Study of correlations of nuclear saturation properties and neutron star f mode oscillations from a machine learning

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The footprint of nuclear saturation properties on the neutron star f mode oscillation frequencies: a machine learning approach

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(**DK**, Tuhin Malik, and Hiranmaya Mishra)

Outline

- **Introduction**
- **Equations of states (RMF) for NS**
  - Relativistic mean field (RMF) with NL couplings
  - Relativistic mean field (RMF) with DD couplings
- **Pearson correlations coefficients**
- **Relations between NSP and NMP using ML**
- **Summary and conclusions**

Introduction

- Neutron stars (NS)s are the exciting natural astrophysical laboratories to study the behaviour of matter at extreme densities.

- Astrophysical observations suggest the mass, $M = (1-2)M_\odot$ and radius, $R = (10-12)$ km.

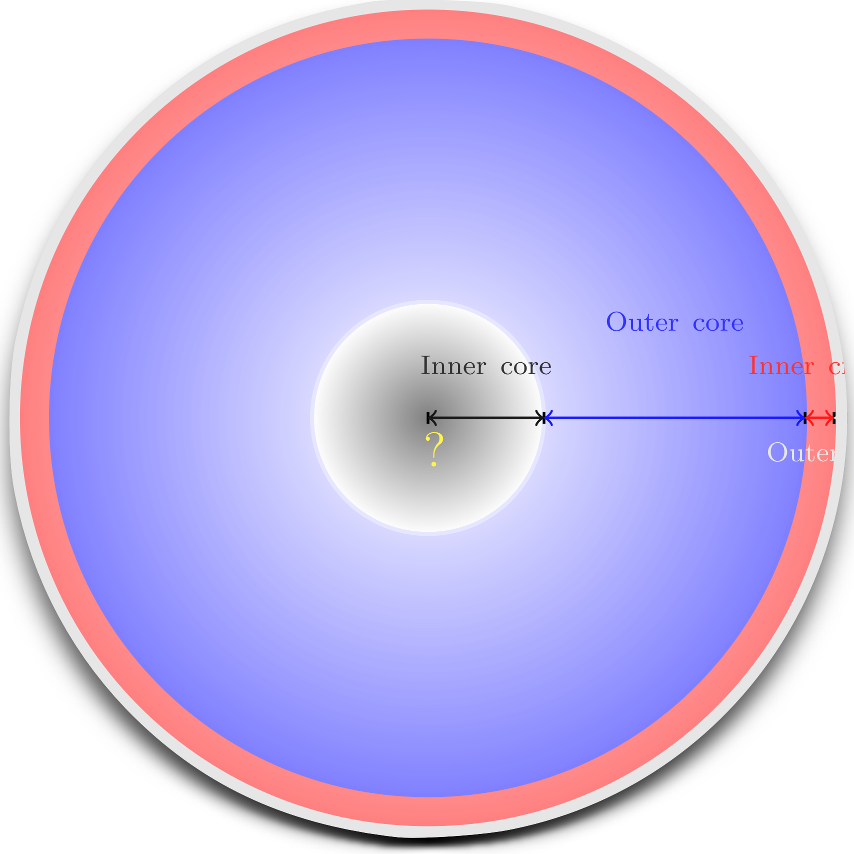
- Density in the core of NSs is few times the nuclear saturation density, $\rho_0 = 0.16\ \text{fm}^{-3}$. At this density, we may have HQPT in the core of NSs.

- Macroscopic properties of such NS like mass, radius, moment of inertia, tidal deformability depend on the equation of state of matter of NS.

$$\frac{dp}{dr} = -\left(\epsilon + p\right)\frac{m + 4\pi r^3 p}{r(r-2m)}, \qquad \frac{dm}{dr} = 4\pi r^2 \epsilon,$$

The boundary conditions:

$$m(r=0) = 0 \quad \text{and} \quad p(r=0) = p_0$$



- **Non-radial oscillations can unveil the neutron star matter.**

Introduction

- Pulsating equations within Cowling Approximations:

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$$Q' - \frac{1}{c_s^2}\left[\omega^2 r^2 e^{\lambda-2\nu} Z + \nu' Q\right] + l(l+1)e^{\lambda} Z = 0$$

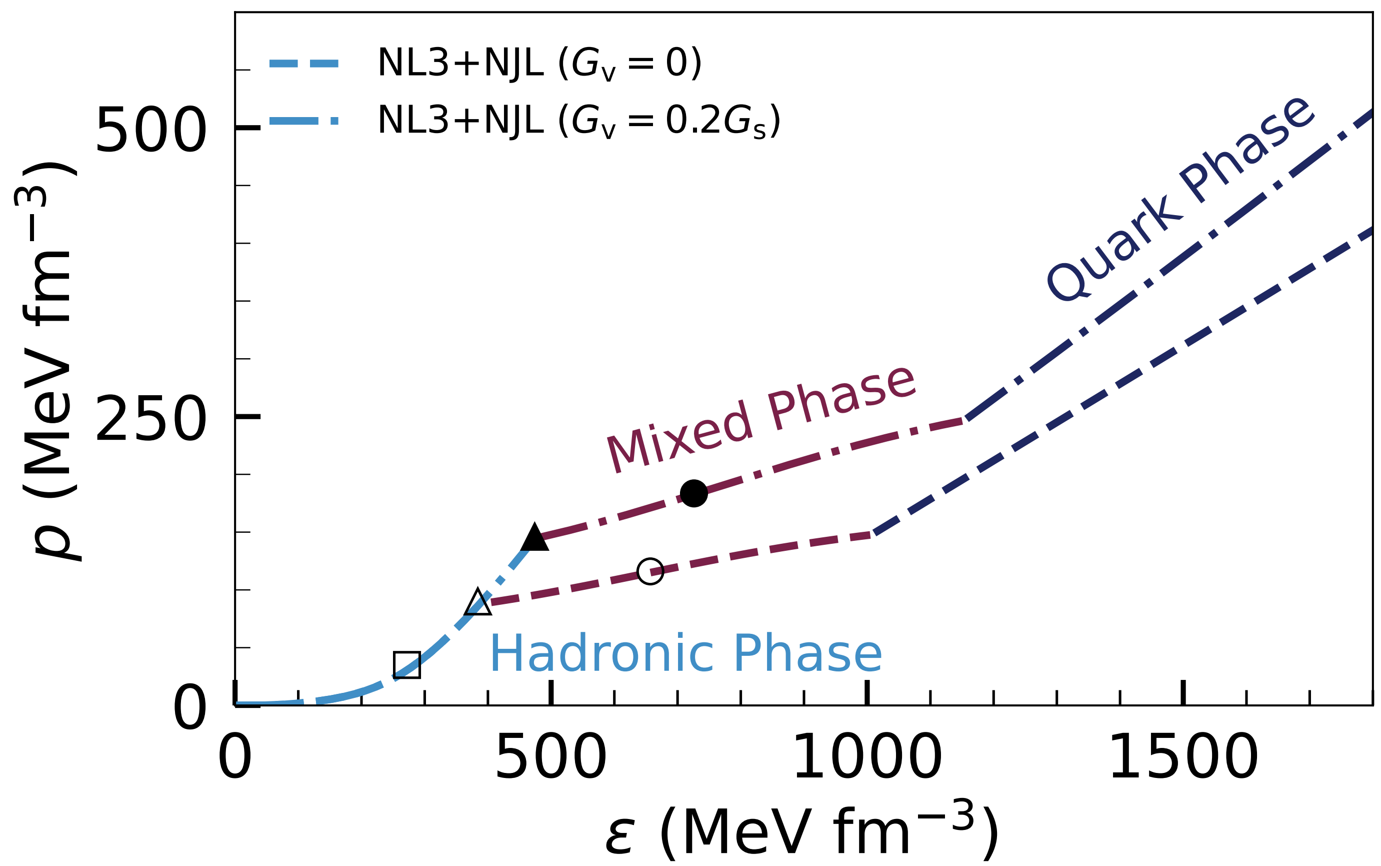
$$Z' - 2\nu' Z + e^{\lambda}\frac{Q}{r^2} - \frac{\omega_{BV}^2 e^{-2\nu}}{\nu'\left(1-\frac{2m}{r}\right)}\left(Z + \nu' e^{-\lambda+2\nu}\frac{Q}{\omega^2 r^2}\right) = 0$$

