Karpacz, May 16-25, 2024

Correlations, Cluster Formation, and Phase Transitions in Dense Fermion Systems

Gerd Röpke, Rostock

Outlook

- Part I: Quantum statistics and the method of Green functions, Coulomb systems
- Part II: Nuclear systems, correlations, bound states and in-medium effects, phase transitions, pairing and quartetting
- Part III: Nonequilibrium processes and cluster formation, freeze-out concept, heavy-ion collisions, fission, astrophysics, transport processes
- TI: Green functions and Feynman diagrams, partial summations, self-energy, polarization function, cluster decomposition
- TII: Separable potentials, bound and scattering states, Pauli blocking and shift of the binding energy

Structure of matter

energy scale	*fermions*	*interaction*	*bound states*	*density effects*	*condensed phase*
$1 \ldots 10$ meV	electrons, holes	Coulomb	excitons	screening	electron-hole liquid
$1 \ldots 10$ eV	electrons, nuclei	Coulomb	ions, atoms	screening	liquid metal
$1 \ldots 10$ MeV	protons, neutrons	$N - N$ int.	nuclei	Pauli blocking	nuclear matter
$0.1 \ldots 1$ GeV	quarks	QCD	hadrons	deconfinement	quark-gluon plasma

Fermion systems: ideal Fermi gases

Interaction - correlations

Low densities: bound states, quantum condensates
High densities: condensed phase

- Plasma physics: Ionization potential depression (IPD)
- Nuclear physics: Weakly bound nuclei in nuclear systems
- QCD: Deconfinement, Quark – Gluon phase transition in neutron-star mergers

nucleon-nucleon interaction potential

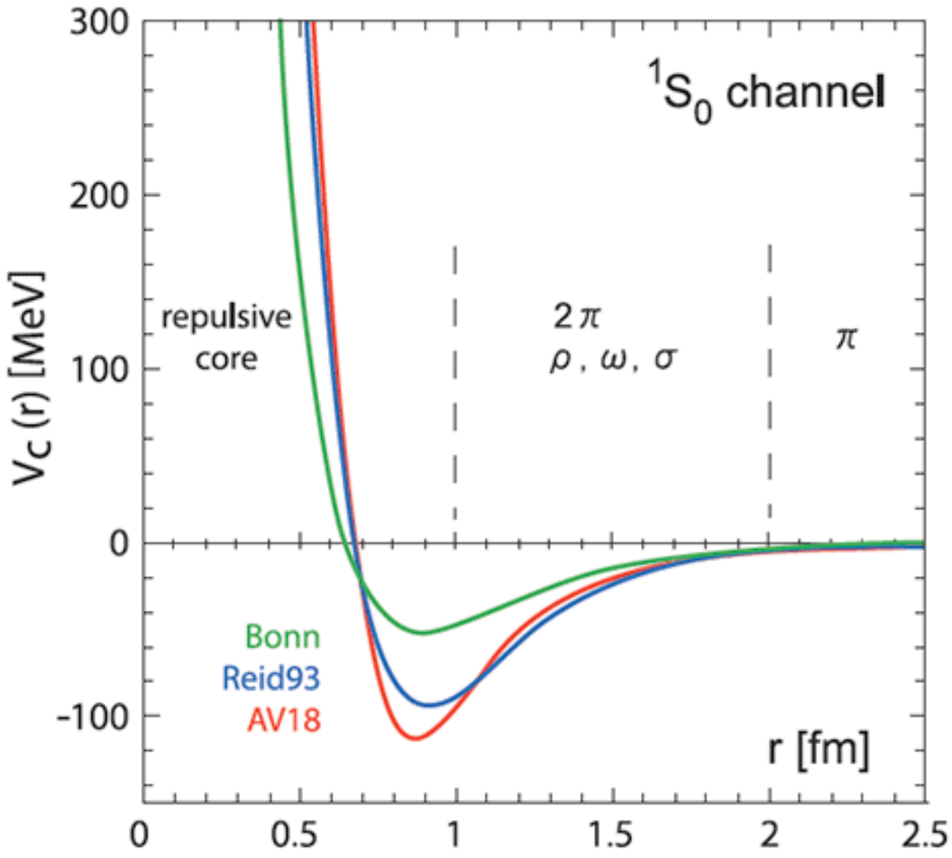
- Effective potentials
  (like atom-atom potential)
  binding energies, scattering

- non-local, energy-dependent?
  QCD?

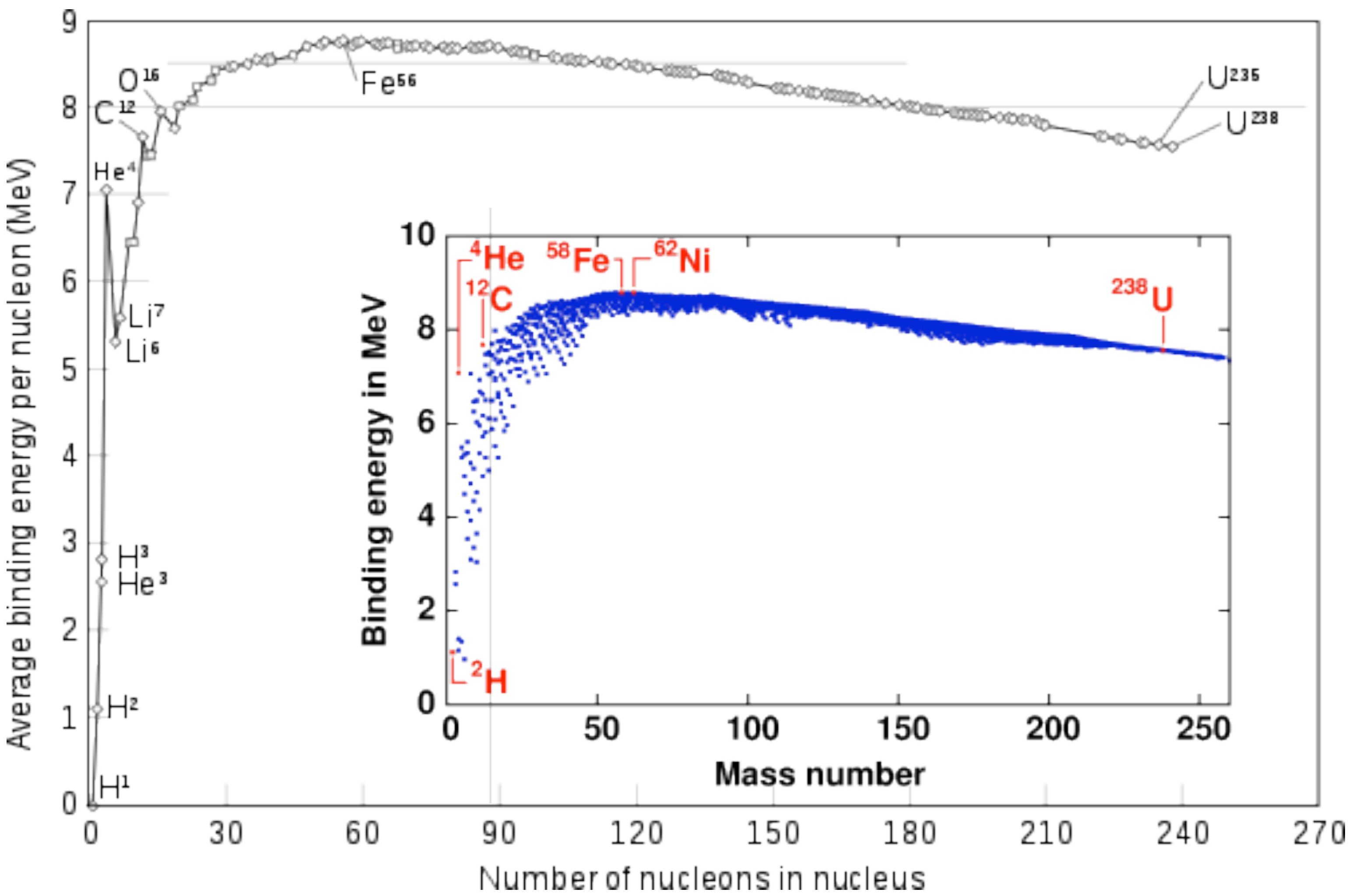
- microscopic calculations
  (AMD, FMD)

- single-particle descriptions:
  Thomas-Fermi approximation
  shell model
  density functional theory (DFT)

- correlations, clustering
  low-density nα nuclei, Volkov



Binding energy per nucleon



Semi-empirical mass formula

Liquid drop model: Bethe-Weizsaecker mass formula

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + a_P \frac{1}{A^{1/2}}$$

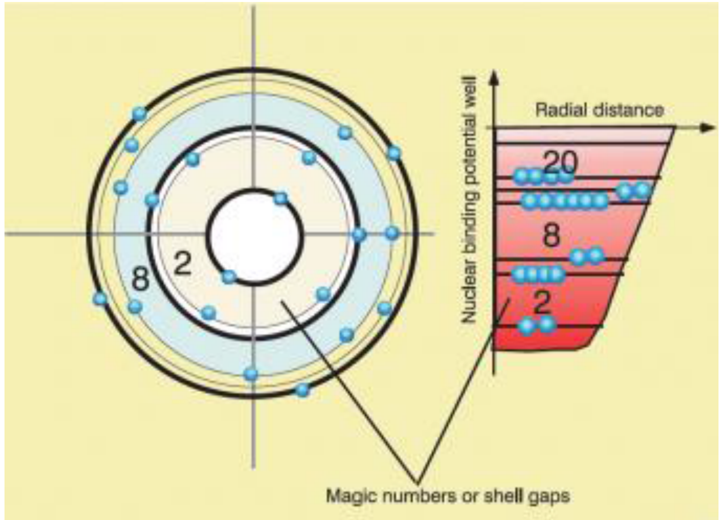
bulk contribution: $a_V = 15.75$ MeV
surface contribution: $a_S = 17.8$ MeV
Coulomb repulsion: $a_C = 0.711$ MeV
asymmetry term: $a_A = 23.7$ MeV
pairing: $a_P = 11.18$ MeV (even-even),
$= 0$ (even-odd), $= -11.18$ MeV (odd-odd)
shell structure and magic numbers

proton fraction $$Y_p = \frac{Z}{A} = \frac{Z}{N+Z}, \quad \frac{N}{Z} = 1 + \frac{a_C}{2a_A} A^{2/3}$$

Models of nuclei

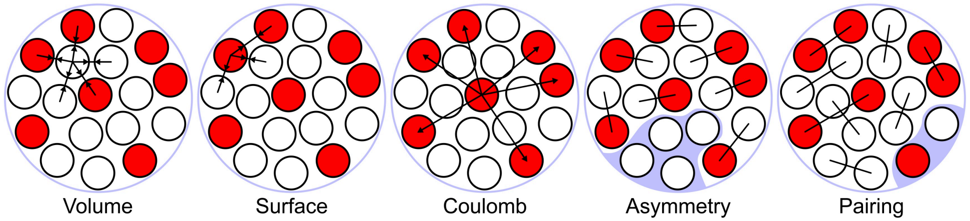
Constituents:
protons, neutrons

Shell model of nucleus:
potential well



Droplet model:
Bethe-Weizsäcker-Formel

C. F. von Weizsäcker:
Zur Theorie der Kernmassen.
In: *Zeitschrift für Physik.* **96** (1935), S. 431–458.



magic numbers:
2; 8; 20; 28; 50; 82; 126

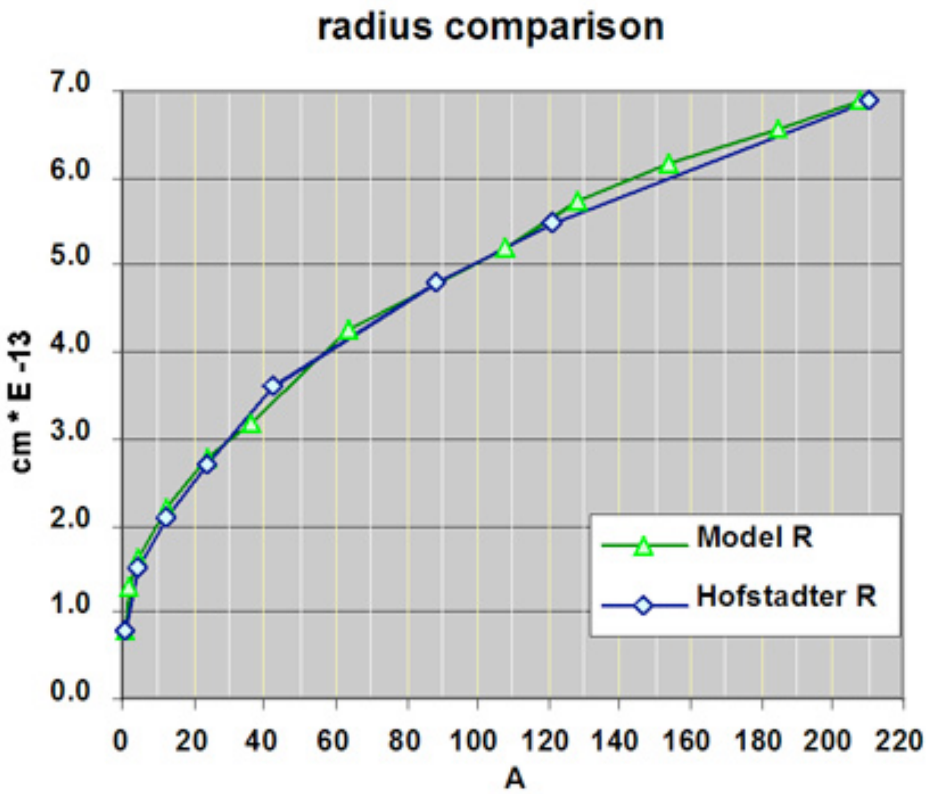
Hans Jensen, Maria Goeppert-Mayer

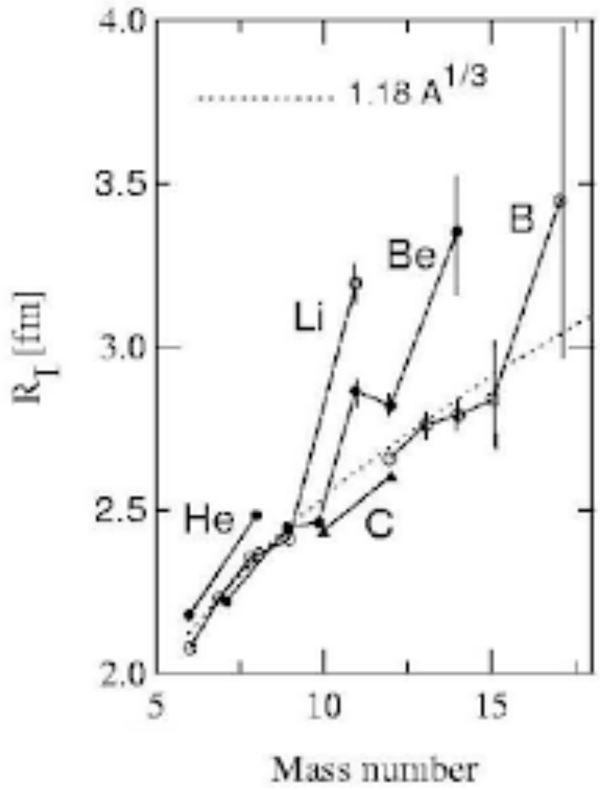
O. Haxel, J.H.D. Jensen, H. E. Suess
Zur Interpretation der ausgezeichneten Nukleonenzahlen im Bau der Atomkerns,
Die Naturwissenschaften, Band **35**, (1949) S.376

