Effect of Colour Superconductivity on QCD Phase diagram

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(Non-Local) Nambu-Jona-Lasinio Model

$$m^*(p) = m - \gamma(p) G_s n_s$$

$$\mu^*(p) = \mu - \gamma(p) G_v n_v$$

Quasi-Particle Model

Dynamical interaction

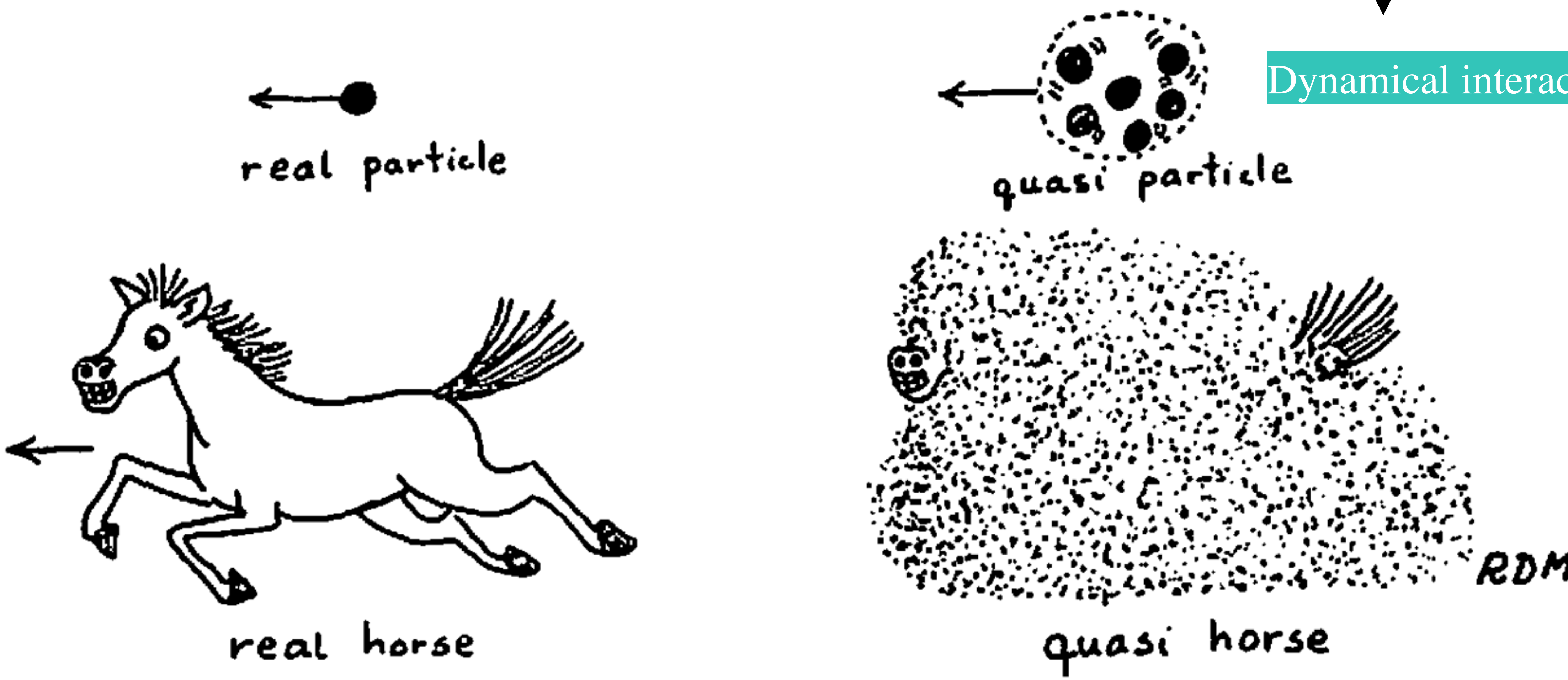


Fig. 0.4 ***Quasi Particle Concept***

Adapted from Richard D. Mattuck, “A Guide to Feynman Diagrams in the Many-Body problem”

(Non-local) Nambu-Jona-Lasinio Model

Quasi-particle mass

$$m^*(p) = m + \gamma(p)\, 2G_s N_c N_f \int \frac{d^3q}{(2\pi)^3} \gamma(q) \frac{4m_q^*}{2E_q} [1 - n(E_q - \mu_q^*) - n(E_q + \mu_q^*)]$$

Quasi-particle chemical potential

$$\mu^*(p) = \mu - \gamma(p)\, 2G_v N_c N_f \int \frac{d^3q}{(2\pi)^3} \gamma(q) \frac{4}{2} [n(E_q - \mu_q^*) - n(E_q + \mu_q^*)]$$