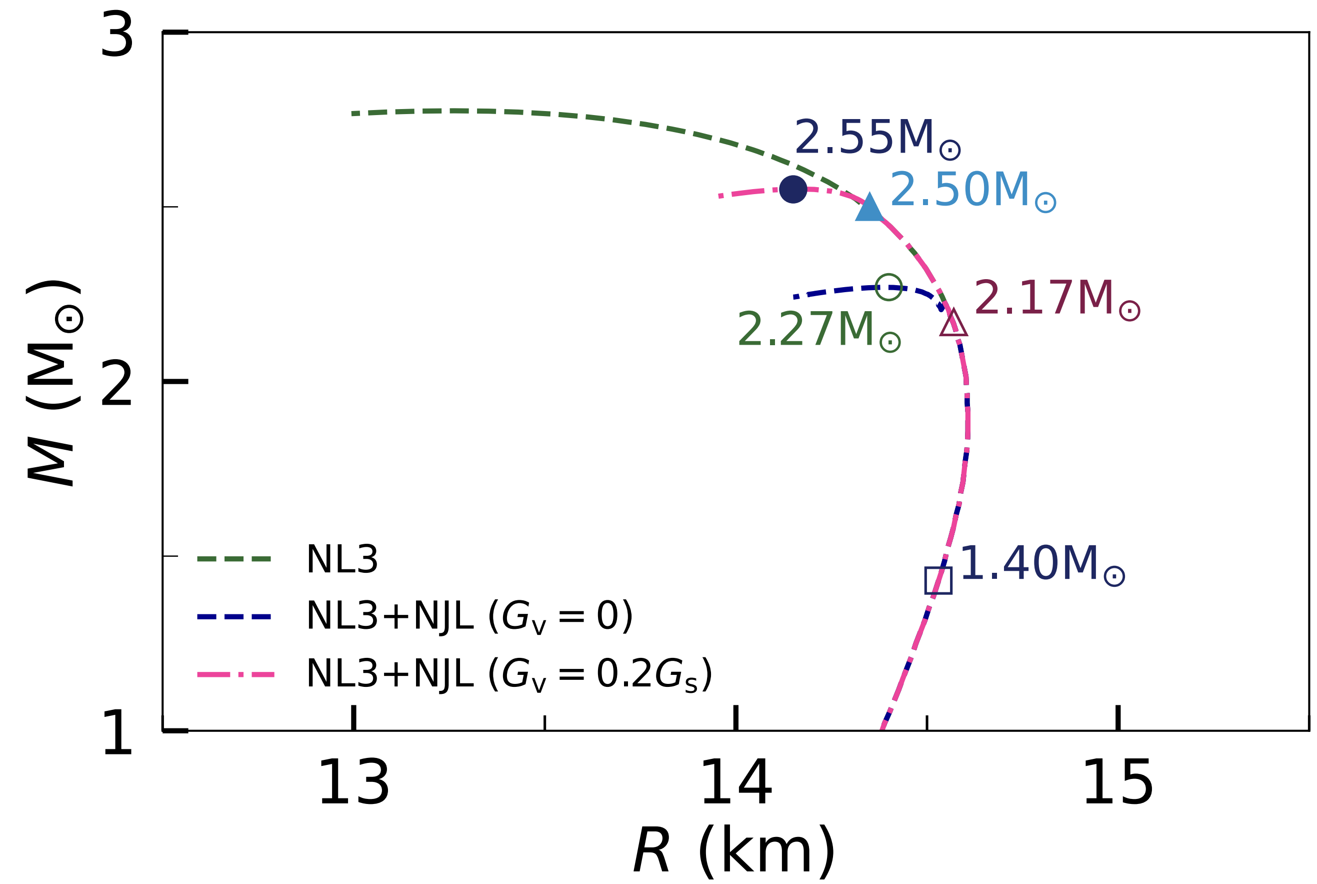
✓ Near the centre:

$$Q(r) = Cr^{l+1}, \quad Z(r) = -Cr^{l}/l$$

✓ At the surface:

$$\left.\omega^2 r^2 e^{\lambda-2\nu} Z + \nu' Q\right|_{r=R} = 0$$





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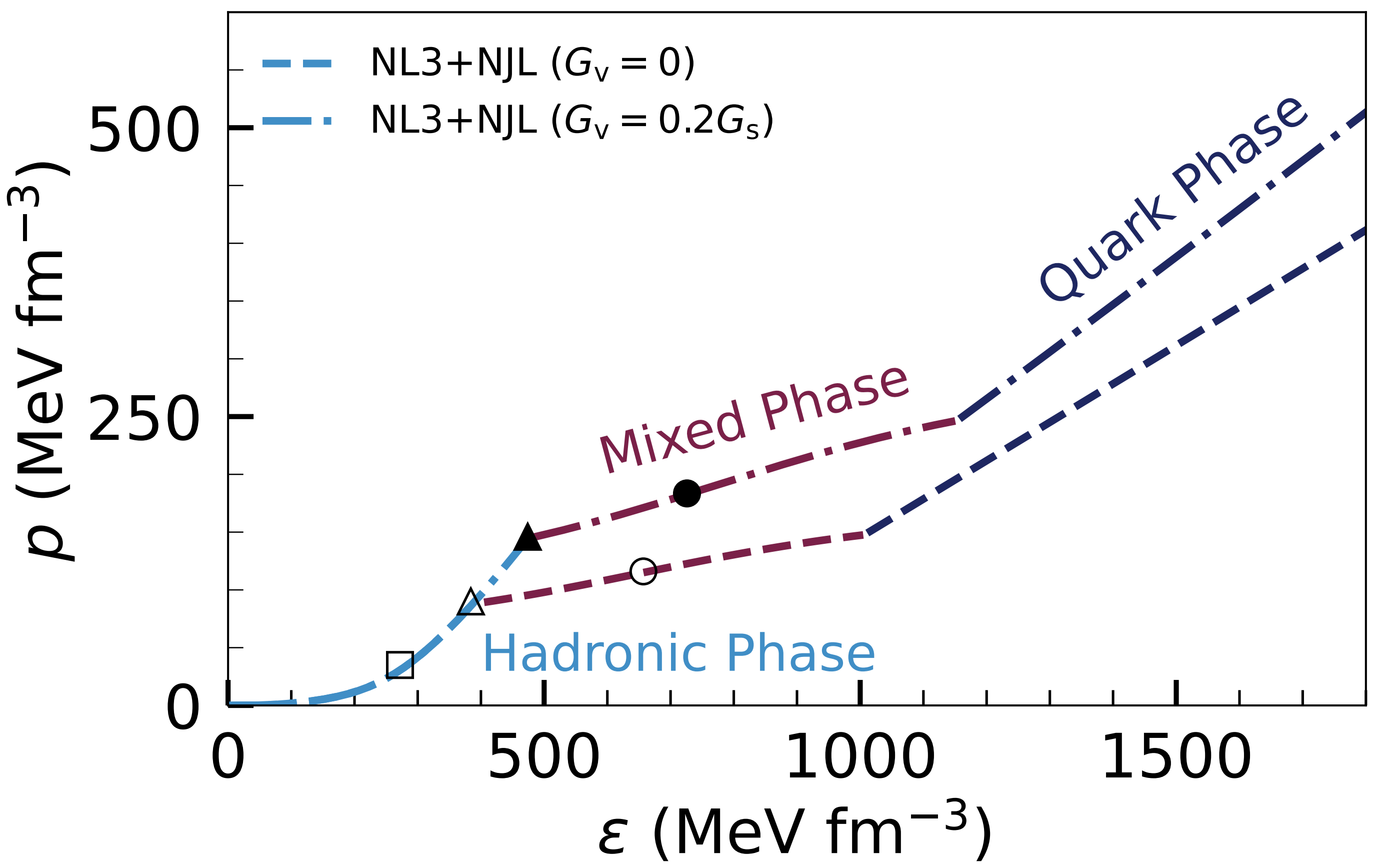
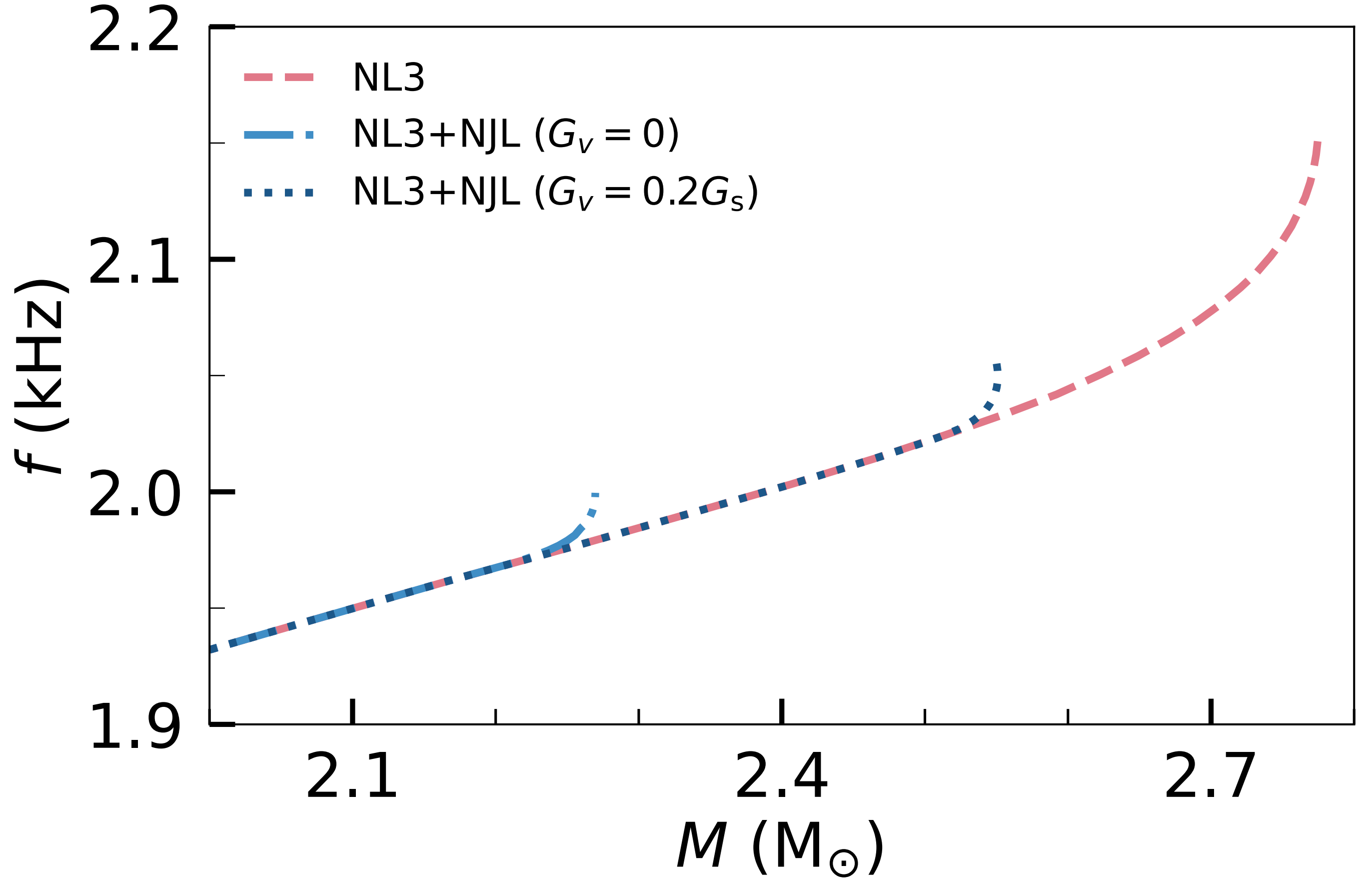




Equations of states (RMF): NL and DD couplings

Nuclear matter (RMF model) EOS

Annals Phys 83 (1974) pp 491-529

$$\mathcal{L} = \sum_b \bar{\Psi}_b \left(i\gamma_\mu \partial^\mu - m_b + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \vec{I}_b \vec{\rho}^\mu \right) \Psi_b + \mathcal{L}_{\text{mes}}$$

$$\mathcal{L}_{\text{mes}} = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{\kappa}{3!}(g_{\sigma N}\sigma)^3 - \frac{\lambda}{4!}(g_{\sigma N}\sigma)^4 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu$$

$$\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$$

$$\vec{R}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$$

- At the mean field level i.e. $\langle \sigma \rangle = \sigma_0$, $\langle \omega_\mu \rangle = \omega_0 \delta_{\mu 0}$ and $\langle \rho_\mu^a \rangle = \delta_{\mu 0} \delta_3^a \rho_3^0$

$$m_b^* = m_b - g_\sigma \sigma_0 \quad \text{and} \quad \mu_b^* = \mu_b - g_\omega \omega_0 - g_\rho I_{3b} \rho_3^0$$

- Equation of state:

$$\epsilon = \frac{1}{\pi^2} \sum_{i=n,p,e} H(m^*/k_F^i) + \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{3} g_2 \sigma_0^3 + \frac{1}{4} g_3 \sigma_0^4 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_3^{02}$$

where, $$H(x) = \frac{1}{8}\left(\sqrt{1+x^2}(2+x^2) - x^4 \ln\left(\frac{x+\sqrt{1+x^2}}{x} \right) \right)$$

$$p = \sum_{i=n,p,e} \mu_i n_i - \epsilon$$

Equations of states (RMF): NL and DD couplings

☑ Nuclear matter properties (NMP)

$$\epsilon(\rho,\delta) \simeq \epsilon(\rho,0) + S_{\rm sym}(\rho)\delta^2, \qquad \text{where} \qquad \delta = \frac{\rho_n - \rho_p}{\rho}$$

- Symmetric matter properties:

$$X_0^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n \epsilon(\rho,0)}{\partial \rho^n}\right)_{\rho_0}, \qquad n = 2,3,4;$$

- Symmetry energy properties:

$$X_{\rm sym,0}^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n S(\rho)}{\partial \rho^n}\right)_{\rho_0}, \qquad n = 1,2,3,4.$$

