Cooking nuclear pasta at zero temperature: from semi-classical to fully microscopic recipes

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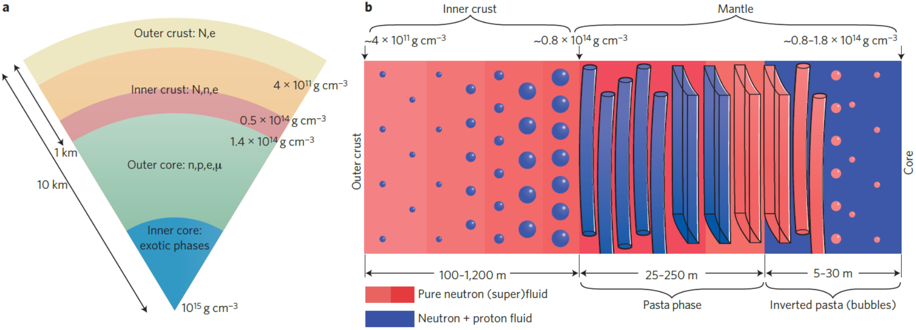
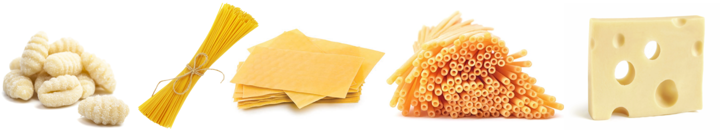
Neutron star schematic structure



[Newton+'13]



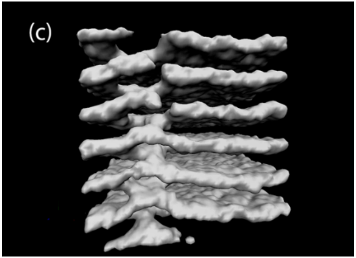
Gnocchi *Spaghetti* *Lasagna* *Bucatini* *Swiss cheese*

Role of pasta phases

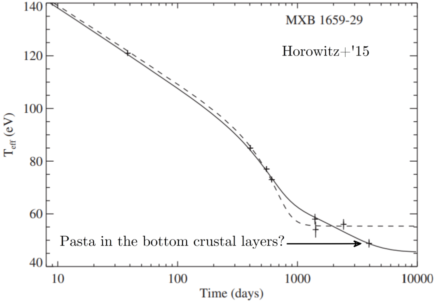
- **Transport properties**

  Thermal and electrical conductivities are determined by electron scattering on pasta structures.

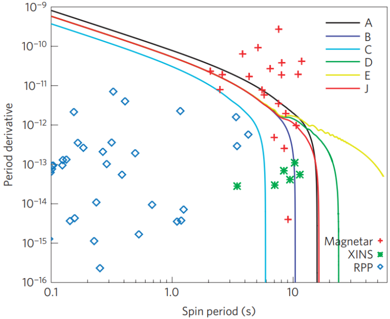
  Possible manifestations:
  - NS thermal evolution
  - Magnetic field evolution



Topological defects in pasta [Horowitz+'15]



[Horowitz+'15], [Pons+'13]



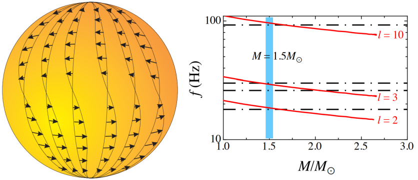
Role of pasta phases

- Mechanical properties

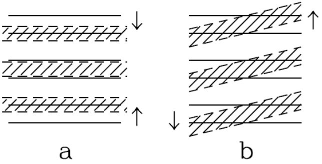
  In response to deformations pasta behave like liquid crystals.

  Possible manifestations:

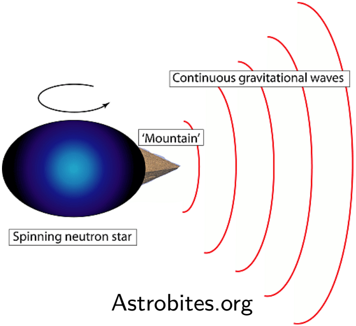
  - NS oscillation frequencies
  - Gravitational waves emission



[Bastrukov+'09], [Sotani'11]

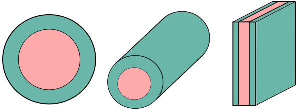


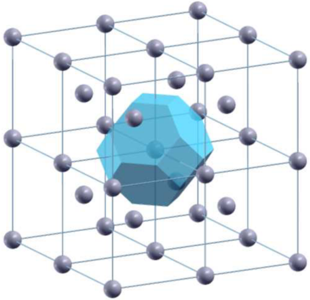
Pasta deformations [Pethick&Potekhin'98]



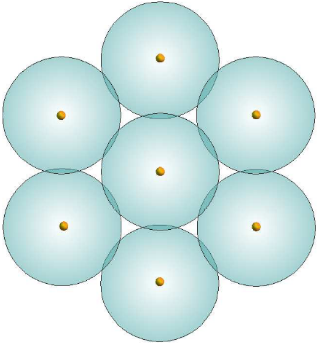
Modelling pasta

- It is assumed that matter is in its ground state at $T = 0 \Rightarrow$ cold catalyzed matter
- Matter is divided into identical cells
- Energy per particle is minimized at given density



