Searching for Sub-GeV dark matter using the Migdal effect

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In collaboration with Alejandro Ibarra, Merlin Reichard, Stefano Scopel, Sunghyun Kang

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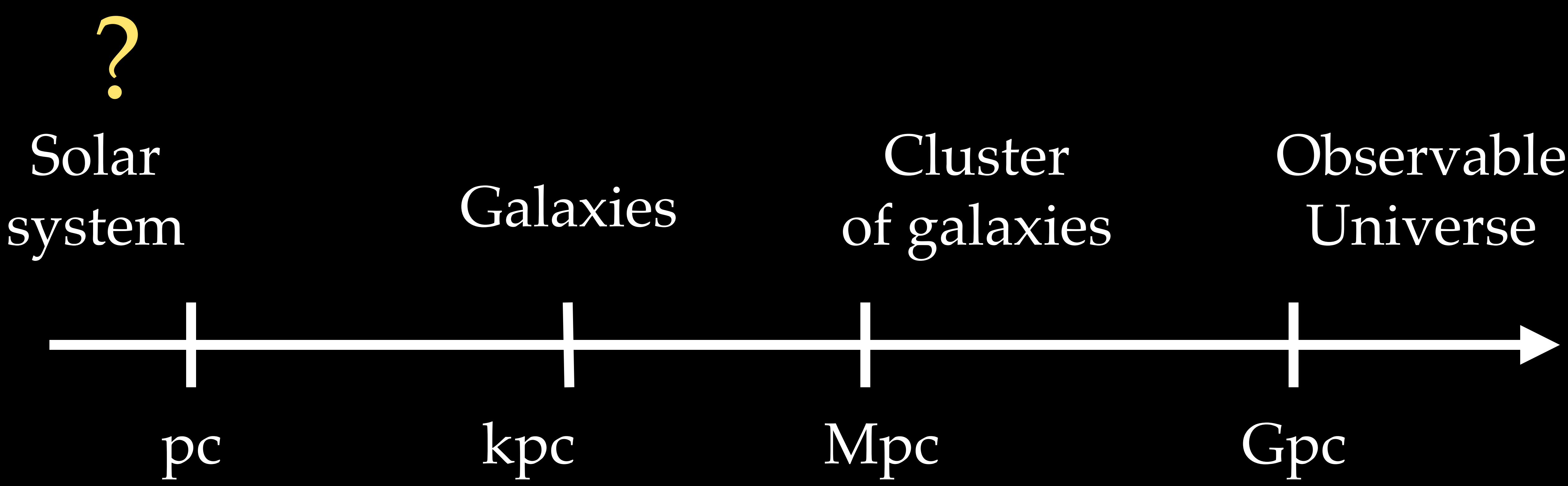
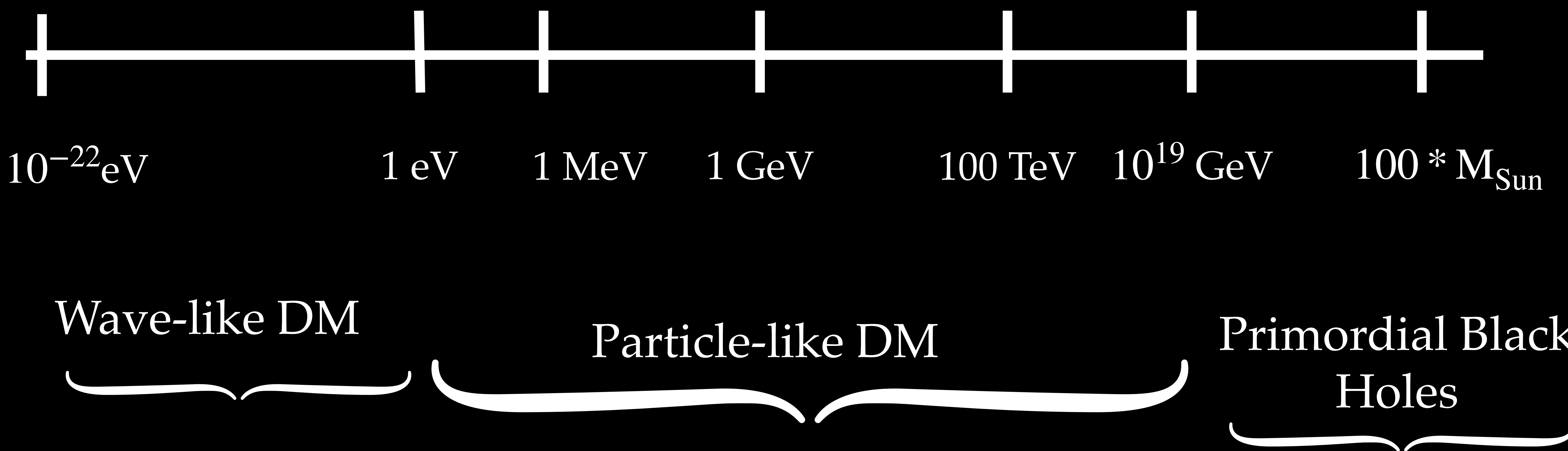
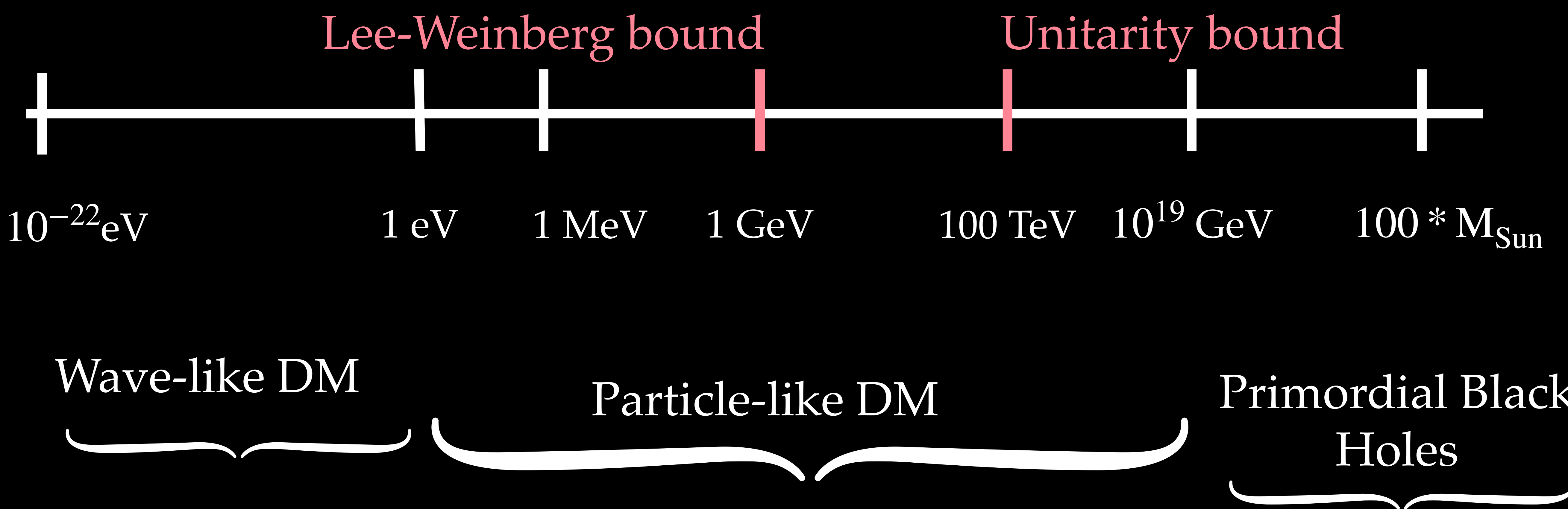
There are evidence for dark matter in a wide range of distance scales



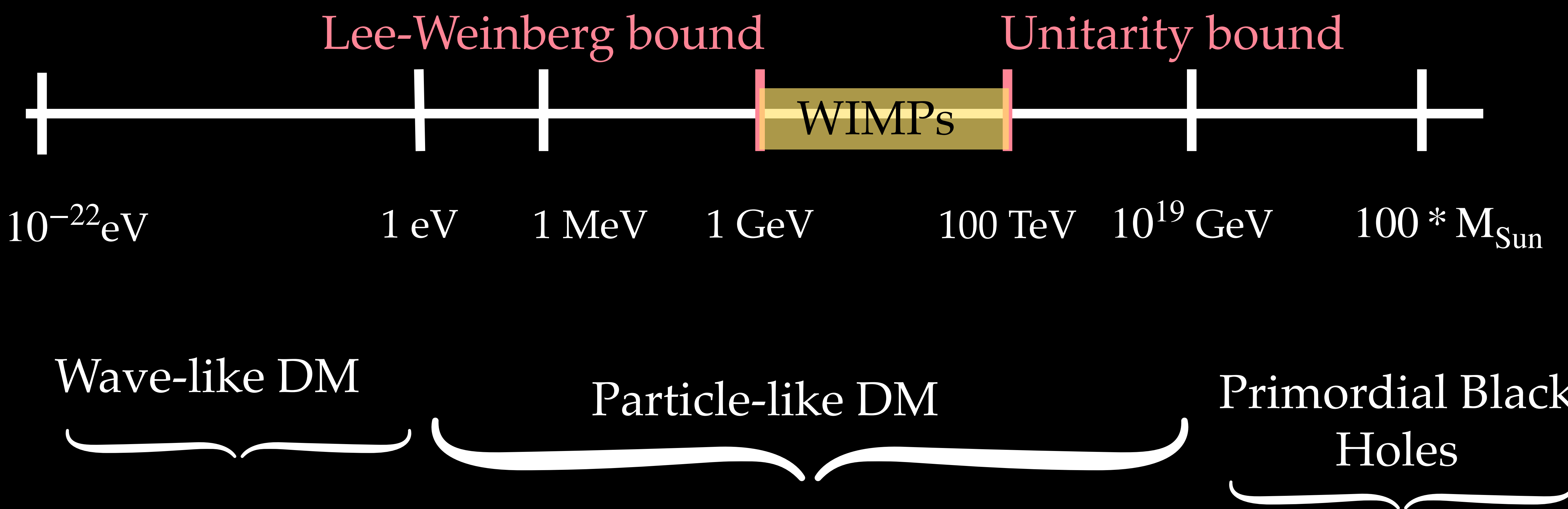
What is dark matter mass?



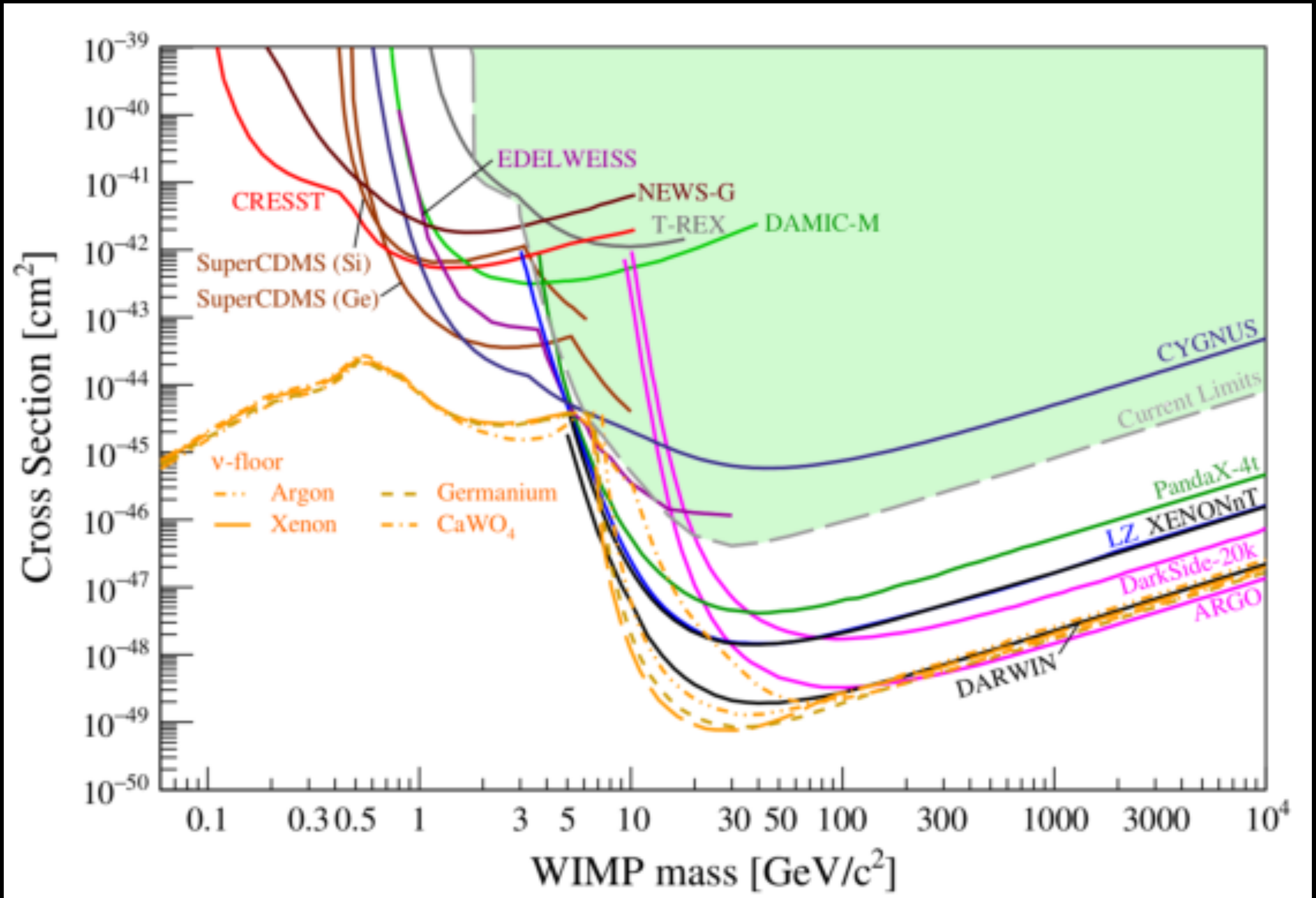
What is dark matter mass?



What is dark matter mass?

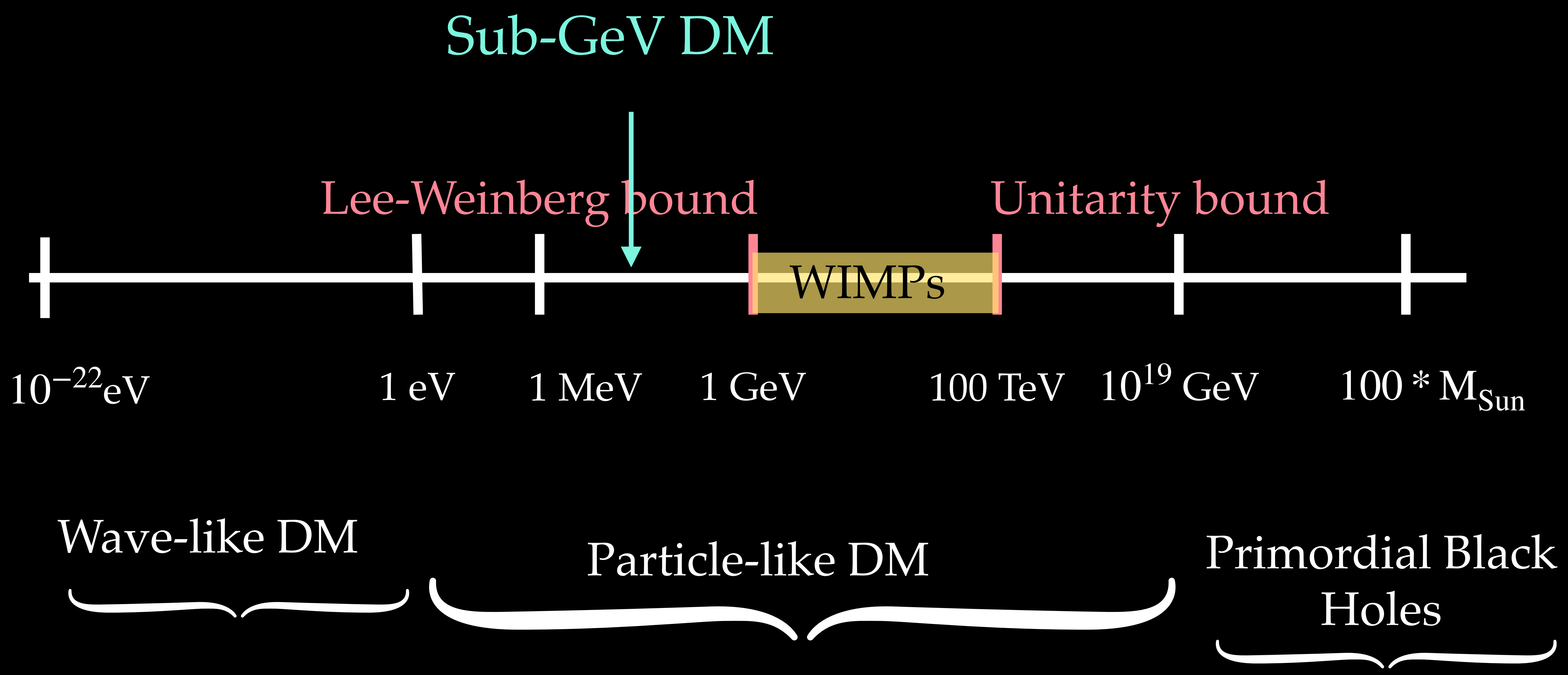


WIMP searches in DD Experiments

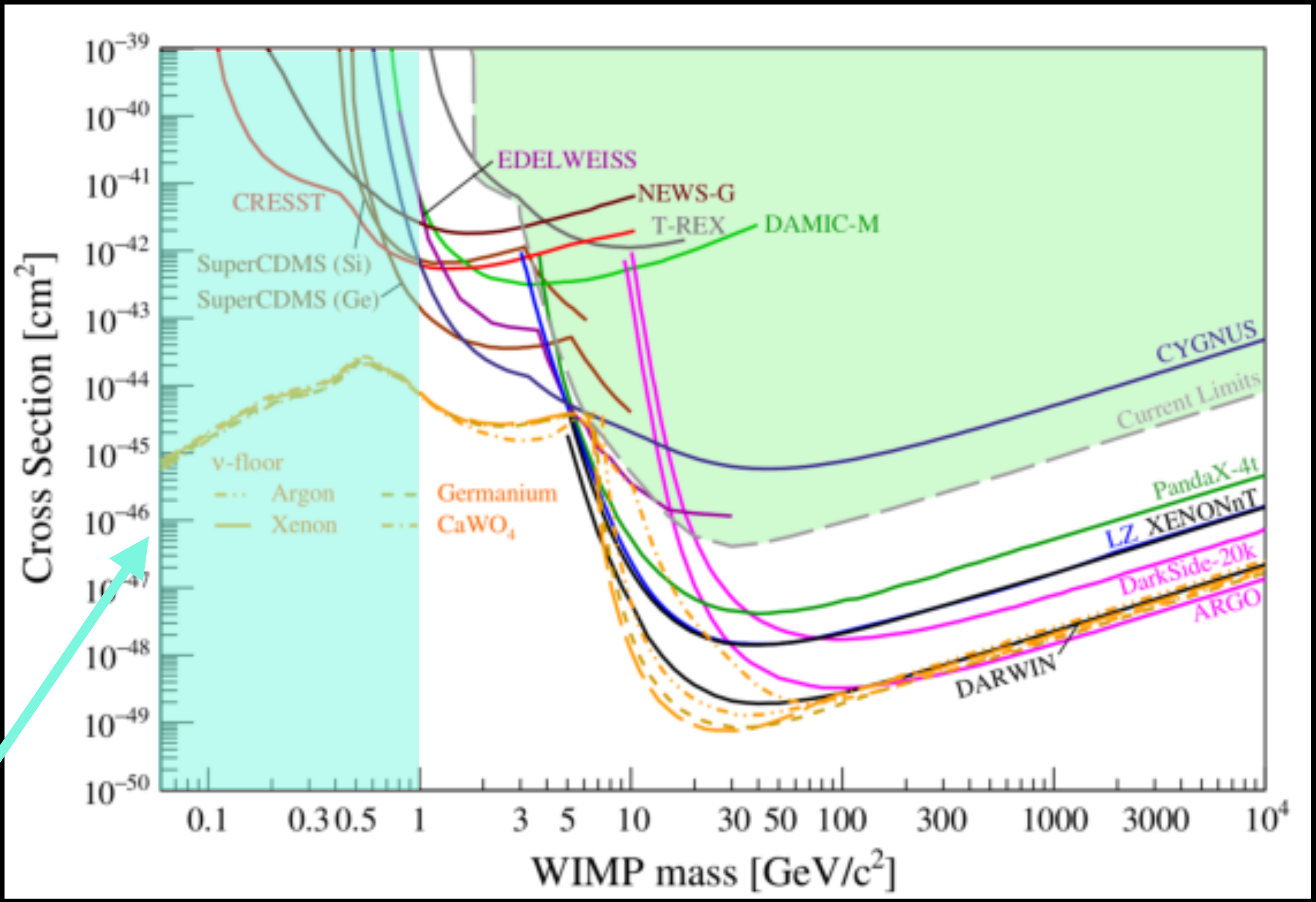


APPEC: arXiv: 2104.07634

What is dark matter mass?



Motivation



APPEC: arXiv: 2104.07634

How to explore Sub-GeV DM?

Challenges

- Nuclear recoil Energy of Sub-GeV dark matter: below the experimental threshold. Most detectors are insufficient.

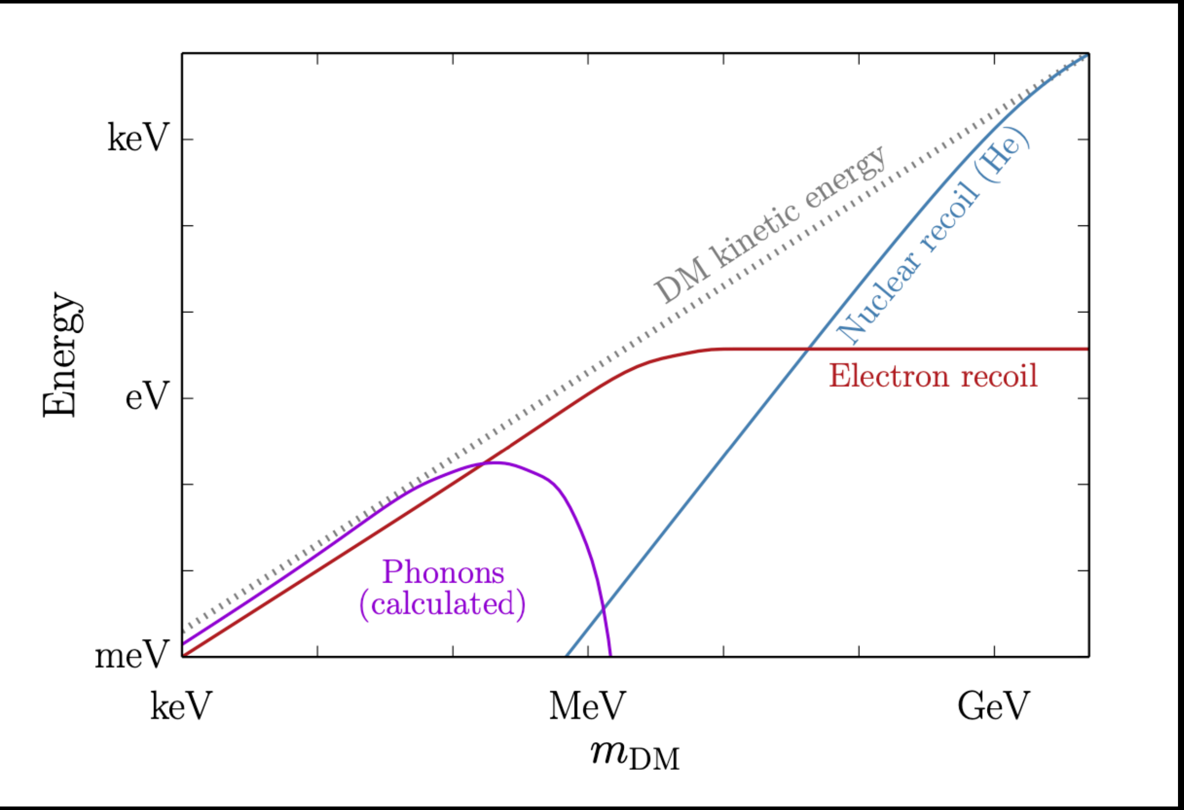
$$E_R \sim m_{\rm DM}^2 v_{\rm DM}^2 / m_T$$

Challenges

- Nuclear recoil Energy of Sub-GeV dark matter: below the experimental threshold. Most detectors are insufficient.

$$E_R \sim m_{\mathrm{DM}}^2 v_{\mathrm{DM}}^2 / m_T$$

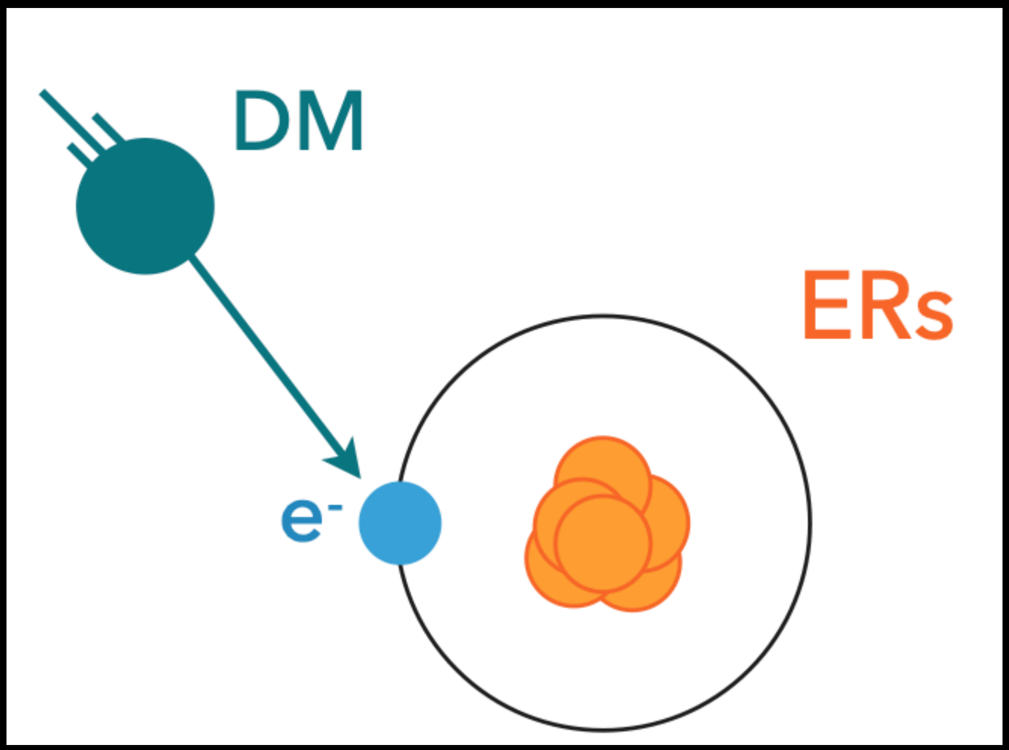
- Sub-GeV Dark Matter necessitates new detection methods: Electron recoils, phonons, or the Migdal Effect.