Nuclear radii

root mean square radius (charge or point): $$\mathrm{rms}^2 = \frac{\int_0^\infty dr\; r^4\, \rho(r)}{\int_0^\infty dr\; r^2\, \rho(r)}$$

mass – radius relation: $R = 1.18\, A^{1/3}$ [fm] → $n_B = 0.15\ \mathrm{fm}^{-3} = \rho_{sat}$



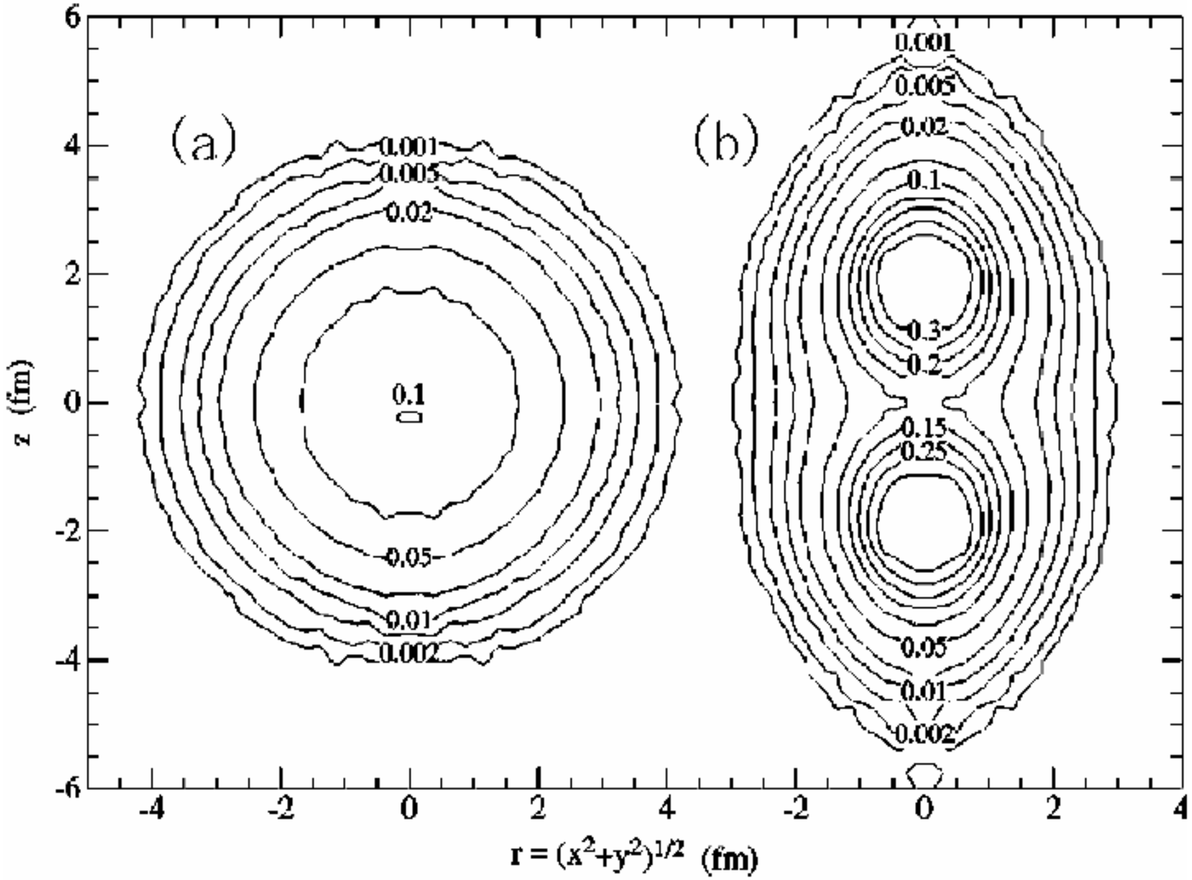


I. Angeli, Atomic Data and Nuclear Data Tables **87**, (2004)

Correlations in nuclei

- Liquid droplet (Bethe – Weizsaecker)
- Shell model (Jensen)
- Pairing (odd-even staggering) - quartetting
- Hoyle state in ^{12}C
- α – formation and α - decay

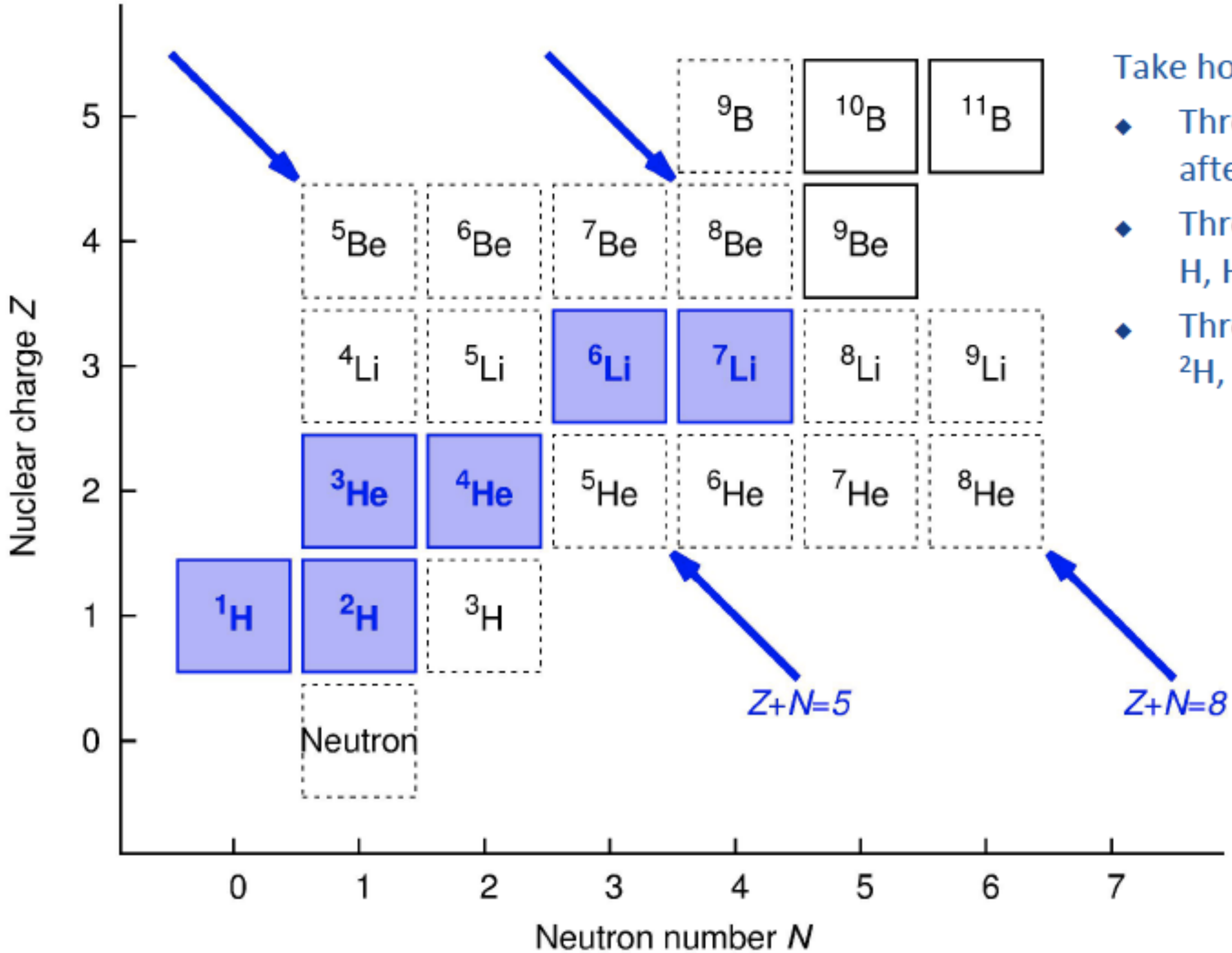
α cluster structure of ^{8}Be



R.B. Wiringa et al.,
PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for ^{8}Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

Big-Bang nucleosynthesis: H, He, Li, __________



Take home textbook knowledge

- Three minutes after the Big Bang
- Three chemical elements: H, He, Li
- Three observed abundances: ^{2}H, ^{4}He, ^{7}Li

Courtesy: D. Bemmerer, lecture at Karpacz-2019

The Hoyle state in ^{12}C

^{12}C: from astrophysics: excited state predicted near the 3 α threshold energy (F. Hoyle).

a 0^+ state at 0.39 MeV above the 3 α threshold energy has been found.

not described by shell structure calculations,
3 α cluster interact predominantly in relative S waves,
gas-like structure, THSR state

A. Tohsaki et al., PRL **87**, 192501 (2001)

α−particle condensation in low-density nuclear matter,
ρ below $\rho_{sat}/5$

nα nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, …
cluster type structures near the n α breakup threshold energy

Excited light nuclei

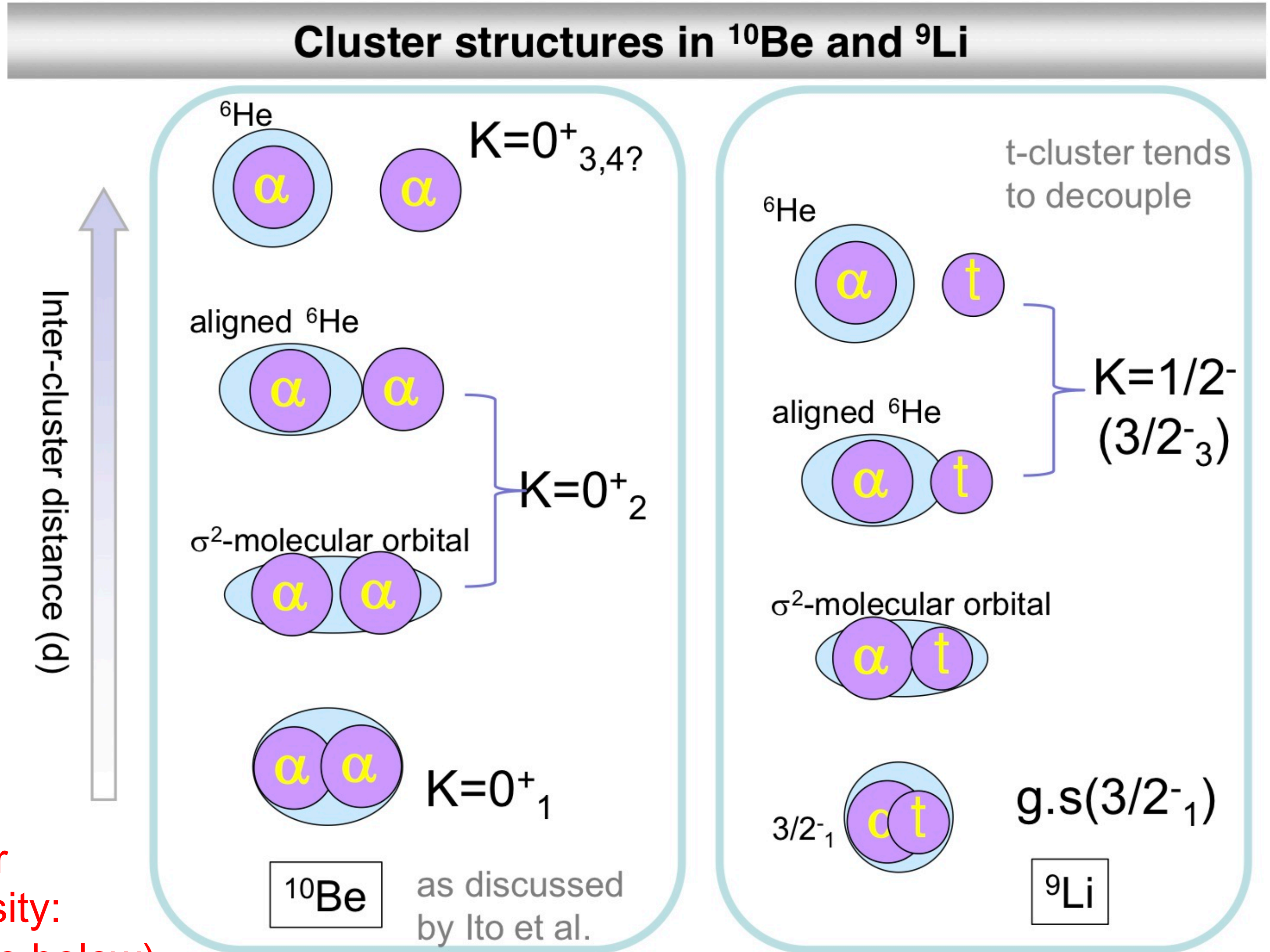
Yoshiko Kanada-En'yo
Cluster2012,Debrecen