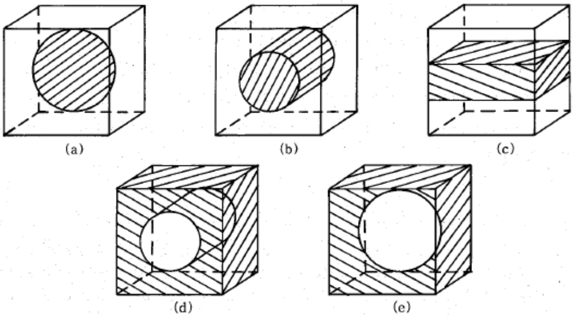


Body-centered cubic lattice of quasi-spherical nuclei [Chamel&Haensel'08]



Spherical Wigner-Seitz approximation [Chamel&Haensel'08]

Recipe I. Liquid drop models



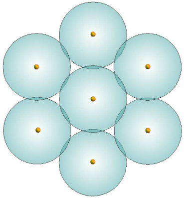
Pasta shapes [Hashimoto+'84]

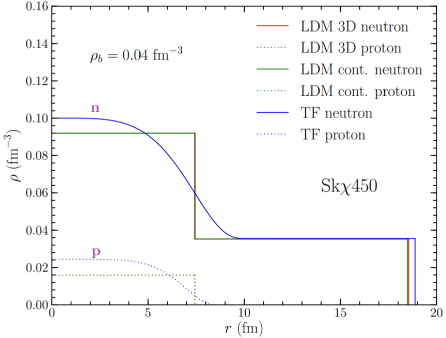
- First predicted in liquid-drop models [Ravenhall+'83], [Hashimoto+'84]
- $E_{\rm tot} = E_{\rm bulk} + E_{\rm surf} + E_{\rm coul}$
- At certain filling factor $u = V_{\rm cl}/V$, $E_{\rm surf}$ is reduced by shape rearrangements

Recipe II. Semi-classical models

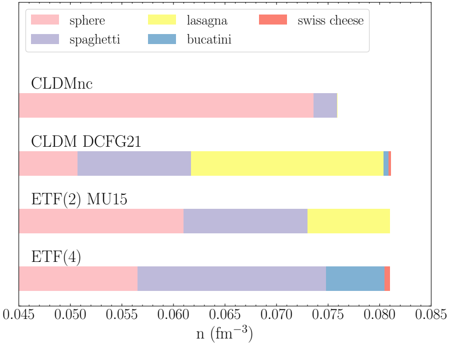
Extended Thomas-Fermi (ETF) method

- Describes smooth nucleons distributions ρ
- Surface energy is included self-consistently
- lack of shell effects





Density profiles [Lim&Holt+'17]



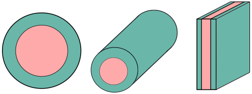
Pasta phases sequences [Shchechilin+'22]

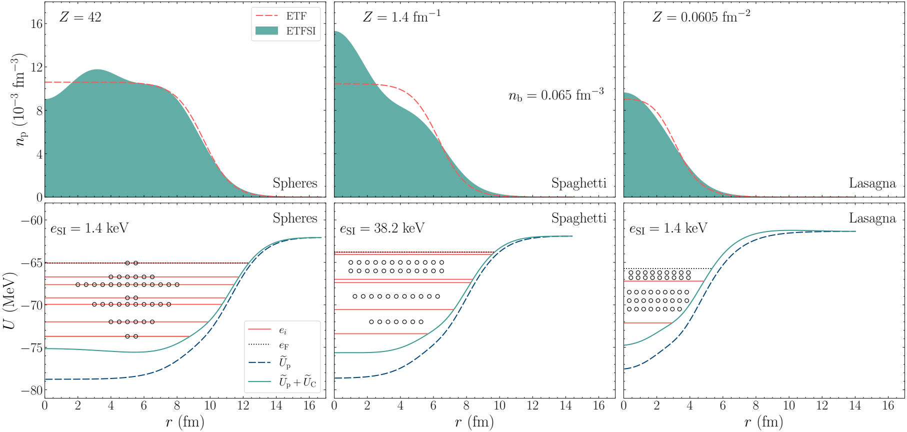
Recipe II*. Semi-classical with microscopic corrections

Extended Thomas-Fermi + Strutinsky Integral (ETFSI)

- Construct central and Coulomb fields U_{ETF} from the ETF density profiles ρ
- Solve Hartree-Fock equations for ψ with U_{ETF}
- Add microscopic (shell) effects pertubatively

