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Non-Hermitian control of Hermitian waveguide arrays

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Introduction

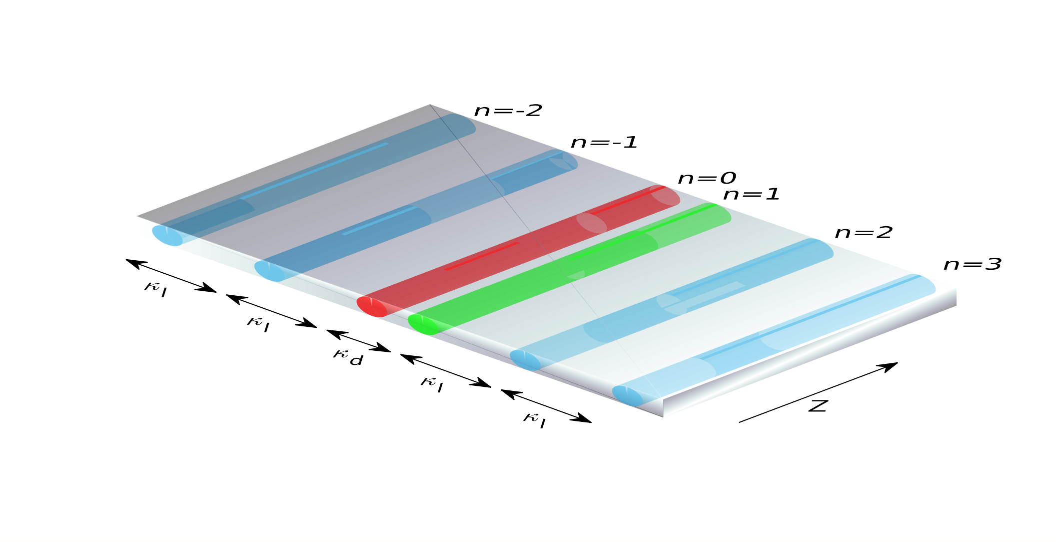
- ✓ Dynamical control of wave propagation in optical lattices *execution of optical op tions*

- *Optical waveguides' arrays* beam propagates under the influence of a spatially periodic, refractive index distribution

- *in our case, wave dynamics approximated considering only amplitude/phase of each individual waveguide, simulating dynamical behavior, characteristic of discrete systems*

Objective

Device under investigation: array of optical waveguides with embedded, active, nonlinear elements

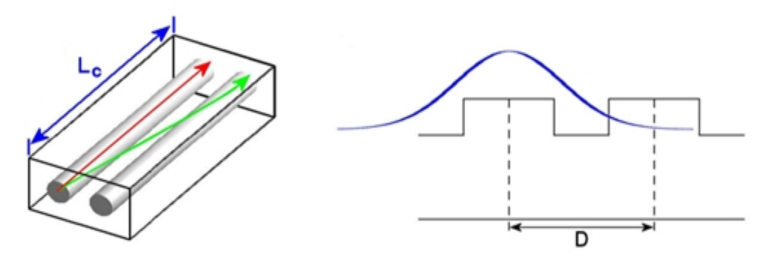


Analysis performed by following 3 primary
steps:

- theoretical background of wave propagation in individual elements
- verification through numerical analysis in the aforementioned elements
- numerical analysis on the device under investigation

Coupled mode theory

✓ *Modelling wave propagation in coupled waveguides* ***(CMT)***



- **Single-mode waveguide fields**

$$\vec{E}_n = \sum_r E_{nr}\hat{\mathbf{r}} = \sum_r e_{nr}(x,y)e^{ik_{nr}z}\hat{\mathbf{r}}$$
$$\vec{H}_n = \sum_r H_{nr}\hat{\mathbf{r}} = \sum_r h_{nr}(x,y)e^{ik_{nr}z}\hat{\mathbf{r}}$$

- **Perturbed fields**

$$\vec{E}' = \sum_{m,r} u_{mr}(z)e_{mr}(x,y)e^{ik_{mr}z}\hat{\mathbf{r}}$$
$$\vec{H}' = \sum_{m,r} u'_{mr}(z)h_{mr}(x,y)e^{ik_{mr}z}\hat{\mathbf{r}}$$

general Maxwell equations

$$\nabla \times \vec{E}_n = i\omega\mu_0\vec{H}_n$$
$$\nabla \times \vec{H}_n = -i\omega\varepsilon_n\vec{E}_n$$
$$\nabla \times \vec{E}' = i\omega\mu_0\vec{H}'$$
$$\nabla \times \vec{H}' = -i\omega(\varepsilon'\vec{E}' + \vec{P}'^{NL})$$

nonlinear term- Kerr effect

General coupled mode equations:

$$i\frac{dA_{nx}}{dz} + k_{nx}A_{nx} + i\sum_{m\neq n} c_{mnxy}\frac{dA_{mx}}{dz} + \sum_{m\neq n} c_{mnxy}k_{mx}A_{mx}$$
$$+ \chi_{nx}A_{nx} + \sum_{m\neq n}\kappa_{nmx}A_{mx} + \gamma_{nx}A_{nx}|A_{nx}|^2 = 0$$

Discrete nonlinear Schrodinger equation (DNLS)

assumptions

$$i\frac{dA_n}{dz} + \beta A_n + \kappa(A_{n+1} + A_{n-1}) + \gamma_n|A_n|^2A_n = 0$$

Hermitian lattice dynamics

➢ Linear waveguide array (γ_n=0)

- plane wave:

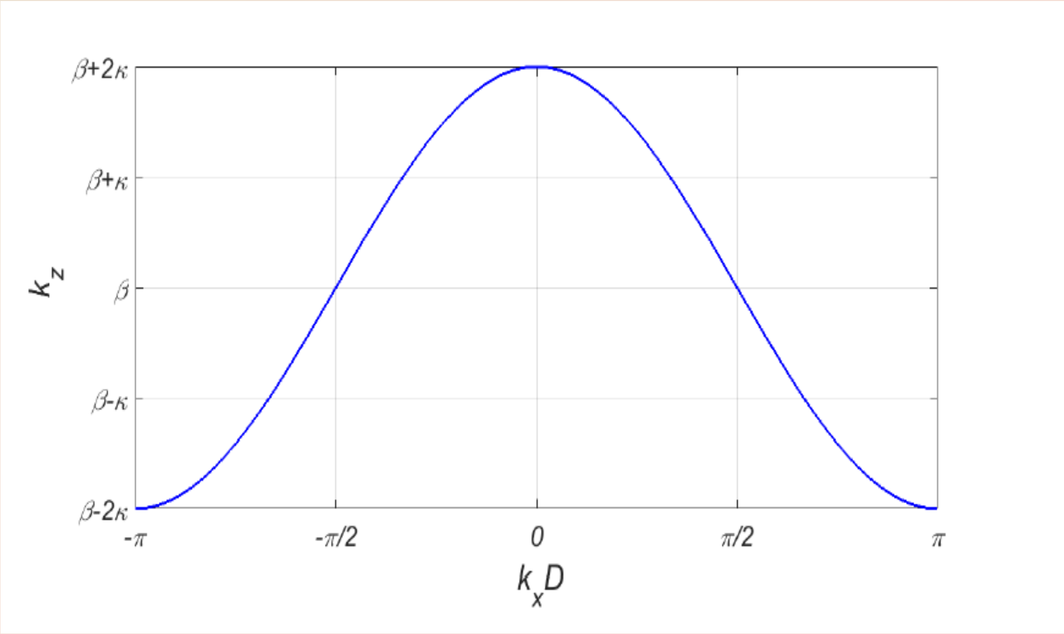
$$E_n(z) = \varphi_n e^{ink_xD} e^{ik_zz}$$

