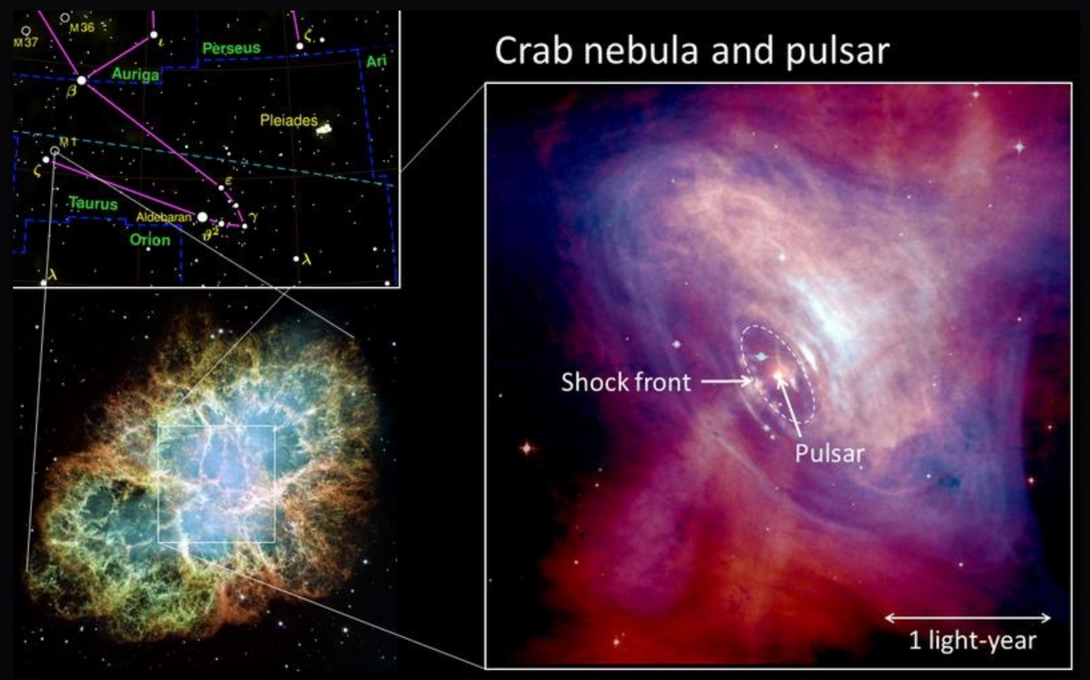
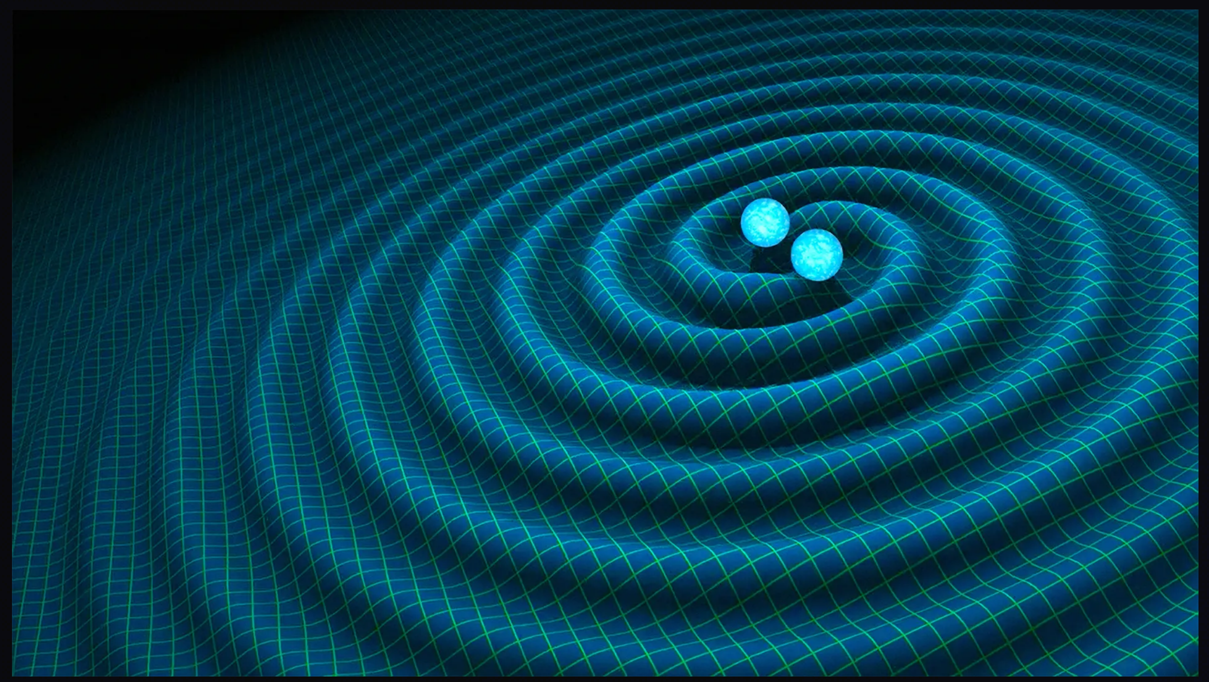




The Neutron Star Matter EoS : Where do we stand ?

INFN CATANIA
Istituto Nazionale di Fisica Nucleare
Sezione di Catania

Fiorella Burgio

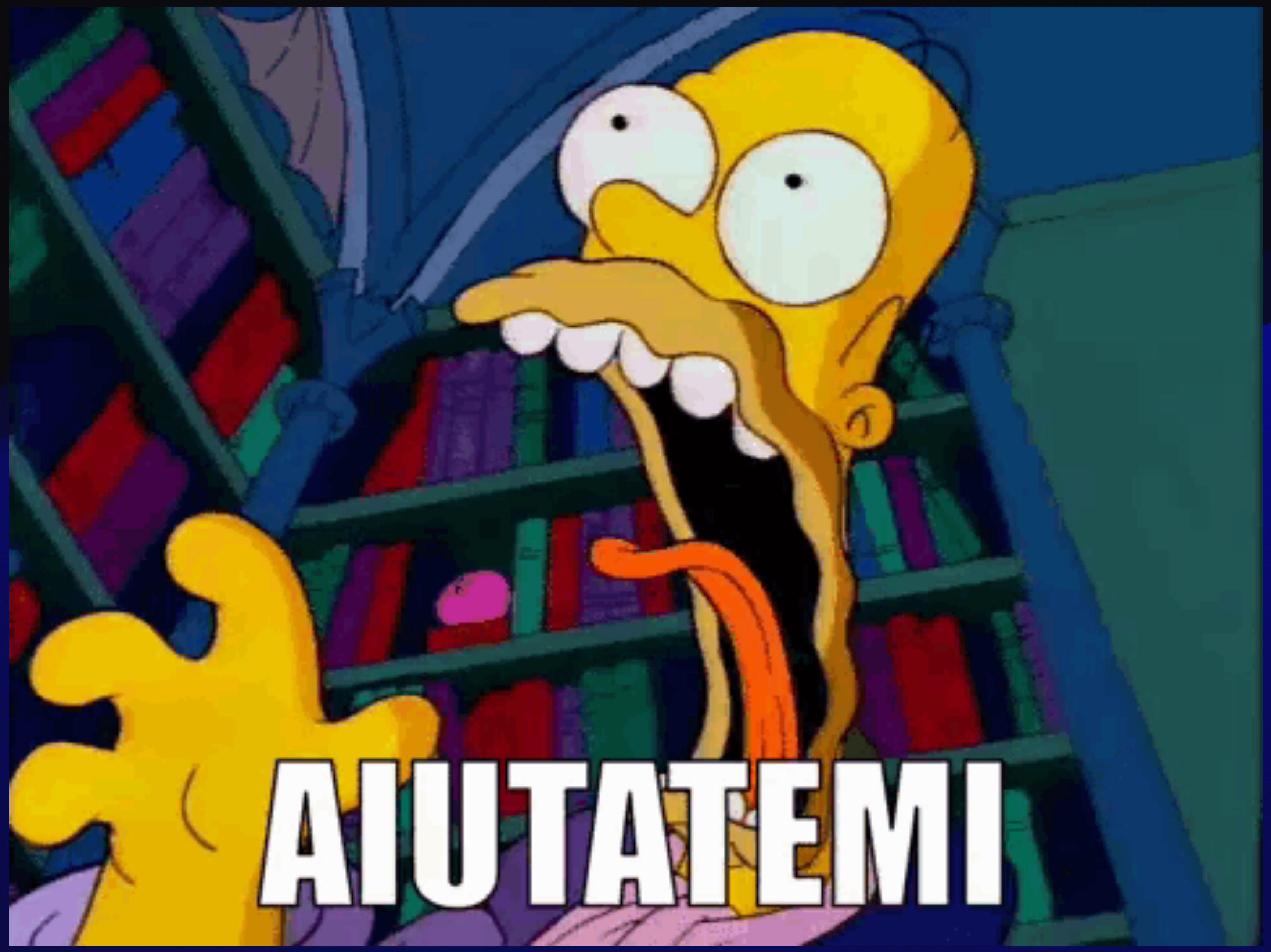
Università di Catania

60th Karpacz Winter School on Theoretical Physics & WE-Heraeus Physics School,

16-25 May 2024, University of Wroclaw / Hotel "Artus", Karpacz

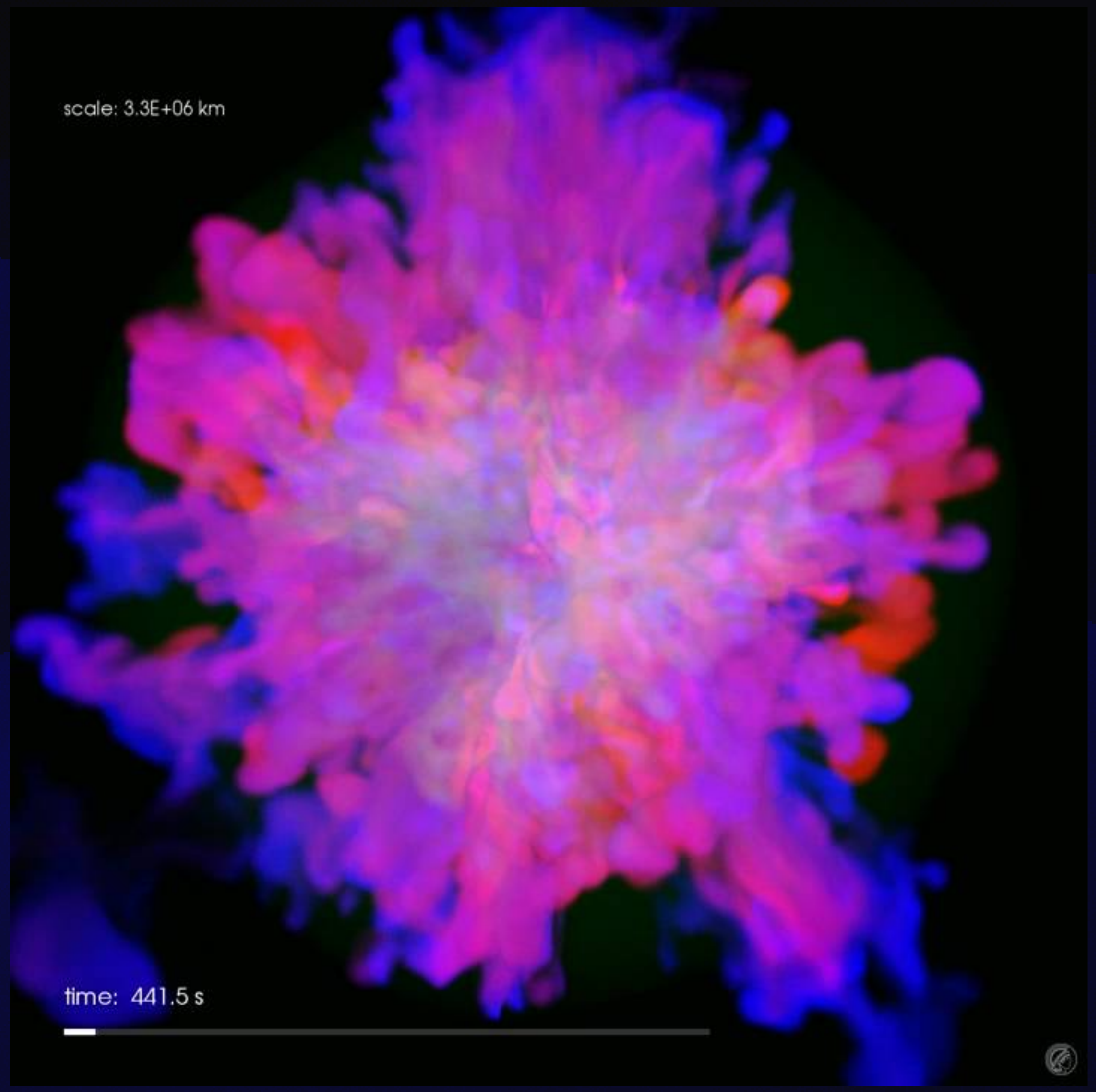
Outline

- NS in a Nutshell
- Relevance of the Equation of State
- EoS Models
- Constraints from HIC and NS Observations
- Onset of Hyperons
- Hyperon puzzle and possible solutions
- Hadron-quark phase transition
- …




M = 1-2 M0 (M0 ~2 x 10^33 g)
R ~ 10 km
ρ ~ M/R^3 ~ 2 x 10^15 g/cm^3
Capo d'Orlando
Barcellona Pozzo di Gotto
Messina
Parco dei Nebrodi
Santa Teresa di Riva
Randazzo
Castiglione di Sicilia
Cesarò
Maletto
Taormina
Parco dell'Etna
Bronte, Sicily
E45
Giarre
Zafferana Etnea
Adrano
Biancavilla
Centuripe
Acireale
San Giovanni La Punta
Paterno
Aci Castello
Terre Nere
Misterbianco
Catania
Sferro
DID YOU KNOW?
A teaspoon of a Neutron star would weigh more than 300 Great Pyramids of Giza.

Everything starts from a supernova explosion

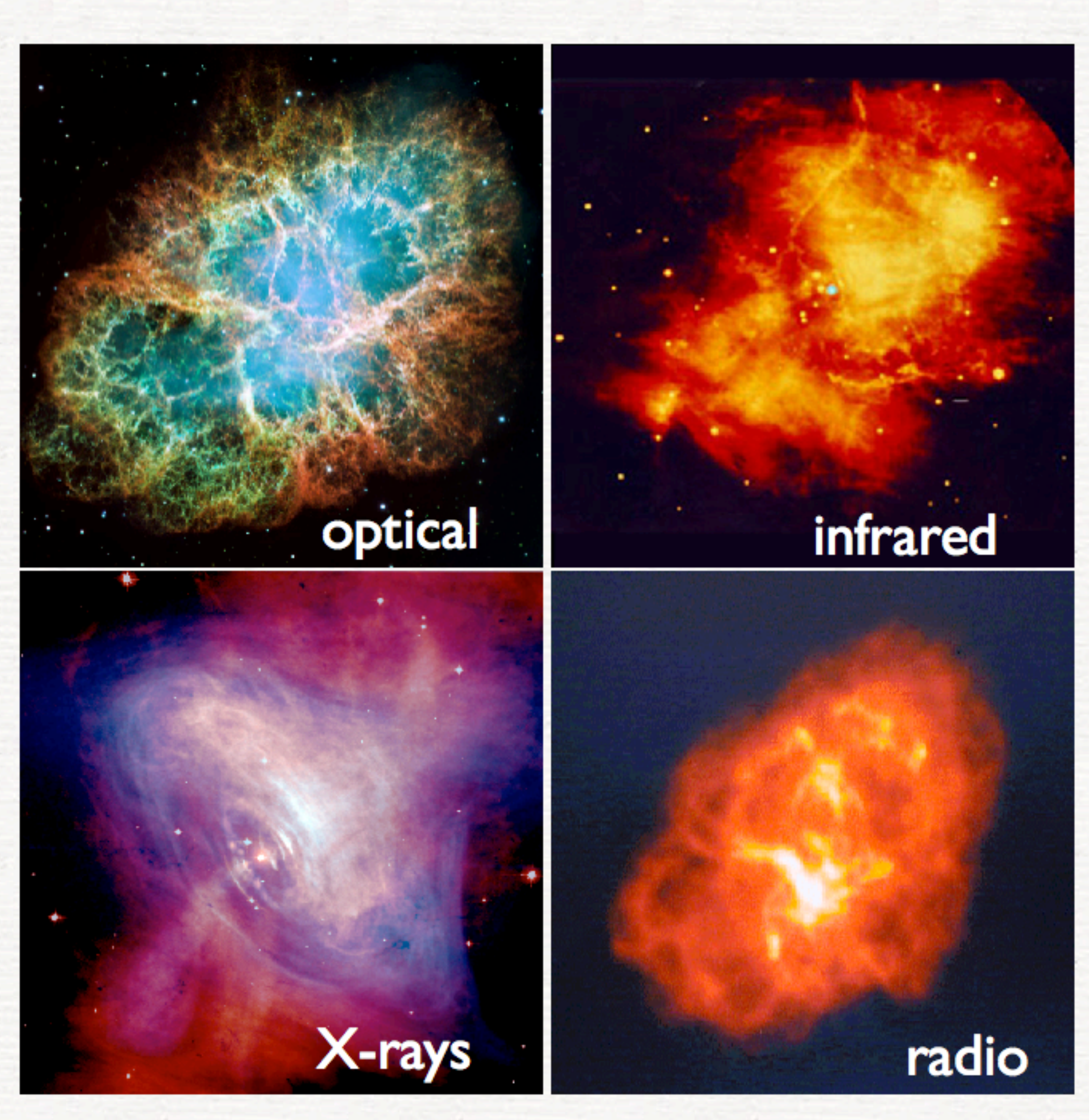


Computer simulation up to 9000 seconds
N.J. Hammer, H.-Th. Janka, E. Müller, ApJ 714 (2010) 1371-1385

A famous example

China, **4 July 1054** : a new star appears in the sky, visible even in daylight in the Crab constellation. Visible for two years !

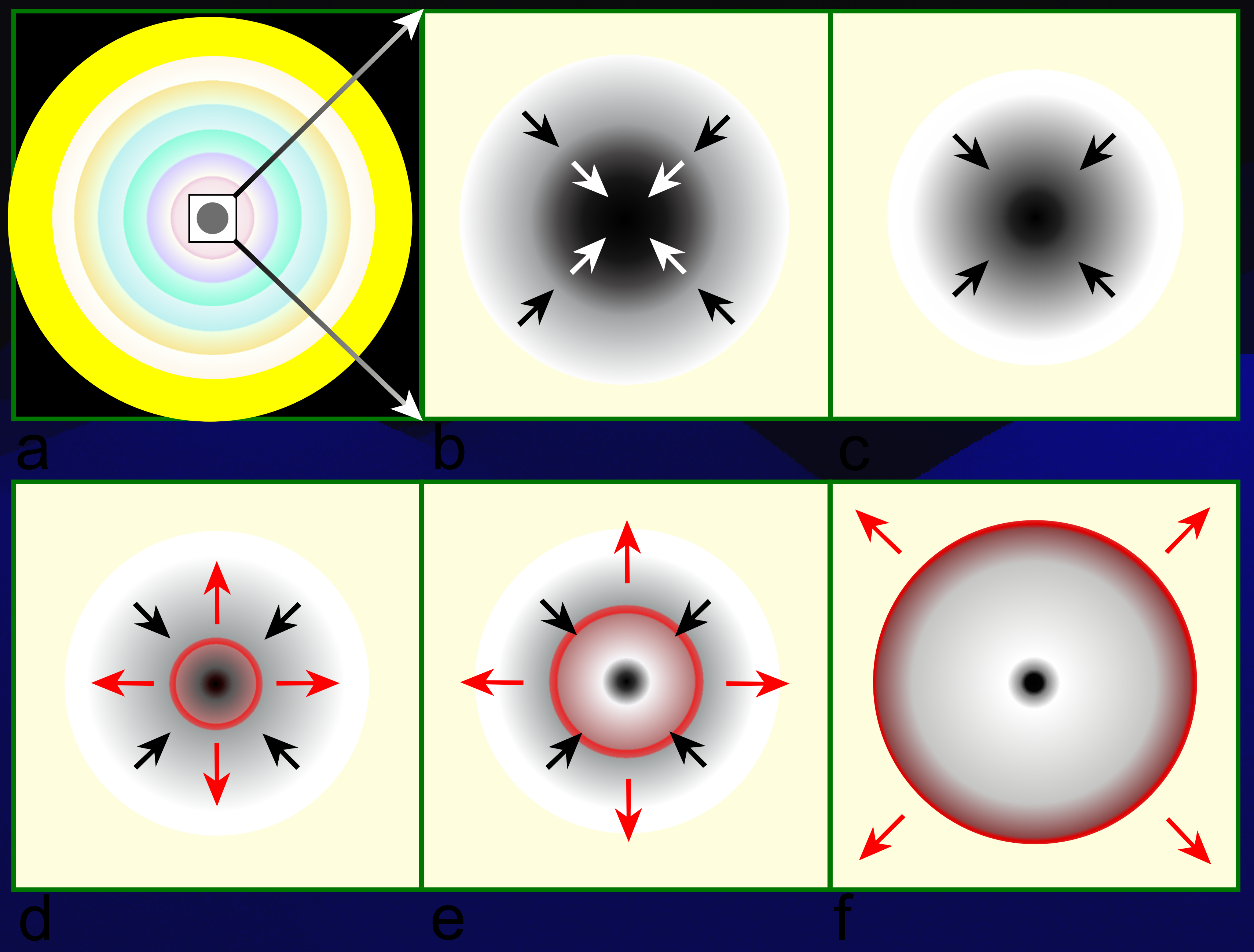
Actually it was a supernova that had produced a **neutron star** : the ***Crab pulsar***.



Evolution of a massive star → neutron star formation

- Massive star => He fusion to C and O
- For heavy enough stars, fusion reactions up to Ne, Mg and Si
- Sequence of fusion reactions ends with Fe

- Once the Fe core reaches the Chandrasekhar mass, the core cannot sustain its own mass and does not withstand gravitational collapse any longer.
- Inner part of the core compressed into neutrons, causing infalling material to bounce.
- Formation of an outward-propagating shock front (red), which is re-invigorad by **neutrino heating**.
- The surrounding material is blasted away, leaving only a degenerate remnant.



"Theory of core-collapse supernovae"
2.
3. **Janka, H.-T.; Langanke, K.; Marek, A.; Martínez-Pinedo, G.; Müller, B.**
4. **Physics Reports. 442 (1–6): 38–74. (2007).**

Observational Facts about NS

- Binary pulsars
- Isolated neutron stars : thermal emission
- Glitches from pulsars
- Quasi-periodic oscillations from accreting neutron stars.

Mass: $M \sim 1 - 2M_{\odot}$, the most precise Hulse-Taylor pulsar: PSR 1913+16 = 1.4411±0.0035 $M_{\odot}$

Radius: $R \sim 10 - 12\,km$

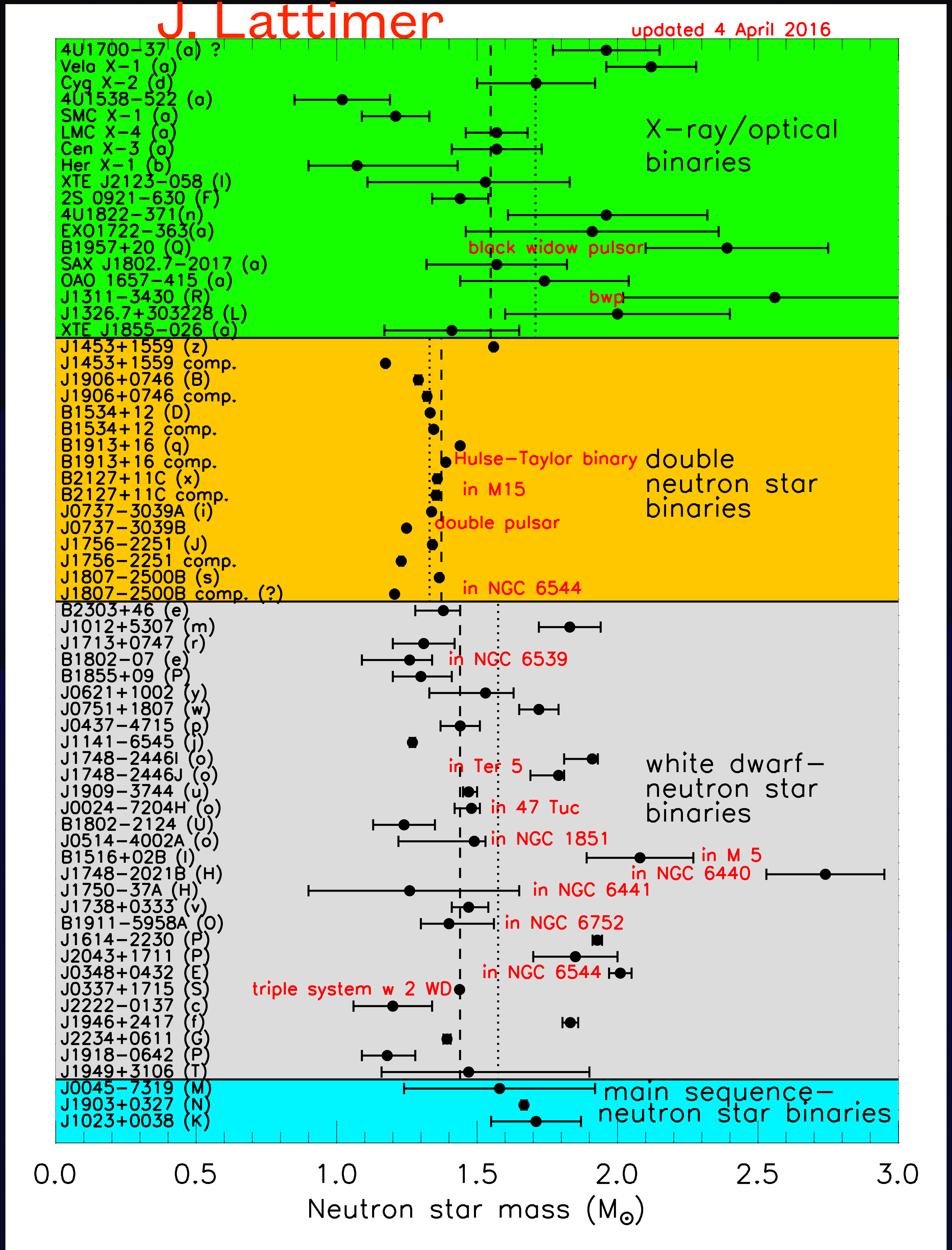
Density : $\rho \sim 10^{14} - 10^{15}\,g/cm^3$ (a gigantic nucleus !)

Temperature : $T \sim 10^6 - 10^{11}\,K$

Magnetic field : $B \sim 10^8 - 10^{16}\,G$

Rotational period : ms to seconds

Observational facts : the Mass and the Radius



Neutron star masses can be measured when they belong to binary systems, using Kepler's third law coupled to other orbital data.

Observed a mass range $M \sim 1 - 2M_\odot$

Best determined masses lie in a narrow interval $M \sim 1.25 - 1.45\, M_\odot$

Values $M > 2M_\odot$ strong constraints for the theory !

- PSR J1614-2230, $\mathrm{M} = (1.97 \pm 0.04)\, \mathrm{M}_\odot$ (P. Demorest et al., Nature, 2010
- PSR J0348+0432, $\mathrm{M} = (2.01 \pm 0.04)\, \mathrm{M}_\odot$ (J. Antoniadis et al., Science, 2013)
- MSP J0740+6620, $\mathrm{M} = (2.14^{+0.2}_{-0.18})\, \mathrm{M}_\odot$ (H. Cromartie et al., Nature Astronomy, 2019)
- PSR J0952-0607, $\mathrm{M} = (2.35 \pm 0.17)\, \mathrm{M}_\odot$ (R. Romani et al., ApJ Lett. 2022)

Combined analysis for PSR J0030+0451 and PSR J0740+6620 with GW170817 inferred values

$R_{2.08} = 12.35 \pm 0.75\,\mathrm{km}$

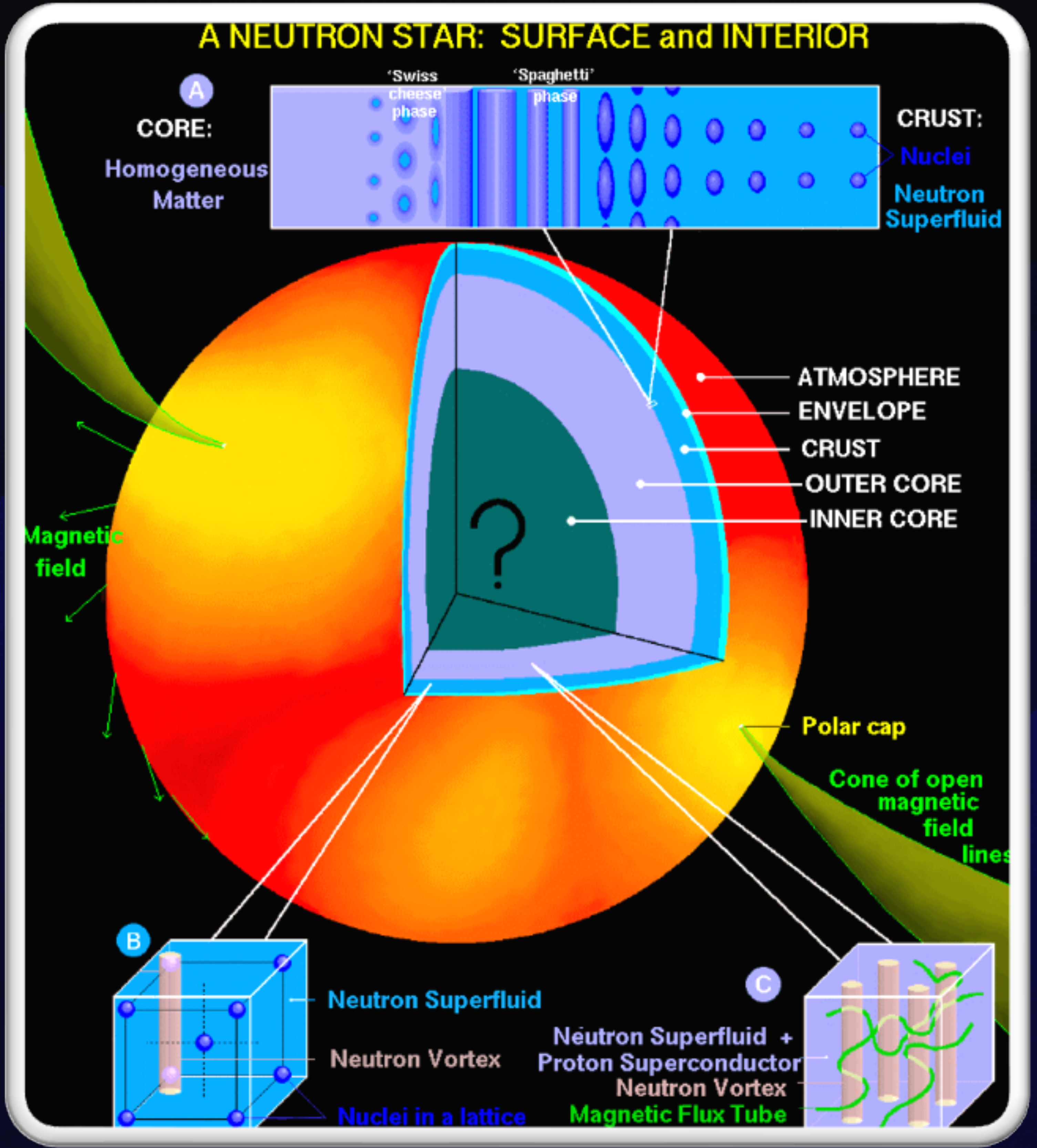
$R_{1.4} = 12.45 \pm 0.65\,\mathrm{km}$ Miller et al., ApJLett. 918, L28(2021)

$R_{1.4} = 11.94^{+0.76}_{-0.87}\,\mathrm{km}$ Pang et al., ApJ 922, 14 (2021)

$R_{1.4} = 12.33^{+0.76}_{-0.81}\,\mathrm{km}$ Raajmakers et al., ApJLett 918, L29 (2021)

Schematic view of a neutron star

by Dany Page, UNAM Mexico City



Atmosphere A few tens of cm, $\rho \leq 10^4 g/cm^3$ made of atoms.

Outer crust A few hundred m's thick, $\rho = (10^4 - 10^{11}) g/cm^3$. Ions immersed in an electron gas.

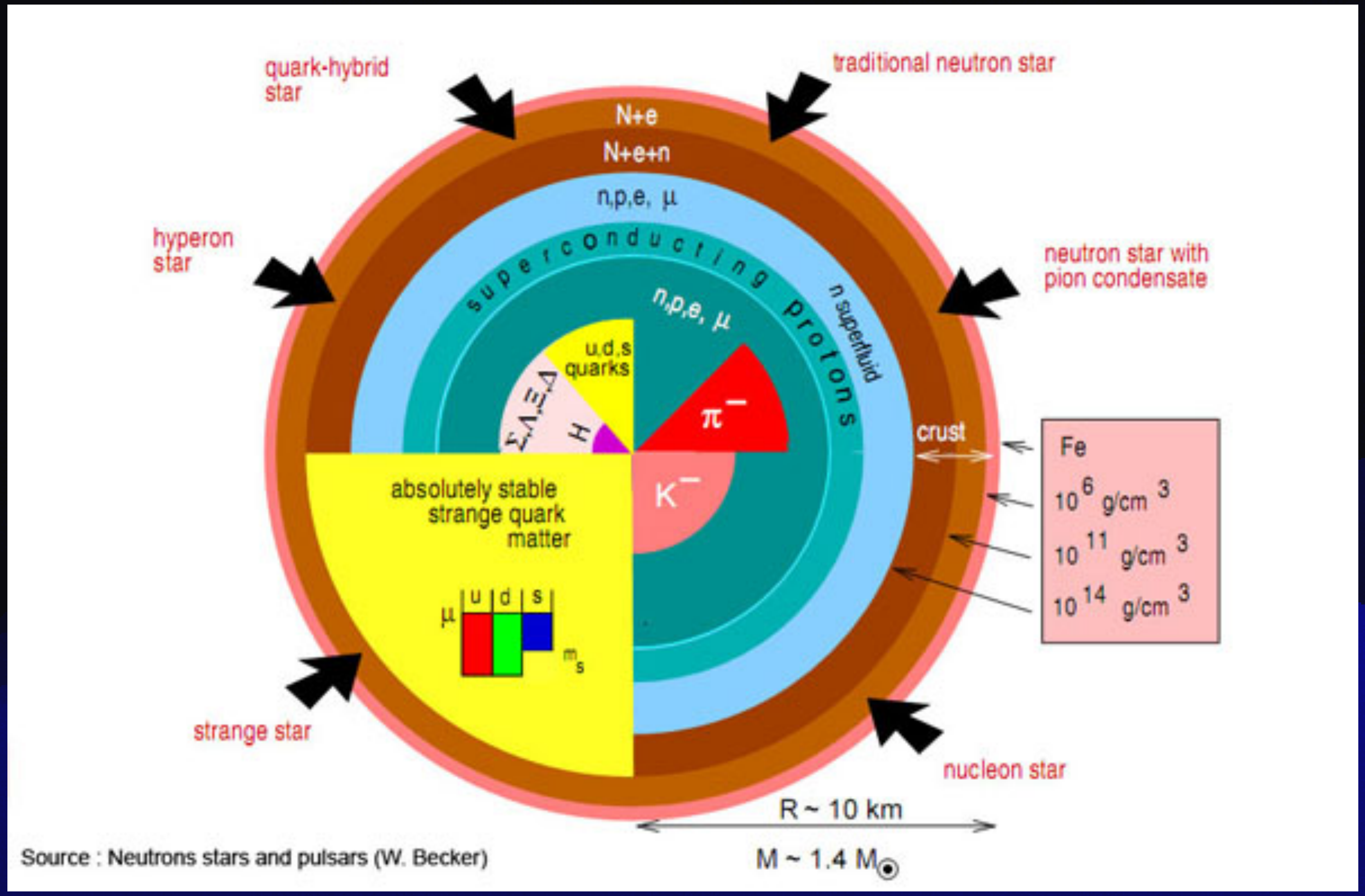
Inner crust 1-2 km, $\rho = 4 \times 10^{11} - 10^{14} g/cm^3$. Electrons beta-captured by nuclei → neutron-rich nuclei → drip point . Gas of free neutrons. Nuclei melt down and nuclear matter sets in from drip up to $\rho \approx \rho_0/2$: uniform fluid of n, p, e^-

Outer core $\rho \approx \rho_0/2 - 2\rho_0$. Asymmetric nuclear matter above saturation. Composition made by neutrons, protons, and leptons.