Tongyan Lin
PoS 333 (2019) 009

Direct Detection of Sub-GeV DM

Detection via scattering off electron



Direct Detection of Sub-GeV DM

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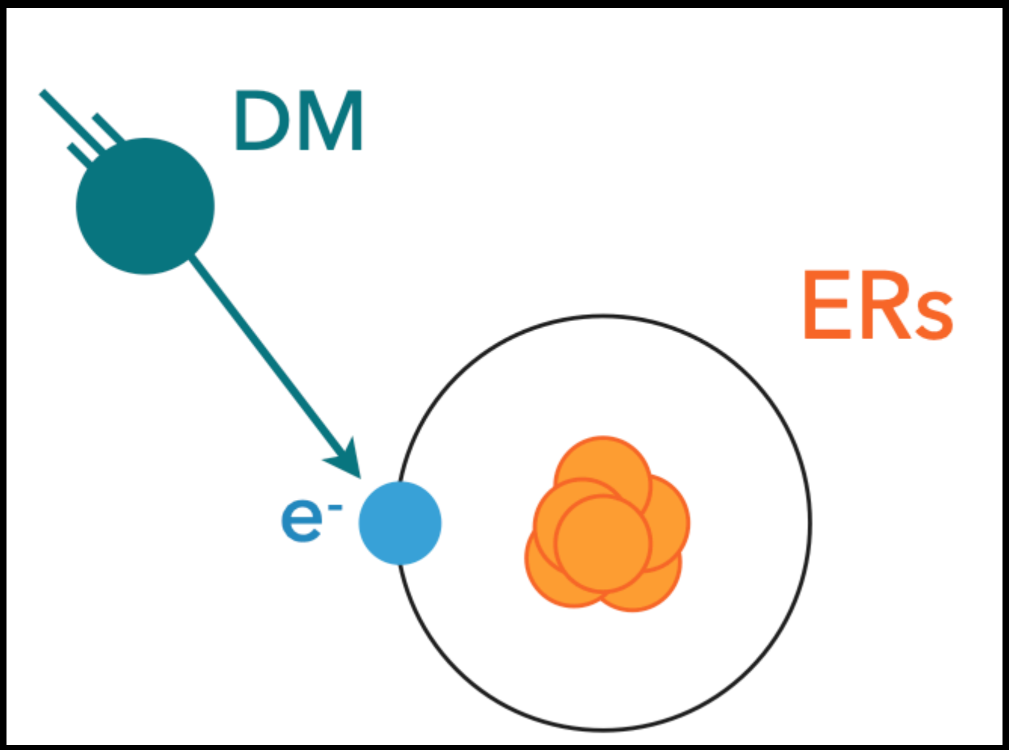
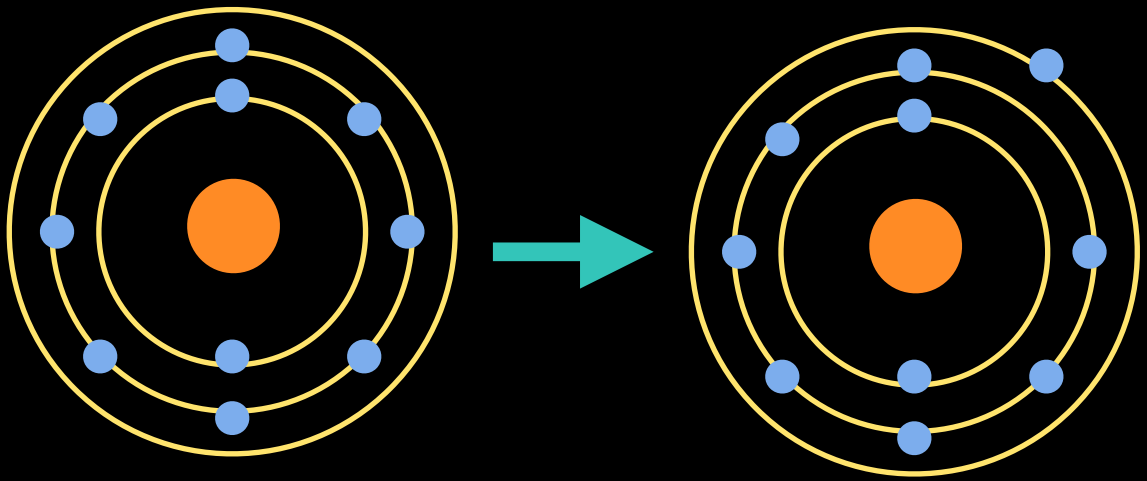


Migdal Effect

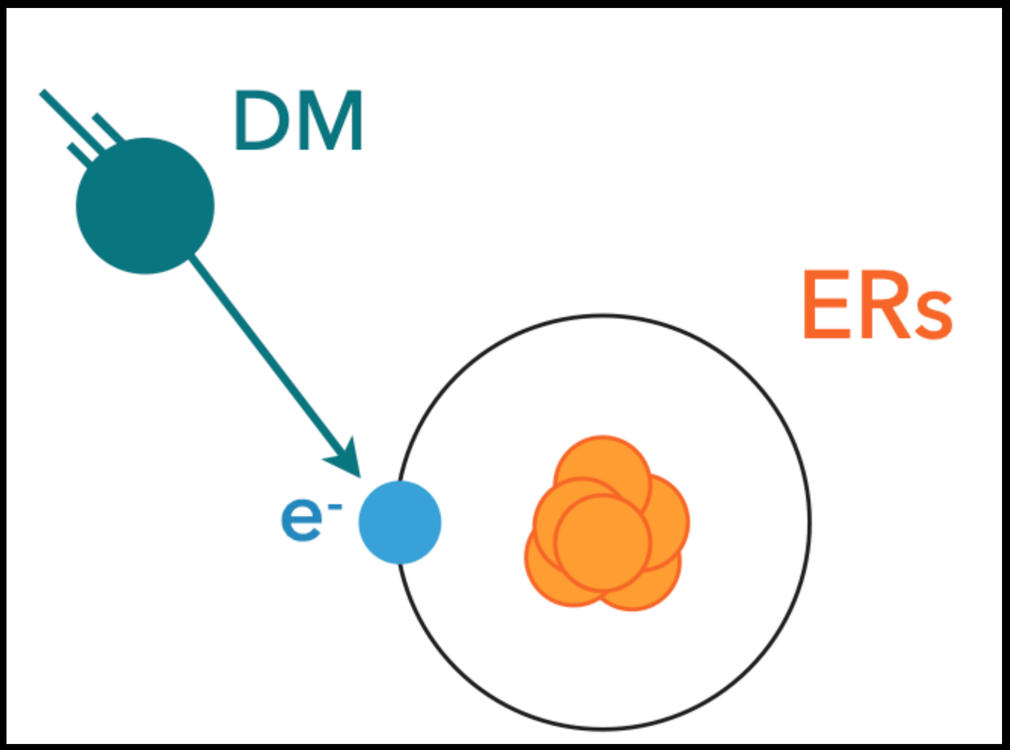


Real state

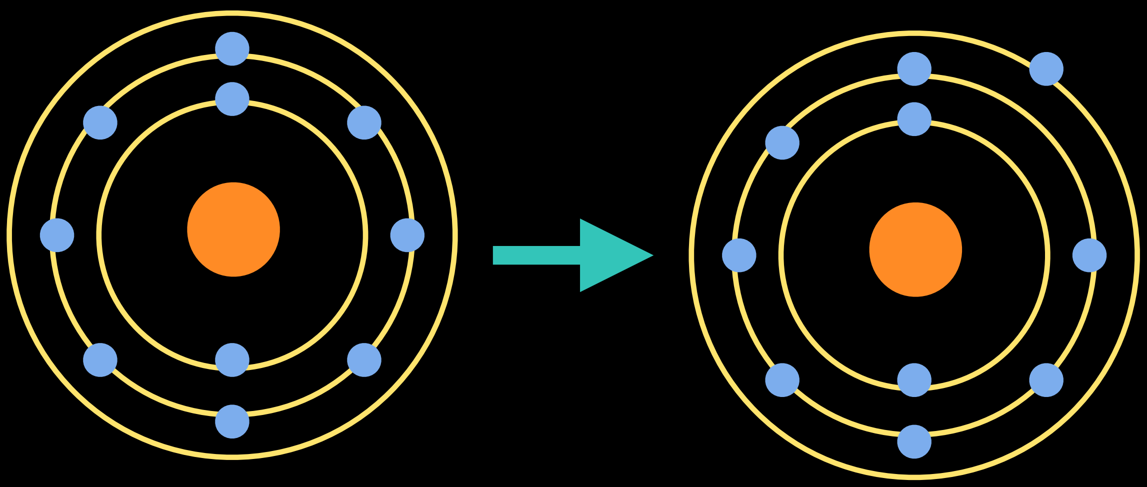
Migdal, J.Phys. USSR 4 (1941) 449

Direct Detection of Sub-GeV DM

Detection via scattering off electron



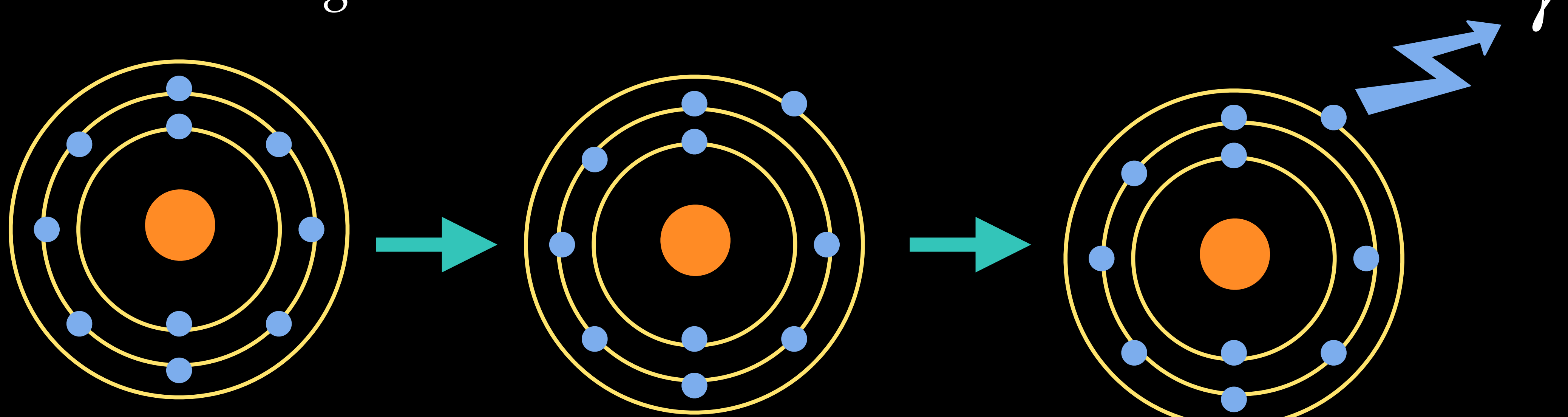
Migdal Effect



Real state

Migdal, J.Phys. USSR 4 (1941) 449

Bremsstrahlung

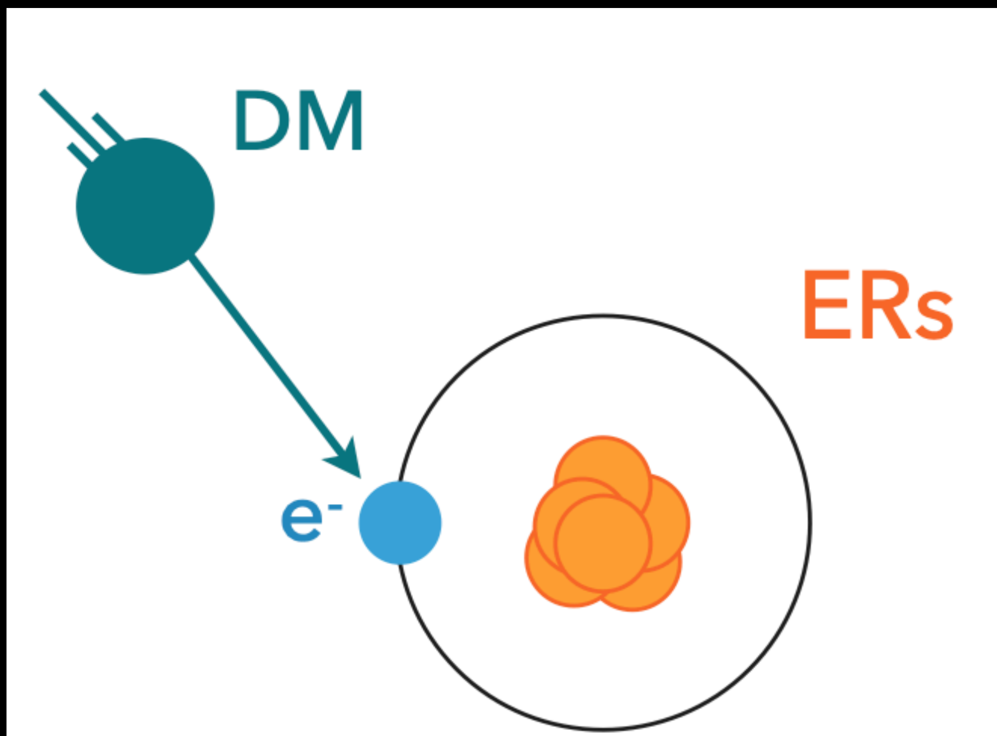


Virtual state

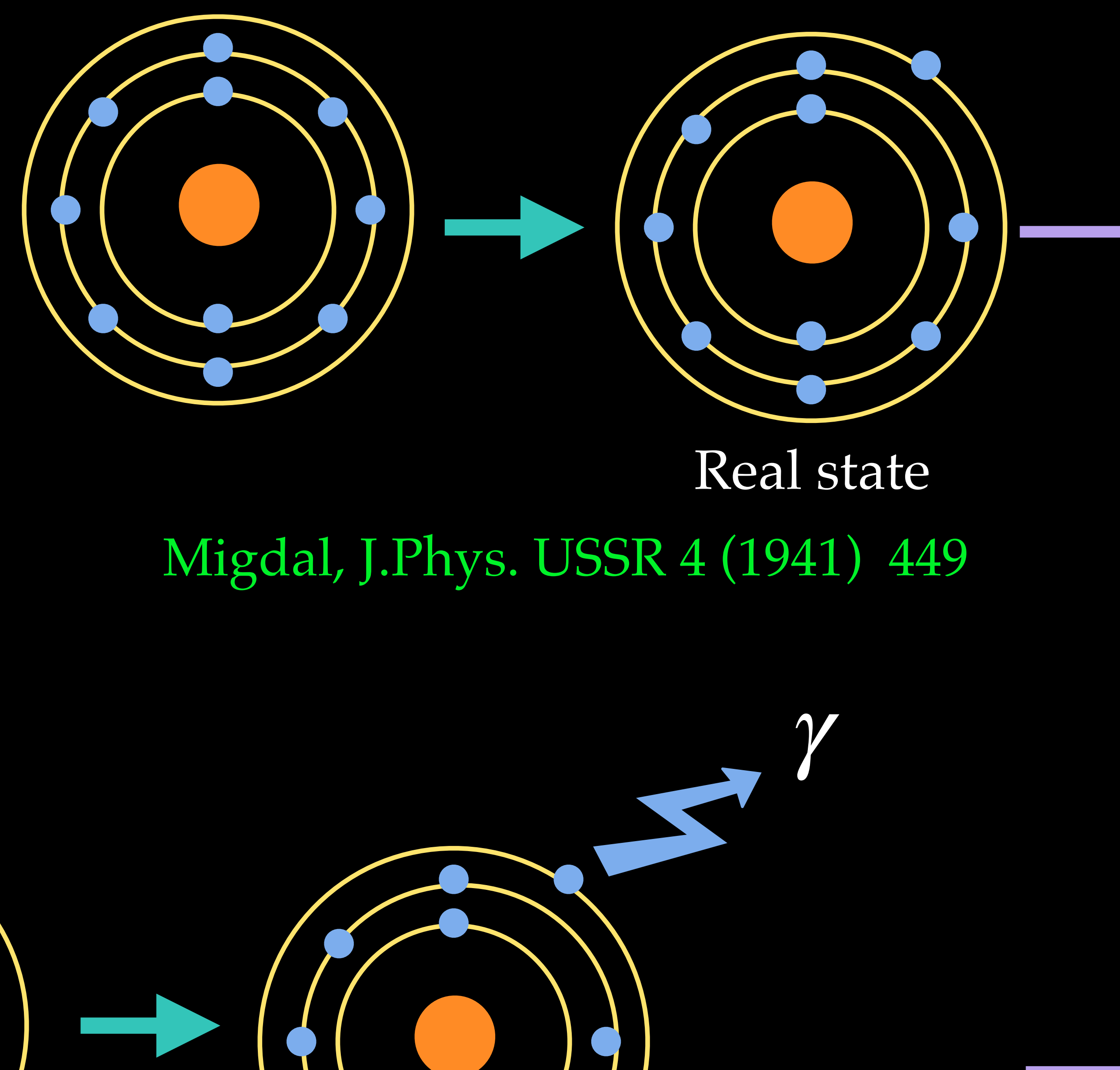
Kouvaris and Pradler, arXiv:1607.01789

Direct Detection of Sub-GeV DM

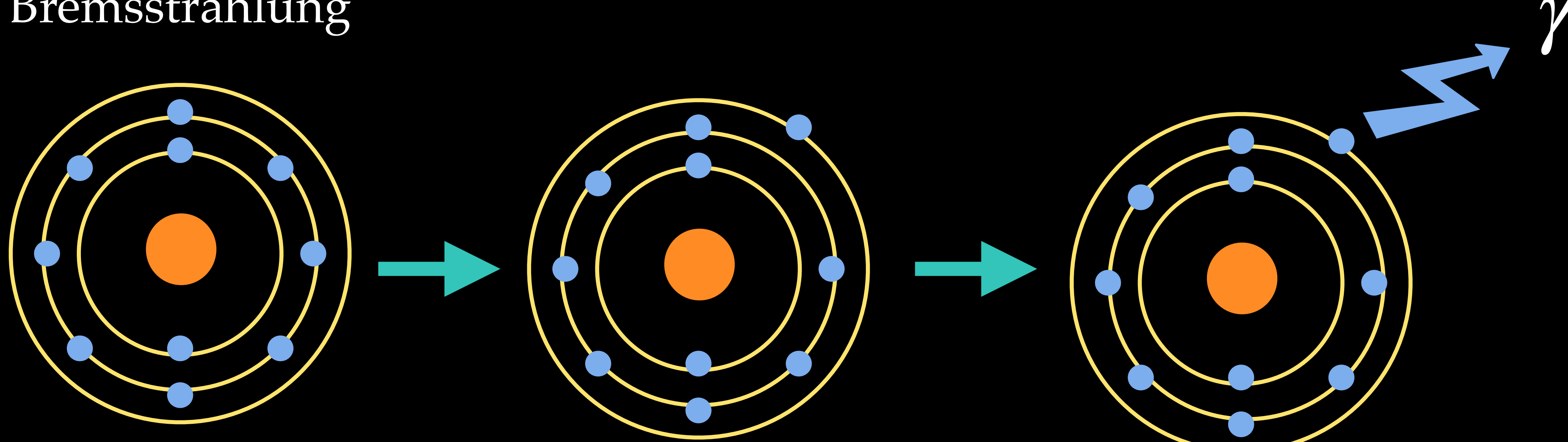
Detection via scattering off electron



Migdal Effect

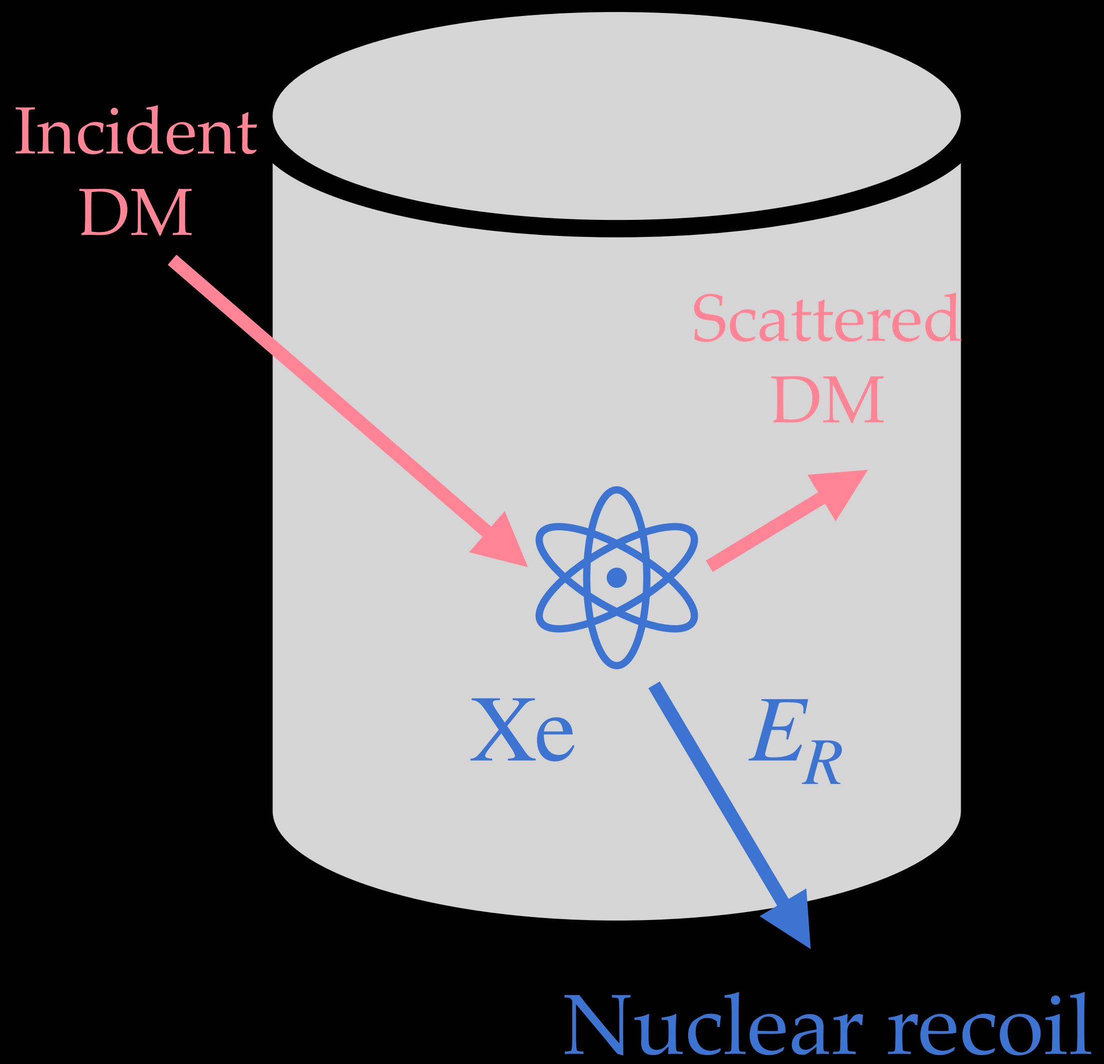


Bremsstrahlung

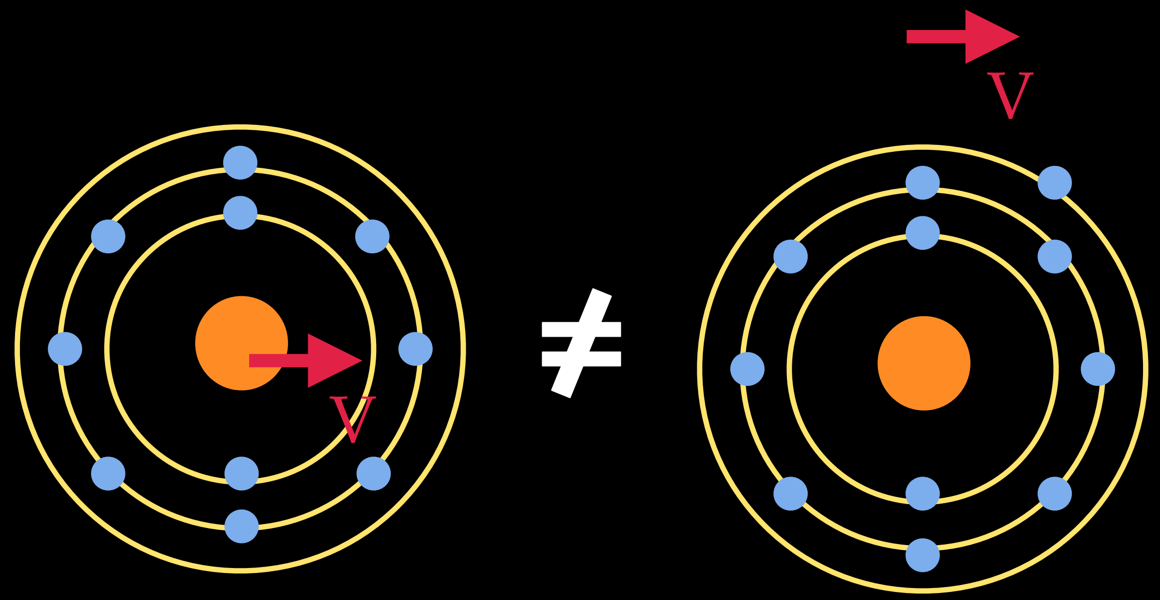


Kouvaris and Pradler, arXiv:1607.01789

Migdal effect



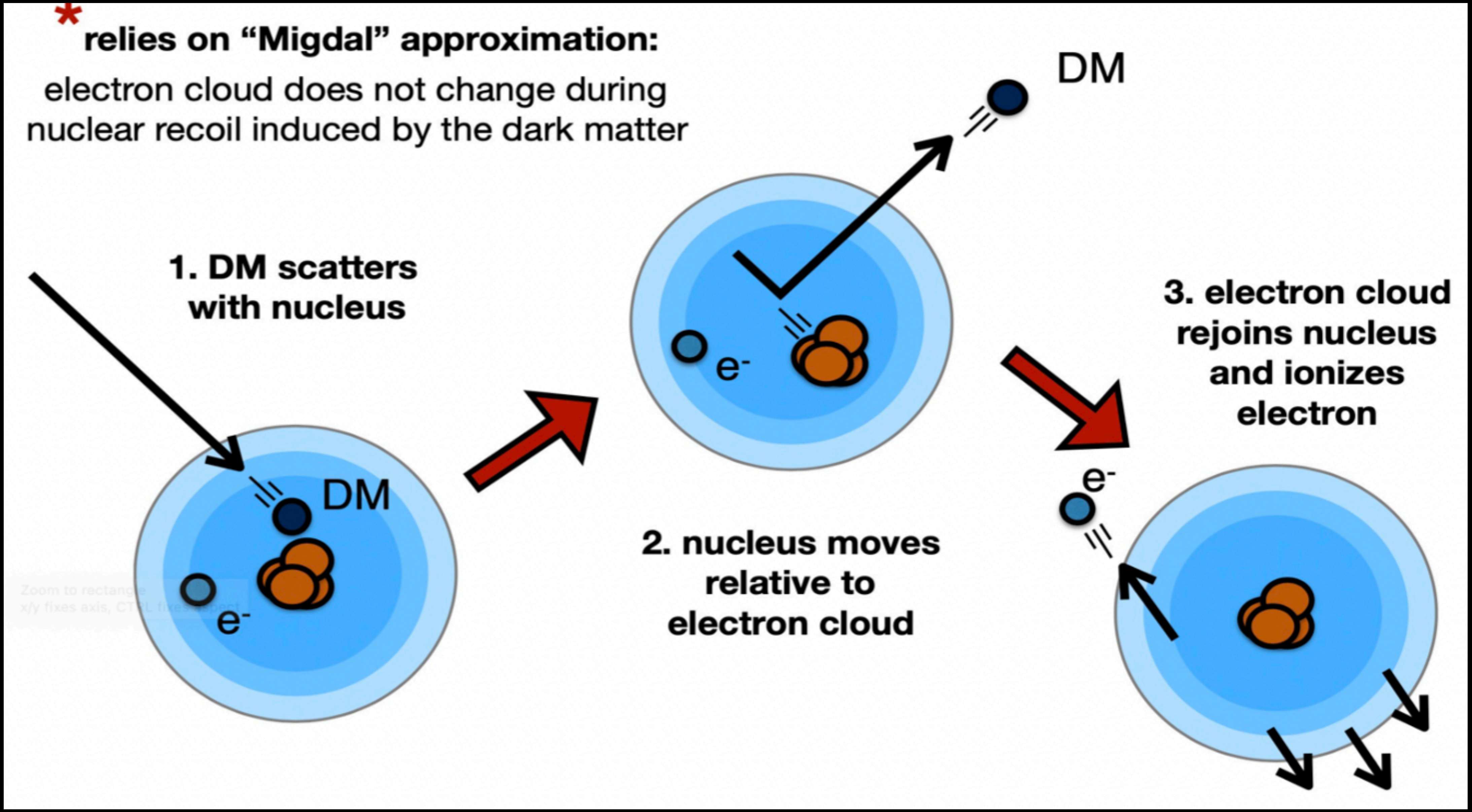
In conventional analysis the *recoiled nucleus* is treated as a *recoiled neutral atom*