$$n(x) = \frac{1}{1 + e^{x/T}} \qquad E_q = \sqrt{q^2 + (m_q^*)^2}$$

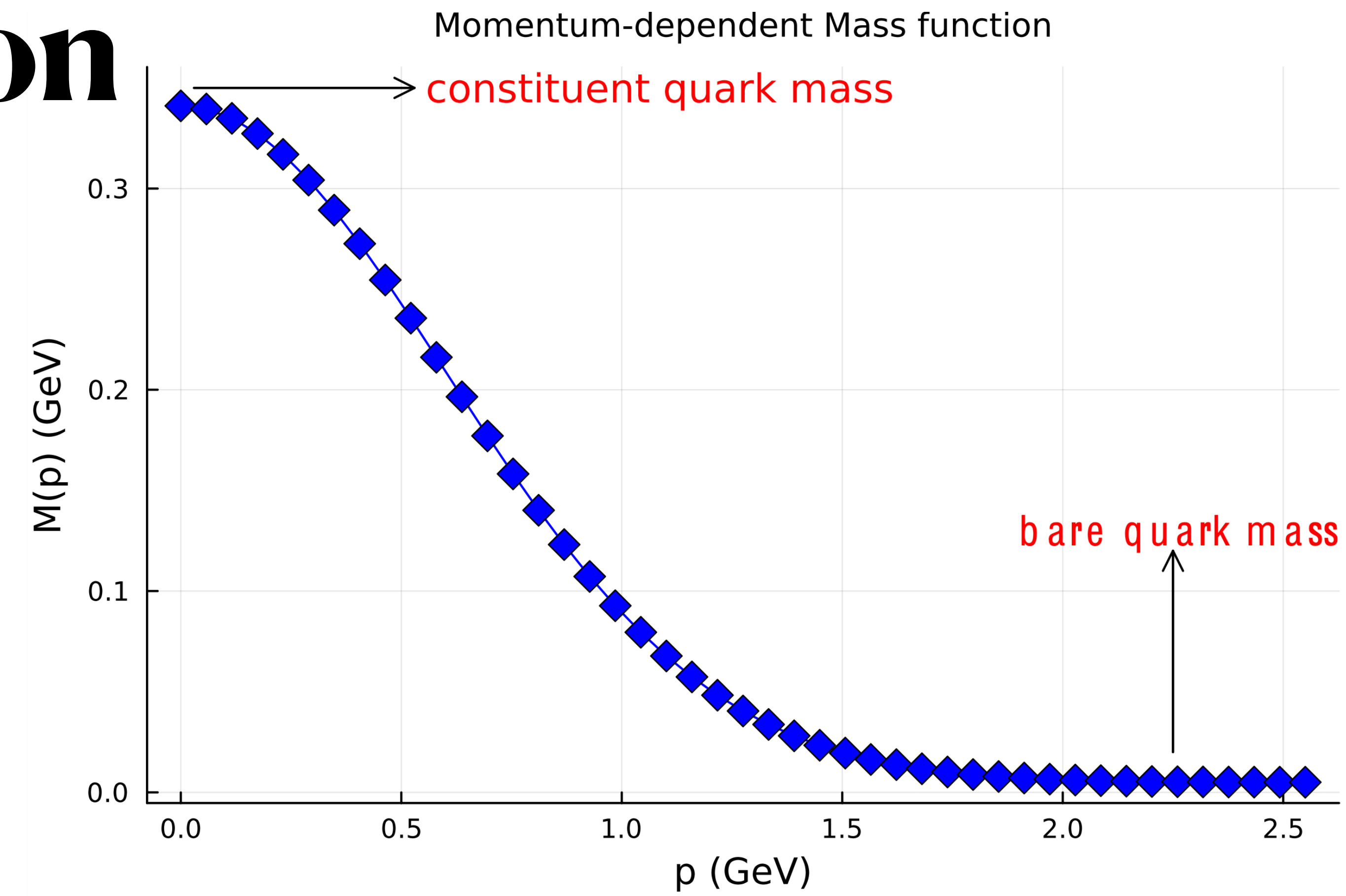
Dynamical interaction

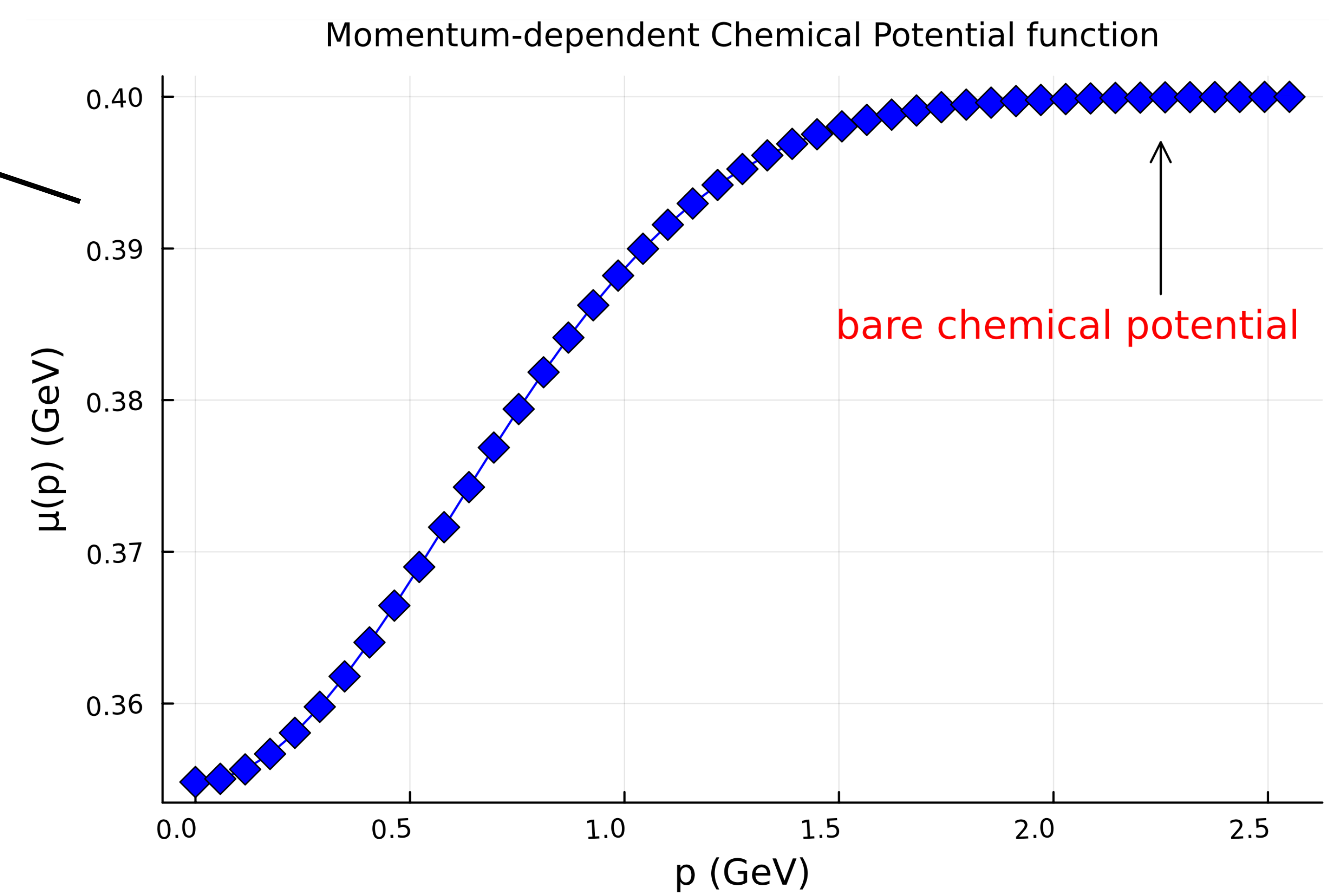
- M(p) saturates asympto- -tically to current quark mass
- μ(p) saturates asympto- -tically to bare μ

?

- Change in dispersion relation -

$$E(p) = \sqrt{p^2 + M(p)^2}$$





Local v/s Dynamical
T → 0

Local Interaction

$$n_v = N_c N_f \frac{1}{\pi^2} \int_0^\infty dq q^2 \Theta(\mu^* - q)$$

$$= N_c N_f \frac{1}{\pi^2} \int_0^{\mu^*} dq q^2$$

$$= N_c N_f \frac{1}{3\pi^2} \mu^*(\mu)^3$$

$$n_v = n_v(\mu^*(\mu))$$

$$\mu^* = \mu - G_v n_v$$

Trivial Fermi surface

Dynamical Interaction

$$n_v = N_c N_f \frac{1}{\pi^2} \int_0^\infty dq q^2 \Theta(\mu^*_q - q)$$

$$= N_c N_f \frac{1}{\pi^2} \int_0^{\mu^*_{p_f(\mu)}} dq q^2$$

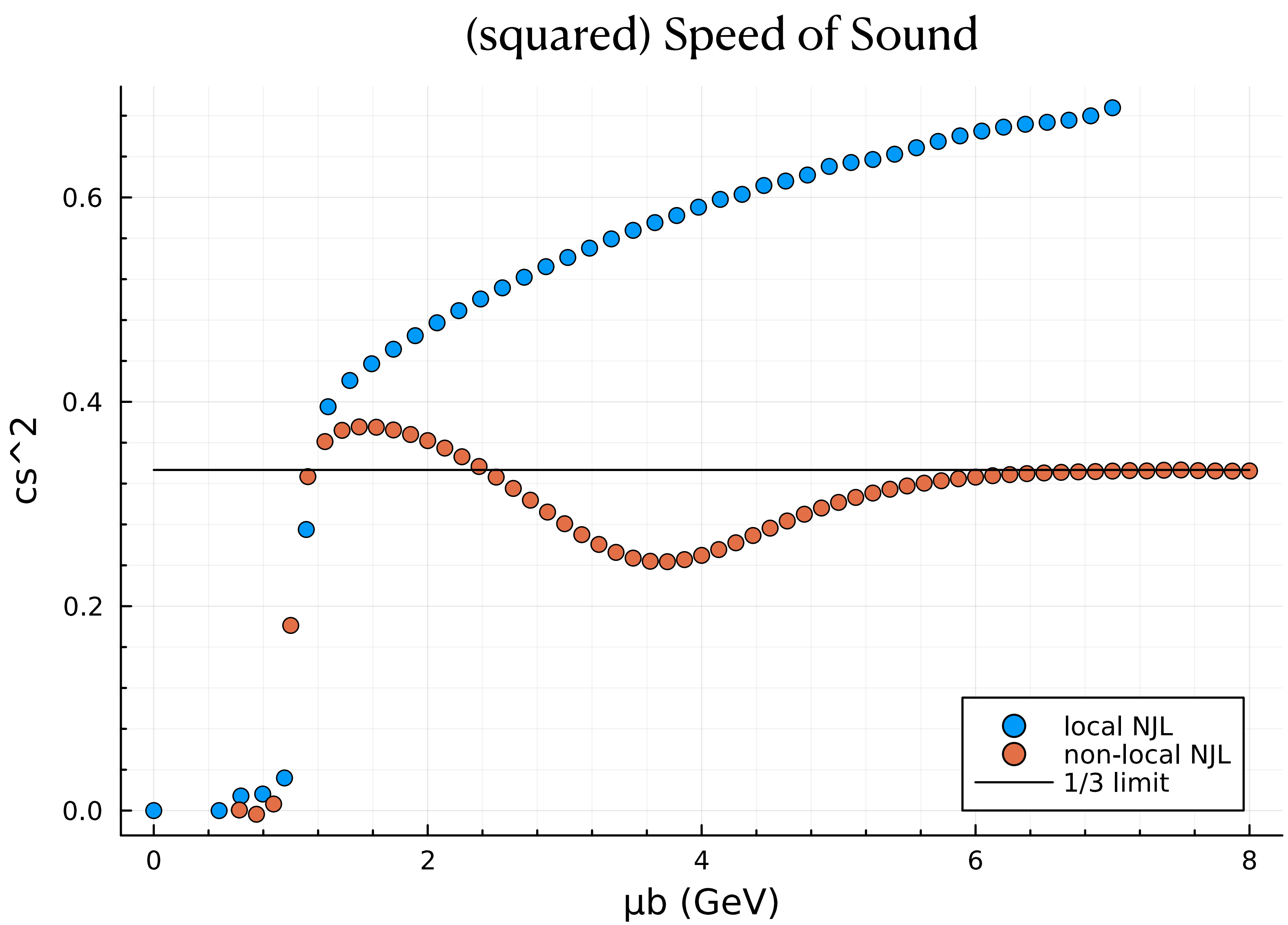
$$n_v = n_v\left[\mu^*_{p_f(\mu)}\right]$$