Inner core $\rho \approx 2\rho_0 - (8-10)\rho_0$ The most unknown region. "Exotic matter" . Hyperons ? Quarks ?

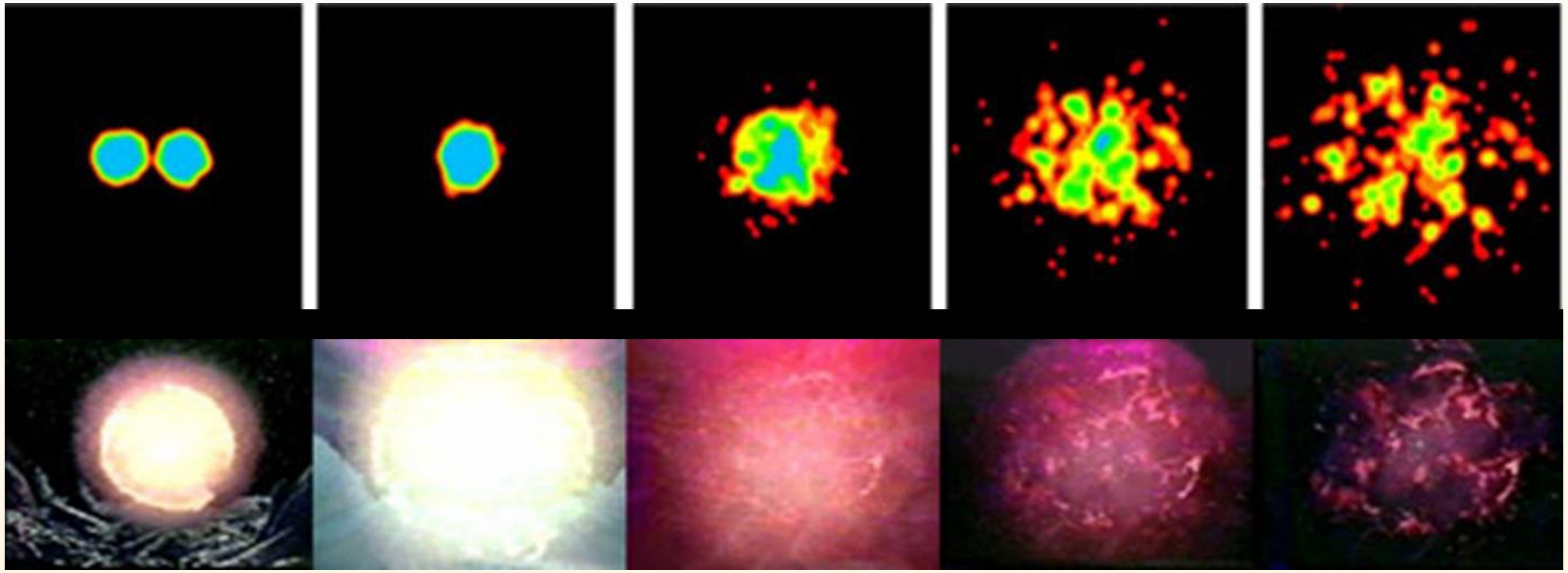
- Maximum possible mass

  (Oppenheimer - Volkoff limit)
- Detailed composition of NS matter
- Maximum rotational frequencies
- (non)-Radial oscillation frequencies
- Thermal evolution



They depend crucially on the Equation of State of nuclear matter !

Relevance of the EoS



1. Heavy ion collisions (small N/Z, high T)

2. Supernovae and Neutron Stars
   (high N/Z, high (small) T in SN (NS))

3. Binary NS merger and GW emission
   (high density, high N/Z and T)

Quite different physical conditions in each case !

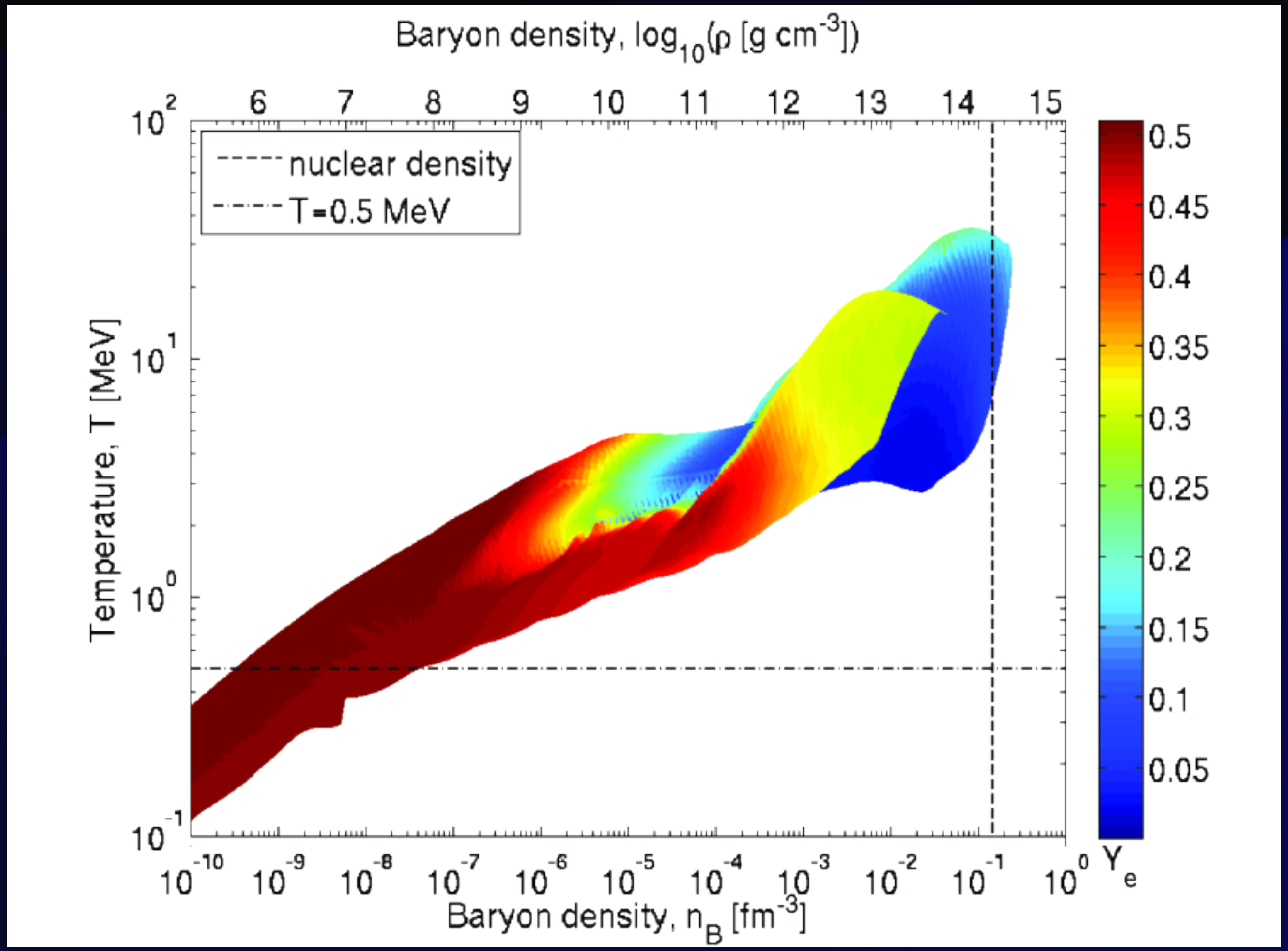
A nuclear matter theory must be able to treat all these physical situations.

The construction of the EoS

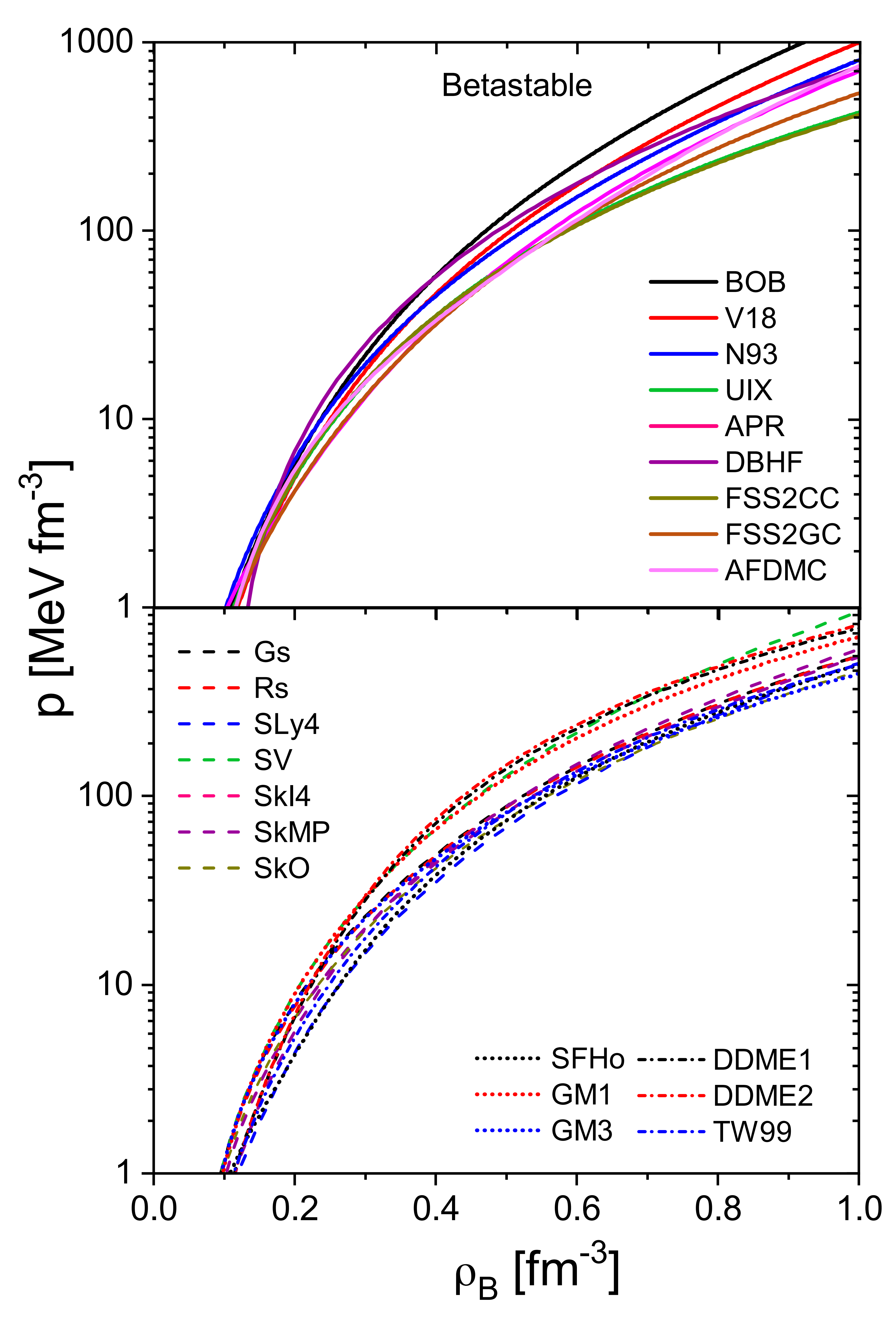
A challenging task

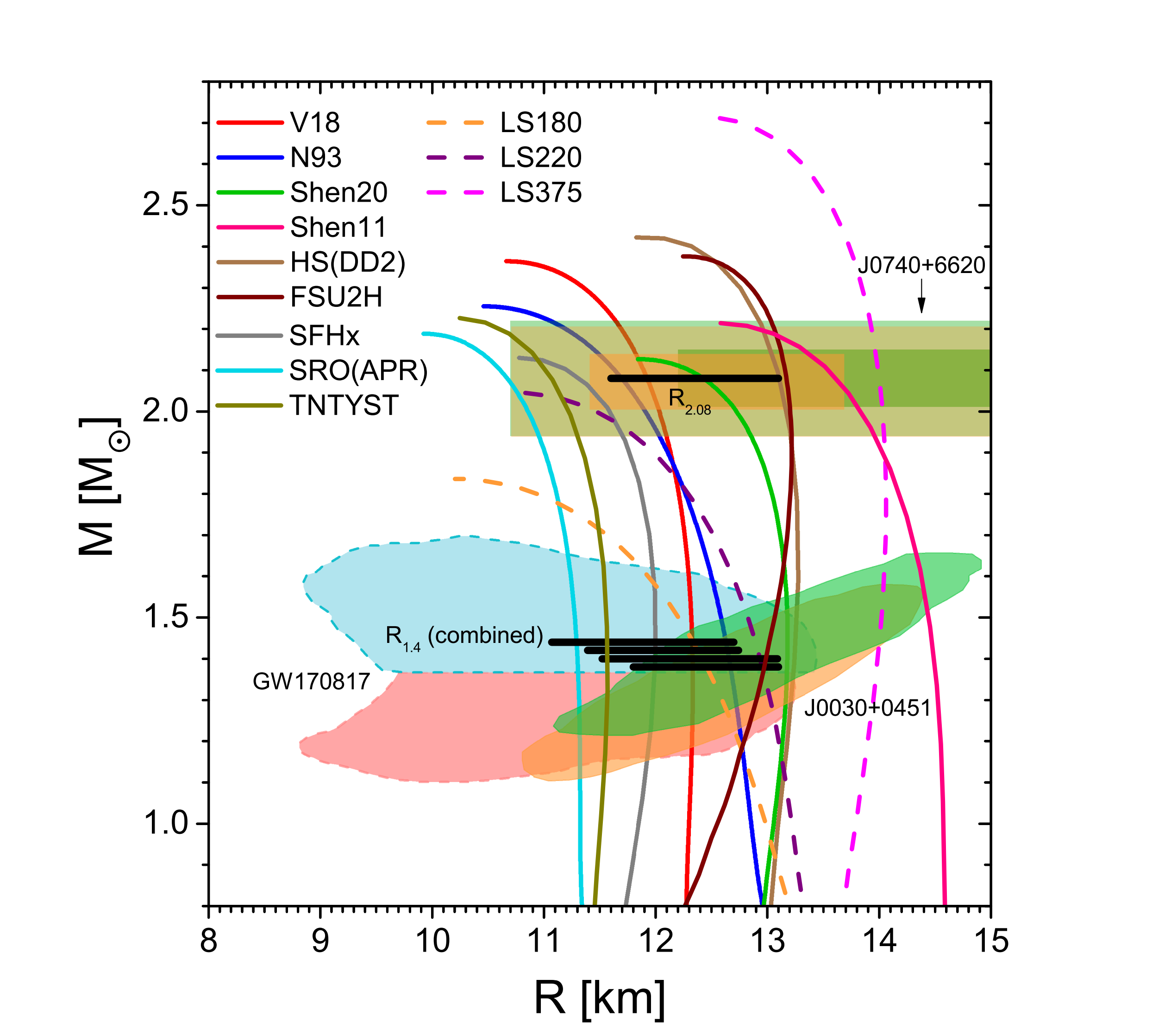
- Wide range of temperature, density and isospin asymmetry reached in astrophysical scenarios.
- Role of the hadronic interaction and its complexity
- Complicated solution of the nuclear many-body problem

Fischer et al., 2021



Temperature and density reached during a standard core-collapse supernova simulation at 100 ms post bounce.





What do we need ?

Exact theory to deal with

(a) Strong interactions of particles of different species

(b) Many-body effects in dense matter

What do we have ?

Many drastically different theoretical models!

Solving the nuclear many-body problem

Overview of the strong interaction in dense matter

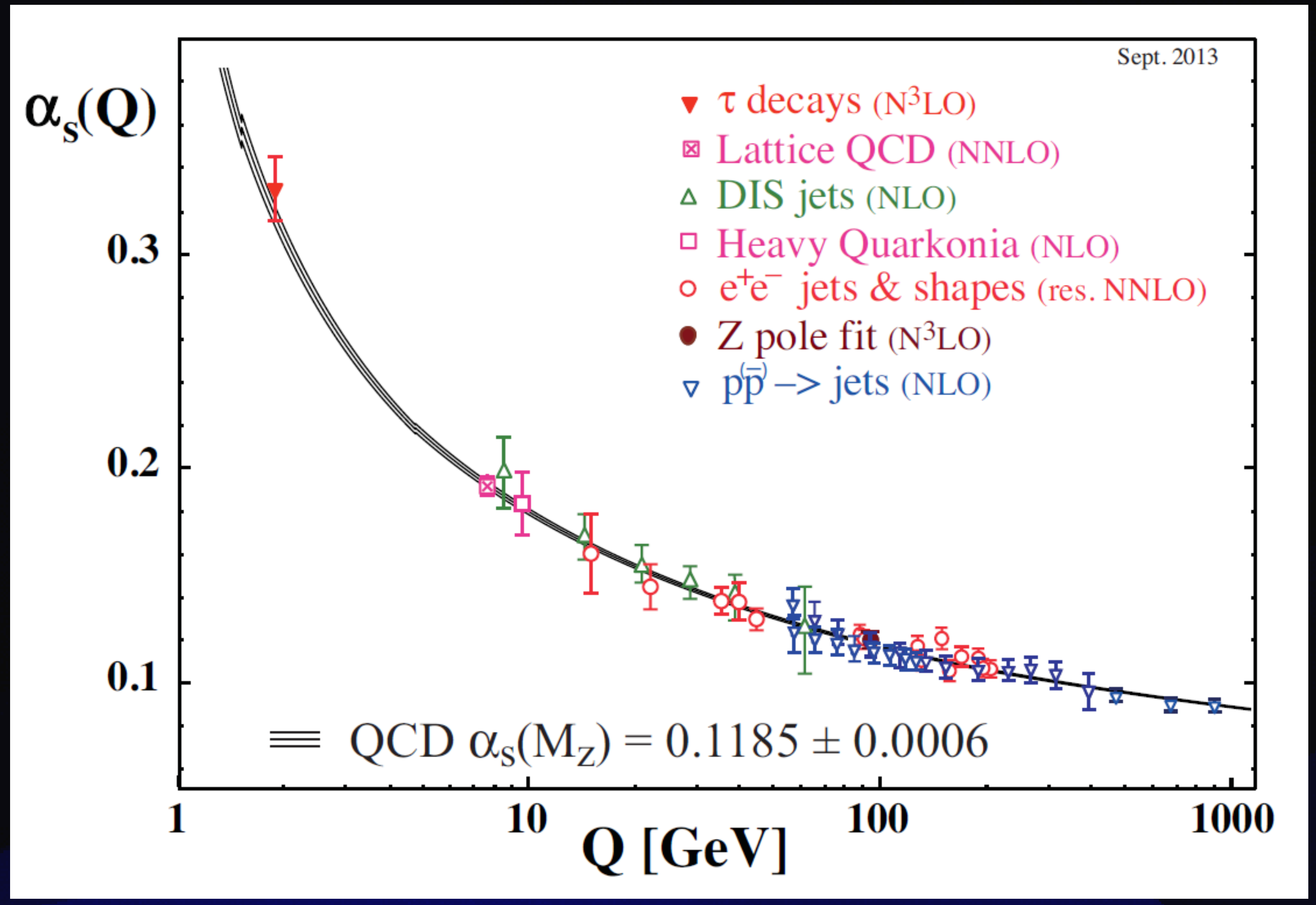
- Hadronic Hamiltonian can, in principle, be derived from the underlying quark-gluon dynamics in QCD.

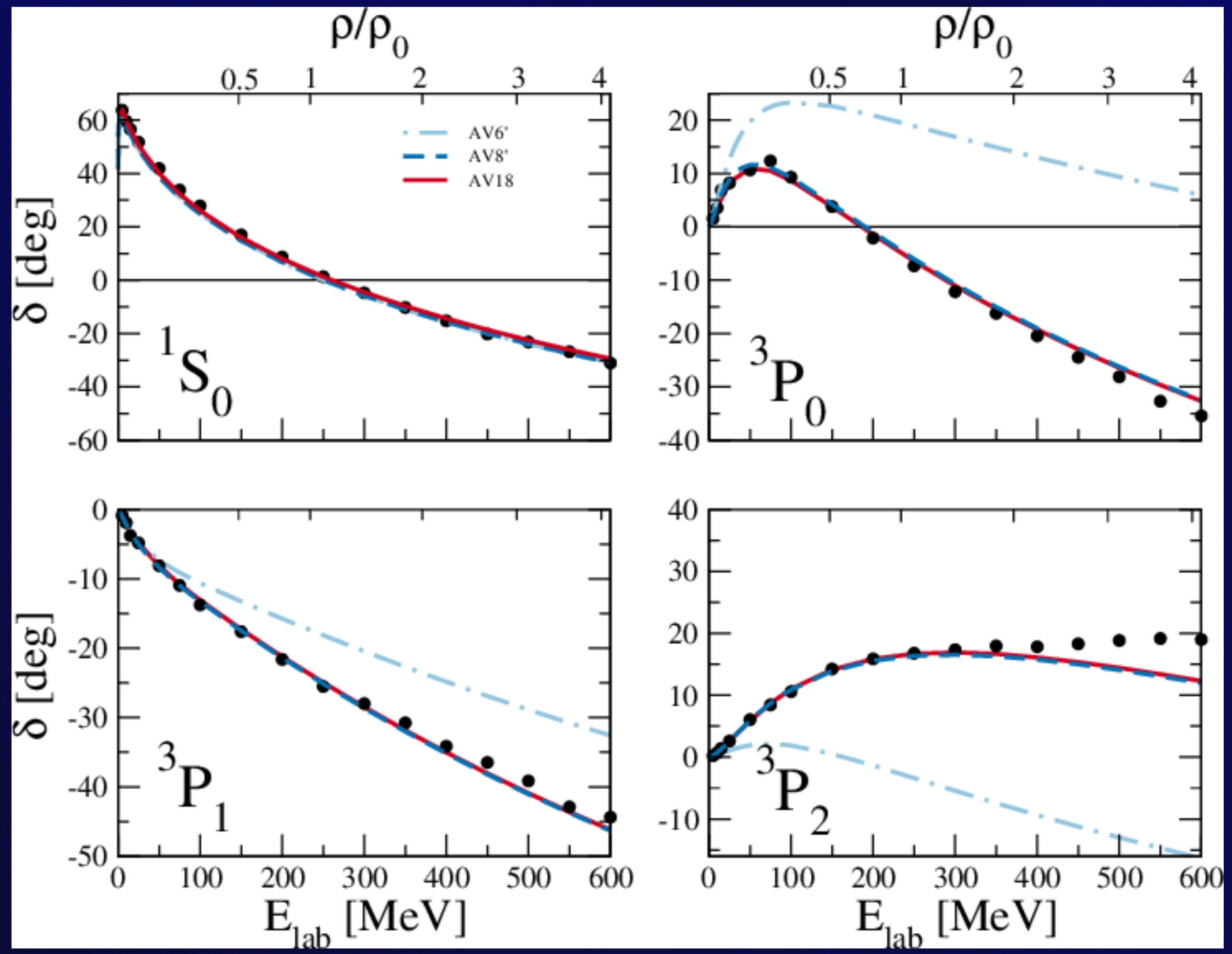
- Non-perturbative character of QCD at low and intermediate energies → far from a quantitative understanding of the baryon-baryon interaction from the QCD point of view.

- Solution : to adopt simplified models where the hadronic degrees of freedom are the relevant ones.

- Use of phenomenological models of the hadronic interaction : meson exchange models and potential models.

- Essential requirement : Fit of the nucleon-nucleon phase shifts.



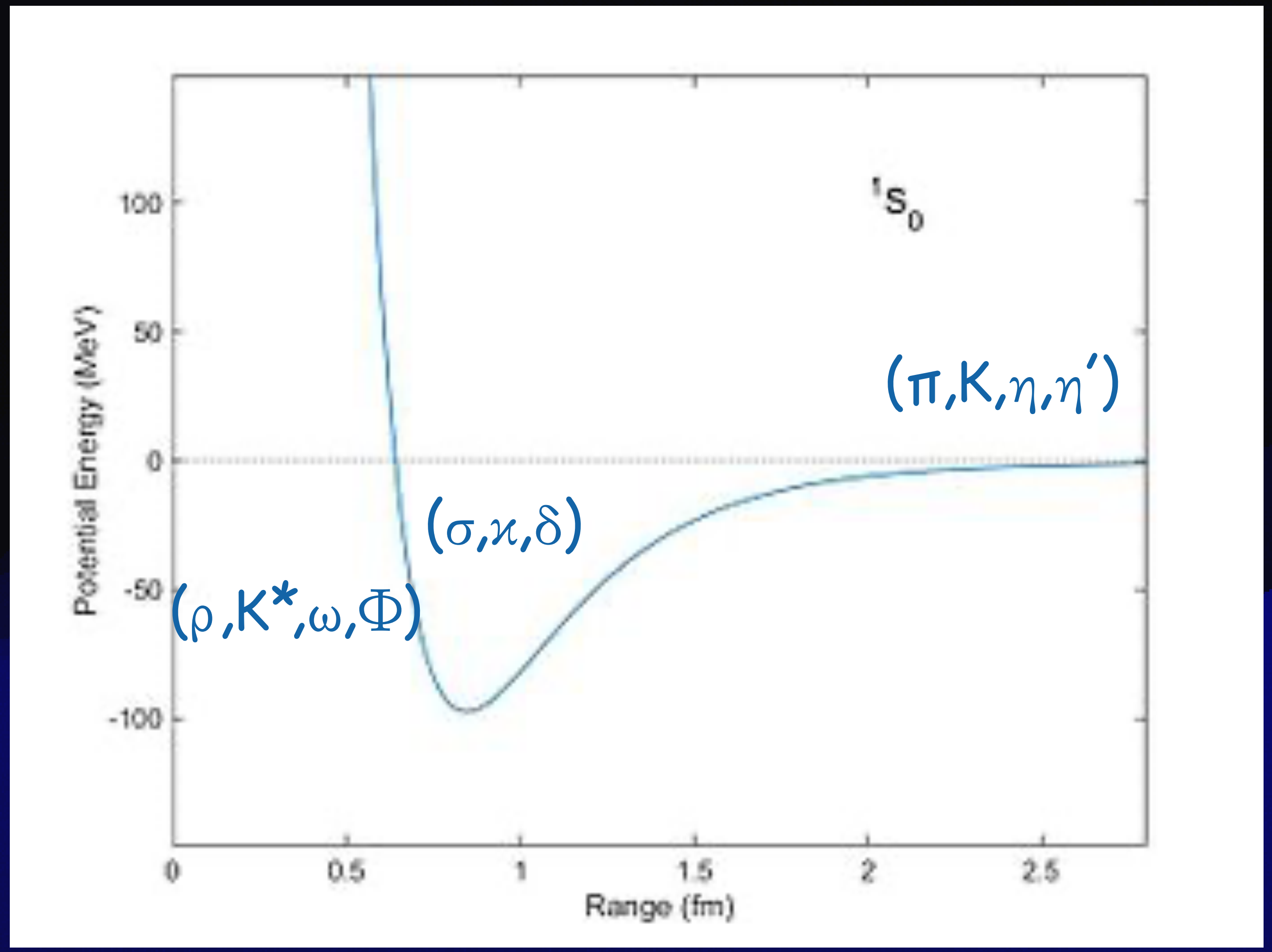


Workman et al., PRC94, 065203 (2016)

Meson-exchange models

Based on the Yukawa theory : baryon-baryon interaction mediated by the exchange of mesons.

- ➡ At large distance, attractive interaction mediated by pseudoscalar mesons (π,K,η,η')
- ➡ At intermediate distance, stronger attraction once an average is made over the different channels. Scalar mesons (σ,$\varkappa$,δ).
- ➡ At short distance, r < 0.5 fm, a strong repulsive core is present. Vector mesons : (ρ,K*,ω,Φ).



Very refined models are constructed for the NN interactions. Tested using thousands of experimental data on NN scattering cross sections supplemented with experimental properties on deuteron. Paris, Bonn, Nijmegen.

CAVEAT ! At short distance, serious divergency problems in many-body calculations. Standard perturbation theory not applicable !

Machleidt et al., Phys. Rep. 149, 1 (1987)
Nagels et al., PRD 17, 768 (1978)

YN and YY meson exchange potentials : Nijmegen, Juelich.

Potential models

A modern NN potential : Argonne v18

A non-relativistic NN potential can be expressed in terms of a set of operators acting on the spin (σ) and isospin (τ) variables of the two nucleons, as well as on the relative angular momentum (L), the total spin operators **S**, and **r** the relative coordinate.

The form of the operators is dictated by symmetry requirements : translational and rotational invariance, charge independence of the nuclear forces, parity and time-reversal symmetry.

1	*central*
$\sigma_1 \cdot \sigma_2$	*spin − spin*
$(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$	*spin − isospin*
$\frac{3(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r})}{r^2} - (\sigma_1 \cdot \sigma_2)$	*tensor*
$(\frac{3(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r})}{r^2} - (\sigma_1 \cdot \sigma_2))(\tau_1 \cdot \tau_2)$	*tensor − isospin*
$\mathbf{l} \cdot \mathbf{S}$	*spin − orbit*
$\mathbf{l}^2$	*ang. mom. square*

In operatorial form the Argonne v18 potential is expressed as :

$$v_{18} = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^p$$

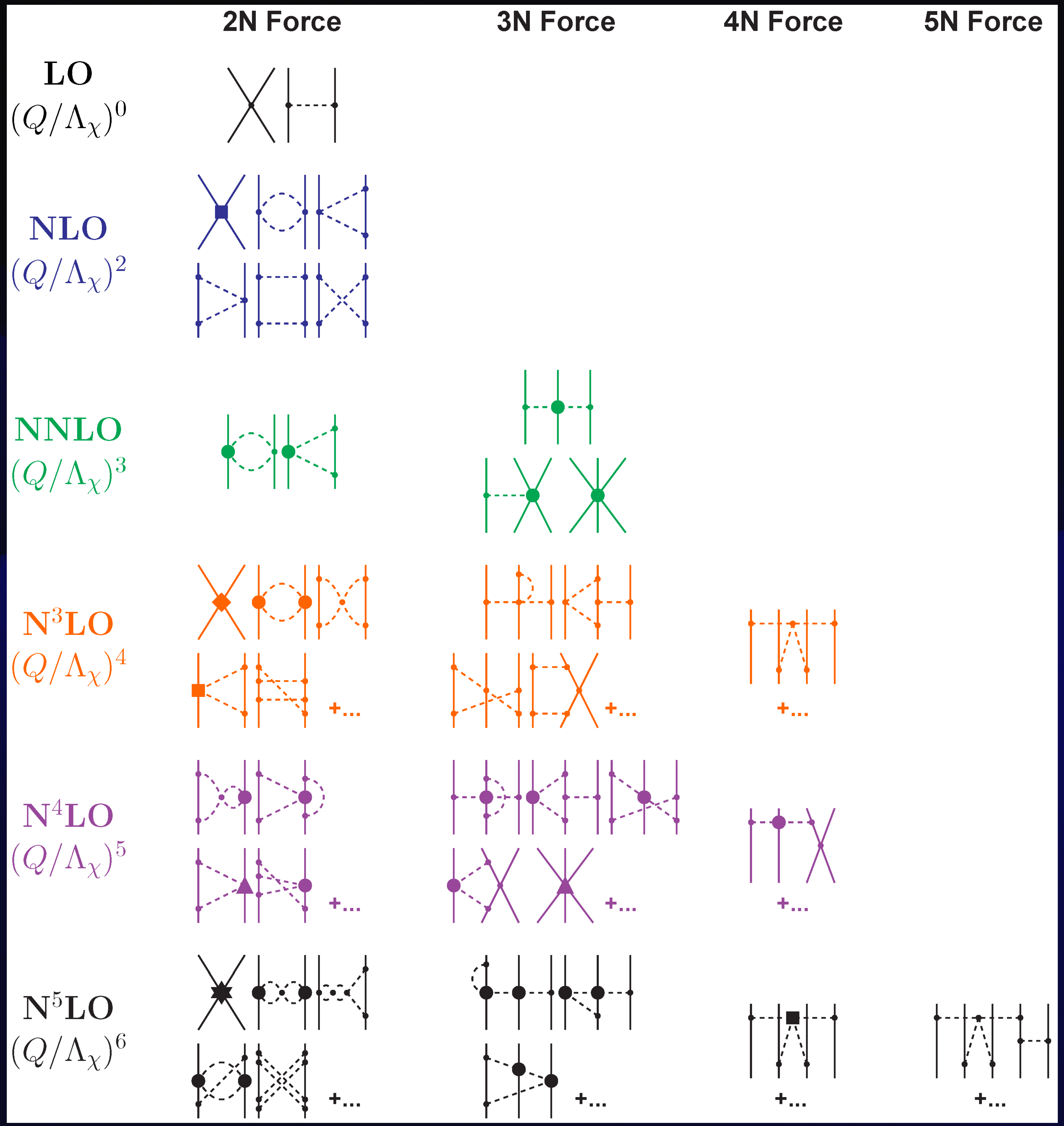
tensor force

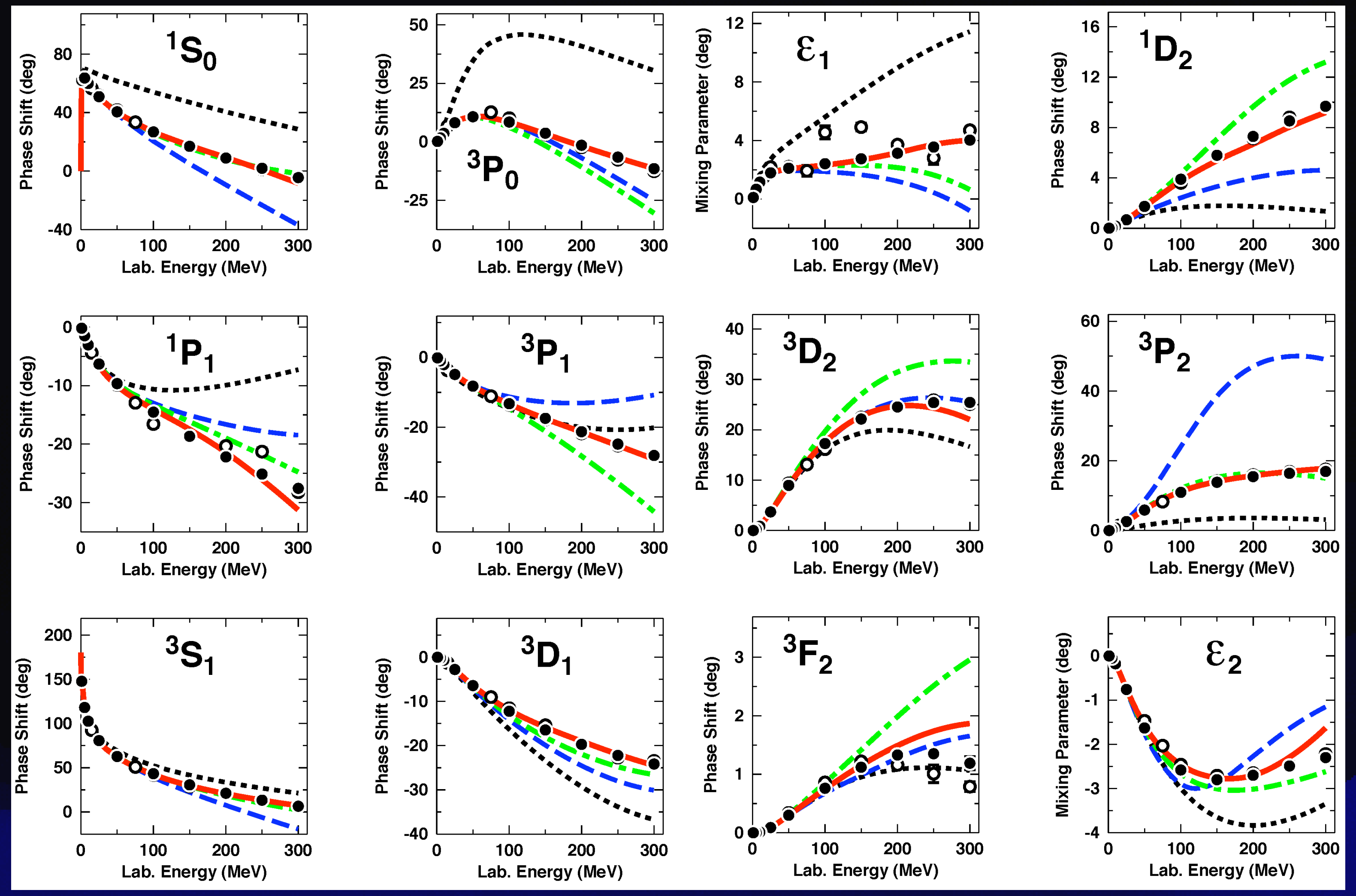
isotensor force

$$O_{ij}^p = [1, \sigma_i \cdot \sigma_j, S_{ij}, L \cdot S, L^2, L^2(\sigma_i \cdot \sigma_j), (L \cdot S)^2] \otimes [1, \tau_i \cdot \tau_j],$$
$$[1, \sigma_i \cdot \sigma_j, S_{ij}] \otimes T_{ij}, \text{ and } (\tau_{zi} + \tau_{zj})$$

Wiringa, Stoks, Schiavilla, PRC 51, (1995) 38

Chiral perturbation expansion (ChPE)





- Starting point : quark and gluons as relevant degrees of freedom. Bridge between the low-energy hadron physics phenomena with the underlying QCD structure of the baryons.
- Weinberg (1990-91) : EFT based on the QCD broken symmetries.
- ChPE used to construct NN interactions of reasonably good quality in reproducing the two-body data.
- Various contributions to the potential systematically calculated order by order. Calculation of two-nucleon and many-nucleon forces in a consistent manner. Method applied also to the hyperon-nucleon case.

