Heavy quark momentum broadening in a non-Abelian plasma off-equilibrium

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May 19, 2024

In collaboration with Sören Schlichting & Sayantan Sharma,
Based on arXiv:2312.12280, accepted in Phys. Rev. Lett.
60th Karpacz Winter School on Theoretical Physics

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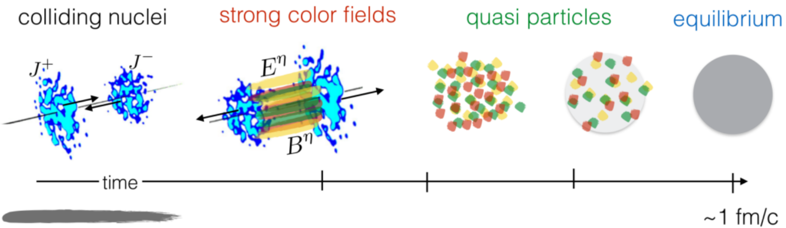
- We have developed a novel lattice technique to study heavy quark momentum broadening.

Initial Conditions: Non-Abelian plasma SU(2) in the self-similar regime

- During the initial stages of HICs, the plasma is primarily gluon dominated with high occupancy ($f \sim \frac{1}{\alpha_s} \gg 1$) hence gauge fields can be evolved classically using Hamilton's equations.

$$H_{YM} = \sum_{\mathbf{x},i} \frac{a_i^2}{g^2 a^3} \frac{E^i_{\mathbf{x}} E^i_{\mathbf{x}}}{2} + \sum_{\mathbf{x},i,j} \frac{a^3}{g^2 a_i^2 a_j^2} \, \mathrm{Re}\,\mathrm{Tr}\left[\mathbf{1} - U^{\square}_{ij}(\mathbf{x})\right]$$



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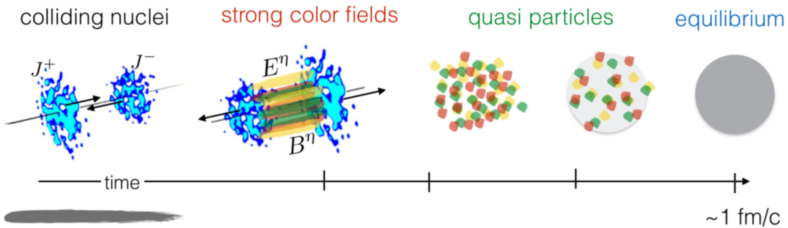
- Such a system undergo a rapid memory loss at late times and enter a self-similar scaling regime. [J. Berges, K. Boguslavski, S. Schlichting, and R. Venugopalan, 14]



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- We have performed our simulations on large volume $N_s^3 = 256^3$ lattice, with lattice spacing $Qa_s = 0.5$, with $N_c = 2$ and $N_f = 1$.

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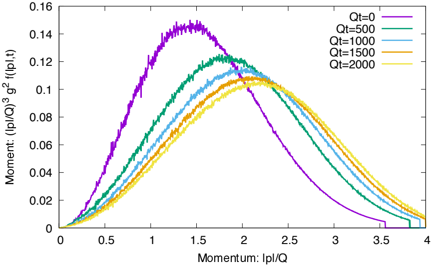
- We consider the following initial phase-space distribution of the gluons, motivated from Color Glass condensate effective theory

[L. McLerran and R. Venugopalan, 94]

$$g^2 f_g(p) = n_0 \frac{Q}{p} e^{-\frac{p^2}{2Q^2}}$$

where $n_0/g^2 >> 1$.
We've chosen n_0 to be 0.2.

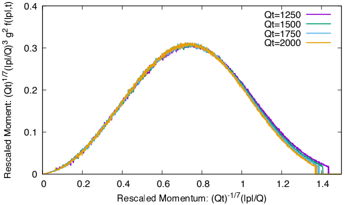


Initial Conditions: Non-Abelian plasma in the self-similar regime

- To be deep within the self-similar scaling regime we have evolved the gauge fields till $Qt = 1500$ where the distribution is:

$$\left(\frac{\tilde{p}}{Q}\right)^3 f_S(\tilde{p}) = (Qt)^{\frac{1}{7}} \left(\frac{|\mathbf{p}|}{Q}\right)^3 f(|\mathbf{p}|, t)$$

(Here, $\tilde{p} = (Qt)^{-\frac{1}{7}} |\mathbf{p}|$.)



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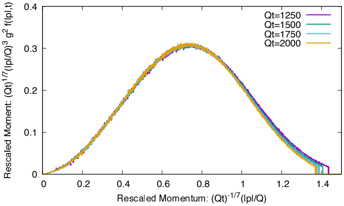

- In analogy to the equilibrium plasma, there is a clear separation of scales here as well

$$\underset{\sim Q(Qt)^{-3/10}}{\sqrt{\sigma(t)}} < \underset{\sim Q(Qt)^{-1/7}}{m_D(t)} \ll \underset{\sim Q(Qt)^{1/7}}{\Lambda(t)}$$

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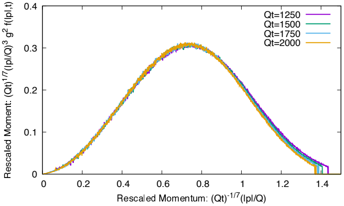

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- Plasma in the self-similar regime represents a characteristic non-equilibrium state which we use as our initial state.