S. Typel 10.3390/particles1010002

T. Malik etal, ApJ 930 17 (2022)

$$g_\sigma = g_{\sigma 0}\ e^{-(x^{a_\sigma}-1)}$$

$$g_\omega = g_{\omega 0}\ e^{-(x^{a_\omega}-1)}$$

$$g_\rho = g_{\rho 0}\ e^{-a_\rho(x-1)}$$

$$\kappa = 0, \quad \lambda = 0$$

$$x = \rho_B/\rho_0$$

Non-linear RMF

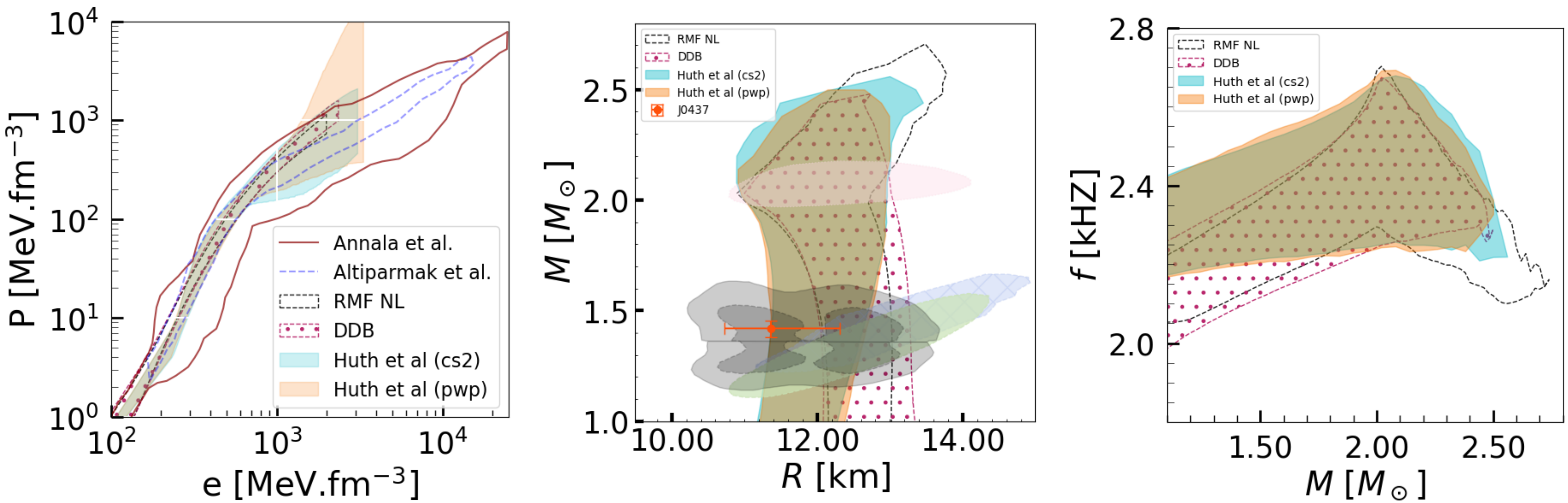
The couplings are constant over the density.

Density dependent RMF

The couplings are density dependent.

Neutron star: EOS's, MR curves and f modes

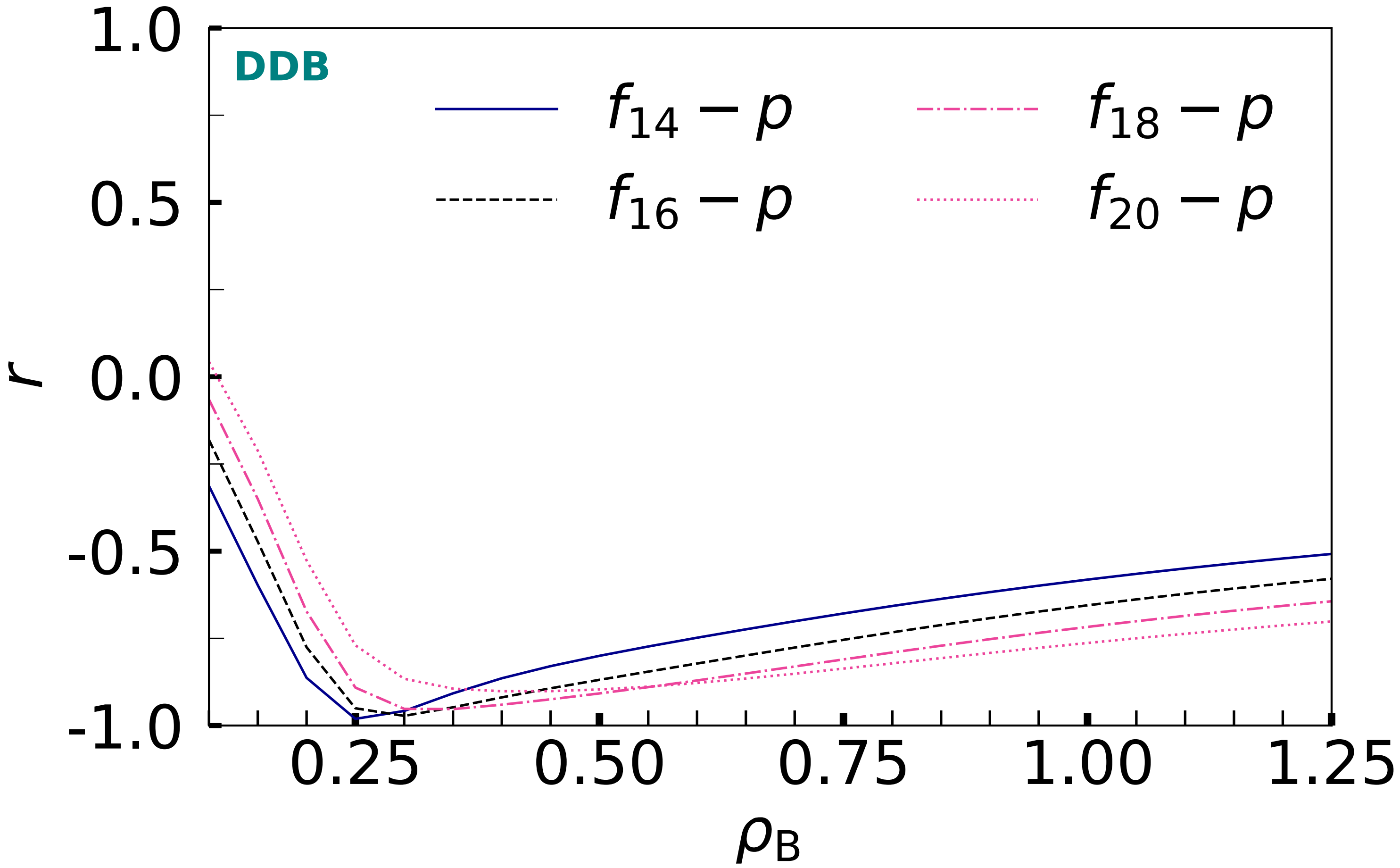
- Equation of states, corresponding mass-radius cloud and the non-radial f-mode frequency-mass cloud