decreasing
density

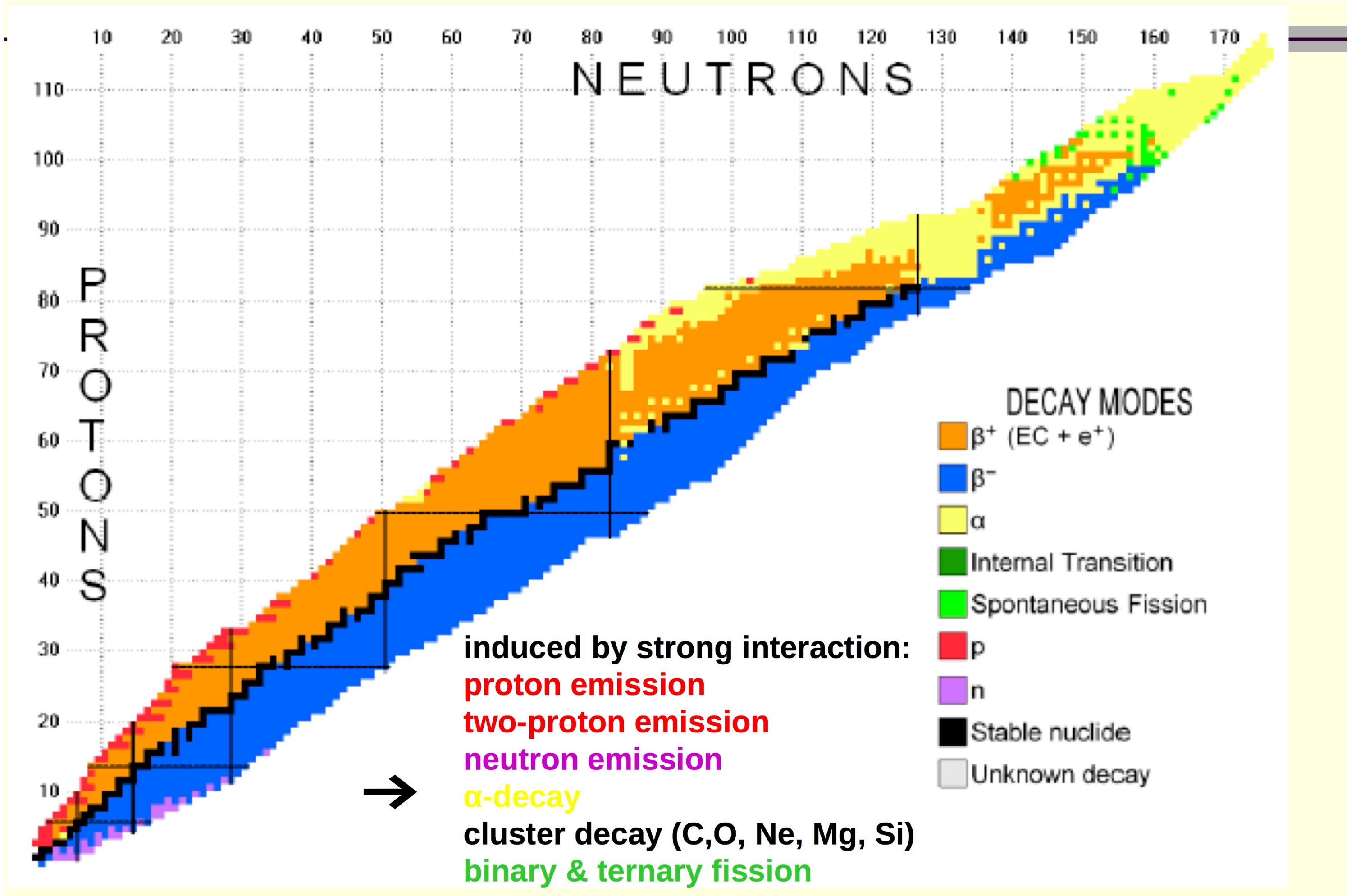
systematics in
weakly bound
light elements

clustering at
low densities

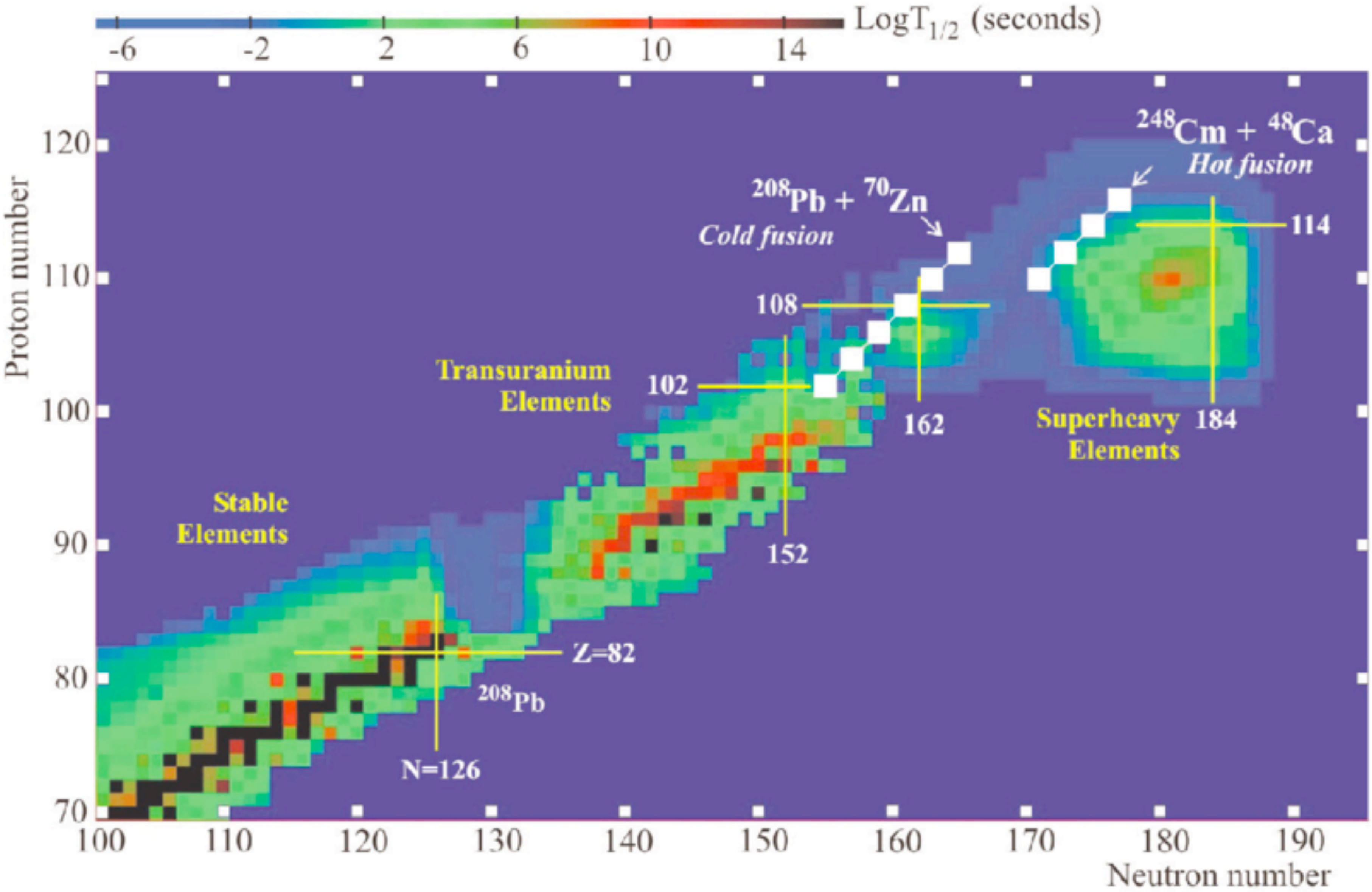
clusters disappear
at increasing density:
Pauli blocking (see below)



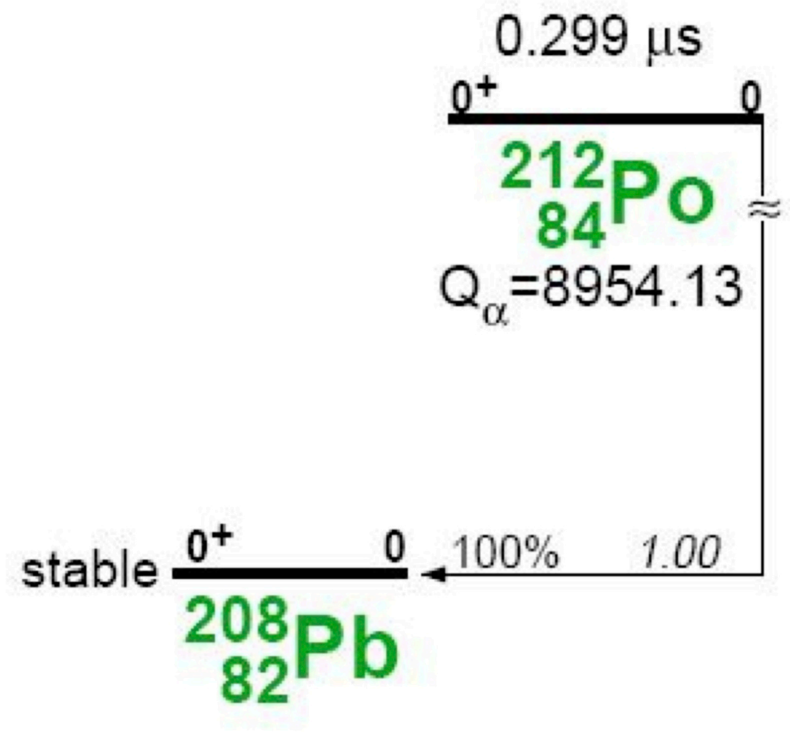
Decay modes of nuclei

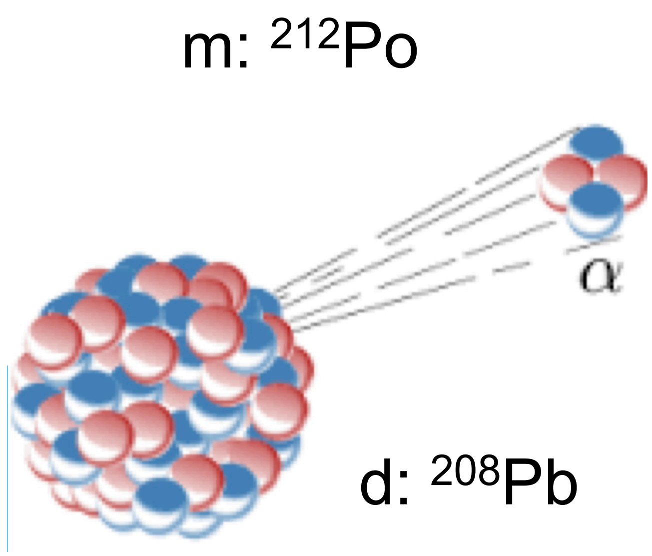


Island of Stability



Preformation: α decay of ^{212}Po





Hot and dense matter

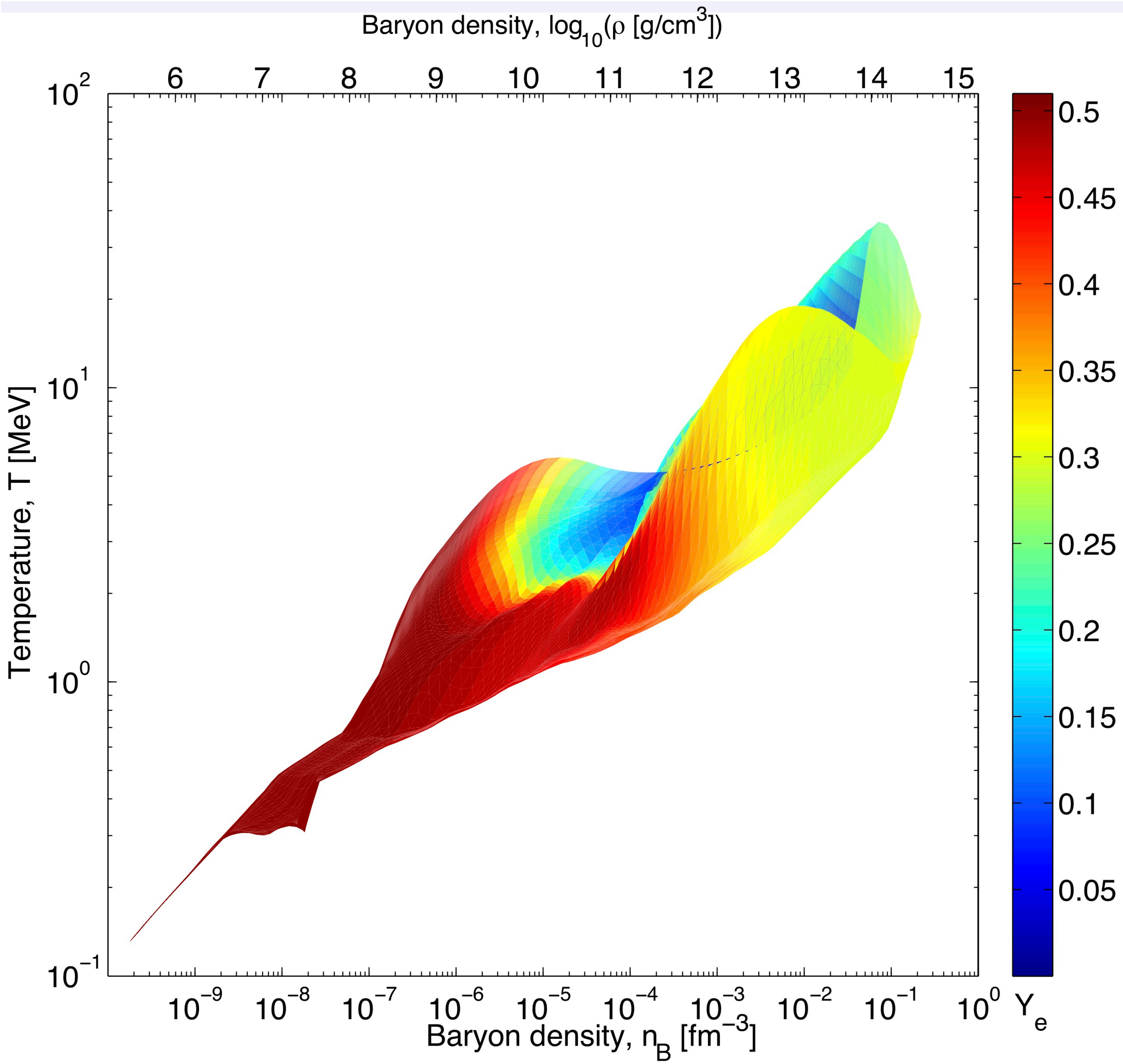
- Early universe
- Compact objects in astrophysics
- Heavy ion collisions
- Spontaneous fission

Nuclear matter phase diagram

Core collapse supernovae

Relevant Parameters:

- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



T. Fischer et al., arXiv 1307.6190, EPJA **50**, 46 (2014)

Quantum statistical approach

The total density as well as the DoS are given by the spectral function A**,**

$$n_e^{\text{total}}(T, \mu_e, \mu_a) = \frac{1}{\Omega} \sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{f}_e(\omega) A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega) D_e(\omega)$$

$$|1\rangle = |\mathbf{p}_1, \sigma_1\rangle$$

which is related to the Green function and the self-energy as

$$A(1,\omega) = 2\,\mathrm{Im}\,G(1,\omega - i0) = 2\,\mathrm{Im}\frac{1}{\omega - E(1) - \Sigma(1,\omega - i0)}$$

$$E(1) = p_1^2/(2m)$$

Quantum statistical approach

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$$E(1) = p_1^2/(2m)$$

A cluster decomposition for the self-energy is possible so that a quasiparticle (free) contribution can be separated,

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\text{quasi}}(1))}{1 - \frac{d}{dz}\mathrm{Re}\,\Sigma_e(1,z)|_{z=E_e^{\text{quasi}}-\mu_e}} - 2\mathrm{Im}\,\Sigma_e(1,\omega + i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega + \mu_e - E_e^{\text{quasi}}(1)}$$

$$E^{\text{quasi}}(1) = p_1^2/(2m) + \mathrm{Re}\Sigma(1,\omega)|_{\omega=E^{\text{quasi}}(1)}$$

Quasiparticle approach

The total density as well as the DoS are given by the spectral function A,

$$n_e^{\mathrm{total}}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\hat{f}_e(\omega)A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega)D_e(\omega)$$

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\mathrm{quasi}}(1))}{1 - \frac{d}{dz}\mathrm{Re}\,\Sigma_e(1,z)|_{z=E_e^{\mathrm{quasi}}-\mu_e}} - 2\mathrm{Im}\,\Sigma_e(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega+\mu_e - E_e^{\mathrm{quasi}}(1)}$$

- quasiparticle concept

$$E^{\mathrm{quasi}}(1) = p_1^2/(2m) + \mathrm{Re}\Sigma(1,\omega)|_{\omega=E^{\mathrm{quasi}}(1)}$$

- generalized Beth-Uhlenbeck formula (quasiparticles)

$$n_e^{\mathrm{total}}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 f_e(E^{\mathrm{quasi}}(1))$$

$$+\frac{1}{\Lambda^3}\sum_{i,\gamma} Z_i e^{\beta\mu_i}\left[\sum_{\nu}^{\mathrm{bound}}(e^{-\beta E_{i,\gamma,\nu}}-1) + \frac{\beta}{\pi}\int_0^{\infty} dE e^{-\beta E}\left\{\delta_{i,\gamma}(E) - \frac{1}{2}\sin[2\delta_{i,\gamma}(E)]\right\}\right]$$

In-medium Schrödinger equation for $E_{i,\gamma,\nu}(T,\mu)$, $\delta_{i,\gamma}(T,\mu)$, channel (spin…) γ

Different approximations

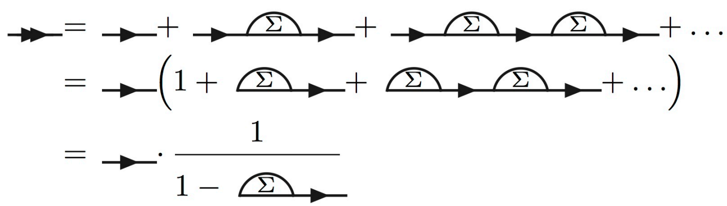
Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Partial summations

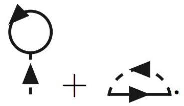
- Dyson equation and self-energy

$$G_1(1, iz_\nu) = \frac{1}{iz_\nu - \epsilon_1 - \Sigma_1(1, iz_\nu)}$$



$$= G_1^0(1, iz_\nu)\frac{1}{1 - \Sigma_1(1, iz_\nu)\, G_1^0(1, iz_\nu)}$$

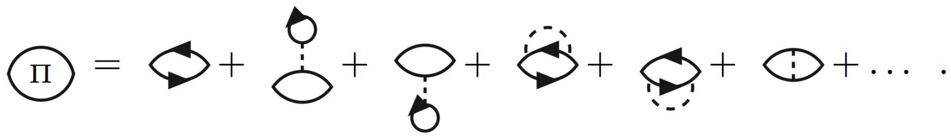
$$= \frac{1}{G_1^0(1, iz_\nu)^{-1} - \Sigma_1(1, iz_\nu)}$$

  - Hartree-Fock $\Sigma_1^{\mathrm{HF}}(1, iz_\nu) =$ 

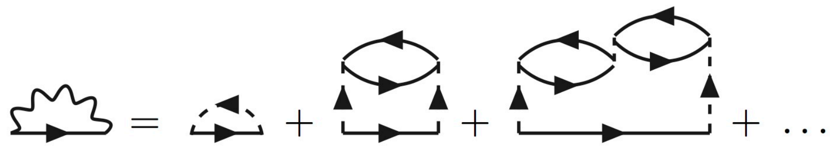
$$\int \frac{\mathrm{d}^3k'}{(2\pi)^3}\left((2s+1)V(0) - V\left(\vec{k}' - \vec{k}\right)\right) f(\epsilon_{k'})$$

$$A_1(1,\omega) = \lim_{\varepsilon \searrow 0} 2\frac{\mathrm{Im}\{\Sigma_1(1, \omega - i\varepsilon)\}}{[\omega - \epsilon_1 - \mathrm{Re}\{\Sigma_1(1, \omega - i\varepsilon)\}]^2 + [\mathrm{Im}\{\Sigma_1(1, \omega - i\varepsilon)\} - \varepsilon]^2}$$

- screening $V_{ab}^s(q, iz_\mu) = \dfrac{V_{ab}(q)}{1 - \sum_c V_{cc}(q)\Pi_{cc}(q, iz_\mu)} \equiv \dfrac{V_{ab}(q)}{\varepsilon(q, iz_\mu)}$



polarization function

$\Sigma_1^{\mathrm{MW}}(1, iz_\nu) =$ 

- Debye potential $V^D(r) = \dfrac{e_1 e_2}{4\pi\varepsilon_0} \cdot \dfrac{\mathrm{e}^{-\kappa r}}{r}$ screening parameter $\kappa^2 = \sum_c \dfrac{e_c^2 n_c}{\varepsilon_0 k_B T}$

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

medium effects

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Medium effects: Quasiparticle approximation

- Skyrme / Gogny

- relativistic mean field (RMF)

  Lagrangian: non-linear sigma, TM1 parameters,
  single particle modifications, energy shift, effective mass

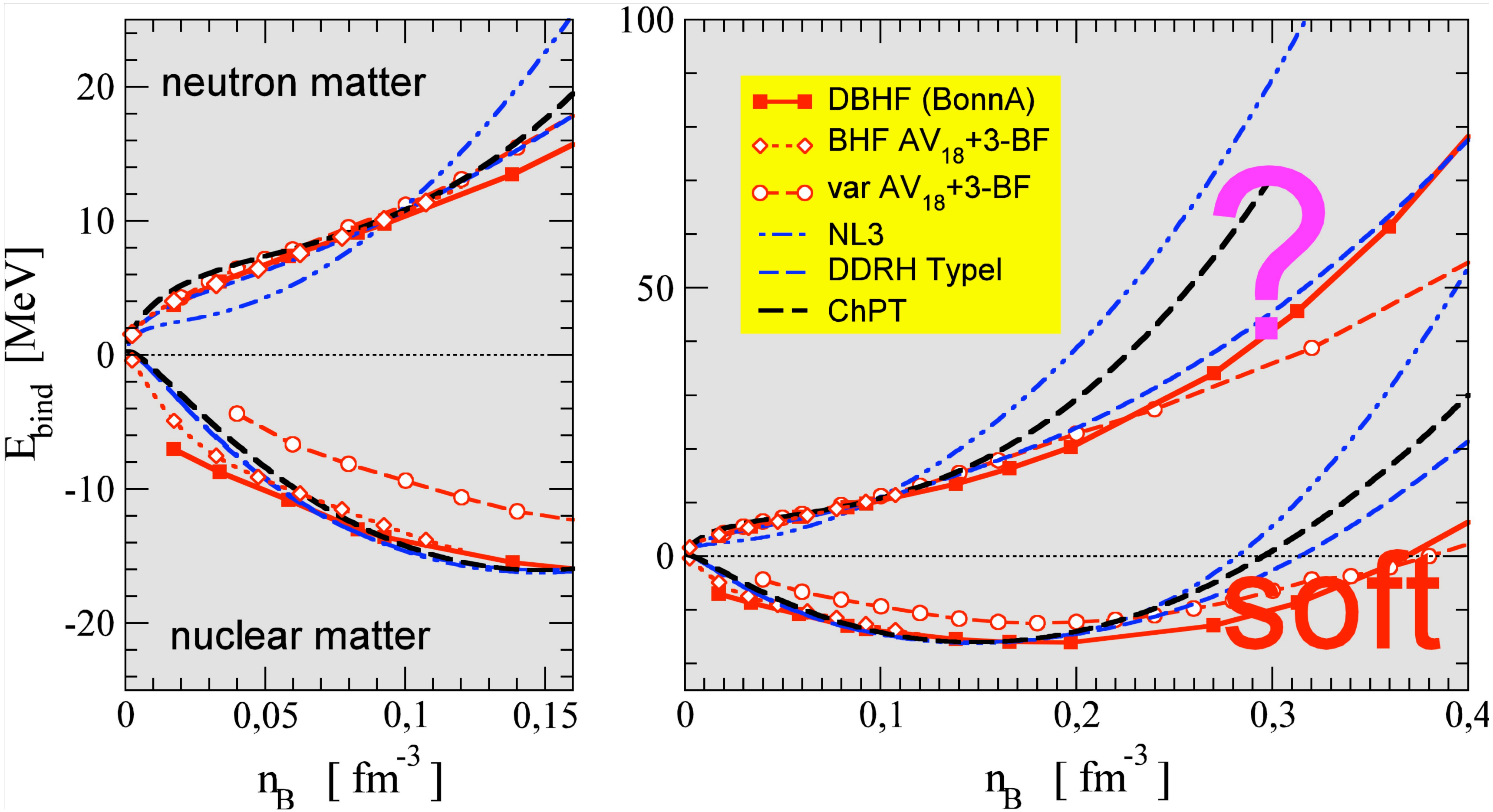
- DD-RMF [S.Typel, Phys. Rev. C **71**, 064301 (2007)]:

  expansion of the scalar field and the vector fields
  in powers of proton/neutron densities

- Dirac-Brueckner Hartree Fock (DBHF)

- Density functional theory

Quasiparticle picture: RMF and DBHF

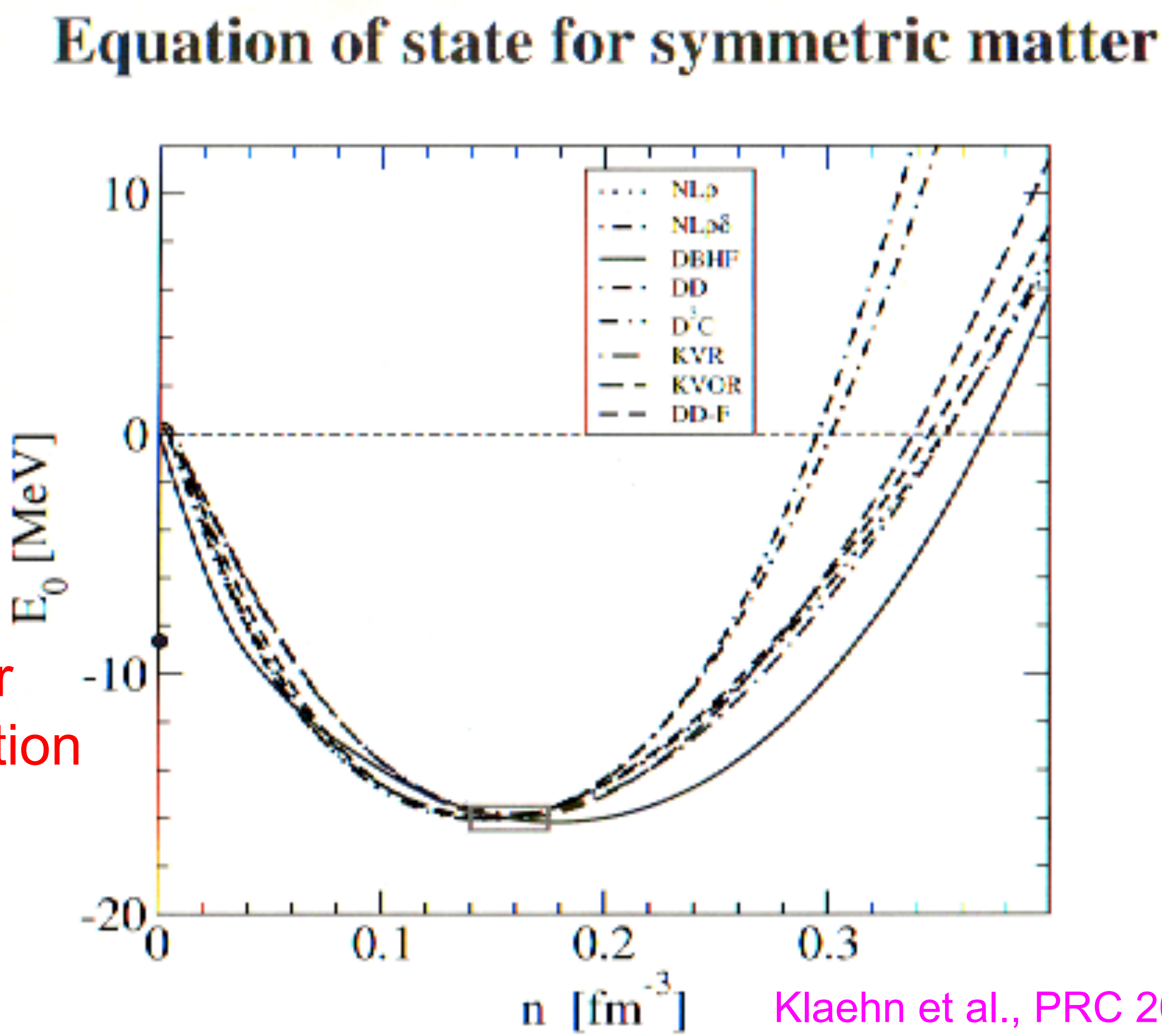


But: cluster formation

Incorrect low-density limit

C. Fuchs et al.;
J.Margueron et al., Phys.Rev.C **76**,034309 (2007)

Quasiparticle approximation for nuclear matter

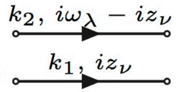


But: cluster formation

Incorrect low-density limit

Klaehn et al., PRC 2006

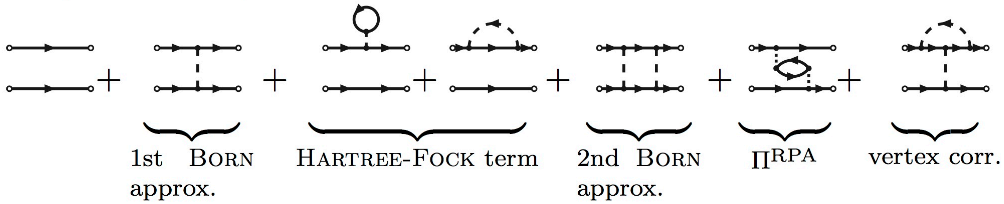
Bethe-Salpeter equation

Free two-particle propagator $G_2^0\left(12,1'2',i\omega_\lambda\right) =$ 

$= \frac{1-f\left(\epsilon_1\right)-f\left(\epsilon_2\right)}{i\omega_\lambda-\epsilon_1-\epsilon_2}\delta_{11'}\delta_{22'}$