DNLS

$$k_z = \beta + 2\kappa cos(k_xD)$$

1D waveguide array dispersion relation

e. phonon-like behaviour



non-Hermitian dimer dynamics

- **Asymmetric, nonlinear, active coupler:**

$$\frac{dA_1}{dz} = i(\beta_1 + i\alpha_1)A_1 + i\gamma(|A_1|^2+\sigma|A_2|^2)A_1 + i\kappa A_2$$
$$\frac{dA_2}{dz} = i(\beta_2 + i\alpha_2)A_2 + i\gamma(|A_2|^2+\sigma|A_1|^2)A_2 + i\kappa A_1$$

Steady states:
Nonlinear Supermodes (NS)

2 supported NS:

$$A_j = |A_j|exp(i(bz + \varphi_j))$$

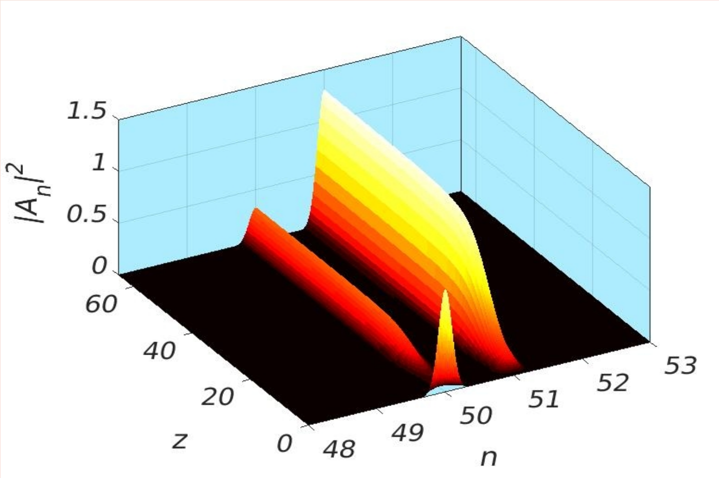
with:

$$|A_1|^2 = \frac{\beta_1\alpha(\beta-1) + \kappa\sqrt{\alpha}(\alpha-1)cos\varphi}{\gamma(1-\sigma)(\alpha-1)}$$
$$|A_2|^2 = \frac{1}{\alpha}|A_1|^2$$

where:

$$b = \frac{1}{2}\left[\beta_1(\beta+1) + \kappa\frac{\alpha+1}{\sqrt{\alpha}}cos\varphi + \gamma(\sigma+1)\frac{\alpha+1}{\alpha}|A_1|^2\right]$$
$$sin\varphi = \frac{|\alpha_1|\sqrt{\alpha}}{\kappa}$$
$$\alpha \equiv -\alpha_2/\alpha_1$$
$$\beta \equiv \beta_2/\beta_1$$

e.g. $\alpha_1=1$, $\alpha_2=-0.25$, $\beta_1=1$, $\beta_2=2$, $\gamma=-1$, $\sigma=0$, $\kappa=0.5$



non-Hermitian dimer dynamics

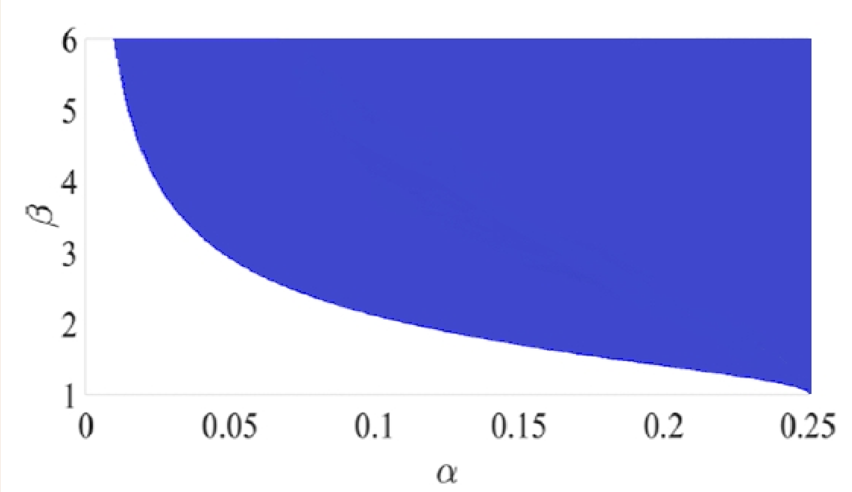
- ***How dimer dynamics determine array's behaviour***

- ***Stability of zero (ground state solution)*** whether di is ***on/off***
- ***Stability of the NS*** in ng ***perturbations***

$$u_{1,2} = \left(A_{1,2} + \varepsilon_{1,2}\right) e^{ibz}$$

$$\varepsilon_{1,2} = c_{1,2}e^{-i\lambda z} + d^*_{1,2}e^{i\lambda^* z}$$

simulating noise in realistic applications



stability region, i.e. eigenvalues of the system with Im(λ)<0

γ=-1, σ=0, κ_d=0.5

Model

- **Coupled mode equations under the assumption of nearest neigbour coupling:**

$$\frac{dA_n}{dz} = i\left((\beta_n + i\alpha_n)A_n + \kappa_{n+1}A_{n+1} + \kappa_{n-1}A_{n-1} + \gamma_n A_n|A_n|^2\right)$$

- discrete array of linear, passive waveguides (γ_n=0, α_n=0, without loss of generality β_n=0)
- asymmetric, active dimer, with self-defocusing nonlinearity ($\gamma_{N/2}$=$\gamma_{N/2+1}$ =-1)
- we investigate wave propagation by variating array's parameters, α_1, α_2, β_1, β_2, κ_l, κ_d
- evaluating array's properties, with the dimer acting as a scatterer, an input/output device and a detector
- scattered beam discrete gaussian with initial amplitude profile:

$$f(n, z=0) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(ik(n-n_i) - \frac{(n-n_i)^2}{2\sigma^2}\right)$$

Scattering dynamics

- **Plane wave** → good approximation for a sufficiently wide gaussian beam

$$A_n = exp(ikn - i\beta z) + R exp(-ikn - i\beta z),\ n \le 0$$
$$A_n = T exp(ikn - i\beta z),\ n \ge 1$$

- reflection/transmission coefficients for the linear case (dimer switched off)

DNLS for dispersionless beam ($k=\pi/2$)

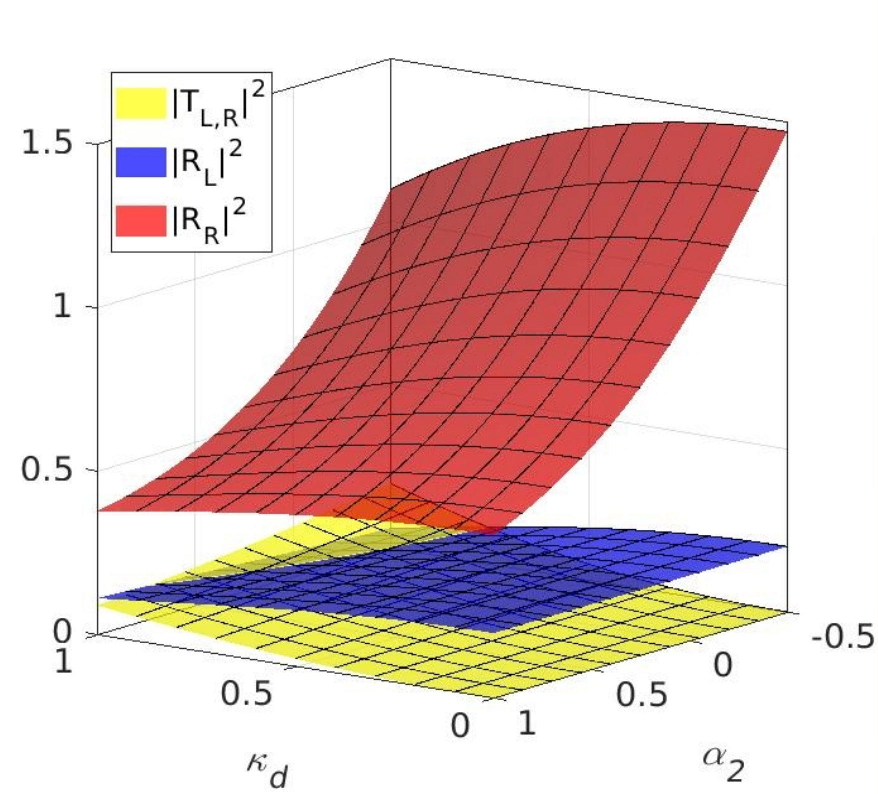
$$|T_L|^2 = \frac{4\bar{\kappa}^2}{D}$$

$$|R_L|^2 = \{\{\bar{\kappa}^2 + \bar{\kappa}(\bar{\alpha}_2 - \bar{\alpha}_1) - 1 + \bar{b}_1\bar{b}_2 - \bar{\alpha}_1\bar{\alpha}_2\}^2 + \{\bar{\kappa}(\bar{b}_2 - \bar{b}_1) - (\bar{\alpha}_1\bar{b}_2 + \bar{b}_1\bar{\alpha}_2)\}^2\}/D$$