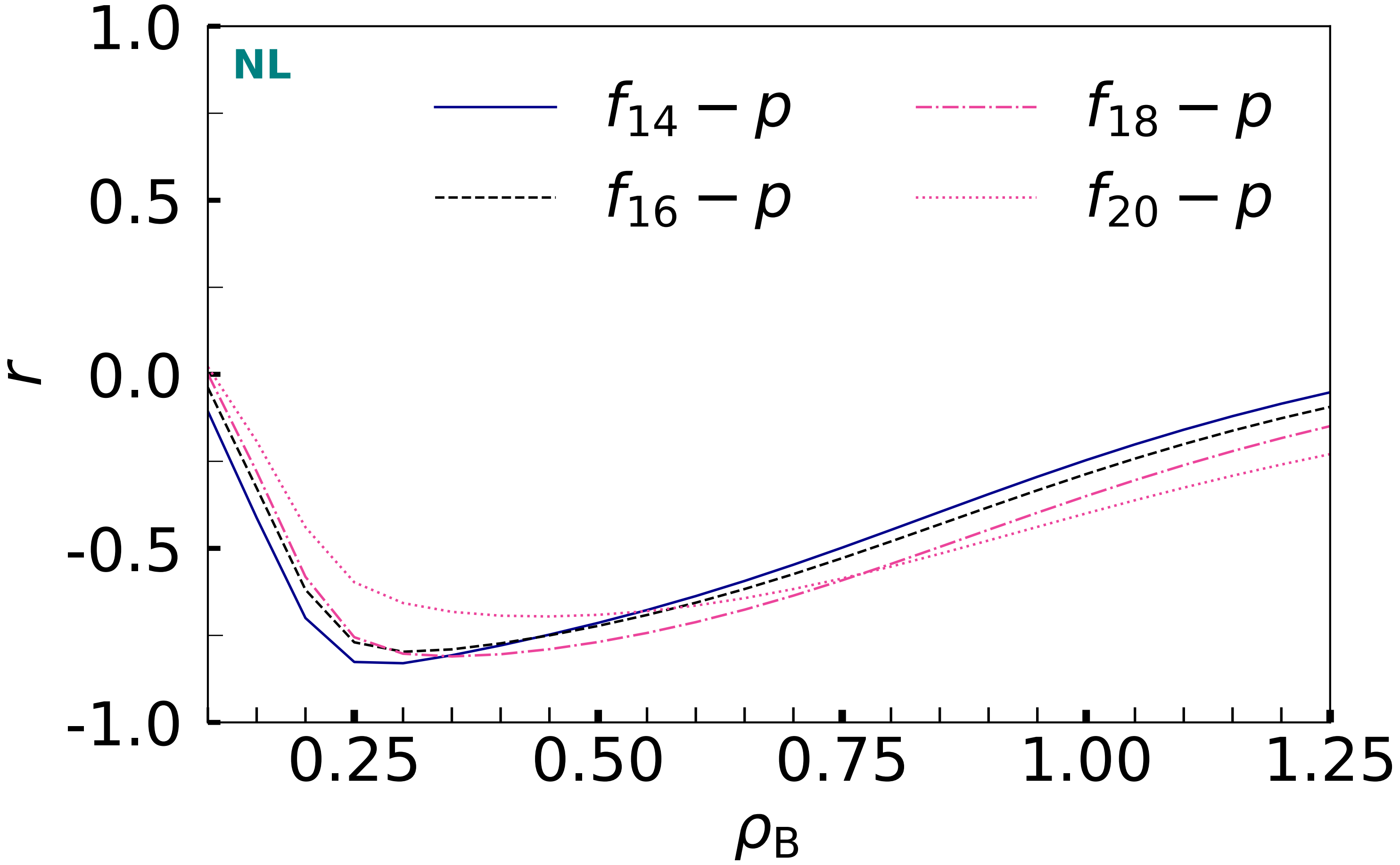


Neutron star: f modes are more sensitive to pressure at low densities

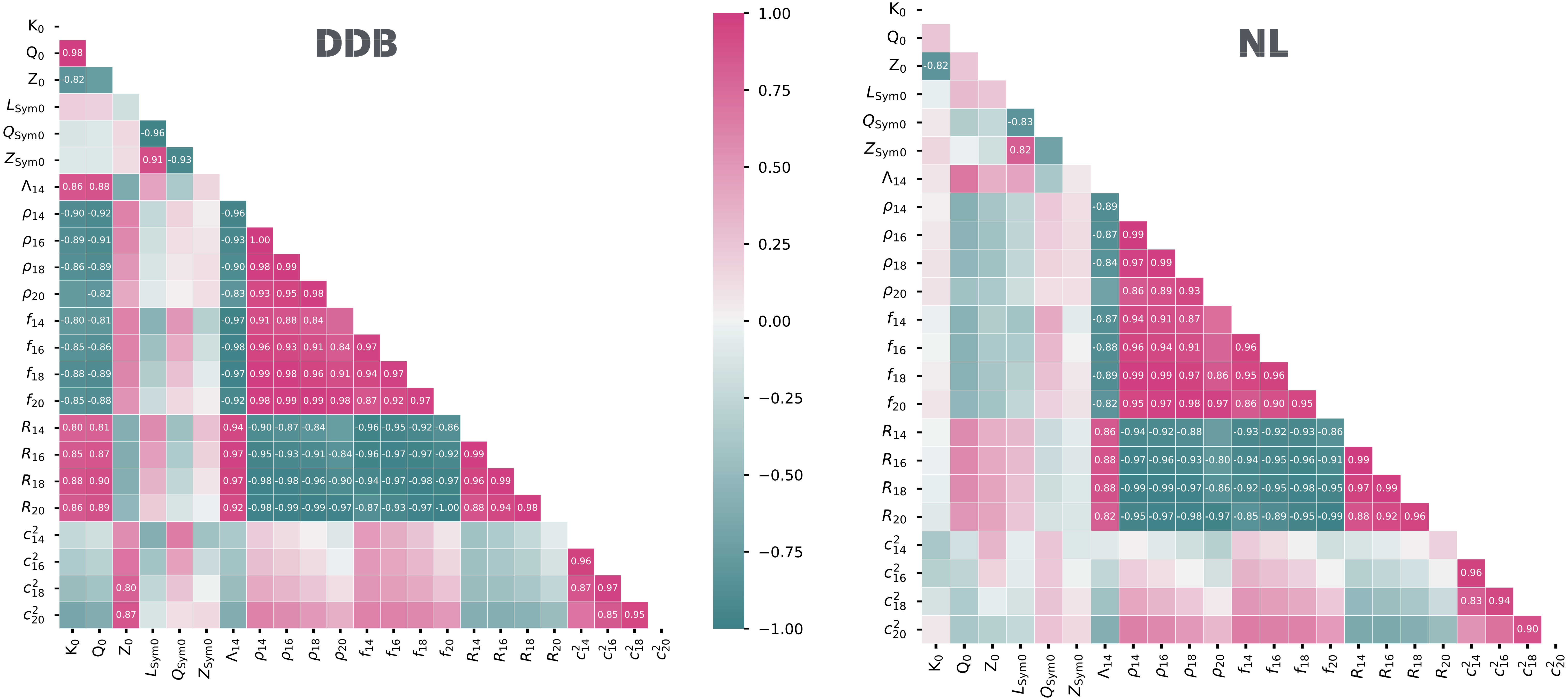
Pearson correlations:

The Pearson correlation coefficient is a descriptive statistic, meaning that it summarises the characteristics of a dataset. Specifically, it describes the strength and direction of the linear relationship between two quantitative variables.