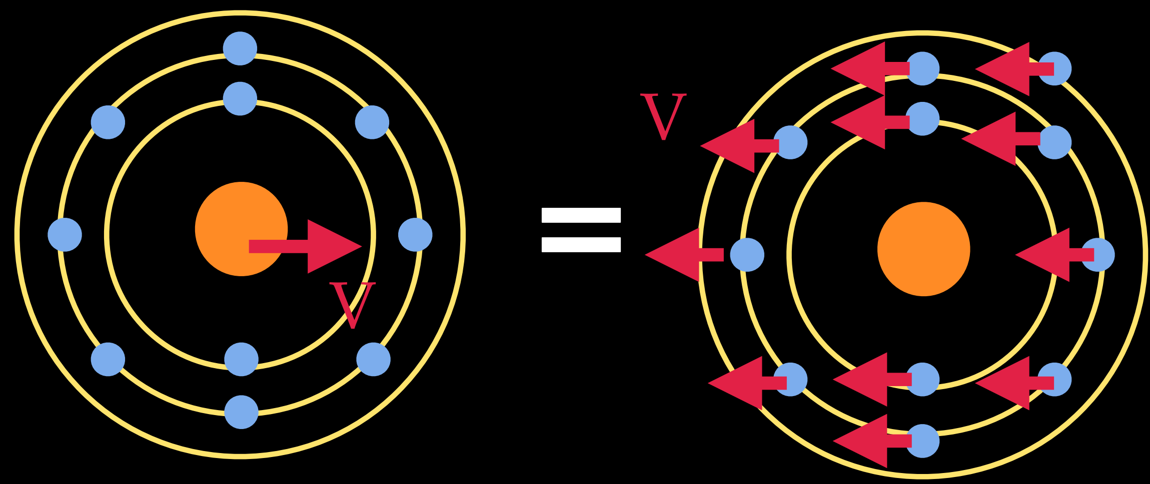
Migdal, J.Phys. USSR 4 (1941) 449

Migdal effect

The electron catch up process to nucleus leads to ionisations



Migdal effect



$$\left|\Psi_{\text{in}}\right\rangle \qquad e^{-im_e v\cdot \sum_i x_i}\left|\Psi_{\text{in}}\right\rangle$$

Electron wave function

The probability of the ionisation is given by

$$P = \left|\left\langle \Psi_F \middle| \Psi_{\text{in}} \right\rangle\right|^2 = \left|\left\langle \Psi_F \middle| e^{-im_e v\cdot \sum_i x_i} \middle| \Psi_{\text{in}} \right\rangle\right|^2$$

Migdal effect

ELSEVIER
Physics Letters B
Volume 606, Issues 3–4, 27 January 2005, Pages 313-322

The role of ionization electrons in direct neutralino detection

J.D. Vergados [a b 1], H. Ejiri [c d]

ELSEVIER
Nuclear Physics B
Volume 727, Issues 1–2, 31 October 2005, Pages 406-420

Direct dark matter search by observing electrons produced in neutralino–nucleus collisions

Ch.C. Moustakidis [a], J.D. Vergados [b], H. Ejiri [c]

ELSEVIER
Physics Letters B
Volume 639, Issues 3–4, 10 August 2006, Pages 218-222

Dark matter search by exclusive studies of X-rays following WIMPs nuclear interactions

H. Ejiri [a b], Ch.C. Moustakidis [c], J.D. Vergados [d]

International Journal of Modern Physics A | Vol. 22, No. 19, pp. 3155-3168 (2007)

| **Research Papers**

ON ELECTROMAGNETIC CONTRIBUTIONS IN WIMP QUESTS

R. BERNABEI, P. BELLI, F. MONTECCHIA, F. NOZZOLI, F. CAPPELLA, A. INCICCHITTI, D. PROSPERI, R. CERULLI, C. J. DAI, H. L. HE, H. H. KUANG, J. M. MA, X. D. SHENG, and Z. P. YE

Migdal effect

Migdal effect in dark matter direct detection experiments

Masahiro Ibe,[a,b] **Wakutaka Nakano,**[a] **Yutaro Shoji**[a] **and Kazumine Suzuki**[a]

[a] *ICRR, The University of Tokyo,*
Kashiwa, Chiba 277-8582, Japan

[b] *Kavli IPMU (WPI), UTIAS, The University of Tokyo,*
Kashiwa, Chiba 277-8583, Japan

E-mail: ibe@icrr.u-tokyo.ac.jp, m156077@icrr.u-tokyo.ac.jp, yshoji@icrr.u-tokyo.ac.jp, ksuzuki@icrr.u-tokyo.ac.jp

The ionisation event rate in an experiment due to the Migdal effect

$$\frac{d^3R}{dE_R\,dE_{det}\,dv_{\chi T}} = \frac{d^2R_{\chi T}}{dE_R\,dv_{\chi T}} \times \frac{1}{2\pi}\sum_{n,l}\frac{d}{dE_e}\,p^c_{q_e}(nl \to (E_e))$$

differential rate

ionisation probability

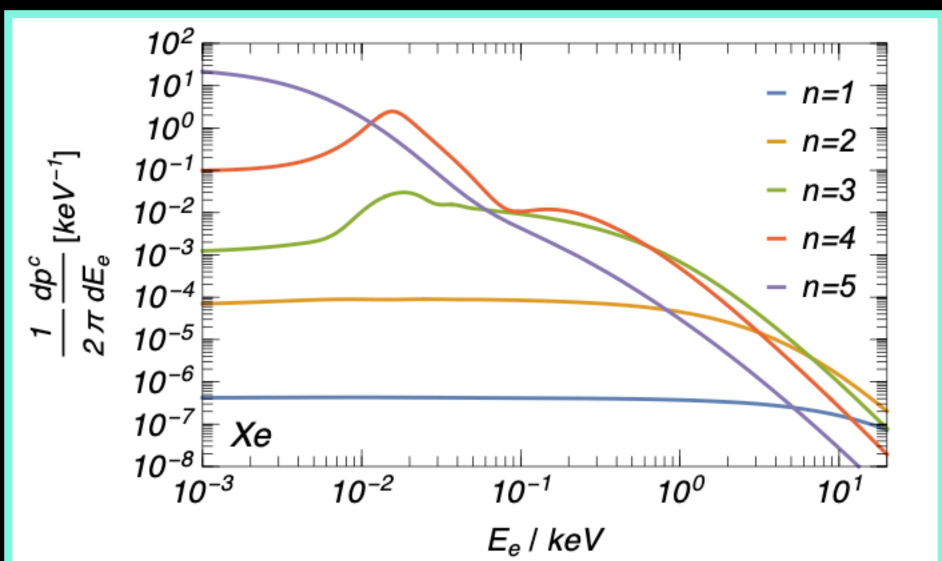
Migdal effect in isolated atoms

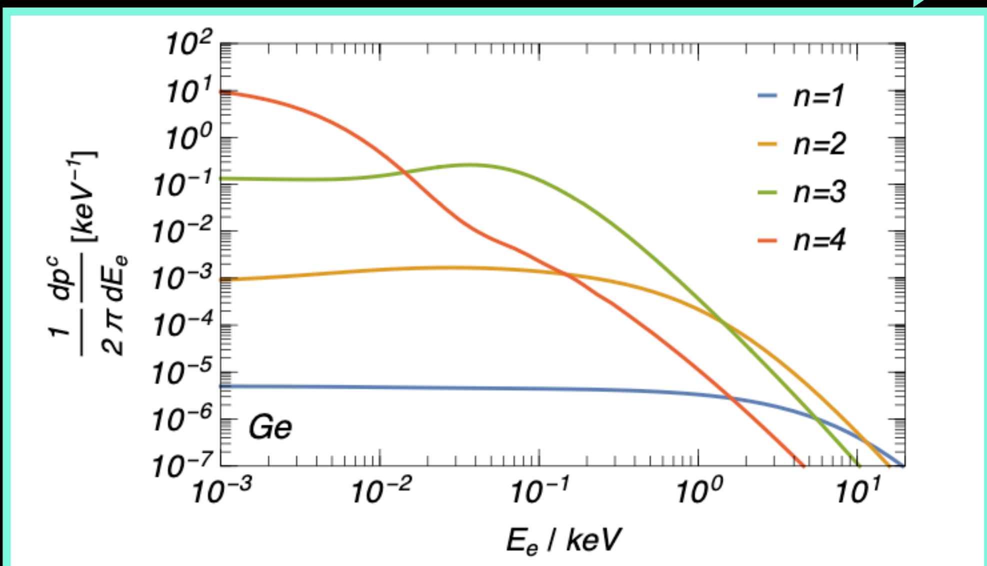
$$\frac{d^3R}{dE_R dE_{det} dv_{\chi T}} = \frac{d^2 R_{\chi T}}{dE_R dv_{\chi T}} \times \frac{1}{2\pi} \sum_{n,l} \frac{d}{dE_e} p^c_{q_e}(nl \to (E_e))$$

differential rate

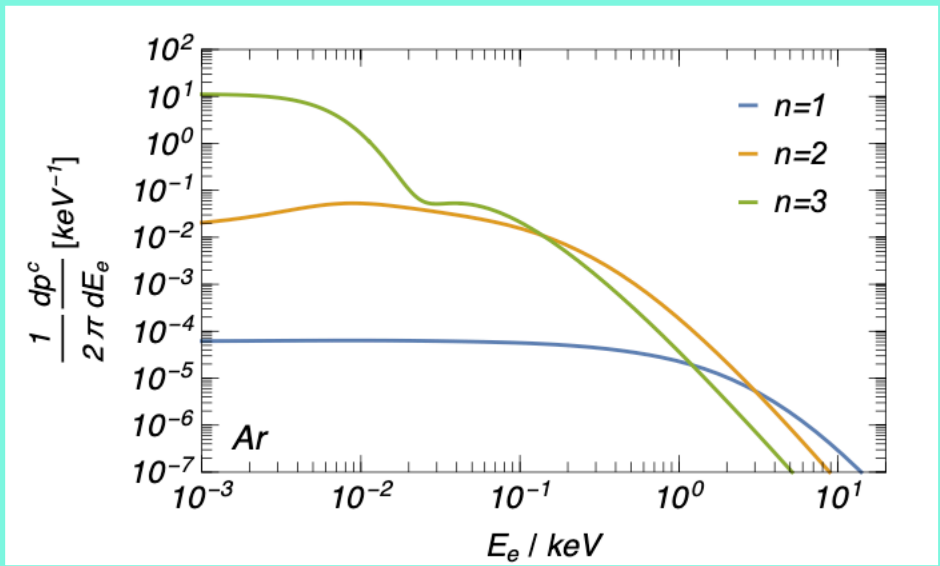
ionisation probability

$q_e = m_e \sqrt{2E_R / m_T}$





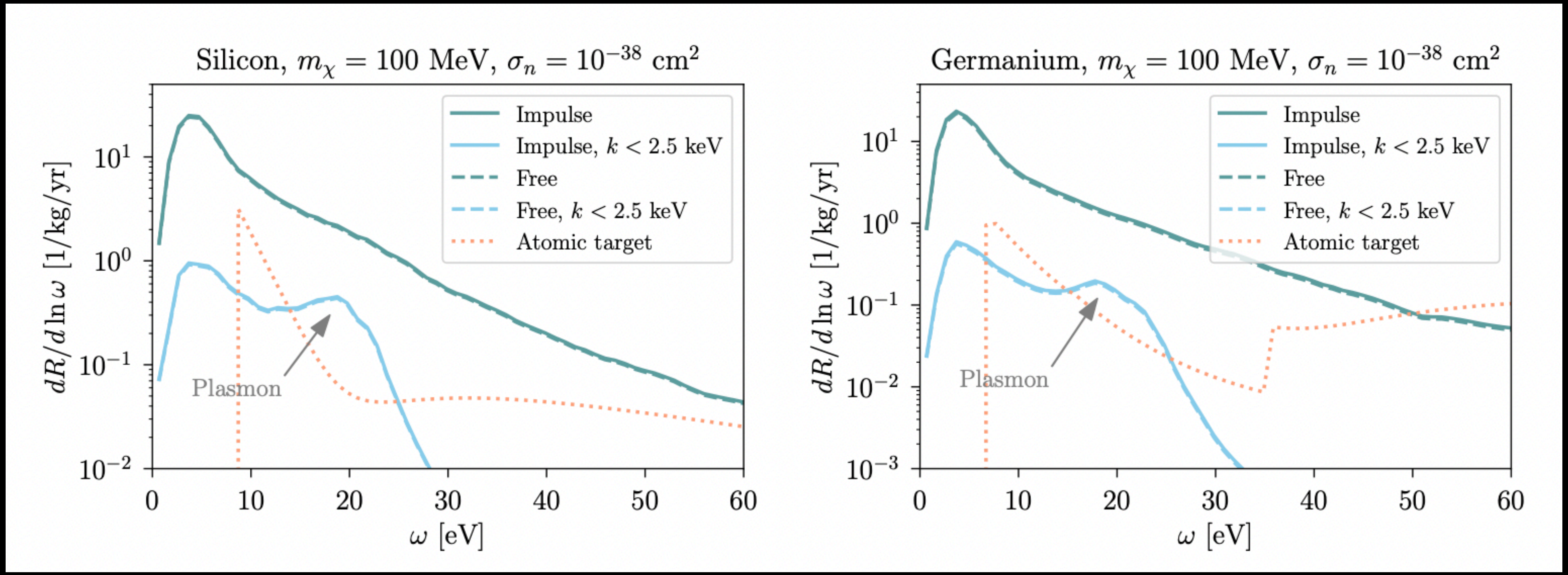
Ibe, Nakano, Shoji, Suzuki *JHEP* 03 (2018) 194



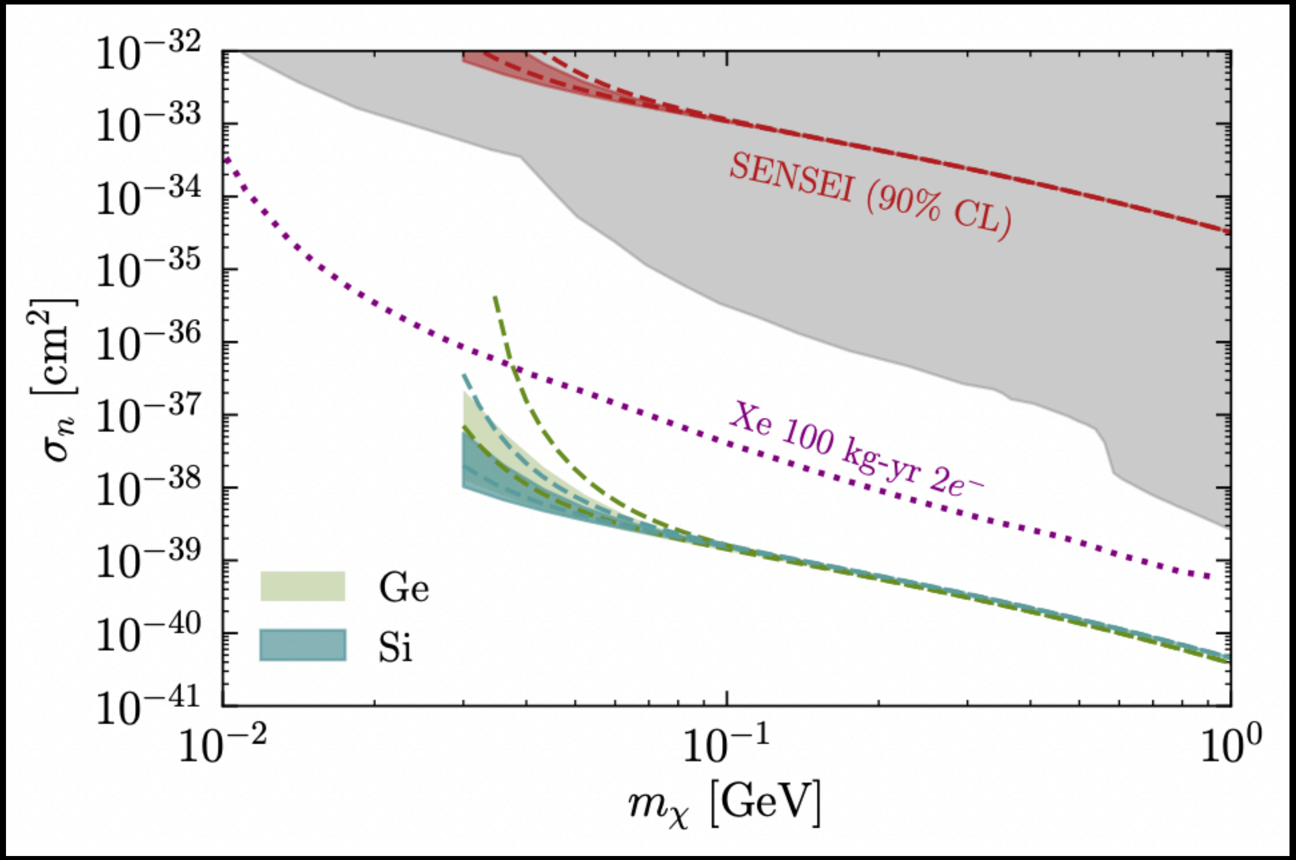
* Atoms in liquid or a crystal: shifts in electronic energy

* Outer shells are specifically affected

Migdal effect in semiconductors



Knapen, Kozaczuk, Lin
PRL 127 (2021) 8, 081805

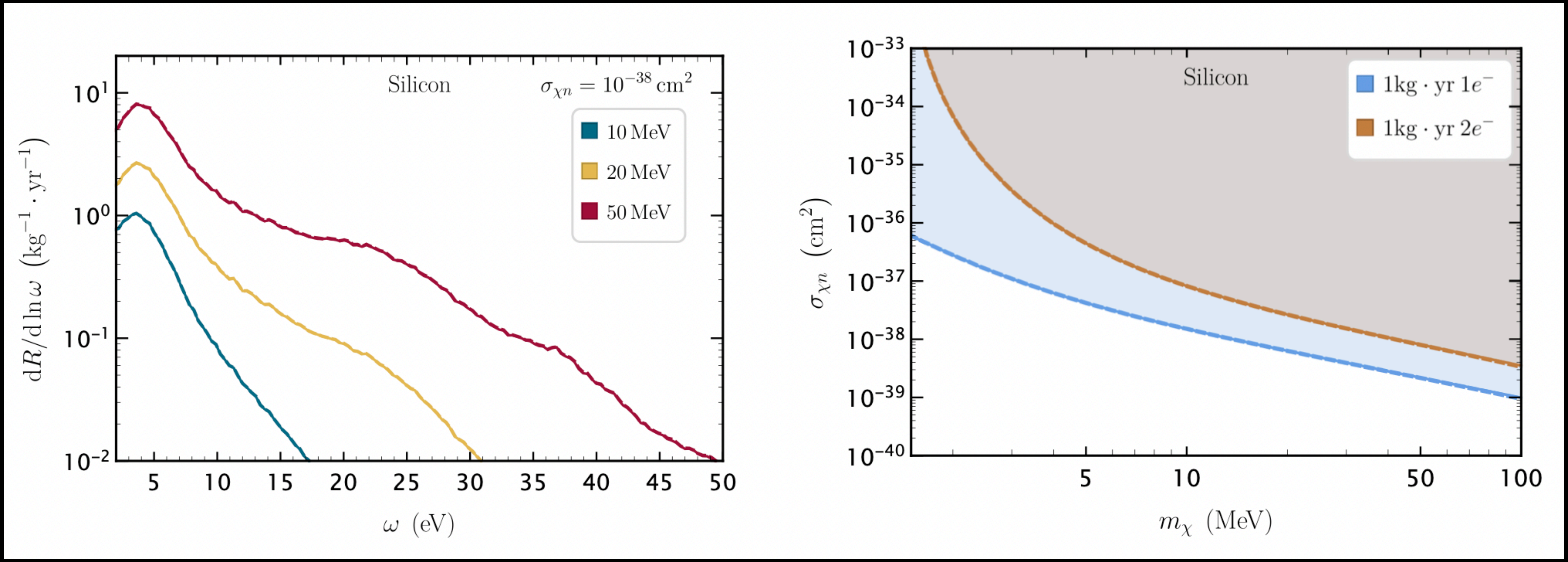


* Crystals share a complicated spectrum of excitations

* Boosting system in the rest frame of the nucleus does not work

* Impulse approximation is used to treat the excited state

Migdal effect in semiconductors using phonon

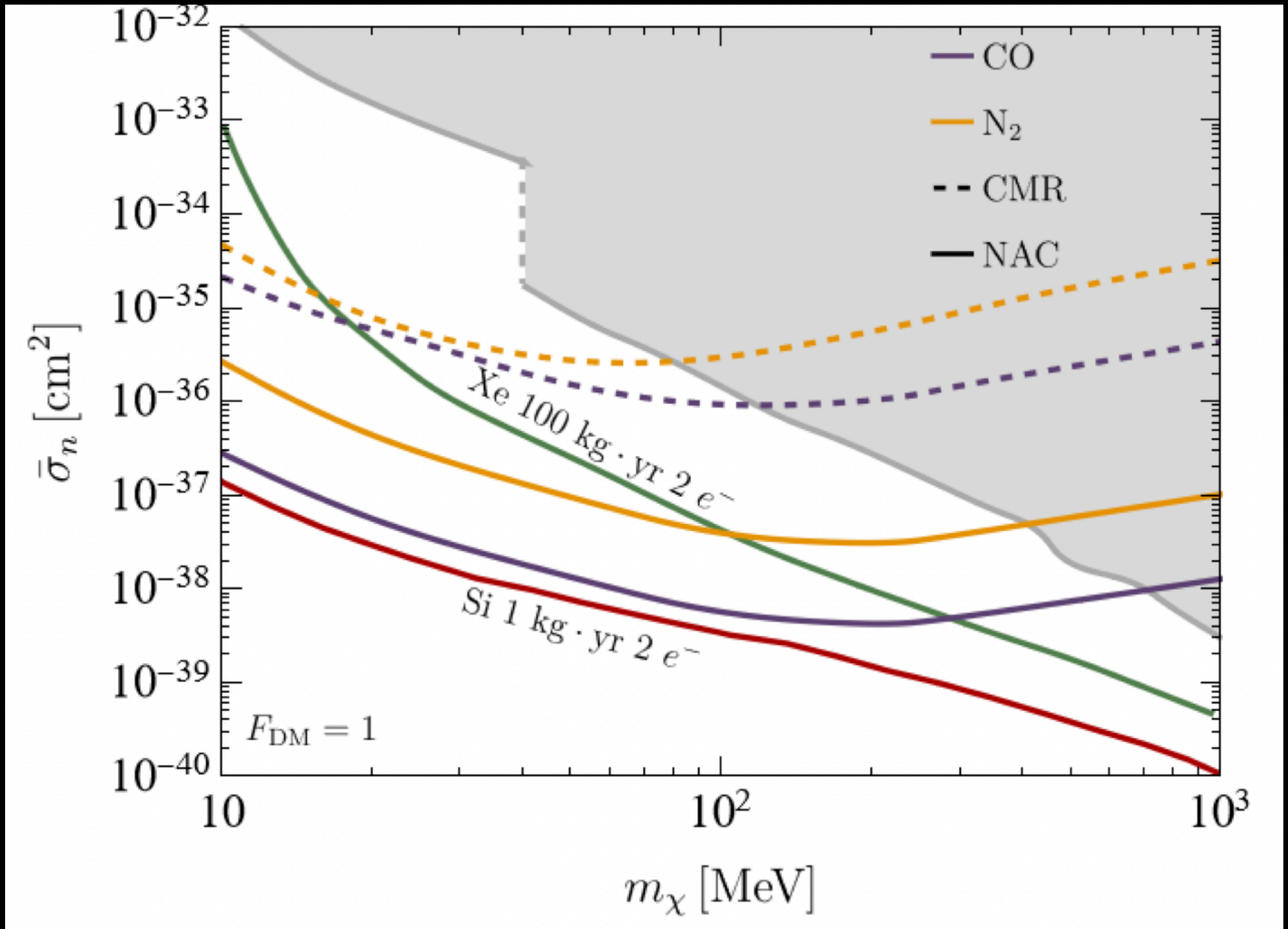


Liang, Mo, Lin et.al.
Phys.Rev.D 106 (2022) 4, 043004

* Below $m_\chi = 50$ MeV the effects of phonon becomes important

* Isotropic material (Silicon, Germanium) is considered

Molecular Migdal effect

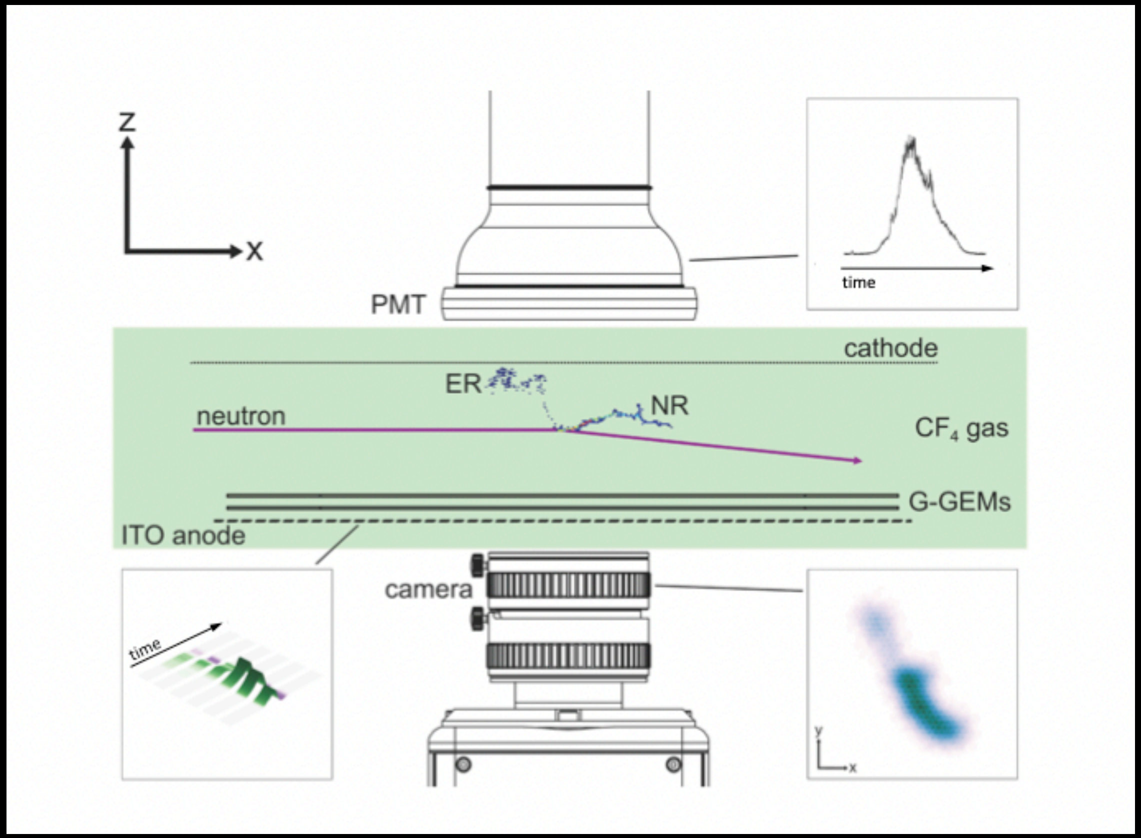


Blanco et. Al.
Phys.Rev.D 106 (2022) 11, 115015

- * Center of Mass Recoil (CMR) where molecule is considered as a rigid body
- * Non Adiabatic Coupling (NAC) non-uniform movement within the molecule. It takes into account rotational and vibrational transitions
- * Diatomic molecules are considered, N_2 and CO
- * Modulation is possible due to structure asymmetry of molecule