where:

$$D = \{\bar{\kappa}^2 + \bar{\kappa}(\bar{\alpha}_1 + \bar{\alpha}_2) + 1 - \bar{b}_1\bar{b}_2 + \bar{\alpha}_1\bar{\alpha}_2\}^2 + \{\bar{\kappa}(\bar{b}_1 + \bar{b}_2) + (\bar{\alpha}_1\bar{b}_2 + \bar{b}_1\bar{\alpha}_2)\}^2$$

$$\bar{\kappa} = \frac{\kappa_l}{\kappa_d},\ \bar{\alpha}_n = \frac{\alpha_n}{\kappa_d},\ \bar{b}_n = \frac{b_n}{\kappa_d}$$



$\alpha_1=1$, $b_1=1$, $b_2=2$, $\kappa_l=1$

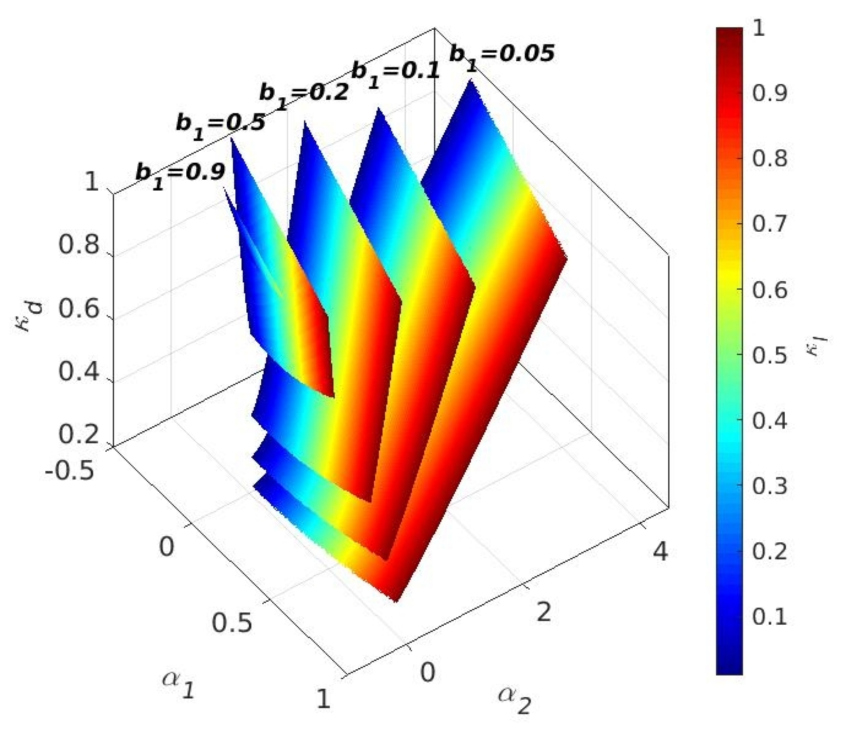
R_R, T_R → interchanging $\bar{\alpha}_1$, $\bar{b}_1$ with $\bar{\alpha}_2$, $\bar{b}_2$

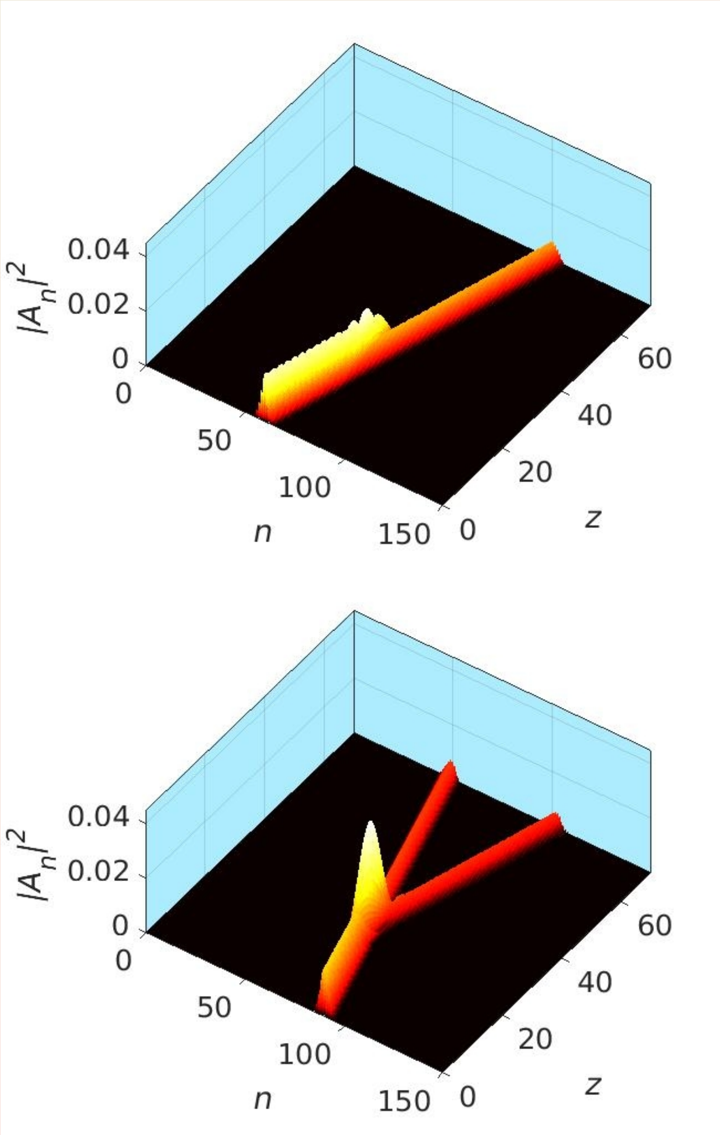
Scattering dynamics

- **Zero reflection** (geometrically dissimilar dimer ($b_1/b_2=1$)):