$$\mu^*(p) = \mu - \gamma(p) G_v n_v$$

NATURAL DENSITY-DEPENDENCE

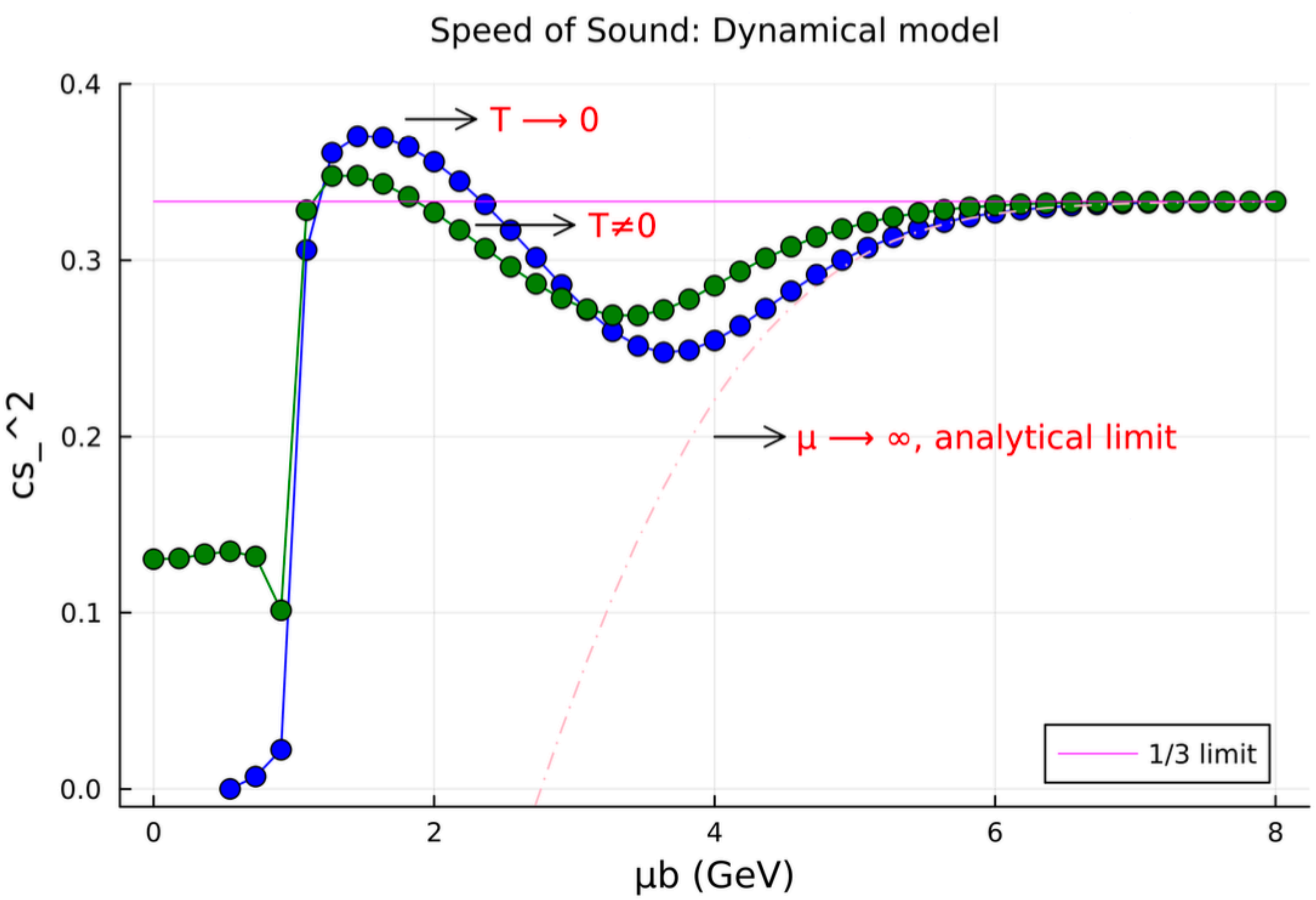
Non-trivial Interacting Fermi surface !

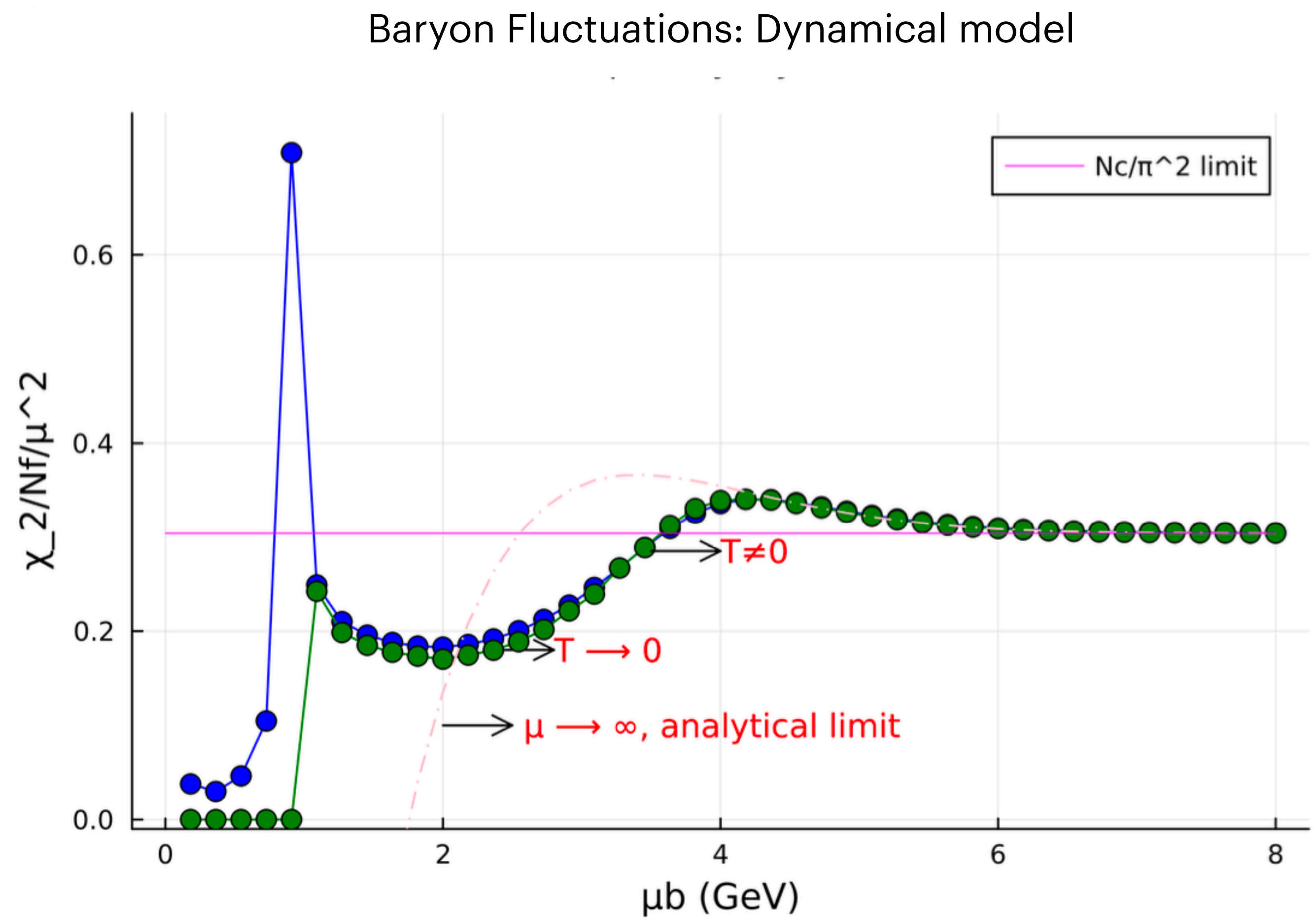
CONSEQUENCE OF DYNAMICAL INTERACTION
$T \longrightarrow 0$



$$cs^2 = \frac{\frac{n_v}{\mu}}{\frac{dn_v}{d\mu}} \longrightarrow cs^2 = \frac{1}{\mu}\frac{n_v}{\chi_2}$$