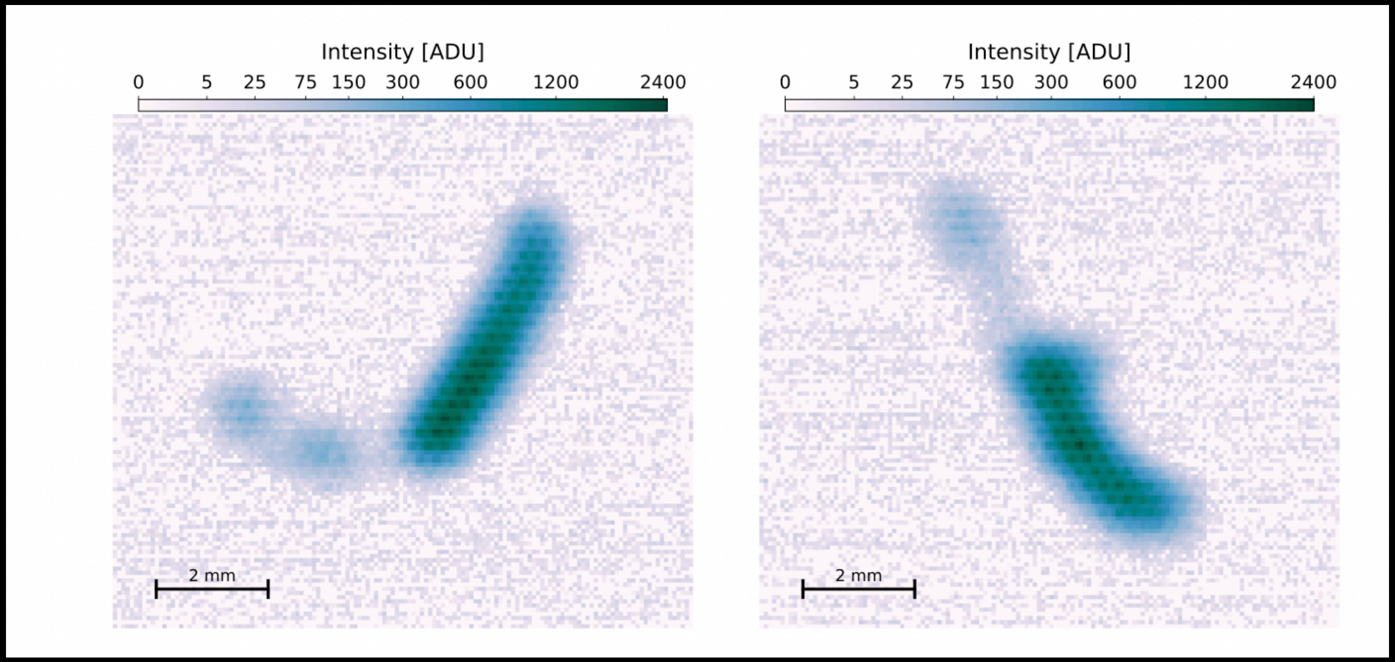
Migdal effect in SM

MIGDAL Experiment



MIGDAL collab.
Astropart.Phys. 151 (2023) 102853

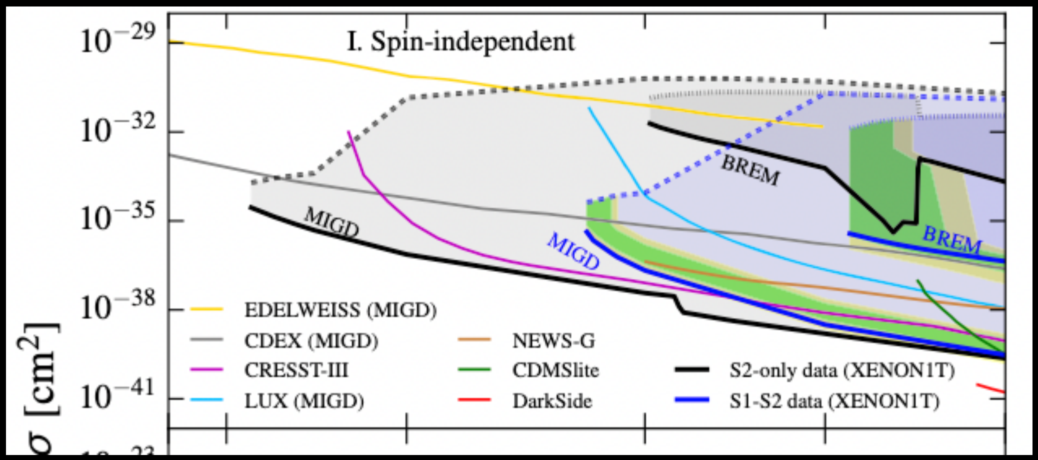
7.5 keV Migdal electron   5.0 keV Migdal electron



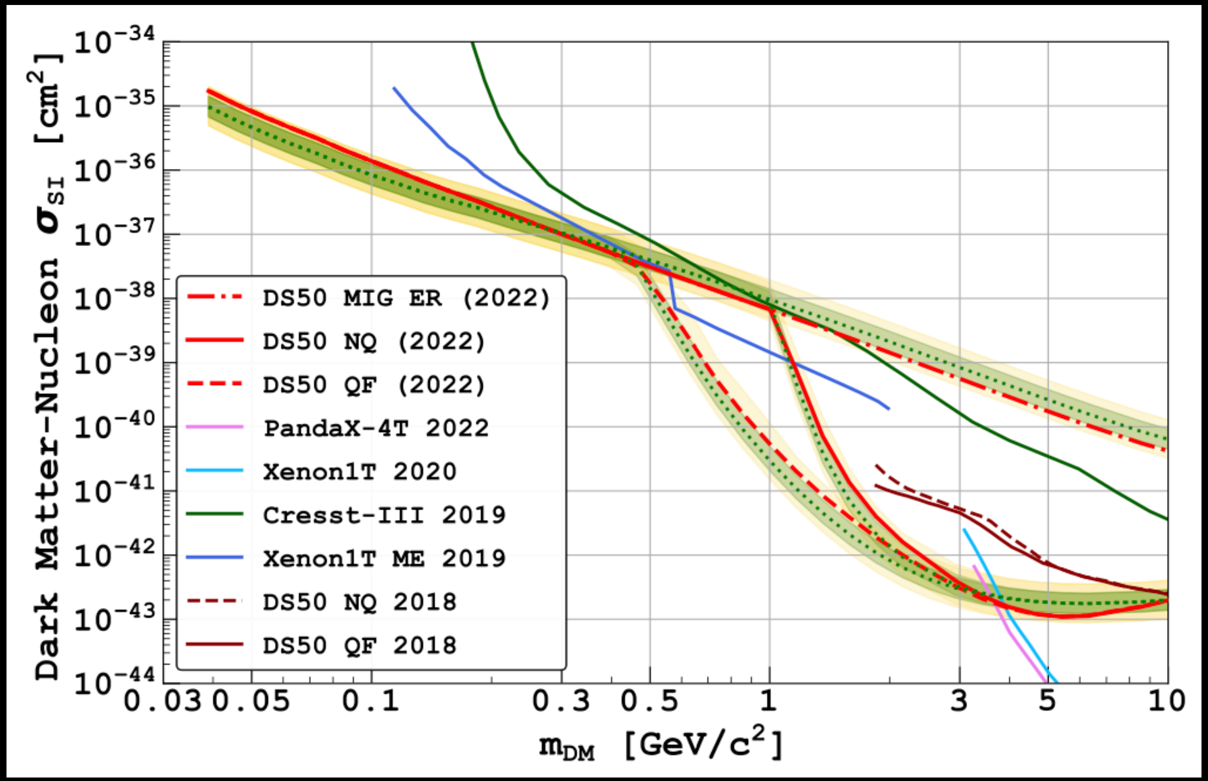
* 3D reconstruction of the characteristic event topology to check two tracks sharing a common vertex

* May 2023 started gathering data@Boulby Underground Laboratory

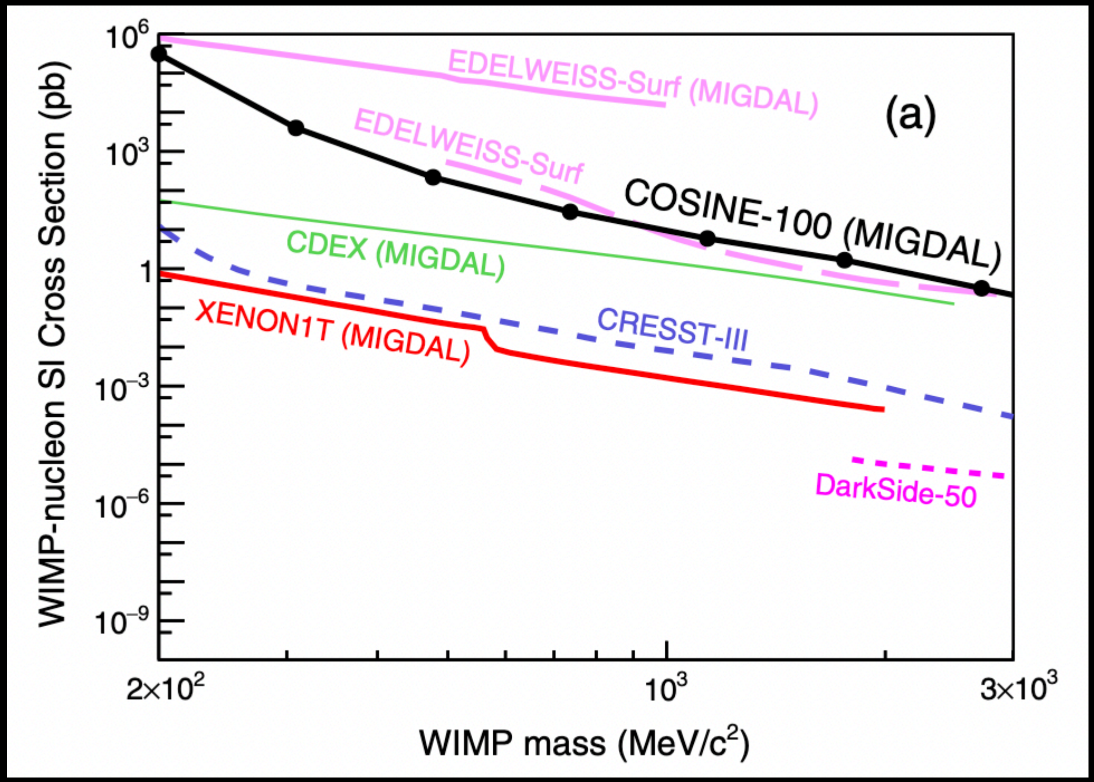
Migdal effect in direct detection experiments



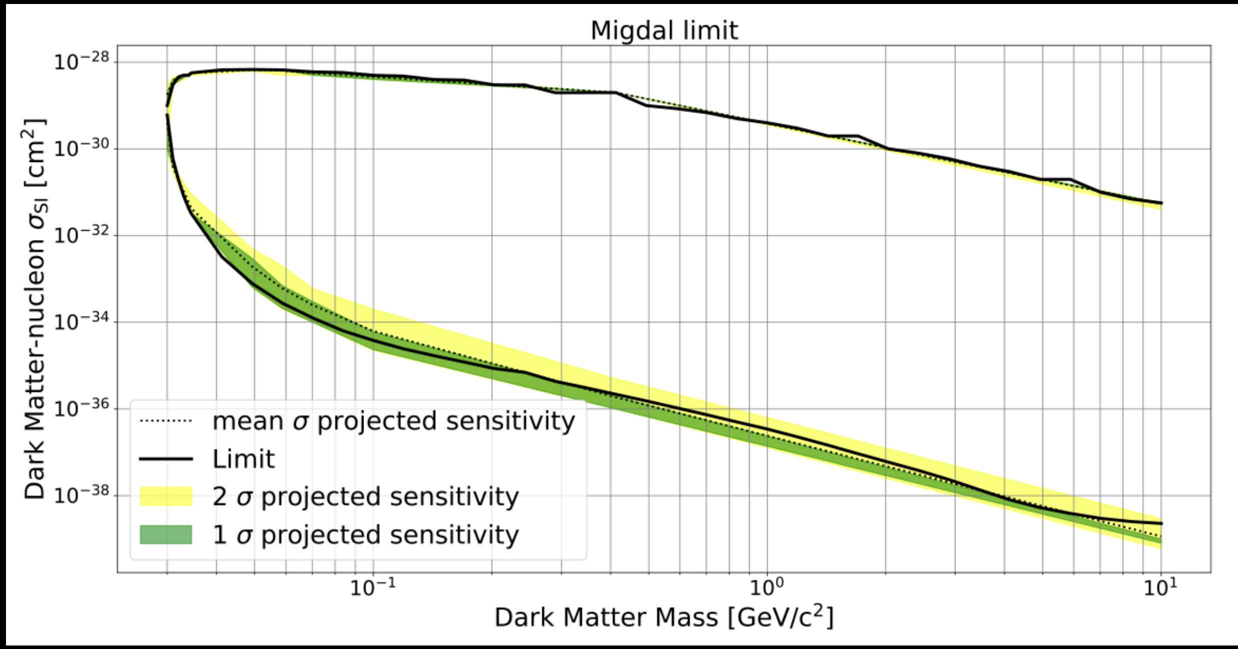
XENON1T, PRL 123 (2019) 241803



DS50, PRL 130 (2023) 10, 10



COSINE-100, PRD 105, 042006



SuperCDMS, PRD 107 (2023) 11, 2023

Migdal effect in models

$$\frac{d^3R}{dE_R dE_{det} dv_{\chi T}} = \frac{d^2R_{\chi T}}{dE_R dv_{\chi T}} \times \frac{1}{2\pi}\sum_{n,l}\frac{d}{dE_e}p^c_{q_e}(nl \to (E_e))$$

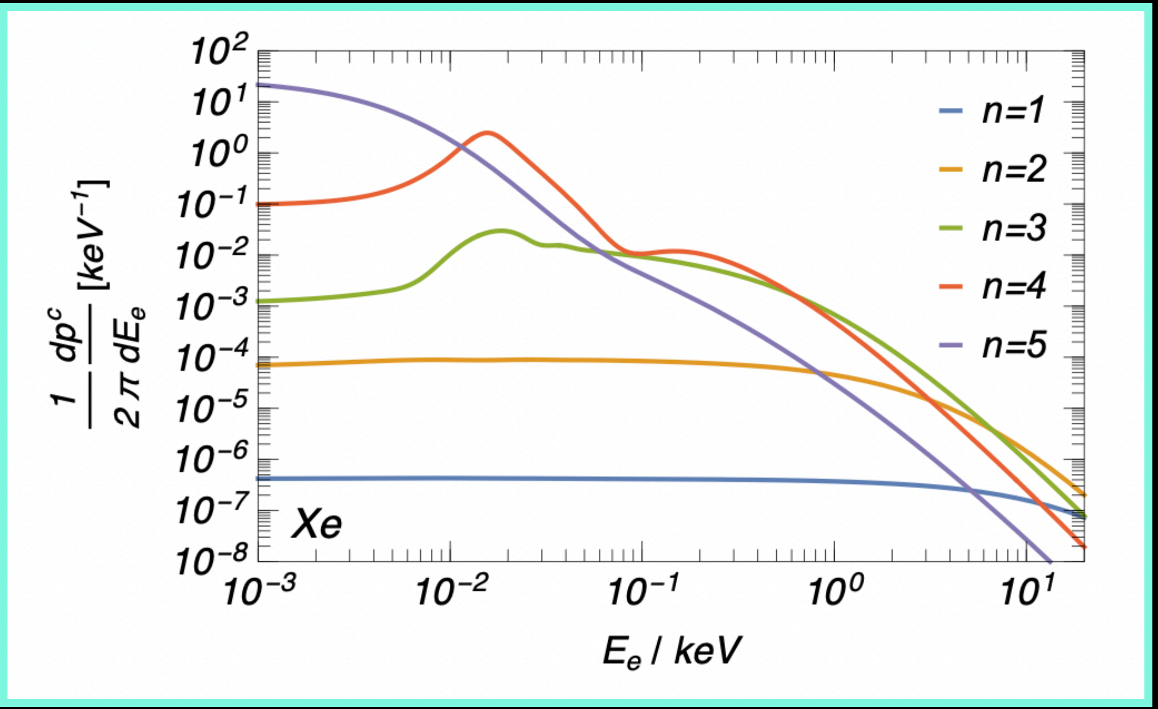
differential rate

ionisation probability

$q_e = m_e\sqrt{2E_R/m_T}$

$$\frac{dR_{\chi T}}{dE_R} = \sum_T N_T \frac{\rho_\chi}{m_\chi}\int_{v_{min}} d^3v_{\chi T} f(v_{\chi T}) v_{\chi T}\frac{d\sigma_T}{dE_R}$$

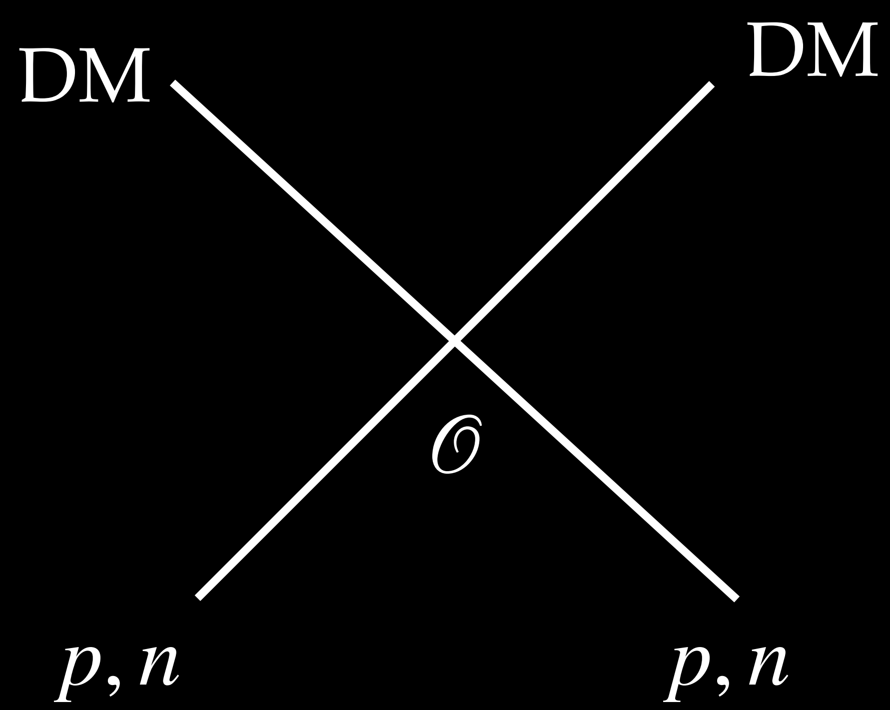
particle physics uncertainties



Ibe, Nakano, Shoji, Suzuki
JHEP 03 (2018) 194

Migdal effect in effective field theory

Mediator mass $\gg$ exchange momentum



Four-particles contact operators

Migdal effect in effective field theory

- Hamiltonian density of DM-nucleon interaction

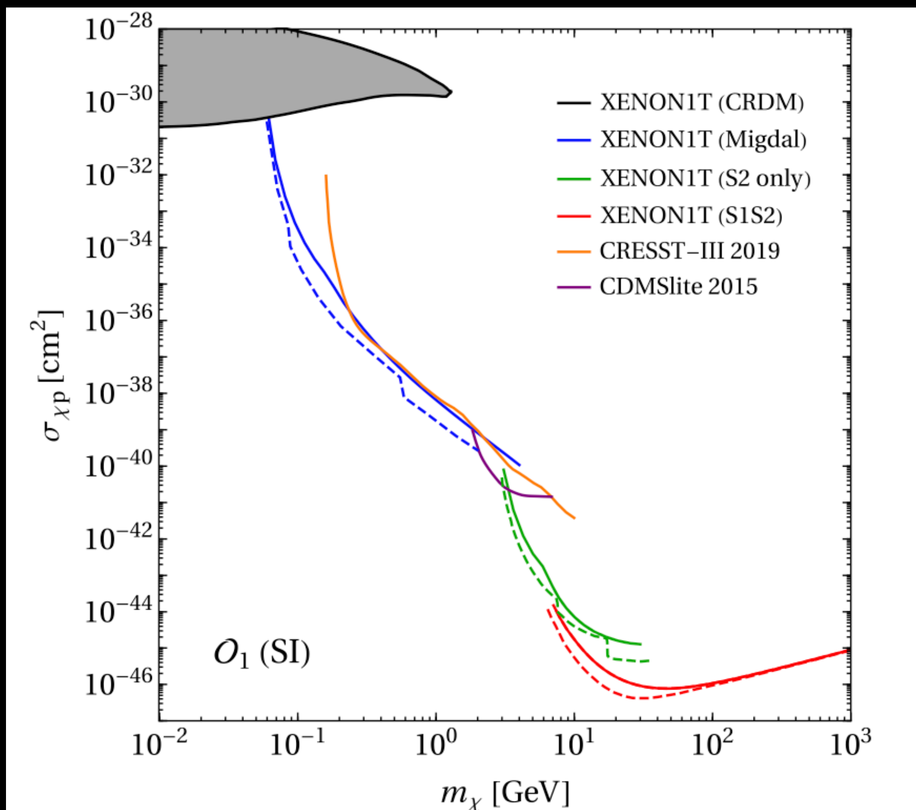
$$H = \sum_{j=1}^{15} (c_j^0 + c_j^1 \tau_3)\mathcal{O}_j \quad \text{with} \quad c_j^0 = c_j^p + c_j^n, \;\; c_j^1 = c_j^p - c_j^n$$

For spin-1/2 DM

$\mathcal{O}_1 = 1_\chi 1_N$	$\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$	$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$
$\mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$
$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp$	$\mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$
$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$	$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})$

Haxton, Phys. Rev. C89, 065501 (2014), 1308.6288
Gondolo, Kang, Scopel, G.Tomar, Phys.Rev.D 104, 063017

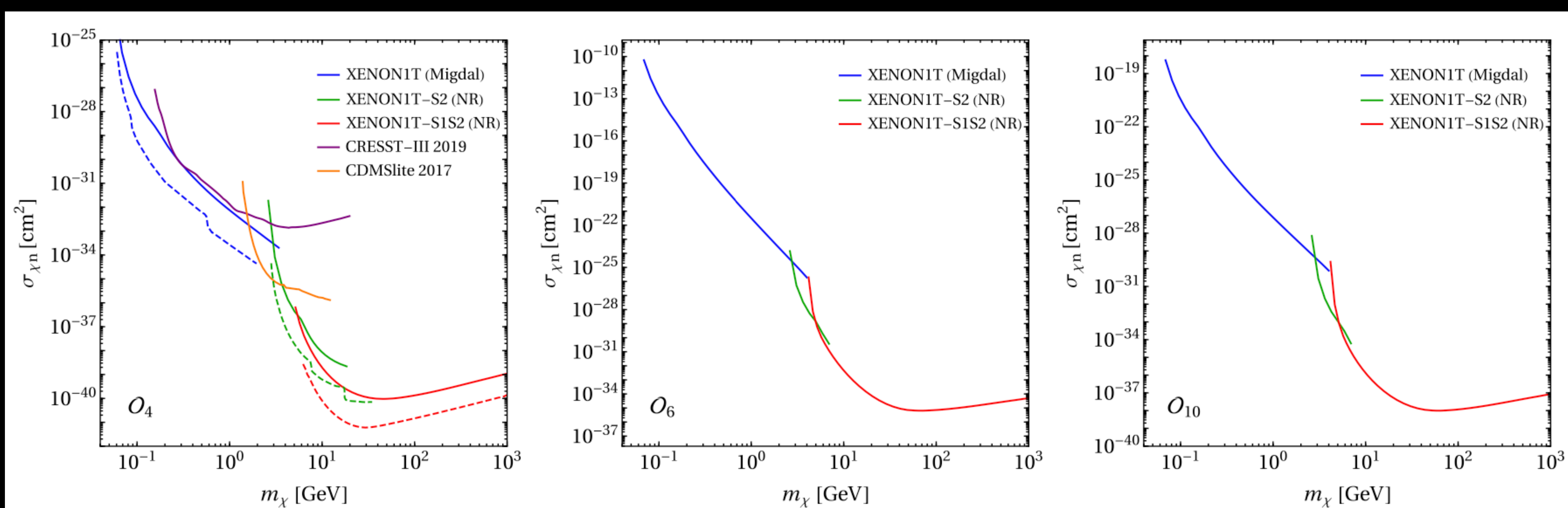
Migdal effect in effective field theory



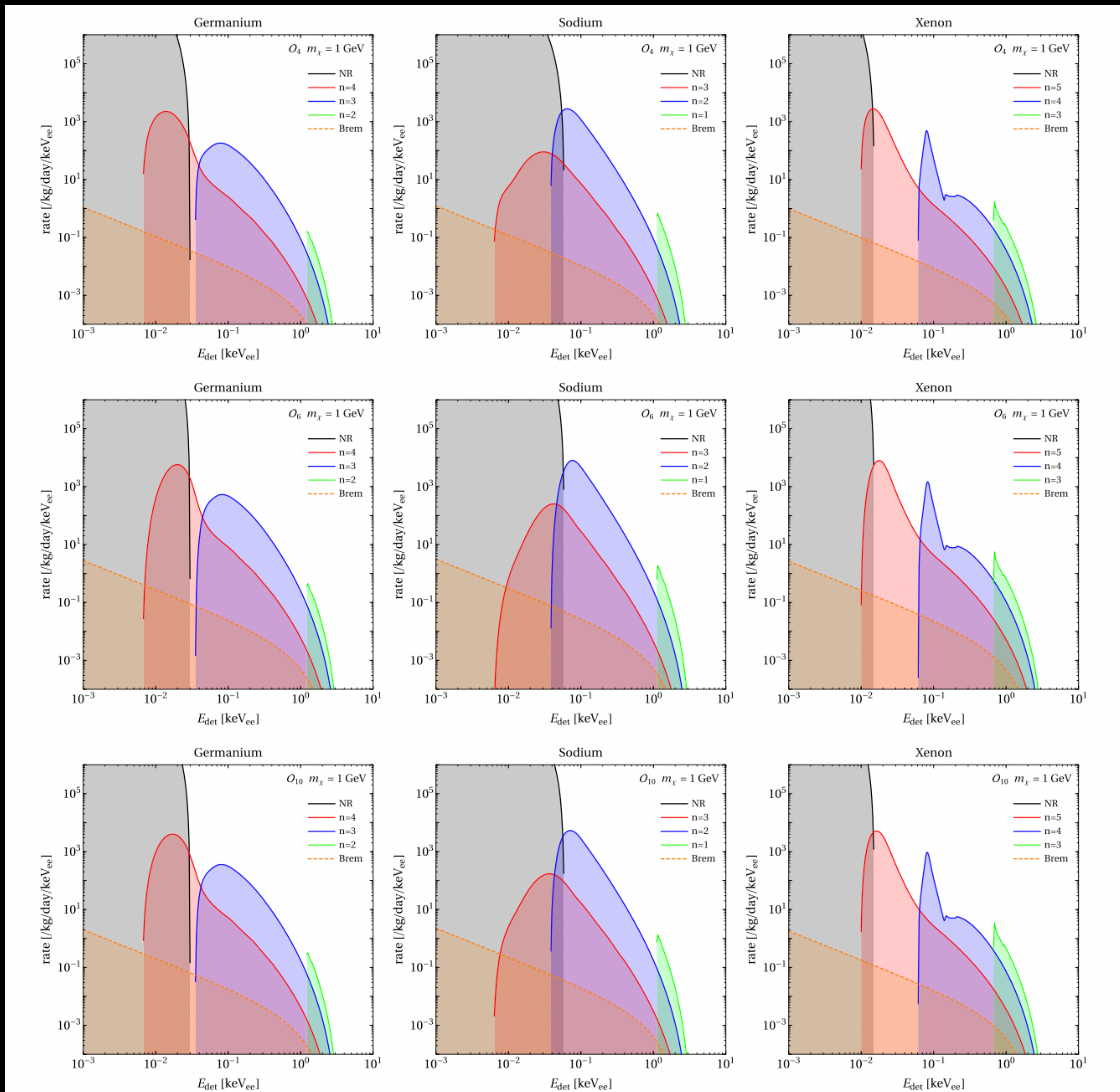
$$
\begin{array}{ll}
\mathcal{O}_1 & \mathbb{1}_\chi \mathbb{1}_N \\
\mathcal{O}_4 & \vec{S}_\chi \cdot \vec{S}_N \\
\mathcal{O}_6 & \left(\frac{\vec{q}}{m_N} \cdot \vec{S}_\chi\right)\left(\frac{\vec{q}}{m_N} \cdot \vec{S}_N\right) \\
\mathcal{O}_{10} & \mathbb{1}_\chi \left(i \frac{\vec{q}}{m_N} \cdot \vec{S}_N\right)
\end{array}
$$

