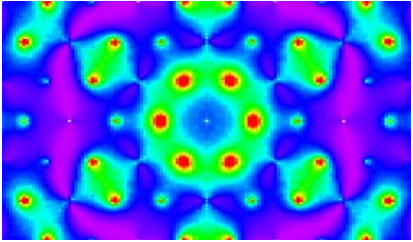
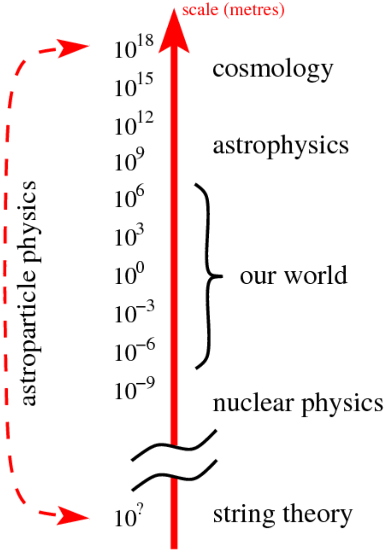
Emergent U(1) gauge field and SU(2) symmetry in an Ising magnet

Roderich Moessner



NIST

The physics landscape



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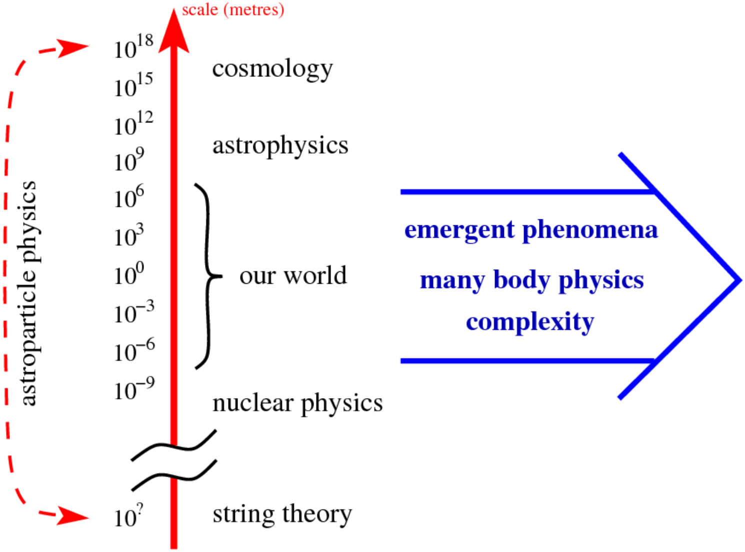
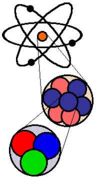


Fundamental questions



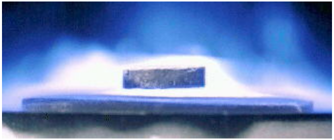
What are *building blocks and interactions* of matter?

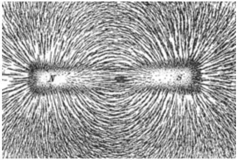
$\Rightarrow$ high energy + particle physics

What is the origin of variety and complexity

$\Rightarrow$ many-body theory:

- understand individual phenomena
  - $\Rightarrow$ ‘applications’
- understand variety as such
  - $\Rightarrow$ ‘organising principles’





Outline

Spin ice

- history (and material)
- frustration and degeneracy

Emergent Maxwell electromagnetism

- U(1) gauge field from constraint

Strings as degrees of freedom

- Kasteleyn transition in a field
- mapping to quantum problem
  - emergent SU(2) symmetry under strain

Gauge fields and strings

- magnetic monopoles and ‘Dirac strings’
- irrational charge

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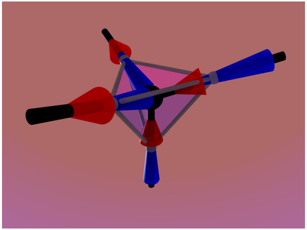
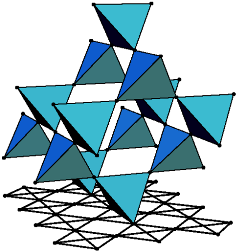
Geometrical Frustration in the Ferromagnetic Pyrochlore $Ho_2Ti_2O_7$

M. J. Harris,[1] S. T. Bramwell,[2] D. F. McMorrow,[3] T. Zeiske,[4] and K. W. Godfrey[5]
[1]*ISIS Facility, Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom*
[2]*Department of Chemistry, University College London, 20 Gordon Street, London, WC1H 0AJ, United Kingdom*

Spin ice compounds $Dy/Ho_2Ti_2O_7$

- local [111] crystal field ~ 200 K
- $\Rightarrow$ Ising spins $\sigma = \pm 1$
- large classical spins (15/2 and 8)
- large magnetic moment $|\vec{\mu}| \approx 10\,\mu_B$

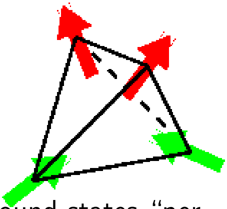




Frustration leads to (classical) degeneracy

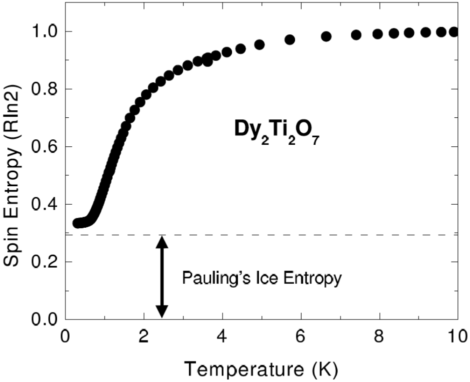
(exchange+dipolar) interactions minimised by
2-in, 2-out ice rules $\Rightarrow$ <u>local constraint</u>