Evolving the heavy quarks

- We have implemented for the first time the evolution of heavy quarks as relativistic particles using Wilson-Dirac Hamiltonian on the lattice.

$$\hat{H}_f = \sum_{\mathbf{x}} \hat{\psi}_{\mathbf{x}}^{\dagger} \gamma^0 (-i\not{D}_W + m) \hat{\psi}_{\mathbf{x}}$$

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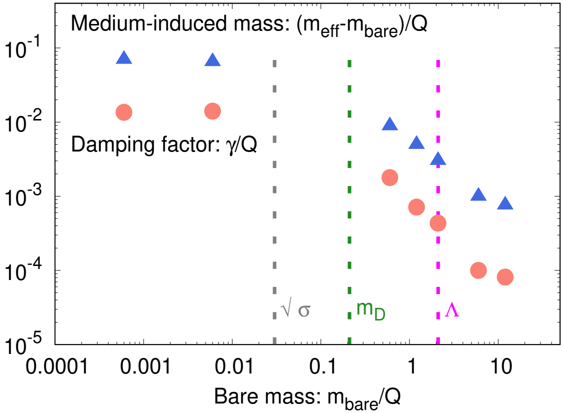
- We have chosen a wide set of quark masses, $m/Q = 1.2,\ 2.1,\ 4.2,\ 6.0,\ 12.0$.
  For $Q \sim 1$ GeV, the choice of $m/Q = 1.2$ represents a particle with mass close to that of the charm quark.

In-Medium effects on the quark quasi-particles

- We obtain effective mass (*m_{eff}*) via fit to spectral function and subtract the bare mass to obtain medium-induced mass



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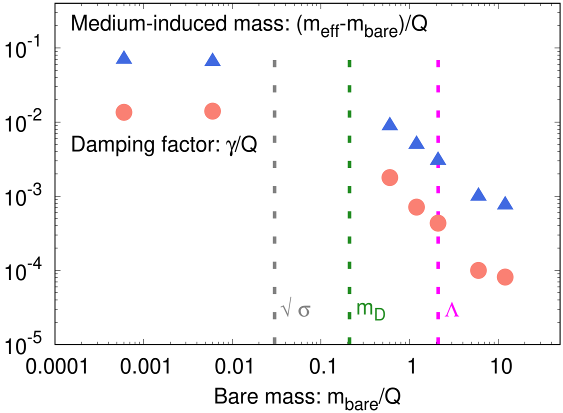

- Width of quasi-particles decreases $> 10\times$ going from light to heavy.

Observations from the analysis of spectral functions

- For $m_{bare}/Q < 0.1$: medium modification effects are similar and large width $\implies$ behaves more like a collective excitation than a weakly-interacting quasi-particle, doesn't make much sense to talk about diffusion.

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- For $m_{bare}/Q > 2.0$: in-medium mass is significantly lesser signifying more stable quasi-particles $\rightarrow$ onset of heavy-quark NR regime. Small width, hence interacts as a quasi-particle with the medium to get kicks from gluons and diffuse.
- Intermediate $m_{bare}/Q \approx 1.2$, i.e. close to charm mass lies in the transient region $\Longrightarrow$ may diffuse similar to heavier quarks but has to be evolved relativistically to capture the correct dynamics.

Momentum Broadening: How do we calculate it?

- We start with a single quark in a fixed momentum ($\mathbf{P}$) and spin polarization (s) mode with initial conditions

$$\langle b_\lambda^\dagger(t=0,\mathbf{p}) b_{\lambda'}(t=0,\mathbf{p}') \rangle = \delta_{\lambda\lambda'}\ \delta_{\mathbf{p}\mathbf{p}'}\ \delta_{\lambda s}\ \delta_{\mathbf{p}\mathbf{P}}$$

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- Evolving the quark fields, at time t' is represented as,

$$\Psi(t',\mathbf{x}) = \frac{1}{\sqrt{N^3}} \sum_{\lambda,\mathbf{p}} \left[\phi^u_{\lambda,\mathbf{p}}(t',\mathbf{x}) b_\lambda(t'=0,\mathbf{p}) + \phi^v_{\lambda,\mathbf{p}}(t',\mathbf{x}) d_\lambda^\dagger(t'=0,\mathbf{p})\right]$$