SPEED OF SOUND V/S CHIRAL SUSCEPTIBILITY





$$cs^2_{\mu\to\infty} \approx \frac{1}{3}\left[1 - \frac{\omega_\infty}{\mu}\left(1 + \frac{2\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

$$\chi 2_{\mu\to\infty} \approx \frac{N_c N_f}{\pi^2}\mu^2\left[1 - \frac{2\omega_\infty}{\mu}\left(1 - \frac{\mu^2}{UV^2}\right)e^{-\frac{\mu^2}{UV^2}}\right]$$

Non-local cut-off is inherent gluon interaction scale

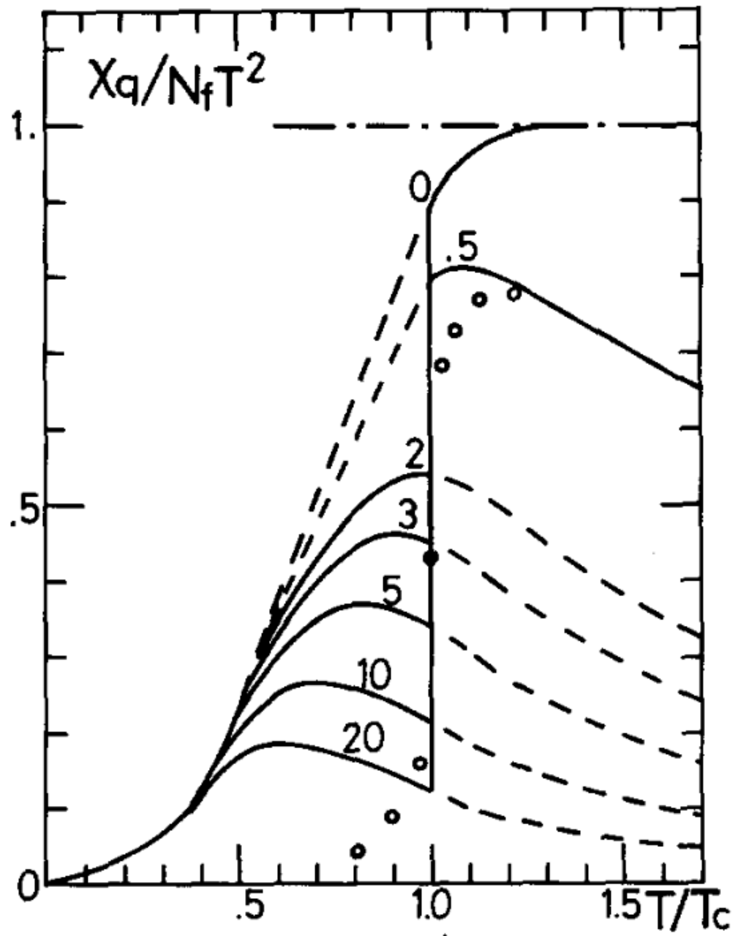


Fig. 1. The temperature dependence of the quark-number susceptibility χ_q in the unit of $N_f T^2$ with some of the vector coupling $g_V \Lambda^2$: $g_V \Lambda^2 = 0$, 0.5, 2, 3, 5, 10, 20, which are indicated with the numbers attached to the respective curves. The dash-dotted line shows the free massless case. The small circles are the lattice result on an $8^3 \times 4$ lattice with the quark mass $m/T = 0.2$ [7] compiled in ref. [9].

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Quark-number susceptibility and fluctuations in the vector channel at high temperatures ☆

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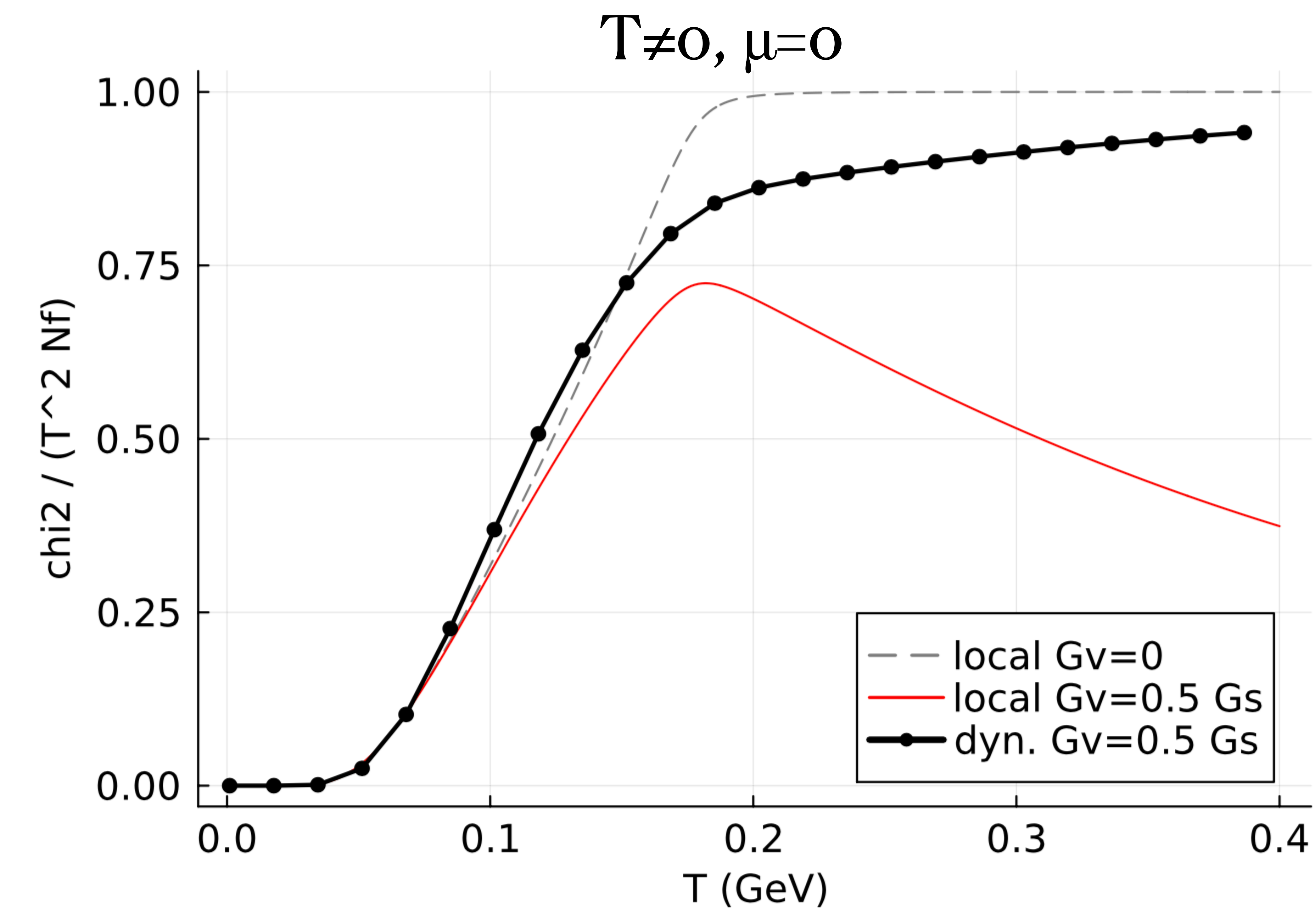
The quark-number susceptibility χ_q is examined as an observable which may help to reveal the physical picture of the high-temperature phase of QCD. It is emphasized that χ_q is intimately related with the fluctuations in the vector channel of the system. It is shown that the results of the recent lattice simulations of χ_q can be understood in terms of a possible change of the interactions between quark and anti-quarks in the vector channel, and imply that the fluctuations in the vector channel is greatly suppressed in the high-temperature phase in contrast with those in the scalar and pseudo-scalar ones.