Full two-particle propagator

$G_2\left(12,1'2',i\omega_\lambda\right) =$

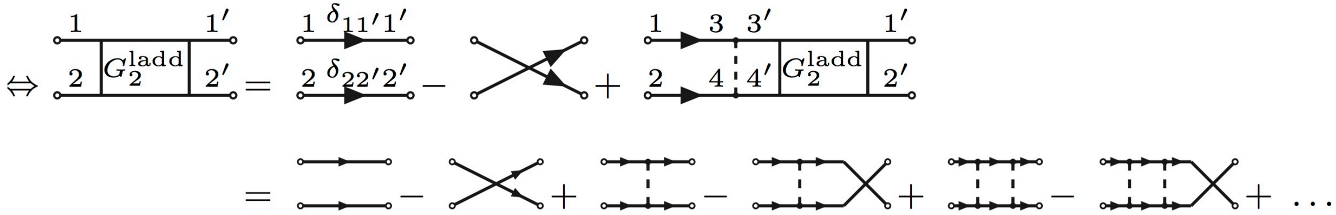


$+\mathcal{O}\left(V^3\right)$

Bethe-Salpeter equation

$$G_2^{\mathrm{ladd.}}\left(12,1'2',i\omega_\lambda\right) = G_2^0\left(12,1'2',i\omega_\lambda\right) + \sum_{\substack{34\\3'4'}} G_2^0\left(12,34,i\omega_\lambda\right)V\left(34,3'4'\right)G_2^{\mathrm{ladd.}}\left(3'4',1'2',i\omega_\lambda\right)$$

Ladder summation



Solution low-density limit

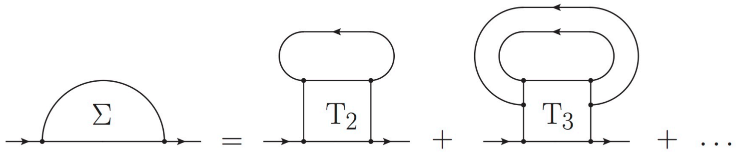
$$G_2^{\mathrm{ladd.}}\left(12,1'2',i\omega_\lambda\right) = \sum_{nP}\psi_{nP}(12)\frac{1}{i\omega_\lambda - E_{nP}+\mu_{12}}\psi_{nP}^*(1'2')$$

Schroedinger equation

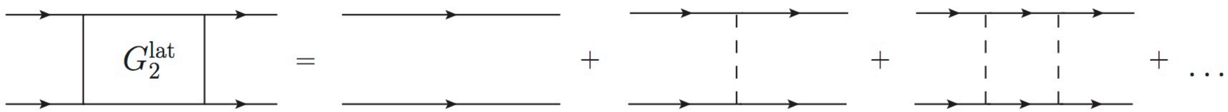
$$\left(E_1+E_2-E_{nP}\right)\psi_{nP}(12) + \sum_{1'2'}V\left(12,1'2'\right)\psi_{nP}\left(1'2'\right) = 0$$

Beth-Uhlenbeck formula

cluster decomposition of the self-energy



Two-particle correlations



cluster propagator $\langle \nu, \mathbf{P} | G_2(z) | \nu', \mathbf{P}' \rangle = \frac{1}{z - E^0_{\nu,P}} \delta_{\nu\nu'} \delta_{\mathbf{P},\mathbf{P}'}$

Beth-Uhlenbeck formula: second virial coefficient, $f_2(E) = \frac{1}{e^{\beta(E-2\mu)} - 1} \approx e^{\beta(2\mu - E)}$

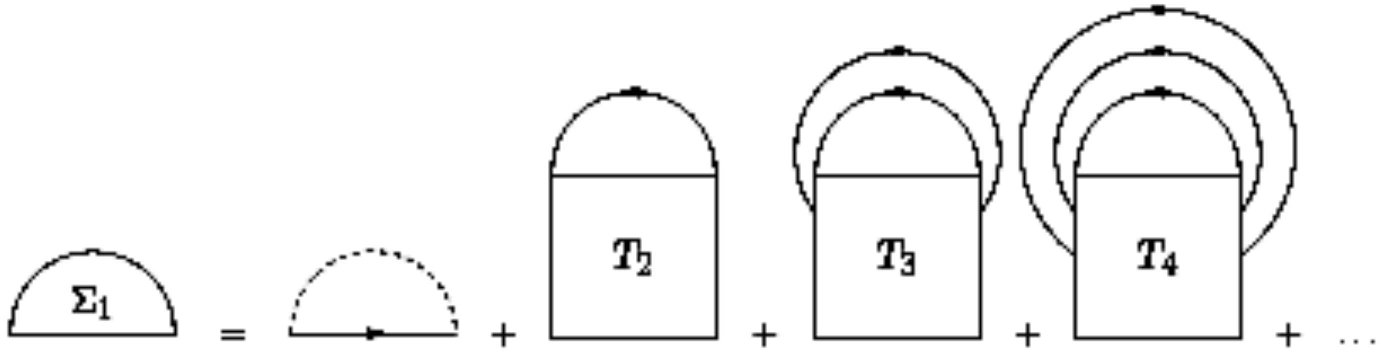
$$n_B^{\mathrm{BU}}(\beta, \mu) = \frac{1}{\Omega_0} \sum_{\mathbf{p}} f_p^0 + \frac{2}{\Omega_0} \sum_{\alpha, \mathbf{P}} \int_{-\infty}^{\infty} \frac{dE_{\mathrm{rel}}}{\pi} f_2 \left(E_{\mathrm{rel}} + \frac{P^2}{4m} \right) D^{\mathrm{BU}}_{\alpha, \mathbf{P}}(E_{\mathrm{rel}}),$$

$$f_p^0 = \frac{1}{e^{\beta(E_p - \mu)} + 1} \approx e^{\beta(\mu - E_p)}$$

$$D^{\mathrm{BU}}_{\alpha, \mathbf{P}}(E_{\mathrm{rel}}) = g_\alpha \left(\sum_\nu \pi \delta(E_{\mathrm{rel}} - E^0_{\alpha\nu, \mathbf{P}}) + \frac{\partial}{\partial E_{\mathrm{rel}}} \delta_{\alpha, \mathbf{P}}(E_{\mathrm{rel}}) \right)$$

degeneracy bound states scattering phase shifts

Cluster decomposition of the self-energy



T_n-matrices: n-particle Schroedinger equation,
n-particle bound states, (we neglect here scattering states).

Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$
$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

mass number *A*,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
~*K* center of mass momentum

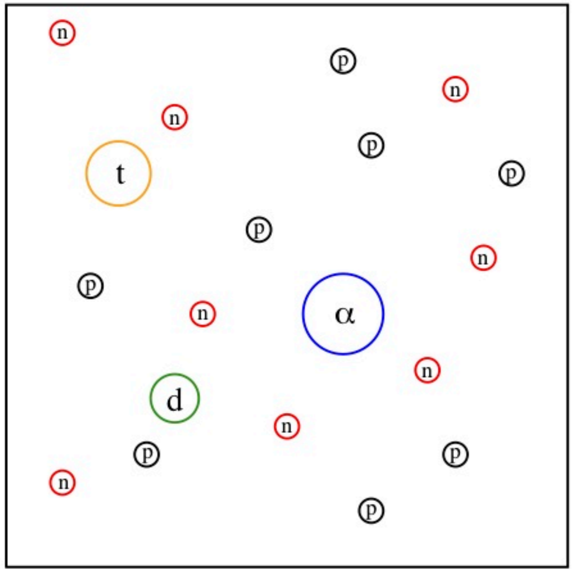
Chemical equilibrium, mass action law,
Nuclear Statistical Equilibrium (NSE)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

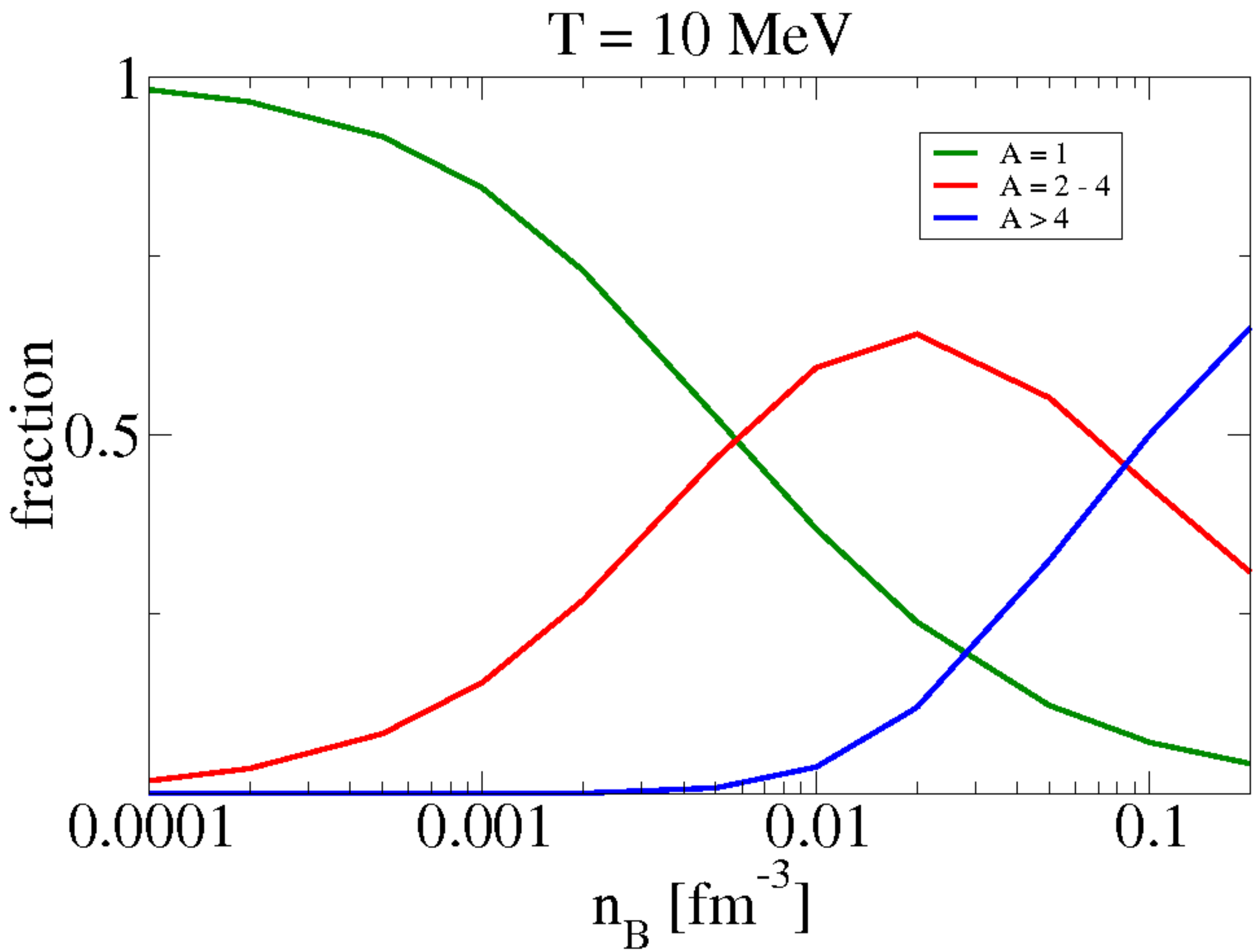
bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Inclusion of the light clusters (d,t,^{3}He,^{4}He)

Composition of symmetric matter
Ideal mixture of nuclides



Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

low density limit

saturation density

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

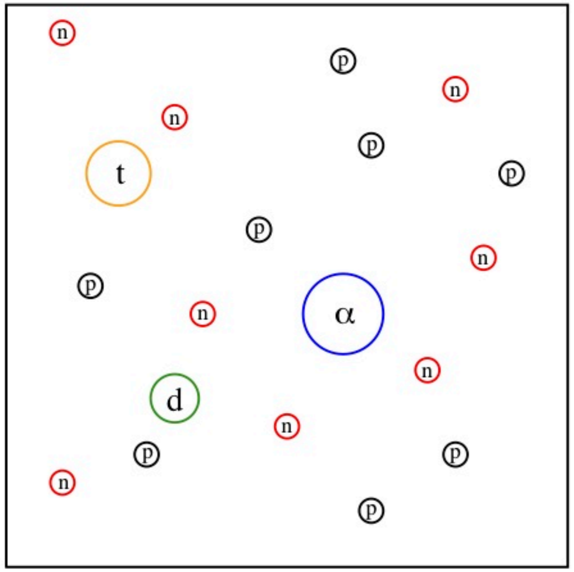
self-energy and Pauli blocking

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law

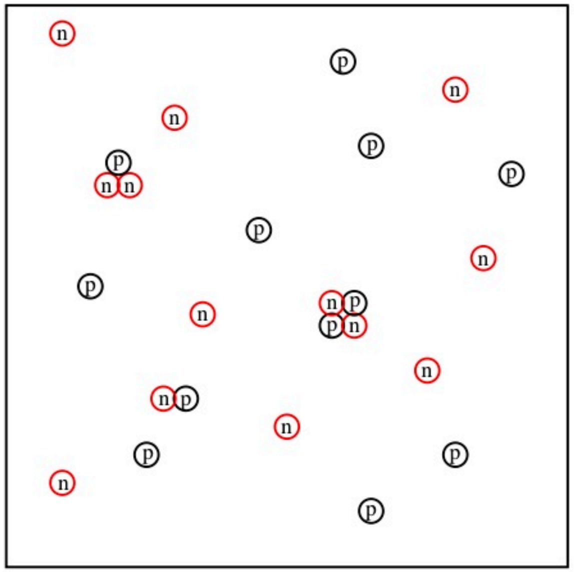


Interaction between the components
internal structure: Pauli principle

Physical picture:

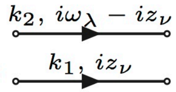
"elementary" constituents

and their interaction



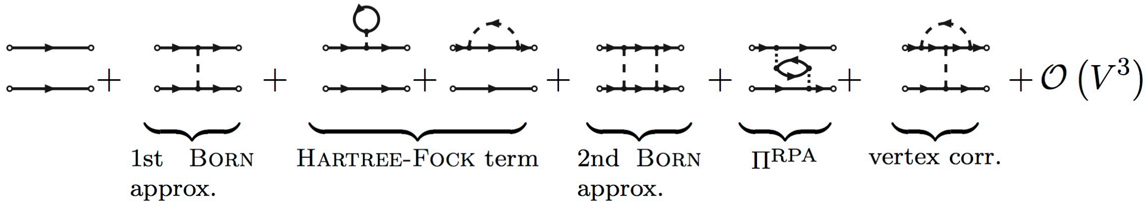
Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Bethe-Salpeter equation

Free two-particle propagator $G_2^0\left(12, 1'2', i\omega_\lambda\right) =$  $= \dfrac{1 - f\left(\epsilon_1\right) - f\left(\epsilon_2\right)}{i\omega_\lambda - \epsilon_1 - \epsilon_2}\delta_{11'}\delta_{22'}$

Full two-particle propagator

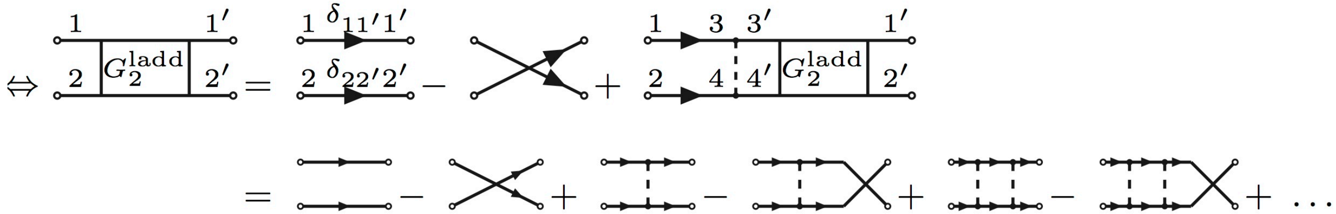
$G_2\left(12, 1'2', i\omega_\lambda\right) =$



Bethe-Salpeter equation

$$G_2^{\mathrm{ladd.}}\left(12, 1'2', i\omega_\lambda\right) = G_2^0\left(12, 1'2', i\omega_\lambda\right) + \sum_{\substack{34\\3'4'}} G_2^0\left(12, 34, i\omega_\lambda\right) V\left(34, 3'4'\right) G_2^{\mathrm{ladd.}}\left(3'4', 1'2', i\omega_\lambda\right)$$

Ladder summation



Solution low-density limit

$$G_2^{\mathrm{ladd.}}\left(12, 1'2', i\omega_\lambda\right) = \sum_{nP} \psi_{nP}(12) \frac{1}{i\omega_\lambda - E_{nP} + \mu_{12}} \psi_{nP}^*(1'2')$$

Schroedinger equation

$$\left(E_1 + E_2 - E_{nP}\right)\psi_{nP}(12) + \sum_{1'2'} V\left(12, 1'2'\right)\psi_{nP}\left(1'2'\right) = 0$$

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2') = E_{d,P}\Psi_{d,P}(p_1,p_2)$$

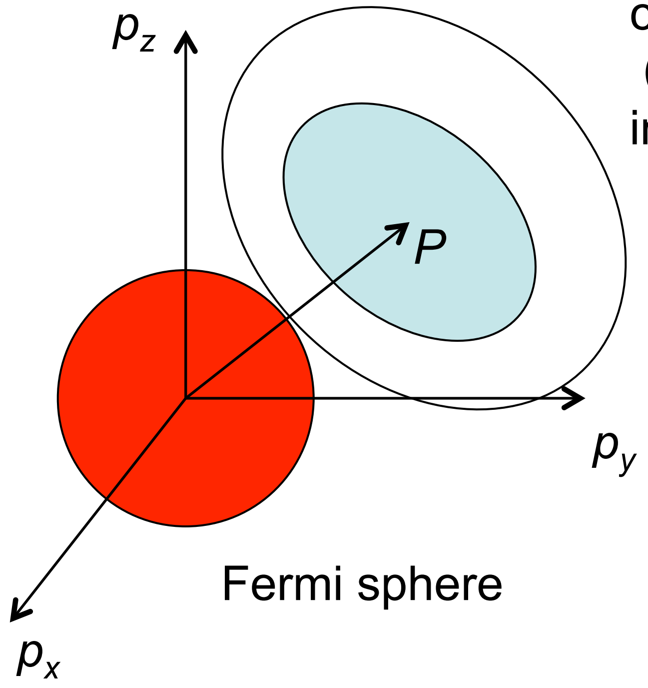
self-energy Pauli-blocking

phase space occupation:
Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

(screening and vertex correction neglected)

Pauli blocking – phase space occupation



cluster wave function
(atoms, ions, …deuteron, alpha,…)
in momentum space

P - center of mass momentum

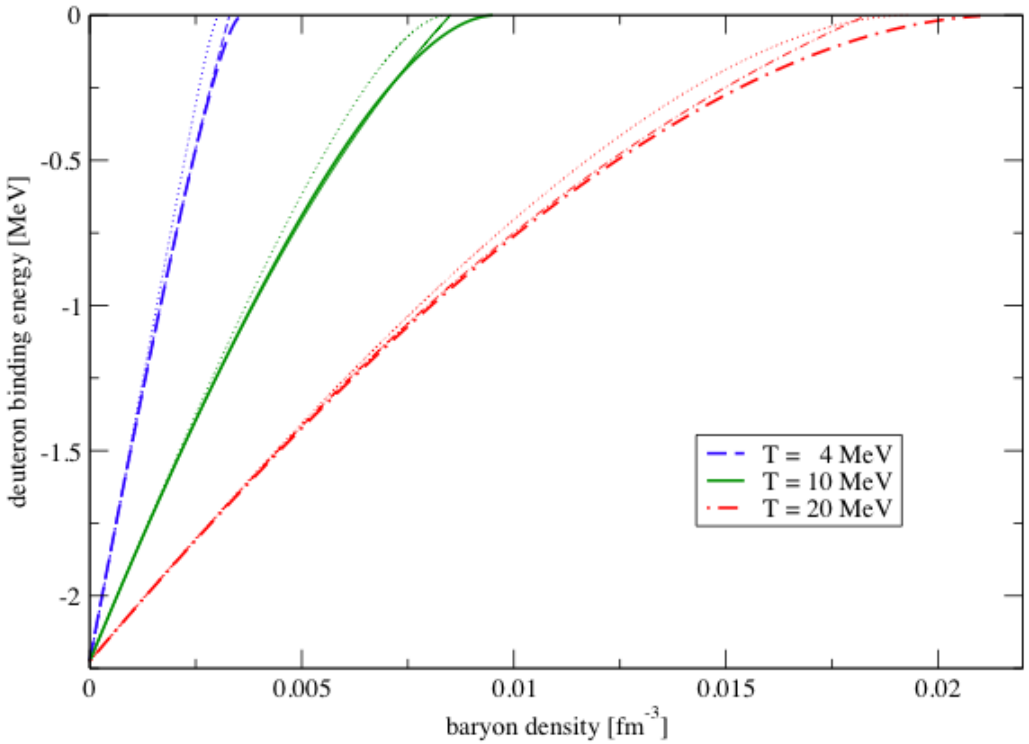
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

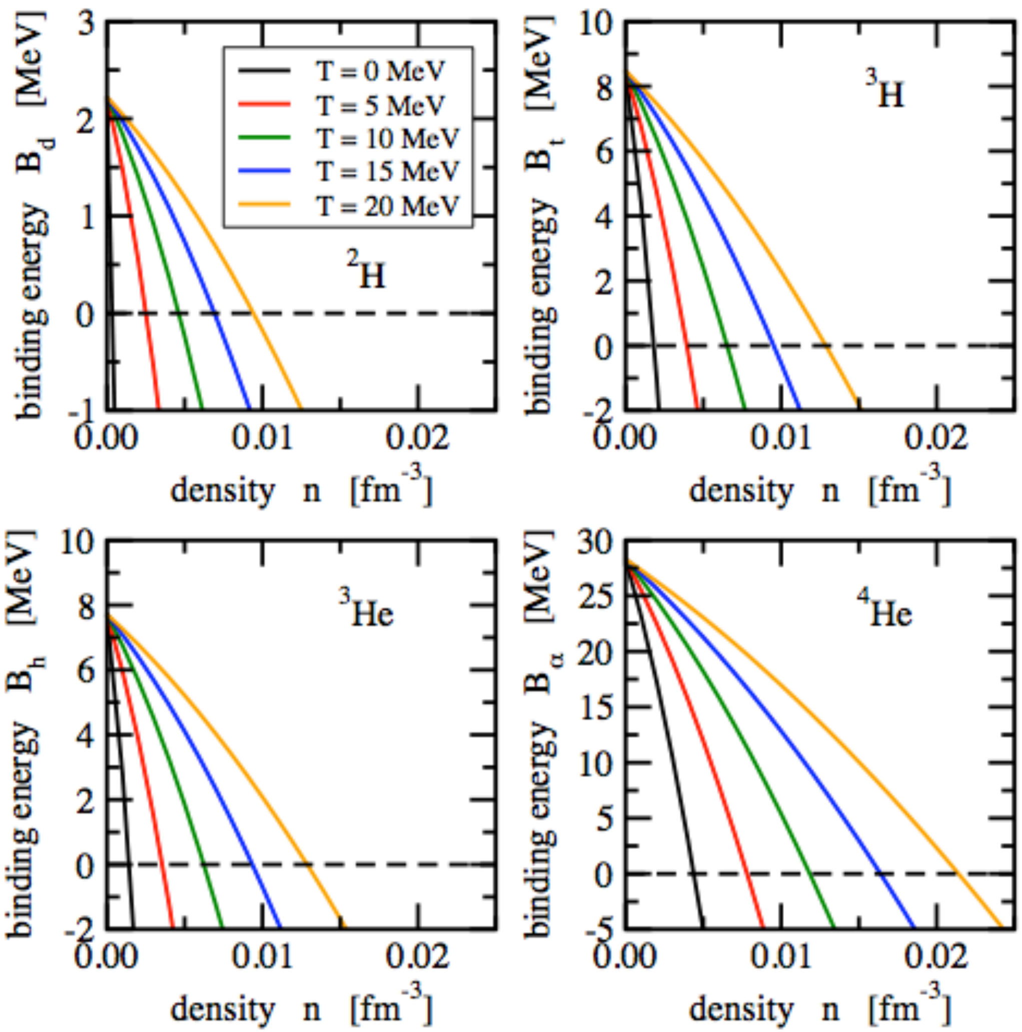


thin lines:

fit formula

G.R., Nucl. Phys. A **867**, 66 (2011)

Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Full virial expansion

- Excited states, resonances, scattering states
- Full expression for the second virial coefficient
- Scattering phase shifts
- Exact in second order of density
- Beth-Uhlenbeck equation, Dashen-Ma-Bernstein: S-matrix

Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

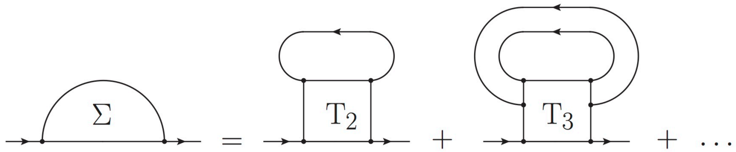
medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

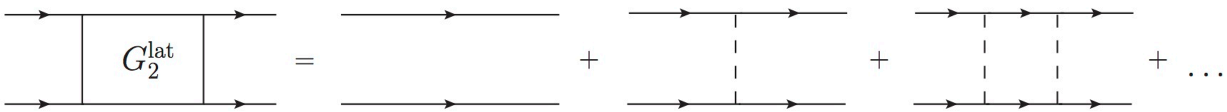
Chemical equilibrium
with quasiparticle clusters:
self-energy and Pauli blocking

Beth-Uhlenbeck formula

cluster decomposition of the self-energy



Two-particle correlations



cluster propagator $\langle \nu, \mathbf{P} | G_2(z) | \nu', \mathbf{P}' \rangle = \frac{1}{z - E^0_{\nu,P}} \delta_{\nu\nu'} \delta_{\mathbf{P},\mathbf{P}'}$

Beth-Uhlenbeck formula: second virial coefficient, $f_2(E) = \frac{1}{e^{\beta(E-2\mu)} - 1} \approx e^{\beta(2\mu - E)}$

$$n_B^{\mathrm{BU}}(\beta, \mu) = \frac{1}{\Omega_0} \sum_{\mathbf{p}} f_p^0 + \frac{2}{\Omega_0} \sum_{\alpha, \mathbf{P}} \int_{-\infty}^{\infty} \frac{dE_{\mathrm{rel}}}{\pi} f_2\left(E_{\mathrm{rel}} + \frac{P^2}{4m}\right) D^{\mathrm{BU}}_{\alpha,\mathbf{P}}(E_{\mathrm{rel}}),$$