Siddharthan+Shastry 1999, Gingras *et al.* 2000^{+}



six ground states "per tetrahedron" $\Rightarrow$ <u>degeneracy</u>
nonzero residual entropy
$S_p = \ln 2 - \int_{T_0}^{\infty} (C/T) dT$

Anderson 1956; Ramirez *et al.* 1999

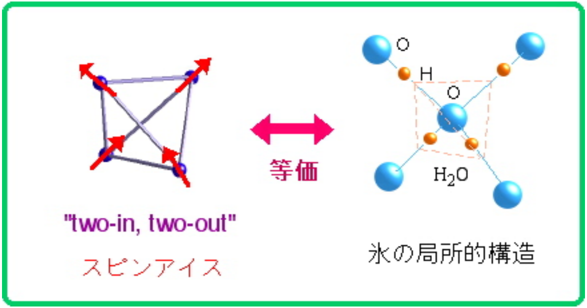


Pauling's estimate: $\mathcal{S}_p = \frac{1}{2} \ln \frac{3}{2}$

- discrete version of Maxwellian constraint counting

Mapping from ice to spin ice

- In ice, water molecules retain their identity
- Hydrogen near oxygen $\leftrightarrow$ spin pointing in



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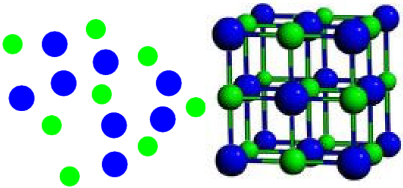
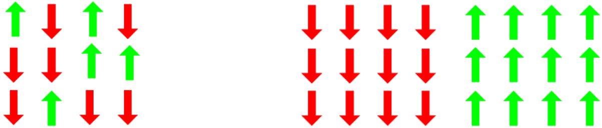
Conventional order and disorder

Gas-crystal (e.g. rock salt):



Paramagnet-ferromagnet (e.g. fridge magnet)



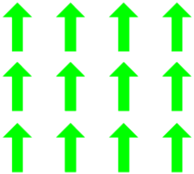
In between: critical points

Anything else???

Is spin ice ordered or not? Henley; Huse et al.; Hermele et al.

No order as in ferromagnet

- extensive degeneracy



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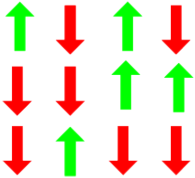

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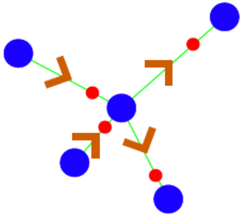

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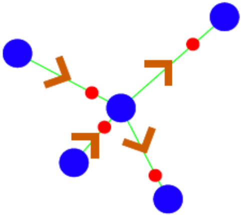
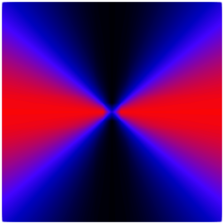


Magnetic moments $\vec{\mu}_i \Leftrightarrow$ (lattice) 'flux'

- Ice rules $\Leftrightarrow \nabla \cdot \vec{\mu} = 0 \;\Rightarrow\; \vec{\mu} = \nabla \times \vec{A}$
- Local constraint
  $\Rightarrow$ emergent gauge structure
  $\rightarrow$ algebraic spin correlations
  $\rightarrow$ 'bow-tie' structure factor

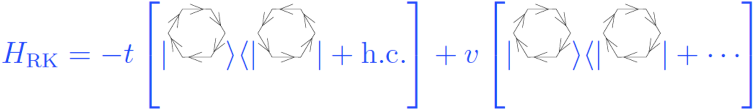


Effective action: $\mathcal{S} = (K/2) \int d^3 r |\nabla \times \vec{A}|^2$

Emergent Maxwell electromagnetism + photons

Hilbert space: classical ground states of spin ice

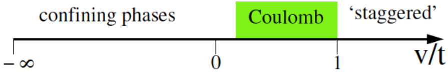
- add quantum dynamics: hexagonal loop resonance



Effective long-wavelength theory: $\mathcal{S}_q = \int \vec{E}^2 - \vec{B}^2$ Maxwell

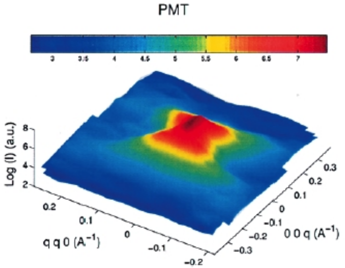
Coulomb phase of U(1) gauge theory

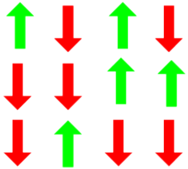
- gapless photons, speed of light $c^2 \sim t - v$
- deconfinement

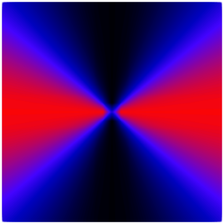


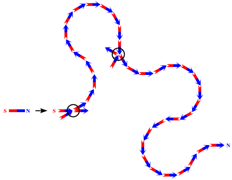
Emergent electrodynamics with frustrated system as 'ether'

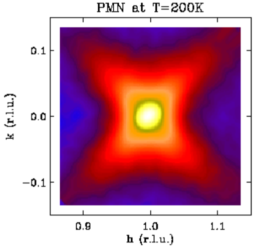
Disorder vs. spin ice vs. order in neutron scattering