gaussian wavepacket simulation

PW calculated parameters



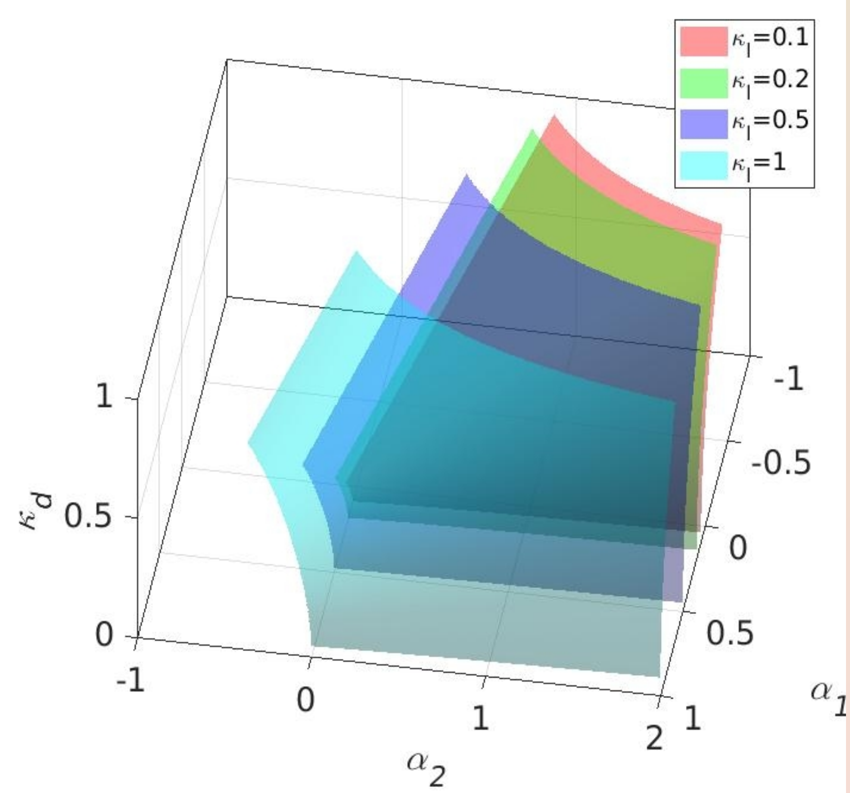


$a_1=-0.124$
$a_2=0.548$
$\kappa_l=0.300$
$\kappa_d=0.927$
$b_1=0.5$ $b_2=1$

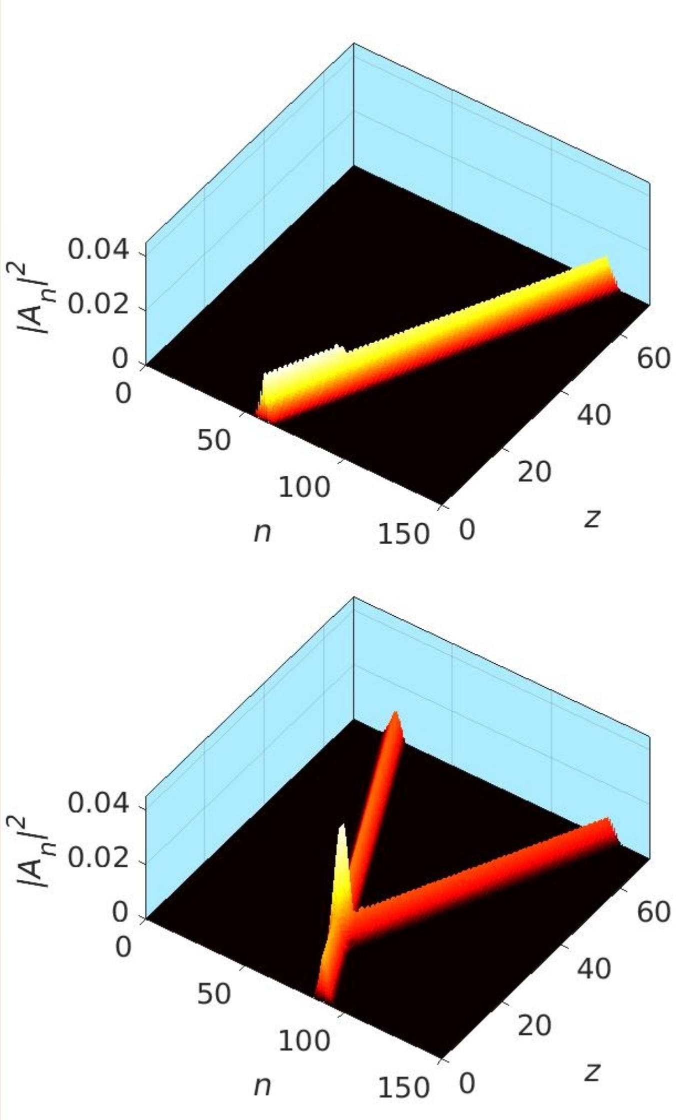
Scattering dynamics

- ***Zero reflection*** ***(geometrically identical waveguides (b_n=0)):***

PW calculated parameters



gaussian wavepacket simulation

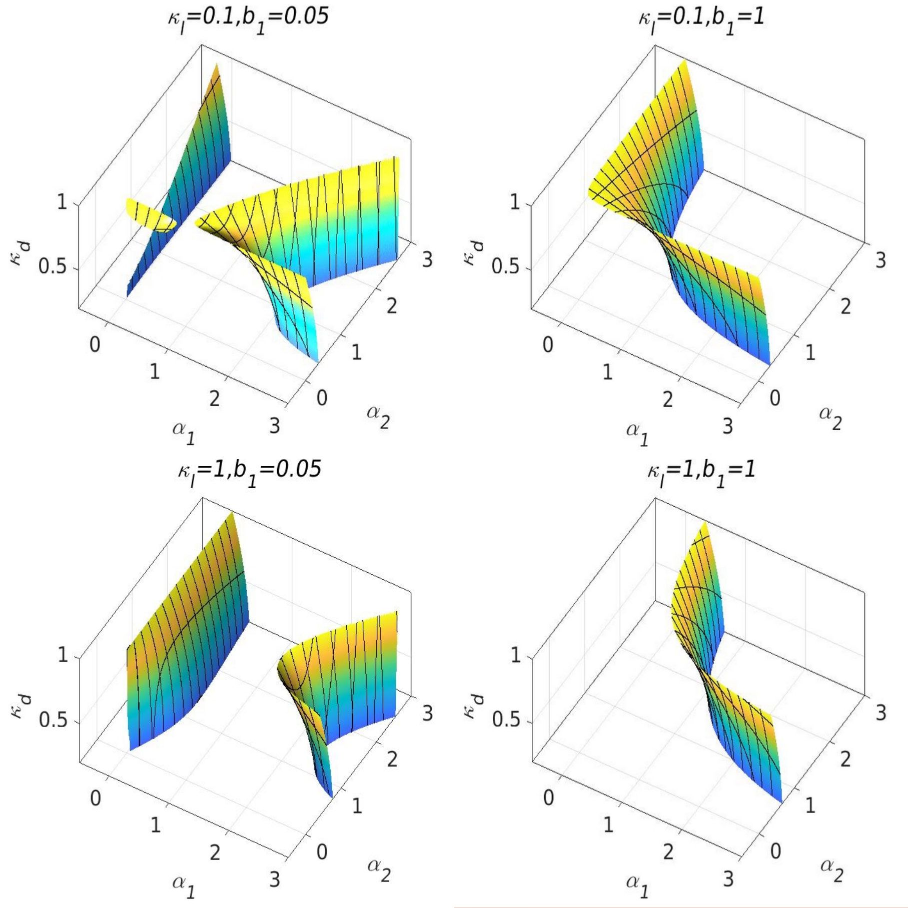


a_1=−0.301
a_2=0.472
κ_l=0.500
κ_d=0.882