BARYON FLUCTUATIONS: DYNAMICAL MODEL



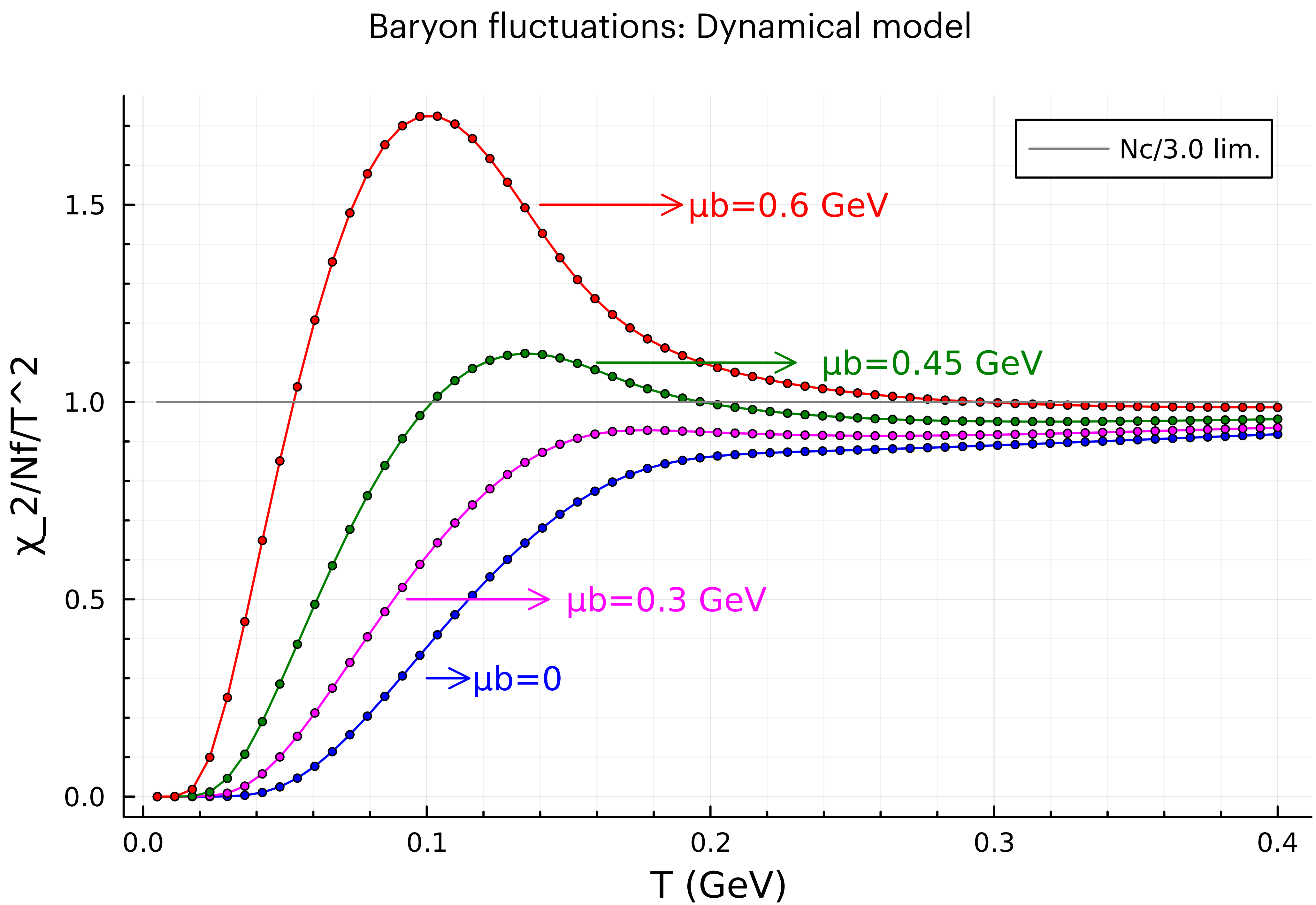
local

$$\chi_2 = \frac{dn_v}{d\mu} \propto \frac{T^2}{1 + CT^2}$$

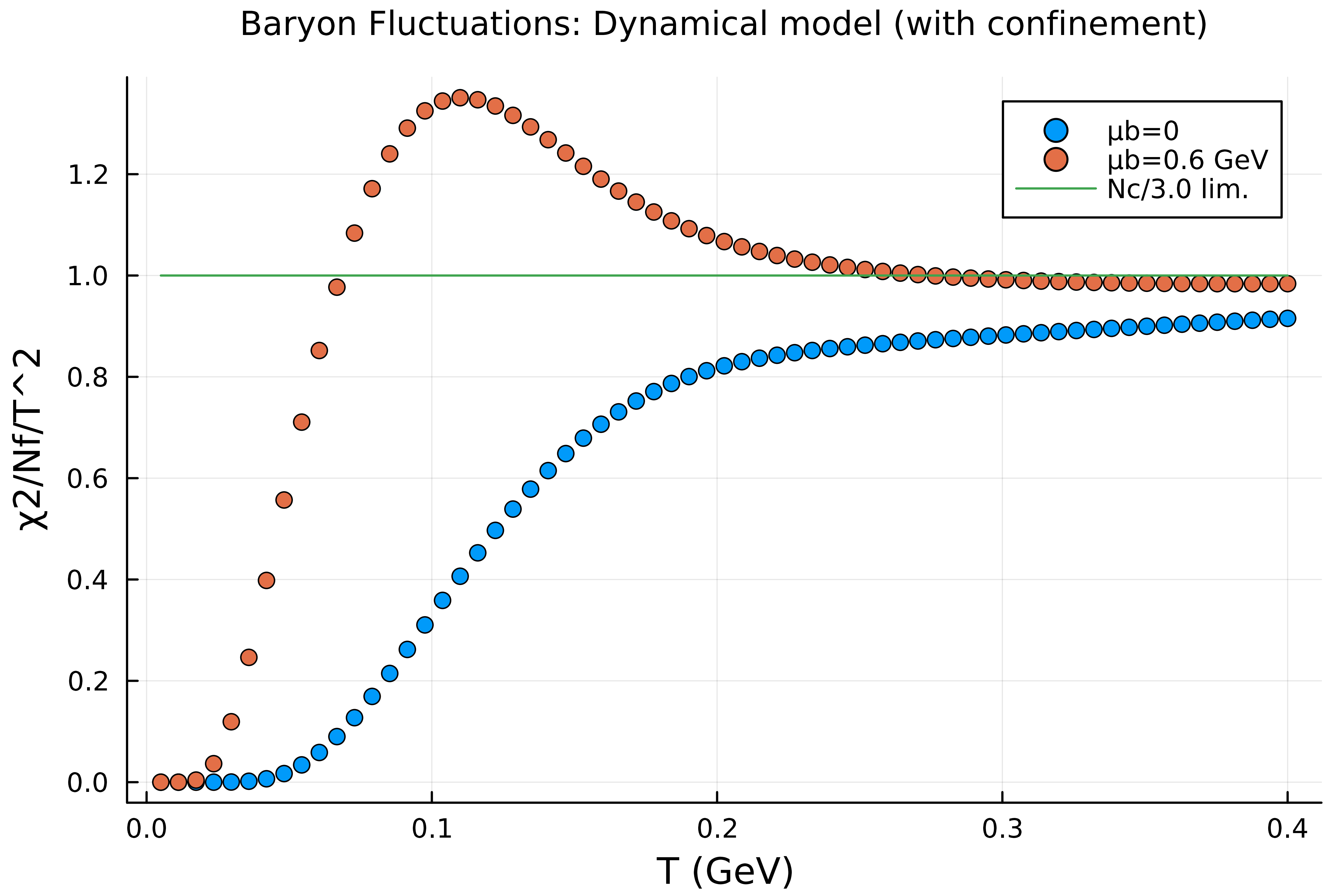
dynamical

$$\chi_2 = \frac{dn_v}{d\mu} \propto T^2$$

CONSEQUENCE OF DYNAMICAL INTERACTION
Finite Temperature



PRELIMINARY



(Non-Local) NJL with Diquark

$$m_p^* = m + \gamma_p 2G_s N_f \int \frac{d^3q}{(2\pi)^3} \gamma_q \frac{4m_q^*}{2E_q}[1 - n(E_q^-) - n(E_q^+) + \frac{E_q^+}{\epsilon_d^+(q)}(1 - 2n(\epsilon_d^+(q))) + \frac{E_q^-}{\epsilon_d^-(q)}(1 - 2n(\epsilon_d^-(q)))]$$

$$\mu_p^* = \mu - \gamma_p 2G_v N_f \int \frac{d^3q}{(2\pi)^3} \gamma_q \frac{4}{2}[n(E_q^-) - n(E_q^+) + \frac{E_q^+}{\epsilon_d^+(q)}(1 - 2n(\epsilon_d^+(q))) - \frac{E_q^-}{\epsilon_d^-(q)}(1 - 2n(\epsilon_d^-(q)))]$$

$$\Delta_p = \gamma_p 2G_d N_f \int \frac{d^3q}{(2\pi)^3} \gamma_q \Delta_q \frac{4}{2}[\frac{1 - 2n(\epsilon_d^+(q))}{\epsilon_d^+(q)} + \frac{1 - 2n(\epsilon_d^-(q))}{\epsilon_d^-(q)}]$$