CAVEAT ! ChPE valid for not too large momenta (i.e.density) of nuclear matter. Safe maximum density around the saturation value.

Weinberg, PLB 251, 288 (1990); NPB 363, 3 (1991)
Entem & Machleidt, PRC 68, 041001(R) (2003)
Epelbaum et al., NPA 747, 363 (2005)

The construction of the nuclear EoS

Two different philosophies toward the construction of the nuclear EoS : Phenomenological vs. Microscopic approaches

Phenomenological approaches

Based on effective density-dependent NN force with parameters fitted to reproduce nuclear properties in the g.s. and compact stars observables.

- Non-relativistic models: Skyrme and Gogny
- Relativistic mean-field models (RMF)

For clusterized matter

- SN approximation models : Liquid Drop models, Thomas-Fermi models, Self-consistent mean field models.
- NSE models.

Microscopic approaches

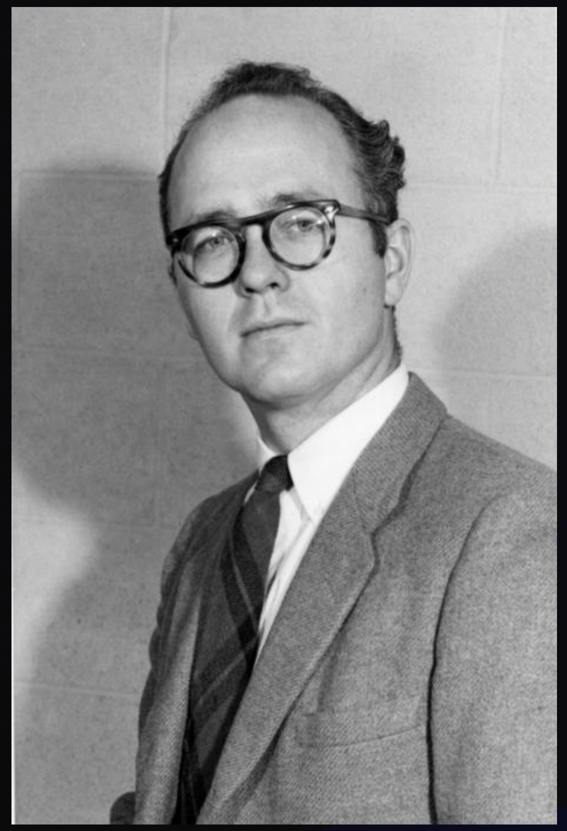
Based on two- and three-nucleon realistic interaction which reproduces scattering data and deuteron properties. The EoS is found by solving the complicated many-body problem.

- Diagrammatic: (Dirac)-Brueckner-Hartree-Fock, SCGF
- Variational : APR, FHNC, LOCV, CBF.
- Quantum Monte Carlo : VMC, GFMC, AFDMC.
- Chiral approaches : χEFT.

more in : Neutron stars and the nuclear equation of state,
F.B., HJ Schulze, I. Vidana, JB Wei, PPNP 120 (2021) 103879,

Ab-initio approaches

Diagrammatic technique: The (Dirac)-Brueckner theory of nuclear matter



K. Brueckner

The (Dirac)-Brueckner-Hartree-Fock theory is based on the Goldstone expansion, which is a perturbation series for the ground-state energy of a many-body system. The theory amounts to ordinary perturbation theory expressed in a tractable form.

The Bethe-Goldstone equation
For the reaction matrix G

$$G(\rho;\omega) = V + \sum_{k_a k_b} V \frac{|k_a k_b\rangle Q \langle k_a k_b|}{\omega - e(k_a) - e(k_b)} G(\rho;\omega)$$

ω=starting energy

single-particle energy

$$e(k;\rho) = \frac{k^2}{2m} + U(k;\rho)$$

$$U(k;\rho) = \mathrm{Re} \sum_{k' \le k_F} \langle kk'|G(\rho;\omega)|kk'\rangle_a$$

The G-matrix is well-behaved even for a singular two-body force, all terms in this new perturbation series are finite and of reasonable size.

Stopping the perturbative series at first order (keep the two-body correlations only), one gets the Brueckner-Hartree-Fock approximation for the binding energy.

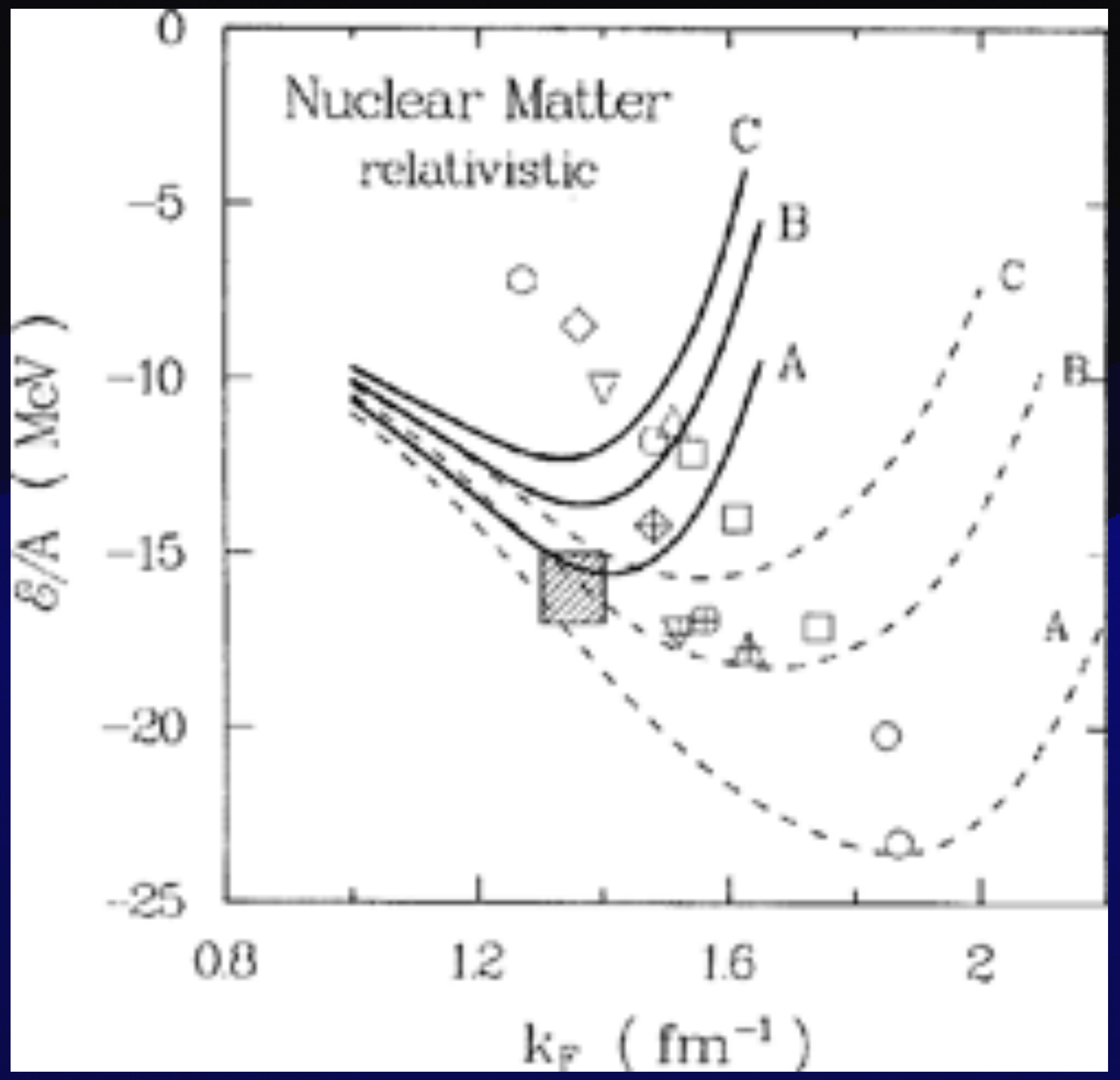
$$\frac{E}{A}(\rho) = \frac{3}{5}\frac{k_F^2}{2m} + \frac{1}{2\rho} \sum_{k,k' \le k_F} \langle kk'|G[\rho;\omega]|kk'\rangle_a$$

The perturbative expansion is convergent !

The relativistic Dirac-BHF

- Introducing the in-medium relativistic G-matrix.
- Nuclear mean field in terms of scalar and vector components
- Use of spinor formalism, equivalent to introduce a special TBF, the Z-diagram, nucleon-anti nucleon pair which gives a repulsive contribution.
- Correct saturation point of nuclear matter.
- Stiffer EoS than the non-relativistic case.
- Superluminal EoS at larger density than in the non-relativistic case.

Brockmann and Machleidt, 1996.



Solid lines: relativistic, dashed lines: non-relativistic calculations.

A few calculations in literature :

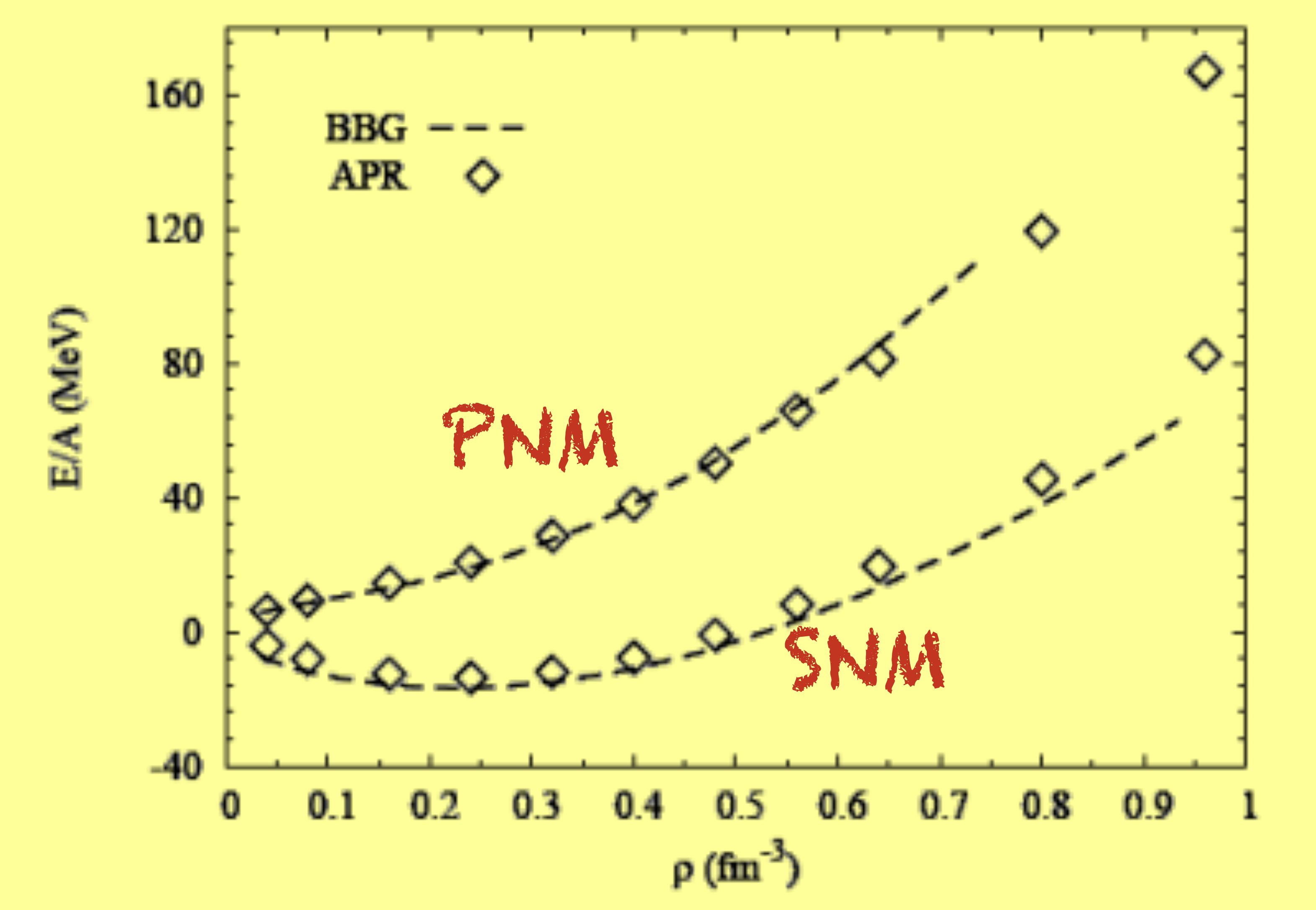
R. Machleidt in Negele, Vogt, 1989, pp.189
G. E. Brown et al., Comments Nuclear Part. Phys. 17 (1987) 39
Ter Haar, Malfliet, PRL 56 (1986) 1237
Huber, Weber, Weigel, PRC57, (1998) 3484
Gross-Boelting, Fuchs, Faessler, NPA648, (1999) 105

Dependence on the many-body scheme: BHF vs. APR

Main differences :

a) In BHF the kinetic energy contribution is kept at its unperturbed value at all orders of the expansion, while all correlations are embodied in the interaction energy part. In the variational, both kinetic and interaction parts are directly modified by the correlation factors.

b) In BHF the s.p. potential is introduced in the expansion and improves the rate of convergence. In the variational, no single particle potential is introduced.

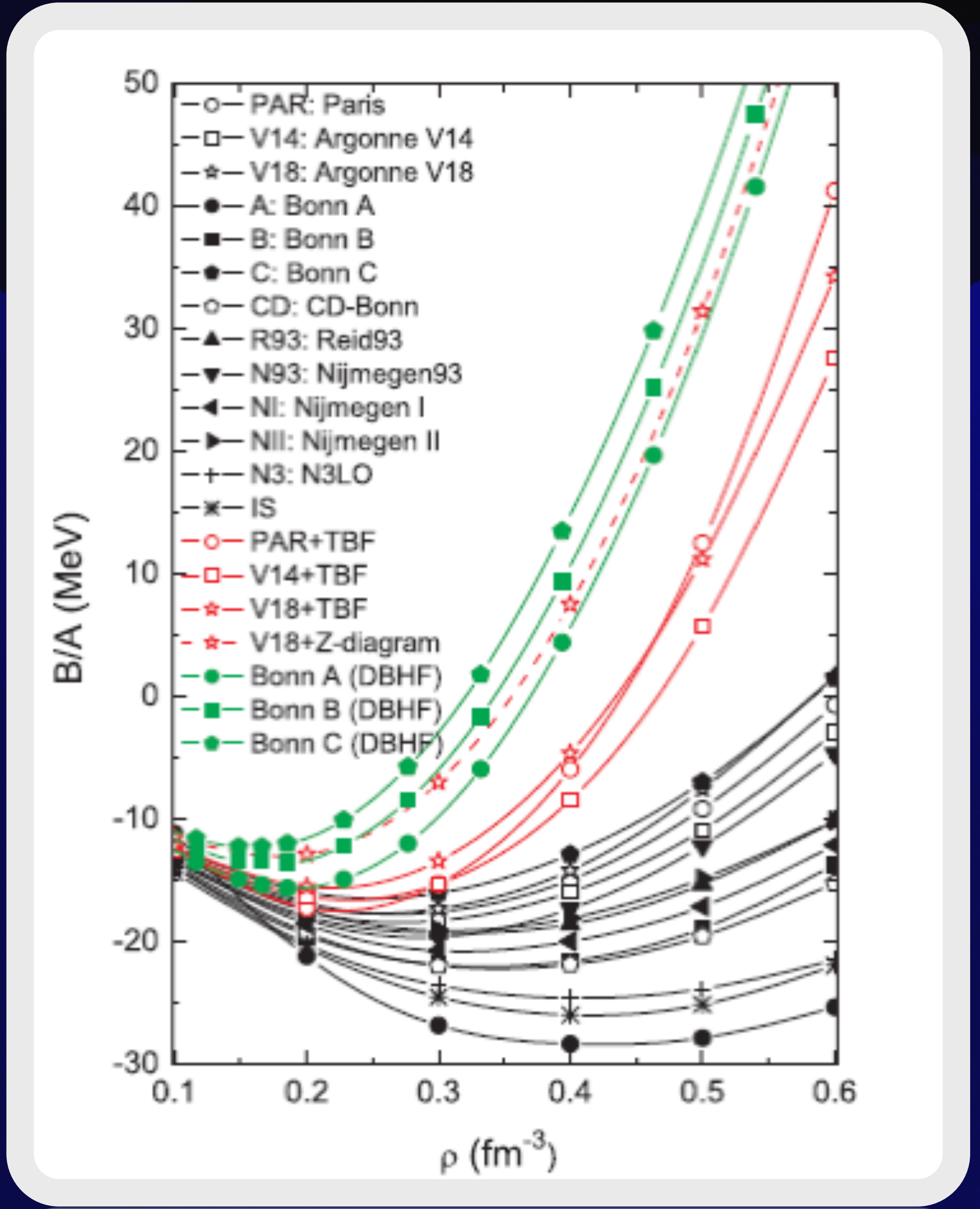


At two-body level, both methods give quite similar results.

Three-body forces

Two-body hadronic interactions yield only a part of the hadronic Hamiltonian of dense matter. At densities typical of NS core, interactions involving three and more hadrons might be important. Our experimental knowledge of three-body interaction is restricted to nucleons. The three-nucleon (NNN) force is necessary to reproduce properties of ^{3}H and ^{3}He, and to obtain correct parameters of symmetric nuclear matter at saturation.

- No complete theory available yet.
- Phenomenological Urbana IX and microscopic approaches.
- TBF needed to improve saturation point.
- Dependence on NN potential.
- TBF unknown at high density.



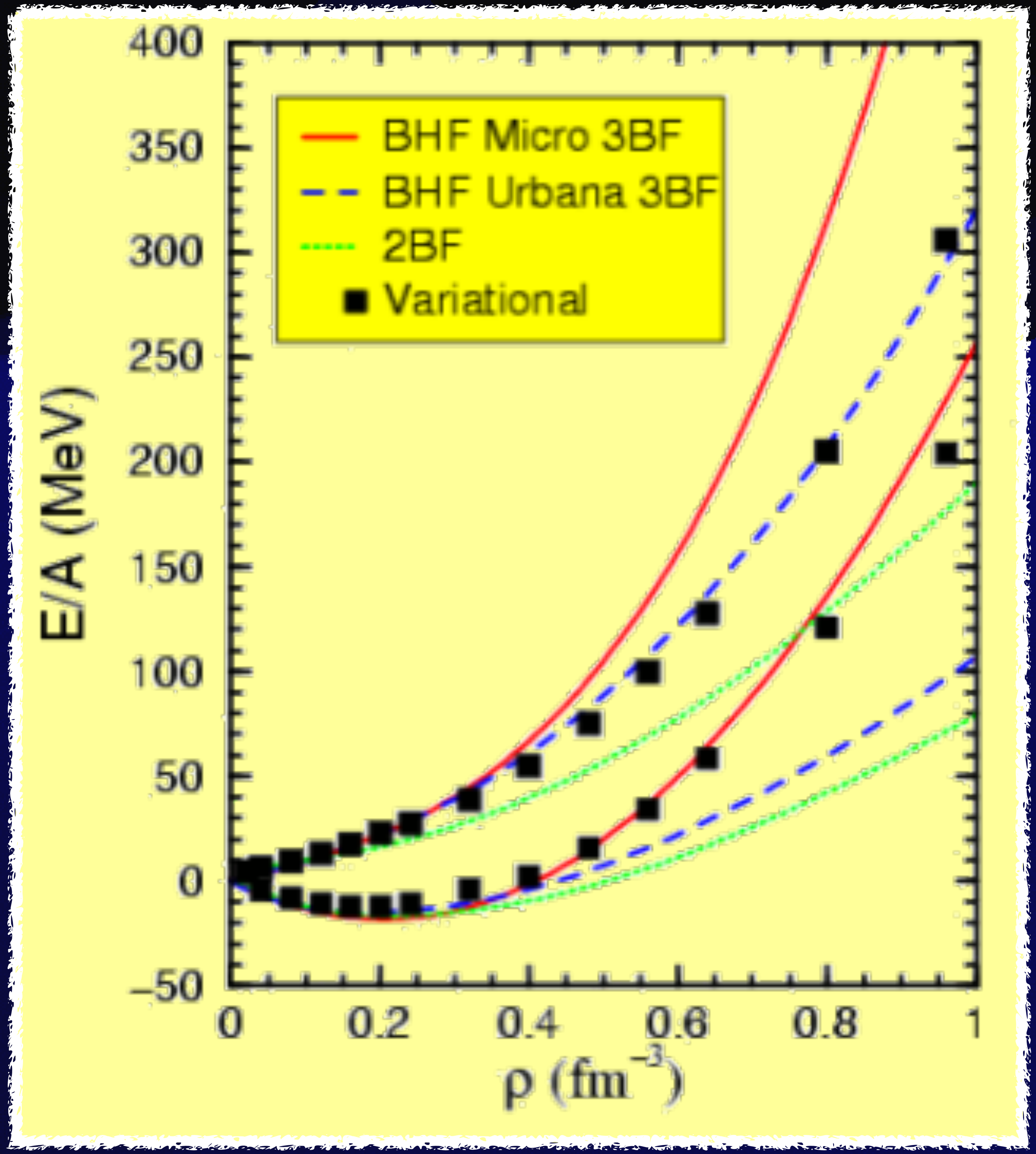
Z.H. Li, U. Lombardo, H.-J. Schulze, W. Zuo, PRC 74, 047304 (2006)

✓ Urbana IX model
Carlson et al., NP A401,(1983) 59

✓ Microscopic model
P. Grange' et al, PR C40, (1989) 1040

Including TBF's and comparing up to high density

- TBF's parameters fitted either to NM saturation point or to finite nuclei in their g.s.
- TBF's are different in either methods.
- Good agreement in SNM up to 0.4 fm-3
- Large discrepancy at the high density typical of a NS core.



Phenomenological approaches

Phenomenological models

Class I : Skyrme interactions

- ✦ Use of effective interactions : simpler structure than realistic interactions used in ab initio approaches.

- ✦ Dependence on a number of parameters (10-15) fitted to different properties of several nuclei and nuclear matter properties.

- ✦ Caveat : extrapolation to high density conditions has to be considered with caution.

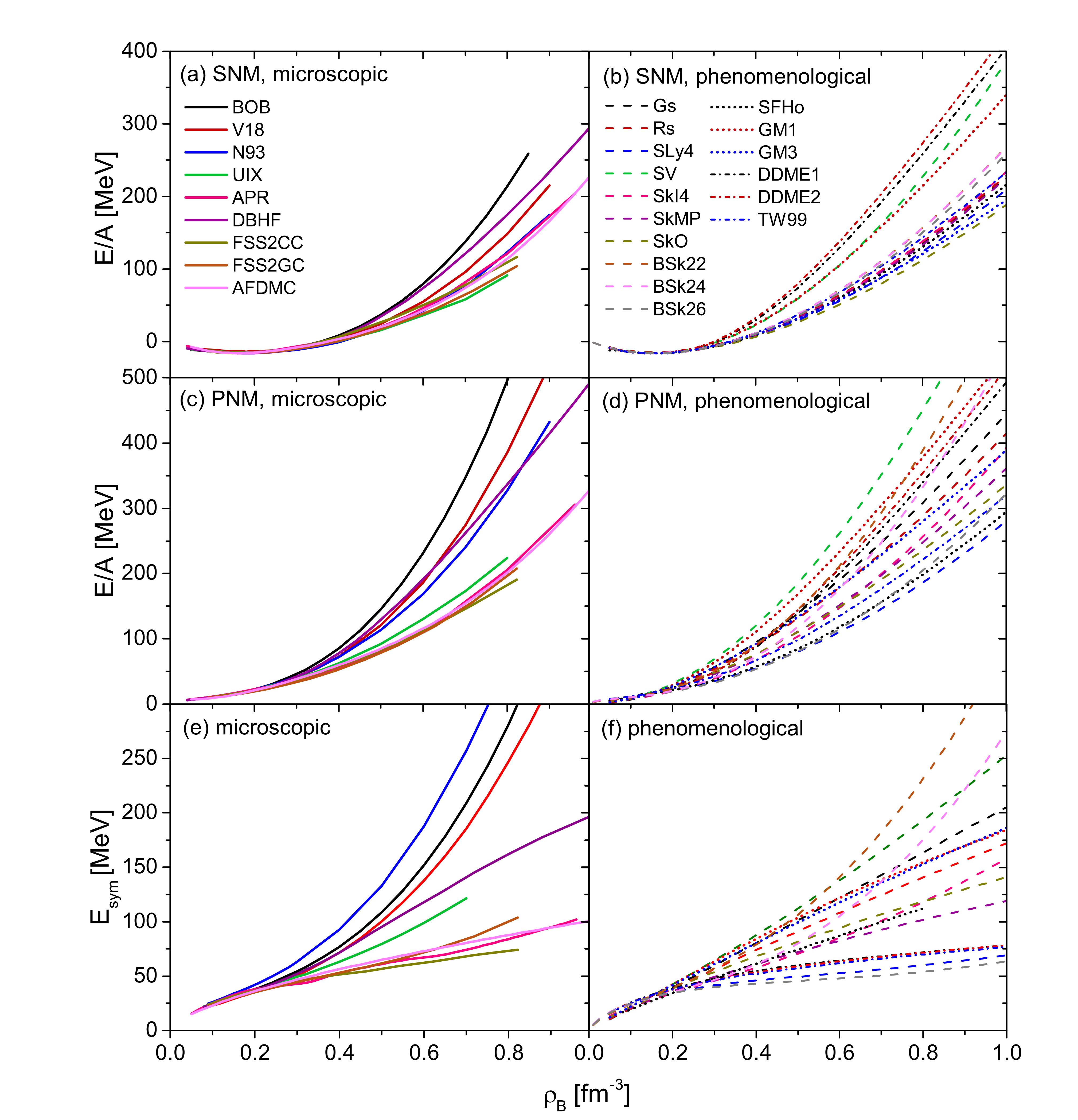
Class II: Relativistic mean field models (RMF)

Effective Lagrangian density in which the baryon-baryon interaction is expressed in terms of mesons exchange. EoS obtained in the mean field approximation.

$$\mathcal{L} = \mathcal{L}_{\rm nuc} + \mathcal{L}_{\rm mes} + \mathcal{L}_{\rm int}$$

Phenomenological approaches are the most widely used methods to construct EoSs for astrophysical applications.

Comparing ab-initio and phenomenological approaches : Binding and Symmetry energy



Large variations over the density range

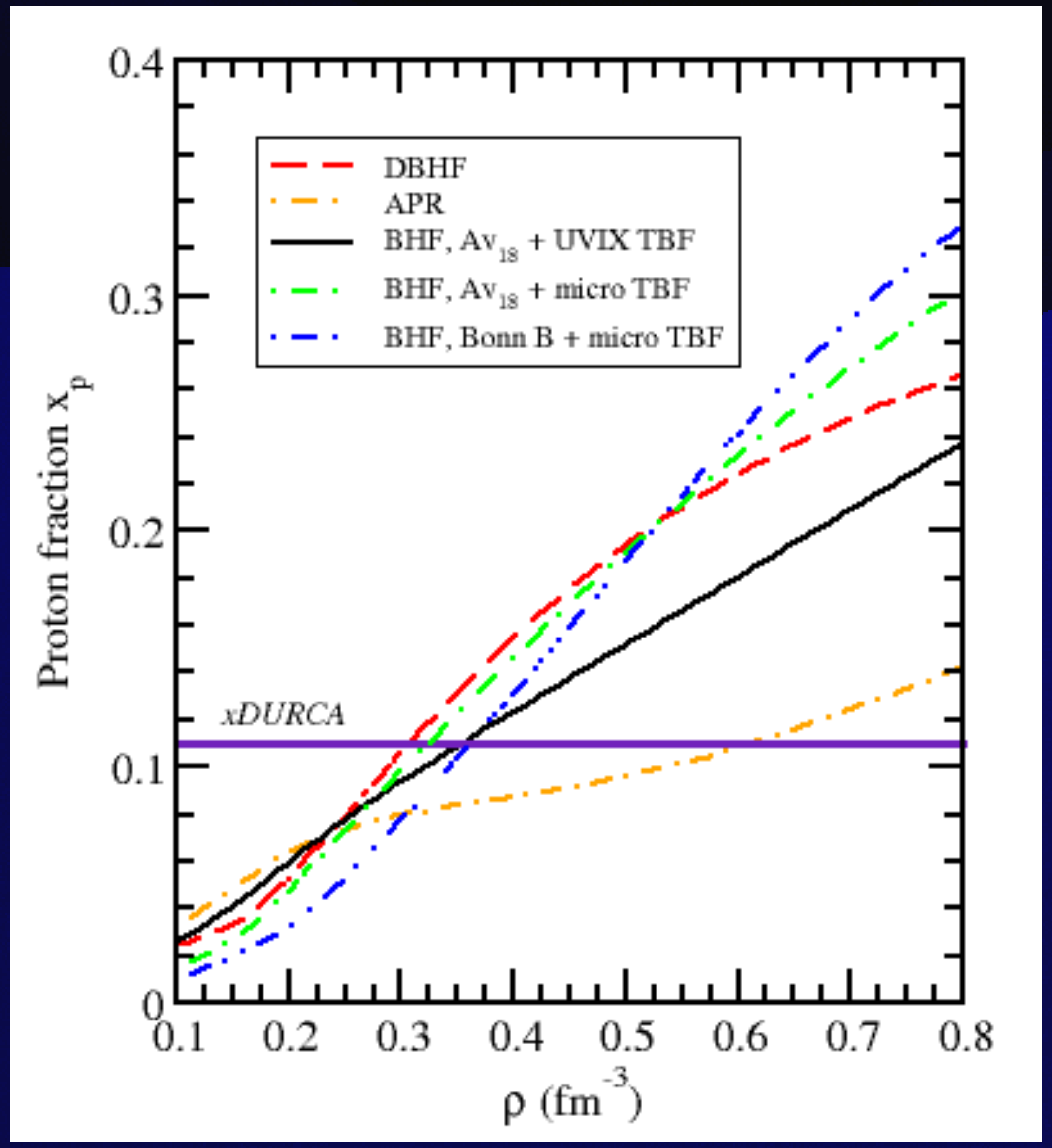
Symmetry energy

$$E_{sym}(\rho) = E_{PNM}(\rho) - E_{SNM}(\rho)$$

Direct URCA processes in NS

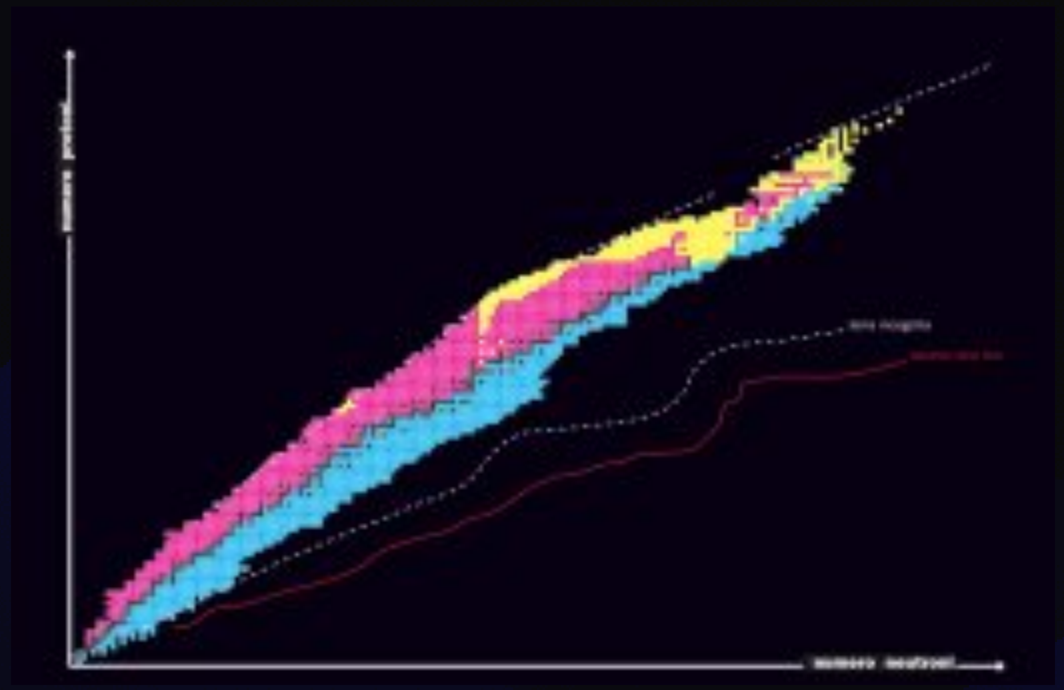
$$n \longrightarrow p + e + \overline{\nu}_e \ , \qquad p + e \longrightarrow n + \nu_e \ ,$$
$$n \longrightarrow p + \mu + \overline{\nu}_\mu \ , \qquad p + \mu \longrightarrow n + \nu_\mu \ .$$

- They are allowed only at a rather high density at which the proton fraction $x_D > 0.11$-0.14 (Lattimer et al. 1991).
- If Direct URCA operate, then a non-superfluid NS core cools to 10^9 K in a minute. If they are not allowed, the time scales will be one year, more or less ...
- The symmetry energy is crucial for determining the proton fraction.



Data from laboratory experiments

Close to saturation point ...



