2 = \frac{4}{5}g_5^2 \ , \quad |g_{222}|^2 = \frac{2}{5}g_5^2 \ , \quad |g_{231}|^2 = |g_{321}|^2 = \frac{1}{10}\left(8g_5^2 - 5\,|g_{111}|^2\right) \ ,$$

$$|g_{333}|^2 = \frac{6}{5}g_5^2 \ , \quad |\bar{g}_{111}|^2 = |\bar{g}_{124}|^2 = \frac{3}{20}\left(8g_5^2 - 5\,|g_{111}|^2\right) \ ,$$

down-type Yukawa

$$|\bar{g}_{214}|^2 = \frac{3}{4}\,|g_{111}|^2 \ , \quad |\bar{g}_{222}|^2 = |\bar{g}_{231}|^2 = \frac{3}{10}g_5^2 \ , \quad |\bar{g}_{321}|^2 = -\frac{3}{20}\left(2g_5^2 - 5\,|g_{111}|^2\right) \ ,$$

$$|\bar{g}_{333}|^2 = \frac{9}{10}g_5^2 \ , \quad |f_{22}|^2 = \frac{3}{4}g_5^2 \ , \quad |f_{33}|^2 = \frac{g_5^2}{4} \ , \quad |p|^2 = \frac{15}{7}g_5^2 \ ,$$

$$|g_{132}|^2 = |g_{312}|^2 = |\bar{g}_{132}|^2 = |\bar{g}_{312}|^2 = |f_{11}|^2 = |f_{44}|^2 = 0 \ .$$

By imposing the positivity conditon to the squared norm of the couplings, we find the following constraint for $|g_{111}|^2$:

$$\frac{2}{5}g_5^2 \leq |g_{111}|^2 \leq \frac{8}{5}g_5^2 \ .$$

evaluating at the end points implies more symmetry = more zeroes

ALL-LOOP FINITE MODEL — V_4^2

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

Z_n	$\bar{\Psi}_1$	$\bar{\Psi}_2$	$\bar{\Psi}_3$	X_1	X_2	X_3	$\mathcal{H}_1$	$\mathcal{H}_2$	$\mathcal{H}_3$	$\mathcal{H}_4$	$\bar{\mathcal{H}}_1$	$\bar{\mathcal{H}}_2$	$\bar{\mathcal{H}}_3$	$\bar{\mathcal{H}}_4$	Σ
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_3	0	2	0	0	2	0	1	1	0	0	1	1	0	0	0
Z_4	3	3	2	3	3	2	2	3	0	2	2	3	0	2	0

- We find the following symmetries ⇒ isolated solution unique relation among couplings ⇒ all-loop finite solution

$$|g_{114}|^2 = |g_{121}|^2 = |g_{211}|^2 = |g_{232}|^2 = |g_{322}|^2 = |g_{333}|^2 = \frac{4}{5}g_5^2$$

$$|\bar{g}_{114}|^2 = |\bar{g}_{121}|^2 = |\bar{g}_{211}|^2 = |\bar{g}_{232}|^2 = |\bar{g}_{322}|^2 = |\bar{g}_{333}|^2 = \frac{3}{5}g_5^2 \quad ,$$

$$|f_{33}|^2 = |f_{44}|^2 = \frac{1}{2}g_5^2 \quad , \quad |p|^2 = \frac{15}{7}g_5^2 \quad .$$

- For the SSB ⇒ sum rule ⇒ 3 free parameters

$$m^2_{\tilde{\bar{\psi}}_1} = m^2_{\tilde{\bar{\psi}}_3} = \frac{1}{6}\left(-MM^\dagger + 9m^2_{H_3}\right) \quad , \quad m^2_{\tilde{\bar{\psi}}_2} = \frac{1}{6}\left(-MM^\dagger - 6m^2_{H_1} + 15m^2_{H_3}\right) ,$$

$$m^2_{\tilde{\chi}_1} = m^2_{\tilde{\chi}_3} = \frac{1}{2}\left(MM^\dagger - m^2_{H_3}\right) \quad , \quad m^2_{\tilde{\chi}_2} = \frac{1}{2}\left(MM^\dagger - 2m^2_{H_1} + m^2_{H_3}\right) ,$$

$$m^2_{\bar{H}_1} = m^2_{\bar{H}_2} = \frac{1}{3}\left(2MM^\dagger + 3m^2_{H_1} - 6m^2_{H_3}\right) \quad , \quad m^2_{\bar{H}_3} = m^2_{\bar{H}_4} = \frac{1}{3}\left(2MM^\dagger - 3m^2_{H_3}\right) ,$$

$$m^2_{H_2} = m^2_{H_1} \quad ; \quad m^2_{H_4} = m^2_{H_3} \quad , \quad m^2_{\phi_\Sigma} = \frac{1}{3}MM^\dagger \quad . \qquad (89)$$