DIQAURK PAIRING GAP
<qq>

$$E_q = \sqrt{p^2 + (m_q^*)^2}$$

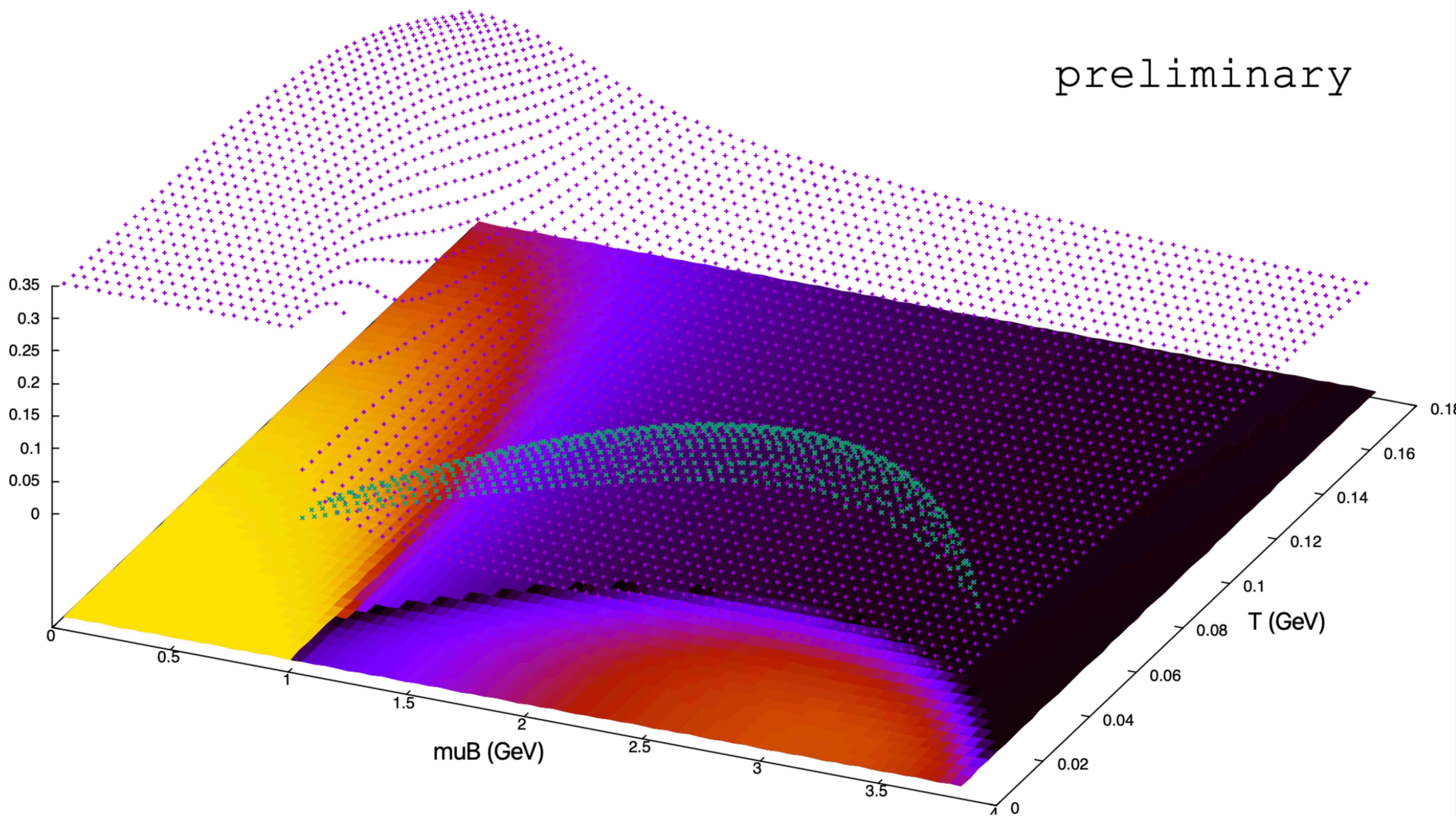
$$E_q^\pm = E_q \pm \mu_q^*$$

$$\epsilon_d^\pm(q) = \sqrt{(E_q^\pm)^2 + \Delta_q^2}$$

Quasi-particle property modified

Non-locality of constituent mass, chemical potential and diquark pairing gap

QCD Phase diagram

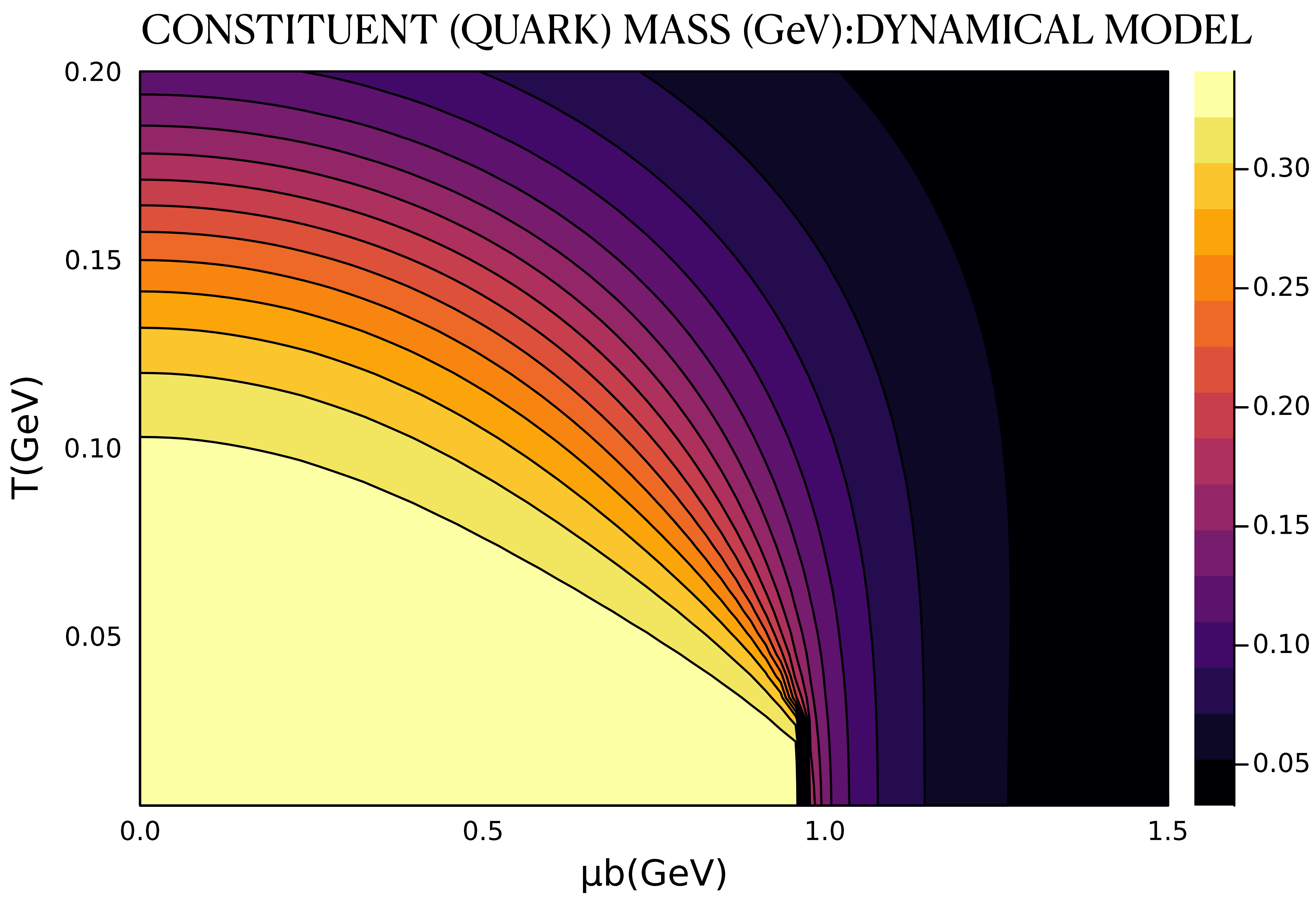


SUMMARY & CONCLUSIONS

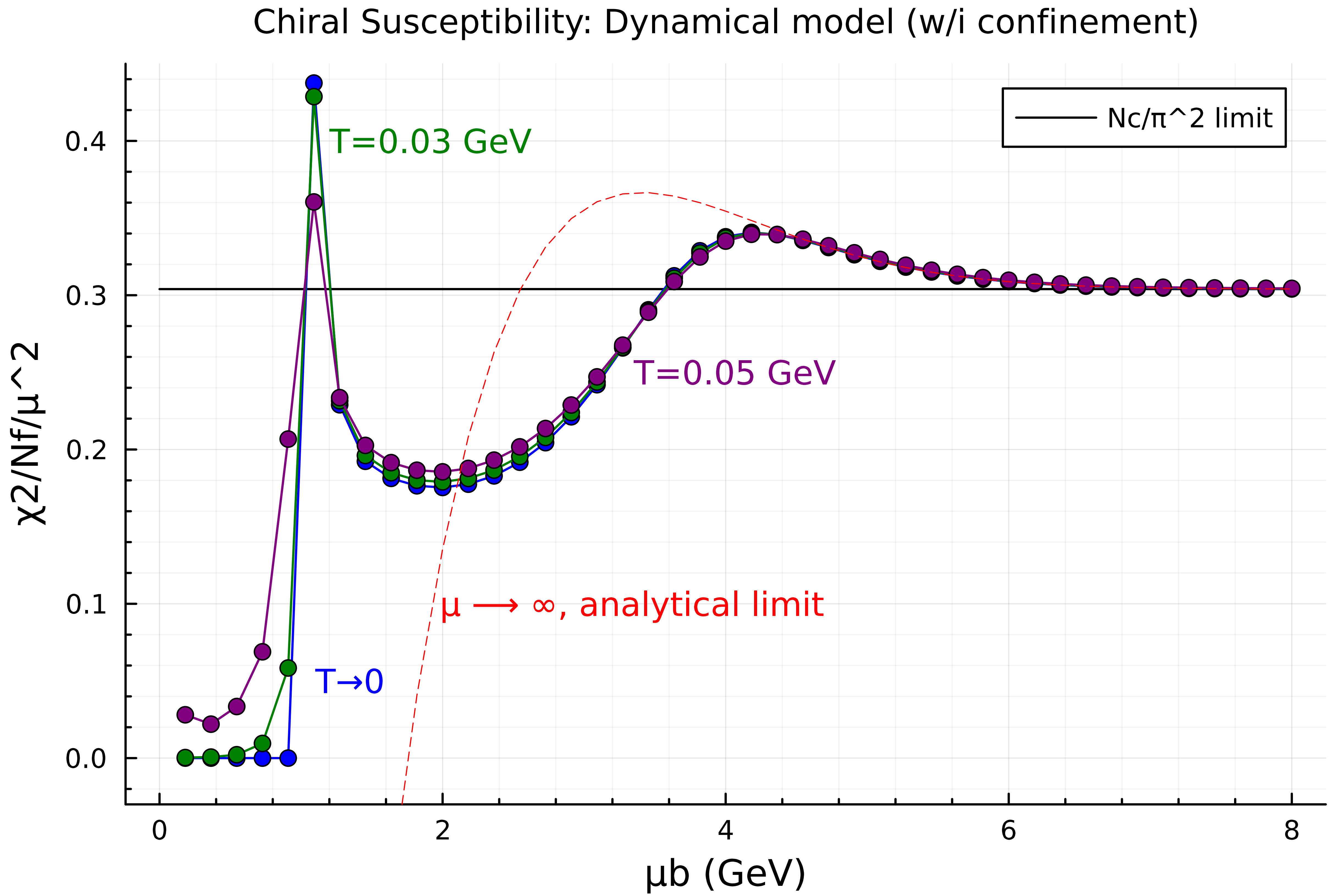
- Inherent gluon scale essential for modelling interacting Fermi surface —> natural density-dependence.
