Bayesian analysis of the dense matter equation of state

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Bayes' theorem:

$$\underbrace{p(H_1 \mid D, I)}_{\text{Posterior}} = \frac{\overbrace{p(D \mid H_1, I)}^{\text{Likelihood}}\;\overbrace{p(H_1 \mid I)}^{\text{Prior}}}{\underbrace{p(D \mid I)}_{\text{Evidence}}}$$

Hypothesis (H1 = EoS parameter set 1)

Prior: knowledge before experiment (logically)

Likelihood: Probability for data if the hypothesis was true

Posterior: Probability that the hypothesis is true given the data

Evidence: normalization; important for model comparison

Prior
EoS parameters
- Coupling constants
- CEP location
- Onset density
- etc.

→ **Likelihood**
Simulation of HIC and Astro phenomena
Comparison results with available data

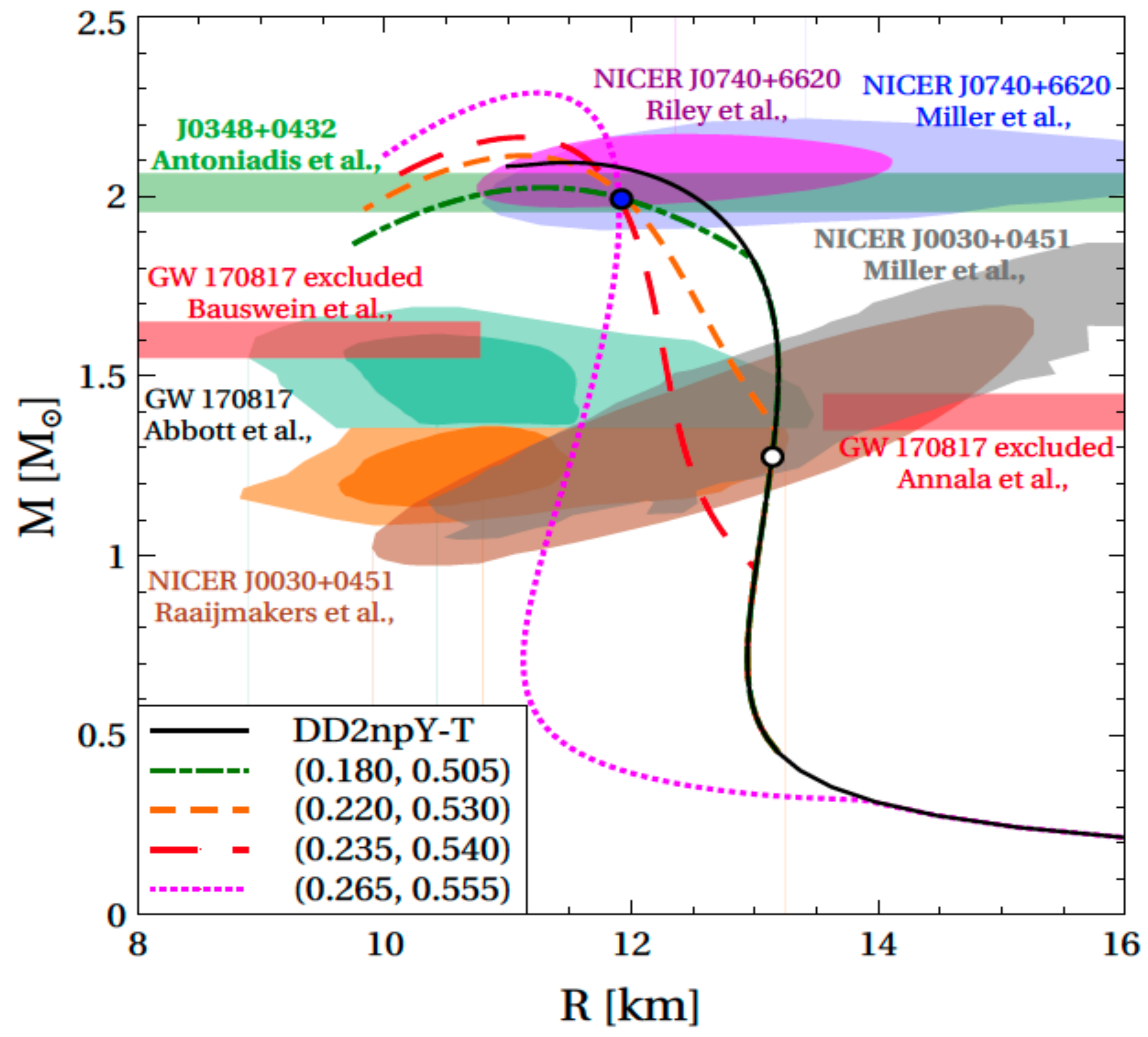
→ Bayes theorem

→ **Posterior**
Refinement of EoS: measure effects on QCD phase diagram phenomenology