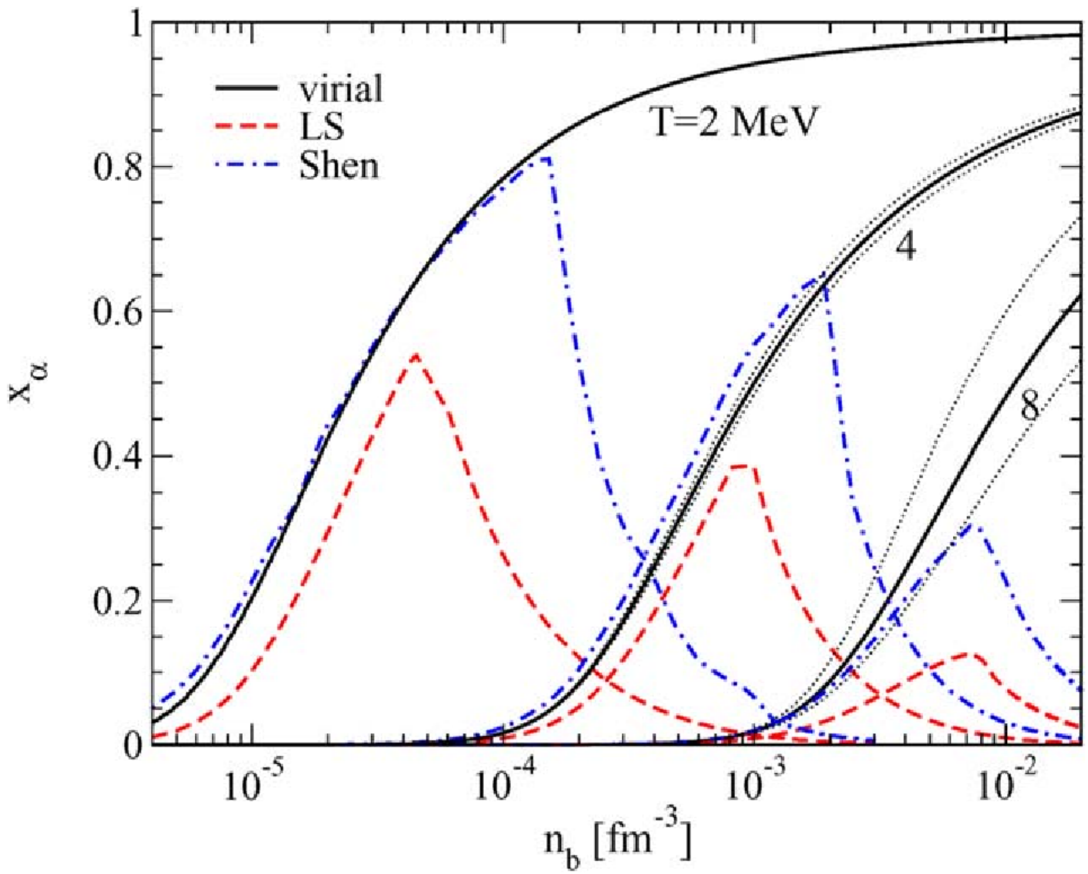
$$f_p^0 = \frac{1}{e^{\beta(E_p - \mu)} + 1} \approx e^{\beta(\mu - E_p)}$$

$$D^{\mathrm{BU}}_{\alpha,\mathbf{P}}(E_{\mathrm{rel}}) = g_\alpha \left(\sum_\nu \pi \delta(E_{\mathrm{rel}} - E^0_{\alpha\nu,\mathbf{P}}) + \frac{\partial}{\partial E_{\mathrm{rel}}} \delta_{\alpha,\mathbf{P}}(E_{\mathrm{rel}}) \right)$$

degeneracy bound states scattering phase shifts

Alpha-particle fraction in the low-density limit

symmetric matter, T=2, 4, 8 MeV



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

self-energy

Pauli-blocking

phase space occupation:
Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

solution for bound and scattering states

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium
of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

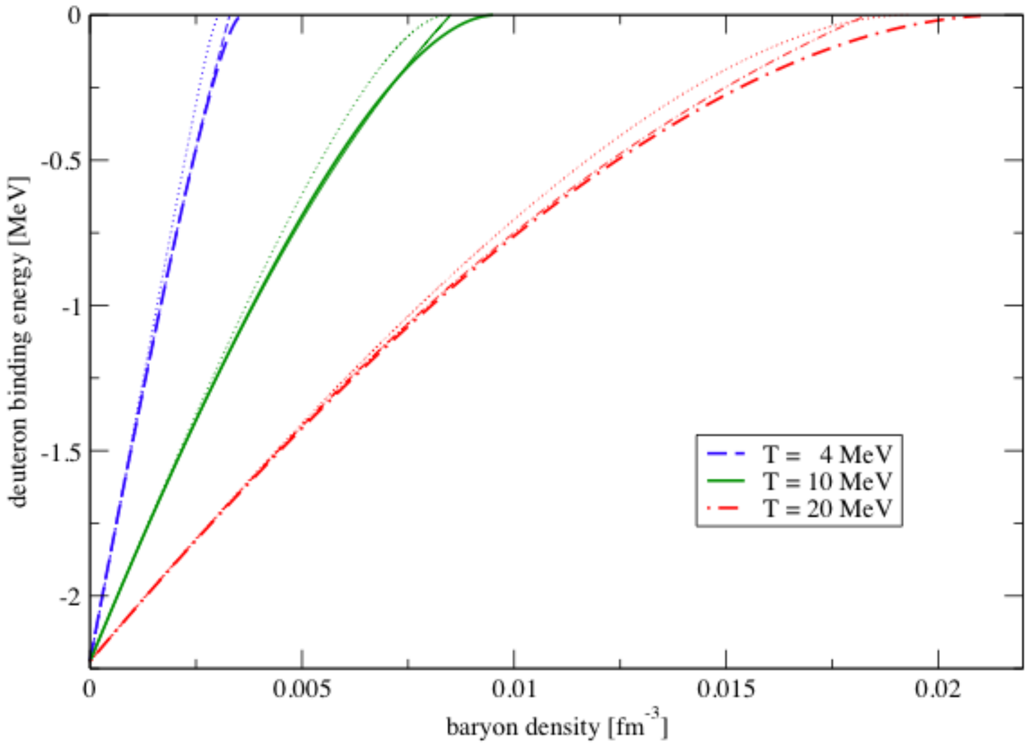
Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

Levinson's theorem

- Bound states disappear at increasing density, merge with the continuum
- No discontinuity in the partition function, jump in the bound state contribution is compensated by a jump in the scattering phase contribution
- Levinson's theorem: scattering phase shift at zero energy is given by the number of bound states multiplied by pi

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

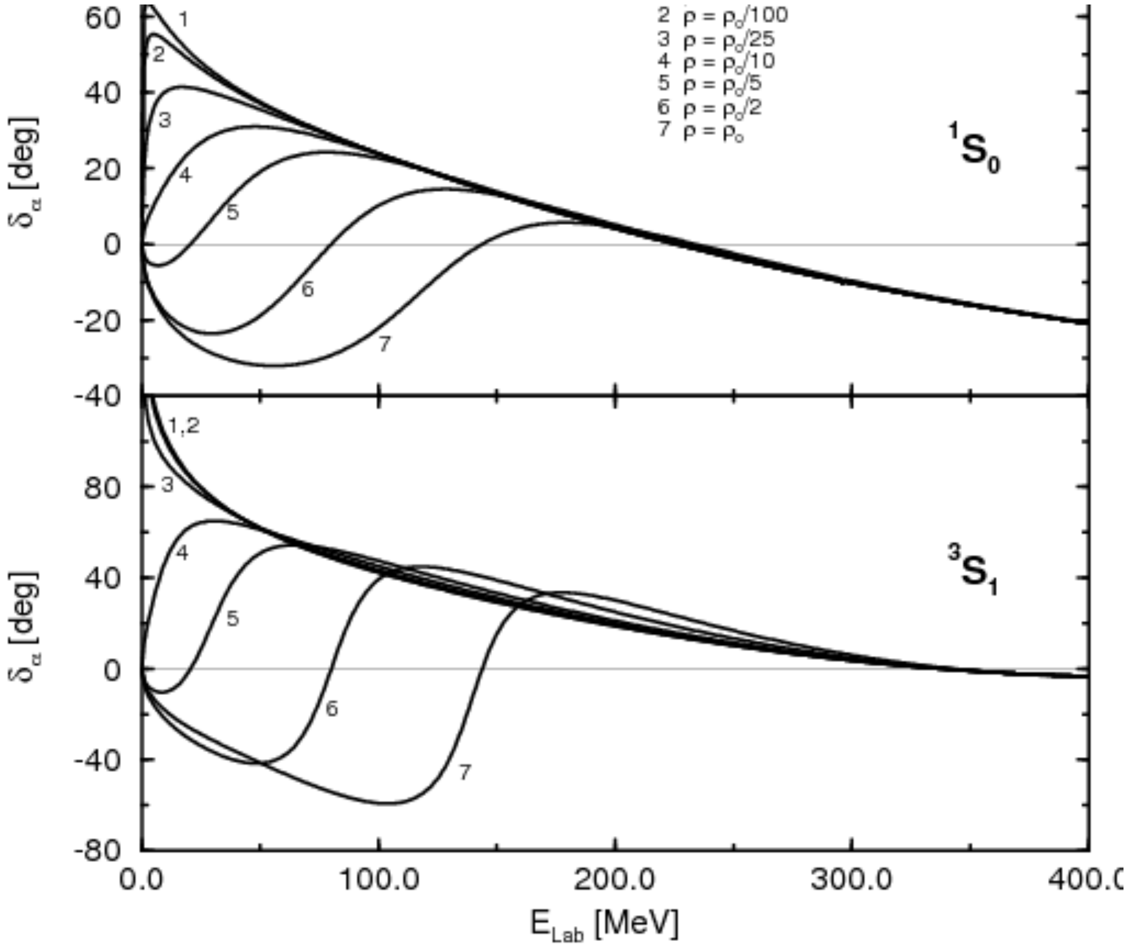


thin lines:

fit formula

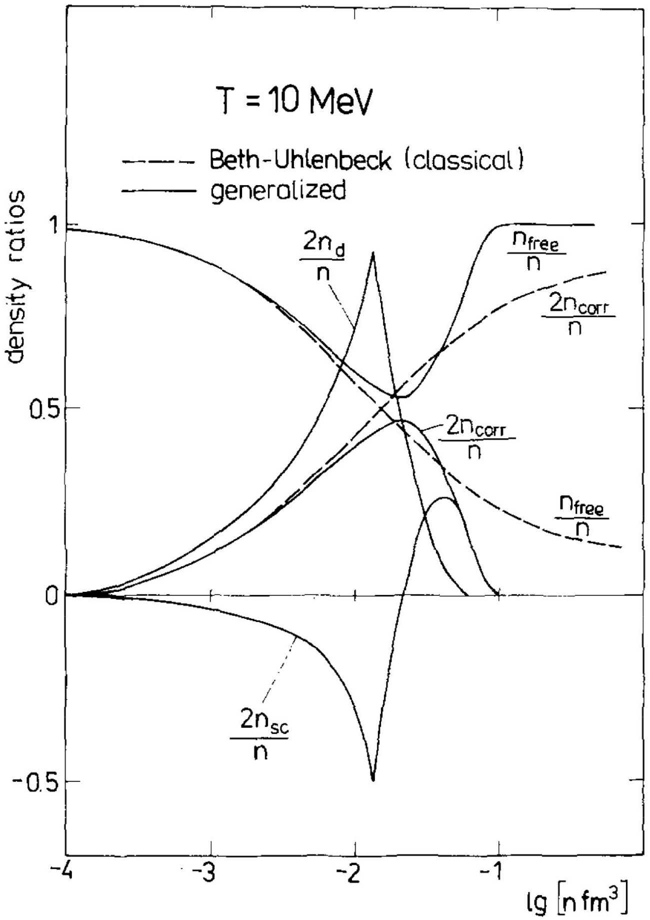
G.R., Nucl. Phys. A **867**, 66 (2011)

Scattering phase shifts in matter



Two-particle correlations

Generalized
Beth-Uhlenbeck Approach
for Hot Nuclear Matter



M. Schmidt, G.R., H. Schulz
Ann. Phys. **202**, 57 (1990)

FIG. 7. The composition of nuclear matter as a function of the density n for given temperature $T = 10$ MeV. The solid and dashed lines show the results of the generalized and classical Beth–Uhlenbeck approach, respectively. Note the distinct behavior of n_{free} and n_{corr} predicted by the two approaches in the low and high density limit!

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$

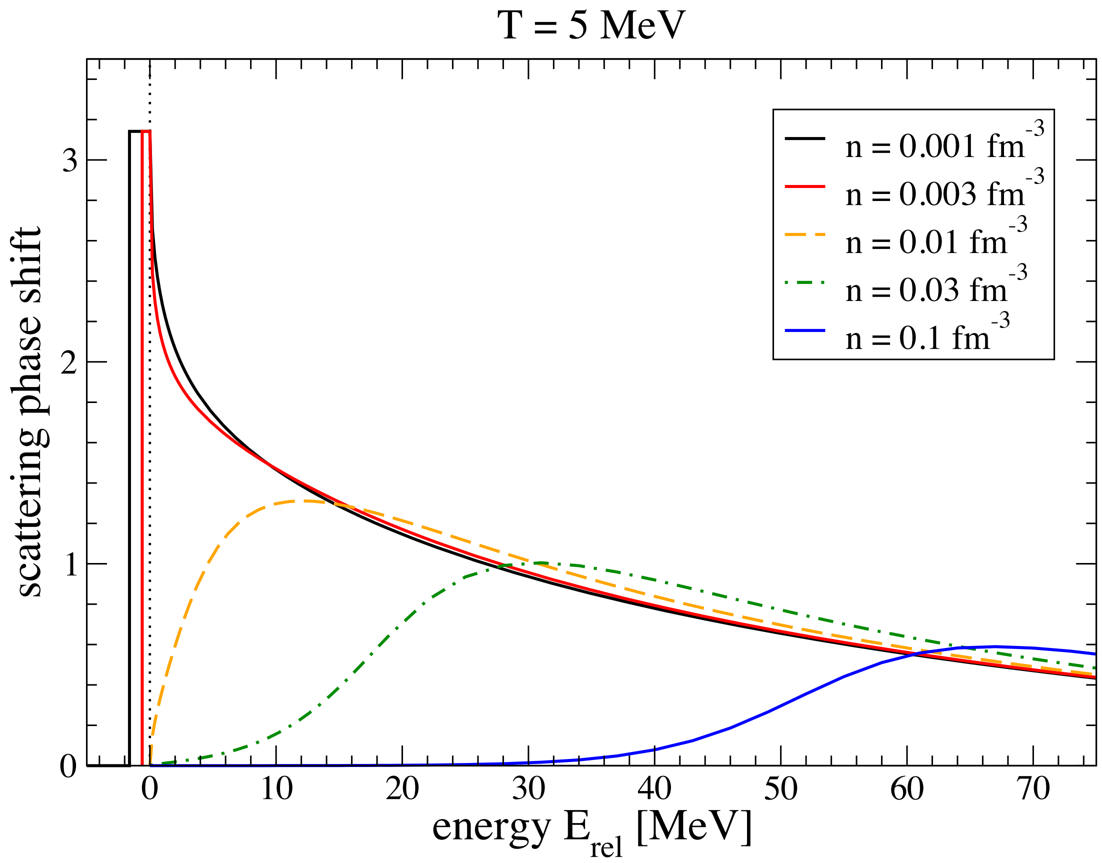
$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