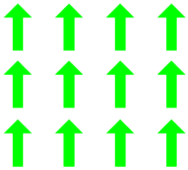




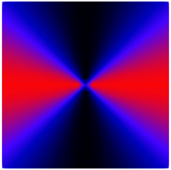






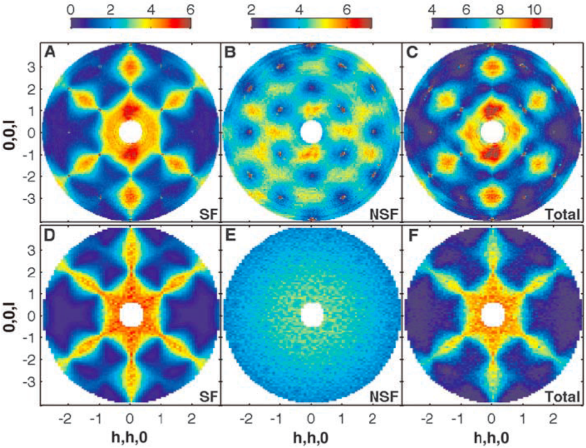


Pinch points in neutron scattering



Isakov, RM, Sondhi 2004

Tom Fennell



Fennell+Bramwell *et al.* 2009

Kasteleyn transition out of full polarisation

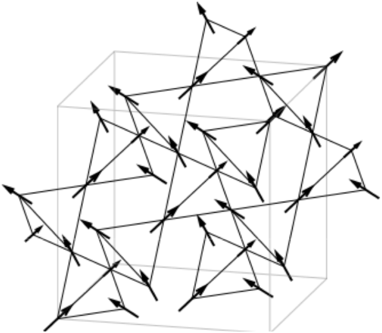
Applying [100] field gives polarised reference configuration

- defect motion $\rightarrow$ string
- strings execute random walk transverse to field cf. Chalker
  - Zeeman energy per step E_z
  - entropy per step $\ln 2$

If strings cannot terminate

- $\mathcal{F} = L\,(E_z/T - \ln 2)$

NO strings at $T < T_c = E_z/\ln 2$



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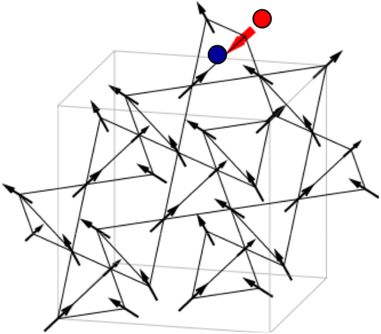

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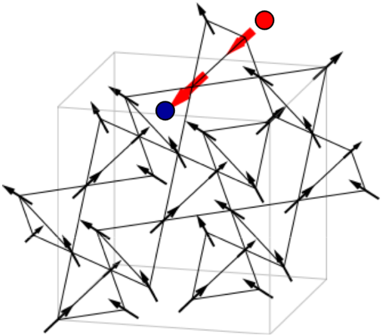

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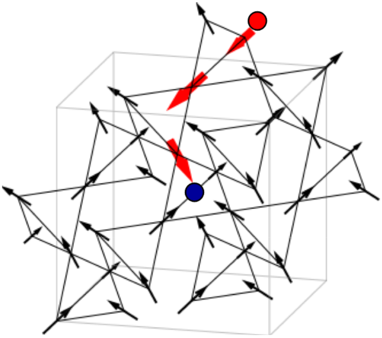

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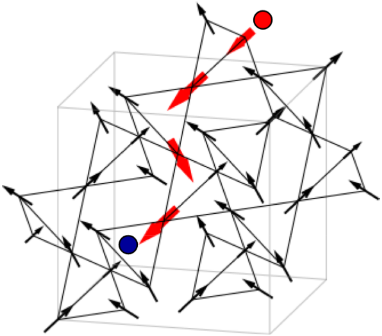

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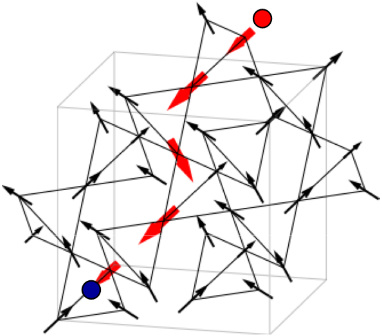

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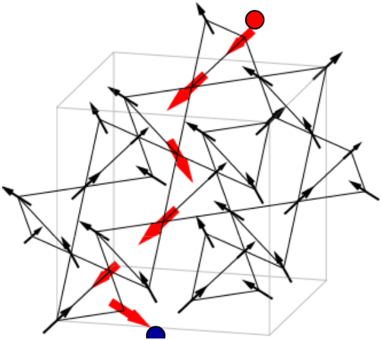

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If strings cannot terminate

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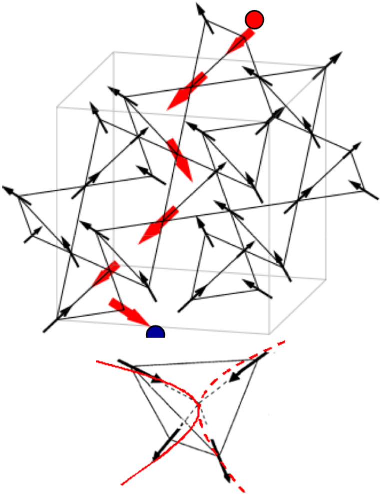
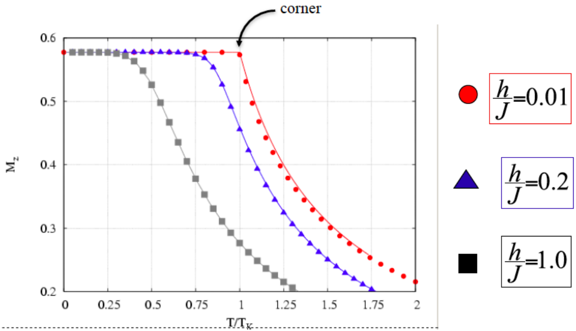
- strings repel entropically
  - continuous onset
  - Kasteleyn transition