- Structure properties known for about 3400 nuclides
- Binding energy in the Liquid Drop Model
- Extrapolating the mass formula for A -> ∞ in the symmetric case, the binding energy close to saturation is usually expanded as

$$\frac{E}{A}(\rho,\beta) = E_0 + \frac{1}{18}K_0\epsilon^2 + \left[S_0 - \frac{1}{3}L\epsilon + \frac{1}{8}K_{sym}\epsilon^2\right]\beta^2$$

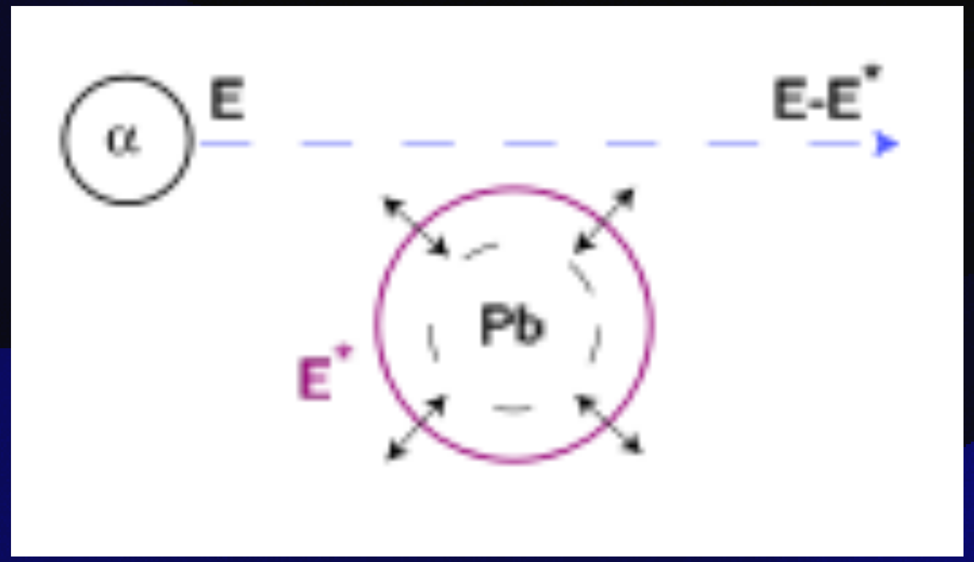
$$\beta = \frac{\rho_n - \rho_p}{\rho}, \qquad \epsilon = \frac{\rho - \rho_0}{\rho_0}$$

Nuclear Incompressibility for symmetric matter K

$$K = 9\rho_0^2 \frac{d^2}{d\rho^2}\left(\frac{E}{A}\right)_{\rho=\rho_0} = R^2 \frac{d^2}{dR^2}\left(\frac{E}{A}\right)_{\rho=\rho_0}$$

In radial oscillations induced by α-particles scattering :

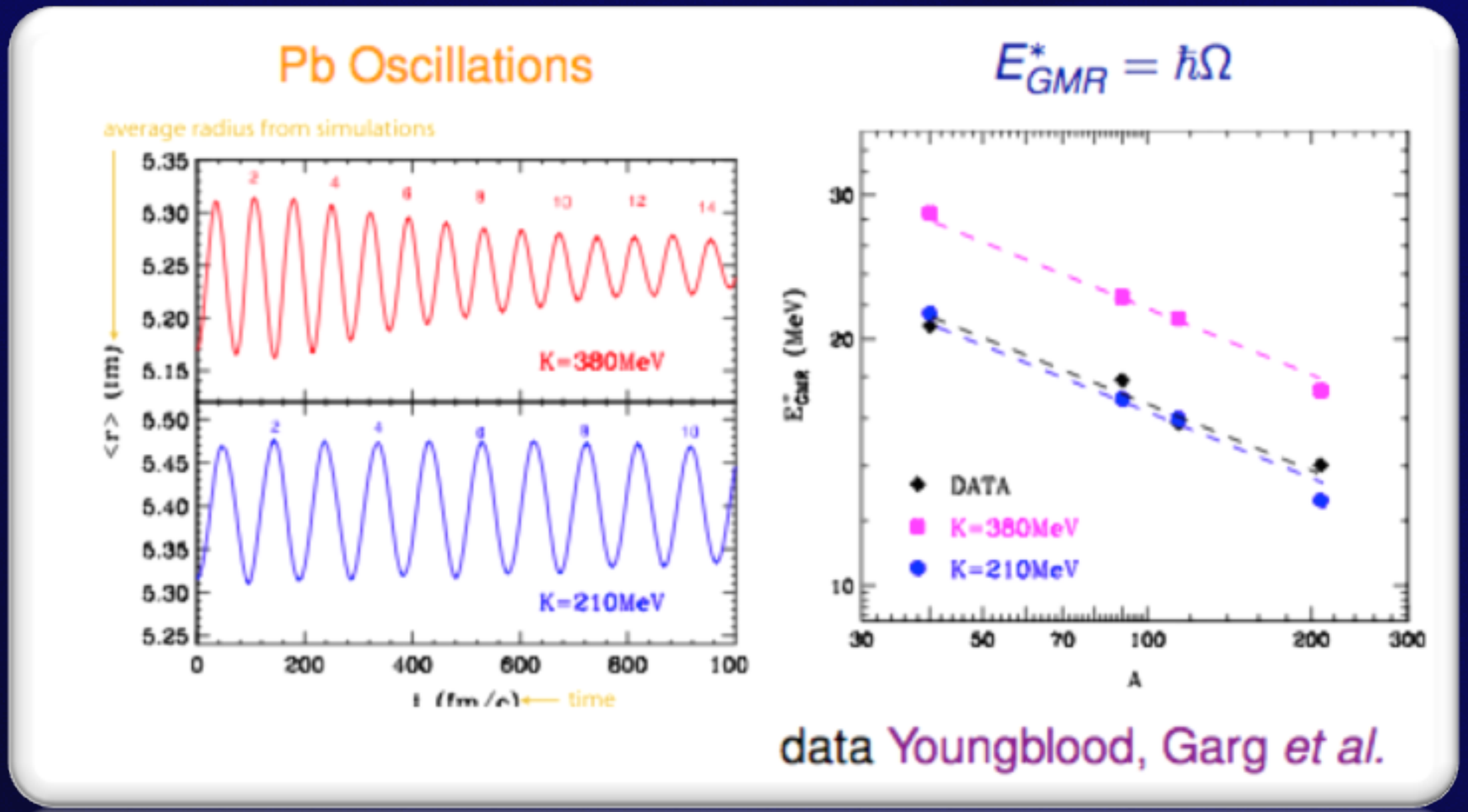
$$E^* = \hbar\sqrt{\frac{K}{m_N < r^2 >_A}}$$



$$K = 240 \pm 10\ MeV$$ (Colo', 2004)

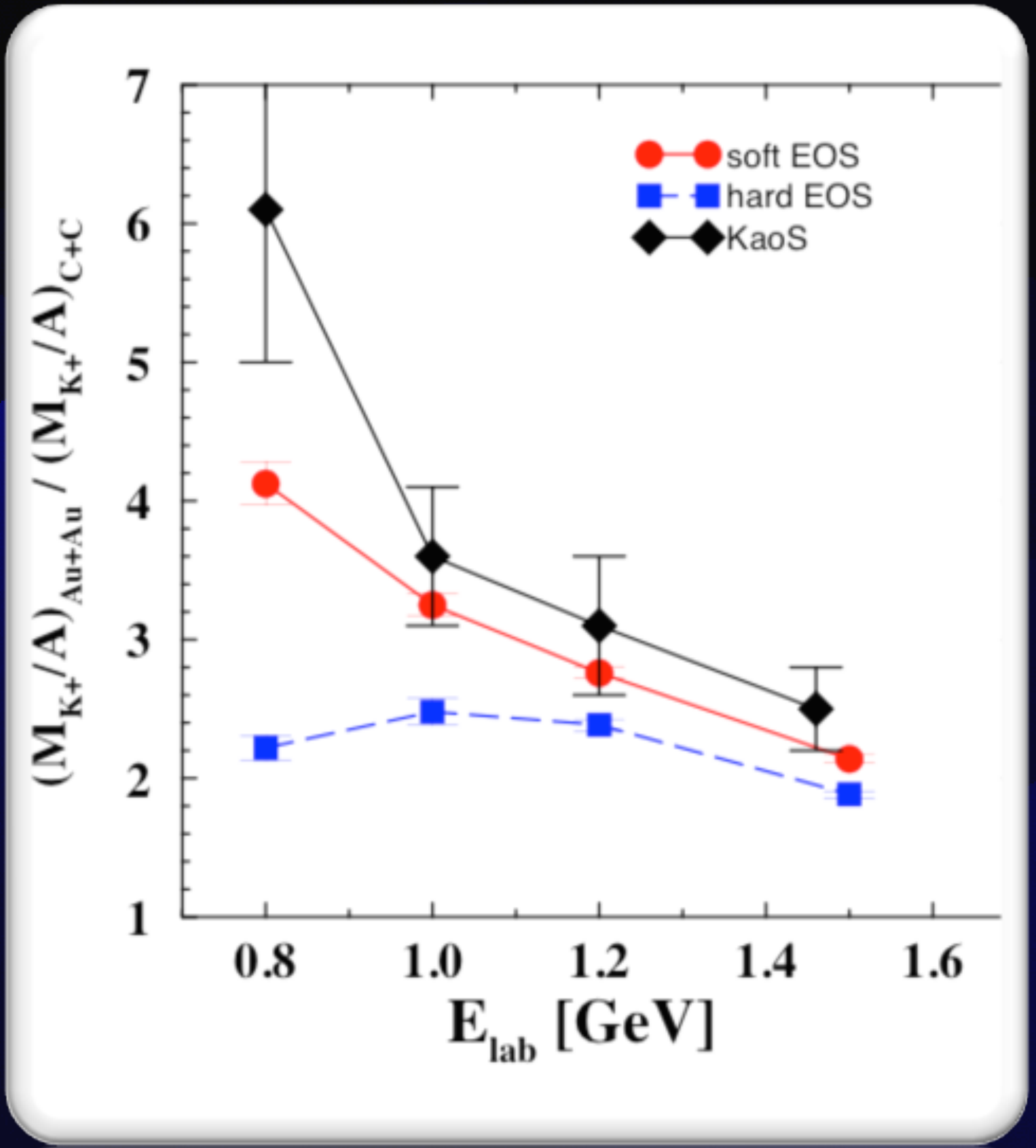
$$= 248 \pm 8\ MeV$$ (Piekarewicz,2004)

A soft EoS is favourite close to saturation density



Kaon production in heavy ion collisions

Near threshold strange particles are produced in the high-density region of the participant fireball. Production rate depends on the maximal density, hence the compressibility.



- Experimental data by the KaoS and FOPI Collaborations.
- Multiplicity per mass number for C+C collisions and Au+Au collisions at 0.8 AGeV and 1.0 AGeV .
- Largest density explored : ρ ≈ 2-3 ρ0
- Only calculations with a compression $180 \leq K_N \leq 250$ MeV can describe the data (Fuchs, 2001)

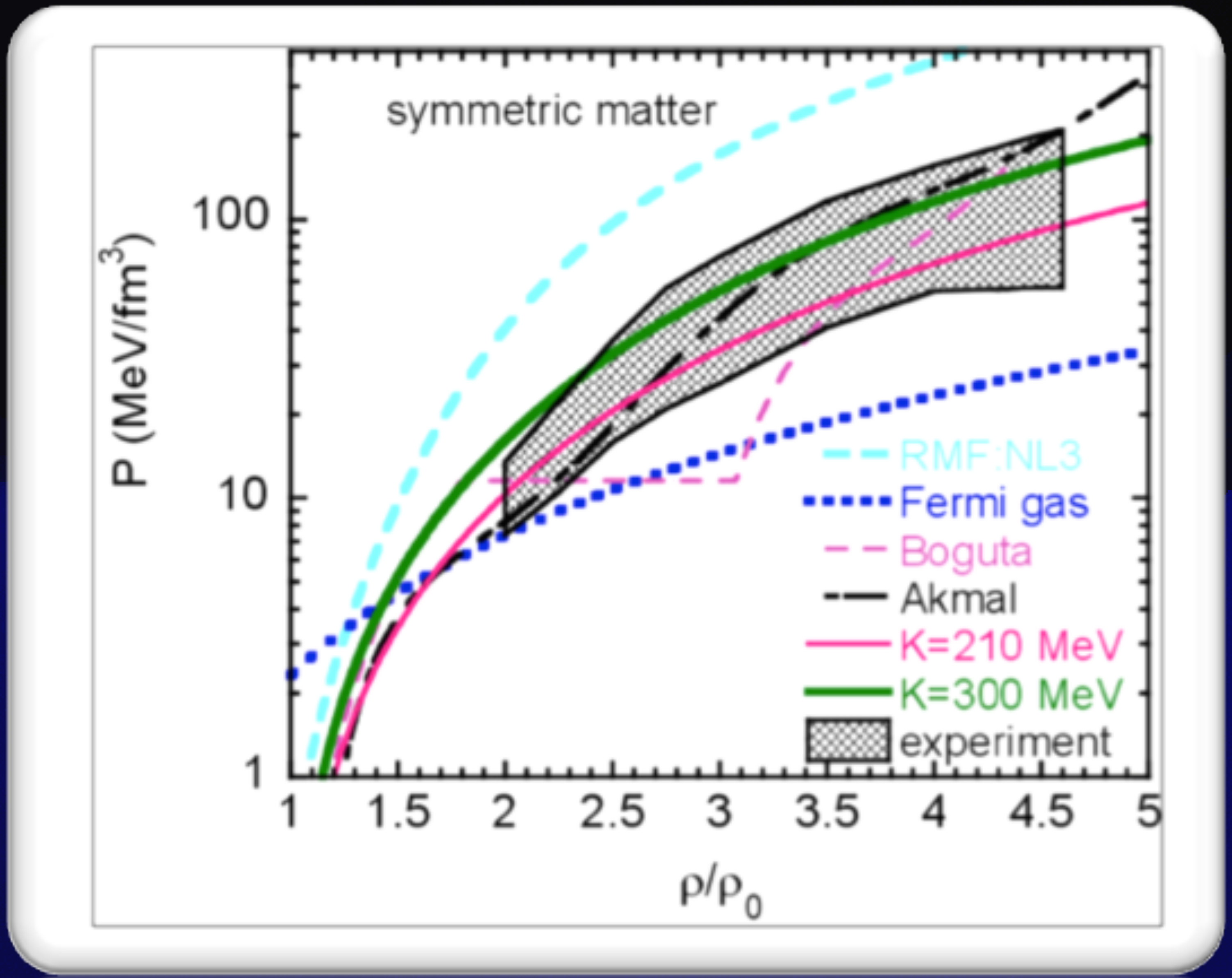
The nuclear equation of state up to $2\text{–}3\rho_0$ is SOFT !

Determination of the Equation of State of Dense Matter

P. Danielewicz, R. Lacey and W. Lynch

Science 298, 1592 (2002)

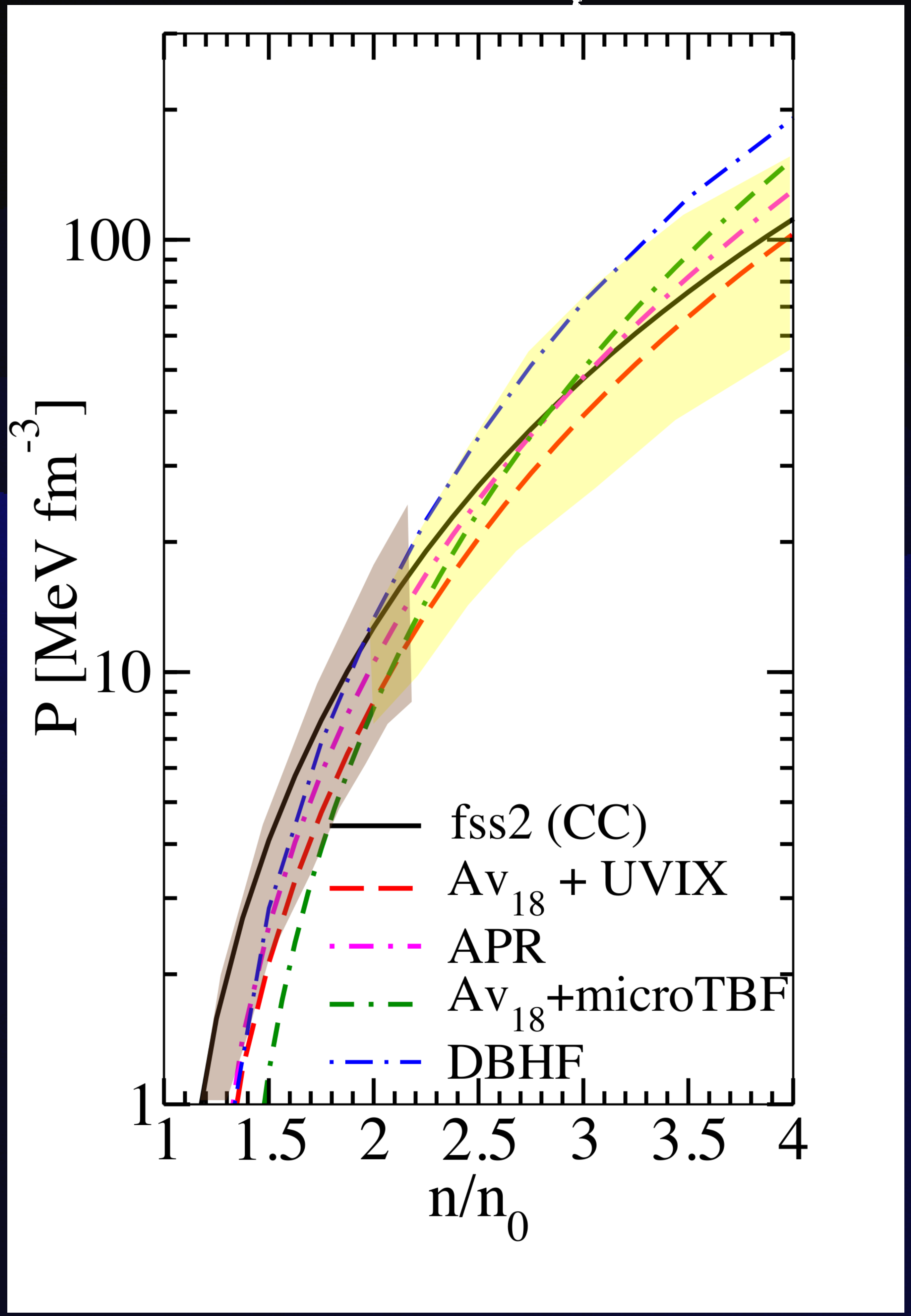
- Transverse flow measurements in Au + Au collisions at E/A=0.5 to 10 GeV
- Pressure determined from simulations based on the Boltzmann-Uehling Uhlenbeck transport theory

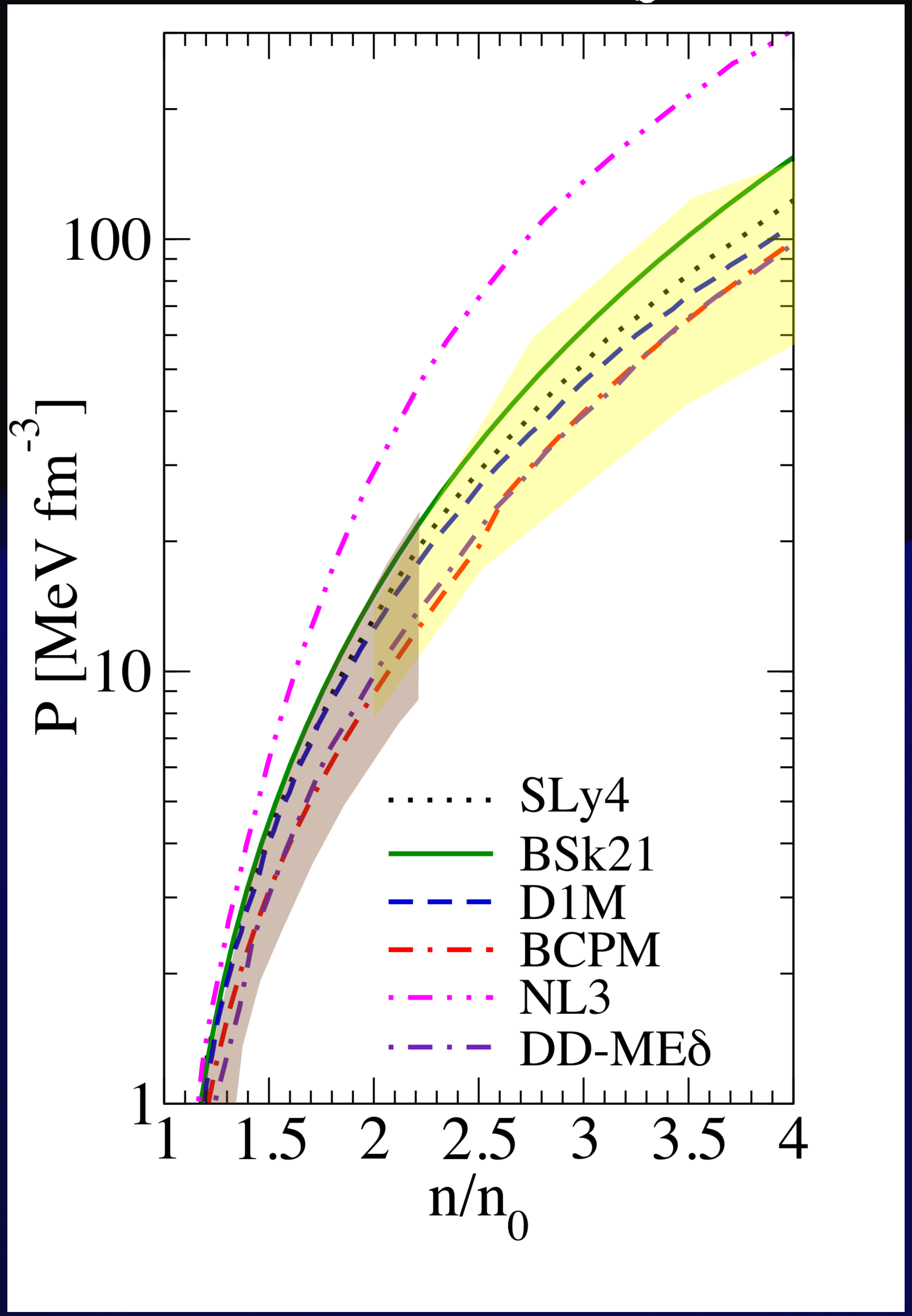


Flow data exclude very repulsive equations of state, but confirm very soft EoS at $\rho < 3\rho_0$

Flow data : do the EoS fit the data ? YES !

Microscopic vs. Phenomenological





Check *wrt* other nuclear physics constraints

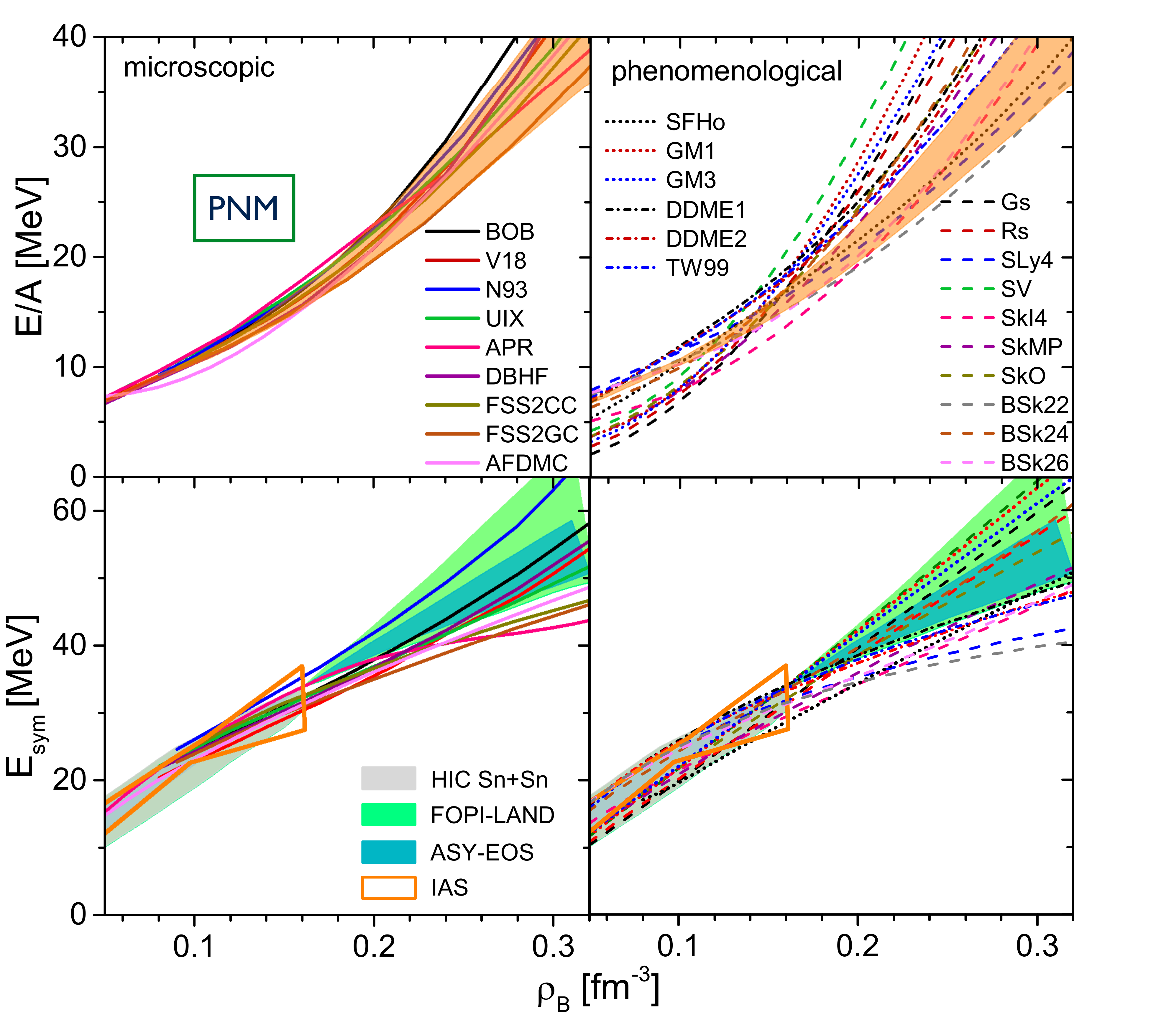
Orange : predictions from the χEFT up to N3LO order
C. Drischler et al., PRL 125, (2020) 202702

Microscopic EoS

- BHF with Argonne V18 or Nijmegen 93 2NF and microscopic 3NF (BOB,V18, N93,UIX)
- BHF with FSS2 NN interaction (quark d.o.f. explicitly taken into account)
- Variational APR with Argonne V18 and 3NF of Urbana UIX type
- Relativistic DBHF (Bonn A)
- AFDMC with modified V18

Phenomenological EoS

- Skyrme forces (Gs,Rs,SLy4,SV etc…)
- Brussels-Montreal group BSk22,24,26
- NLWM (SFHo, GM1,3), RMF models with different parameterizations.
- DDM, RMF model with density dependent coupling constants.



Constraints from Nuclear Physics Experiments

$$L \equiv 3\rho_0 \frac{dE_{\mathrm{sym}}}{d\rho}(\rho_0)$$

- E/A from experimentally measured nuclear masses

$$\rho_0 = 0.16 \pm 0.01 \ \mathrm{fm}^{-3}$$
$$E_0/A = -16.0 \pm 1.0 \ \mathrm{MeV}$$

- K_0 from isoscalar giant monopole resonances in heavy nuclei and HiCs

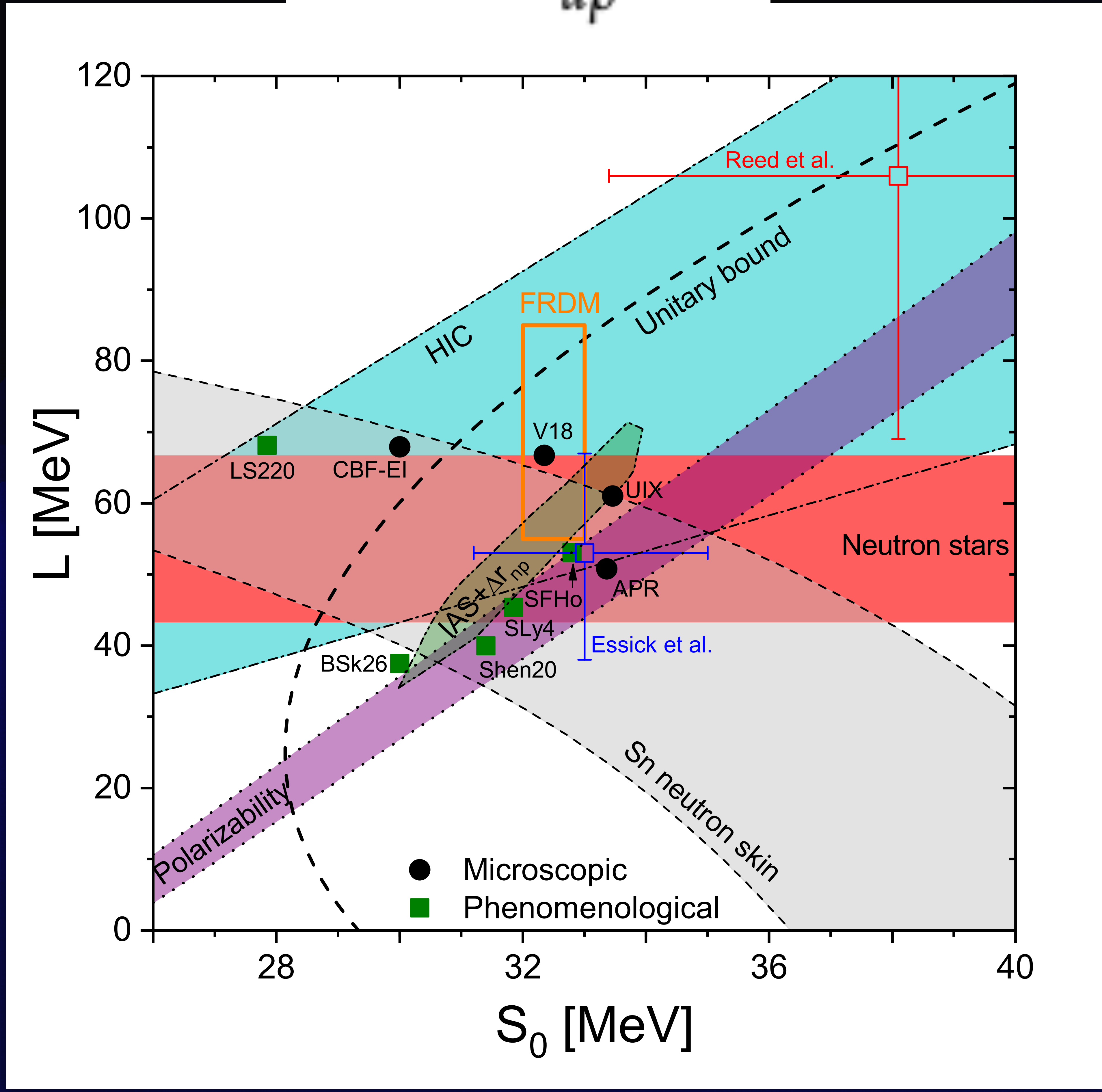
$230 \ \mathrm{MeV} < K_0 < 270 \ \mathrm{MeV}$

- S_0 from nuclear masses, isobaric analog state phenomenology, neutron skin thickness and HiCs isospin diffusion

constrained)

$S_0 \sim 30\text{-}32 \ \mathrm{MeV}$

- L from dipole resonances, electric dipole polarizability and neutron skin thickness. No overlap region ! Too many uncertainties in the experimental measurements and in the models used for the data interpretation.



No theoretical model can be ruled out a priori.

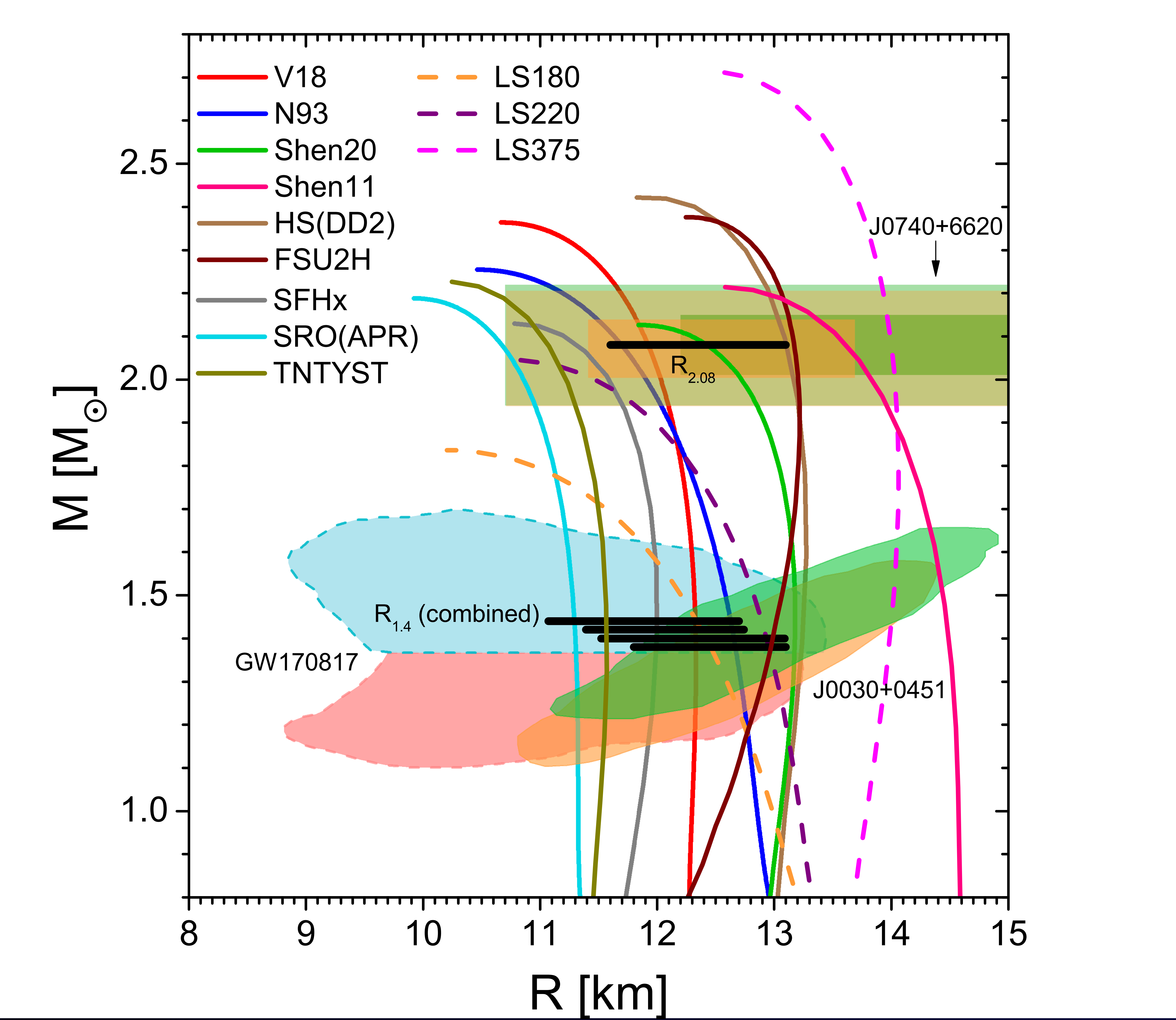
Check wrt NS observations

“Recipe” for neutron star structure calculations

- Energy density : $\epsilon(\rho_i); i = n, p, e^-, \mu^-$
- Chemical potentials : $\mu_i = \dfrac{\partial \epsilon}{\partial \rho_i}$
- Beta-equilibrium : $\mu_i = b_i \mu_n - q_i \mu_e$
- Charge neutrality : $\sum_i x_i q_i = 0$
- Composition : $x_i(\rho)$
- Equation of State : $p(\rho) = \rho^2 \dfrac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$
- TOV equations :

$$\begin{cases} \dfrac{dP}{dr} = -\dfrac{Gm}{r^2} \dfrac{(\epsilon + P)(1 + 4\pi r^3 P/m)}{1 - \frac{2Gm}{r}} \\ \dfrac{dm}{dr} = 4\pi r^2 \epsilon(r) \end{cases}$$

Mass-Radius relation



Inspiral phase of GW170817 :
Tidal deformability λ and Love numbers

The Newtonian Theory of Tides :

The Love numbers were introduced by August E. H. Love in 1911 : they are a set of dimensionless parameters which measure the rigidity of a planetary body and show how its shape changes in response to an external tidal potential.

These numbers can be generalized for stars in General Relativity. In particular, we are interested in one of these numbers, which connects the tidal field with the quadrupolar deformation of the star.