ν: internal quantum number
excited states, continuum correlations

- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\, e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



Tamm-Dancoff

deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

chemical & physical picture

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

parametrization (d – like):

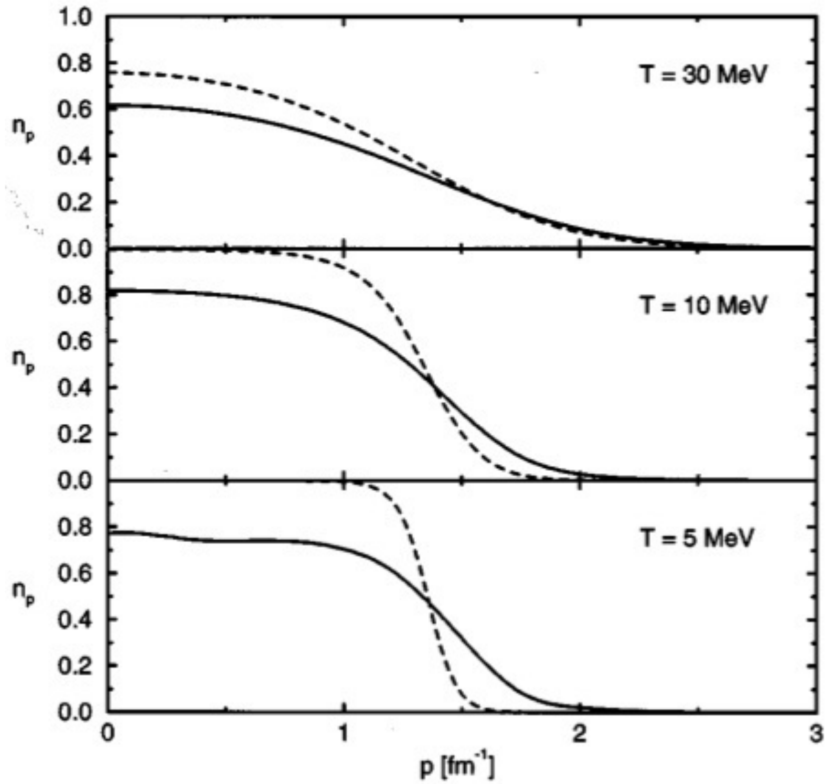
$$v_c(\mathbf{P}=0;T,n_B,Y_p)\approx\left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T)=v_{T_I=0}^0(T)\approx 0.30857+0.65327\;e^{-0.102424\;T/{\rm MeV}}$$

G. Roepke, PRC 92,054001 (2015)

Single nucleon distribution function

Dependence on temperature



saturation density

Alm et al., PRC 53, 2181 (1996)

Pauli blocking, correlated medium

In-medium Schroedinger equation

$$[E_{\tau_1}(\mathbf{p}_1;T,\mu_n,\mu_p)+\cdots+E_{\tau_A}(\mathbf{p}_A;T,\mu_n,\mu_p)-E_{A\nu}(\mathbf{P};T,\mu_n,\mu_p)]\psi_{A\nu\mathbf{P}}(1\dots A)$$
$$+\sum_{1'\dots A'}\sum_{i<j}[1-n(i;T,\mu_n,\mu_p)-n(j;T,\mu_n,\mu_p)]V(ij,i'j')\prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu\mathbf{P}}(1'\dots i'\dots j'\dots A')=0$$

effective occupation numbers

$$n(1)=f_{1,\tau_1}(1)+\sum_{B=2}^{\infty}\sum_{\bar\nu,\bar{\mathbf{P}}}\sum_{2\dots B}Bf_B\left(E_{B,\bar\nu}(\bar{\mathbf{P}};T,\mu_n,\mu_p)\right)|\psi_{B\bar\nu\bar{\mathbf{P}}}(1\dots B)|^2$$

effective Fermi distribution

$$n(1;T,\mu_n,\mu_p)\approx f_{1,\tau_1}(1;T_{\rm eff},\mu_n^{\rm eff},\mu_p^{\rm eff})$$

blocking by all nucleons

$$n(1;T,\mu_n,\mu_p)\approx\tilde f_{1,\tau_1}(1;T_{\rm eff},n_B,Y_p)$$

effective temperature $\quad T_{\rm eff}\approx 5.5\,{\rm MeV}+0.5\;T+60\;n_B\;{\rm MeV\;\;fm}^3$

G. Roepke, PRC 92,054001 (2015)

In-medium effects

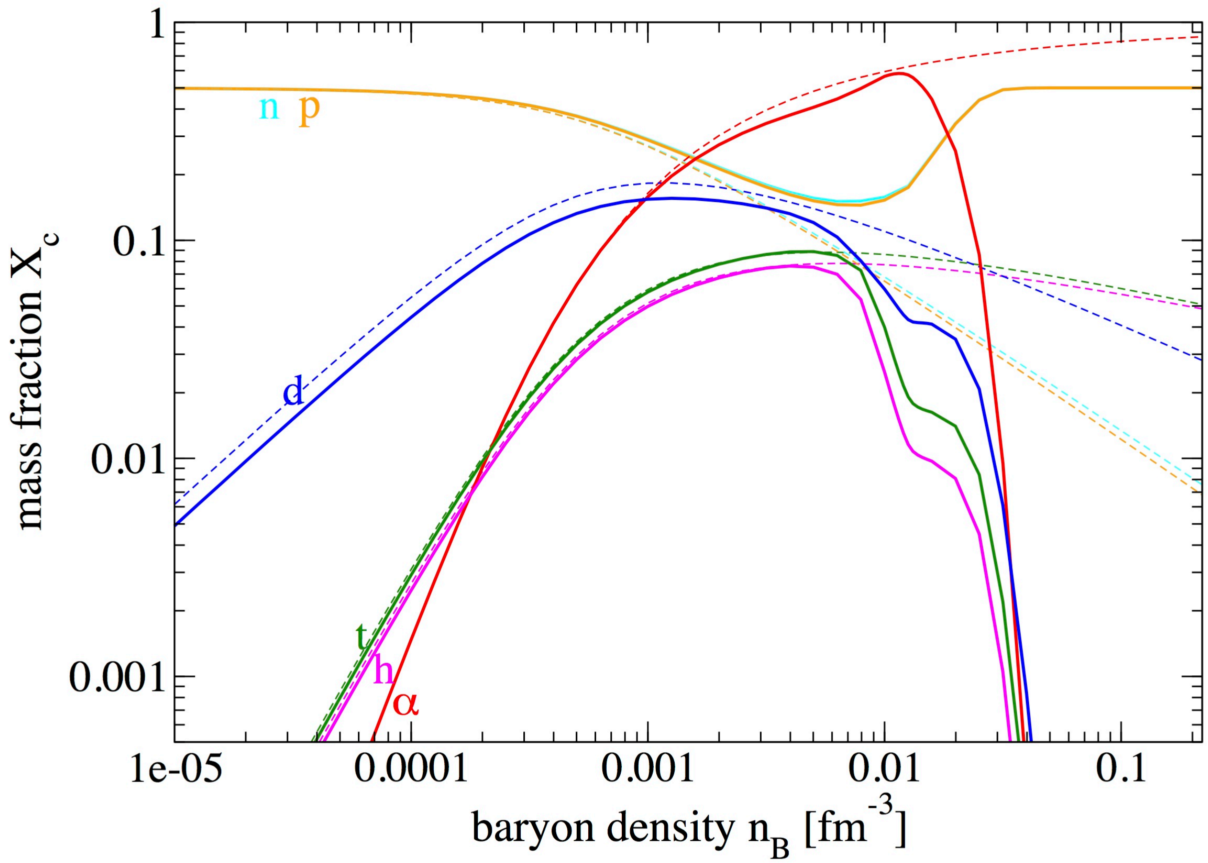
- Self energy, mean-field approximation
- Quasiparticle picture of elementary particles
- Full antisymmetrization: Pauli blocking
- Bound states as new quasiparticles
- Continuum correlations
- Correlated medium

- Quantum statistical approach.
- Excluded volume (Hempel, Schaffner-Bielich,…)
- Generalized relativistic mean field:
  clusters as quasiparticles (Typel, Pais,…)

EoS including correlations

- Composition
- Chemical potential, nuclear matter and stellar matter (β equilibrium)
- Free energy and related quantities, symmetry energy,…
- Phase transition
- Quantum condensates: pairing, quartetting,…

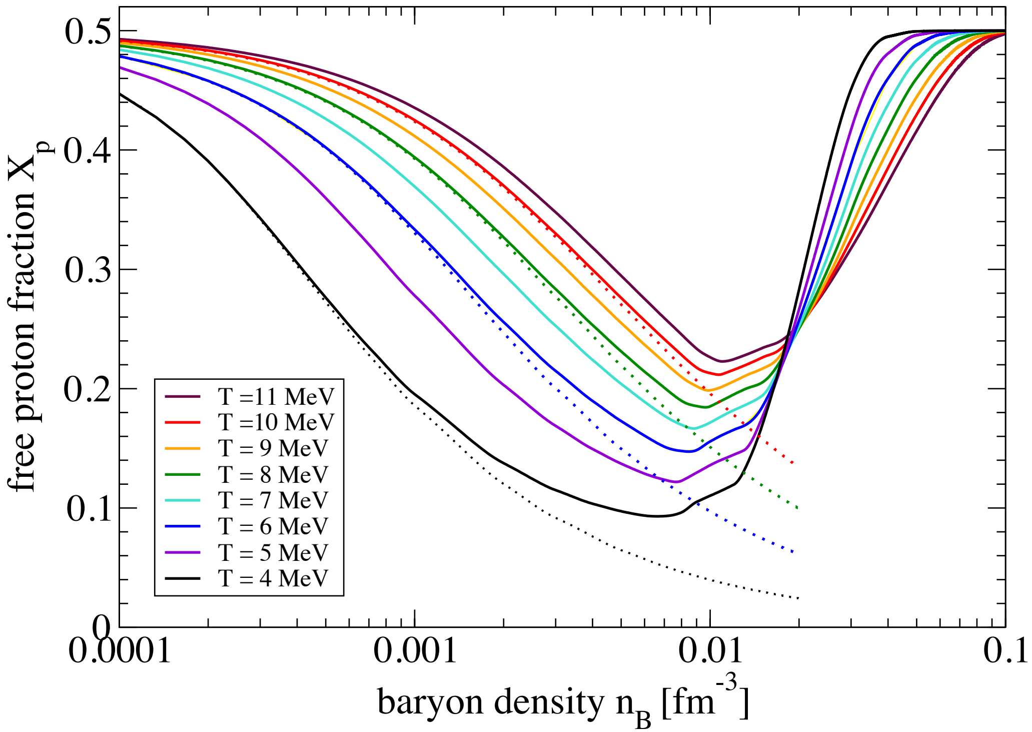
Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

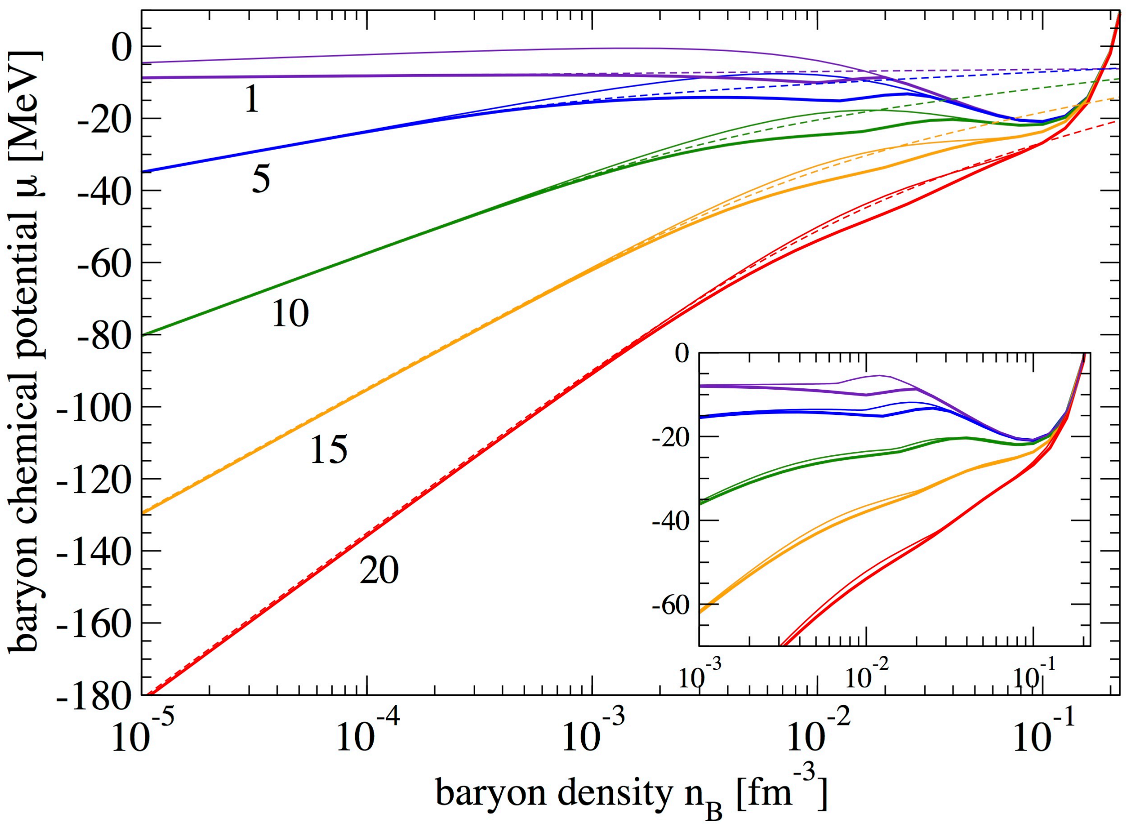
G. R., PRC **92**, 054001 (2015)

Pauli blocking in symmetric matter



Free proton fraction as function of density and temperature in symmetric matter.
QS calculations (solid lines) are compared with the NSE results (dotted lines).
Mott effect in the region $n_{saturation}/5$.

Equation of state: chemical potential



Chemical potential for symmetric matter. T=1, 5, 10, 15, 20 MeV.
QS calculation compared with RMF (thin) and NSE (dashed).
Insert: QS calculation without continuum correlations (thin lines).