Kasteleyn transition

Familiar from other settings

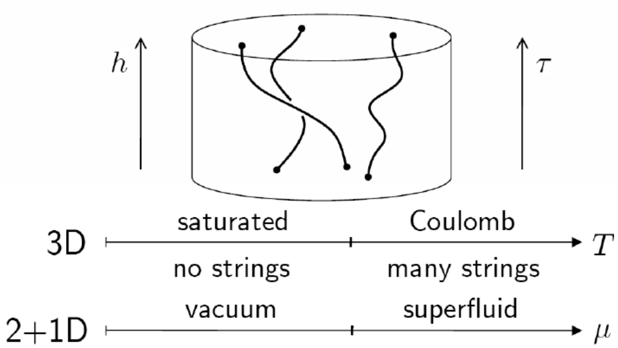
- commensurate/incommensurate transition
- dilute Bose gas



Classical to quantum mapping

Interpret strings as world lines

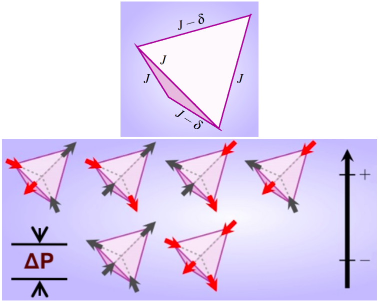
- field picks out 'imaginary time' direction
- provides critical theory



Strained spin ice: Ising symmetry

Six ice states split energetically

- 0 and 2 strings degenerate
- Ising symmetry survives



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Same energy/entropy argument

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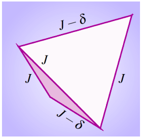
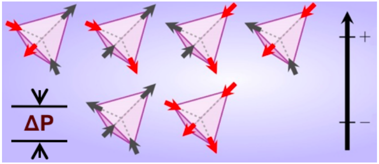
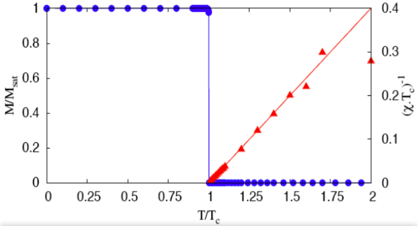
- looks first order







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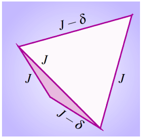
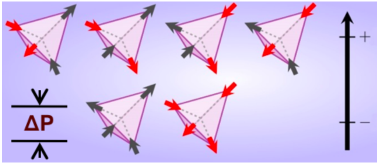
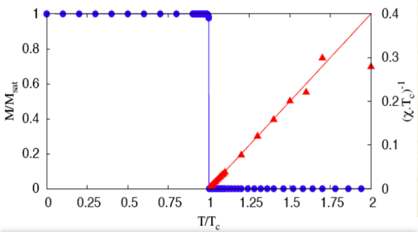
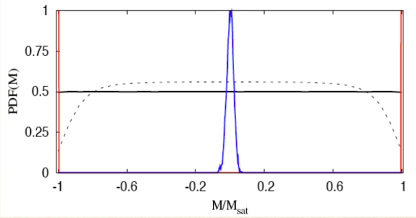
Same energy/entropy argument

- but strings no longer interact
- looks first order
  - but is not: "∞"-order
  - all sectors equiprobable









Emergent SU(2) symmetry

'Transfer matrix' from layer to layer in strain direction

- dominant eigenvalue independent of number n of strings

Corresponds to imaginary time Trotterised quantum Hamiltonian

$$H_{XXZ} = -J \sum_{\langle ij \rangle} s_i^x s_j^x + s_i^y s_j^y + \Delta s_i^z s_j^z$$

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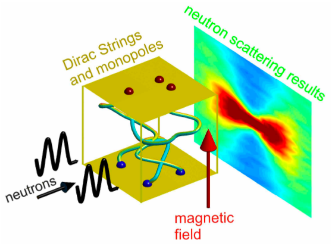
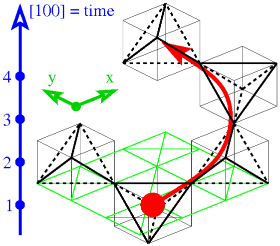
Phase transition at $\Delta = 1$

- H'berg: SU(2) symmetry
  - transition from Ising to XY
  - degeneracy between $N+1$ values of S^z_{tot}
- exhibited by full transfer matrix
  - commutes with $S^{\pm}_{tot}$
- many unusual consequences
  - soft domain walls: $l_W^{-1} \sim \sqrt{1 - T/T_c}$
  - 'random walk' correlations: $C(r,z) = (1/z)\exp\left[-r^2/(\rho z)\right]$

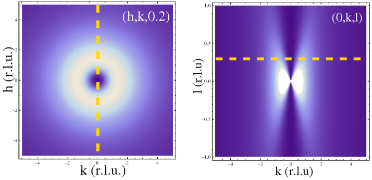
Imaging ‘Dirac strings’