The Love number k_2

Solve in GR together with the TOV eqs. for the pressure p and the enclosed mass m

$$k_2 = \frac{8}{5}\frac{\beta^5 z}{6\beta(2-y_R)+6\beta^2(5y_R-8)+4\beta^3(13-11y_R)+4\beta^4(3y_R-2)+8\beta^5(1+y_R)+3z\log(1-2\beta)}$$

$$z \equiv (1-2\beta^2)[2-y_R+2\beta(y_R-1)]$$

$$\frac{dp}{dr} = -\frac{m\epsilon}{r^2}\frac{(1+p/\epsilon)\,(1+4\pi r^3 p/m)}{1-m/r},$$

$$\frac{dm}{dr} = 4\pi r^2\epsilon,$$

ε being the mass-energy density

$$\frac{dy}{dr} = -\frac{y^2}{r} - \frac{y-6}{r-2m} - rQ,$$

$$Q \equiv 4\pi\frac{(5-y)\epsilon+(9+y)p+(\epsilon+p)/c_s^2}{1-2m/r} - \left[\frac{2(m+4\pi r^3 p)}{r(r-2m)}\right]^2,$$

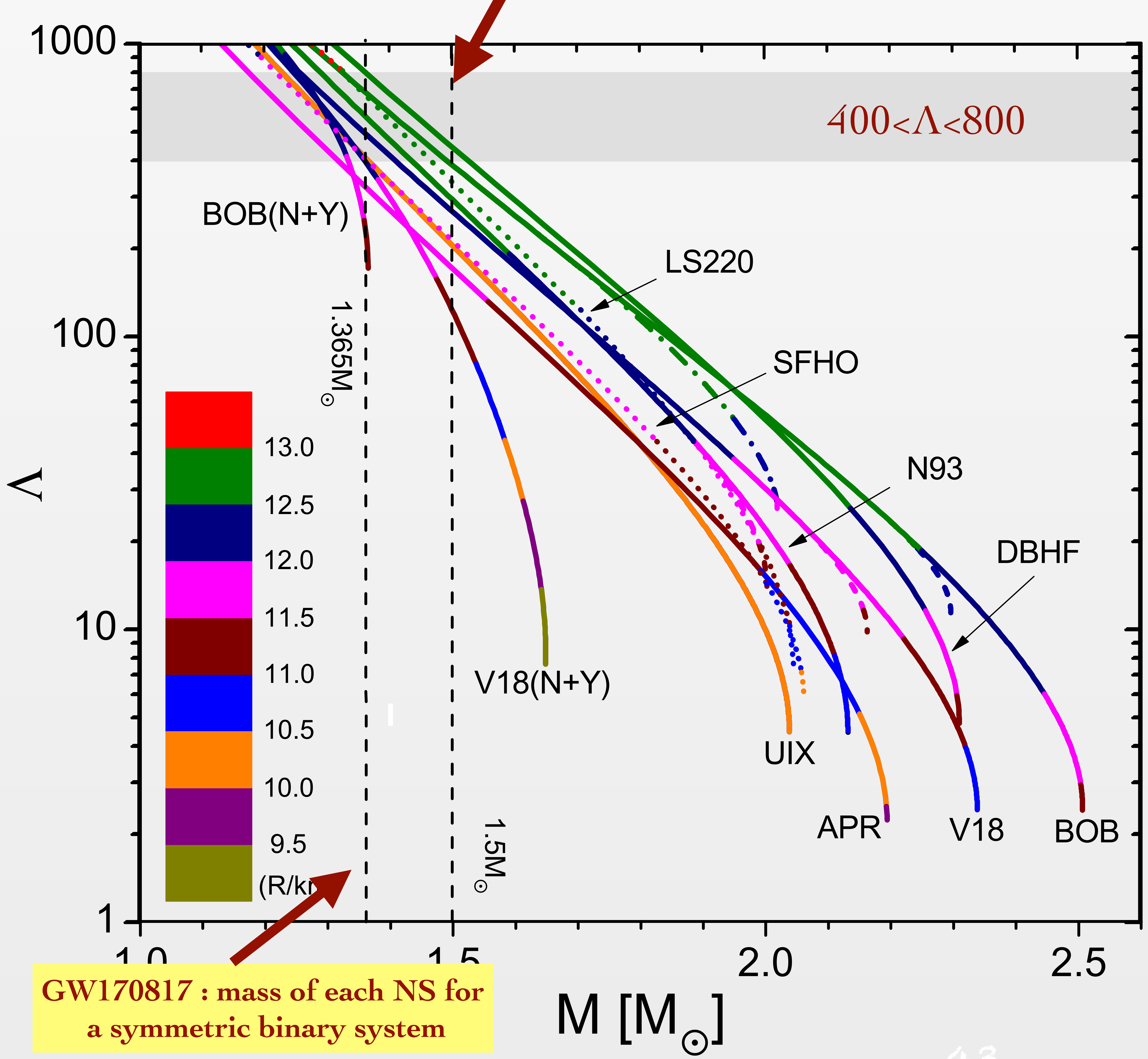
with $c_s^2 = d\epsilon/dp$ and the EOS $\epsilon(p)$ as input.

$$k_2 = \frac{3G}{2R^5}\lambda$$

The Love number k_2

depends crucially on the compactness β=M/R, hence on the EoS.

Correlations between M, R and Λ



- Fixed chirp mass

$$\mathcal{M}_c = \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}} = 1.188\, M_\odot$$

$$q = \frac{M_2}{M_1} = 0.7 - 1$$

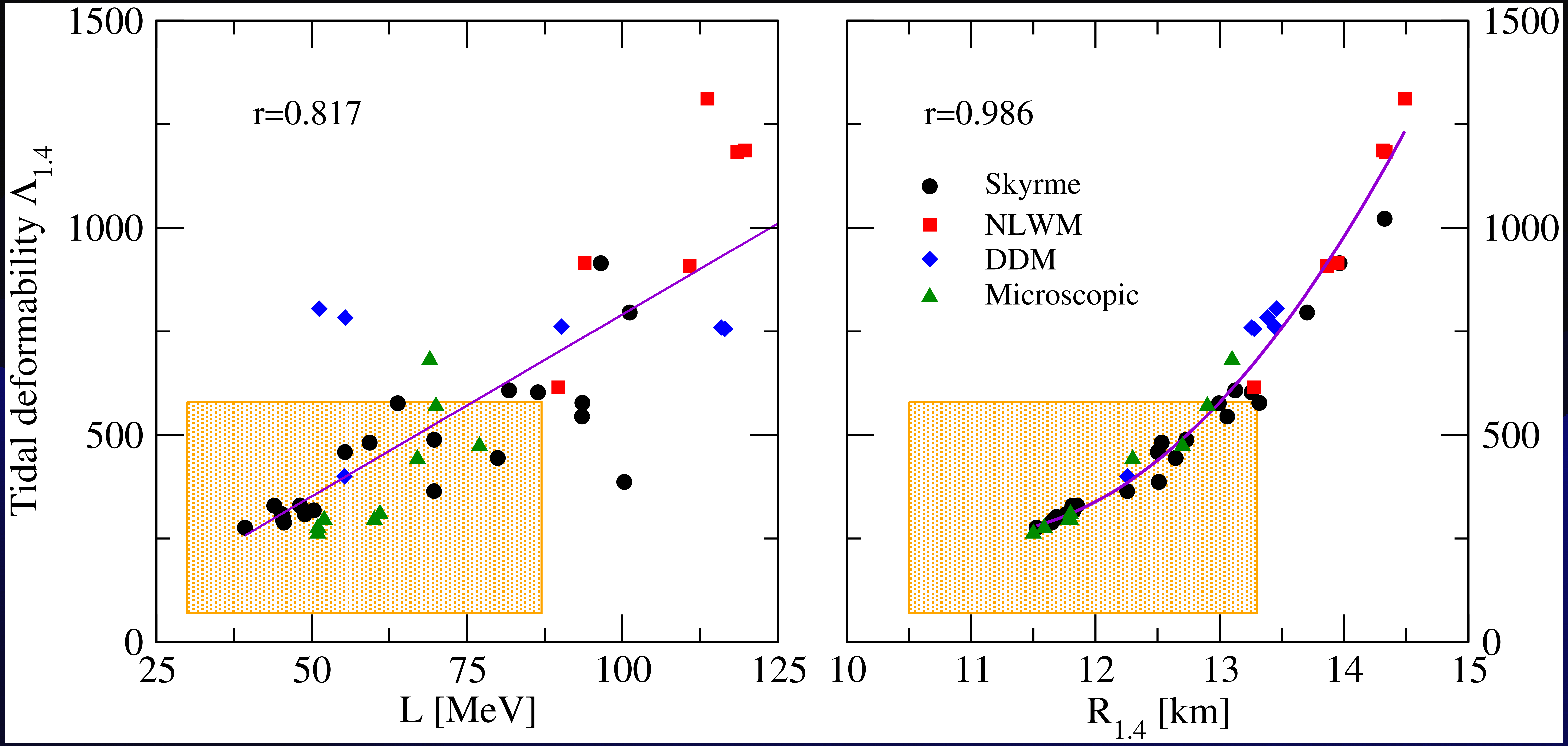
- The conditions $M_1=M_2=1.365$ M_0 and 400<Λ<800 imply **12<R<13 km**

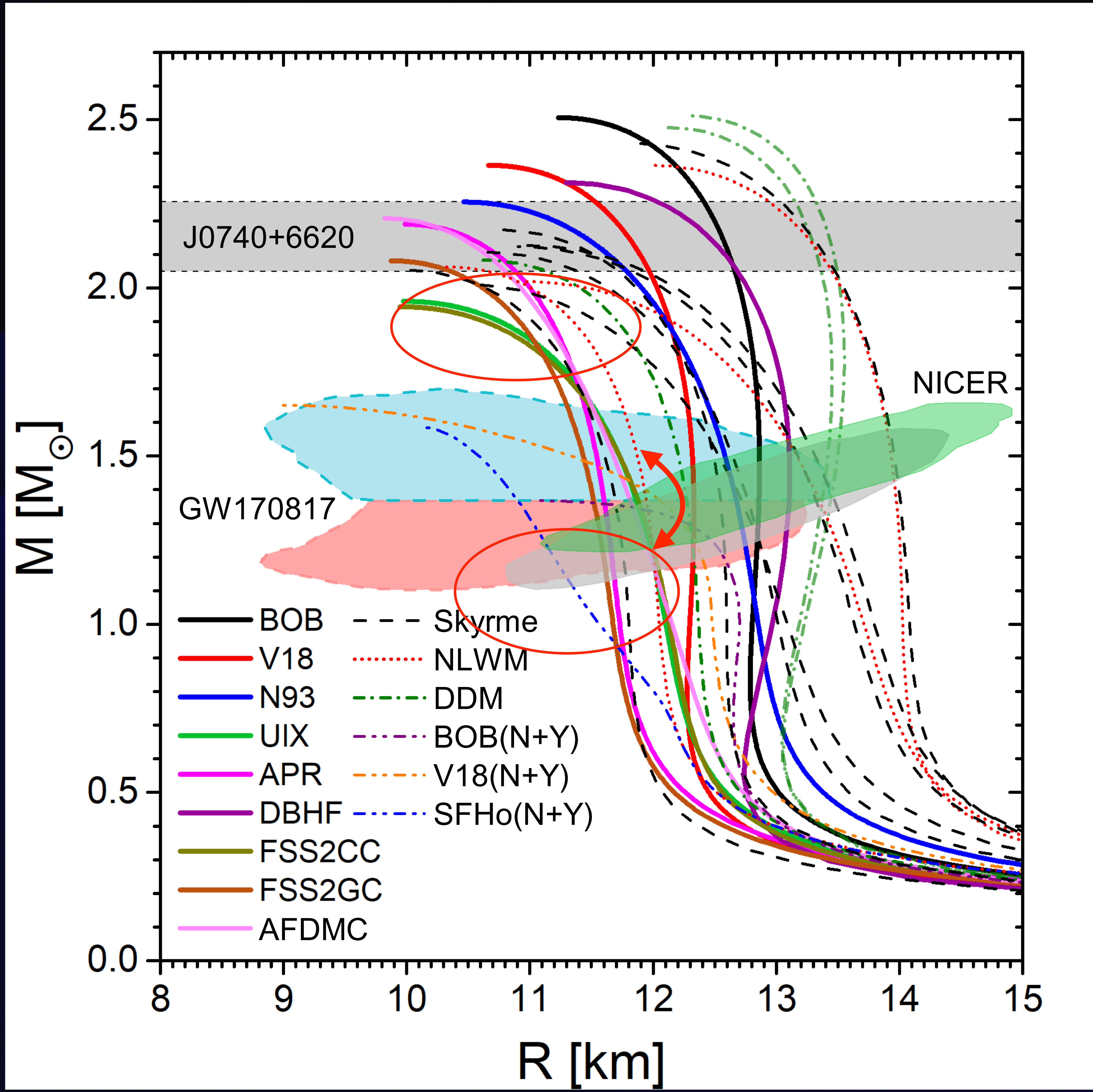
- Compatible EoS : V18(N+Y), UIX, V18,N93, BOB(N), DBHF, LS220, DS1, DS2.

- Not compatible : APR, BOB(N+Y), and SFHO (marginally).

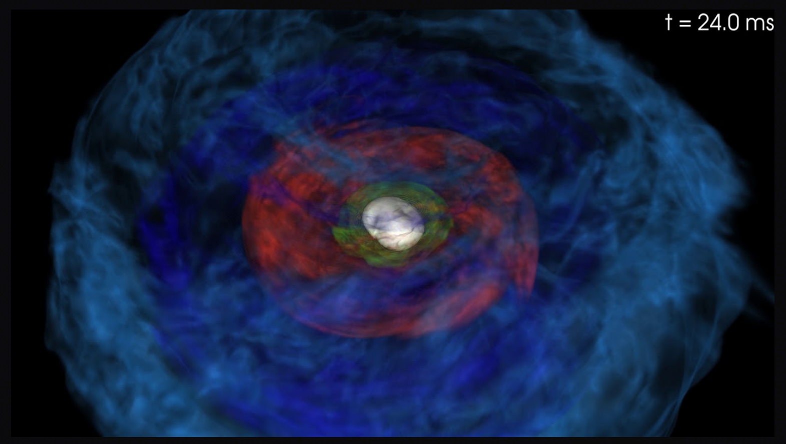
Selection of the EoS !

Correlations between $\Lambda_{1.4}$ and $R_{1.4}$





- *GW170817 : tidal deformability in binary NS, limits on the compactness M/R.*



www.ligo.caltech.edu

- *EoS with Hyperons (N+Y) : maximum mass around 1.6-1.7 solar mass.*
- *Not compatible with observational data.*

Observation of $\sim 2\,M_{\odot}$ neutron stars

Given that constrain, can hyperons, or strangeness in general, still be present in neutron stars interiors ?

Probably yes, due to the high value of density at the center and the rapid increase of the nucleon chemical potential with density.

What do we know to include hyperons in the EoS ?

Unfortunately much less than in the nucleonic sector.
Hard to draw strong conclusions given our ignorance of the nucleon-hyperon (NY) and hyperon-hyperon (YY) interaction.

A bit of experimental data …

- Most of data for the Λ hyperon. Single particle energies of hypernuclei from spectroscopy.
- Λ attractive potential at saturation density:

$$U_\Lambda(n = n_0) = U_{\Lambda,0} = -30 \text{ MeV}$$

- A nontrivial density dependence of the Λ potential as a function of the baryon number density.

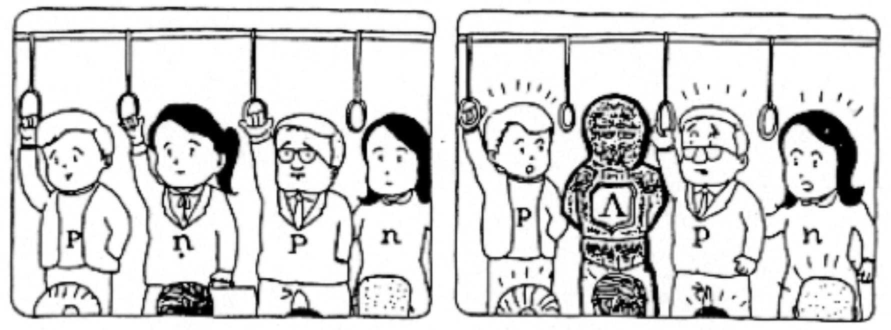
For heavier hyperons data much less certain.

Σ hyperons : strongly repulsive potential in nuclear matter with a likely value of U = 30 ± 20 MeV.

Ξ hyperons : a few old emulsion data suggesting bound Ξ hypernuclear states, which hint at an attractive potential in nuclear matter. U_Ξ=-24 MeV ?

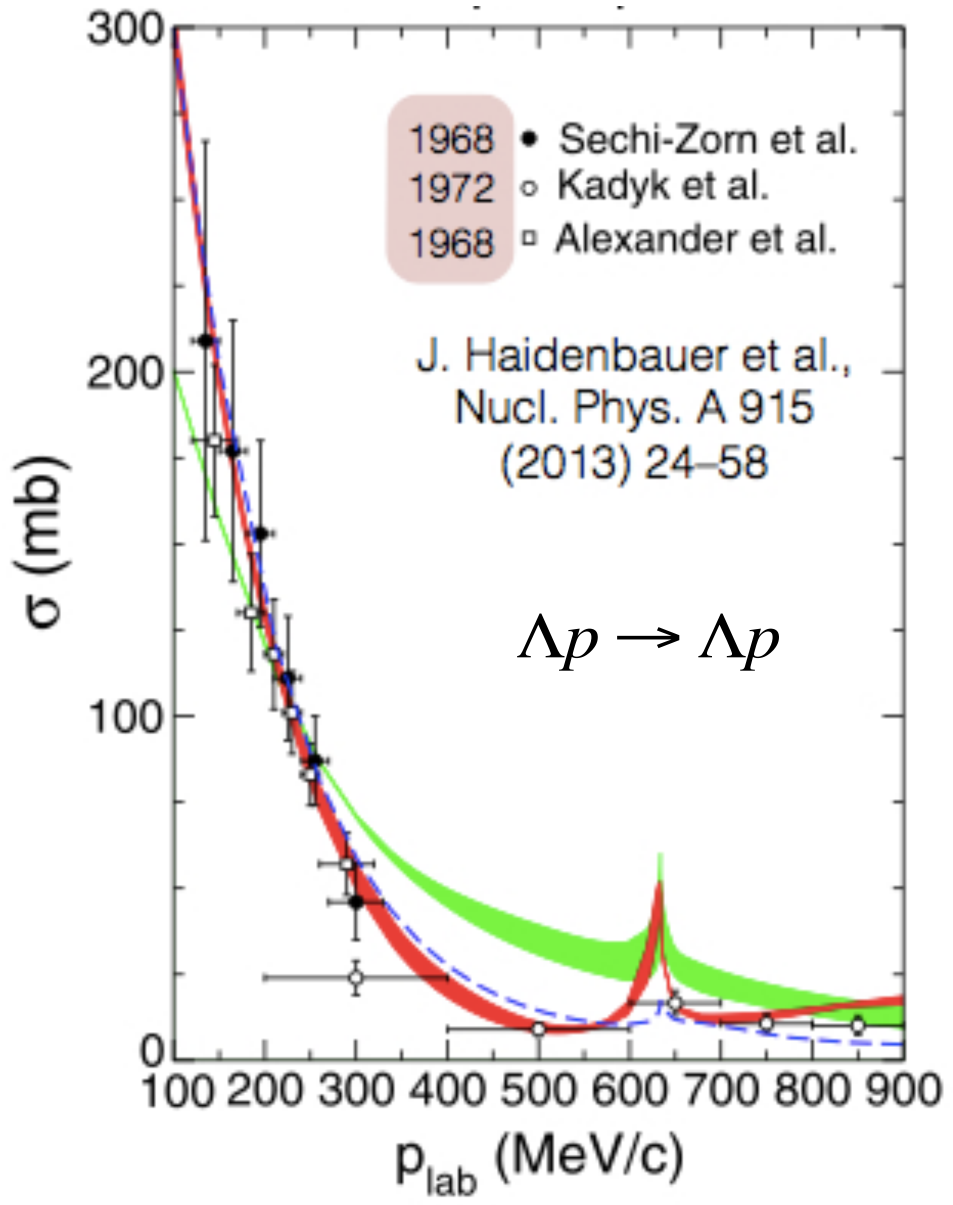
$\Lambda\Lambda$: few $\Lambda\Lambda$ hypernuclear events, slightly attractive ?

YY: Y=Λ, Σ, Ξ unknown!



Ordinary nucleus — With a strange particle

H. Bando, PARITY 1, 54 (1986)



- Very few YN scattering data due to short lifetime of hyperons & low intensity beam fluxes
  - ~ 35 data points, all from the 1960s
  - 10 new data points, from KEK-PS E251 collaboration (2000)
- No YY scattering data exists

(cf. > 4000 NN data for E_{lab} < 350 MeV)

from I. Vidaña

Theoretical models

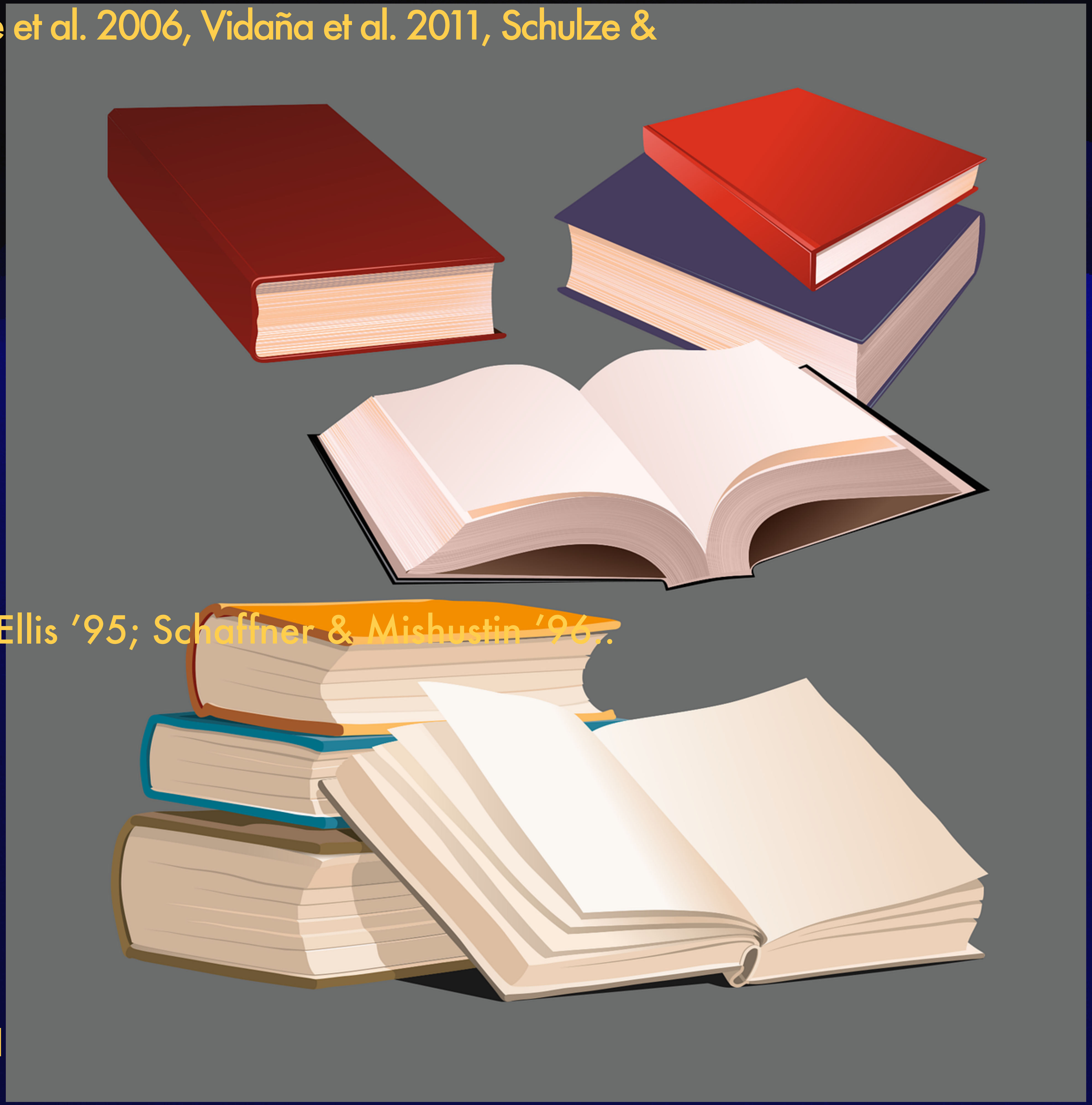
First considered by Ambartsumyan and Saakyan (1960)

➡Microscopic approaches

- Brueckner-Hartree-Fock theory : Baldo et al. 1998, Vidaña et al. 2000, Schulze et al. 2006, Vidaña et al. 2011, Schulze & Rjiken 2011
- DBHF : Sammarruca (2009), Katayama and Saito (2014)
- $V_{low\text{-}k}$: Djapo, Schaefer & Wambach (2010)
- Quantum Monte Carlo : Lonardoni et al., (2014)

➡Phenomenological approaches

- Relativistic mean field models : Glendenning '85; Knorren, Prakash & Ellis '95; Schaffner & Mishustin '96.
- Non-relativistic potential model : Balberg & Gal '97
- Quark-meson coupling model : Pal et al. '99
- Chiral effective Lagrangians : Hanauske et al 2000
- Density dependent hadron field models : Hofmann Keil & Lenske 2001



Including hyperons in BHF approach

$$G_{ab}[W] = V_{ab} + \sum_c \sum_{p,p'} V_{ac}\big|pp'\big\rangle \frac{Q_c}{W - E_c + i\epsilon}\big\langle pp'\big|G_{cb}[W]\,, \qquad (1)$$

where the indices a, b, c indicate pairs of baryons and the angle-averaged Pauli operator Q and energy E determine the propagation of intermediate baryon pairs. In a given nucleon-hyperon channels $c = (NY)$ one has, for example,

$$E_{(NY)} = m_N + m_Y + \frac{k_N^2}{2m_N} + \frac{k_Y^2}{2m_Y} + U_N(k_N) + U_Y(k_Y)\,. \qquad (2)$$

The hyperon single-particle potentials within the continuous choice are given by

$$U_Y(k) = \sum_{N=n,p} U_Y^{(N)}(k) = \mathrm{Re} \sum_{N=n,p} \sum_{k'<k_F^{(N)}} \big\langle kk'\big|G_{(NY)(NY)}\left[E_{(NY)}(k,k')\right]\big|kk'\big\rangle \qquad (3)$$

and similar expressions of the form

$$U_N(k) = \sum_{N'=n,p} U_N^{(N')}(k) + \sum_{Y=\Sigma^-,\Lambda} U_N^{(Y)}(k) \qquad (4)$$

$$\frac{B}{A} = \frac{\epsilon}{\rho_n + \rho_p + \rho_{\Sigma^-} + \rho_\Lambda}\,,$$

$$\epsilon = \sum_{i=n,p,\Sigma^-,\Lambda} \int_0^{k_F^{(i)}} \frac{dk\,k^2}{\pi^2} \left(m_i + \frac{k^2}{2m_i} + \frac{1}{2}U_i(k) \right) = \epsilon_{NN} + \epsilon_{NY}$$

with

$$\epsilon_{NN} = \sum_{N=n,p} \int_0^{k_F^{(N)}} \frac{dk\,k^2}{\pi^2} \left(m_N + \frac{k^2}{2m_N} + \frac{1}{2}\left[U_N^{(n)}(k) + U_N^{(p)}(k)\right] \right),$$

$$\epsilon_{NY} = \sum_{Y=\Sigma^-,\Lambda} \int_0^{k_F^{(Y)}} \frac{dk\,k^2}{\pi^2} \left(m_Y + \frac{k^2}{2m_Y} \right) + \sum_{N=n,p} \int_0^{k_F^{(N)}} \frac{dk\,k^2}{\pi^2} \left[U_N^{(\Sigma^-)}(k) + U_N^{(\Lambda)}(k)\right]$$

$$= \sum_{Y=\Sigma^-,\Lambda} \int_0^{k_F^{(Y)}} \frac{dk\,k^2}{\pi^2} \left(m_Y + \frac{k^2}{2m_Y} + \left[U_Y^{(n)}(k) + U_Y^{(p)}(k)\right] \right).$$

Technical difficulty :

coupled channel calculation !

Only NN and NY interactions are included. No YY potentials.

The nucleons feel direct effects of the other nucleons and the hyperons.

For the hyperons there are only nucleonic contributions, because of the missing hyperon-hyperon potentials.

Baldo et al., Phys.Rev.C61:055801,2000

Including hyperons in BHF approach

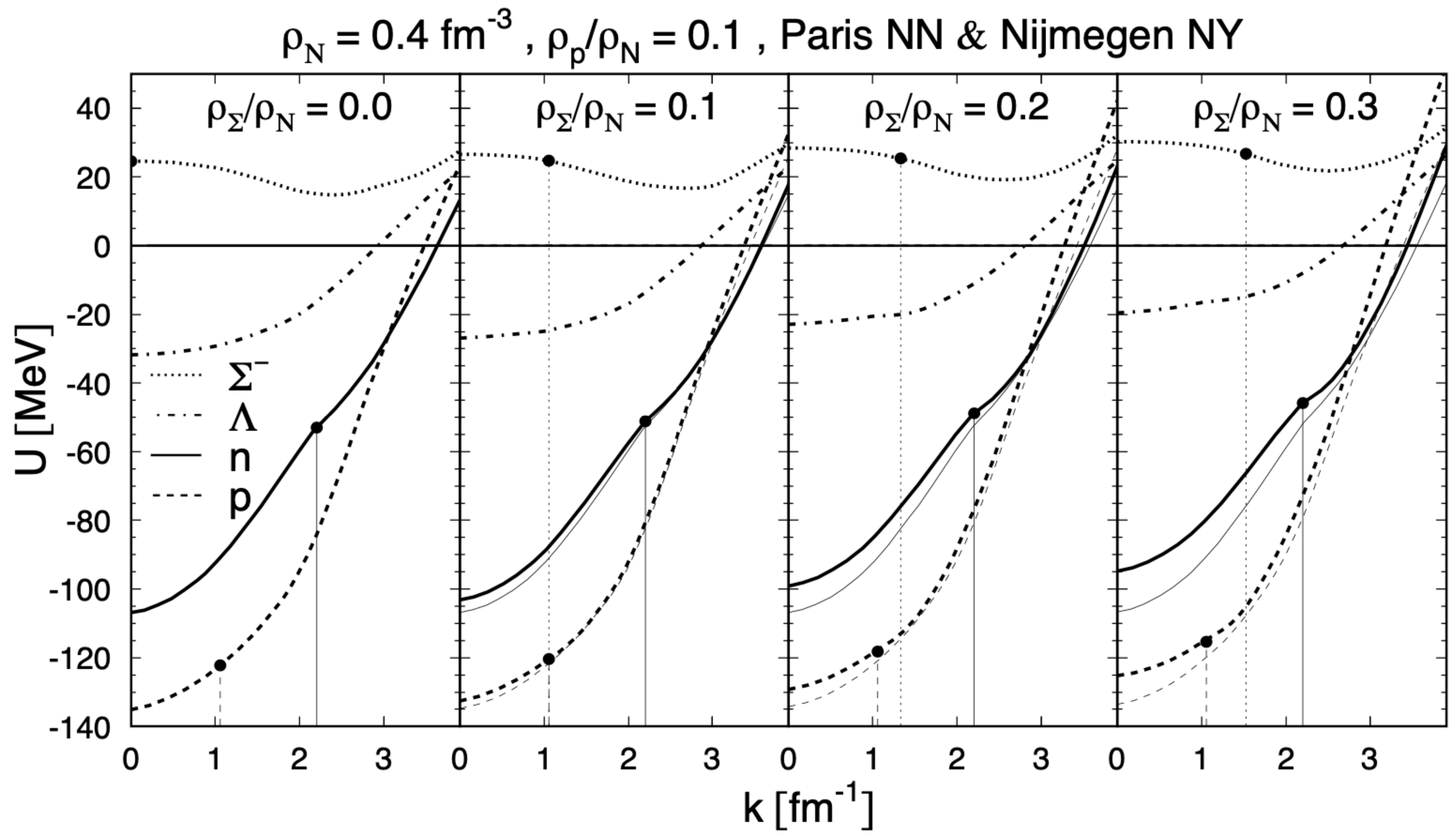
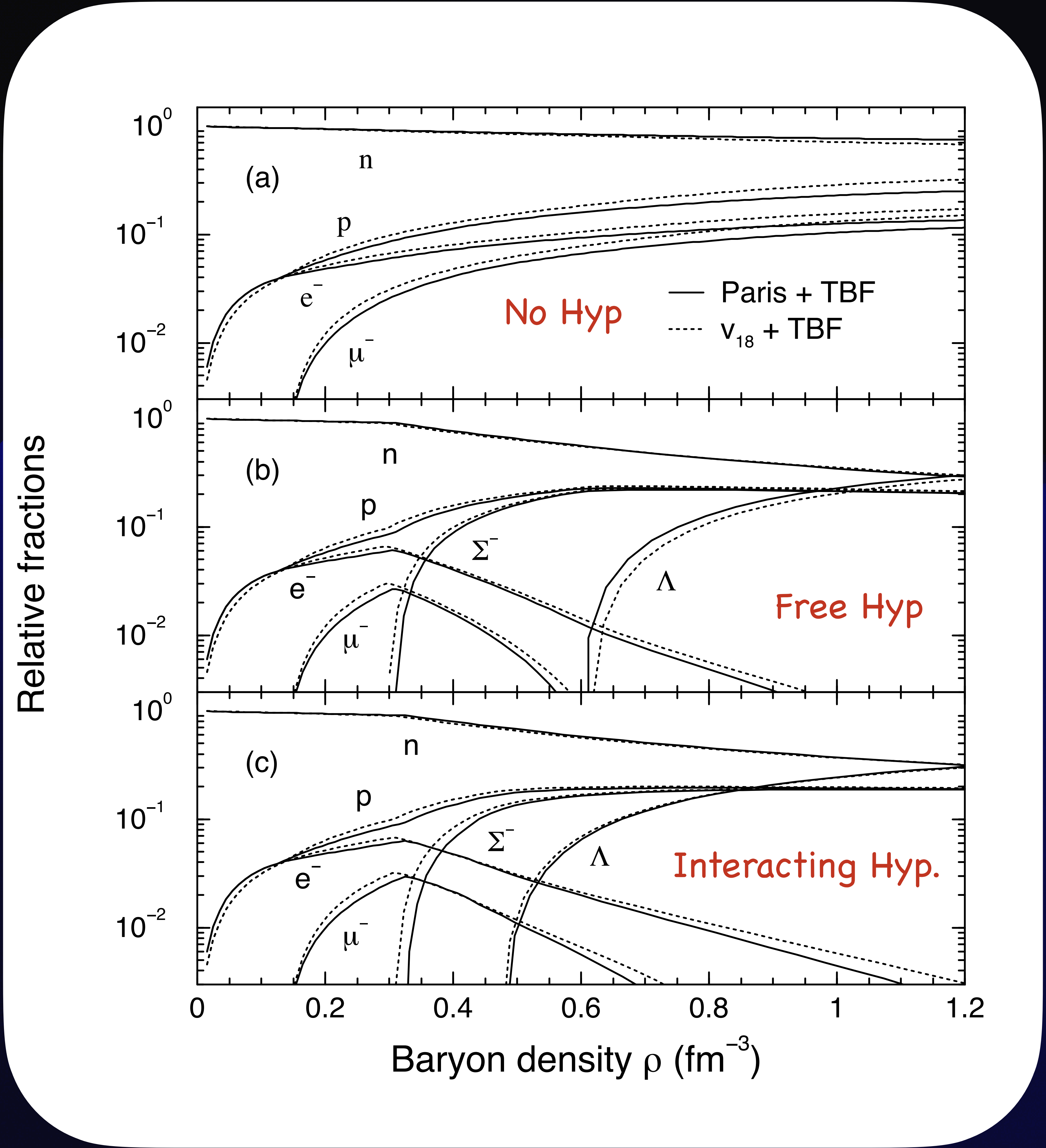
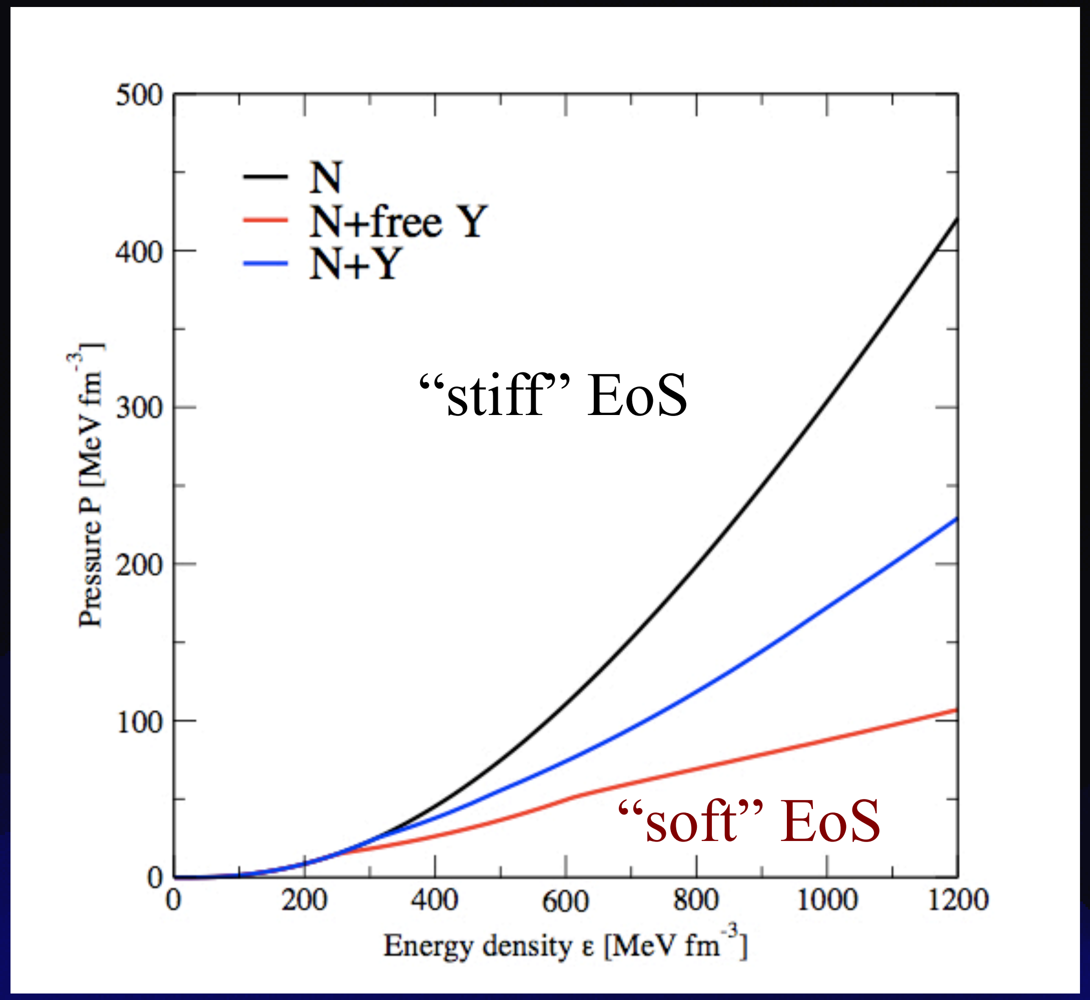