Neutron star: Correlations between NMP and NSP



Machine Learning: Dataset and Algorithm

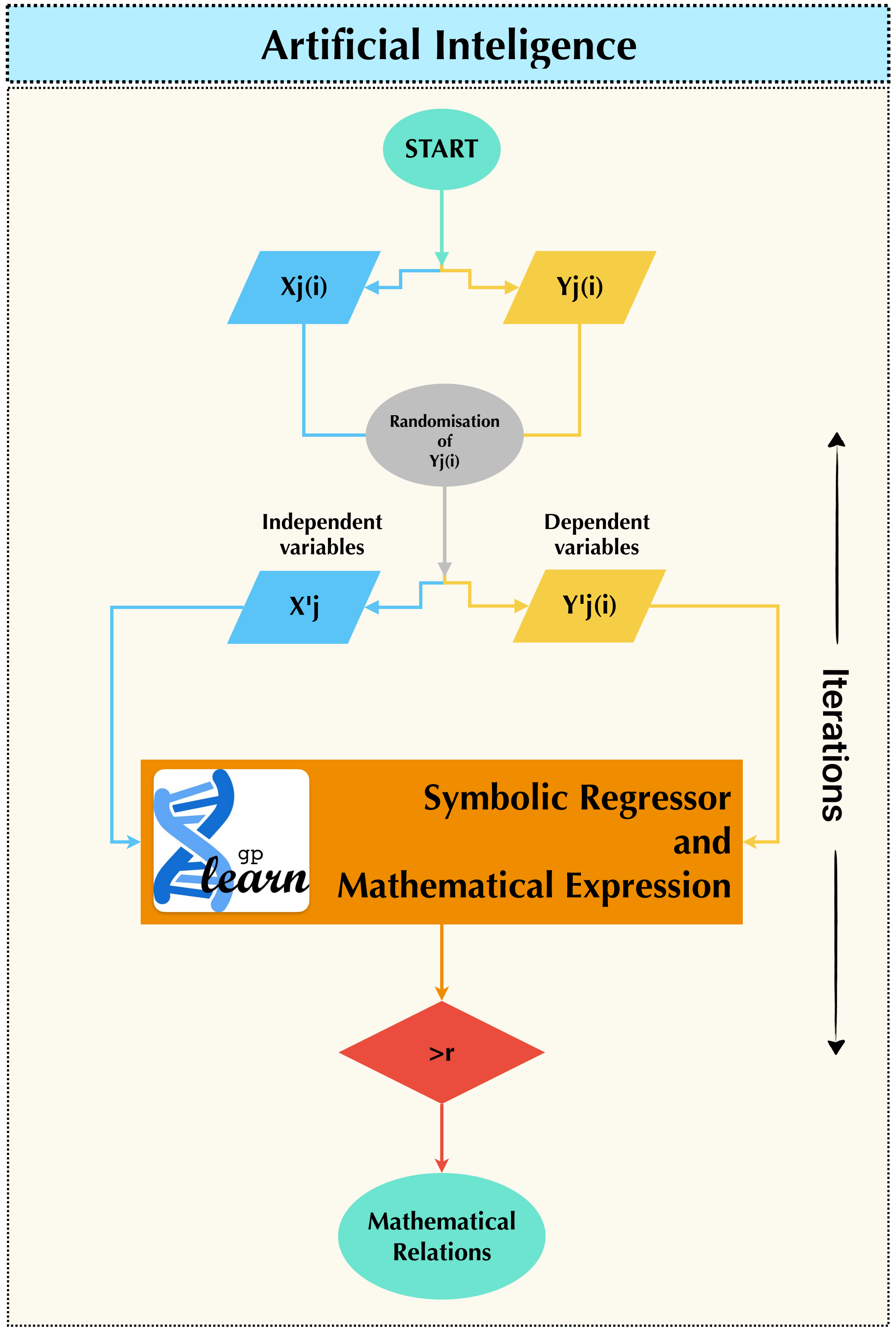
- ✓ From both datasets, we generated one dataset containing nuclear saturation properties and neutron star properties.

- ✓ Nuclear saturation properties:
  - ☑ Binding energy per nucleon (E)
  - ☑ Incompressibility (K)
  - ☑ Symmetry energy (J)
  - ☑ Slope parameter (L)

  etc.

- ✓ Neutron star properties.
  - ☑ Mass ($M = (1.0, 1.2, 1.4, 1.6, 1.8, 2.0, M_{\mathrm{MAX}})$)
  - ☑ Radius (corresponding to above star masses)
  - ☑ Square of sound velocity at the centre of corresponding stars
  - ☑ central baryon density
  - ☑ f mode frequencies

  etc.