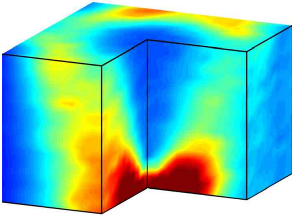


⇔



⇒ random walk in 2 dimensions + time



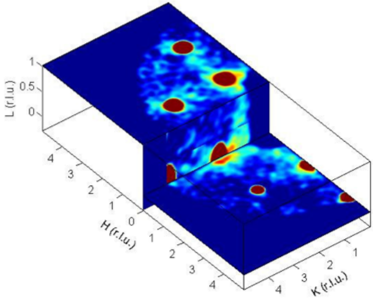


H in the [001] direction

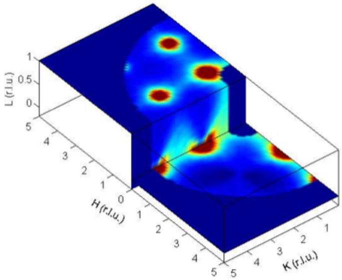
Dirac strings in neutron scattering Morris et al. 2009

Neutrons in fields of order 1T HZB–Tennant group

- compared to random-walk model



Data

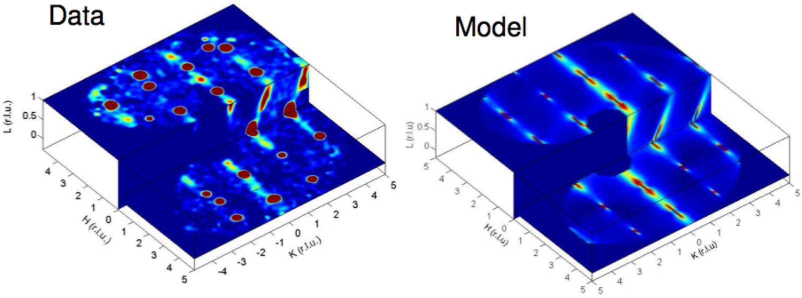


Model

Dirac strings in neutron scattering Morris et al. 2009

Neutrons in fields of order 1T HZB–Tennant group

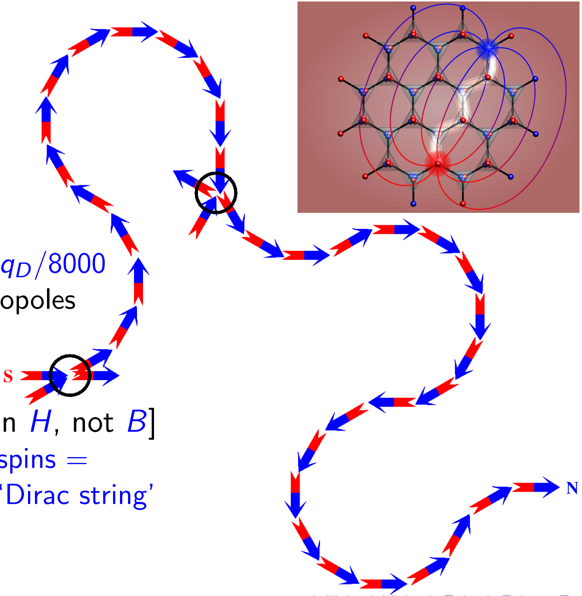
- compared to random-walk model
- tilted field: biased random walk



‘Dirac strings’ and emergent magnetic monopoles

magnetic Coulomb interaction

$$E(r) = -\frac{\mu_0}{4\pi}\frac{q_m^2}{r}$$



- $q_m = 2|\vec{\mu}|/a_d \approx q_D/8000$
- **deconfined** monopoles

[monopoles in H, not B]

flipped spins =
(observable) ‘Dirac string’

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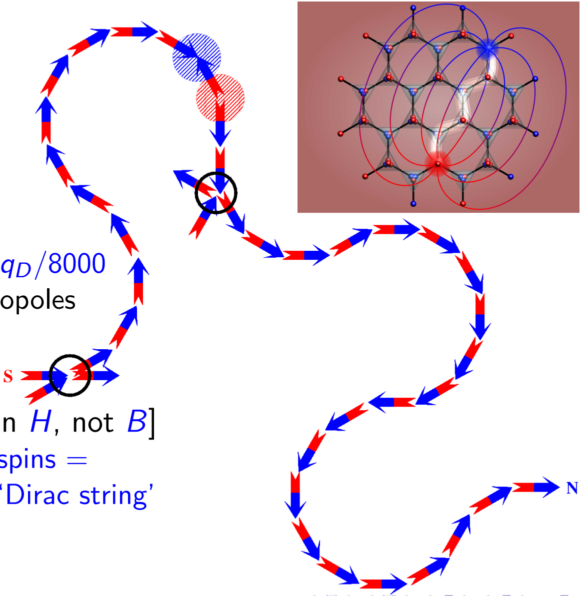

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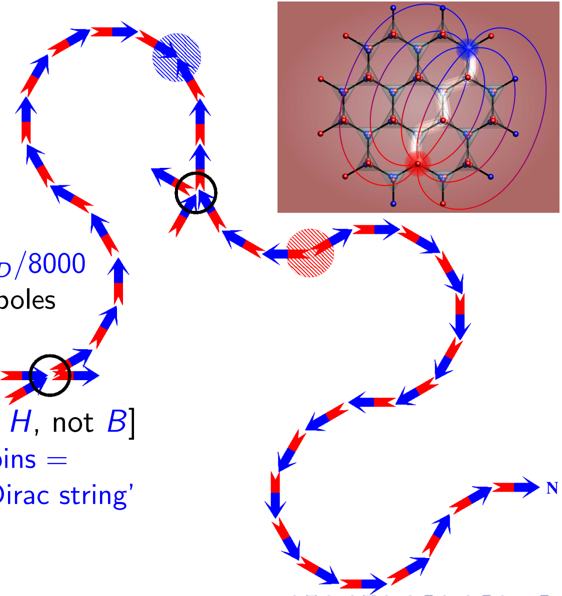
‘Dirac strings’ and emergent magnetic monopoles