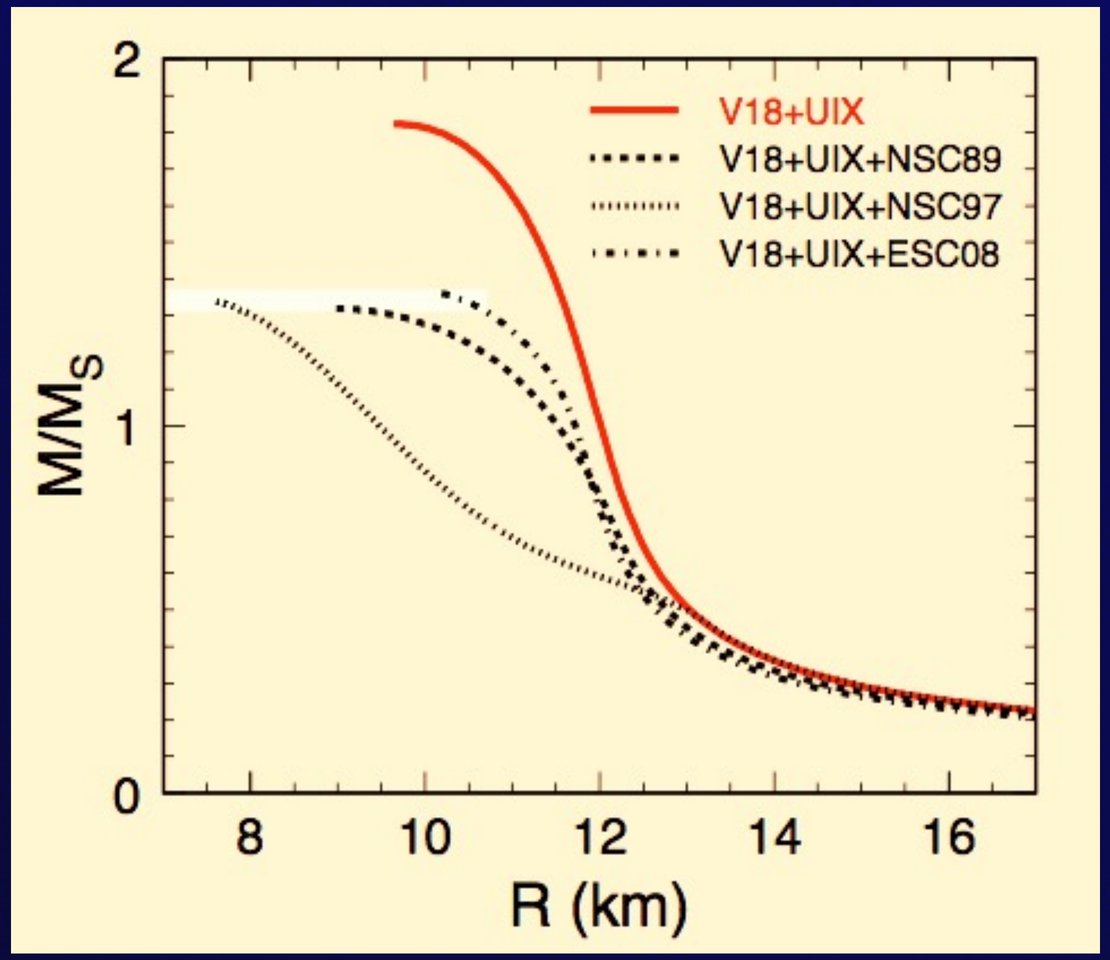
FIG. 1. The single-particle potentials of nucleons n, p and hyperons Σ^-, Λ in baryonic matter of fixed nucleonic density $\rho_N = 0.4\,\mathrm{fm}^{-3}$, proton density $\rho_p/\rho_N = 0.1$, and varying Σ^- density $\rho_{\Sigma^-}/\rho_N = 0.0, 0.1, 0.2, 0.3$. The vertical lines represent the corresponding Fermi momenta of n, p, and Σ^-. For the nucleonic curves, the thick lines represent the complete single-particle potentials U_N, whereas the thin lines show the values excluding the Σ^- contribution, i.e., $U_N^{(n)} + U_N^{(p)}$.

Baldo et al., Phys.Rev.C61:055801,2000

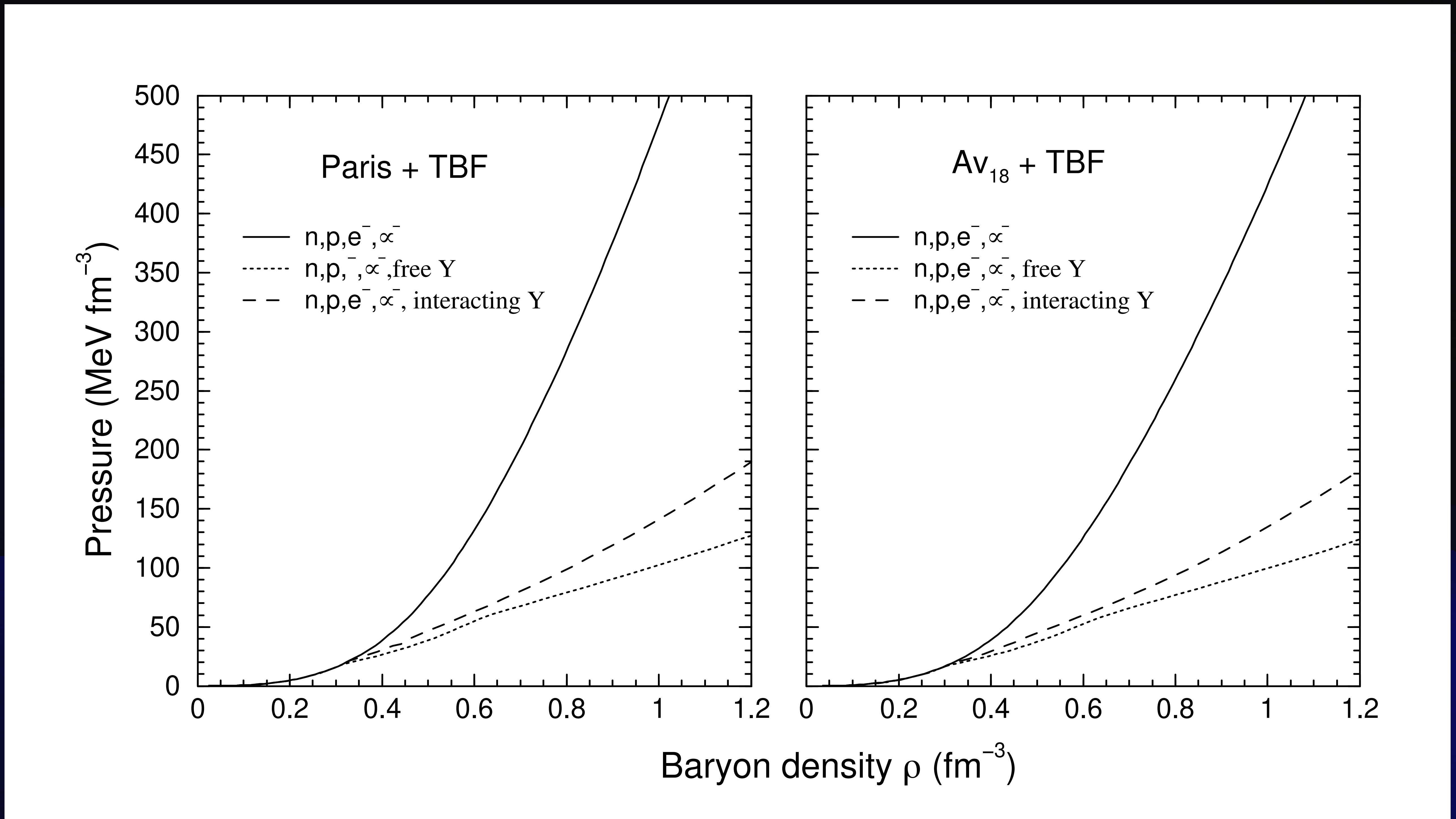
The composition of hypernuclear matter







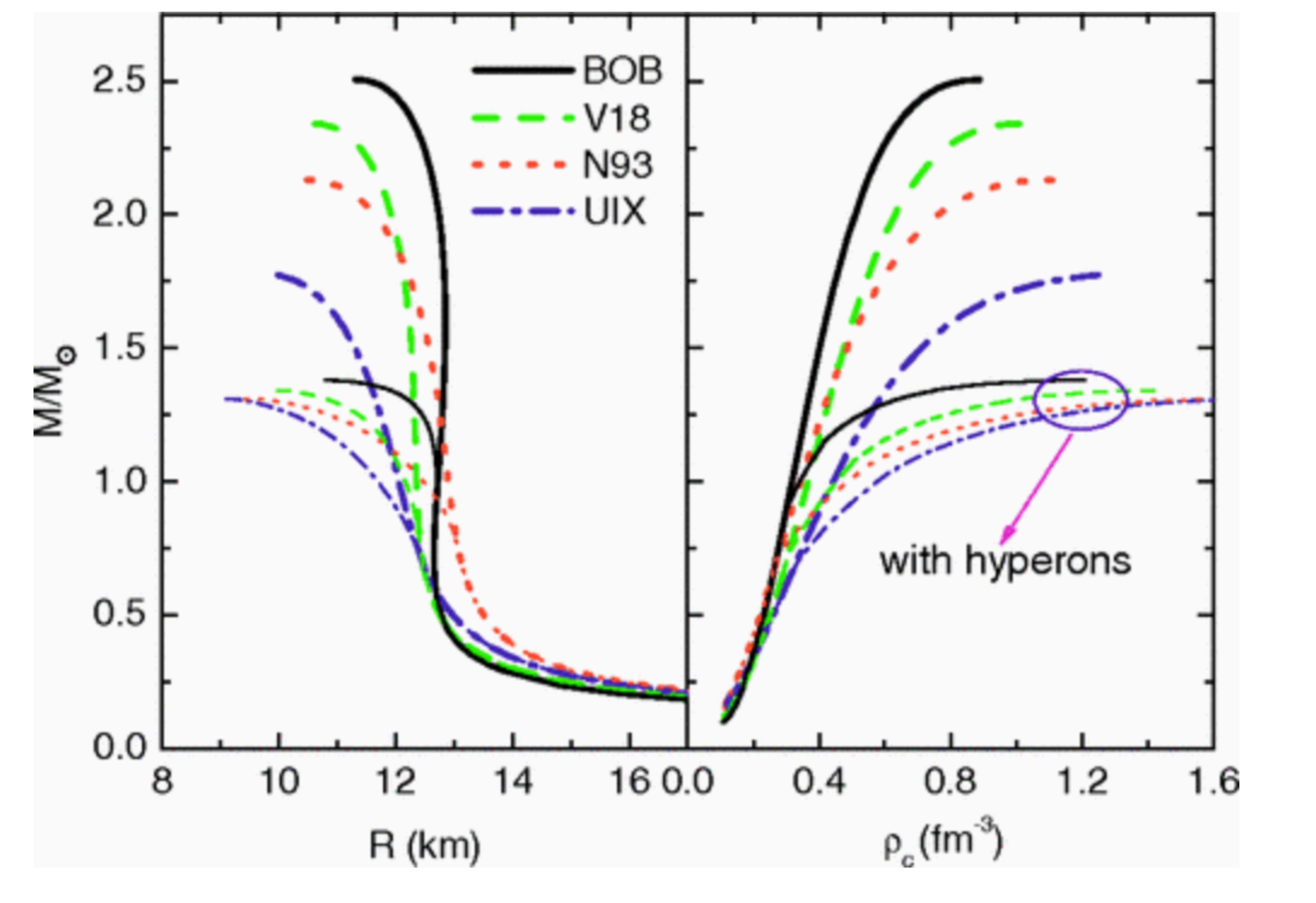
… check the dependence on the NN potential



Strong softening due to hyperon onset

(more Fermi seas available)

Mass-Radius relation with different NN interactions



NSC89 NY potential

No YY interaction

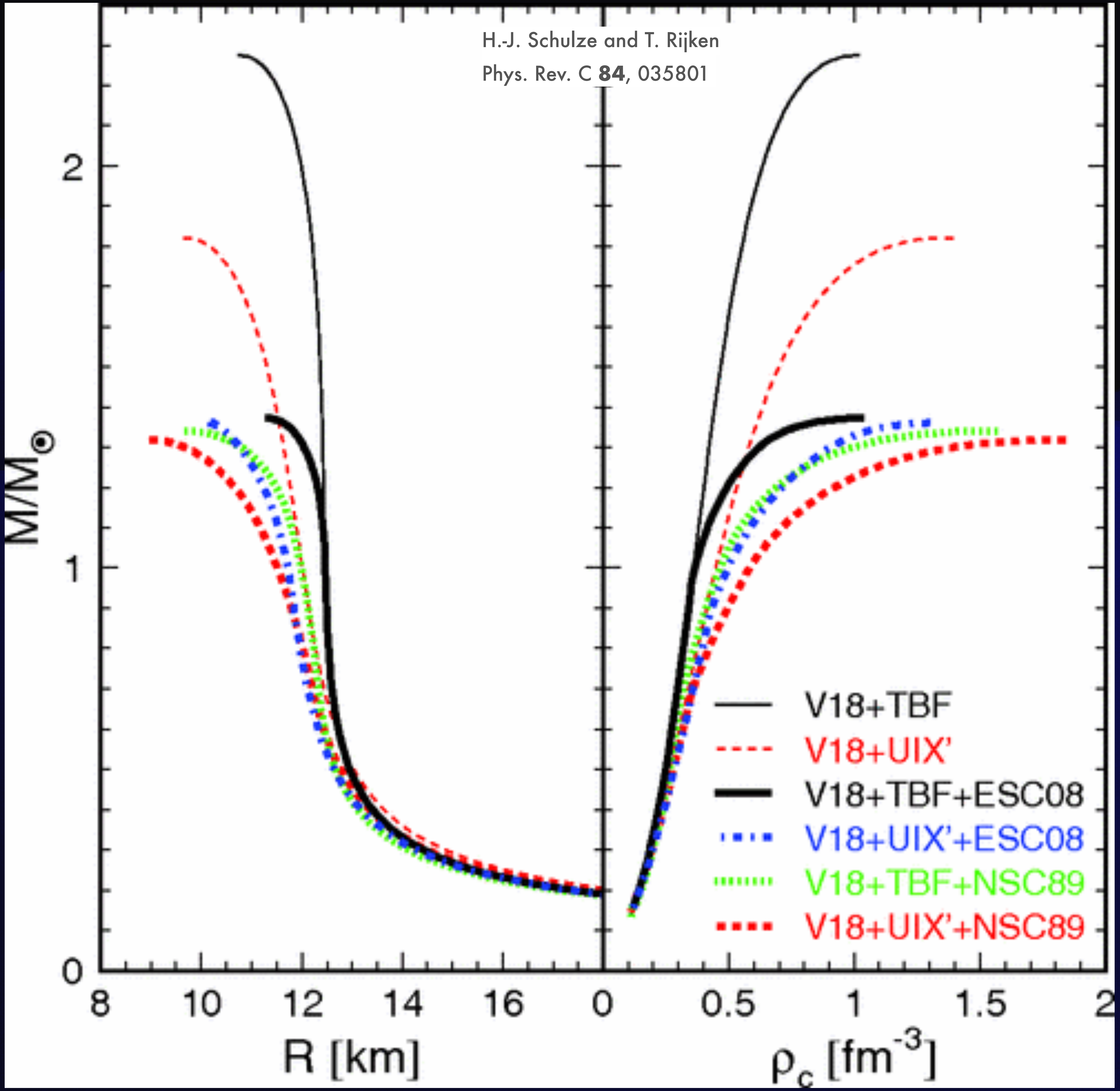
No hyperon TBF

Large variation of M_{max} with the NN interaction

Softening due to hyperon appearance

(Stiffer EoS → earlier hyperon onset)

Mass-Radius relation with different NY potentials

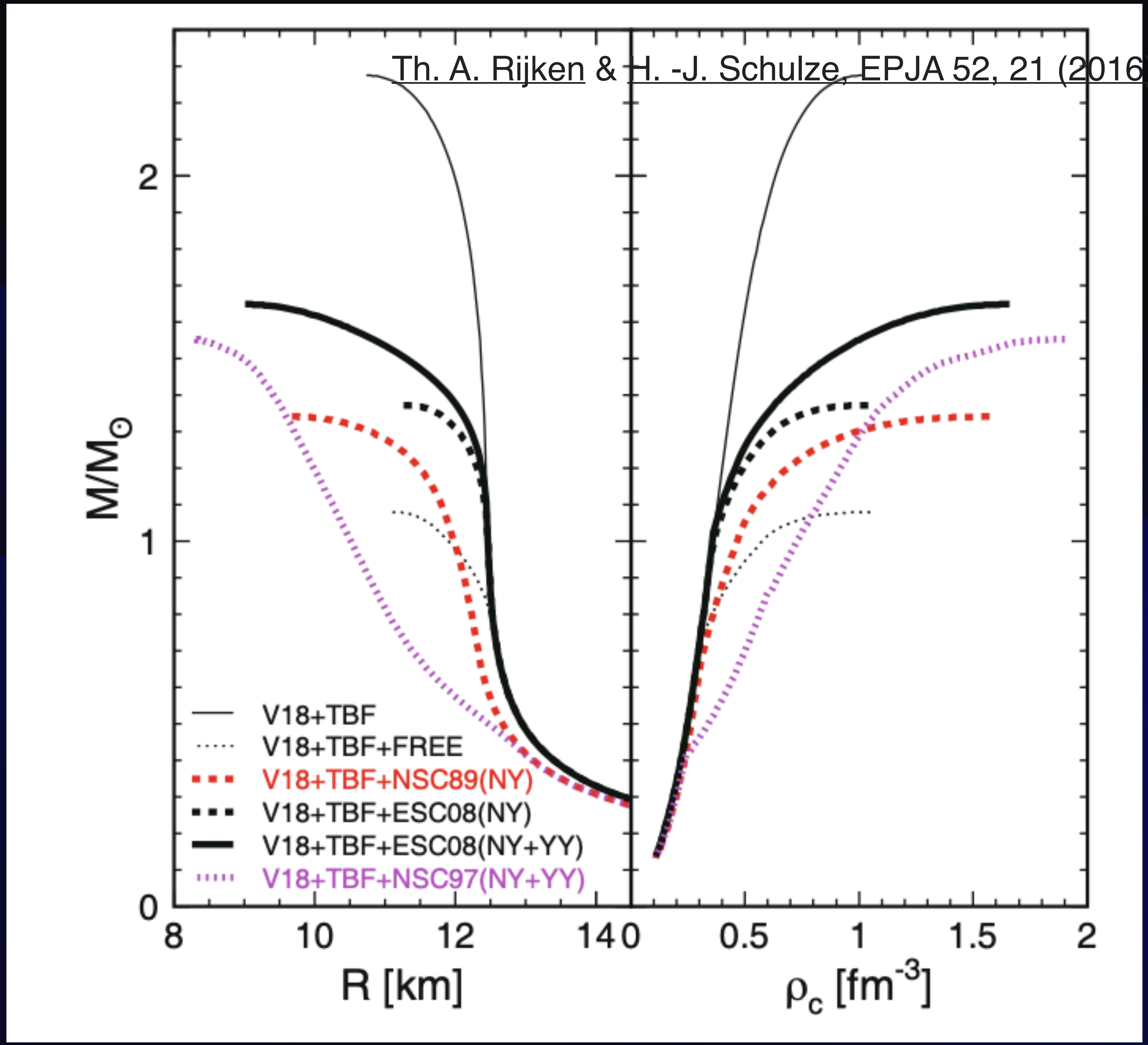


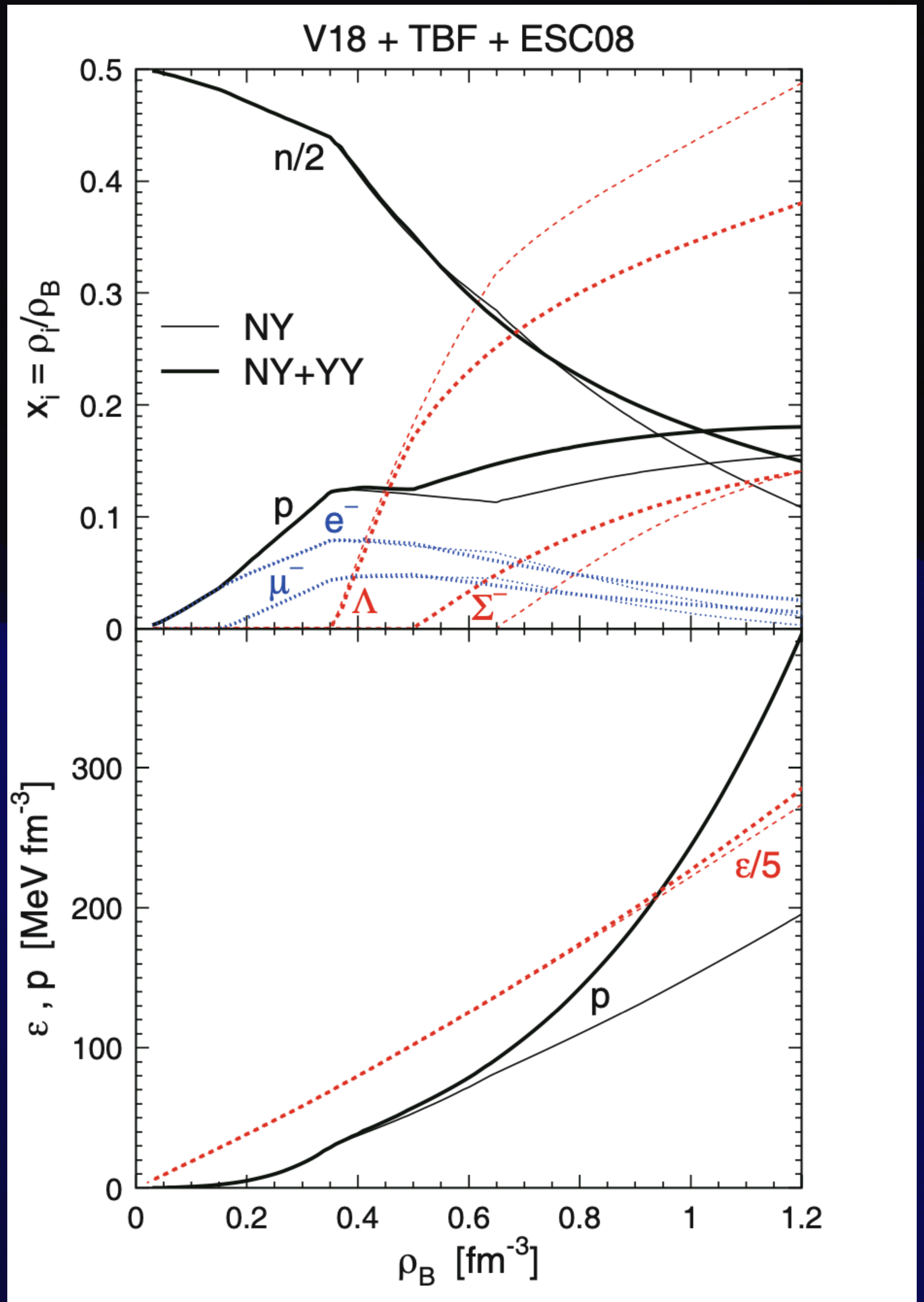
M_{max} independent of potentials !

M_{max} too low (< 1.4 Mo) !

Proof for quark matter inside NS ?

Possible effects of the YY potentials





ΛΛ, Σ-Σ- repulsive

ΛΣ- attractive

M_{max} increases to about 1.7 M_o

Hyperon TBF (YNN, YYN, YYY) unknown (exp. and theor.) !

Hyperon puzzle

HYPERONS → IN MICROSCOPIC APPROACHES
A TOO SOFT EOS NOT COMPATIBLE WITH MEASURED NS MASSES.
CAVEAT : THE PRESENCE OF HYPERONS IN THE NS CORE SEEMS TO BE UNAVOIDABLE !

Possible solutions of the hyperon puzzle

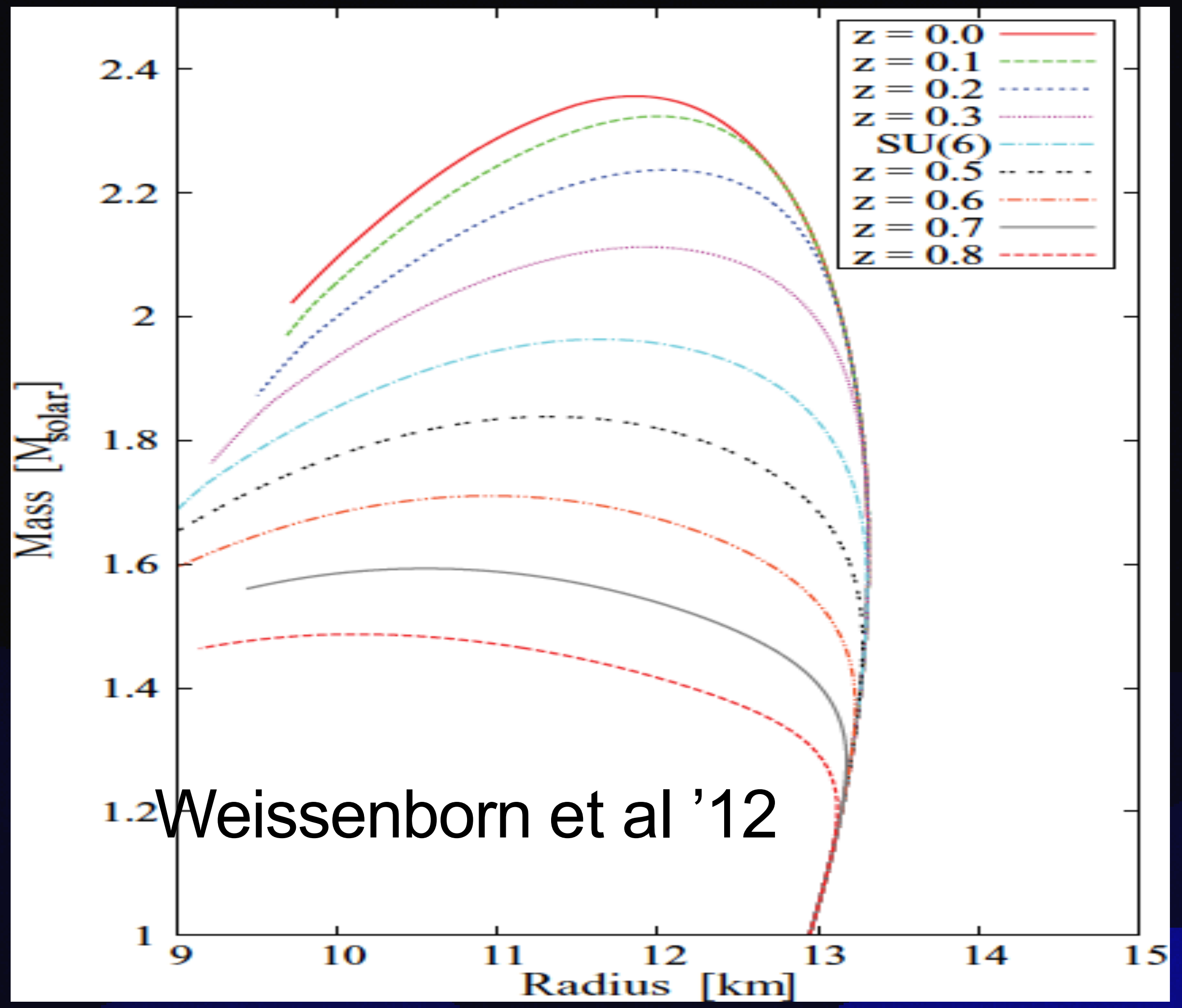
- One excludes hyperons in the nuclear models **by hand,** **thus** ignoring experimental data from hypernuclei.

- One pushes up the critical density for the onset of hyperon formation in neutron star matter beyond the maximum density in neutron stars. Arbitrary !

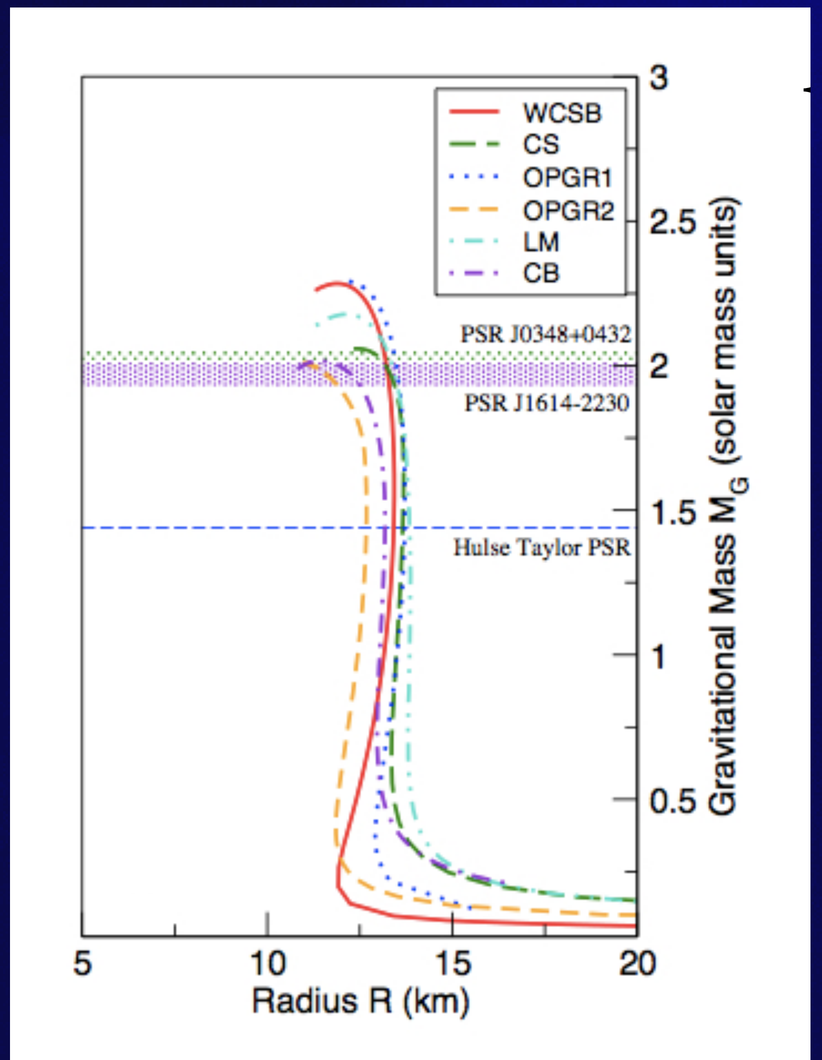
Solution I :YY and NY vector meson repulsion

- Mainly studied in RMF models
- Coupling of Φ to hyperons in order to shift their onset to higher density

Bednarek et al '12; Weissenborn et al '12; Oertel et al '15; Maslov et al '15..



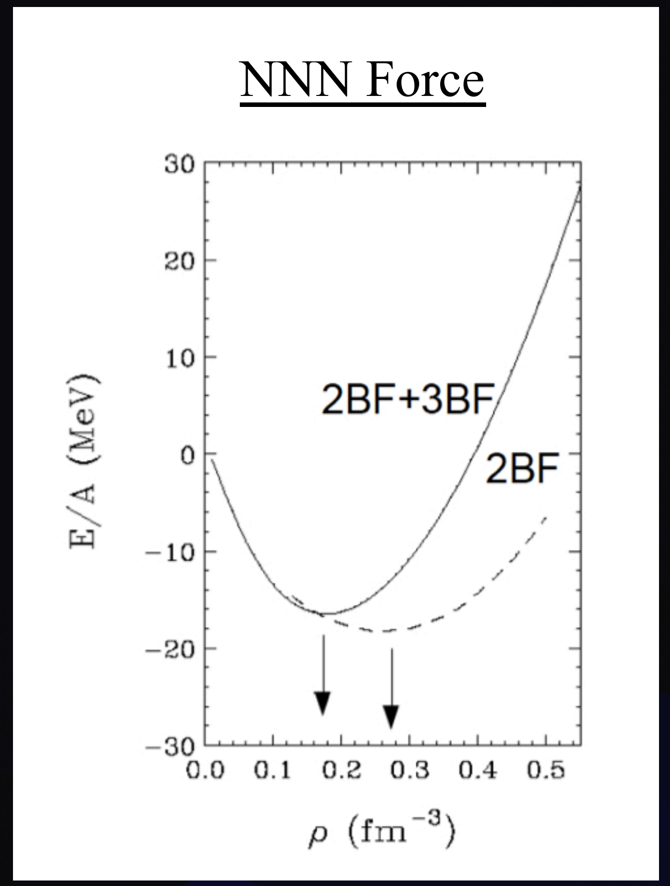
Some of these models are able to reconcile the presence of hyperons with the 2Mo limit, they contain several free parameters which very often are arbitrarily chosen.