b_1=0
b_2=0

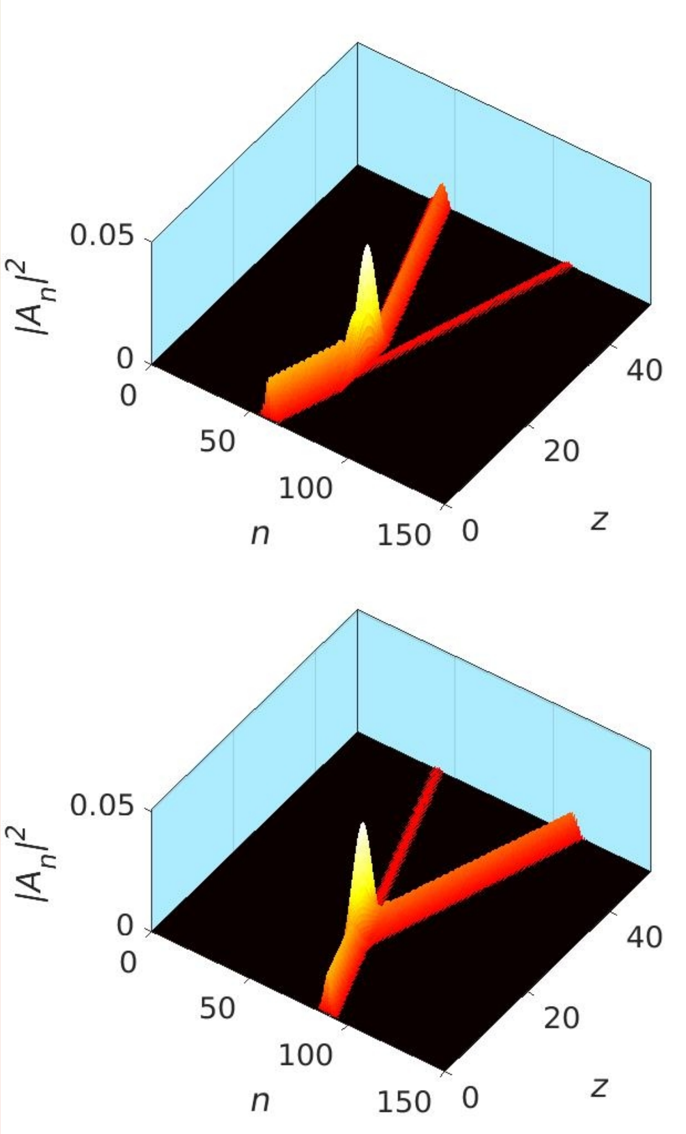
Scattering dynamics

- **Symmetrical reflection:**

PW calculated parameters



gaussian wavepacket simulation

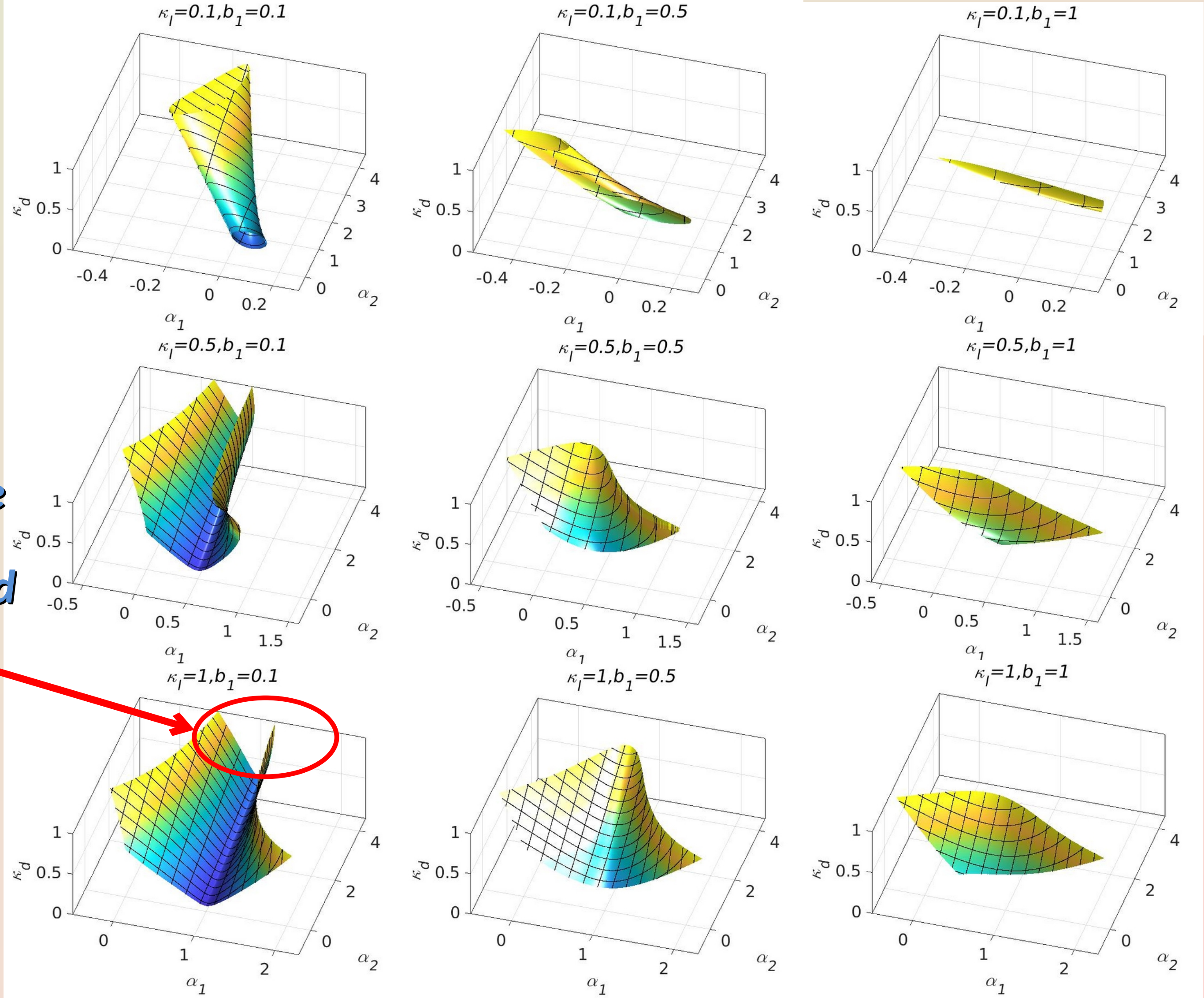


$a_1=-0.100$
$a_2=3.43$
$\kappa_l=0.500$
$\kappa_d=0.795$
$b_1=0.1$
$b_2=1$

Scattering dynamics

- **Reflection equal to transmission:**

PW calculated parameters



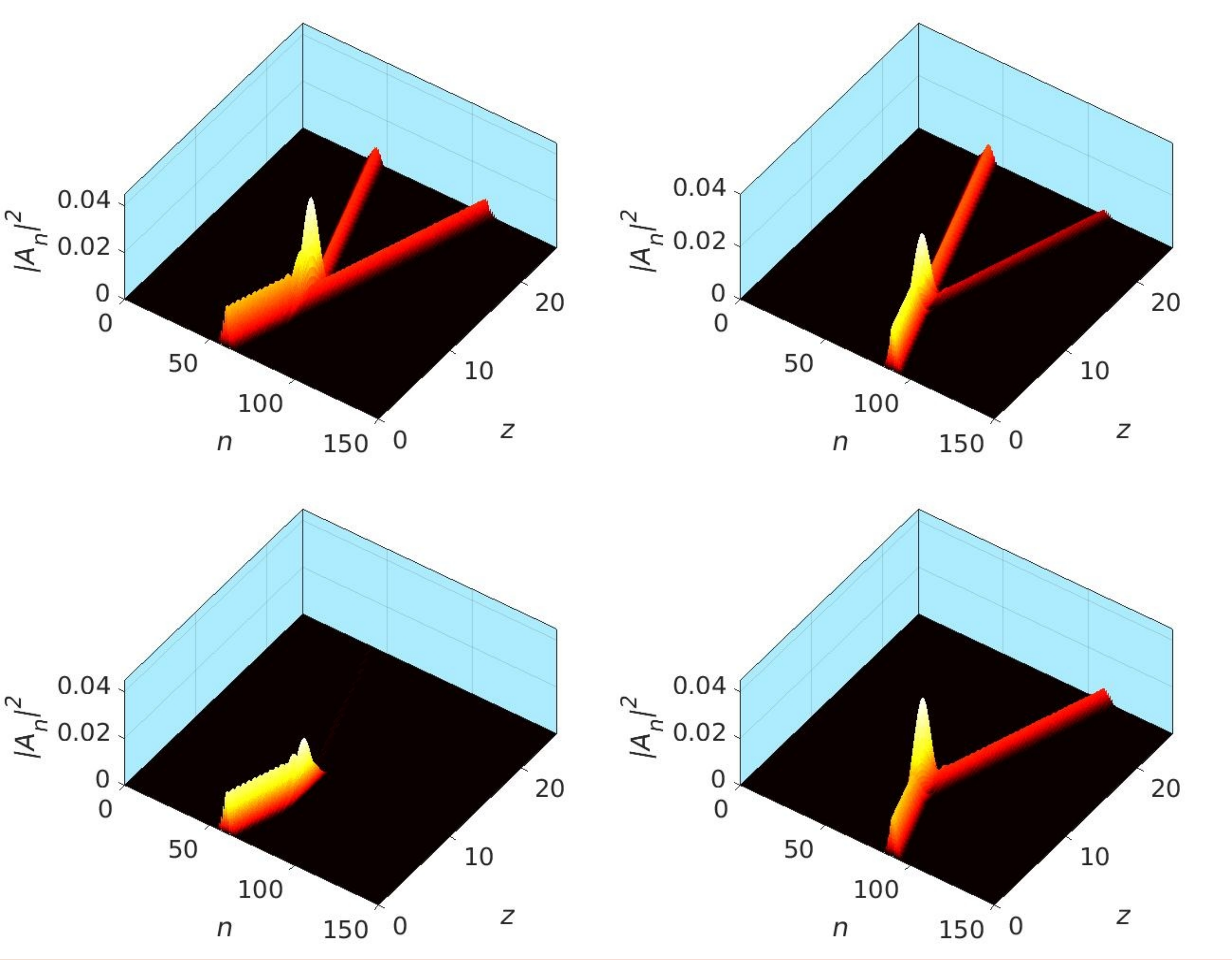
areas with large a_1, a_2, additionally lead to