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[monopoles in H, not B]

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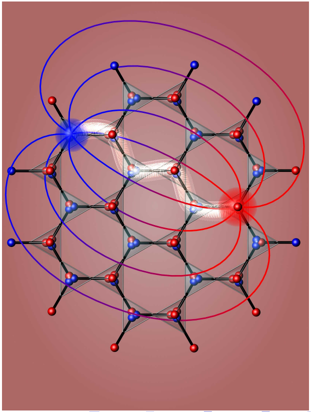
(observable) ‘Dirac string’

Monopole charge from inverting dipole string

$$V(r) = \frac{|\vec{\mu}|}{a} \int_\Lambda d\vec{r'} \cdot \vec{\nabla} \frac{1}{|r - r'|} = q_m \left(\frac{1}{|r - r_a|} - \frac{1}{|r - r_b|} \right)$$

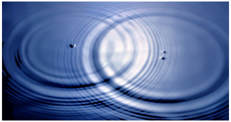
Potential due to a string of dipoles

- same as charges at ends of string
- charge $q_m = |\vec{\mu}|/a =$ moment per unit length of string
- reversing string of dipoles creates (tunable irrational) charges
- fractionalisation/deconfinement



Emergent versus intrinsic gauge charge

Emergence of qualitatively new degrees of freedom is common phenomenon



- low-energy d.o.f. $\neq$ high energy d.o.f.

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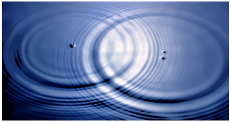
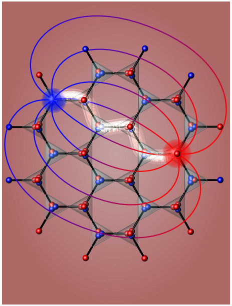


Here: emergent d.o.f. is gauge field

- bow-ties in neutron scattering



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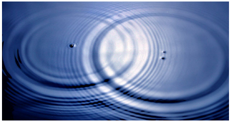
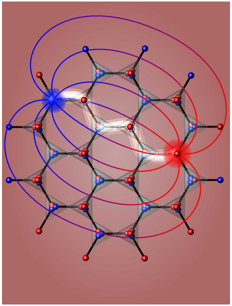
But: we also have high-energy gauge structure

- magnetic dipole moment of spins
- 'intrinsic' magnetic charge of monopole





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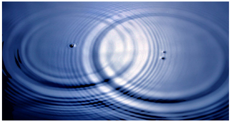

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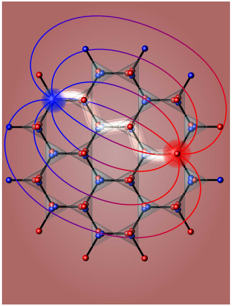

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Emergent and intrinsic gauge charges are

- distinct
- (partially) independent

Emergent versus intrinsic gauge charge

Emergence of qualitatively new degrees of freedom is common phenomenon

- low-energy d.o.f. $\neq$ high energy d.o.f.

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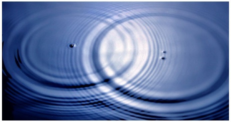
- bow-ties in neutron scattering

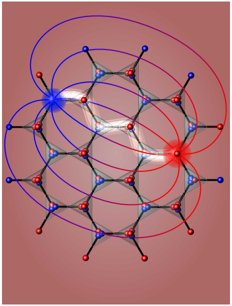
But: we also have high-energy gauge structure

- magnetic dipole moment of spins
- 'intrinsic' magnetic charge of monopole

Emergent and intrinsic gauge charges are

- distinct but mathematically identical
- (partially) independent





Dimensional reduction of emergent gauge theory

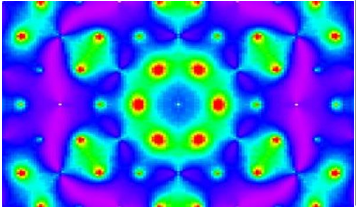
[111] field pins spins in triangular layer

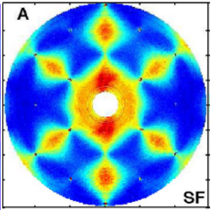
Effective action in $d = 2$ vs. $d = 3$:

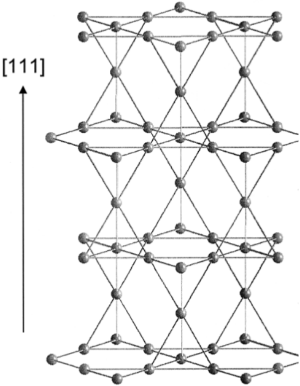
$$3d\ :\ \mathcal{S} = (K/2)\int d^3r|\nabla \times \vec{A}|^2$$

$$2d\ :\ \mathcal{S} = (K/2)\int d^2r|\nabla \times h|^2 + \lambda\cos(2\pi h)$$

$\Rightarrow$kagome ice







Kadowaki *et al.* 2009

Fennell *et al.* 2009

Additional terms permitted in $2d$ RM+Sondhi 2003

$\Rightarrow$ additional peaks in structure factor

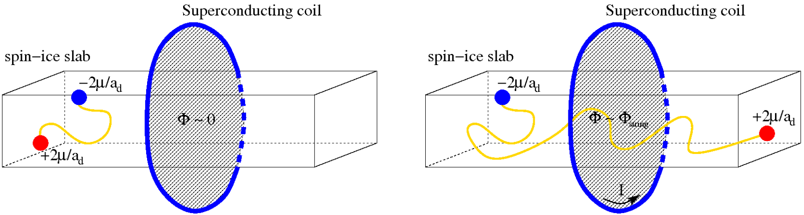
magnetic interaction remains $3d$

Single monopole search: Stanford experiment Cabrera 1982

Monopole passes through superconducting ring

$\Rightarrow$ magnetic flux through ring changes

$\Rightarrow$ e.m.f. induced in the ring $\Rightarrow$ countercurrent $\propto q_m$ is set up