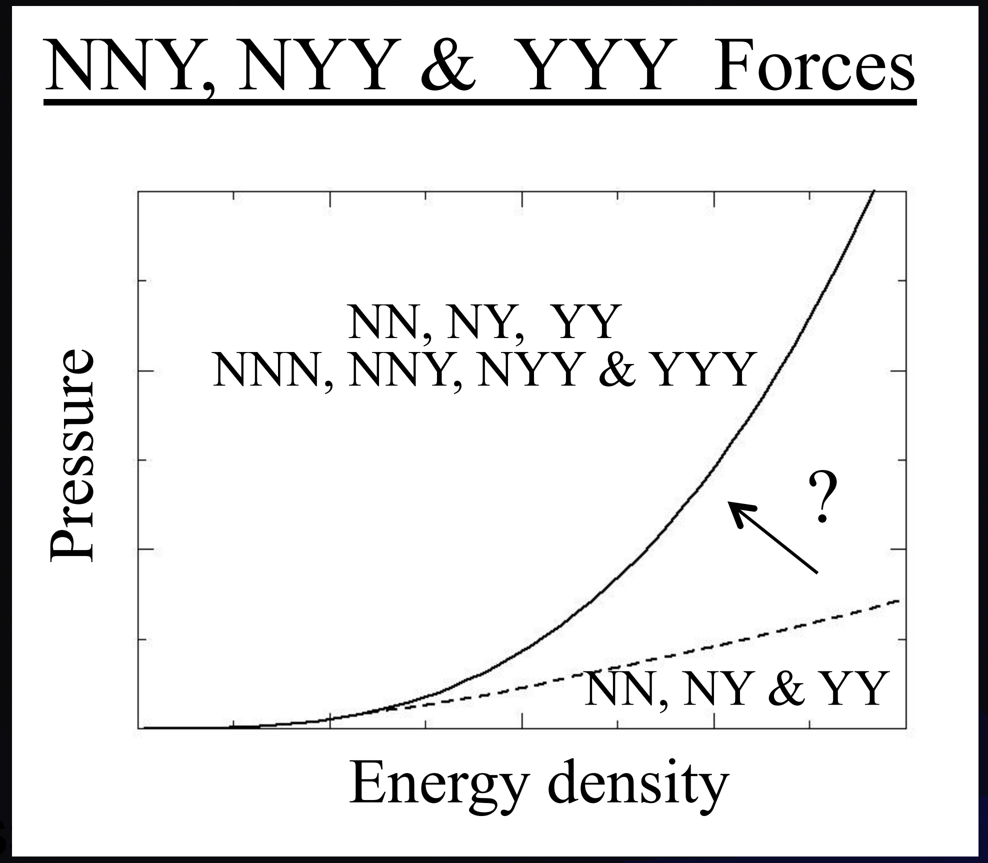


D. Chatterjee & I. V. (2016)

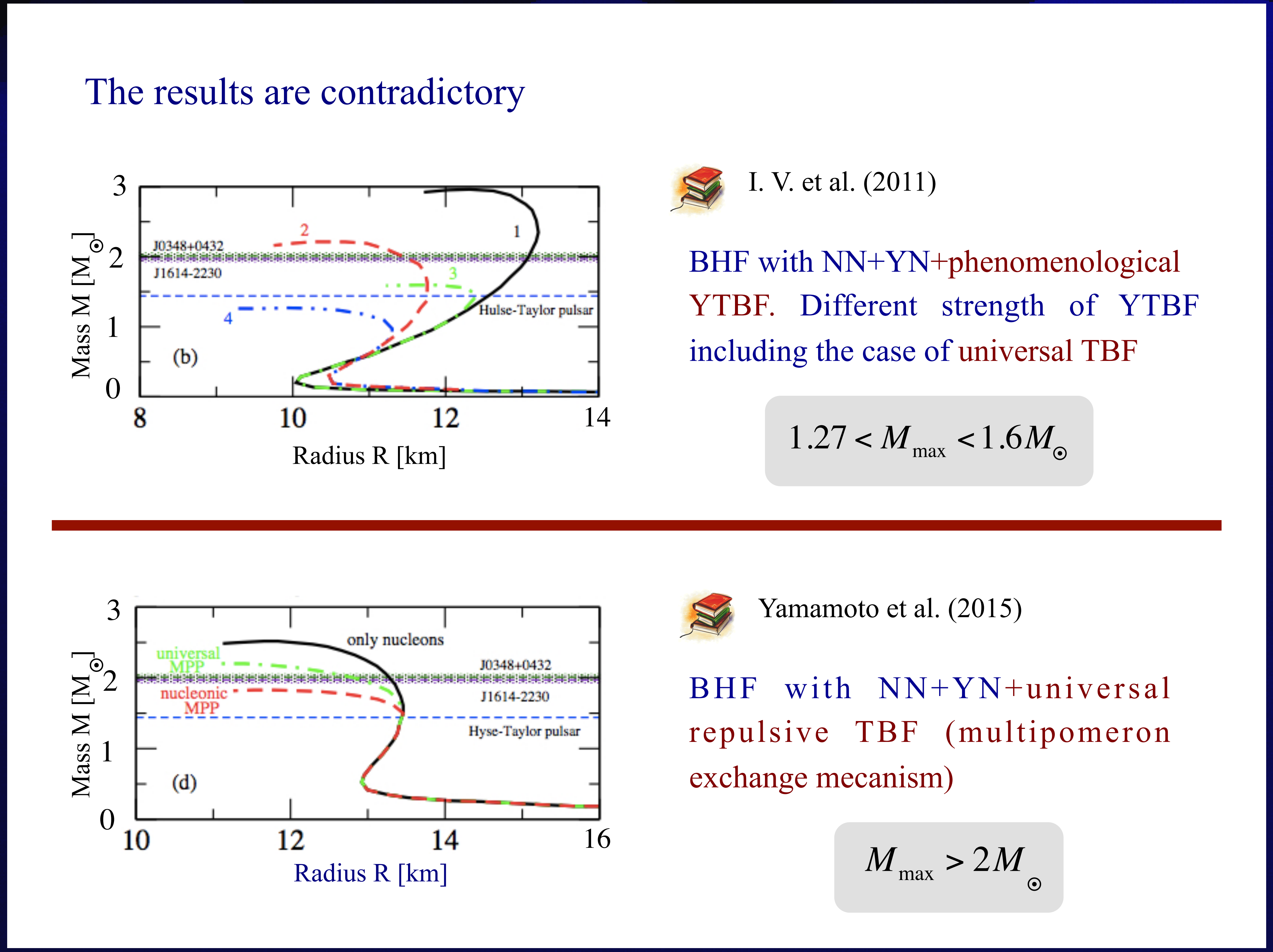
Validity of these models is questionable.

Solution II : Hyperonic TBF





- Importance of TBF in Nuclear Physics
- Correct saturation point in microscopic approaches
- Can hyperonic TBF solve the puzzle ?
- No general consensus regarding the results.
- Hyperonic 3-body forces in χEFT might solve the hyperon puzzle ??



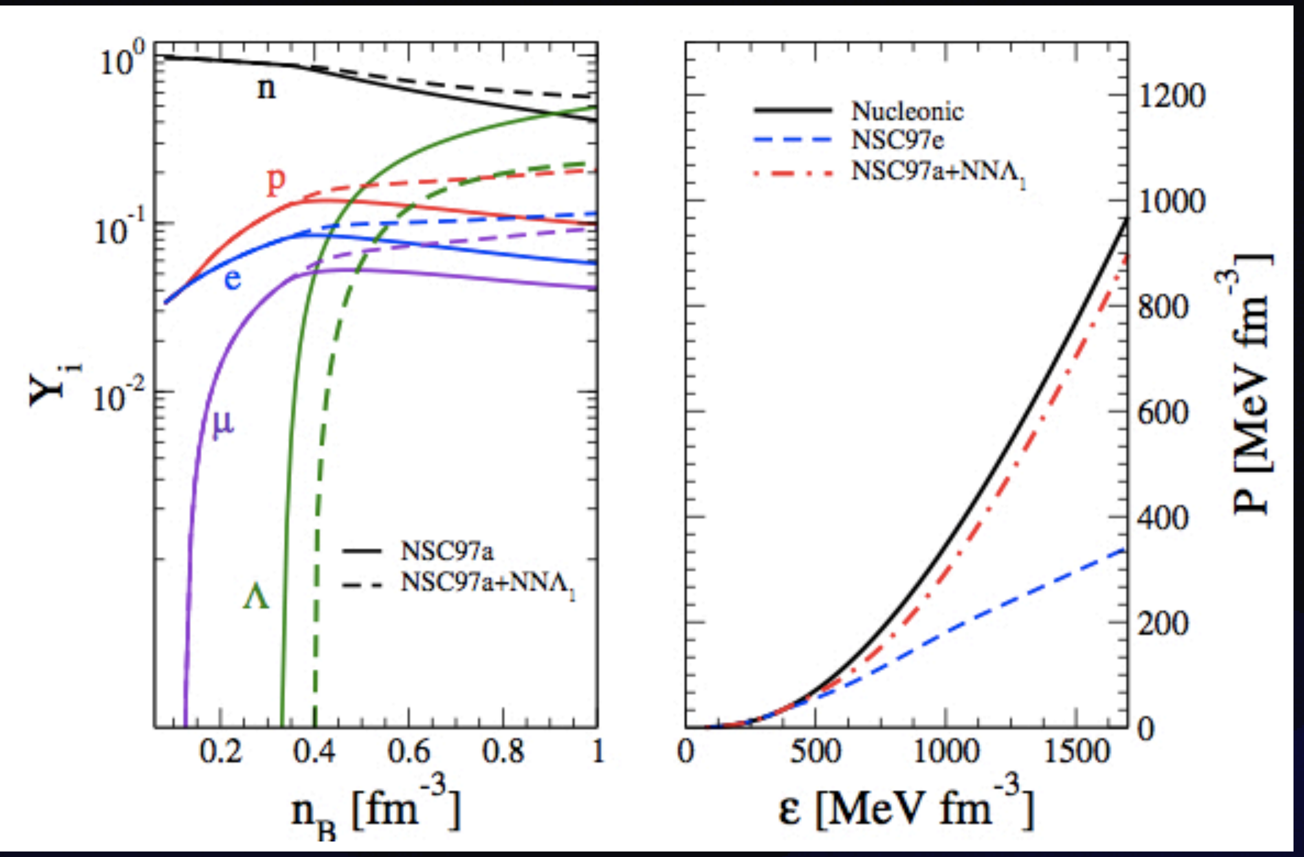
Solution III : the NNΛ chiral forces

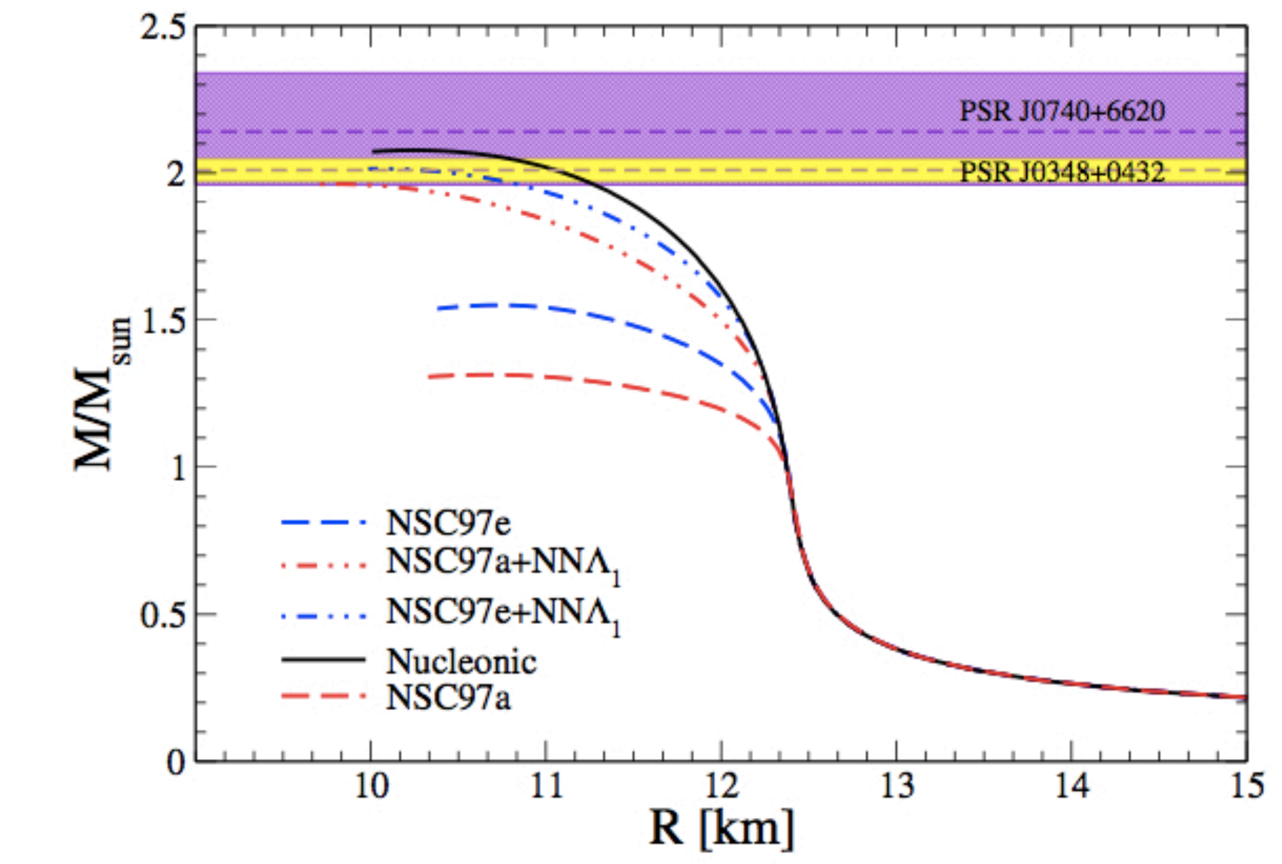
- Preliminary exploratory work by Logoteta, Vidana & Bombaci on the role of NNΛ interaction. Eur. Phys. J. A (2019) 55:207.

- The chiral NNΛ shifts the onset of Λ to slightly larger densities and largely reduces the concentration, thus stiffening the EoS.

- Maximum mass "almost" compatible with the 2 M_o limit, but other hyperons should be included.

Hyperon puzzle cannot be considered as solved !



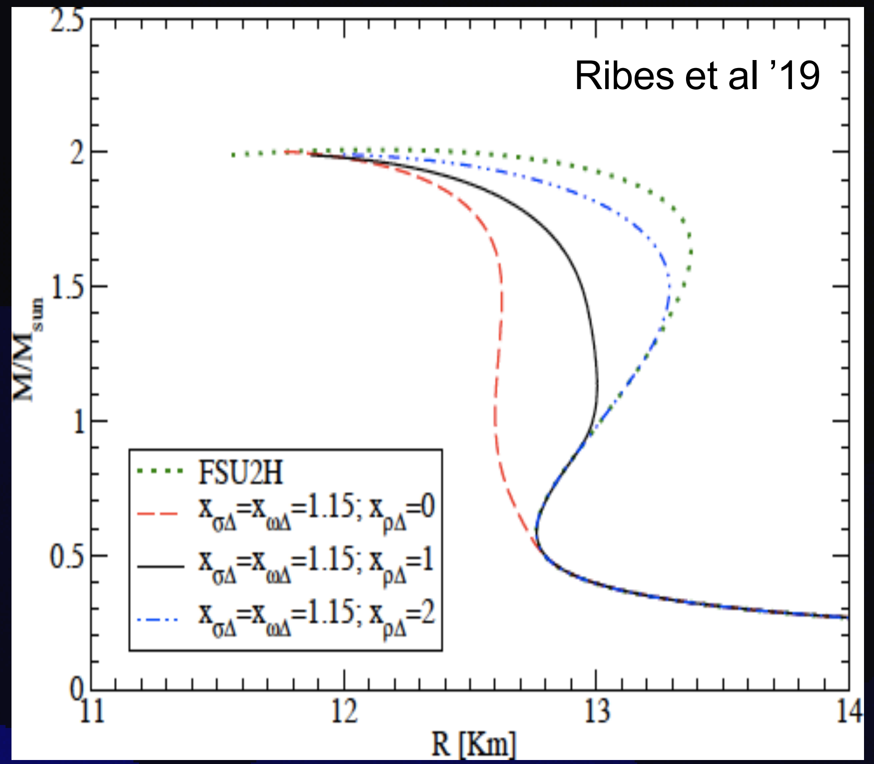


	$M_{max}(M_\odot)$	R (km)	n_c (fm^{-3})
Nucleonic	2.08	10.26	1.15
NSC97a	1.31	10.60	1.40
NSC97a+NNΛ$_1$	1.96	9.80	1.30
NSC97a+NNΛ$_2$	1.97	9.87	1.28
NSC97e	1.54	10.81	1.18
NSC97e+NNΛ$_1$	2.01	10.10	1.20
NSC97e+NNΛ$_2$	2.02	10.15	1.19

Solution IV : Appearance of Δ-isobars

- It would push the Y onset to higher densities
- This might or not reach the 2Mo limit

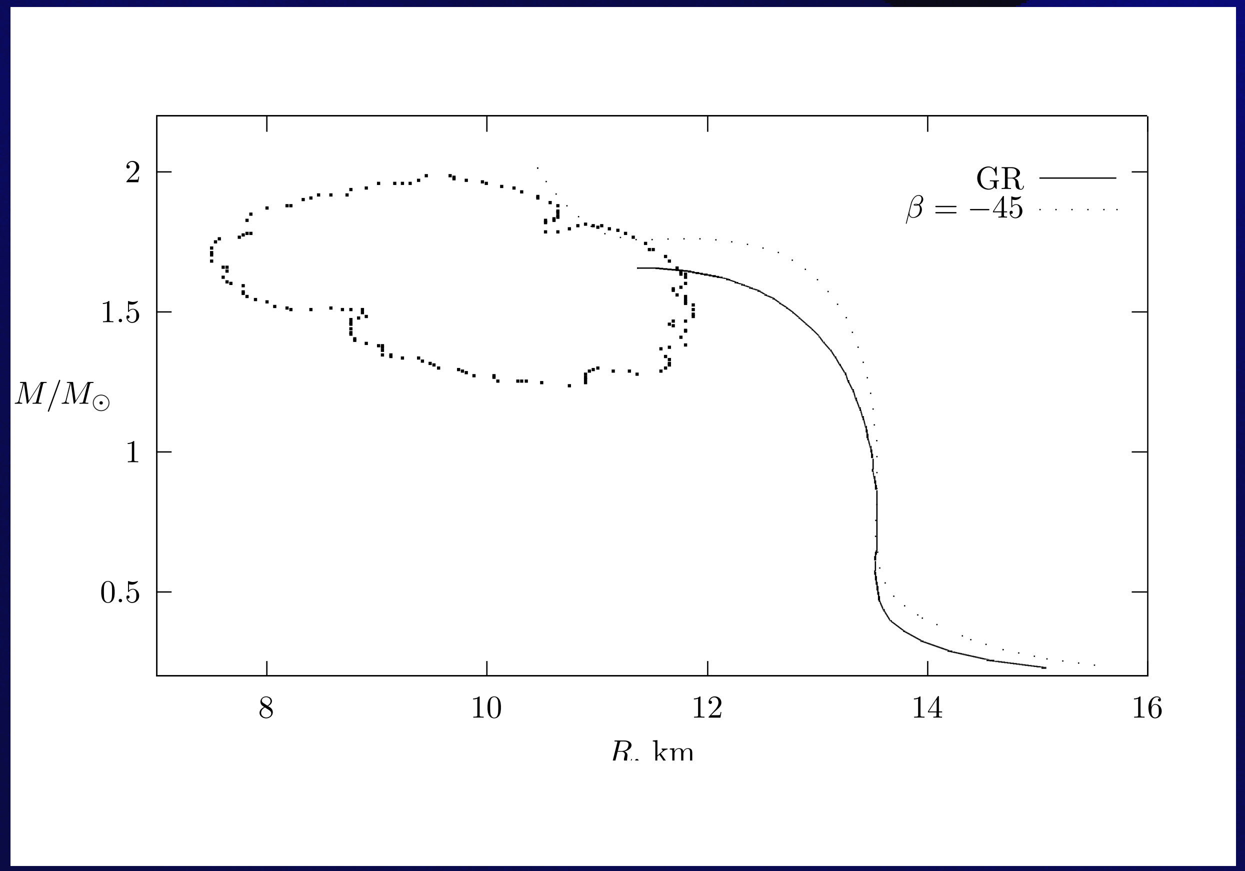
Drago et al '14 '15, Jie Li et al '19 ; Ribes et al '19…



Solution V : Modified gravity

- Modified TOV eqs. in f(R) gravity.
- It depends on the chosen f(R).

Astashenok, Capozziello, Odintsov '10, '11, '14 ...



Solution VI : Dark matter ...

Solution VII : Quark matter core

- No commonly accepted solution to the problem. A particular striking one: Hyperons appear but before they can destabilize the neutron star a new phase appears at high density with a stiff EOS supporting a $2M_\odot$ compact star. That new phase would be not based on hadronic degrees of freedom, nucleons, and hyperons, but on a new degree of freedom in the form of the constituents of hadrons, that is, quarks, forming a quark matter core.

As better expressed by F. Wilczek:

"The behaviour of QCD at large net baryon density (and low temperature) is also of obvious interest. It answers yet another childlike question : What will happen when you keep squeezing things harder and harder ?" (Wilczek, Phys. Today, August 2000.)

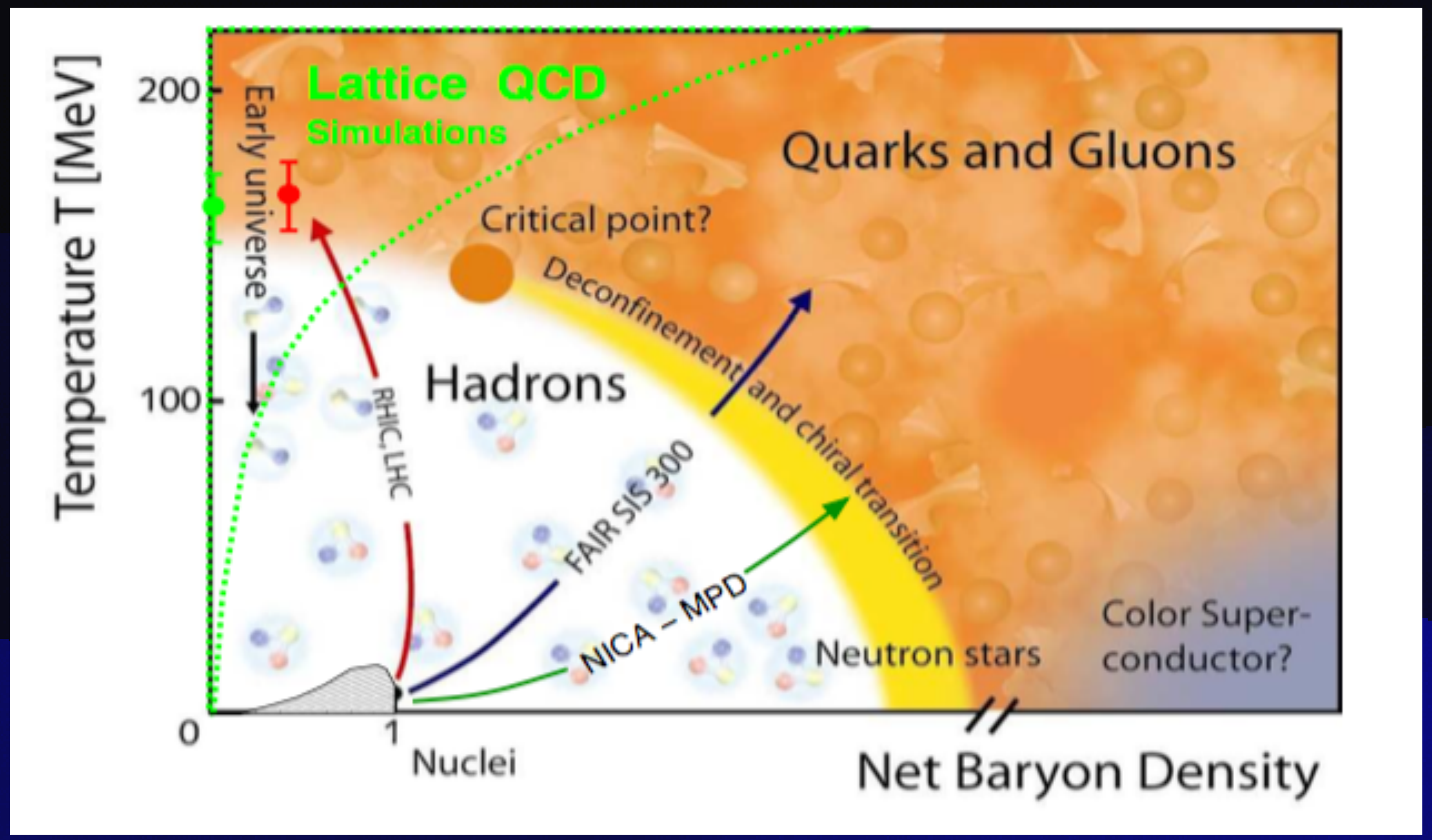
Quark : the building blocks of matter

- The theory is, in principle, known : **QCD**.

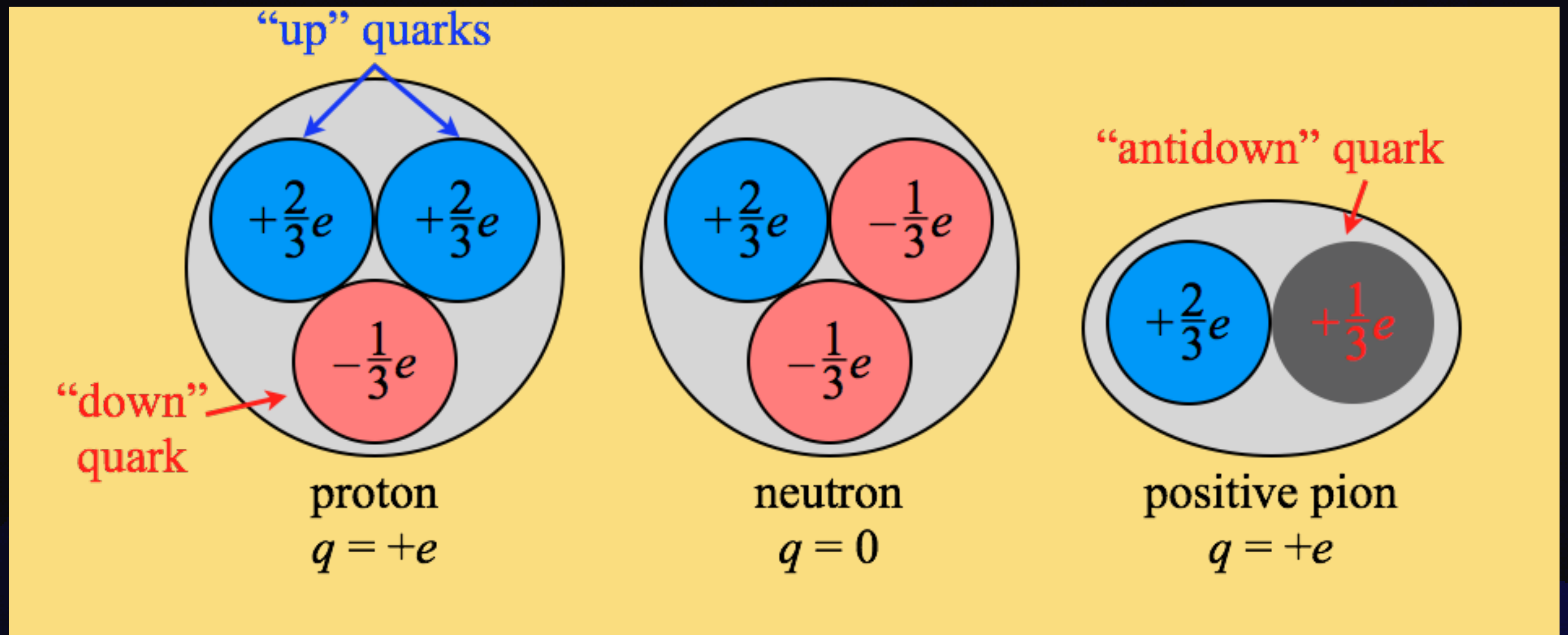
- Problem : differently from the hot and dilute case, no "exact" results at finite density → "Sign Problem". No Lattice guidance. Large theoretical uncertainties and limited predictive power.

- Current strategy :
  Use available effective quark models (MIT, NJL, DSM, FCM, CDM ...) in combination with the hadronic EoS.

- An important constraint : in symmetric matter phase transition not below $(2-3)\rho_0$ (no evidence of quark from HIC at intermediate energy).



The MIT Bag Model : the most popular approach to QM



Asymptotic freedom & Confinement

In the simplest fashion : bags of perturbative vacuum in a confining medium + eventual corrections $\approx \alpha_s$

- Asymptotic freedom : free quarks and gluons inside color singlet bags
- Confinement : vector current vanishing at the boundary

$$\epsilon_Q = B + \epsilon_{kin} + \alpha_S \times \ldots, B = 57.5 - 400 \text{ MeV fm}^{-3}$$

For avoiding the transition at 2-3 ρ_0 the MIT model requires a density-dependent bag "constant":

$$B(\rho) = B_\infty + (B_0 - B_\infty) exp[-\beta(\frac{\rho}{\rho_0})^2]$$

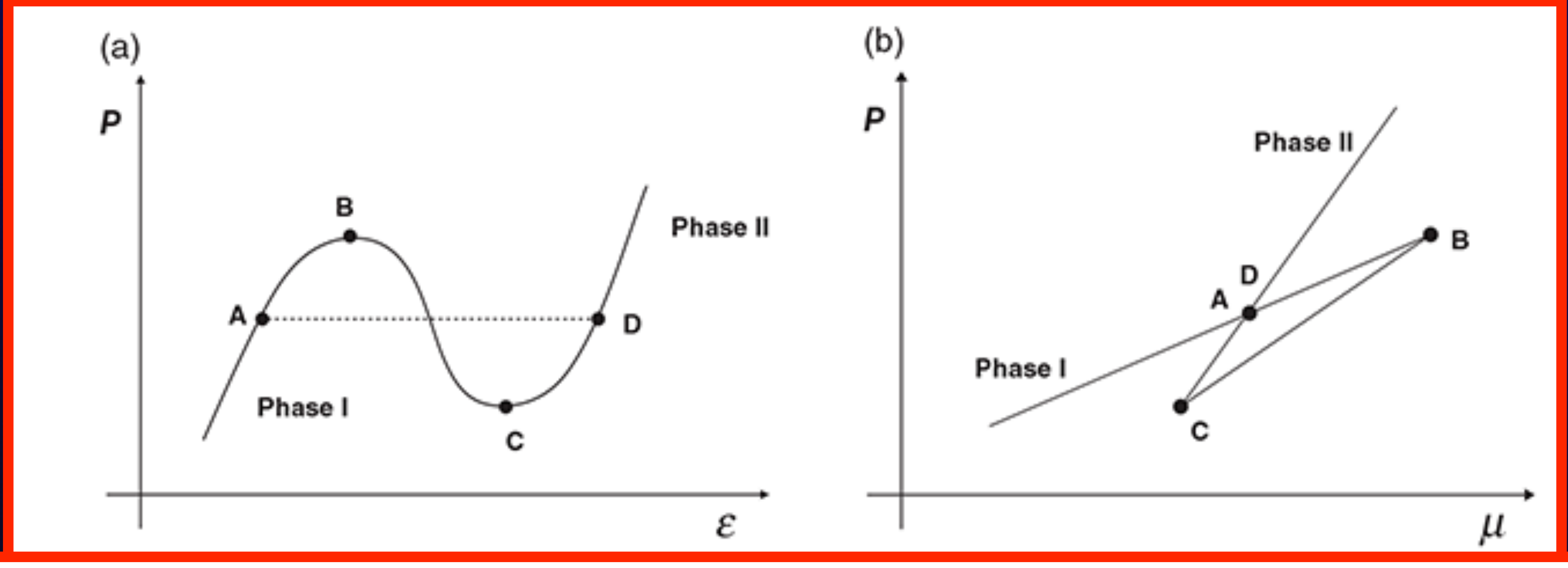
Combining Neutron and Quark Matter

Maxwell vs. Gibbs construction

The possibility of a phase transition in dense matter opens new features of the mass-radius relation, which can be formulated quite generically and model independently.

Conservation of the baryon number.

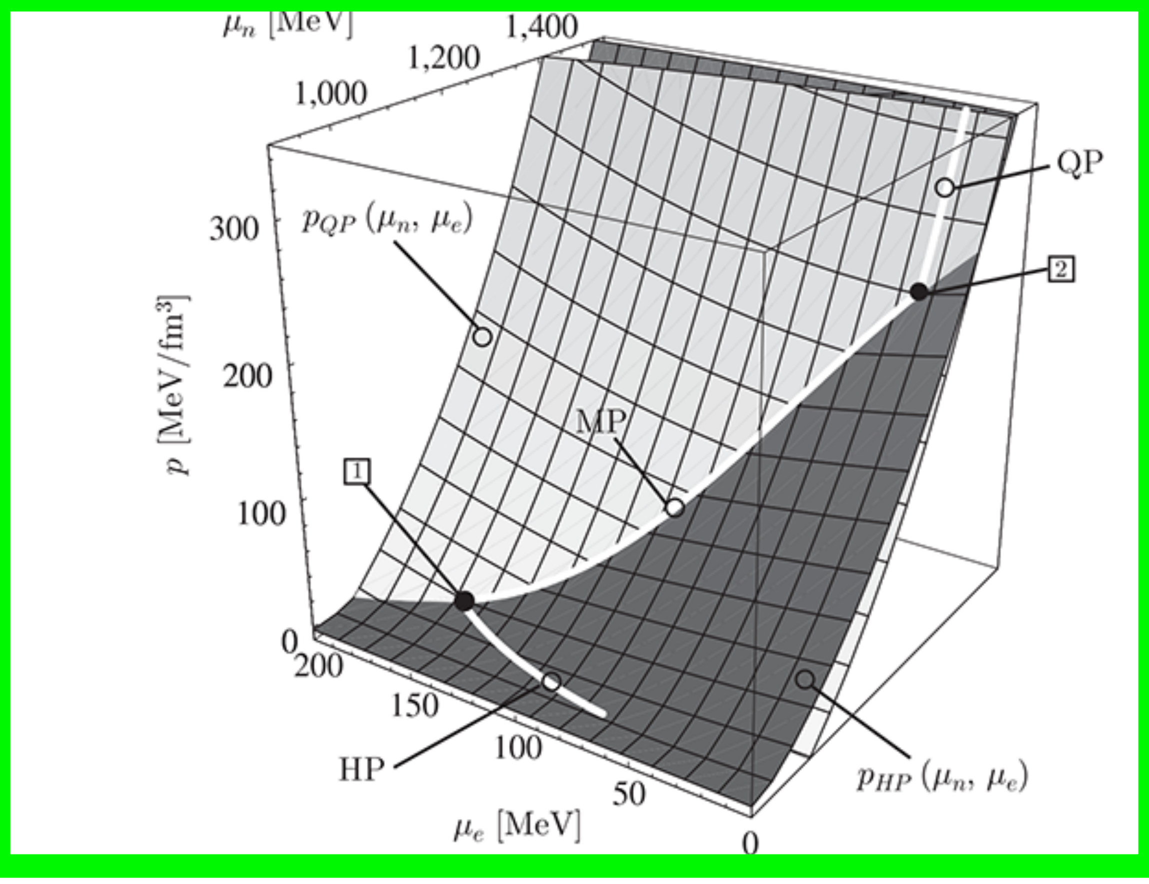
One chemical potential is required, μ_B



$$P_{\mathrm{I}}\left(\mu_B^{(\mathrm{I})}\right) = P_{\mathrm{II}}\left(\mu_B^{(\mathrm{II})}\right) \qquad \text{with} \qquad \mu_B^{(\mathrm{I})} = \mu_B^{(\mathrm{II})}.$$

Conservation of the baryon number and global charge..