- Non-local cut-off —> essential, quantifiable contributions to cs^2 and χ_2 & not a scale to be removed.
- cs^2 and χ_2 reach asymptotic limit in a dynamical model.
- χ_2 along finite T (zero μ) reaches asymptotic limit in a dynamical model.

Thank you for your attention.

BACK UP SLIDES

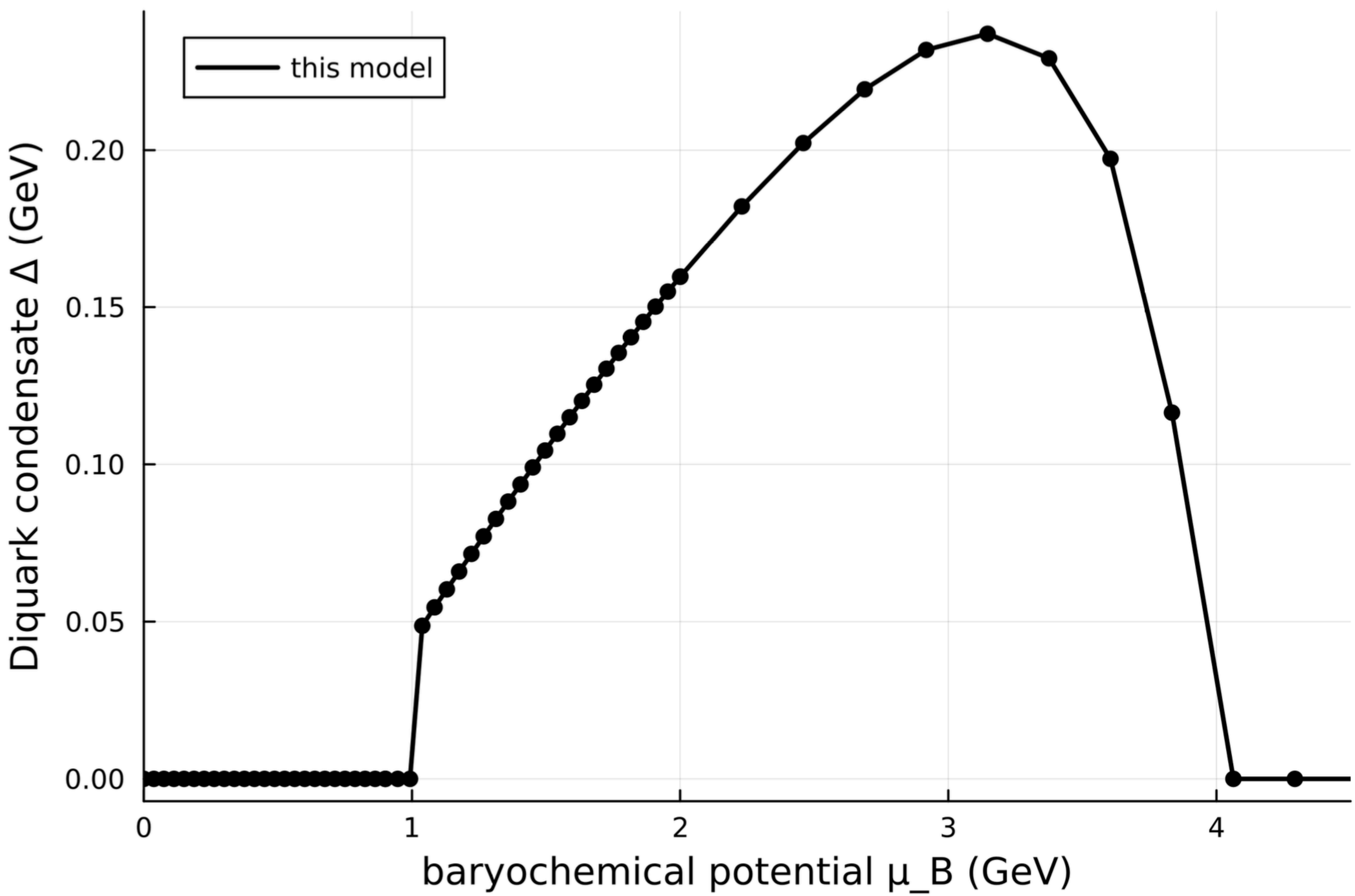

CONSTITUENT (QUARK) MASS (GeV):DYNAMICAL MODEL
T(GeV)
0.20
0.15
0.10
0.05
0.0
0.5
1.0
1.5
μb(GeV)
0.30
0.25
0.20
0.15
0.10
0.05