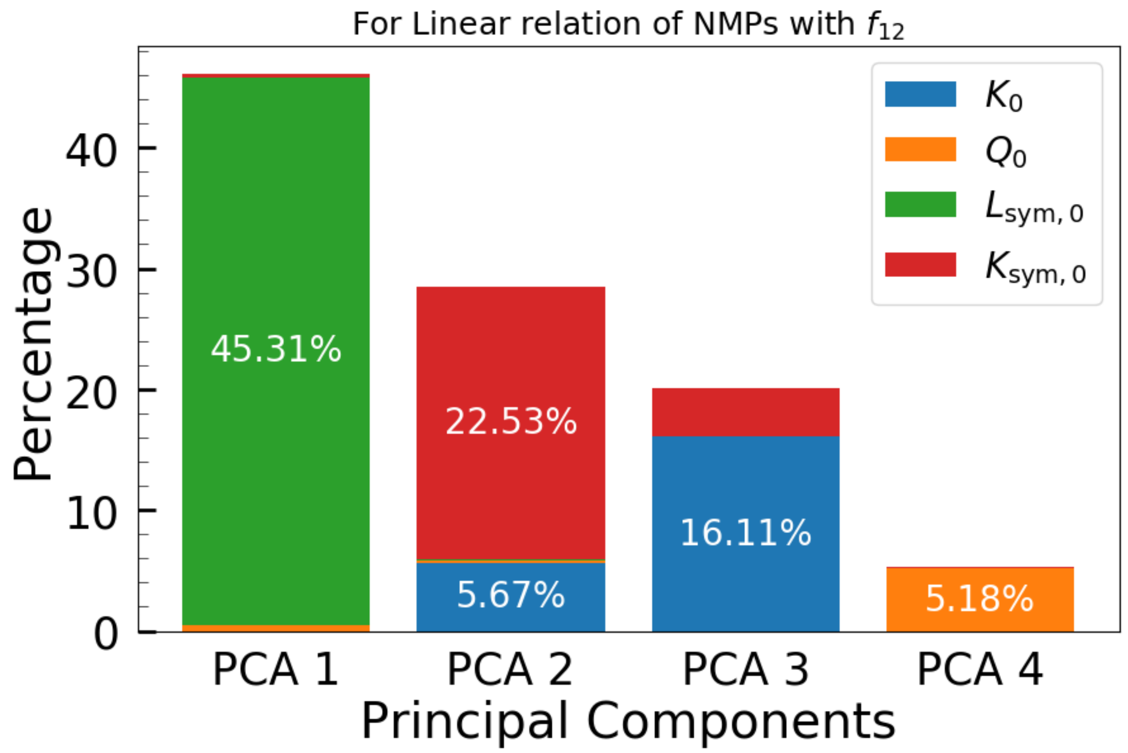


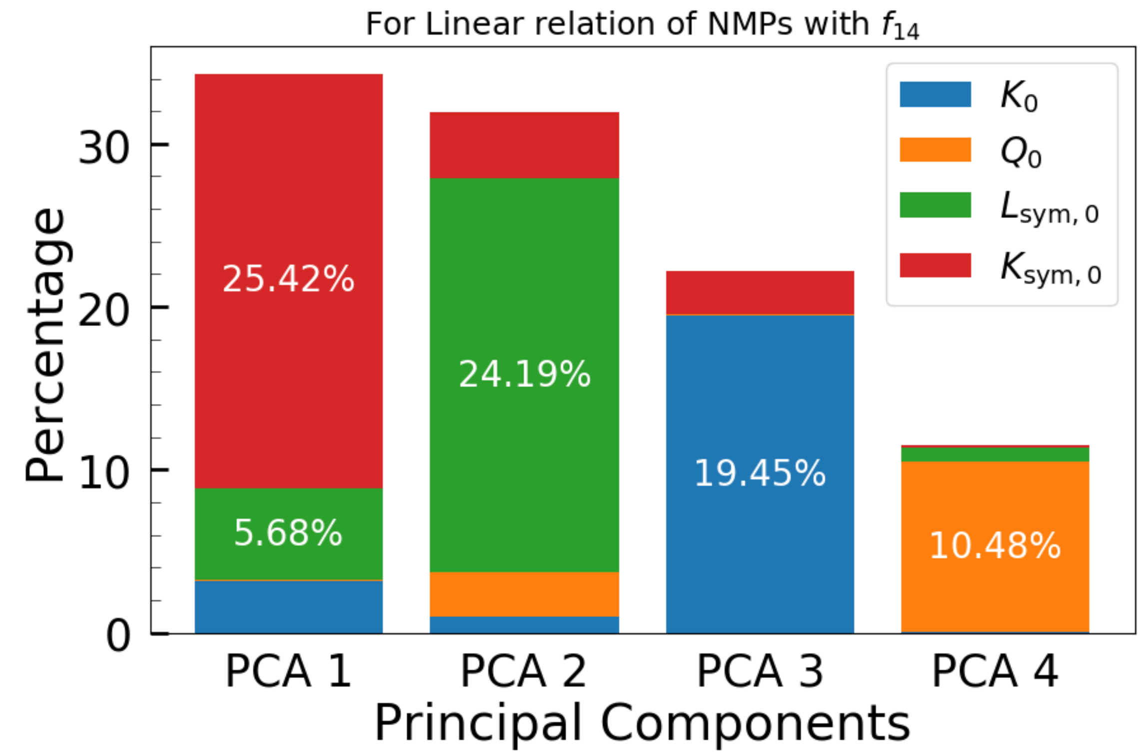
Machine Learning: Universal relations

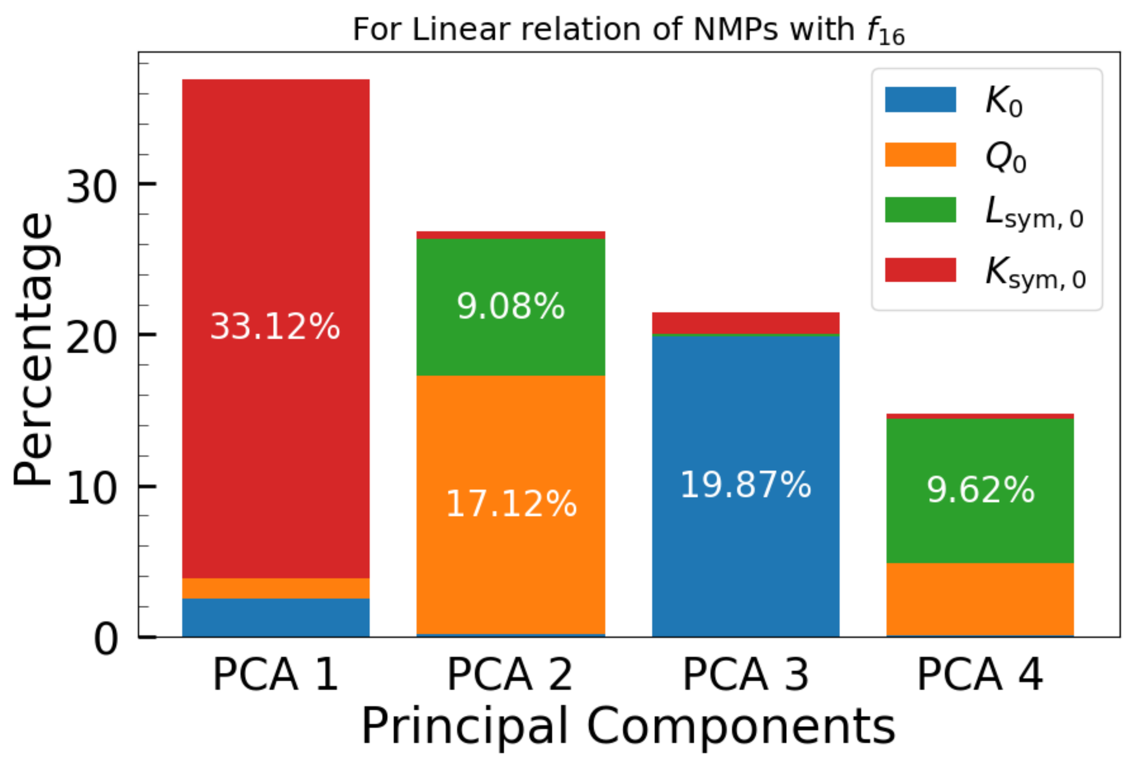
Sl No	NS mass ($M_\odot$)	Frequency (kHz)	Universal Relationship	Corr	RE %	Huth etal (CSE) Corr	Huth etal (CSE) RE %	Huth etal (PWP) Corr	Huth etal (PWP) RE %
1	1.2	f_{12}	$0.19 + 17.65 \cdot \frac{p_{c,12}}{\epsilon_{c,12}}$	0.98	0.5	0.94	2.95	0.92	2.8
2		f_{12}	$c_{12}^2 + 2.40 - \frac{23.32}{p_{c,12}}$	0.95	0.7	0.61	3.3	0.64	3.8
3		f_{12}	$2 \cdot \rho_{c,12} - 0.002 \cdot L_{\mathrm{sym},0} + 1.47$	0.94	0.8				
4	1.4	f_{14}	$0.39 + 13.26 \cdot \frac{p_{c,14}}{\epsilon_{c,14}}$	0.98	0.5	0.95	2.4	0.94	2.5
5		f_{14}	$c_{14}^2 + 2.38 - \frac{30.51}{p_{c,14}}$	0.96	0.7	0.73	3.5	0.68	4.6
6		f_{14}	$1.74 \cdot \rho_{c,14} + c_{14}^2 + 1.08 + \frac{1.29}{L_{\mathrm{sym},0}}$	0.97	0.6				
7	1.6	f_{16}	$0.65 + 9.64 \cdot \frac{p_{c,16}}{\epsilon_{c,16}}$	0.97	0.5	0.97	2.2	0.94	2.5
8		f_{16}	$0.006 \cdot p_{c,16} + 1.77$	0.95	0.7	0.96	2.6	0.94	2.5
9		f_{16}	$1.55 \cdot \rho_{c,16} + c_{16}^2 + 1.13$	0.96	0.7	0.82	3.1	0.77	4.7
10	1.8	f_{18}	$\frac{5.69 \cdot p_{c,18}}{\epsilon_{c,18} + L_{\mathrm{sym},0} + 236.08} + 1.53$	0.99	0.3				
11		f_{18}	$0.004 \cdot p_{c,18} + 1.90$	0.97	0.6	0.97	1.1	0.95	1.2
12		f_{18}	$1.19 + \frac{0.005 \cdot p_{c,18}}{\rho_{c,18}}$	0.99	0.5	0.98	2.0	0.96	2.0
13	2.0	f_{20}	$0.84 \cdot \rho_{c,20} + 0.45 \cdot c_{20}^2 + 1.62$	0.98	0.7	0.89	1.9	0.83	2.9
14		f_{20}	$\rho_{c,20} + 1.75$	0.97	1	0.94	3.7	0.88	3.4
15		f_{20}	$0.001 \cdot p_{c,20} + 2.12$	0.96	1	0.94	5.7	0.92	5.5