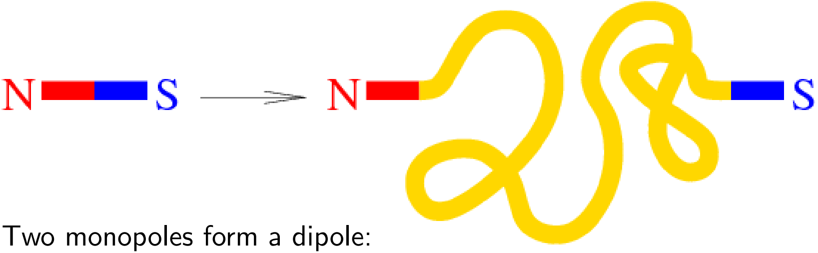


- 'Works' for both fundamental cosmic and spin ice monopoles
- signal-noise ratio a problem

Intuitive picture for monopoles

Simplest picture does not work: disconnect monopoles

Next best thing: no string tension between monopoles:



Two monopoles form a dipole:

- connected by tensionless 'Dirac string'
- Dirac string is observable

$\Rightarrow$ $q_m \approx q_D/8000$ not in conflict with quantisation of e

Loops and strings/worms in the ice model

Corner-sharing square/tetrahedra

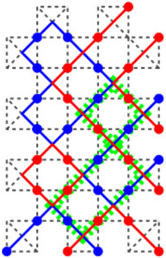
- Ising spins as basic d.o.f.

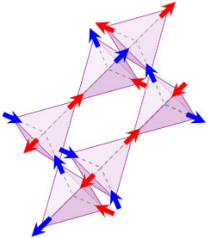
Each square/tetrahedral unit

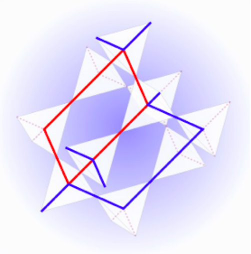
- two up/two down spins
- realises six-vertex model

Two red and two blue sites each

- strings = alternating red/blue
  - emergent gauge flux = spins
- adjacent red (blue) spins form red (blue) loops
  - fully-packed two-color loop model Kondev+Henley







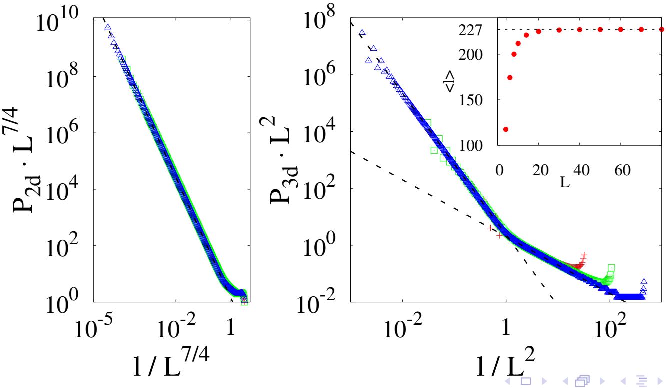
Statistics of strings in spin ice Jacobsen 90s; Jaubert, Haque, RM 2011

Algebraic length distribution, finite average length (24 vs. 227)

- 2d Kondev vs. 3d are different: two populations in 3d cf. random walk

Different effective descriptions

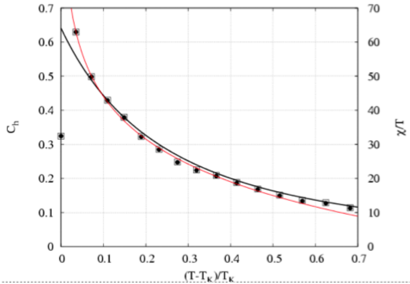
- 2d critical percolation; 3d Brownian motion
  - topological phase!



Use for numerical simulations Newman+Barkema; Gingras et al; Isakov et al; ...

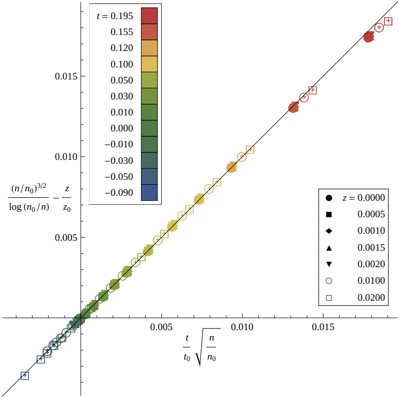
Algorithm flips worms – weighted by length of worm

- in $d = 3$, each MC move flips finite fraction of sample
- can simulate unconventional phase transition very accurately
  - log-corrections at upper critical dim. of Kasteleyn transition



$$\frac{t}{t_0}\left(\frac{n}{n_0}\right)^{1/2} = \frac{1}{\ln(n_0/n)}\left(\frac{n}{n_0}\right)^{3/2} - \frac{z}{z_0}$$

Powell, unpub (2012)