Verify this

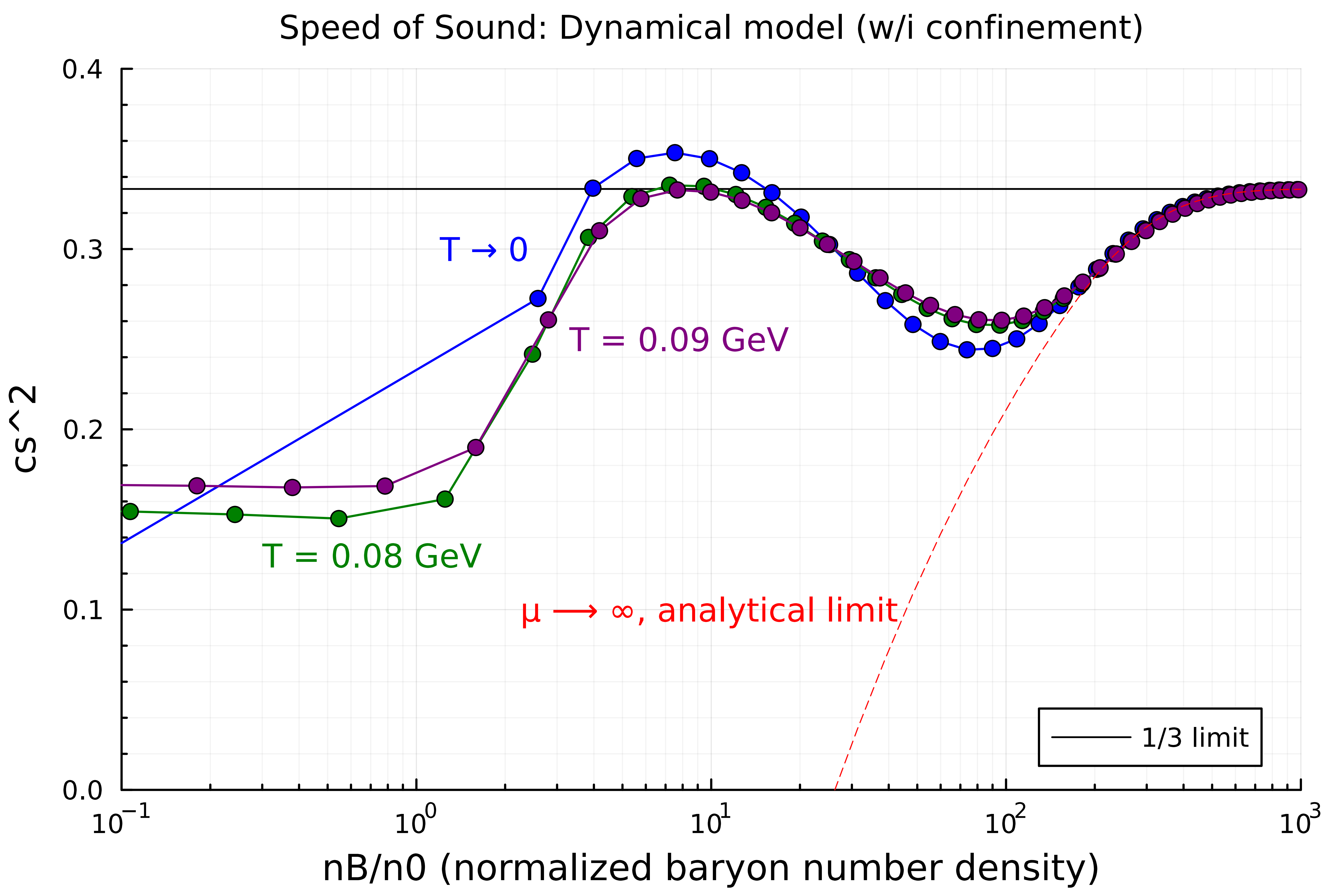


Chiral Susceptibility: Dynamical model (w/i confinement)
Nc/π^2 limit
T=0.03 GeV
T=0.05 GeV
μ ⟶ ∞, analytical limit
T→0
χ2/Nf/μ^2
μb (GeV)
0.0
0.1
0.2
0.3
0.4
0
2
4
6
8

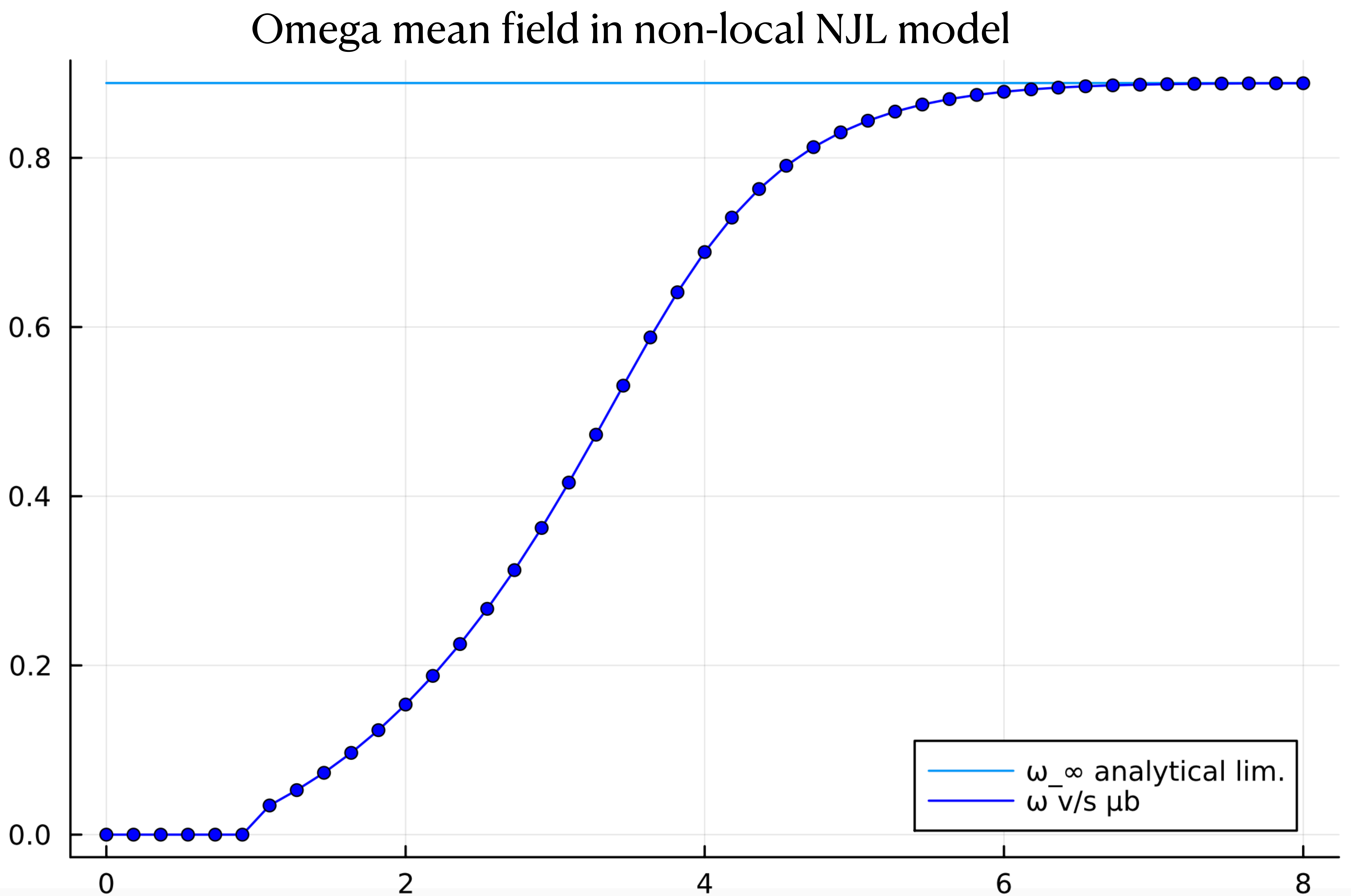
Diquark Gap



this model
Diquark condensate Δ (GeV)
0.00
0.05
0.10
0.15
0.20
0
1
2
3
4
baryochemical potential μ_B (GeV)



Speed of Sound: Dynamical model (w/i confinement)
cs^2
nB/n0 (normalized baryon number density)
0.0
0.1
0.2
0.3
0.4
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
T → 0
T = 0.09 GeV
T = 0.08 GeV
μ ⟶ ∞, analytical limit
1/3 limit


Omega mean field in non-local NJL model
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
ω_∞ analytical lim.
ω v/s μb