ALL-LOOP FINITE MASS MATRICES

Estrada, MM, Patellis, Zoupanos, Fortschr. Phys. 2024, 24001

- It is possible to find the minimum amount of phases — rephasing invariants
- The mass matrices are then:

$$M_u = \begin{pmatrix} g_{114}\langle\mathcal{H}_4^5\rangle & g_{121}\langle\mathcal{H}_1^5\rangle & 0 \\ g_{211}\langle\mathcal{H}_1^5\rangle & 0 & g_{232}\langle\mathcal{H}_2^5\rangle \\ 0 & g_{322}\langle\mathcal{H}_2^5\rangle & g_{333}\langle\mathcal{H}_3^5\rangle \end{pmatrix} = \frac{2}{\sqrt{5}}g_5 \begin{pmatrix} \langle\mathcal{H}_4^5\rangle & \langle\mathcal{H}_1^5\rangle & 0 \\ \langle\mathcal{H}_1^5\rangle & 0 & \langle\mathcal{H}_2^5\rangle \\ 0 & \langle\mathcal{H}_2^5\rangle & e^{i\phi_3}\langle\mathcal{H}_3^5\rangle \end{pmatrix},$$

$$M_d = \begin{pmatrix} \bar{g}_{114}\langle\bar{\mathcal{H}}_{45}\rangle & \bar{g}_{121}\langle\bar{\mathcal{H}}_{15}\rangle & 0 \\ \bar{g}_{211}\langle\bar{\mathcal{H}}_{15}\rangle & 0 & \bar{g}_{232}\langle\bar{\mathcal{H}}_{25}\rangle \\ 0 & \bar{g}_{322}\langle\bar{\mathcal{H}}_{25}\rangle & \bar{g}_{333}\langle\bar{\mathcal{H}}_{35}\rangle \end{pmatrix} = \sqrt{\tfrac{3}{5}}g_5 \begin{pmatrix} \langle\bar{\mathcal{H}}_{45}\rangle & \langle\bar{\mathcal{H}}_{15}\rangle & 0 \\ e^{i\bar{\phi}_1}\langle\bar{\mathcal{H}}_{15}\rangle & 0 & \langle\bar{\mathcal{H}}_{25}\rangle \\ 0 & e^{i\bar{\phi}_2}\langle\bar{\mathcal{H}}_{25}\rangle & e^{i\bar{\phi}_3}\langle\bar{\mathcal{H}}_{35}\rangle \end{pmatrix}.$$

- After the rotation in the Higgs sector to the MSSM basis:

$$M_u = \frac{2}{\sqrt{5}}g_5 \begin{pmatrix} \widetilde{\alpha}_4 & \widetilde{\alpha}_1 & 0 \\ \widetilde{\alpha}_1 & 0 & \widetilde{\alpha}_2 \\ 0 & \widetilde{\alpha}_2 & e^{i\phi_3}\widetilde{\alpha}_3 \end{pmatrix} \langle\mathcal{K}_3^5\rangle,$$

$$M_d = \sqrt{\frac{3}{5}}g_5 \begin{pmatrix} \widetilde{\beta}_4 & \widetilde{\beta}_1 & 0 \\ e^{i\bar{\phi}_1}\widetilde{\beta}_1 & 0 & \widetilde{\beta}_2 \\ 0 & e^{i\bar{\phi}_2}\widetilde{\beta}_2 & e^{i\bar{\phi}_3}\widetilde{\beta}_3 \end{pmatrix} \langle\bar{\mathcal{K}}_{35}\rangle.$$

Same solution as FUTB for 3rd generation!
we know it works…

α_i, β_i refer to the rotation angles in up and down sectors respectively, $\Sigma\beta_i = \Sigma\alpha_i = 1$

FINALLY, HOW MANY FREE PARAMETERS?

GUT scale 89 free parameters

Yukawa couplings, soft breaking terms, phases, vev's of the Higgs fields

After Finiteness solutions

33 free parameters

Require doublet-triplet splitting, rotation to MSSM basis with constraints over angles, rephasing invariants

Low energies:

radiative electroweak breaking, fix m_τ^{exp} and SM vev give $\tan\beta$

⇒ 12 parameters left:

The soft breaking terms, the phases, and the rotation angles

$\phi_1, \phi_2, \phi_3, \phi_4, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, M, \mu$

Only one phase is observable

$\Rightarrow \phi_{obs}, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, M, \mu$

only 9 parameters left to fit masses and mixing angles

$SU(3)^3$

- Trinification model beta function

$$b = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N \,.$$

- Finite $\Longleftrightarrow$ 3 generations

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3),$$

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*).$$

- Only $SU(5)$ and $SU(3)^3$ seem to have phenomenological possibilities so far

WHAT ABOUT NEUTRINO MASSES, DARK MATTER, ETC?

- **SU(5) models:**
  Cold DM
  LSP is neutralino
  $\Rightarrow$ overabundance
- Neutrino masses may be incorporated by breaking R symmetry $\Rightarrow$
  gravitino Dark Matter
- Other mechanisms?
  thermal inflation?
- g-2 like in SM

- **$SU(3)^3$ models:**
  ν_R are present
- Neutrino masses may be generated by seesaw or radiatively
- Depending on the breaking of $SU(3)^3$
  DM may be neutralino
  (or scalar?)
- Neutralino DM overabundance

Flavor Structure may change the above!

CONCLUSIONS AND OUTLOOK

- Reduction of couplings finiteness powerful principle implies Gauge Yukawa Unification
- Conformal or scale invariant theory
- SSB terms satisfy a sum rule among soft scalars
- SSB same as anomaly mediated scenario
- Finiteness reduces greatly number of free parameters completely finite theories SU(5)
- Very predictive

- Flavor 3 generation models
  2-loops: Yukawa couplings determined within a range
  All-loops: Yukawa couplings completely determined
- Leads to viable mass textures
- Drastic reduction in number of free parameters
- Free parameters come from Higgs sector, SSB and phases
- More fundamental theory?

How can we restrict phases? CP violation?
Higgs sector? Flavor processes?
Dark matter? Inflation?

Thank you!