Collective behaviour: magnetic Coulomb liquid

Debye-Hückel theory for low temperatures CMS 2008

- sparse charges without strings
- screening of Coulomb interaction

'Magnetolyte' chemistry + 'magnetricity' Bramwell et al. 2009

- Wien effect: nonequilibrium response to changing field
- transient magnetic currents in response to field steps

[111] magnetic field = chemical potential CMS 2008

- liquid gas transition
- dimensional reduction to 2d

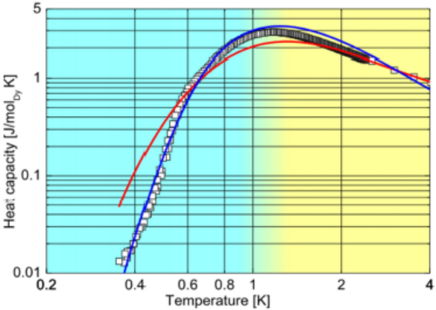
Specific heat of magnetic Coulomb liquid

- Debye-Hückel theory of monopole gas (blue)

  (no free parameters!)

- Bethe lattice calculation (red)

  (tuning J_{eff} to fit the data)



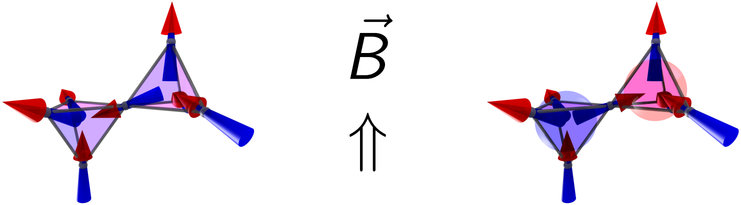
expt by Grigera/Tennant groups 2009

Interacting Coulomb liquid

point-like charged excitations + magnetic Coulomb interaction

(i) interaction strength $\Gamma \propto (q_m^2/\langle r \rangle)/T \sim \exp[-\Delta/T]/T$
vanishes at high and low T

(ii) [111] magnetic field acts as chemical potential
$\Rightarrow$ can tune $\langle r \rangle$ and T separately



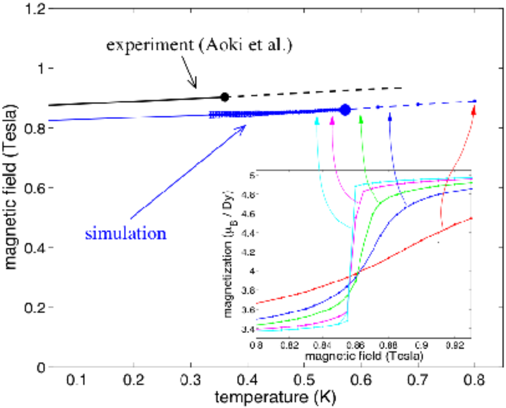
Liquid-gas transition in a [111] field CMS 2008

- first-order transition with critical endpoint — Fisher *et al.*

- observed experimentally
  Sakakibara+Maeno

  "*unprecedented* in localized spin systems"

- confirmed numerically



The Wien effect in a ‘magnetolyte’

Bramwell et al. 2009

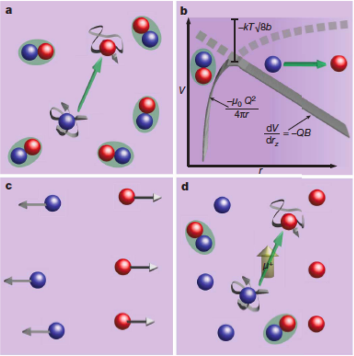
Double equilibrium: vacuum $\leftrightarrow$ bound monopoles $\leftrightarrow$ free monopoles

- applied magnetic field alters bound $\leftrightarrow$ free reaction constant Onsager

$$\frac{K(B)}{K(0)} \simeq 1 + \frac{\mu_0 Q^3 B}{8\pi k_B^2 T^2}$$

- buffering: vacuum $\leftrightarrow$ bound equilibrium unchanged

$\Rightarrow$ free charges increase in field in universal fashion



Expt: magnetic fluctuations/dynamics

Sean Giblin

Collaborators

Coulomb phase:

C. Castelnovo
J. Chalker
K. Gregor
P. Holdsworth
S. Isakov
V. Khemani
S. Parameswaran
S. Sondhi

Loops:

M. Haque
L. Jaubert
S. Piatecki
S. Powell

3D RVB:

A. F. Albuquerque
F. Alet
K. Damle

String expt–HMI:

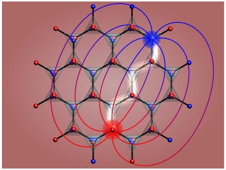
S. Grigera
B. Klemke
J. Morris
A. Tennant

Discussions:

S. Bramwell
P. Fulde
P. McClarty
A. Nahum
F. Pollmann
A. Sen

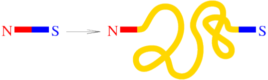

Top 10 Physics Discoveries of the last 10 Years
Majorana Permions
Parity-Time Symmetry in Optics
Magnetic Monopoles ...on (Spin) ICE!
Planck's CoMB Map
Graphene
Scotch Tape, Nobel Prize Edition
The Higgs Bison
Faster Than Light Neutrinos
Oops!
It's Topo-LOGICAL!
Topographical Insulators
Measuring Quantum Effects
NOT Measuring Quantum Effects
In Superposition

From Ising to SU(2)



Emergent gauge field, fractionalisation

- topological physics in $d = 3$
- Maxwell electrodynamics and photons
- deconfined magnetic monopoles



'Dirac string': emergent gauge flux

- topological transitions
  - Kasteleyn
  - SU(2) symmetry under strain
- tensionless and observable; ...