R,T ≈0.

Scattering dynamics

- **Reflection equal to trransmission:**

gaussian wavepacket simulation



$a_1=-0.144$
$a_2=0.868$
$\kappa_l=1$
$\kappa_d=0.877$
$b_1=0.1$
$b_2=1$

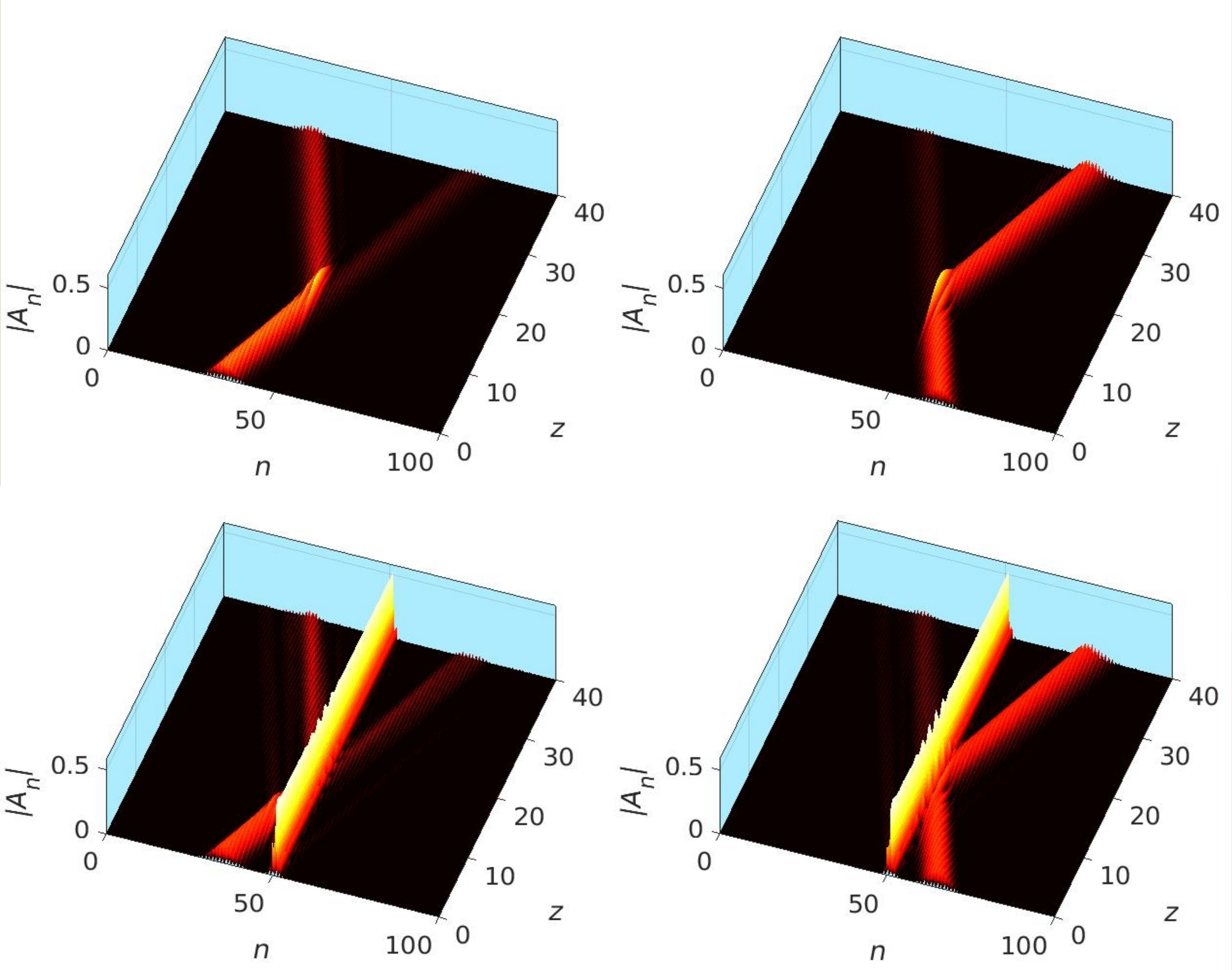
$a_1=1.177$
$a_2=4.097$
$\kappa_l=1$
$\kappa_d=0.799$
$b_1=0.1$
$b_2=1$

Scattering dynamics

- **Nonlinear scattering (dimer on):**

a_1=1, a_2=−0.05
κ_d=0.45, κ_l=0.5
b_1=1, b_2=6

gaussian wavepacket simulation



Dimer off

$|R_L|^2 = 0.25$
$|T_{L,R}|^2 = 0.06$
$|R_R|^2 = 0.92$

no qualitative difference

Dimer on

$|R_L|^2 = 0.21$
$|T_L|^2 = 0.10$
$|R_R|^2 = 0.86$
$|T_R|^2 = 0.12$

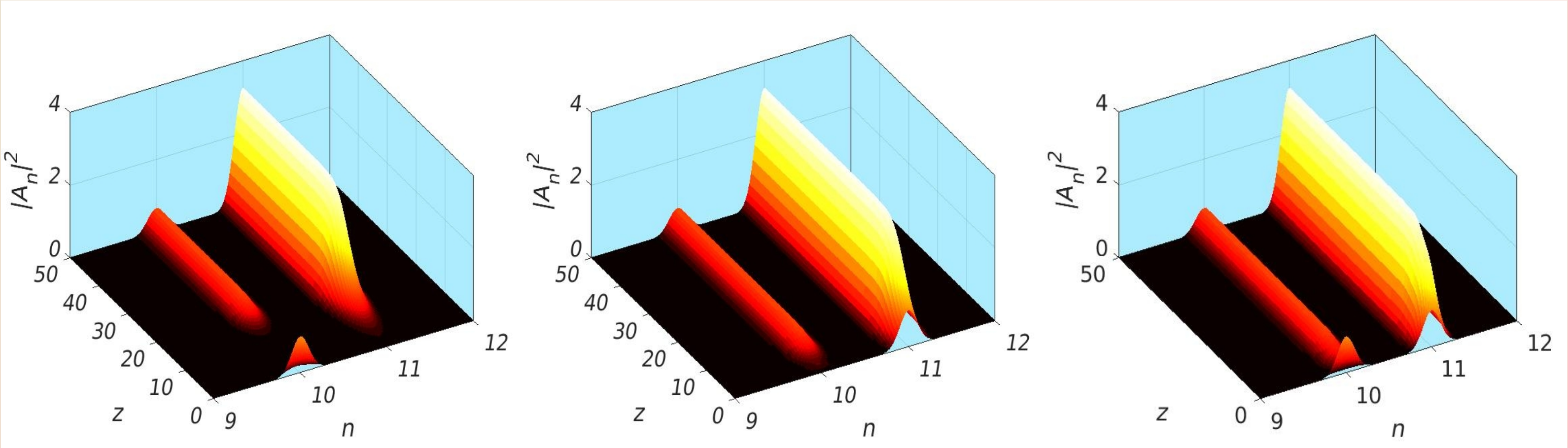
Embedded dimer dynamics

- ***Input/output device:***

Isolated dimer:

- ***test for different initial conditions (IC) i.e. input in loss/gain/both dimer's waveguides***
- ***taking advantage of different convergence speed to final state (NS)***

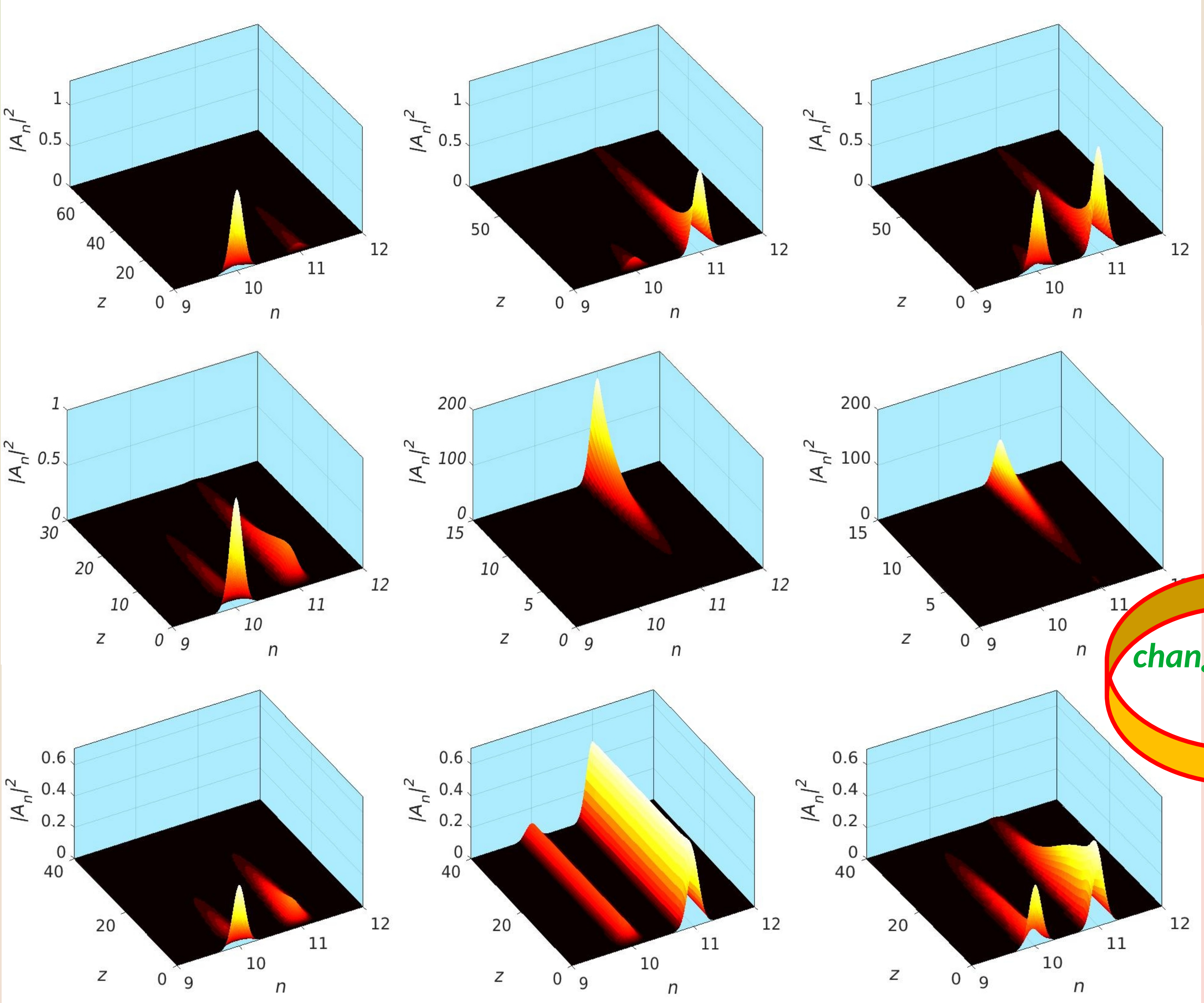
identical output irrespectively of IC
(with $(|A_2|/|A_1|)^2=-a_1/a_2$)



$a_1=1$, $a_2=-0.2$, $b_1=1$, $b_2=4$, $\kappa_d=0.5$, $P_0=1$

Embedded dimer dynamics

- **But….**



a_1=1, a_2=−0.05, b_1=1, b_2=2.5, κ_d=0.5, P_0=1
zero output irrespectively of IC

a_1=1, a_2=−0.2, b_1=1, b_2 =1, κ_d=0.5, P_0=1
infinite/zero output

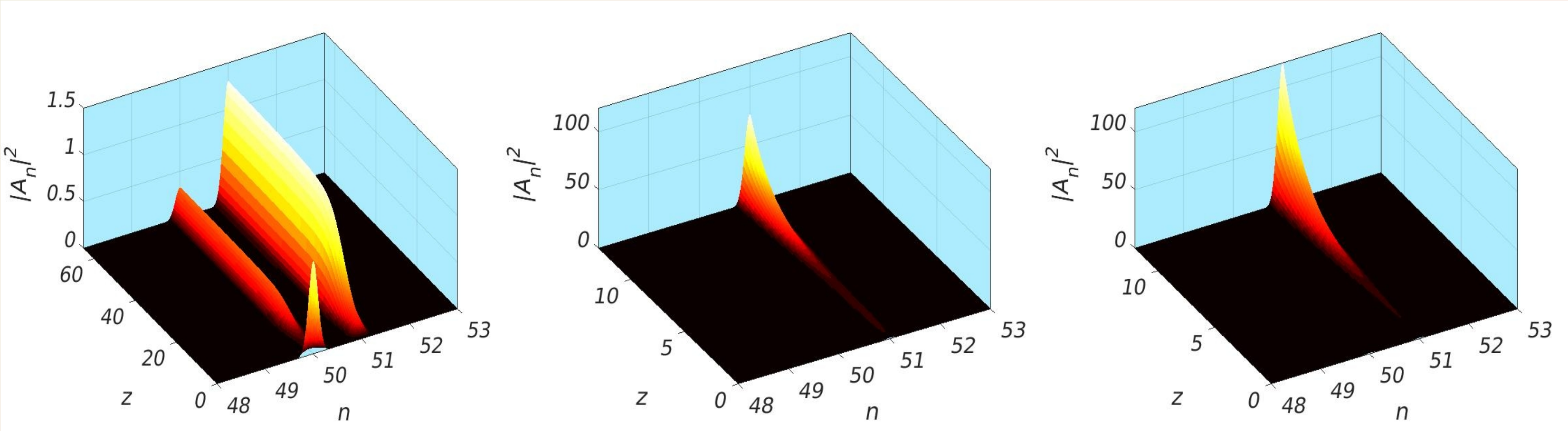
a_1=1, a_2=−0.2, b_1=1, b_2=1, κ_d=0.5, P_0=0.6256
finite/zero output

changing input power

Embedded dimer dynamics

- ***Input/output device*:**
  - ***Treating infinities:*** → ***stabilizing through coupling with the linear lattice ($\kappa_l \neq 0$)***