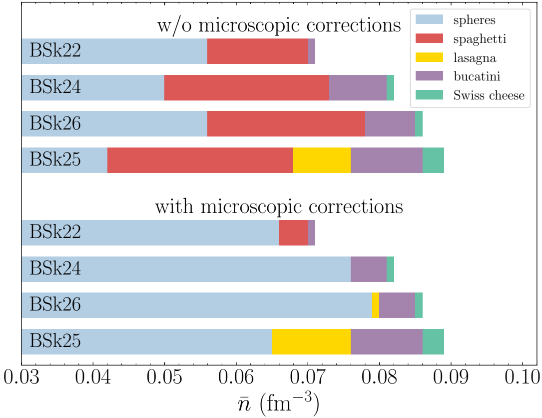
[Pearson&Chamel'22], [Shchechilin+'24]





Semi-classical models with microscopic corrections

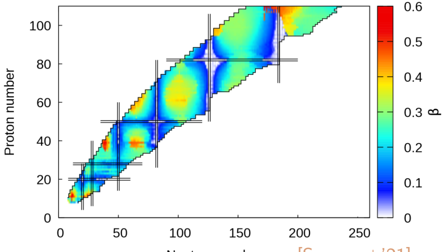
- Allows fast estimation of the role of the nuclear interaction employed
- Microscopic effects are important ⇒ call for the fully quantum calculations



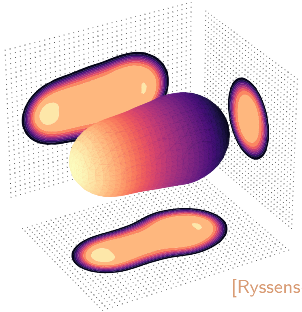
Pasta sequences with nuclear functionals differing in symmetry energy [Shchechilin+'23]

Recipe III. Microscopic models

- Hartree-Fock-Bogoliubov method
  - Mean-field approach for finding self-consistent solution for ψ
  - Pairing correlations are included
- 3D HFB code MOCCa [Ryssens+'15]
  - tool of choice for deformed nuclei
- BSkG3 parametrization of generalized Skyrme force [Grams+'23]
  - rms (exp masses): 0.631 MeV
  - Max NS mass: 2.3 $M_{\odot}$
  - $\Rightarrow$ suitable for unified EOS



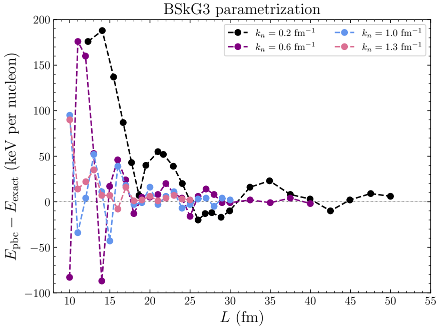
[Scamps+'21]



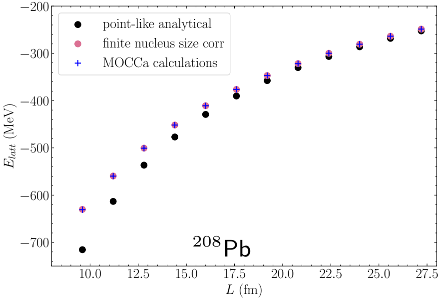
[Ryssens+'23]

MOCCa code generalization

- We implement periodic boundary conditions
  - Pure neutron matter energy tends to analytical result at large box sizes
  - Lattice energy agrees with analytic expression for simple cubic lattice

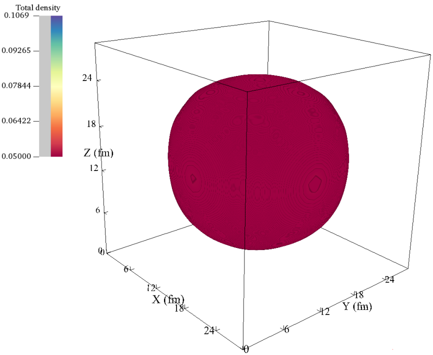


- We scale the problem for the large number of particles $\mathcal{N} \sim 5000$

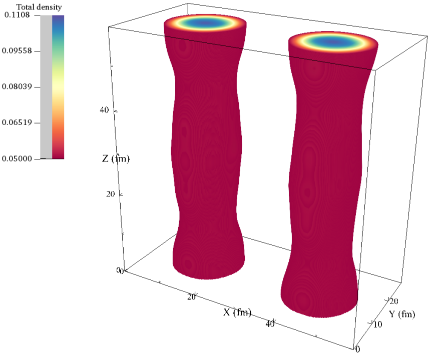


Microscopic models for pasta

- Proof of principle calculations with maximal symmetries and no pairing (Hartree-Fock in 1/8 of the full box).



(Spherical shape) $Z = 40,\ A = 1318,$ $\bar{\rho} = 0.05\,\text{fm}^{-3}$



(Spaghetti shapes) $Z = 160,\ A = 5240,$ $\bar{\rho} = 0.05\,\text{fm}^{-3}$

Conclusions

- Motivation
  - Pasta in neutron stars can impact: cooling, gravitational waves, etc.
  - Reliable calculations of pasta are needed
- Main results
  - We show within the pertubative approach that microscopic effects are important
  - To take them into account fully self-consistently we adapt the 3D Hartree-Fock-Bogoliubov code MOCCa for pasta calculations
- Perspectives
  - Calculation of the mantle composition and equation of state
  - Investigation of dynamics and stability of pasta structures