Bell et al.
PRD 101, 015012 (2020)

G.Tomar, Kang, Scopel
Astropart.Phys. 150 (2023) 102851



Migdal effect vs bremsstrahlung



$\mathcal{O}_1$	$\mathbb{1}_\chi \mathbb{1}_N$
$\mathcal{O}_4$	$\vec{S}_\chi \cdot \vec{S}_N$
$\mathcal{O}_6$	$\left(\frac{\vec{q}}{m_N} \cdot \vec{S}_\chi\right)\left(\frac{\vec{q}}{m_N} \cdot \vec{S}_N\right)$
$\mathcal{O}_{10}$	$\mathbb{1}_\chi \left(i \frac{\vec{q}}{m_N} \cdot \vec{S}_N\right)$

Bell, Dent, Newstead, Sabharwal, Weiler
PHYSICAL REVIEW D 101, 015012 (2020)

Migdal effect vs bremsstrahlung

- Bremsstrahlung

$$\frac{d^3R}{dE_{det}d\omega dv} = \frac{d^2R_{\chi T}}{dE_R dv}\frac{4\alpha Z^2}{3\pi}\boxed{\frac{E_R}{m_T\omega}}$$

suppressed

$$\Delta E = \omega$$

- Migdal effect

$$\frac{d^3R}{dE_{det}dE_e dv} = \frac{d^2R_{\chi T}}{dE_R dv} \times \frac{1}{2\pi}\frac{dp_{q_e}}{dE_e}$$

$$\Delta E = E_{nl} + E_e$$

Migdal effect vs DM-electron scattering

- In model with comparable coupling to electrons and protons

$$\frac{dR_M/dq}{dR_e/dq} > Z^2 \left(\frac{m_e}{m_N}\right)^2 (qr_a)^2$$

Effective atomic radius

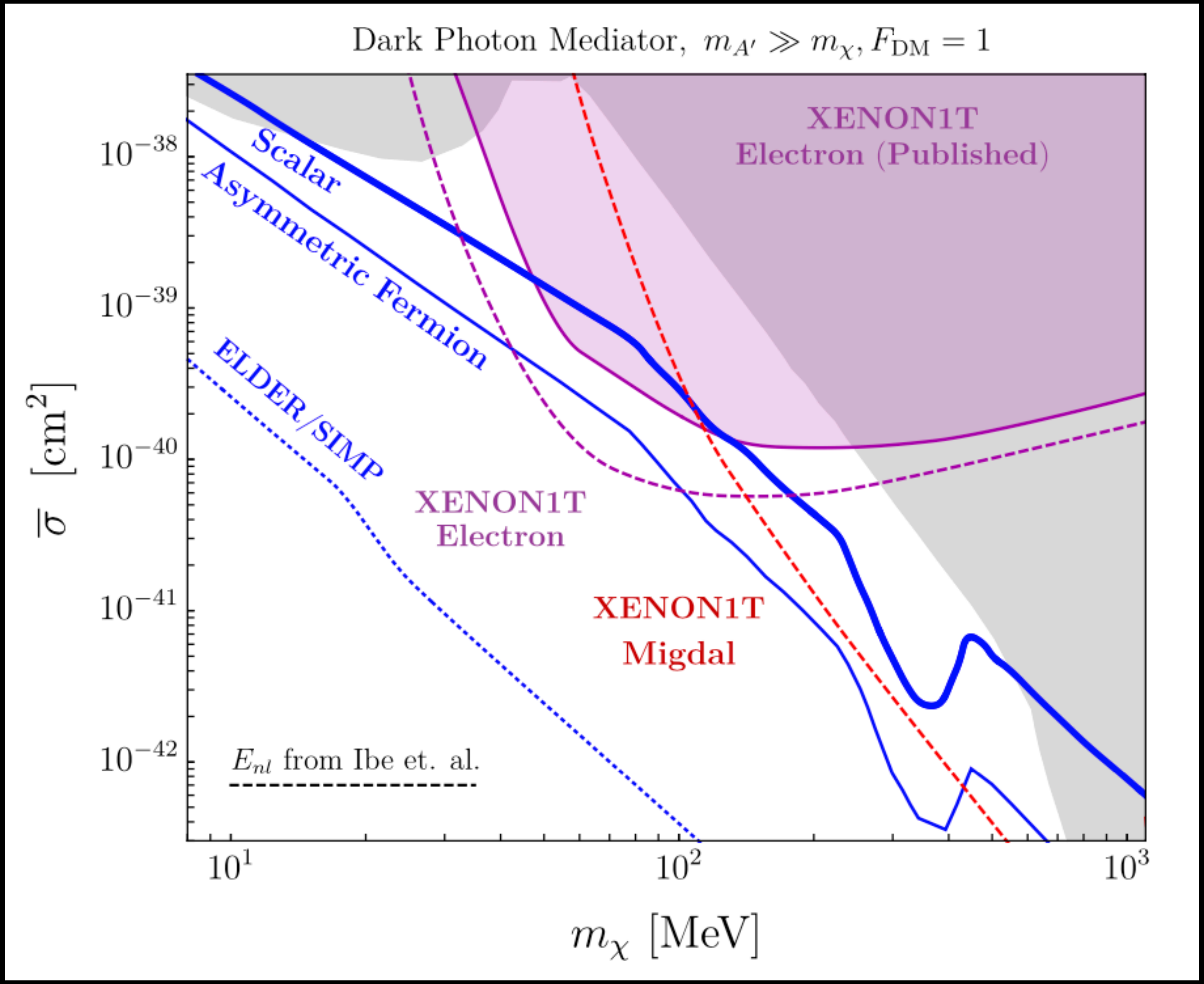
Momentum transfer to atom

Target nucleus mass

Baxter, Kahn, Krnjaic, *arXiv: 1908.00012*

Migdal effect vs DM-electron scattering

- Dark photon model with equal couplings to electrons and protons (heavy mediators)



Baxter, Kahn, Krnjaic, *arXiv: 1908.00012*

Essig, Radler, Sholapurkar, Yu, *arXiv: 1908.10881*

Probing Electromagnetic interactions by Migdal Effect

$\mathscr{L}_{\text{int}}$

$Q_\chi e \bar{\chi}\gamma^\mu \chi A_\mu$	$\frac{\mu_\chi}{2}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$	$\frac{d_\chi}{2} i\bar{\chi}\sigma^{\mu\nu}\gamma^5\chi F_{\mu\nu}$	$A_\chi \bar{\chi}\gamma^\mu\gamma^5\chi\partial^\nu F_{\mu\nu}$	$b_\chi \bar{\chi}\gamma^\mu\chi\partial^\nu F_{\mu\nu}$
Millicharge	Magnetic Dipole	Electric Dipole	Anapole	Charge radius

Probing Electromagnetic interactions by Migdal Effect

$\mathscr{L}_{\text{int}}$

↓

Millicharge	Magnetic Dipole	Electric Dipole	Anapole	Charge radius
$Q_\chi e\bar{\chi}\gamma^\mu\chi A_\mu$	$\frac{\mu_\chi}{2}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$	$\frac{d_\chi}{2}i\bar{\chi}\sigma^{\mu\nu}\gamma^5\chi F_{\mu\nu}$	$A_\chi\bar{\chi}\gamma^\mu\gamma^5\chi\partial^\nu F_{\mu\nu}$	$b_\chi\bar{\chi}\gamma^\mu\chi\partial^\nu F_{\mu\nu}$
$\propto \frac{\mathcal{O}_1}{q^2}$	$\propto \mathcal{O}_1 + \frac{\mathcal{O}_5}{q^2} + \mathcal{O}_4 + \frac{\mathcal{O}_6}{q^2}$	$\propto \frac{\mathcal{O}_{11}}{q^2}$	$\propto \mathcal{O}_8 + \mathcal{O}_9$	$\propto \mathcal{O}_1$

Non-relativistic operators

WimPyDD with Migdal

- To handle Particle physics as well as Astrophysical uncertainties in DM direct detection

* WimPyDD: object-oriented Python code

A. Uses most general non-relativistic Effective Field Theory

B. Valid for any velocity distribution

C. Includes DM of arbitrary spin

D. Handles inelastic scattering

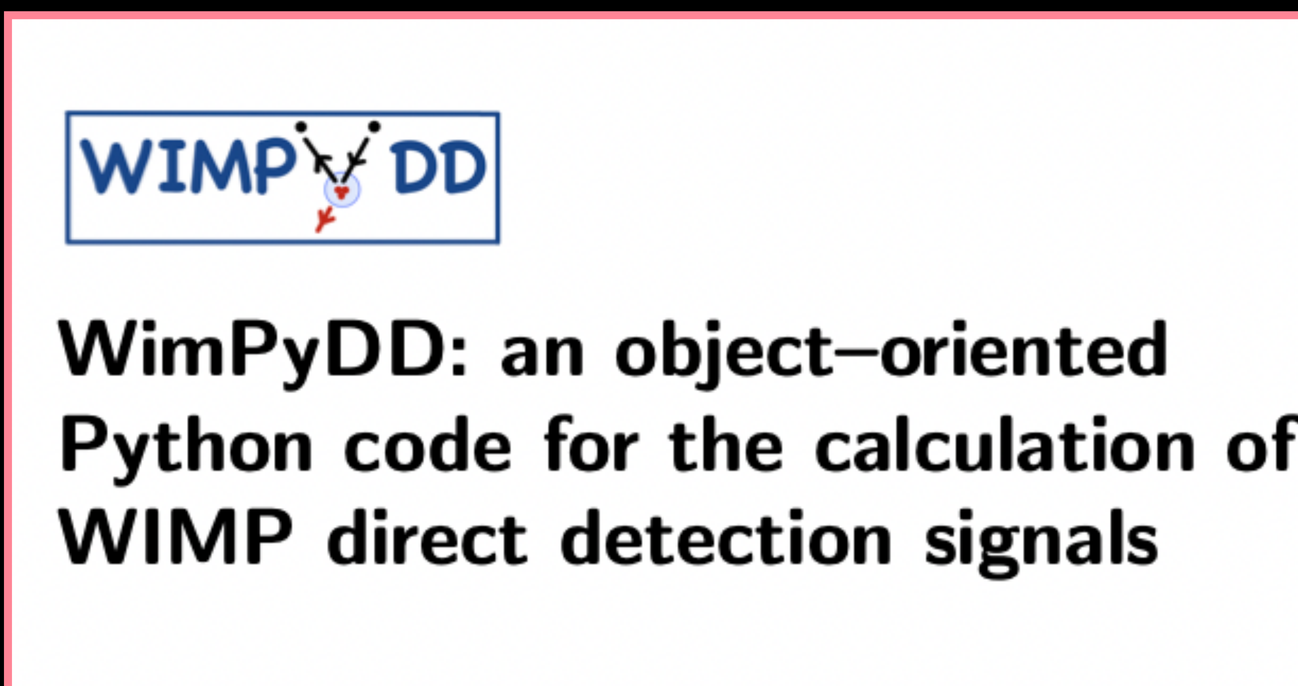
Jeong, Kang, Scopel, G.Tomar,
Computer Physics Communication, 2022

Migdal effect in relativistic effective models

$$\frac{d^3R}{dE_R dE_{det} dv_{\chi T}} = \frac{d^2R_{\chi T}}{dE_R dv_{\chi T}} \times \frac{1}{2\pi}\sum_{n,l}\frac{d}{dE_e}p^c_{q_e}(nl \to (E_e))$$

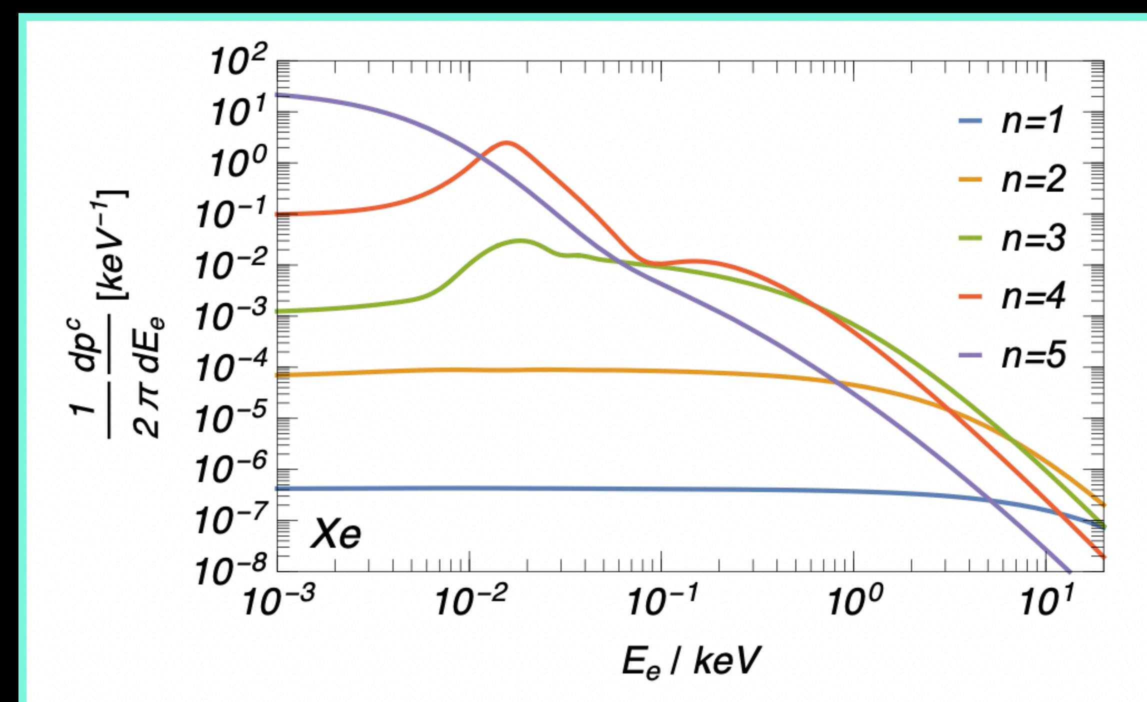
differential rate

ionisation probability


WIMPyDD

WimPyDD: an object-oriented Python code for the calculation of WIMP direct detection signals

Jeong, Kang, Scopel, G.Tomar,
Computer Physics Communication



Ibe, Nakano, Shoji, Suzuki
JHEP 03 (2018) 194

Relativistic effective models

Any High Energy Physics model can be studied

High Energy Physics models

NR limit

NREFT building block

$\mathcal{O}_1 + \mathcal{O}_3 \cdots\cdots \mathcal{O}_{15}$

$$\frac{\mu_\chi}{2}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$$

NREFT building block

$A\,\mathcal{O}_1 + B\,\mathcal{O}_5 + C\,\mathcal{O}_4 + D\,\mathcal{O}_6$

WimPyDD arXiv:2106.06207

Probing Electromagnetic interactions by the Migdal Effect

$\mathscr{L}_{\text{int}}$

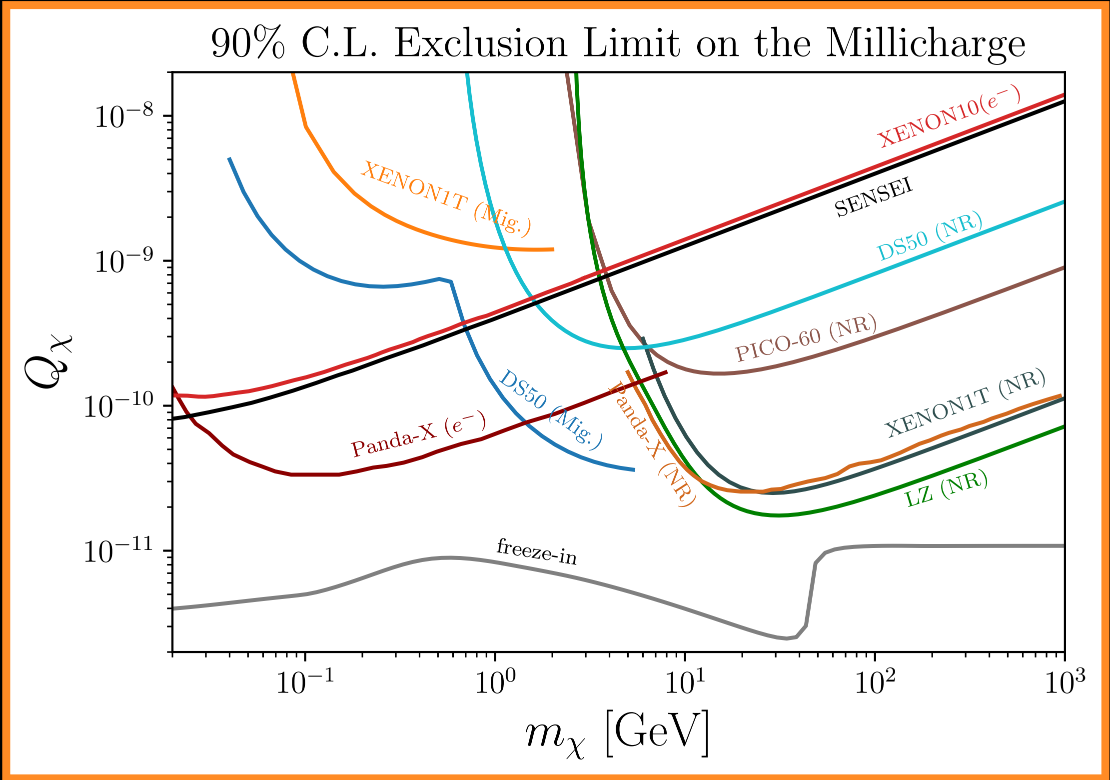
$Q_\chi e \bar{\chi}\gamma^\mu \chi A_\mu$ — Millicharge

$\frac{\mu_\chi}{2}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$ — Magnetic Dipole

$\frac{d_\chi}{2} i\bar{\chi}\sigma^{\mu\nu}\gamma^5\chi F_{\mu\nu}$ — Electric Dipole

$A_\chi \bar{\chi}\gamma^\mu\gamma^5\chi\partial^\nu F_{\mu\nu}$ — Anapole

$b_\chi \bar{\chi}\gamma^\mu\chi\partial^\nu F_{\mu\nu}$ — Charge radius



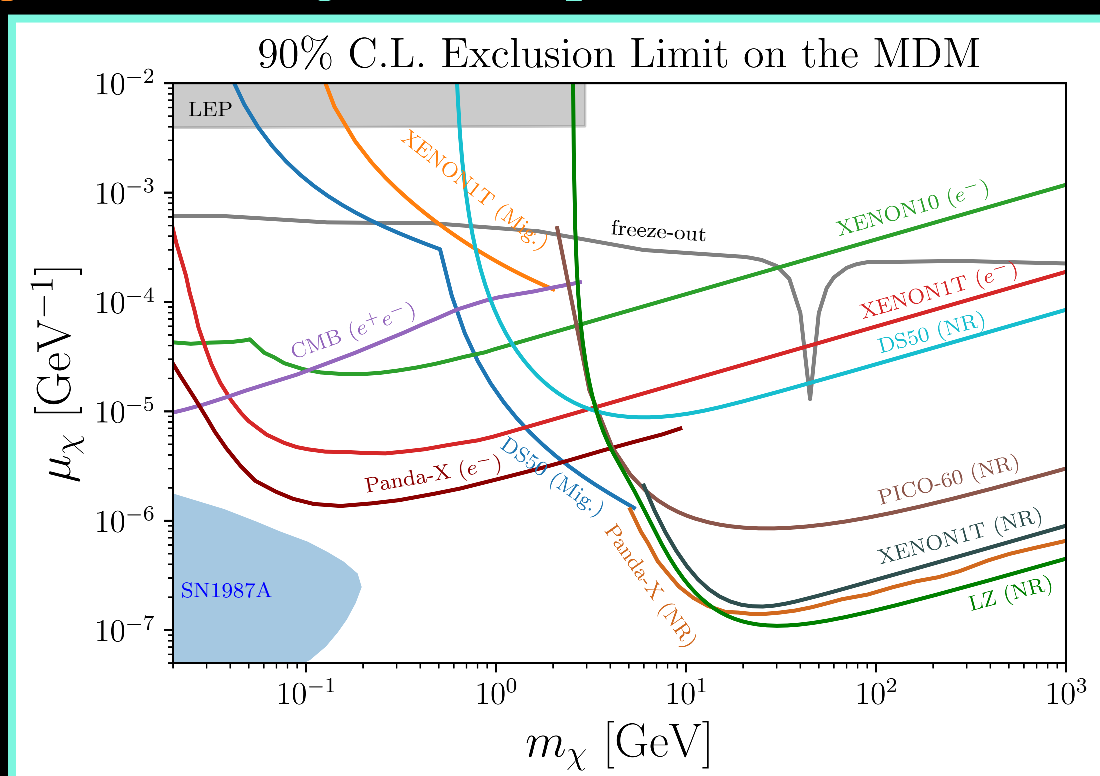
A. Ibarra, M. Reichard, G.Tomar,
arXiv: 2408.15760

Probing Electromagnetic interactions by the Migdal Effect

$\mathscr{L}_{\text{int}}$

$Q_\chi e \bar{\chi}\gamma^\mu \chi A_\mu$ | $\frac{\mu_\chi}{2}\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$ | $\frac{d_\chi}{2} i\bar{\chi}\sigma^{\mu\nu}\gamma^5\chi F_{\mu\nu}$ | $A_\chi \bar{\chi}\gamma^\mu\gamma^5\chi\partial^\nu F_{\mu\nu}$ | $b_\chi \bar{\chi}\gamma^\mu\chi\partial^\nu F_{\mu\nu}$

Millicharge | Magnetic Dipole | Electric Dipole | Anapole | Charge radius



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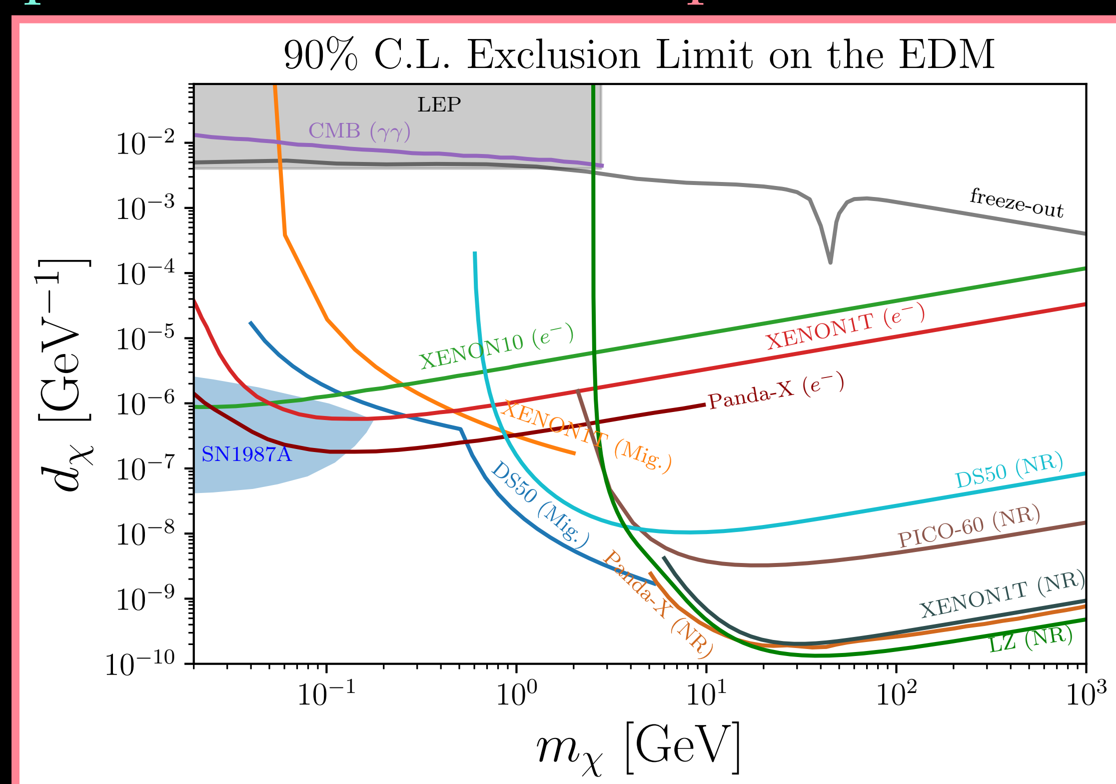

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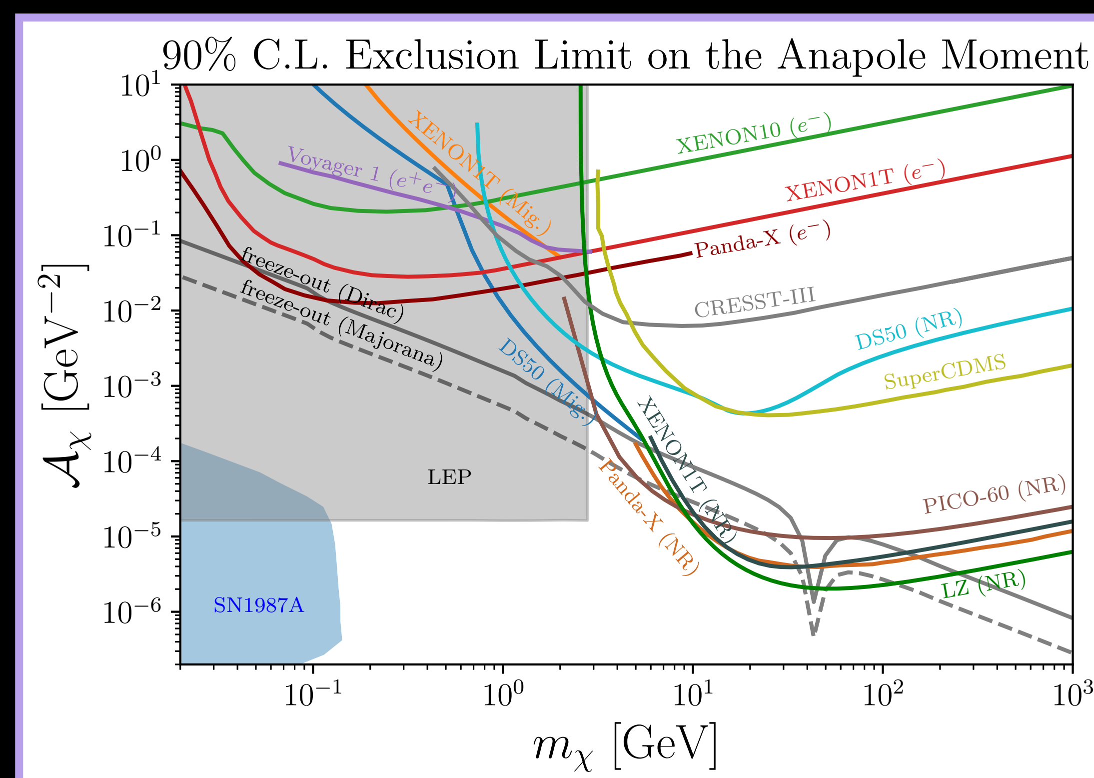

