UNIVERSITY OF MINNESOTA
School of Physics and Astronomy

# Physics 8702 – Statistical Mechanics and Transport Theory

Homework 13 due Thursday April 25 in class.

**1.** (10 pts) Consider the Schofield parametric scaling equation of state as given in lecture. Show that $P(R,\theta)$ as given there is consistent with the thermodynamic identity $n = (\partial P/\partial \mu)_T$. You do not need to solve the differential equation for $g(\theta)$ but just consider $h(\theta)$ as a given function which acts as a source for $g(\theta)$. Feel free to use the two relations among the critical exponents $\alpha, \beta, \gamma, \delta$.

**2.** (10 pts) In the context of the XY model and the BKT transition, calculate the total energy of a vortex centered at $x = \frac{1}{2}R$ and $y = 0$ and an antivortex centered at $x = -\frac{1}{2}R$ and $y = 0$. This arises from $\theta(\mathbf{x}) = \theta_+(\mathbf{x}) + \theta_-(\mathbf{x})$ where

$$\theta_+(\mathbf{x}) = \tan^{-1}\left(\frac{y}{x - \frac{1}{2}R}\right)$$
$$\theta_-(\mathbf{x}) = -\tan^{-1}\left(\frac{y}{x + \frac{1}{2}R}\right)$$

It amounts to calculating the cross terms between the vortex and antivortex in the Hamiltonian and adding them to the known energies of individual vortices and antivortices. For mathematical simplicity I suggest working in polar coordinates. However, removing the core energies is not trivial. To first approximation you could remove the interval $\frac{1}{2}R - a < r < \frac{1}{2}R + a$, although this removes a ring in the plane rather than disks centered at the vortex and antivortex. Show that this approximation leads to a net energy proportional to $\ln R$ versus $\ln L$ which implies that vortex-antivortex pairs have finite energy below $T_{KT}$ versus logarithmically diverging energies for individual vortices and antivorticies. You may assume that $L \gg R \gg a$.