

$$\textcircled{1} \quad \frac{T - T_c}{T_c} = R(1 - \theta^2) \quad \frac{u - u_c}{u_c} = h_0 R^{\beta\delta} h(\theta)$$

$$\frac{n - n_c}{n_c} = m_0 R^\beta \theta \quad \text{Claim is that } \left(\frac{\partial P}{\partial m}\right)_T = n \text{ with}$$

$$P - P_c = [u(R, \theta) - u_c]n(R, \theta) - m_0 h_0 u_c n_c R^{2-\alpha} g(\theta)$$

The independent variables are  $R$  and  $\theta$ .

Calculate  $dP$  and divide by  $dm$  with  $T$  fixed.

$$\frac{dT}{T_c} = (1 - \theta^2) dR - 2R\theta d\theta = 0$$

$$dm = h_0 u_c \left[ \beta\delta R^{\beta\delta-1} h(\theta) dR + R^{\beta\delta} h'(\theta) d\theta \right]$$

$$= h_0 u_c \left\{ \beta\delta R^{\beta\delta-1} h(\theta) \left[ \frac{2R\theta d\theta}{1 - \theta^2} \right] - R^{\beta\delta} h'(\theta) d\theta \right\}$$

$$= \frac{h_0 u_c R^{\beta\delta}}{1 - \theta^2} \left[ 2\beta\delta \theta h(\theta) - (1 - \theta^2) h'(\theta) \right] d\theta$$

$$dn = m_0 n_c \left[ \beta R^{\beta-1} \theta dR + R^\beta d\theta \right]$$

$$= m_0 n_c \left\{ \beta R^{\beta-1} \theta \left[ \frac{2R\theta d\theta}{1 - \theta^2} \right] + R^\beta d\theta \right\} =$$

$$= \frac{m_0 h_c R^\beta}{1-\theta^2} \int [2\beta\theta^2 + (1-\theta^2)] d\theta$$

$$dP = n d\mu + (\mu - \mu_c) dn - m_0 h_0 \mu_c h_c \left[ (2-\alpha) R^{1-\alpha} g(\theta) dR \right. \\ \left. + R^{2-\alpha} g'(\theta) d\theta \right] \quad \uparrow \quad \frac{2R\theta d\theta}{1-\theta^2}$$

$$= n d\mu + h_0 \mu_c R^{\beta\delta} h(\theta) dn - \frac{m_0 h_0 \mu_c h_c R^{2-\alpha}}{1-\theta^2} \cdot$$

$$\cdot \int [2(2-\alpha)\theta g(\theta) + (1-\theta^2)g'(\theta)] d\theta$$

To have  $\left(\frac{\partial P}{\partial n}\right)_T = \mu$  requires that  $g(\theta)$  satisfies

$$0 = R^{\beta\delta} h(\theta) \left[ \frac{m_0 h_c R^\beta}{1-\theta^2} \right] \int [2\beta\theta^2 + (1-\theta^2)] d\theta$$

$$- \frac{m_0 h_c R^{2-\alpha}}{1-\theta^2} \int [(1-\theta^2)g'(\theta) + 2(2-\alpha)\theta g(\theta)] d\theta$$

Use  $\alpha + 2\beta + \delta = 2$  and  $\beta(\delta-1) = \delta$  to get

$\beta(\delta+1) = 2-\alpha$  so the  $R$ 's drop out.

We require that

$$0 = (1 + 2\beta\theta^2 - \theta^2)h(\theta) - (1 - \theta^2)g'(\theta) + 2(2 - \alpha)\theta g(\theta)$$

$$\Rightarrow (1 - \theta^2)g'(\theta) + 2(2 - \alpha)\theta g(\theta) = (1 + 2\beta\theta^2 - \theta^2)h(\theta)$$