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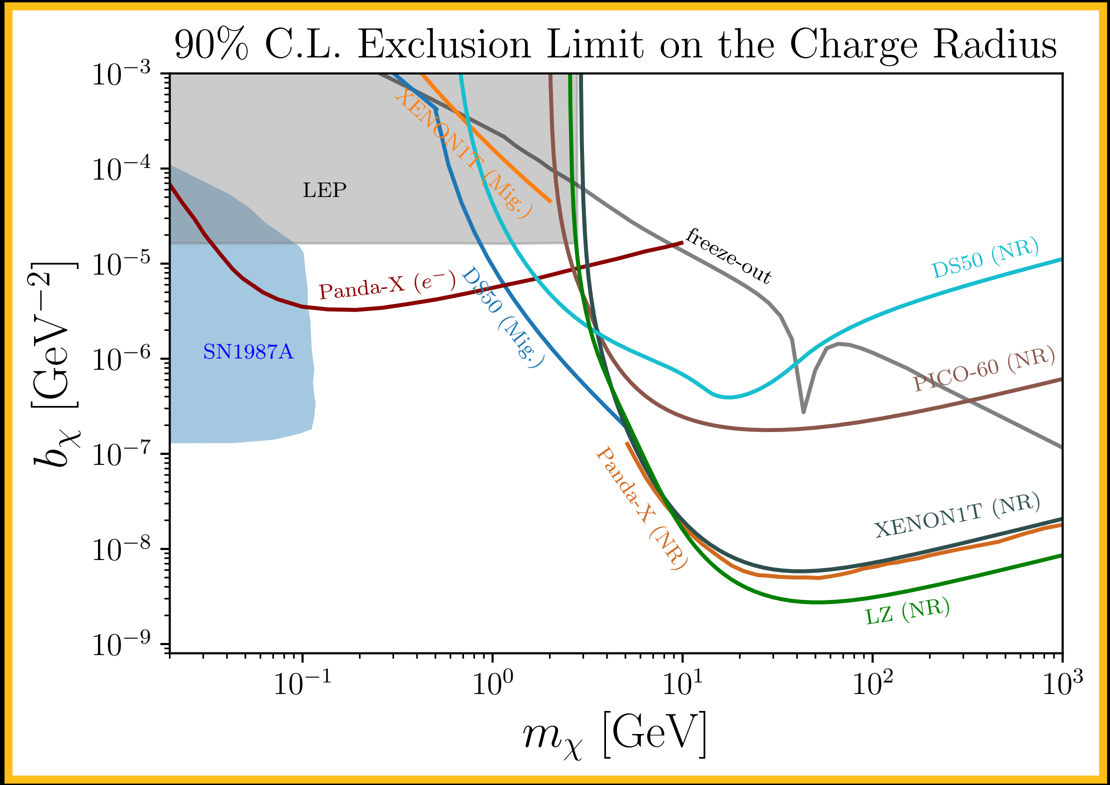
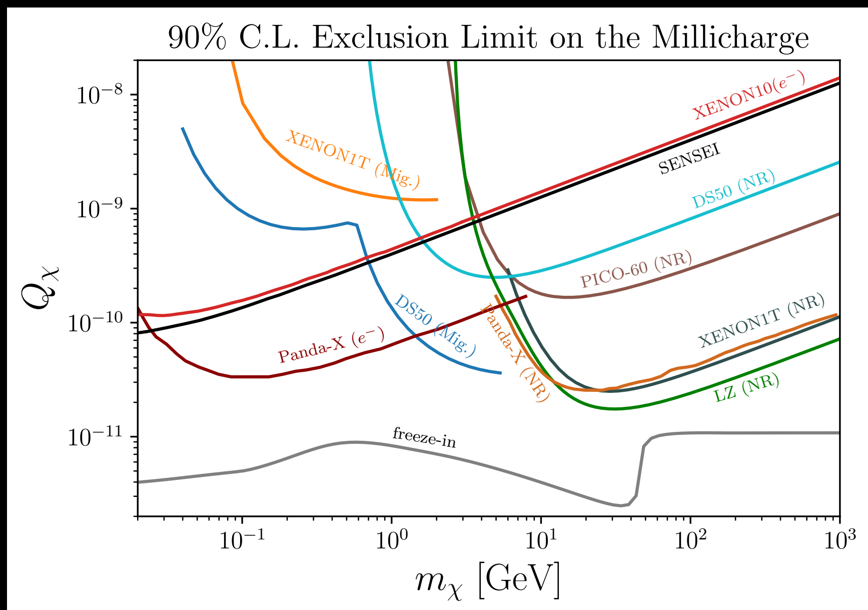
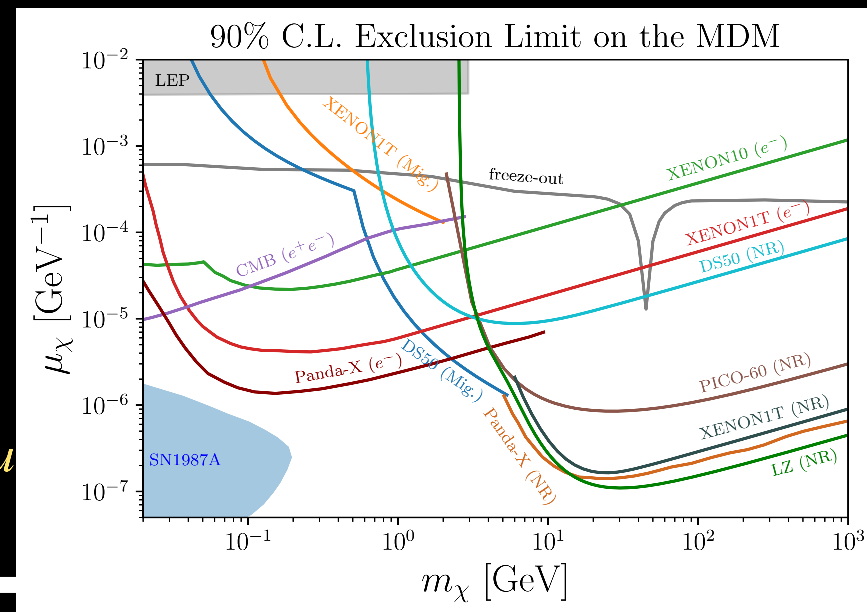
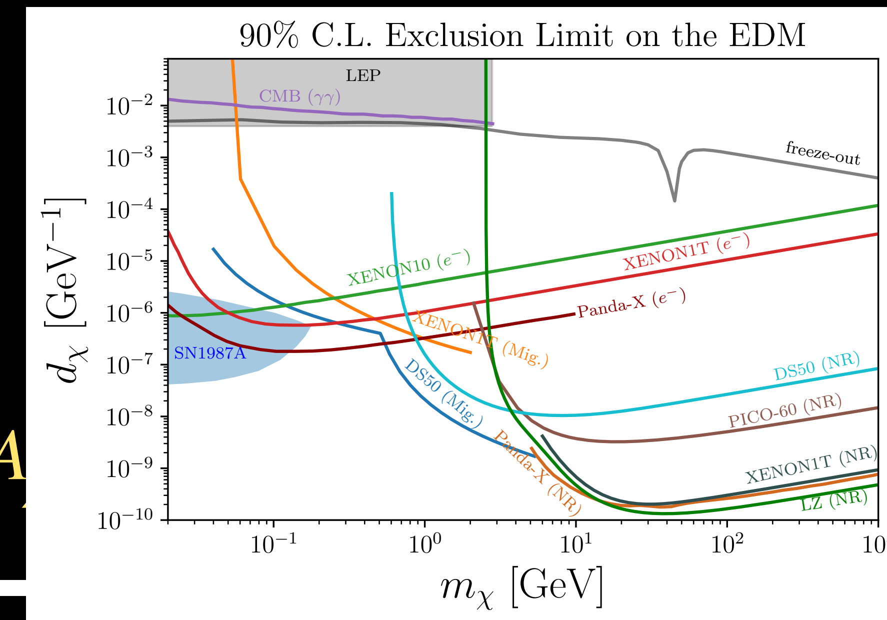


A. Ibarra, M. Reichard, G.Tomar, arXiv: 2408.15760

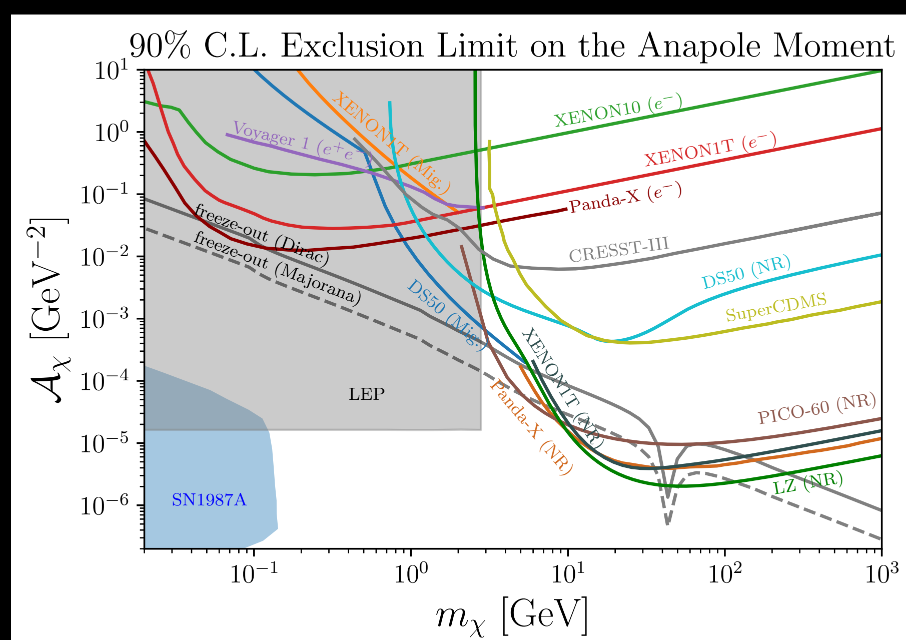
Probing Electromagnetic interactions by the Migdal Effect

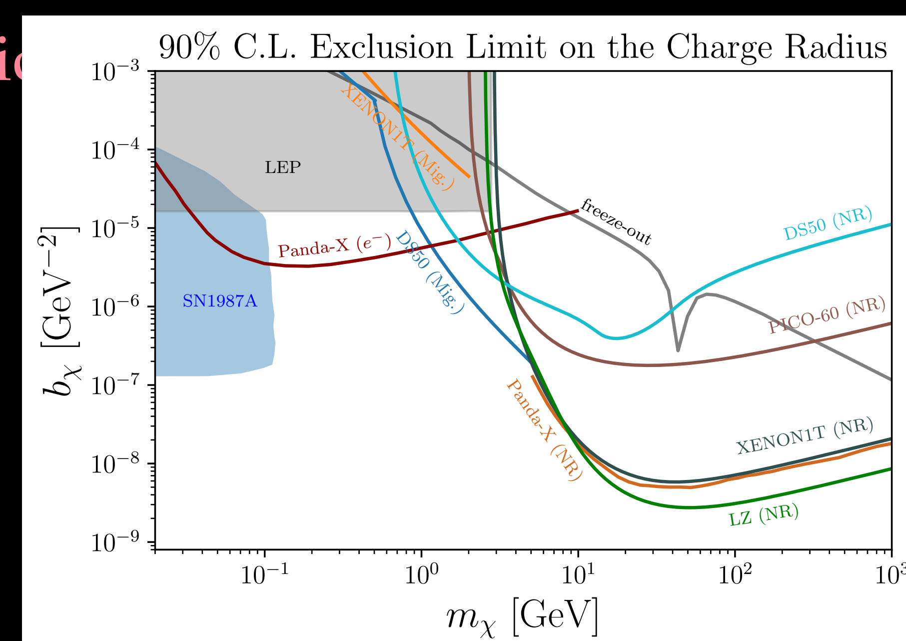






Millicharge





arge radius

A. Ibarra, M. Reichard, G.Tomar,
arXiv: 2408.15760

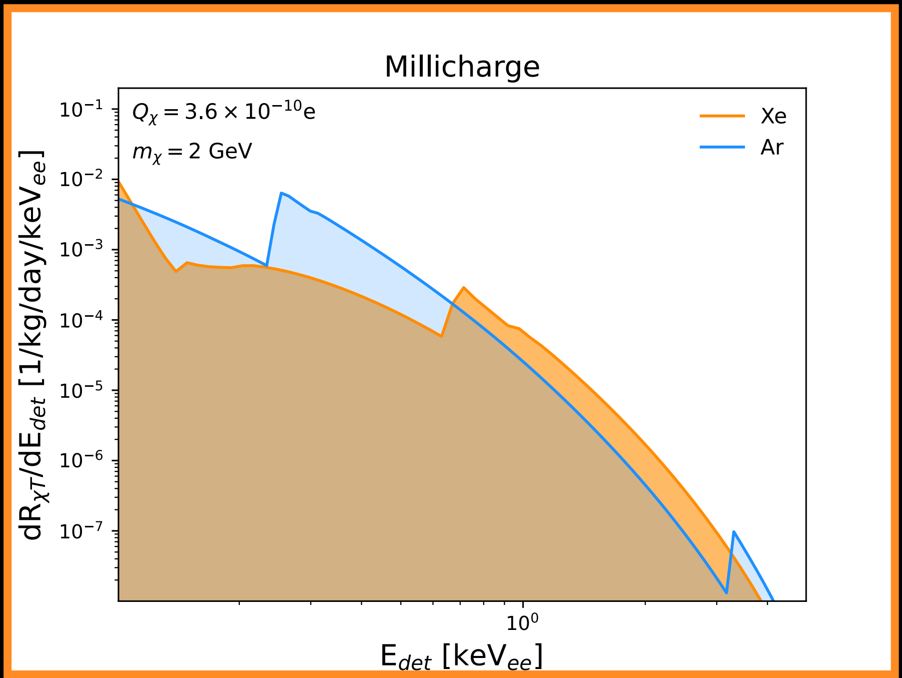
Future prospects

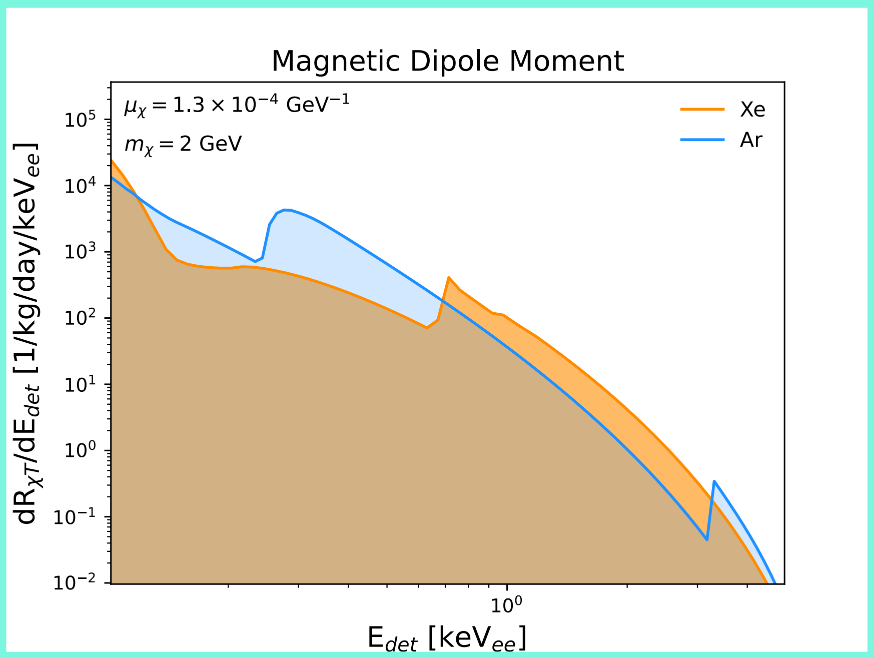
- Increased Sensitivity: SuperCDMS and SENSEI will focus on lowering detection thresholds, aiming to detect dark matter masses as low as tens of MeV.
- Technological Advancements: Detector technologies like cryogenic detectors, low-noise amplifiers and noble gases are being optimized for better Migdal effect detection.
- Complementary Approaches: Combining DM-electron, DM-nucleus, and Migdal effect signals will improve constraints on sub-GeV DM.
- Use of multi-target materials across experiments (e.g., semiconductors, noble liquids) creates a more comprehensive exploration range.
- Theoretical Refinements: Improved theoretical calculations on Migdal effect cross-sections and dark matter interaction rates can help guide future experiments and enhance their sensitivity

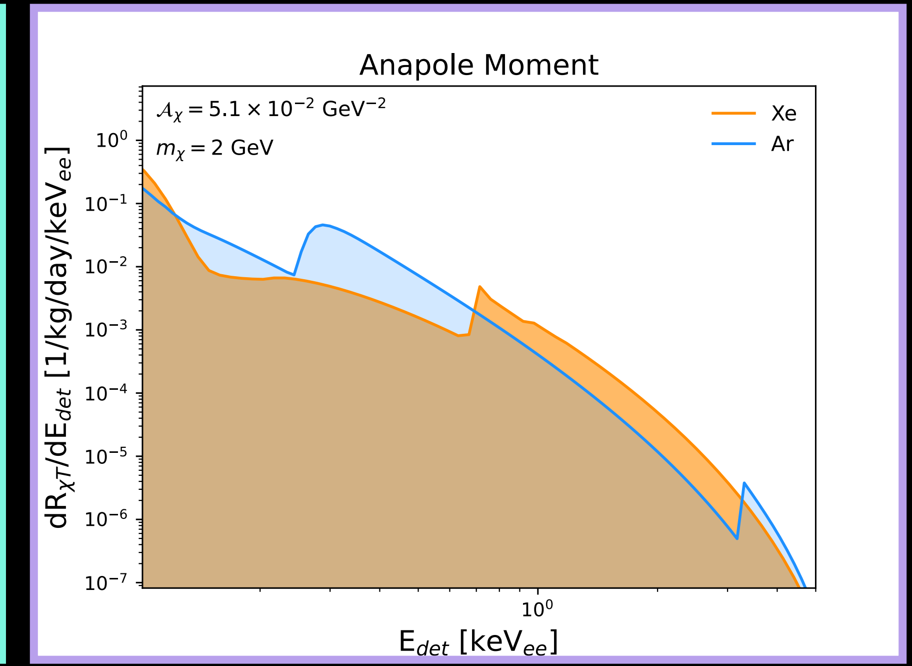
Summary

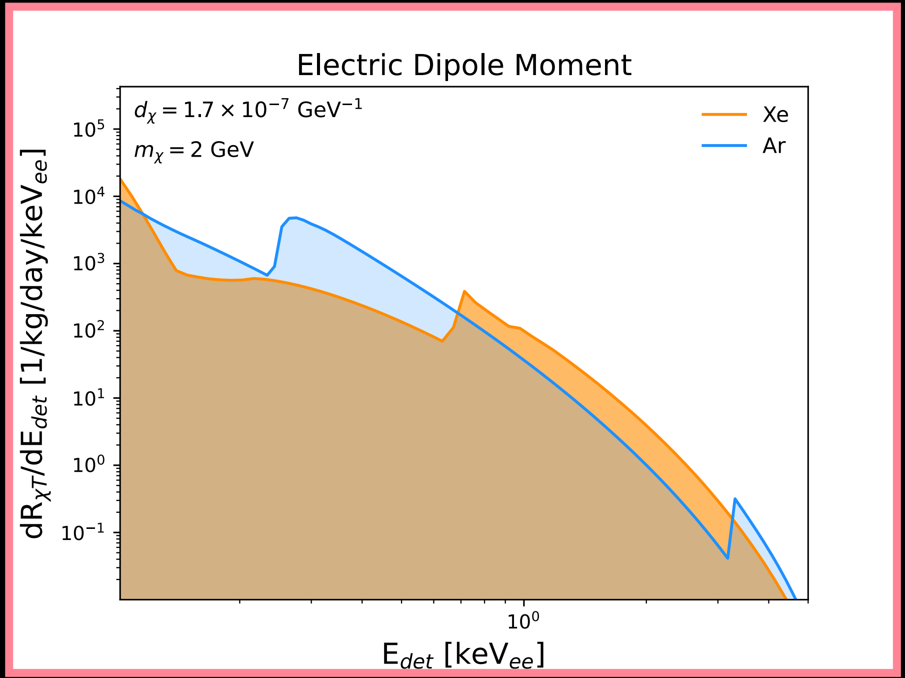
- Absence of DM signal in GeV-TeV range has prompt motivation for Sub-GeV DM
- The analysis of DM-nuclear elastic scattering is limited to DM mass of ~GeV mass range due to experimental threshold limitation
- Ionisation signal from DM-electron scattering (primary signal), and *Migdal effect*, photon bremsstrahlung (secondary signal) can lift this restriction
- We extended the Migdal analysis to the electromagnetic interactions
- The complementarity among DM-electron, DM-nucleus, and the Migdal effect is is observed
- The Migdal effect is an important tool to look for Sub-GeV dark matter

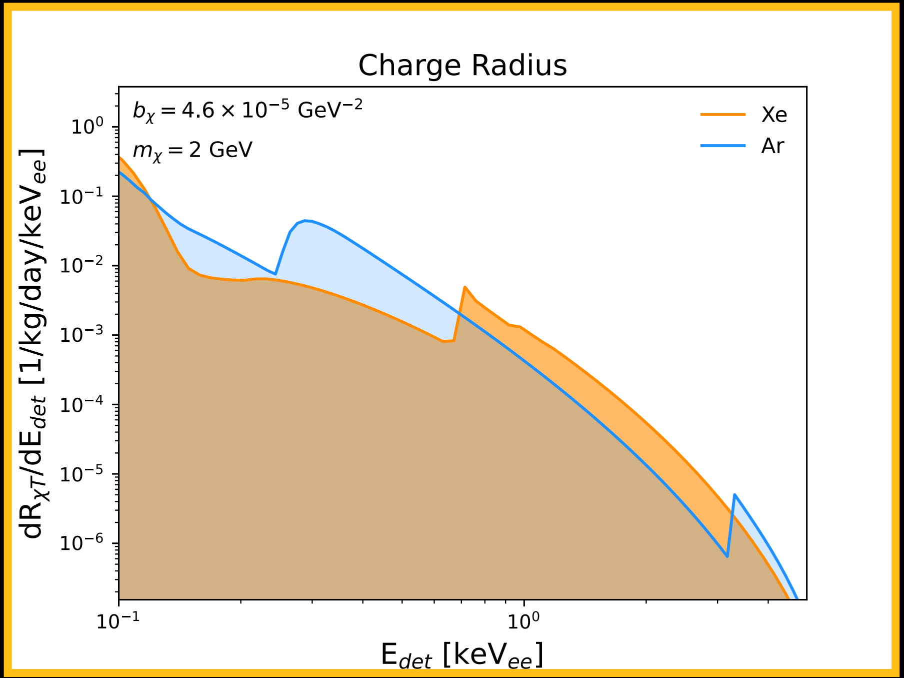
Migdal Spectrum with EM interactions











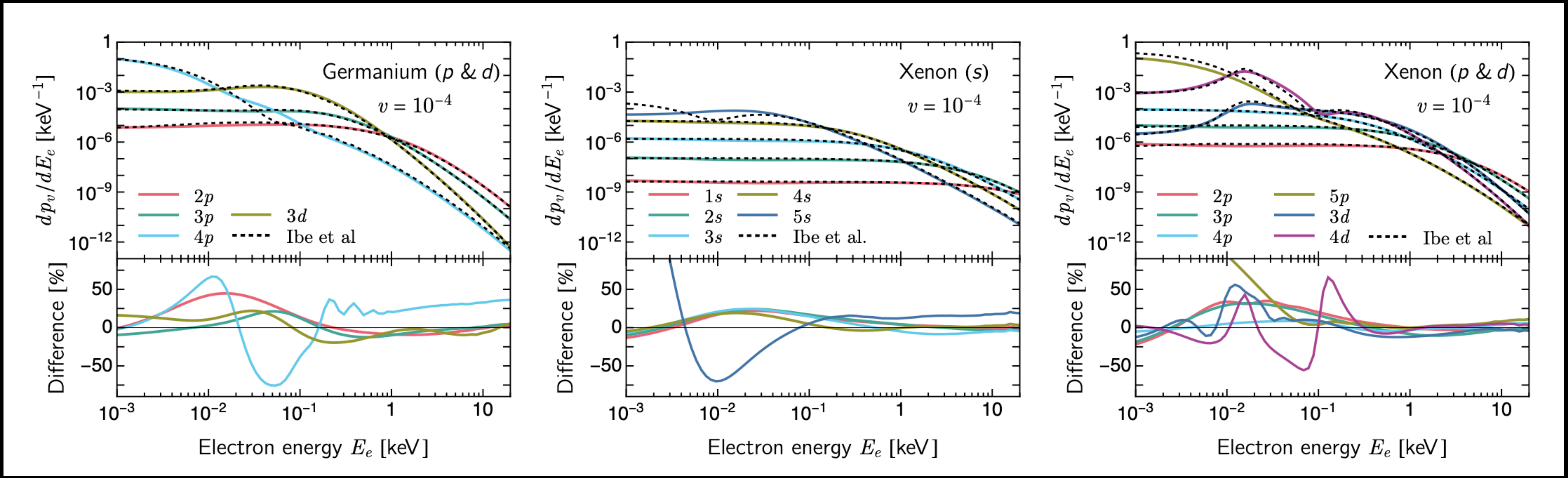
A. Ibarra, M. Reichard, G.Tomar,
arXiv: 2408.15760

Migdal effect

* Precise predictions for atomic ionisation form the Migdal effect

* Dirac-Hartree-Fock method used to calculate the atomic wave function

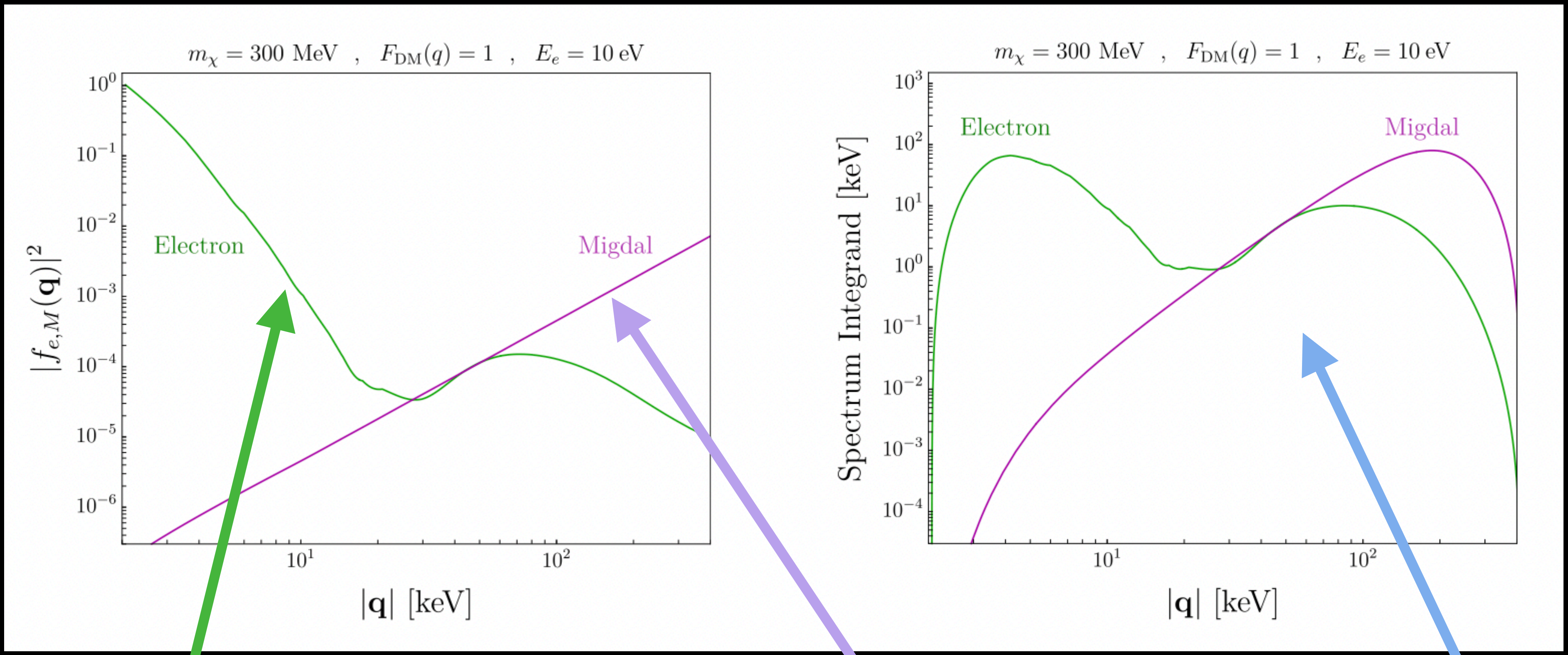
$$P = \left| \langle \Psi_F | \Psi_{in} \rangle \right|^2 = \left| \langle \Psi_F | e^{-iq_e \cdot \sum_i x_i} | \Psi_{in} \rangle \right|^2$$