e.g. $a_1=1$, $a_2=-0.25$, $b_1=1$, $b_2=2$, $\kappa_d=0.5$, $P_0=1$

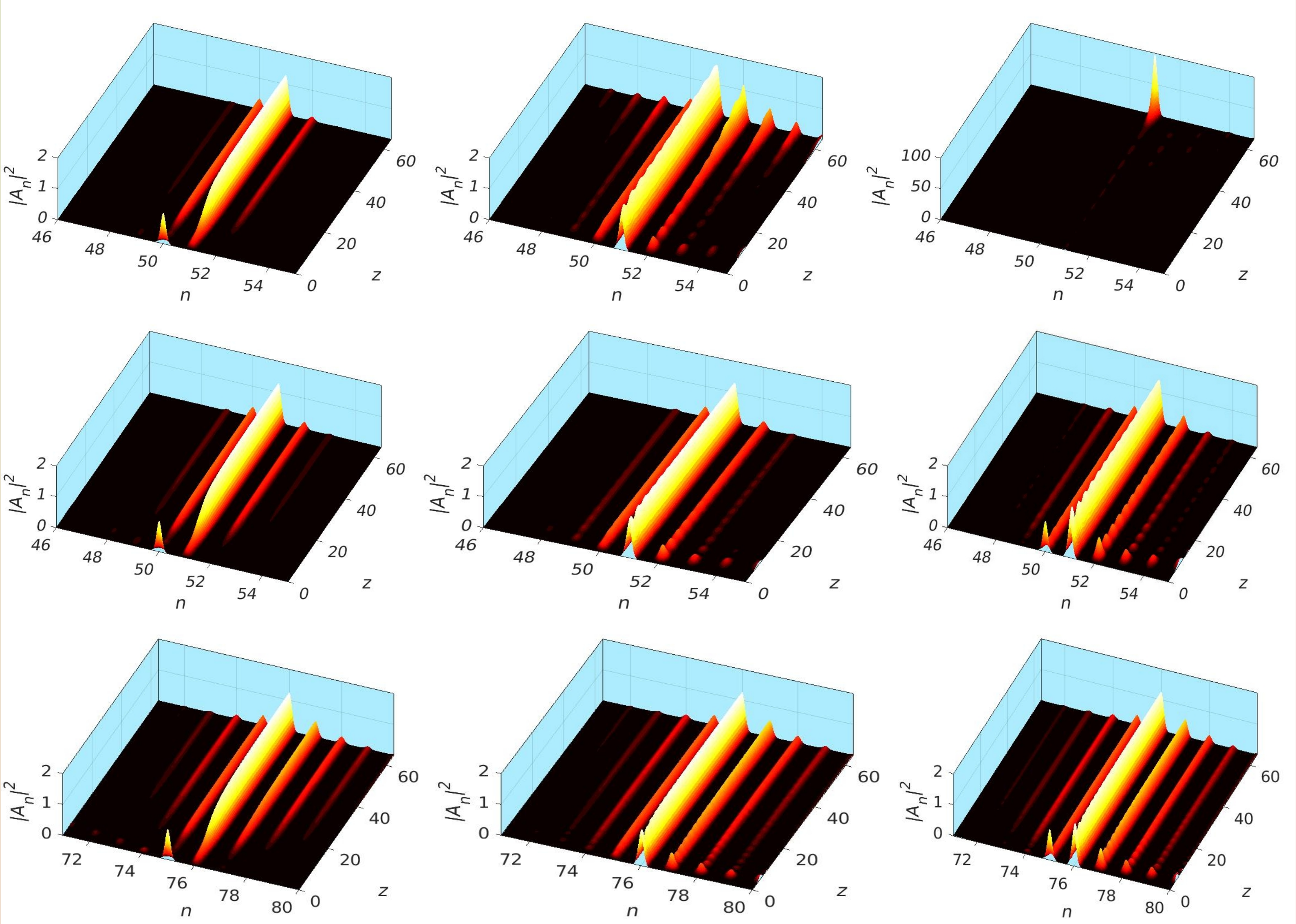


$\kappa_l = 0$

uncoupled dimer (only loss input stable)

Embedded dimer dynamics

- **But….**



κ$_l$ =0.225
gain input stabilized

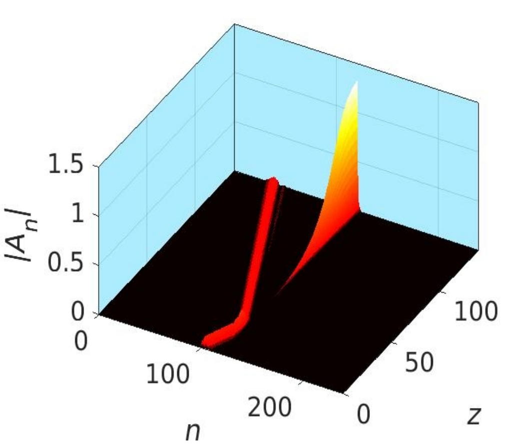
κ$_l$ =0.3
all inputs stabilized

κ$_l$ =0.5

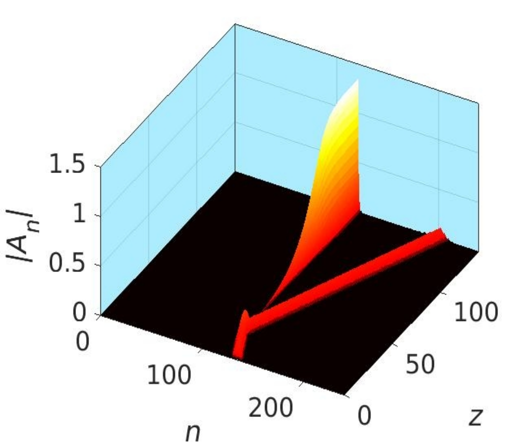
Embedded dimer dynamics

- **Array as a detector:**

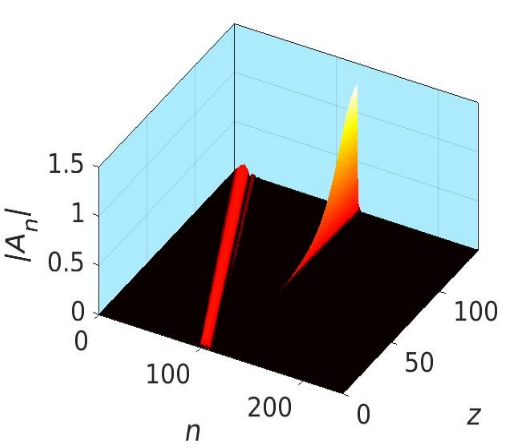
e.g. $a_1=1$, $a_2=-0.08$, $b_1=1$, $b_2=3$, $\kappa_l=0.35$, $\kappa_d = 0.3$



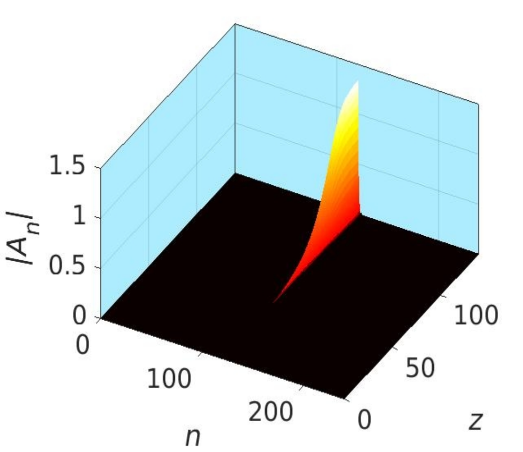
beam scattered by 'loss' waveguide



beam scattered by 'gain' waveguide



non-interacting beam



rng input

Conclusions

- <u>***In total, embedding an active, nonlinear, asymmetric dimer in a waveguide array:***</u>

i. *offers the ability to adjust asymmetric scattering of beams*

ii. *offers the ability to adjust power output during dimer excitation by controlling IC, gain/loss and κ_d*

iii. *has a stabilizing effect on propagating waveforms*

iv. *can be used for detection of propagating waves*

↓

<u>*Dimer controls scattering properties*</u>

<u>*Lattice modifies dimer dynamics*</u>

The End

Thank you for your attention!