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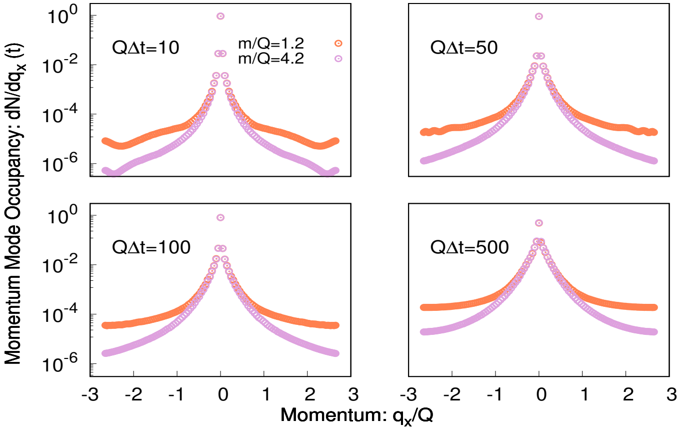
- Momentum mode occupancy is then calculated as,

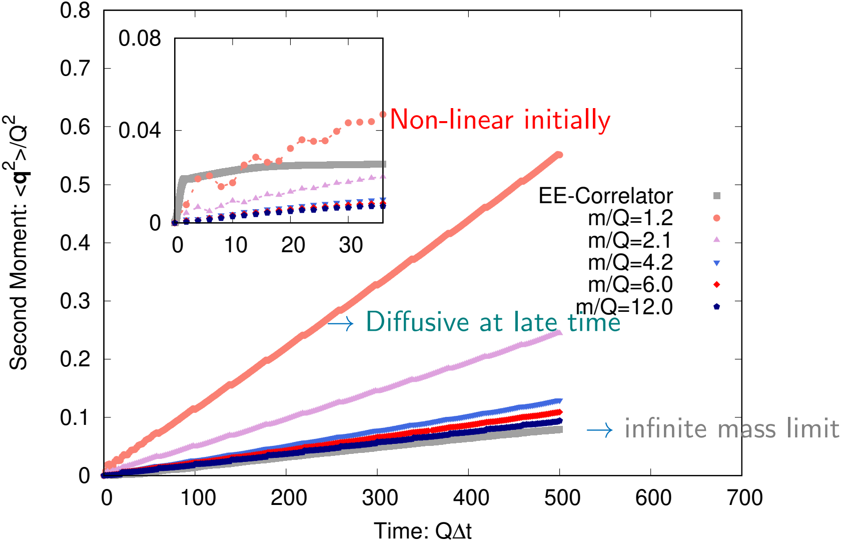
$$\frac{dN}{d^3\mathbf{q}} = \sum_{\lambda'} |u^\dagger_{\lambda'}(\mathbf{q})\tilde{\phi}^u_s(t',\mathbf{P})|^2$$

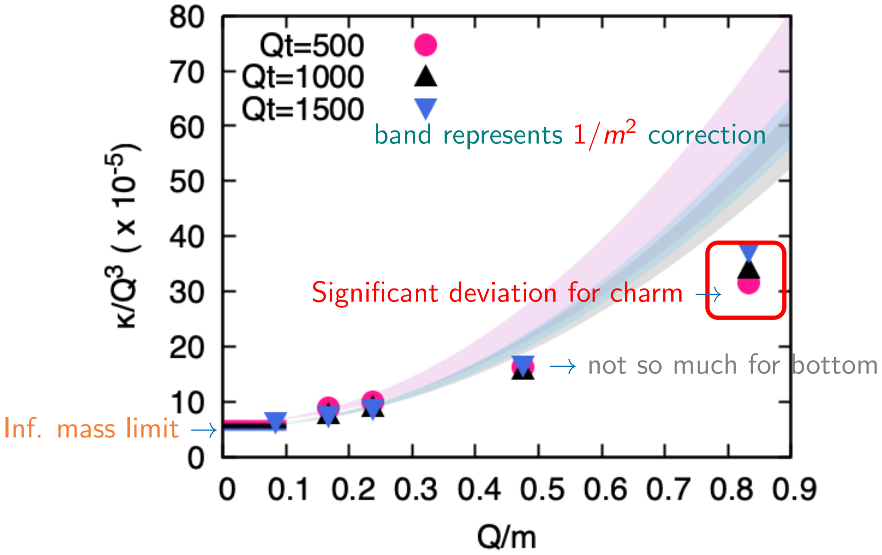
Momentum Broadening of heavy quarks: This is what it looks like!

- Starting with zero initial momenta, the momentum distribution broadens due to kicks it receives from the gluon plasma.



Quantifying broadening through second moment of the momentum distribution




Qt=500
Qt=1000
Qt=1500
band represents $1/m^2$ correction
Significant deviation for charm →
→ not so much for bottom
Inf. mass limit →
κ/Q³ (x 10⁻⁵)
Q/m
0
10
20
30
40
50
60
70
80
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

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- We find that there are large corrections to momentum broadening of charm quarks in full relativistic treatment without resorting to expansion in $1/m^2$ about infinite mass limit.
- Currently, we are studying the drift and broadening phenomena for finite initial momenta and also looking for possible extension of this technique in a thermal quark-gluon plasma.

Thanks :)

Back-up slide: Value of κ with varying m_{bare}/Q

Quark mass, m/Q	κ/Q^3 ($\times 10^{-5}$)
1.2	37.13 (5)
2.1	16.54 (4)
4.2	8.64 (2)
6.0	7.24 (1)
12.0	6.20 (2)
∞ (EE-correlator)	5.227 (3)