Cox, Dolan, McCabe, Quiney Phys. Rev. D 107, 035032 (2023)

Migdal effect vs electron scattering

- Dark photon model with equal couplings to electrons and protons (heavy mediators)



$$|f_e(E_e,\mathbf{q})|^2 = \frac{k'^3}{4\pi^3} \times 2 \sum_{n,l,l',m'} |\langle \psi^f_{E_e} | e^{i\mathbf{q}\cdot\mathbf{x}} | \psi^i_{E_{nl}} \rangle|^2,$$

$$|f_M(E_e,\mathbf{q})|^2 = \frac{k'^3}{4\pi^3} Z^2 |F_N(q)|^2 \times 2 \sum_{n,l,l',m'} |\langle \psi^f_{E_e} | e^{i\mathbf{q}_e\cdot\mathbf{x}} | \psi^i_{E_{nl}} \rangle|^2,$$

$$\frac{dR_{e,M}}{d\ln E_e} = N_T \frac{\rho_\chi}{m_\chi} \frac{\overline{\sigma}}{8\mu^2_{\chi e}} I_{e,M}(E_e),$$

where N_T is the number of atomic targets and

$$I_{e,M}(E_e) = \int d|\mathbf{q}||\mathbf{q}||F_{\rm DM}(q)|^2 \eta(v_{\rm min}) |f_{e,M}(E_e,\mathbf{q})|^2$$

Baxter, Kahn, Krnjaic, *arXiv: 1908.00012*

Essig, Radler, Sholapurkar, Yu, *arXiv: 1908.10881*

Experimental thresholds

Experiment	Thresholds
XENON1T	0.186-3.8 keVee
DS50	0.083-0.106 keVee
SuperCDMS	0.07-2 keVee
COSINE-100	1-1.25 keVee

Migdal effect

✓ *Reformulation of the Migdal Effect*

✓ Migdal's approach

Initial state of the DM scattering : ***(DM plane wave) x (Nucleus plane wave)***
Final state of the DM scattering : ***(DM plane wave) x (Nucleus plane wave)***
Migdal Effect = ***Final state effects***

The Migdal Effect is treated separately from the nuclear scattering

✓ New approach

Initial state of the DM scattering : ***(DM plane wave) x (Atomic plane wave)***
Final state of the DM scattering : ***(DM plane wave) x (Atomic plane wave)***

Image credit: Masahiro Ibe

Migdal effect vs DM-electron scattering

- In the dark photon model with arbitrary mediator couplings,

$$\frac{dR_M/dq}{dR_e/dq} \gtrsim \left(\frac{Zc_p + (A-Z)c_n}{c_e}\right)^2 \left(\frac{m_e}{m_N}\right)^2 (qr_a)^2$$

Effective atomic radius

Momentum transfer to atom

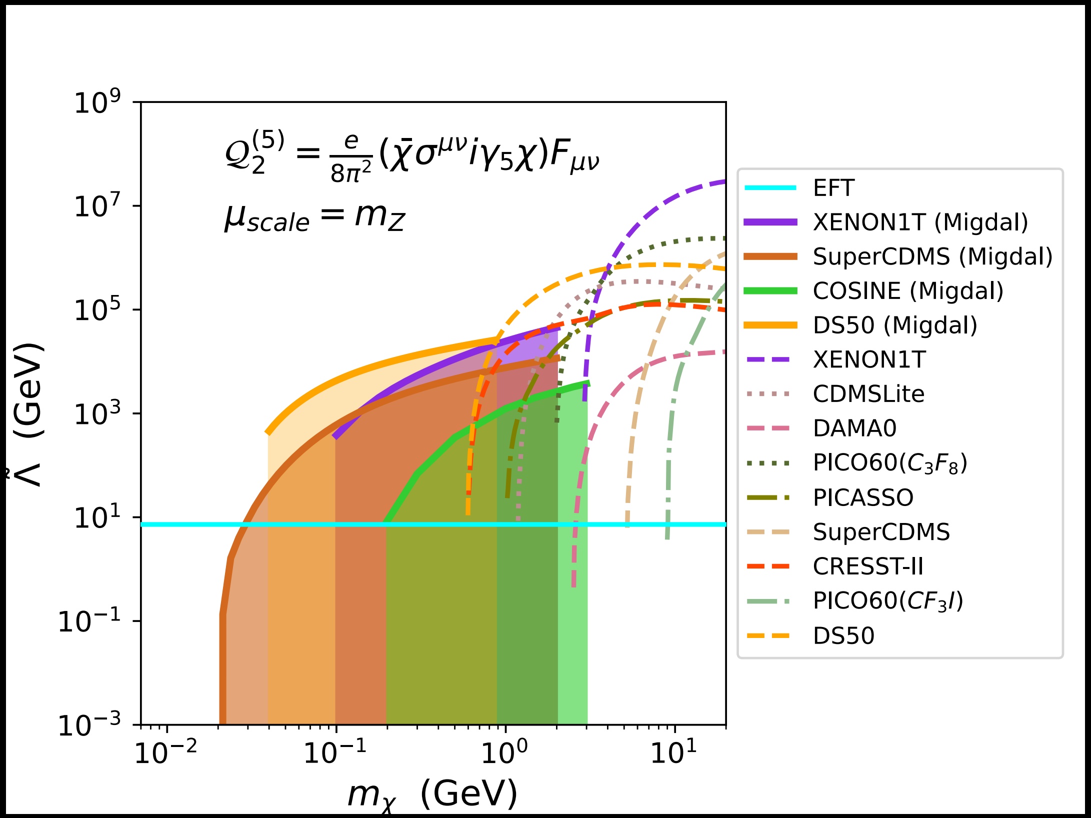
Target nucleus mass

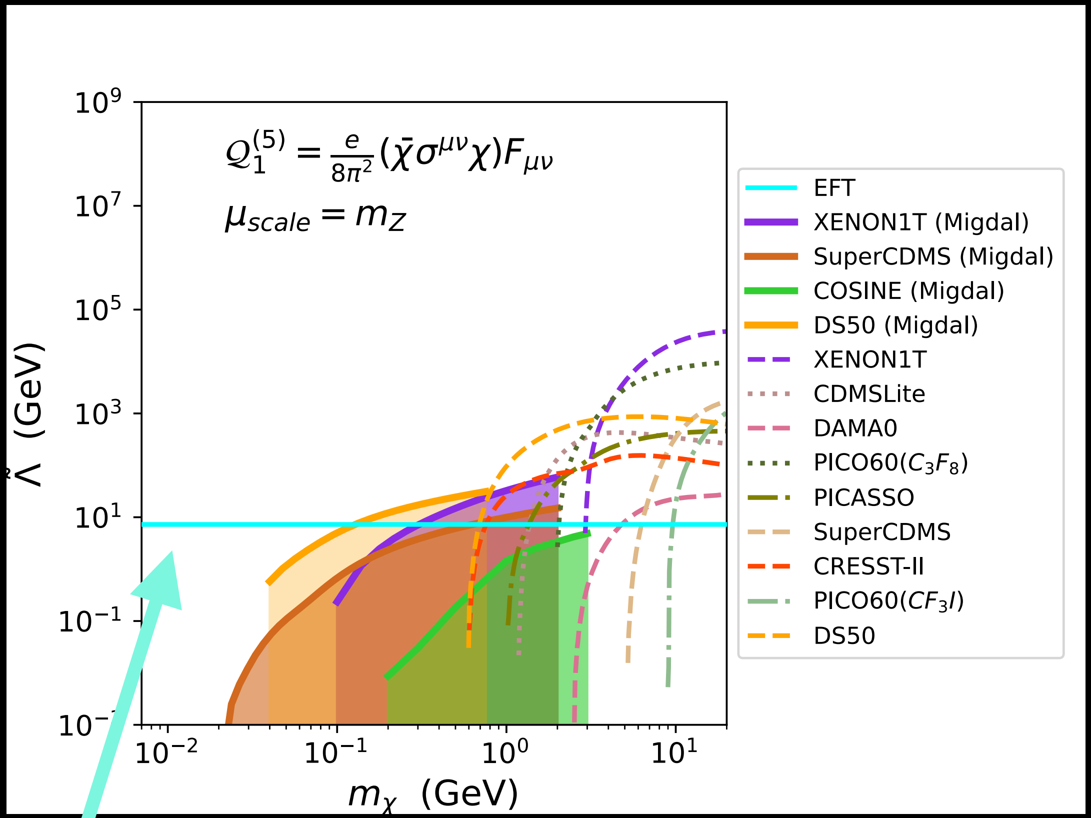
Baxter, Kahn, Krnjaic, *arXiv: 1908.00012*

Migdal effect in relativistic effective models

Dimension-5

G.Tomar, Kang, Scopel, *arXiv: 2210.00199*





EFT validity scale

$$\tilde{\Lambda} > \frac{\mu_{scale}}{(4\pi)^{1/(d-4)}}$$

$$\mu_{scale} = m_Z$$

Spin-1/2 WIMP response functions

$$R_M^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{\vec{q}^{\,2}}{m_N^2}\vec{v}_T^{\perp 2} c_5^\tau c_5^{\tau'} + \vec{v}_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right],$$

$$R_{\Phi''}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = \frac{\vec{q}^{\,2}}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^{\tau'}\right),$$

Interference terms

$$R_{\Phi'' M}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'},$$

$$R_{\tilde{\Phi}'}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{12}\left[c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right],$$

$$R_{\Sigma''}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = \frac{\vec{q}^{\,2}}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} \frac{\vec{q}^{\,2}}{m_N^2}\left(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}\right) + \frac{\vec{q}^{\,4}}{m_N^4} c_6^\tau c_6^{\tau'} + \vec{v}_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2}\vec{v}_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right], \quad (38)$$

$$R_{\Sigma'}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = \frac{1}{8}\left[\frac{\vec{q}^{\,2}}{m_N^2}\vec{v}_T^{\perp 2} c_3^\tau c_3^{\tau'} + \vec{v}_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{\vec{q}^{\,2}}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{\vec{v}_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{\vec{q}^{\,2}}{m_N^2} c_{15}^{\tau'}\right) + \frac{\vec{q}^{\,2}}{2m_N^2}\vec{v}_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right],$$

$$R_{\Delta}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left[\frac{\vec{q}^{\,2}}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right],$$

$$R_{\Delta\Sigma'}^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^{\,2}}{m_N^2}\right) = \frac{j_\chi(j_\chi+1)}{3}\left[c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right].$$

Anand, Fitzpatrick, Haxton, Phys. Rev. C89, 065501 (2014), 1308.6288

NREFT

- Free nucleon operators

$$\bar{\psi}_f \Gamma \psi_i \longrightarrow \chi_f^\dagger \mathcal{O}_X \tau_N^t \chi_i$$

$$\widehat{O}_M = 1, \quad \widehat{\vec{O}}_\Sigma = \vec{\sigma}_N, \quad \widehat{\vec{O}}_\Delta = \widehat{\vec{v}}_N^+, \quad \widehat{\vec{O}}_\Phi = \widehat{\vec{v}}_N^+ \times \vec{\sigma}_N, \quad \widehat{O}_\Omega = \widehat{\vec{v}}_N^+ \cdot \vec{\sigma}_N.$$

- WIMP-nucleon operators

$$\widehat{\mathcal{O}}_M = 1, \quad \widehat{\vec{\mathcal{O}}}_\Sigma = \vec{\sigma}_N, \quad \widehat{\vec{\mathcal{O}}}_\Delta = \vec{v}_{\chi N}^+, \quad \widehat{\vec{\mathcal{O}}}_\Phi = \vec{v}_{\chi N}^+ \times \vec{\sigma}_N, \quad \widehat{\mathcal{O}}_\Omega = \vec{v}_{\chi N}^+ \cdot \vec{\sigma}_N.$$

$$\vec{v}_{\chi N}^+ = \vec{v}_\chi^+ - \vec{v}_N^+.$$

$$\vec{v}_\chi^+ = \vec{v}_\chi - \frac{\vec{q}}{2m_\chi}$$

NREFT

- Nuclear response

  - M : vector-charge (**spin-independent part**, non-zero for all nuclei)
  - Φ'' : vector-longitudinal, related to spin-orbit coupling $\sigma \cdot l$ (also spin-independent, non-zero for all nuclei)
  - Σ', Σ'' : longitudinal and transverse components of nuclear spin, **their sum is the usual spin-dependent interaction**, require $j > 0$
  - Δ : associated to orbital angular momentum operator l, requires $j > 0$
  - $\tilde{\Phi}'$: related to the vector-longitudinal operator, transforms as a tensor under rotation, require $j > 1/2$

DM Velocity from *Gaia*

- DM velocity is a linear combination of the substructures and halo-velocities

$$f(v) = (1 - \eta_{\rm sub}) f_{\rm halo}(v) + \eta_{\rm sub} f_{\rm sub}(v)$$

$\eta_{\rm sub}$: DM fraction in the substructure

Migdal effect in effective field theory

- Differential rate

$$\frac{dR_{\chi T}}{dE_R} = \sum_T N_T \frac{\rho_\chi}{m_\chi} \int_{v_{min}} d^3 v_{\chi T} f(v_{\chi T}) v_{\chi T} \frac{d\sigma_T}{dE_R}$$

- Differential cross-section

$$\frac{d\sigma}{dE_R} = \frac{2m_T}{4\pi v_T^2} \left[\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}_T|^2 \right]$$

- Scattering amplitude

WIMP Response

Nuclear Response

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}_T|^2 = \frac{4\pi}{2j_T + 1} \sum_{\tau,\tau'} \sum_k R_k^{\tau,\tau'} \left[c_j^\tau, (v_T^\perp)^2, \frac{q^2}{m_N^2} \right] W_{Tk}^{\tau\tau'}(y)$$

Here, $k = M, \Phi'', \tilde{\Phi}', \Sigma', \Sigma'', \Delta, \Phi'' M, \Delta\Sigma'$

$y = (qb/2)^2$ with nuclear size b

Anand, Fitzpatrick, Haxton, Phys. Rev. C89, 065501 (2014), 1308.6288

Migdal effect in models

The ionisation event rate in an experiment due to the Migdal effect

$$\frac{dR}{dE_{det}} = \int_0^\infty dE_R \int_{v_{min}}^\infty dv_{\chi T} \frac{d^3R}{dE_R dE_{det} dv_{\chi T}}$$

Minimum DM speed to register the recoil

$$v_{min}(E_R) = \frac{m_T E_R + \mu_{\chi T} \Delta E}{\mu_{\chi T} \sqrt{2 m_T E_R}}$$

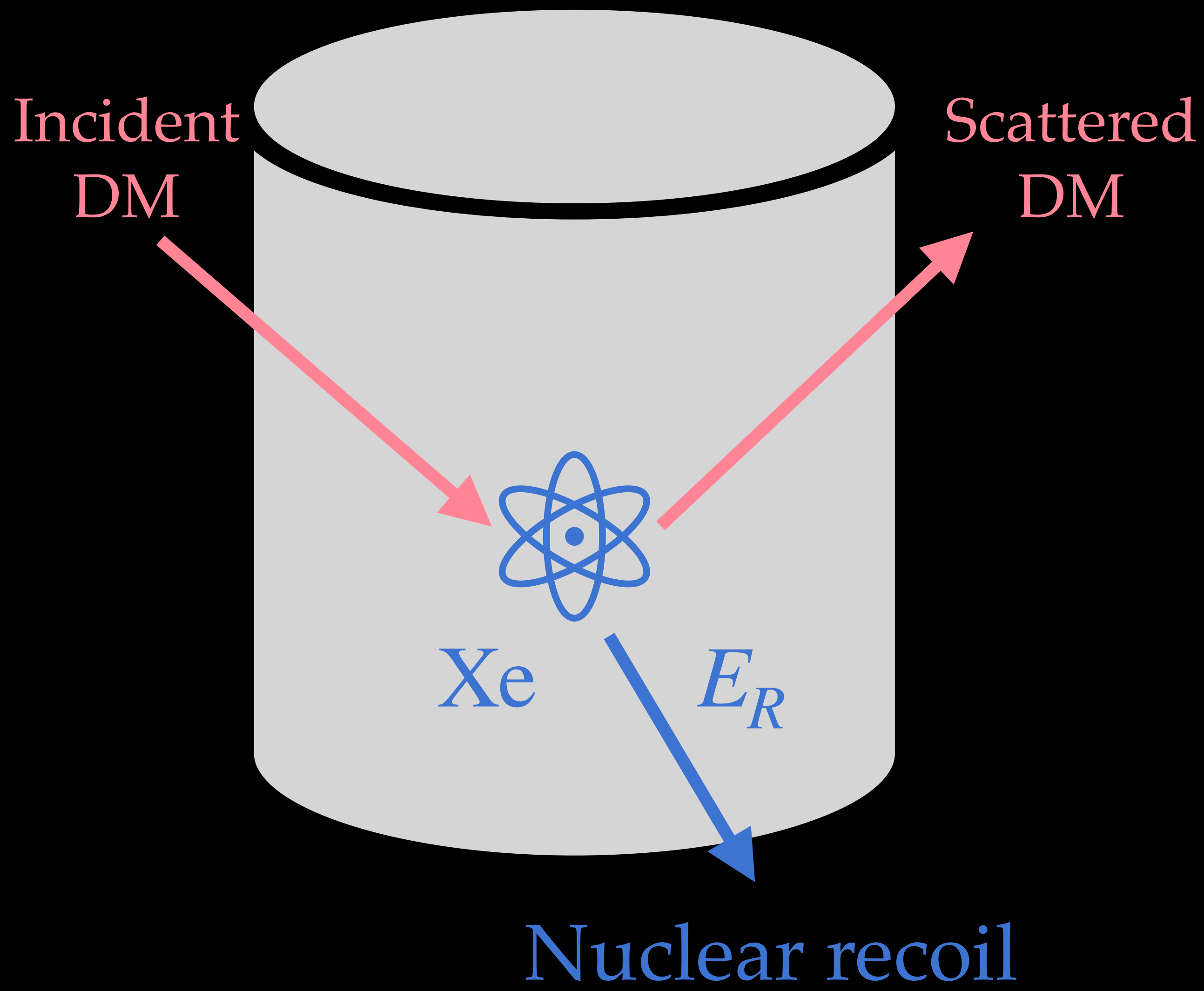
Identical to inelastic DM with $\delta \to \Delta E$