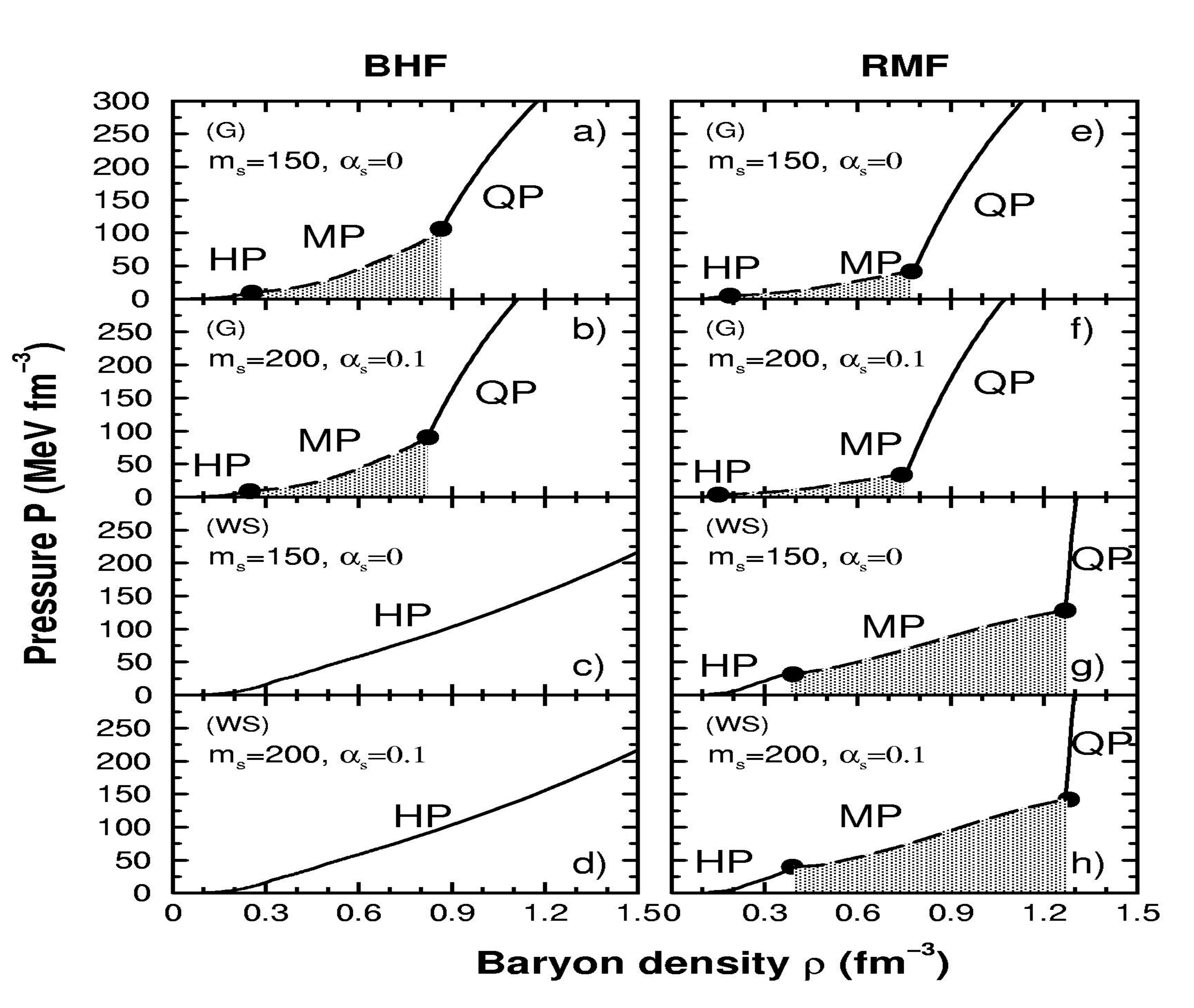
Two chemical potentials are required, μ_B and μ_e

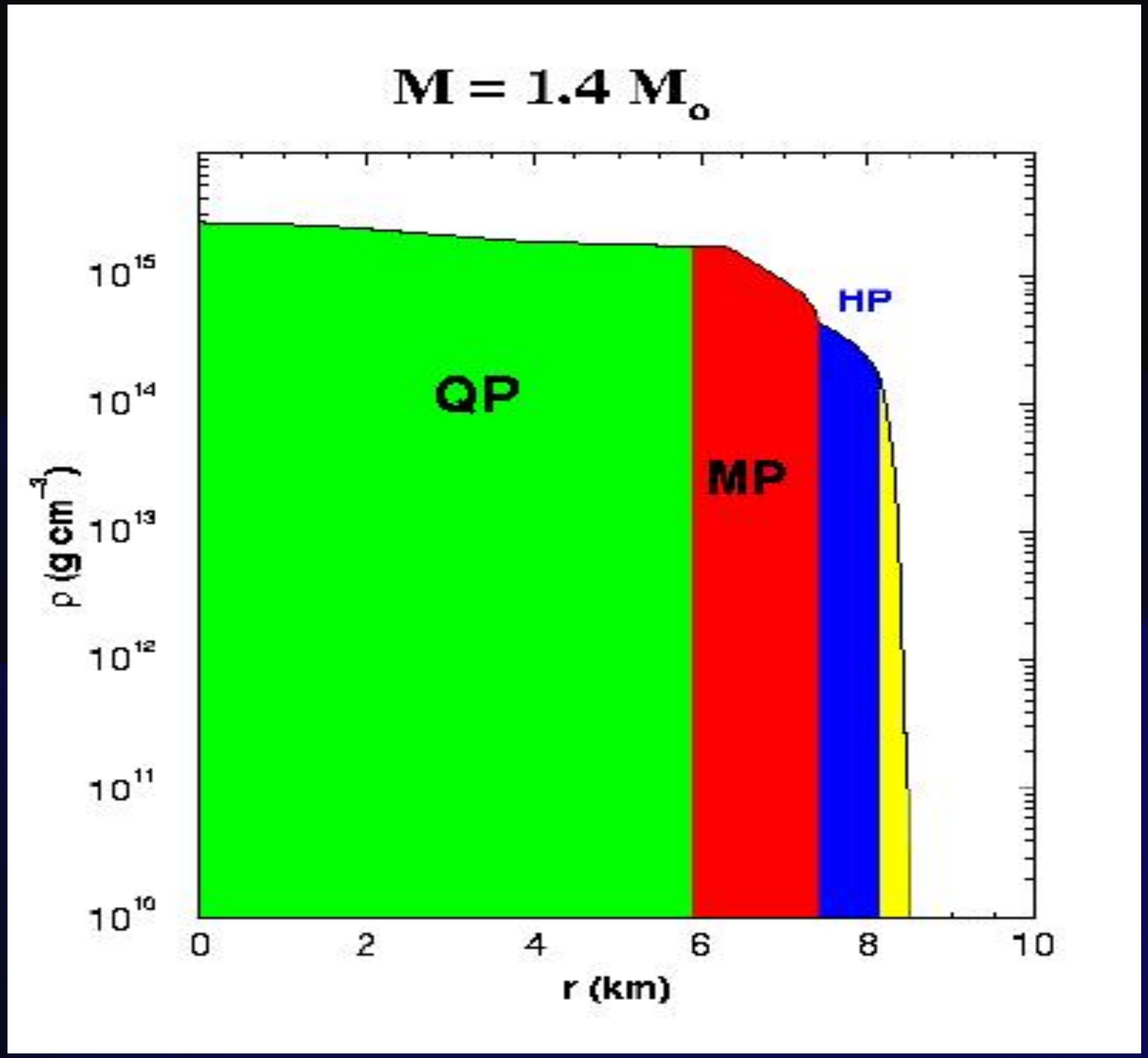


$$P_{\mathrm{I}}\left(\mu_B^{(\mathrm{I})}, \mu_Q^{(\mathrm{I})}\right) = P_{\mathrm{II}}\left(\mu_B^{(\mathrm{II})}, \mu_Q^{(\mathrm{II})}\right) \quad \text{with} \quad \mu_B^{(\mathrm{I})} = \mu_B^{(\mathrm{II})}, \quad \mu_Q^{(\mathrm{I})} = \mu_Q^{(\mathrm{II})}.$$

Glendenning, 1992

HQ phase transition with MIT bag model

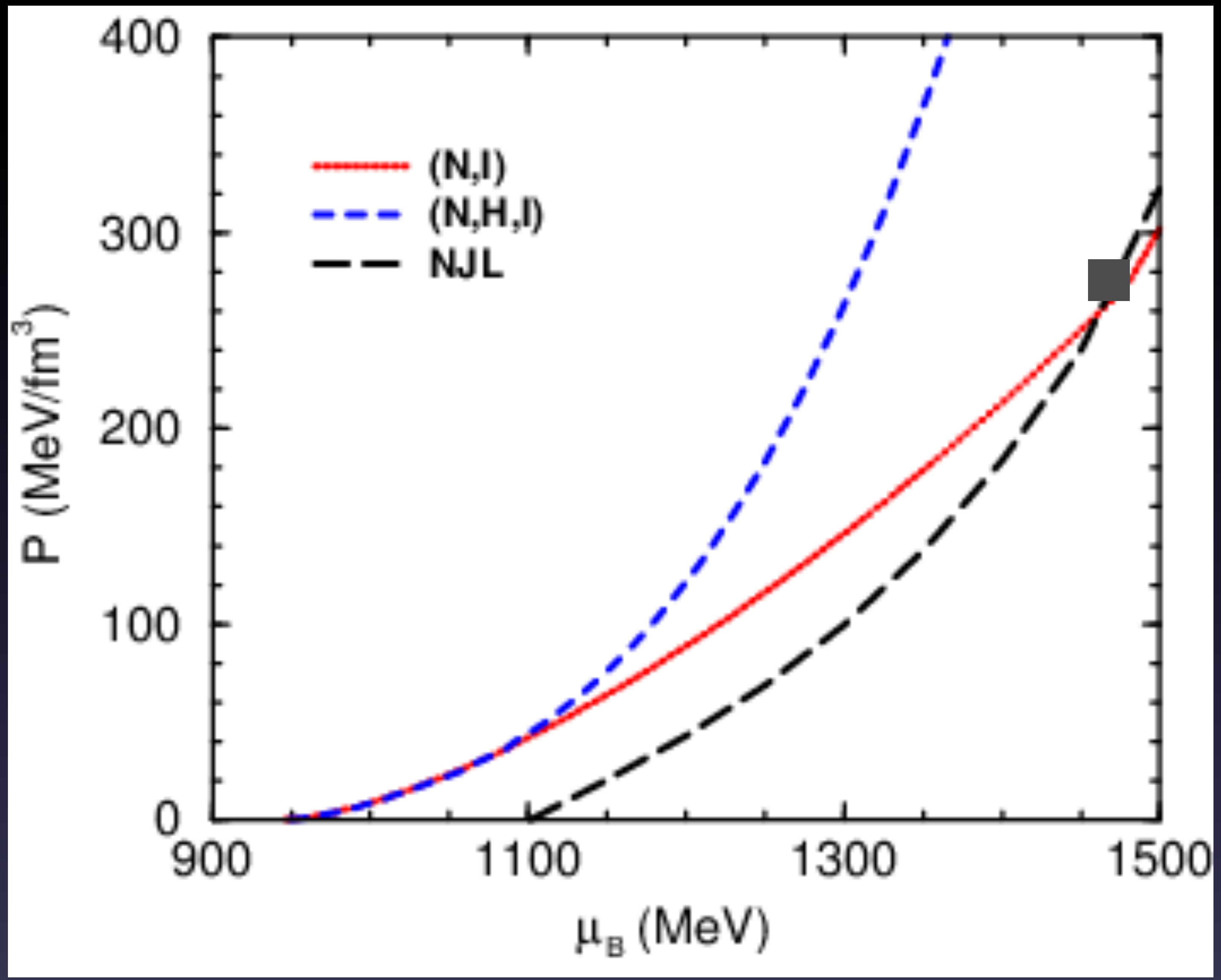


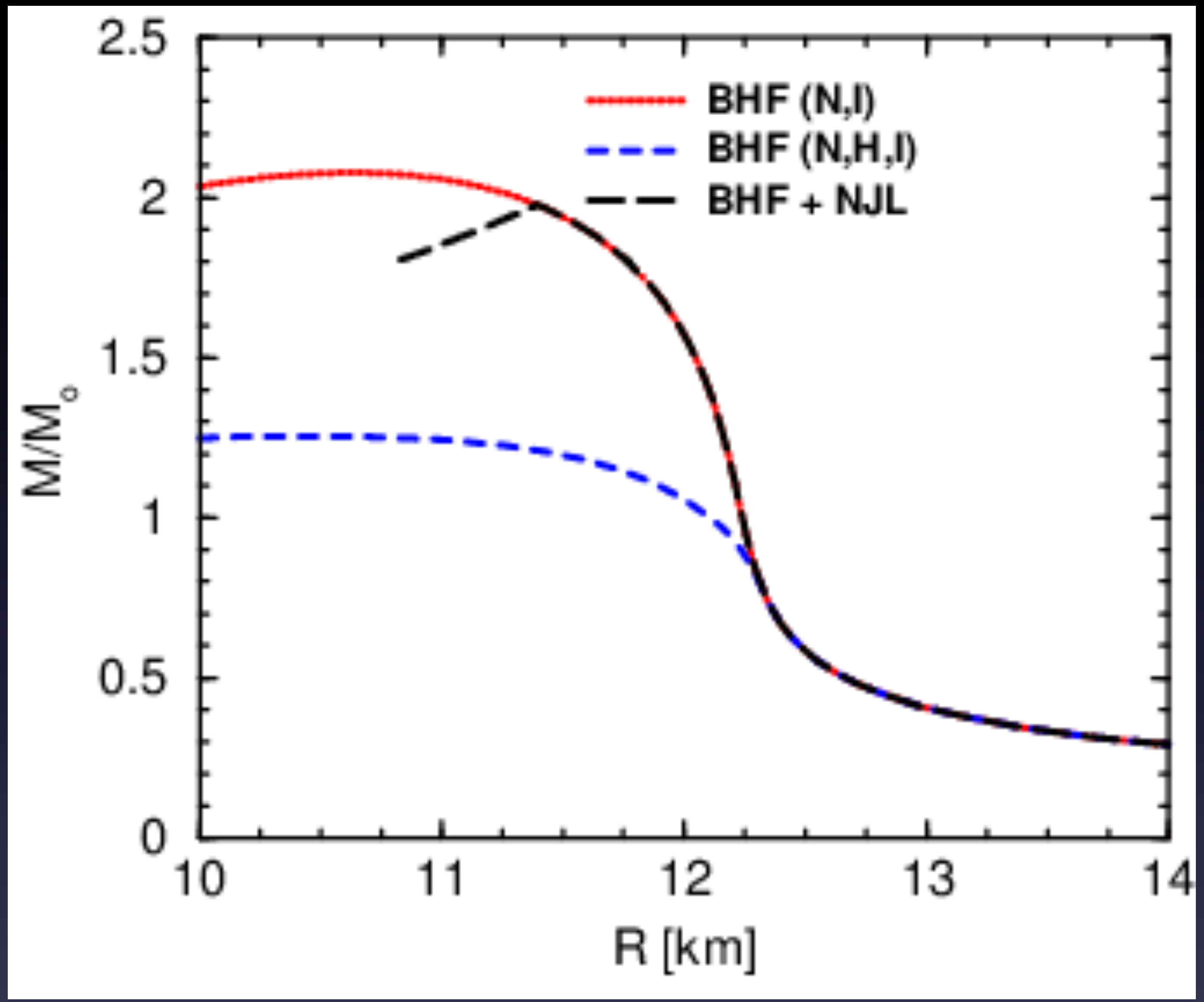


Different profiles of different phases

MIT bag model

BHF + NJL Model





No stable stars with pure quark phase do exist.
Radius above 11 km !

The observational data on the Neutron Stars masses can also be used to constrain the parameters that eventually appear in a microscopic model of the quark phase.

As an example we consider the Field Correlator Method

- It is able to cover the full T- μ plane
- Confinement is introduced ab initio through the QCD field correlators
- The phase transition depends on two parameters, i.e.

  the gluon condensate G_2 and the large distance static $q\bar{q}$ potential V_1

More in :

Di Giacomo, Dosch, Schevchenko, and Simonov, Phys. Rep. **372**, (2002) 319.
Simonov, Ann. Phys. **323**, (2008) 783.

FCM generalized at finite T and μ_B

Yu. Simonov, and M. Trusov, JETP Lett. 85 (2007) 598 ; PLB 650 (2007) 36.

$$P_{qg} = P_g + \sum_{j=u,d,s} P_q^j + \Delta\epsilon_{vac}$$

The quark pressure :

$$\nu = m_q/T$$

$$P_q/T^4 = \frac{1}{\pi^2}\left[\phi_\nu\left(\frac{\mu_q - V_1/2}{T}\right) + \phi_\nu\left(-\frac{\mu_q + V_1/2}{T}\right)\right]$$

$$\phi_\nu(a) = \int_0^\infty du \frac{u^4}{\sqrt{u^2+\nu^2}} \frac{1}{(\exp[\sqrt{u^2+\nu^2} - a] + 1)}$$

The gluon pressure :

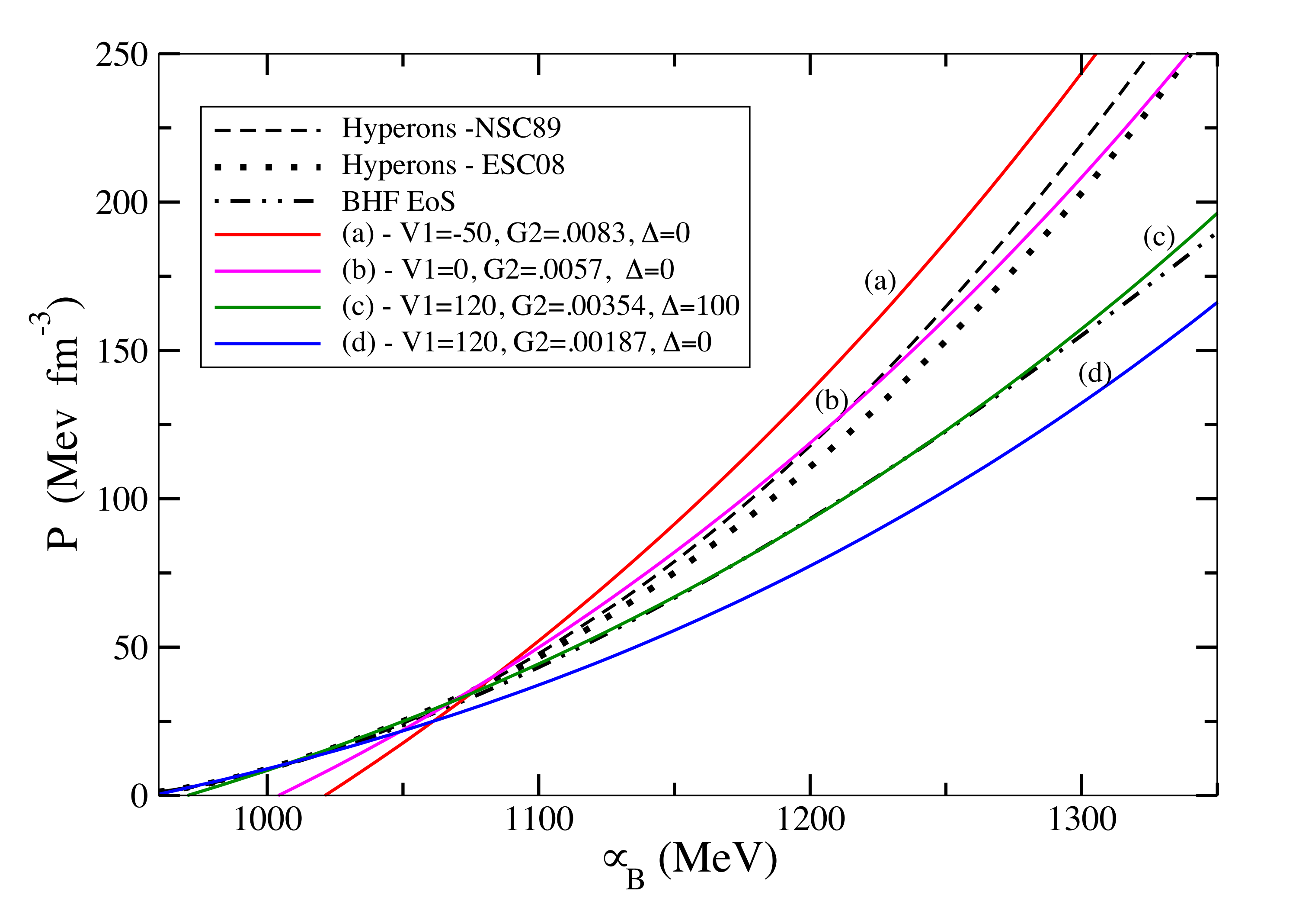
$$P_g/T^4 = \frac{8}{3\pi^2}\int_0^\infty d\chi \chi^3 \frac{1}{\exp(\chi + \frac{9V_1}{8T}) - 1}$$

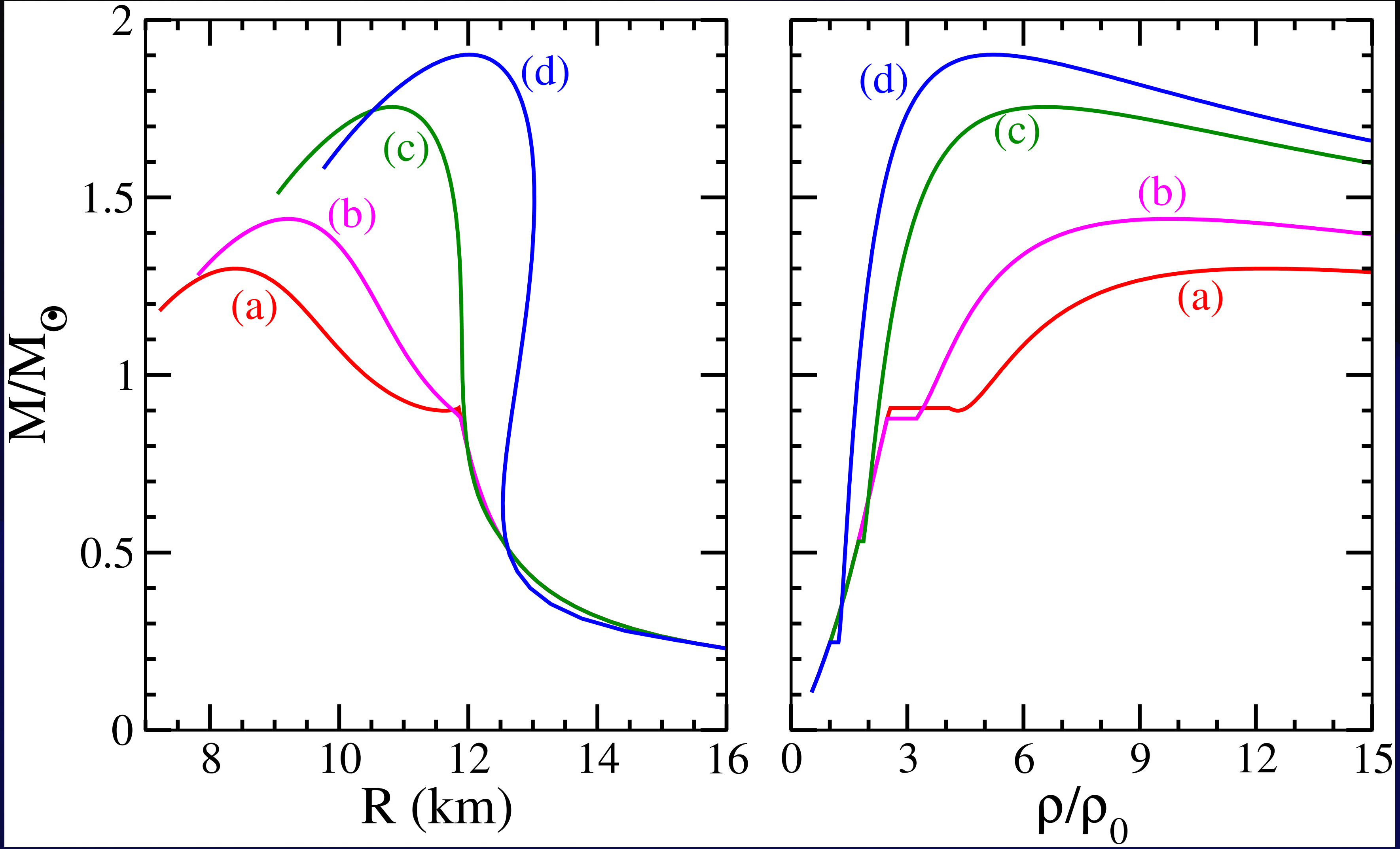
Eff. Bag constant :

$$\Delta\epsilon_{vac} \approx -\frac{(11 - \frac{2}{3}N_f)}{32}\frac{G_2}{2}$$

EoS expressed in terms of V_1 and G_2 only !

Phase transition with BHF+FCM





- Hyperons prevent the crossing in the P-μ plane.
- Maximum mass below the 2 solar mass.

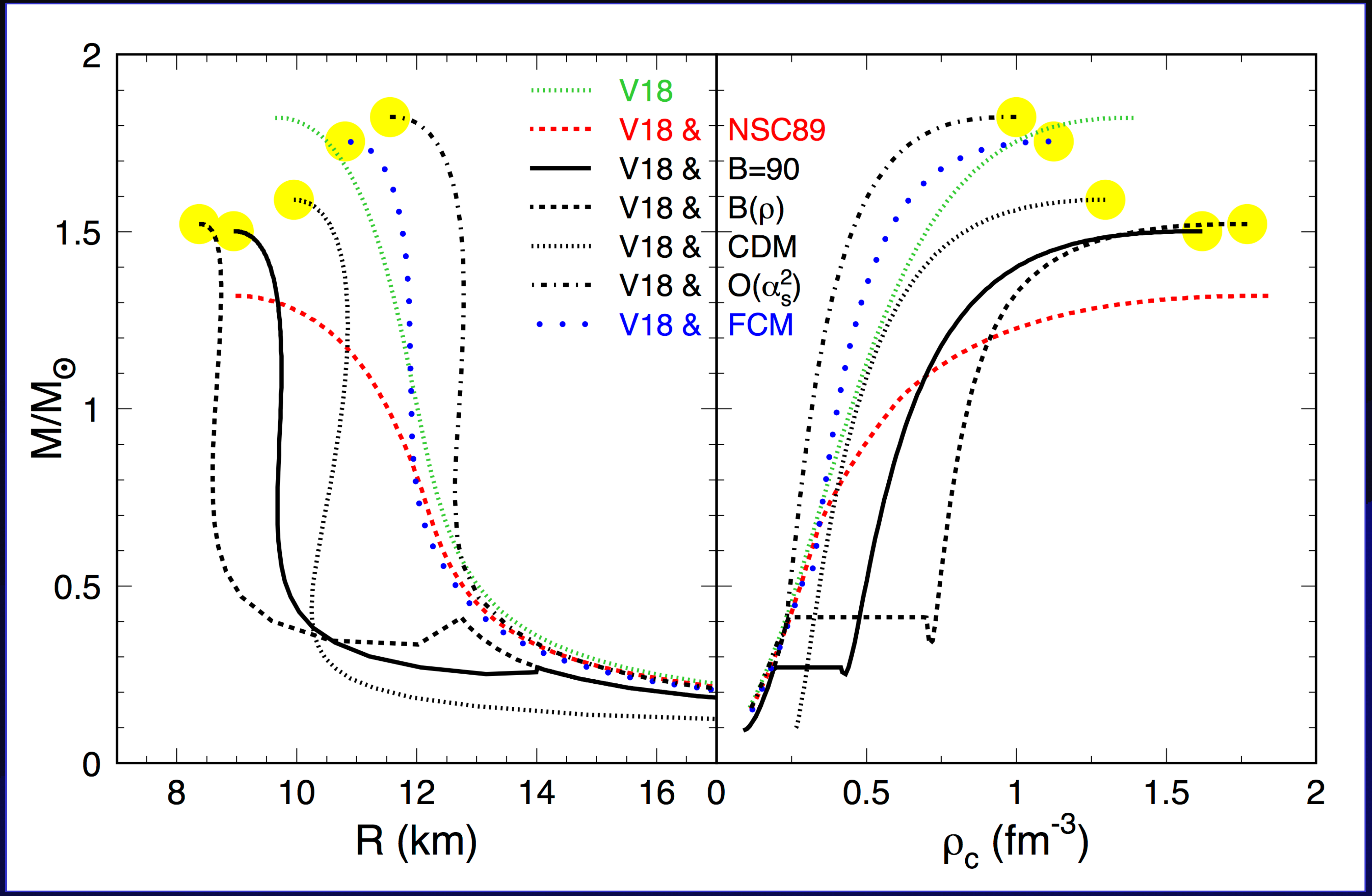
Stable configurations of hybrid stars

The value of the maximum mass lies in a range 1.5…1.9 $M_{\odot}$

Neutron stars with quark matter core have smaller radii than purely hadronic stars.

The value of the maximum mass is mainly determined by the quark component and relative EoS.

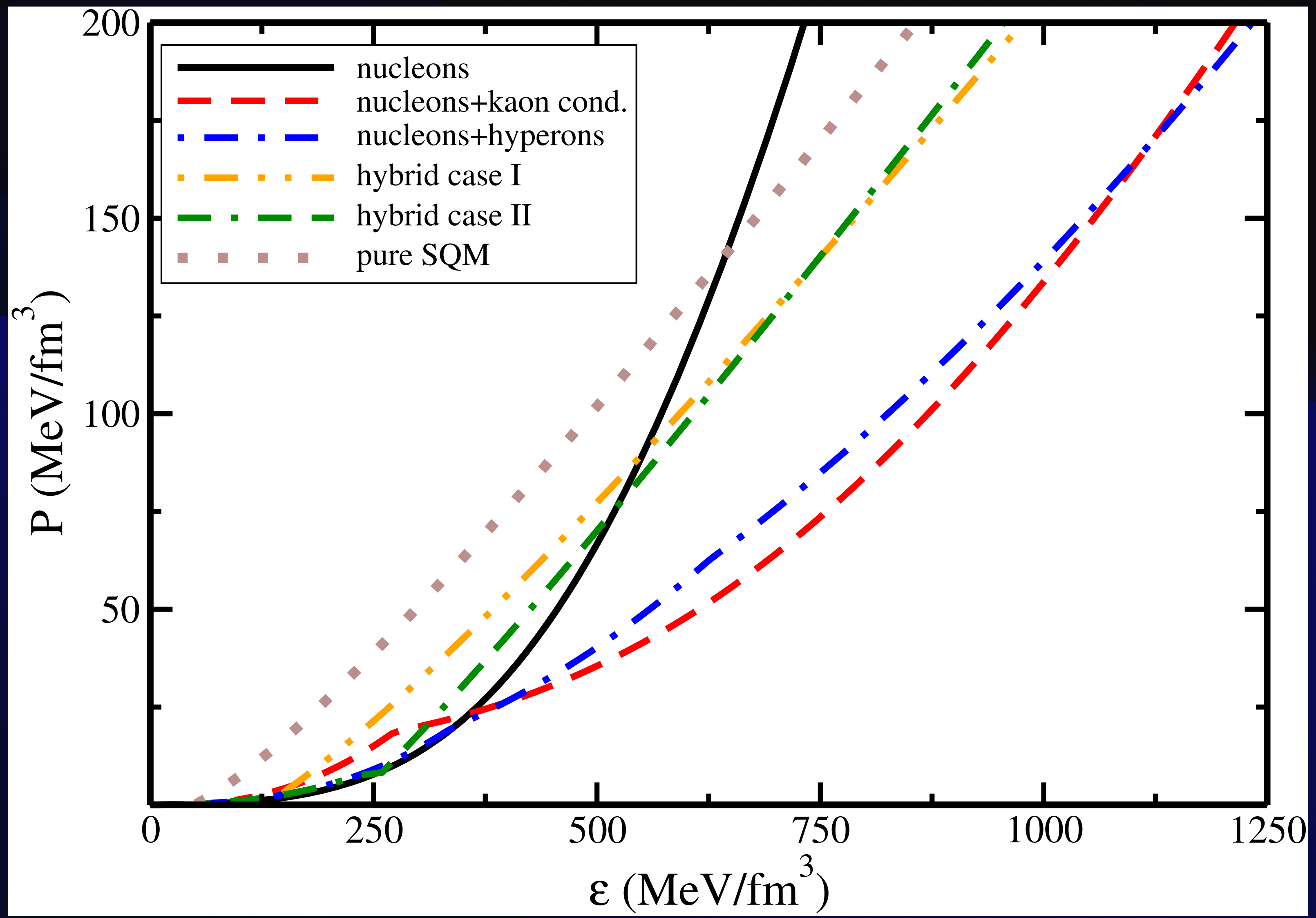
For some models (NJL, FCM and Dyson-Schwinger) hyperons prevent the phase transition.

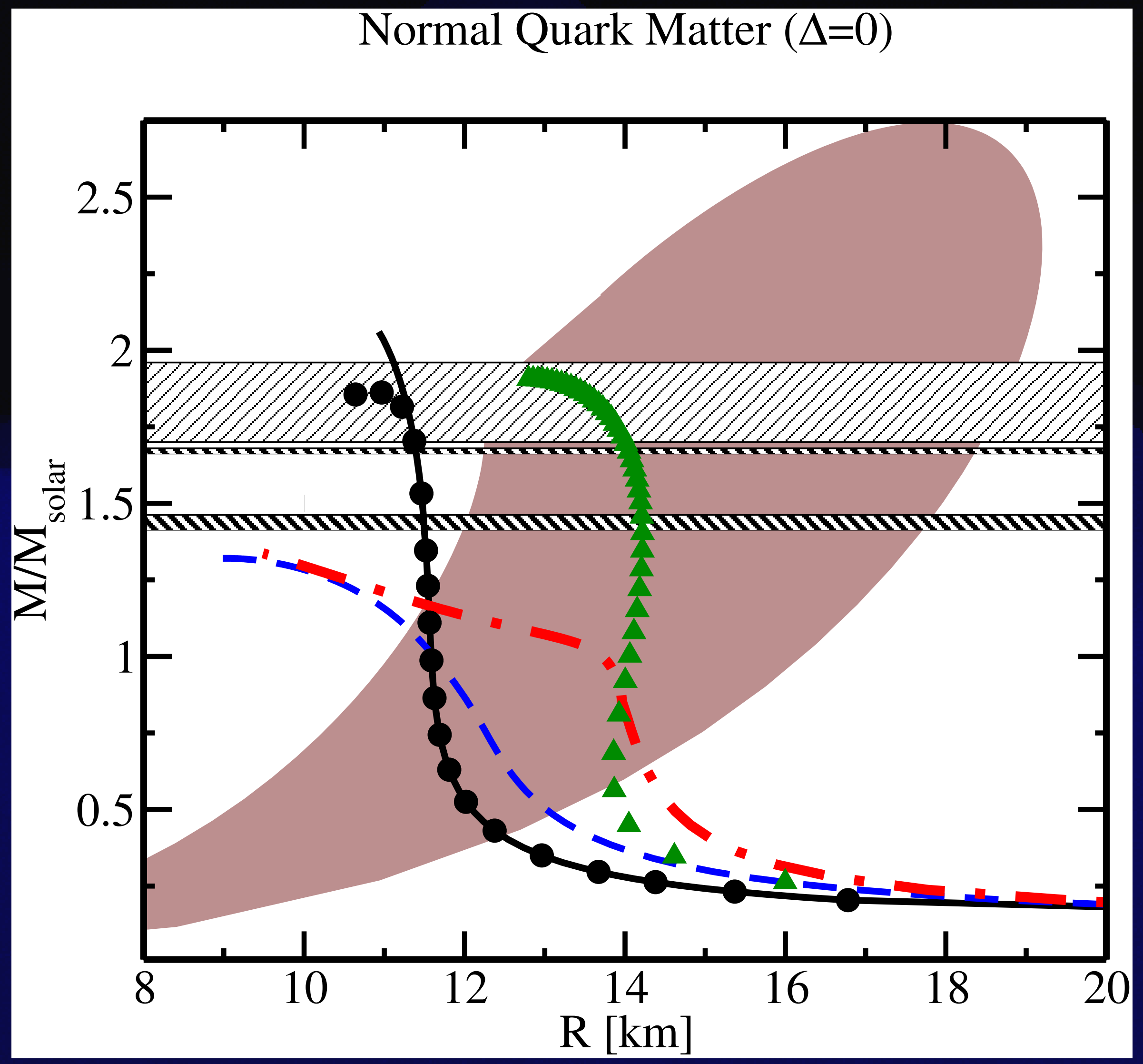


EoS and NS masses from state-of-the-art cold pQCD

Kurkela, Romatschke & Vuorinen (2010)

- $O(\alpha_s^2)$ perturbative calculation of the equation of state of cold but dense QCD matter
- perturbation theory converges reasonably well for quark chemical potentials above 1 GeV.





- Hyperons prevent the crossing in the P-μ plane.
- Maximum mass below the 2 solar mass.

Even including quark matter,
2 solar mass limit is not or hardly reached !
Additional repulsion required...

Back to Oppenheimer-Volkoff, 1939 !

Conclusions

- On the NN + NNN + NY level, the prediction of very low NS maximum masses is rather robust.
- Reliable YY, YNN, YYN, YYY forces are not available and will not be for coming decades (no exp. constraints).
- However, any single less repulsive channel will keep the maximum mass low, such that only simultaneous repulsion in all relevant YY,YNN,... channels could substantially increase the maximum mass.

Need quark matter to reach higher masses of hybrid stars !

A big theoretical challenge for the future.