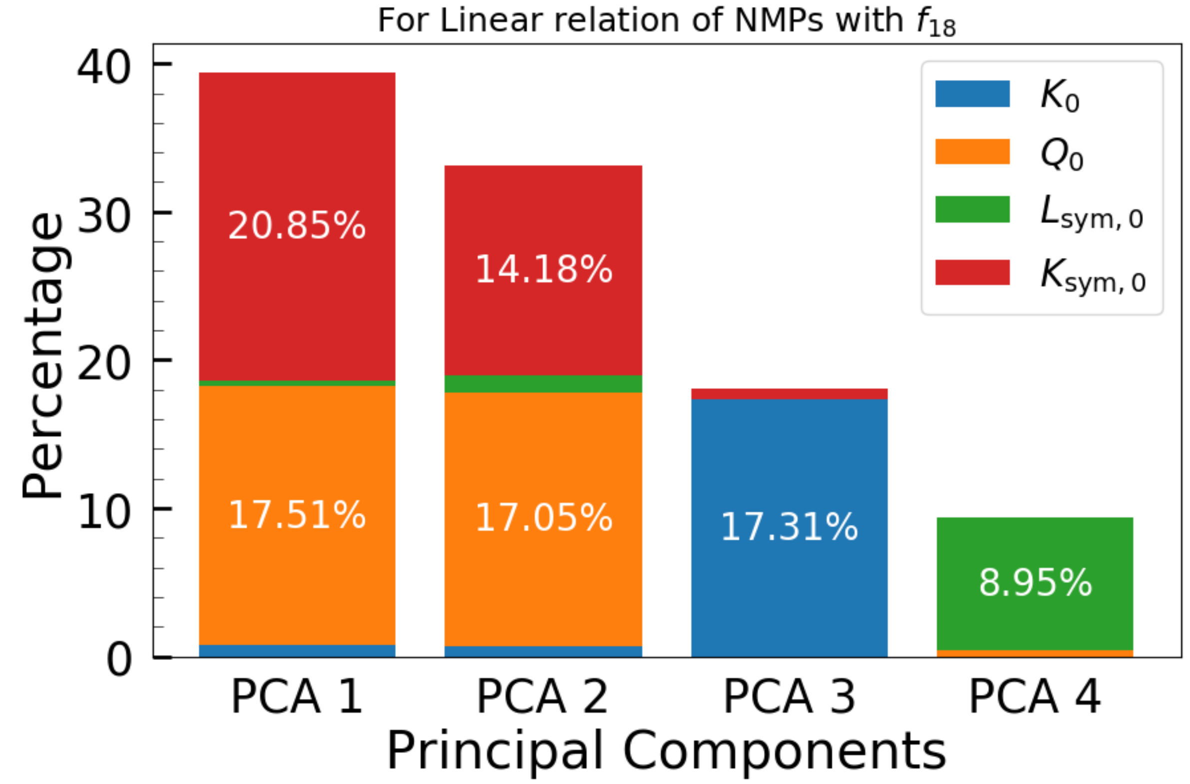
We can also find the footprints of different nuclear saturation properties to the non-radial oscillations of neutron stars

Machine Learning: Footprints

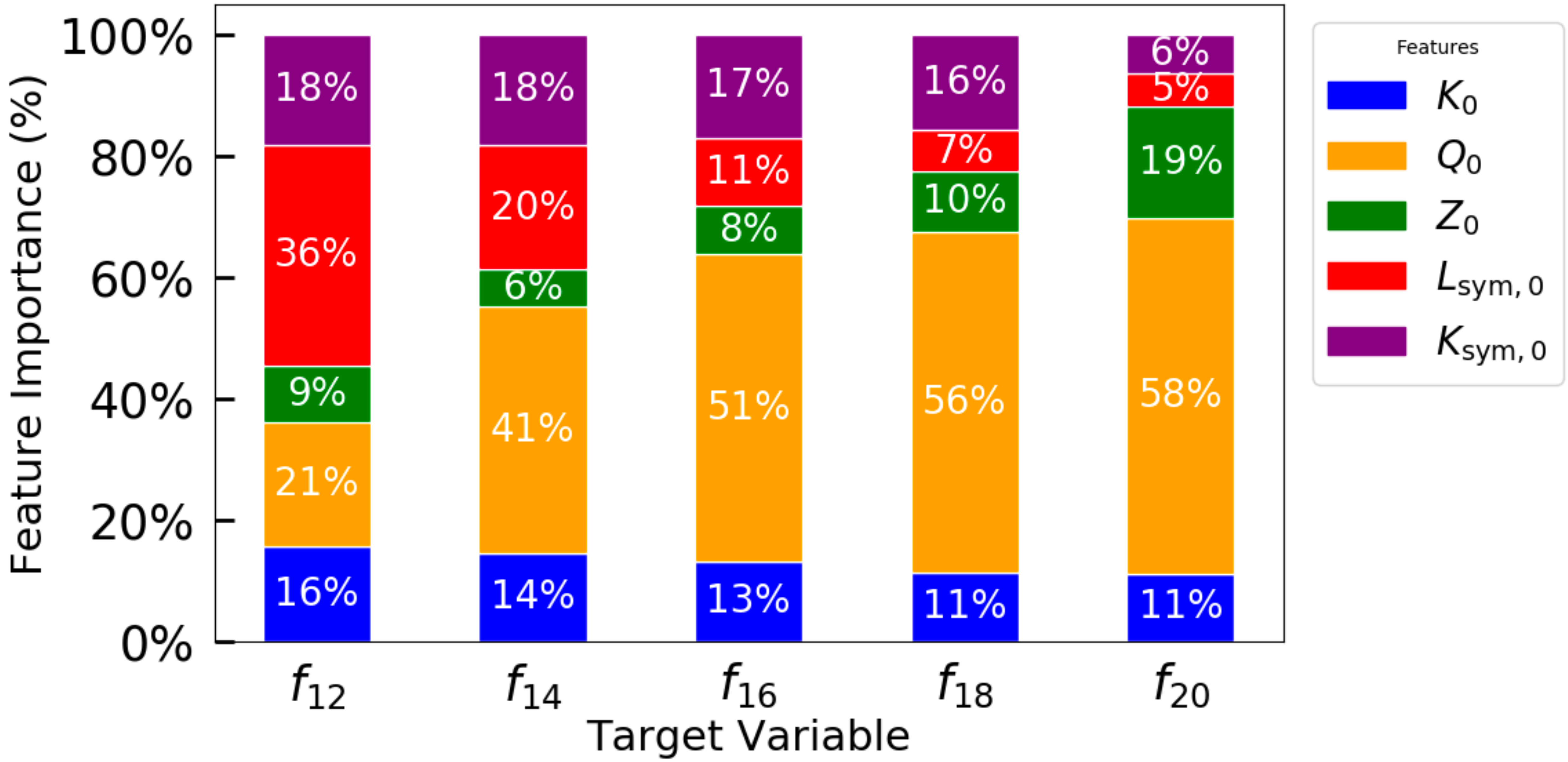








Machine Learning: Footprints



Which says that frequencies of smaller mass NSs are the great observables to constrain the slop parameter ($L_{sym,0}$) of the symmetry energy while the frequencies of high mass NSs are the great observables to constrain the higher moment (Q_0) of symmetric matter at saturation density.

Summary and Conclusions

- NSs are the exciting natural astrophysical laboratories to study the behaviour of matter at extreme densities.
- We discussed that the f mode non-radial oscillation frequencies are more sensitive to the low density part of equation of state.
- We see that both nuclear models (with NL and DD couplings) behave differently.
- We have seen that ML can play very important role to get the hidden universal relations between neutron star and nuclear matter properties.
- We have discussed very important relation between symmetry energy and f mode non-radial oscillation frequency.

Thank YOU