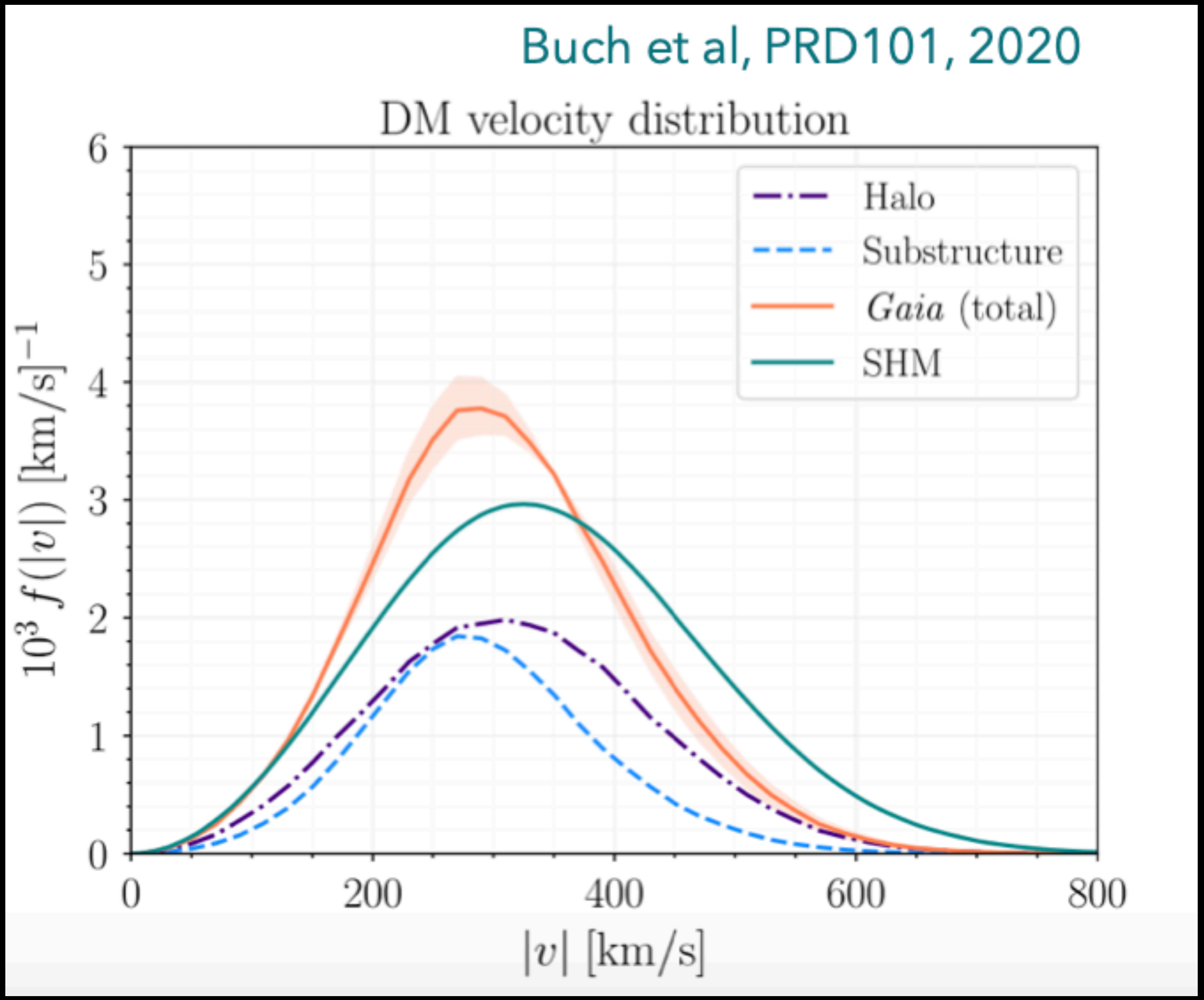
Smith and Weiner, *arXiv: 0101138*

Migdal $(\Delta E = E_e + E_{nl})$

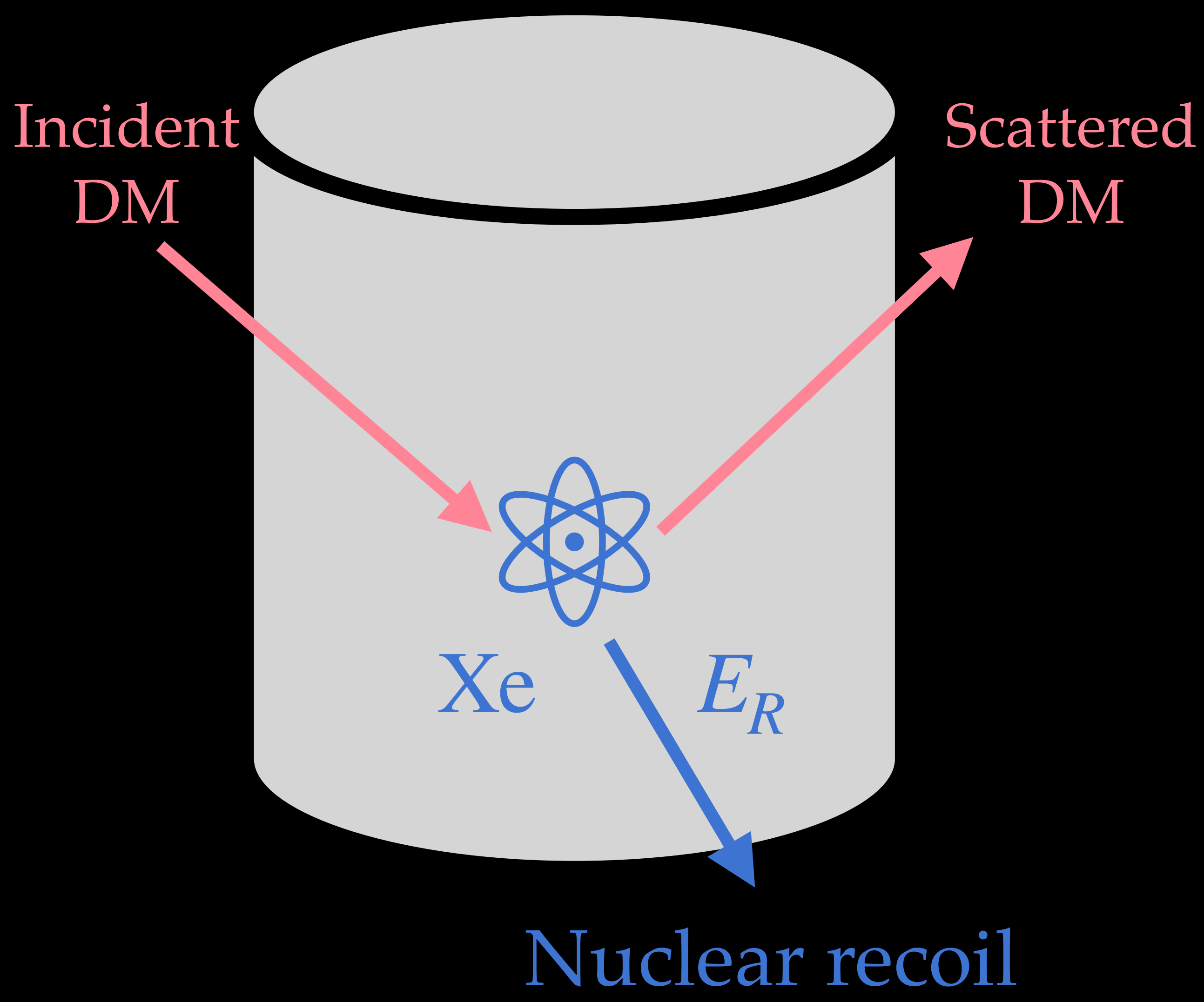
Bremsstrahlung $(\Delta E = \omega)$

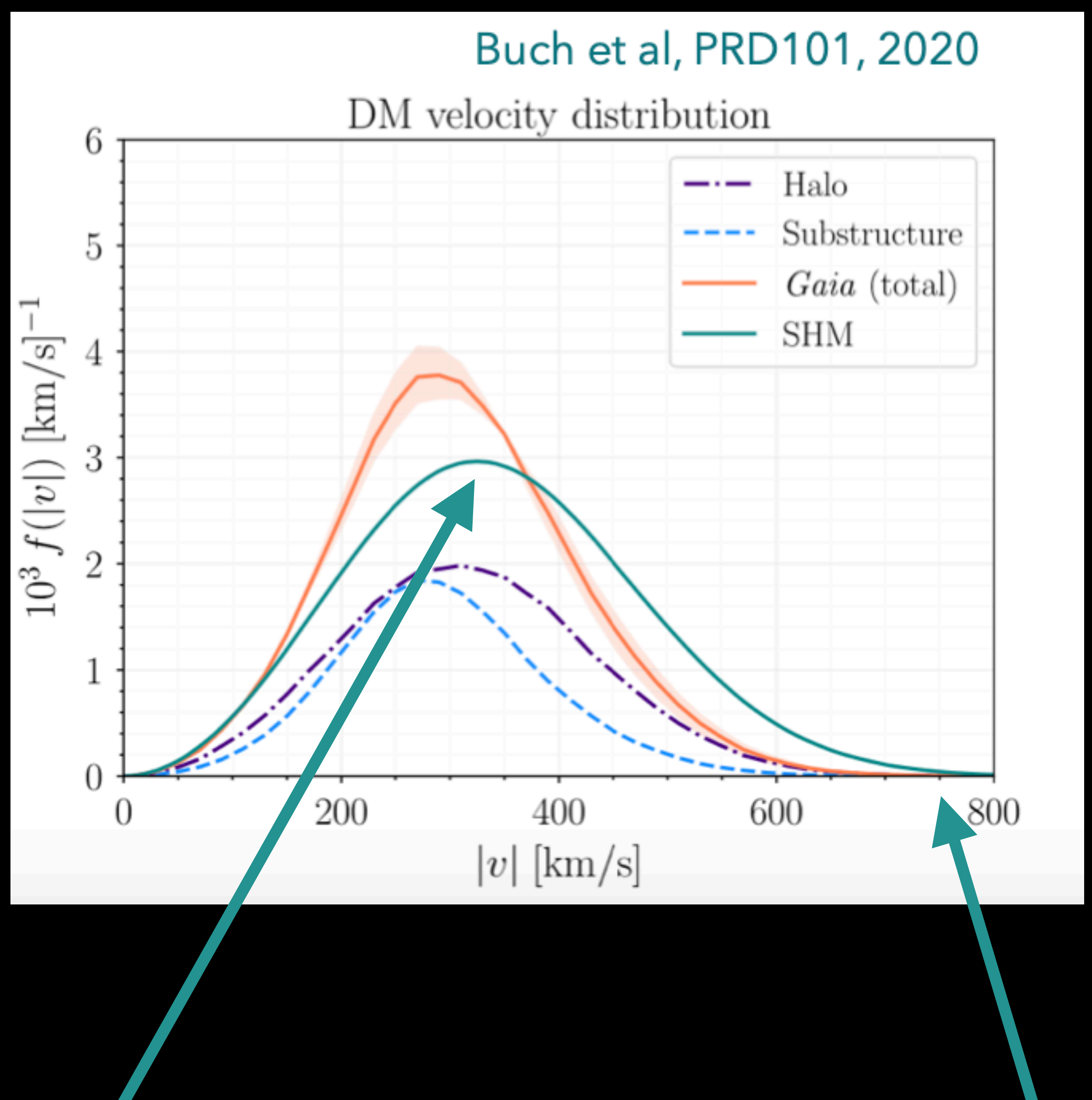
Direct Detection





Direct Detection

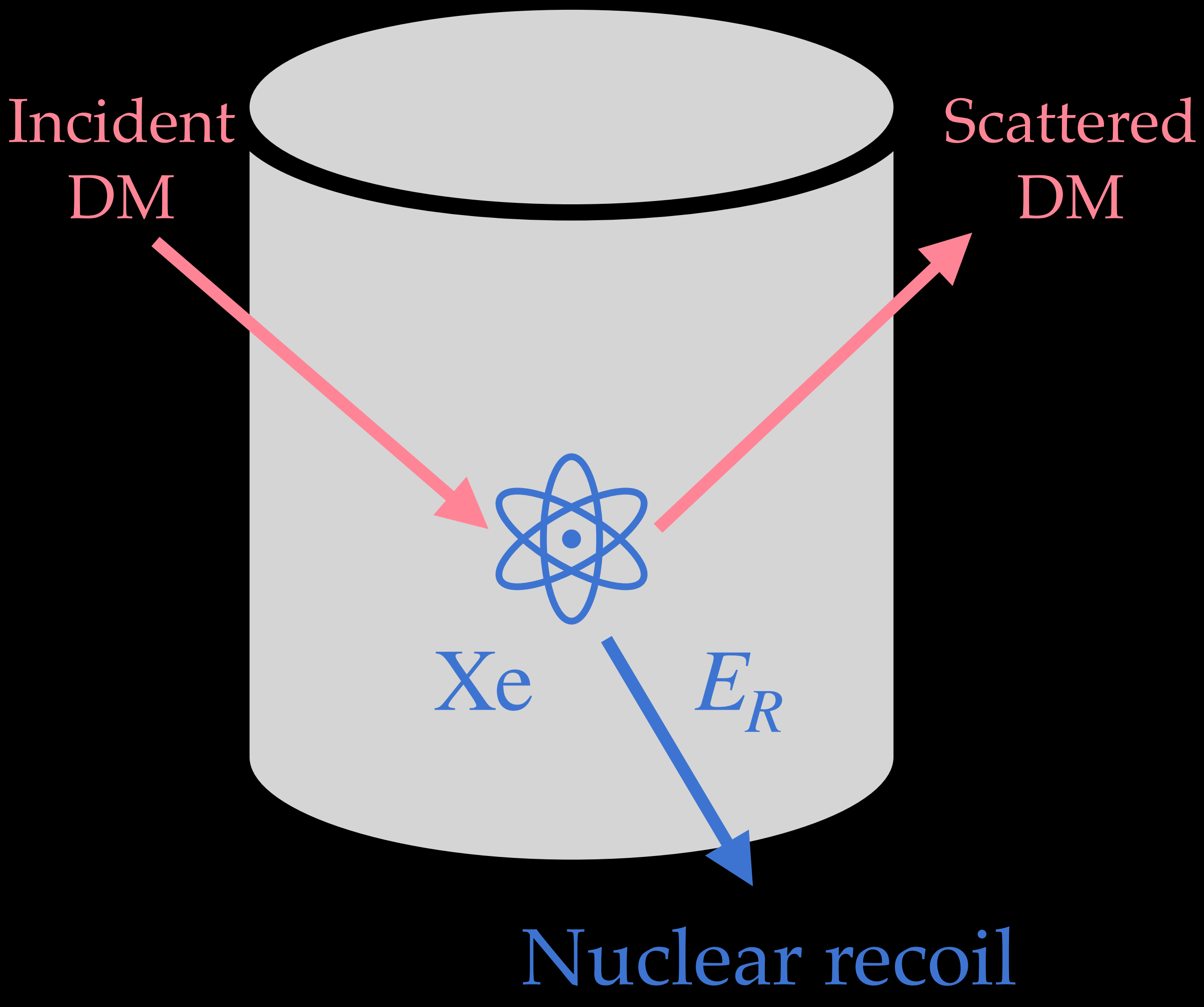




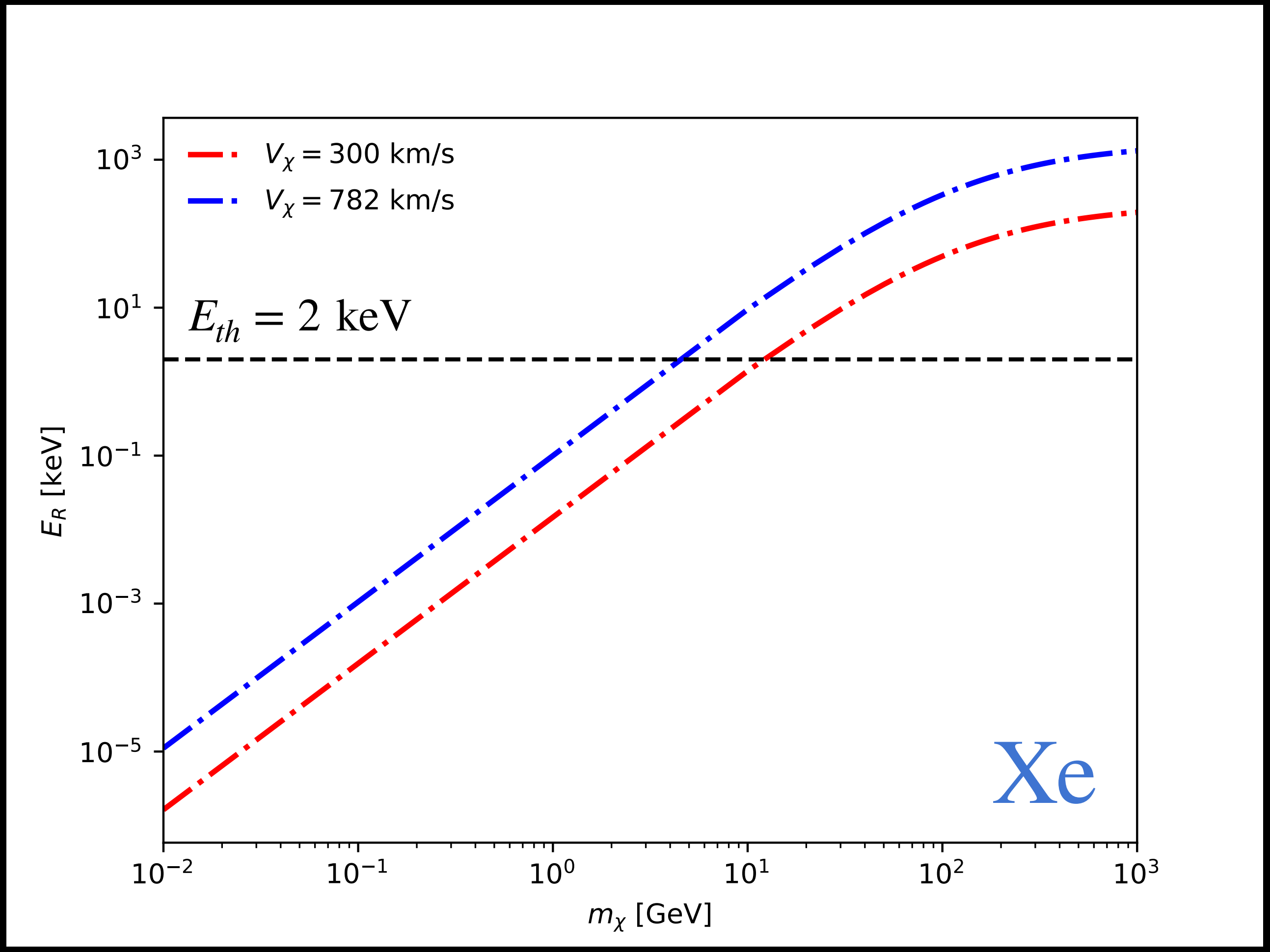
300 km / s

782 km / s

Direct Detection (Elastic Scattering)

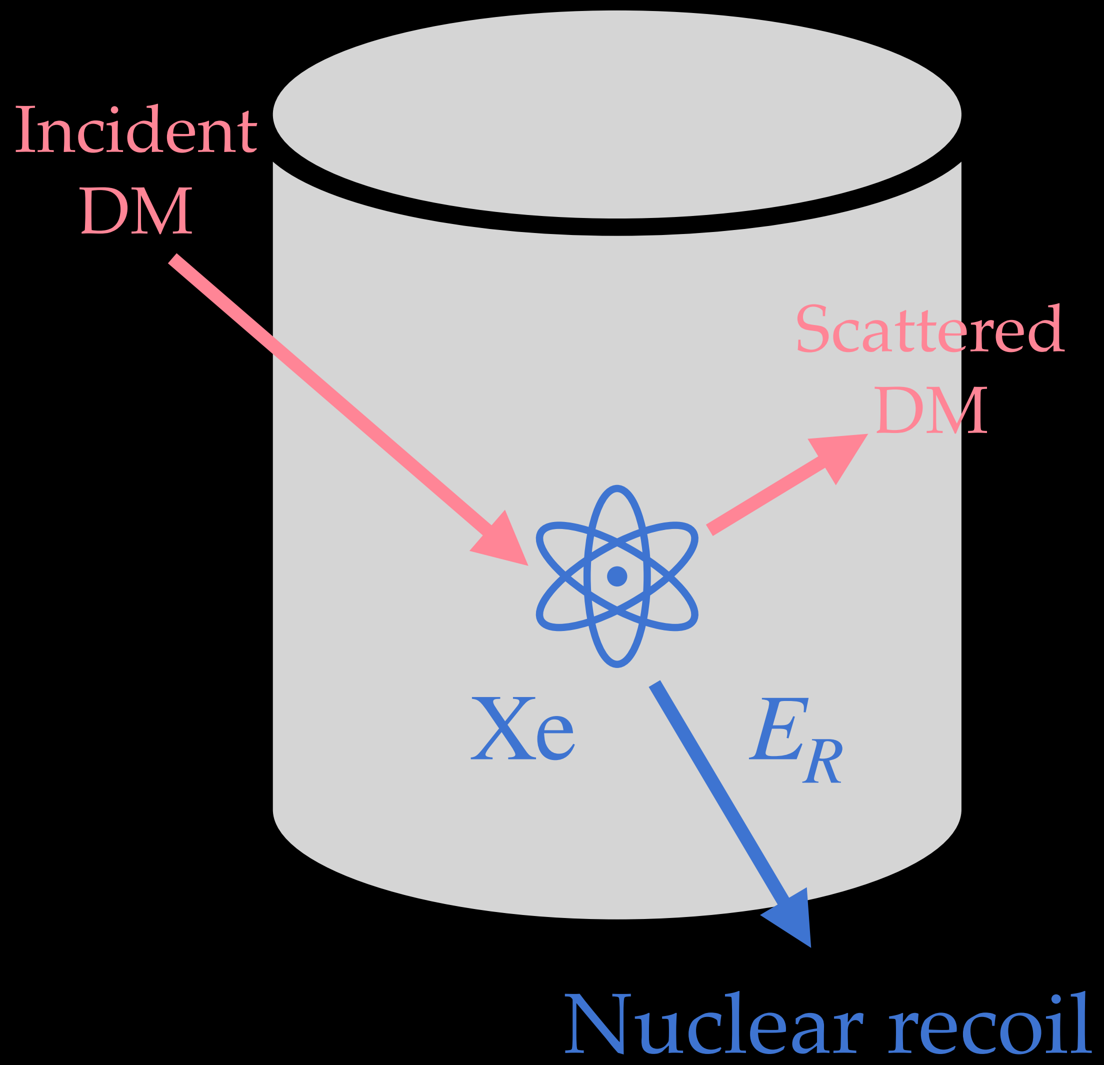


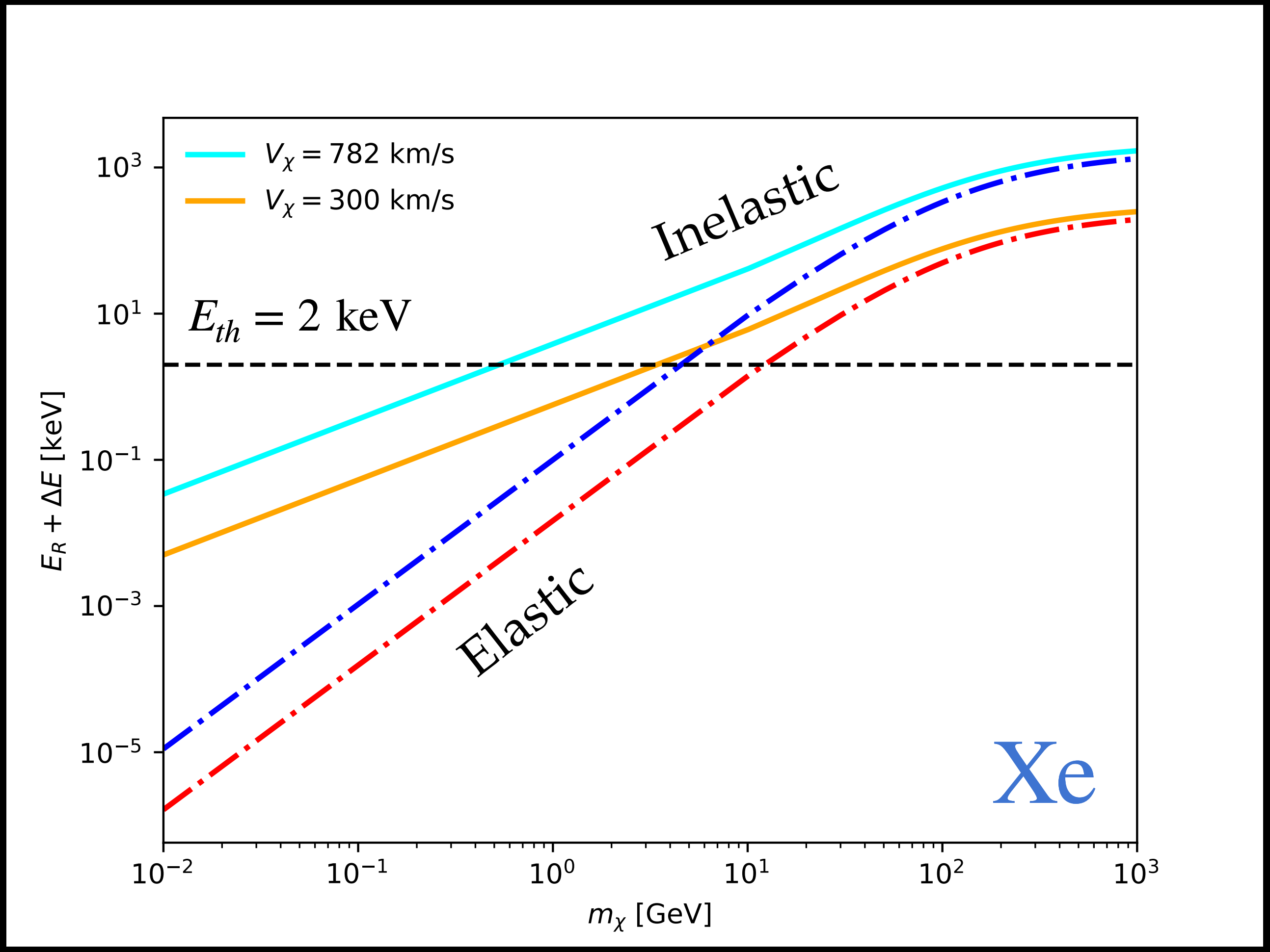
$$E_R^{max} = \frac{2\mu_{\chi T} v_{\chi T}^2}{m_T} \qquad \mu_{\chi T} = \frac{m_\chi m_T}{m_\chi + m_T}$$



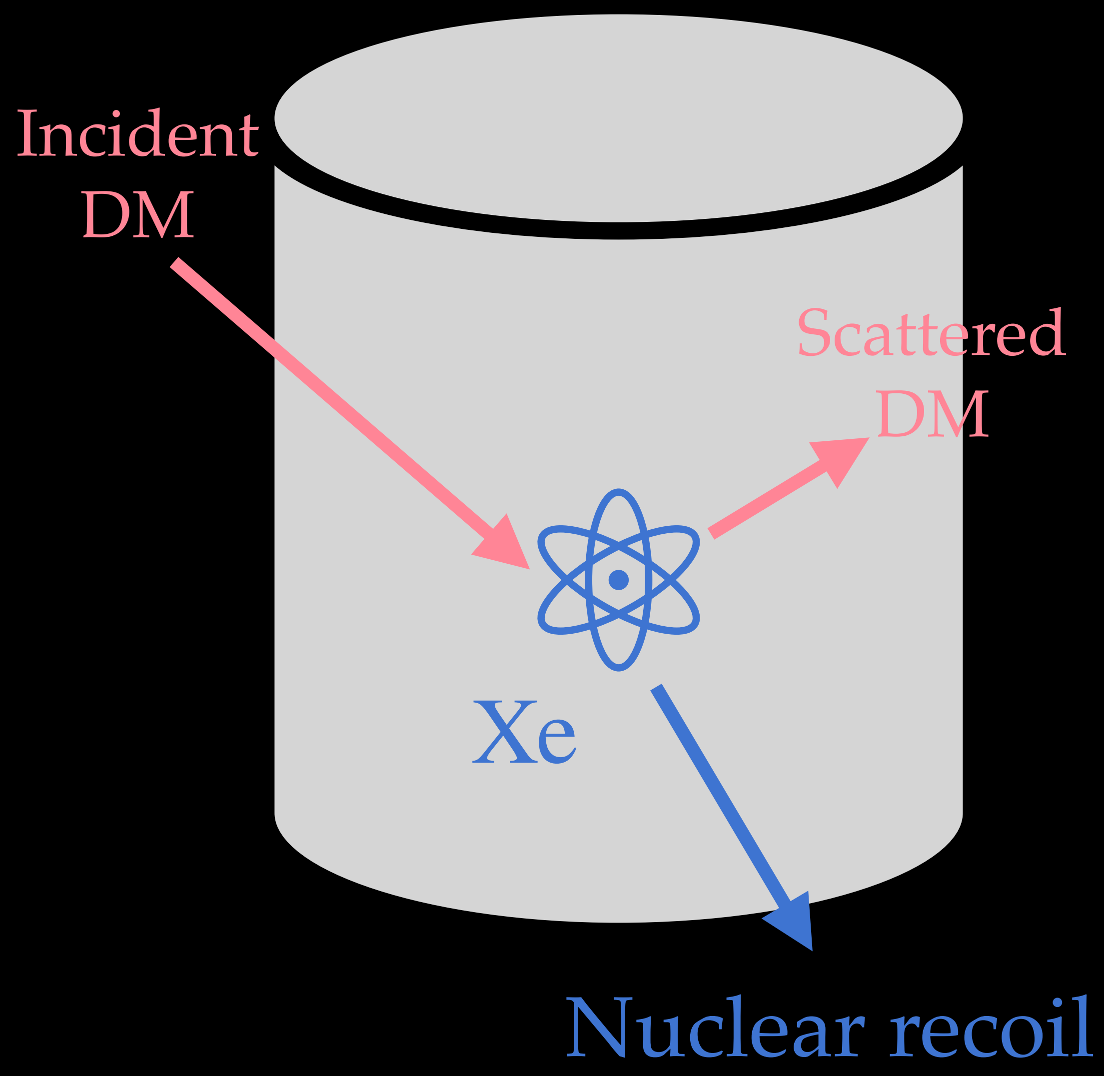
Direct Detection (Inelastic Scattering)

$$E_R^{max} = \frac{2\mu_{\chi T} v_\chi^2}{m_T} \qquad \Delta E = \frac{1}{2}\mu_{\chi T} v_{max}^2$$





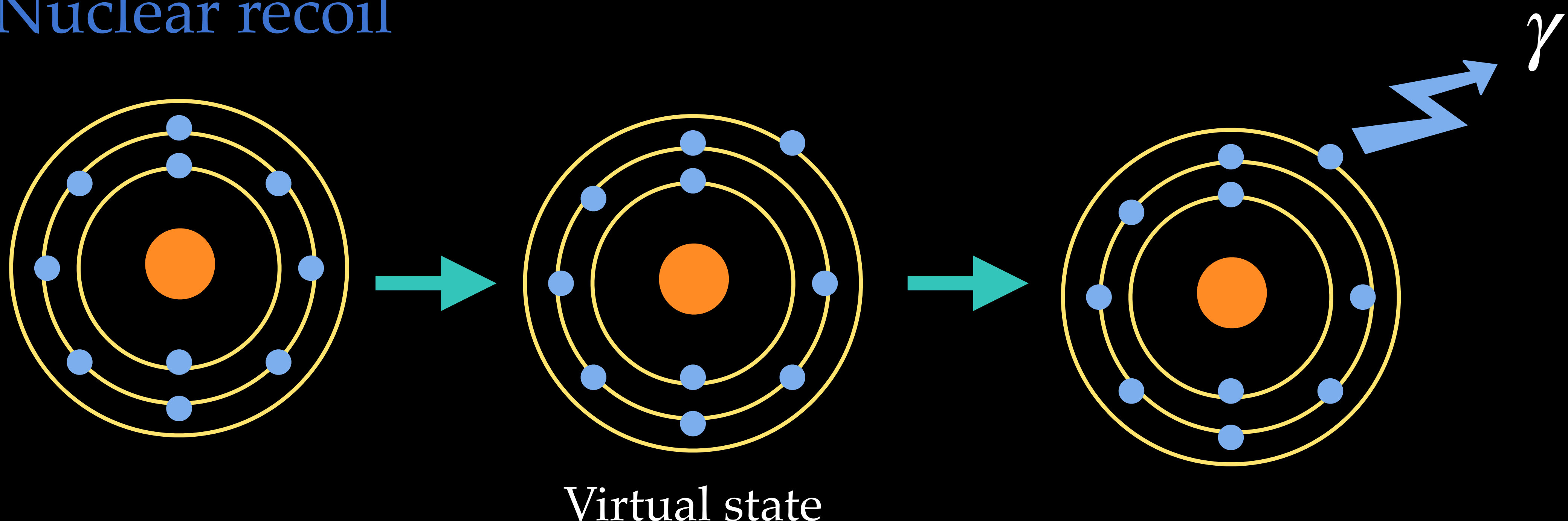
Direct Detection (Inelastic Scattering)



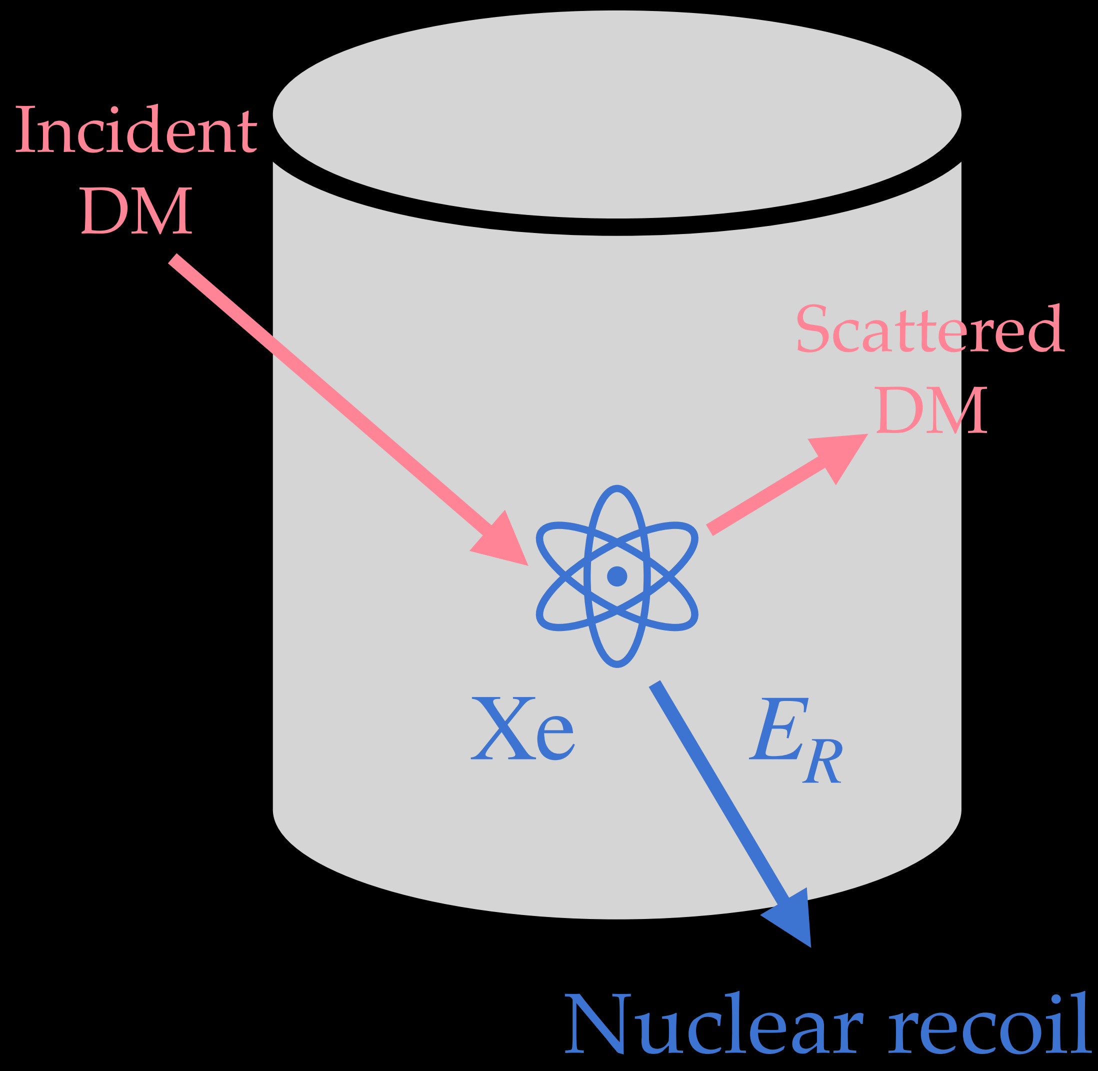
$$E_R^{max} = \frac{2\mu_{\chi T} v_\chi^2}{m_T}$$

$$\Delta E = \frac{1}{2}\mu_{\chi T} v_{max}^2$$

Bremsstrahlung $(\Delta E = \omega)$



Direct Detection (Inelastic Scattering)



$$E_R^{max} = \frac{2\mu_{\chi T} v_\chi^2}{m_T}$$

$$\Delta E = \frac{1}{2}\mu_{\chi T} v_{max}^2$$

Atomic ionisation
(Migdal)

$(\Delta E = E_e + E_{nl})$