In Bayesian inference, two main approaches exist for studying equations of state: the **'agnostic'** and **'educated'** methods:

- The **'agnostic'** approach treats the equation of state merely as a relation between pressure and density, making minimal assumptions about underlying physics. It relies solely on observational or experimental data to inform the probabilistic distribution of possible equations of state.
- Conversely, the **'educated'** approach analyzes the physical parameters defining the equation of state, incorporating prior knowledge from physical laws or empirical observations to constrain possible equations of state. This method is advantageous when theoretical foundations are well-established, and significant empirical evidence is available.

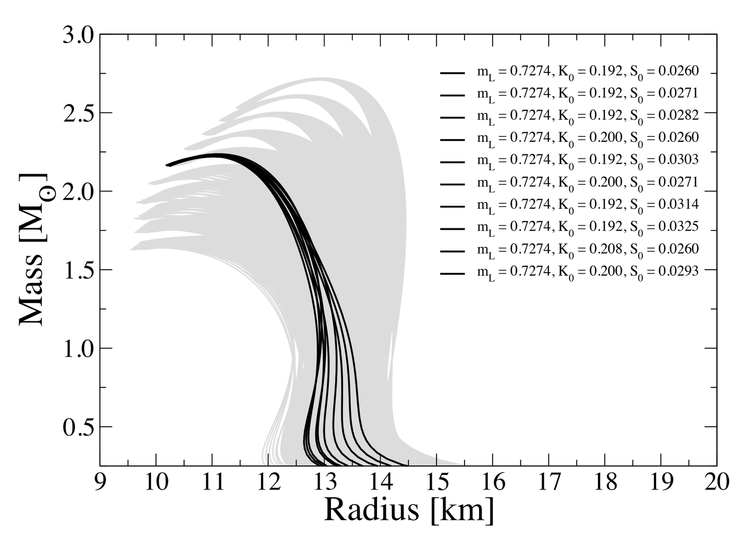
In our Bayesian analysis, we employ the **'educated'** approach. Here, we focus on analyzing the physical parameters that define the equation of state.

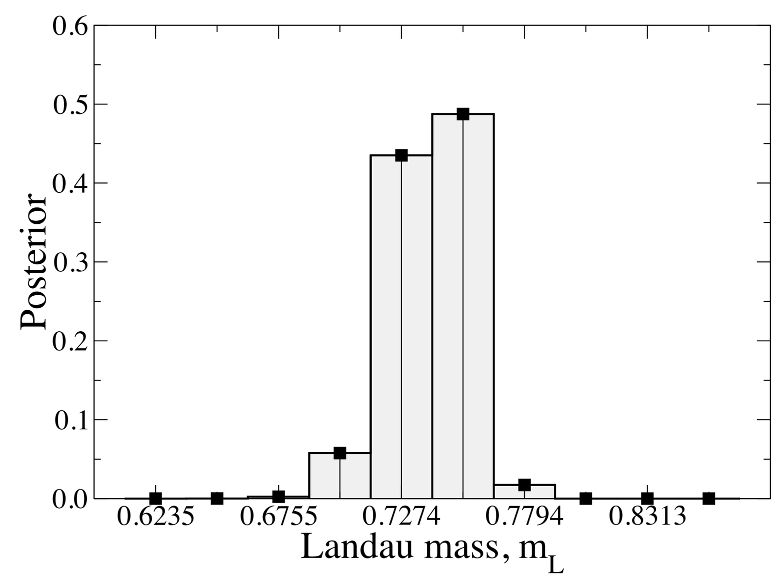


Modern data on the observation of masses and radii of neutron stars

$$\mathcal{L} = N_f \overline{\Psi}\left(i\partial\!\!\!/ - m_N + g_\sigma \sigma - g_\omega \omega\!\!\!/\right)\Psi + \frac{1}{2}\sigma\left(\partial^2 - m_\sigma^2\right)\sigma - U_i(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2$$

$$m_L = \frac{k_F}{v_F} \quad \text{with} \quad v_F = \frac{\partial E_k}{\partial k}\bigg|_{k=k_F} \quad m_L = \sqrt{k_F^2 + m^{*2}}$$



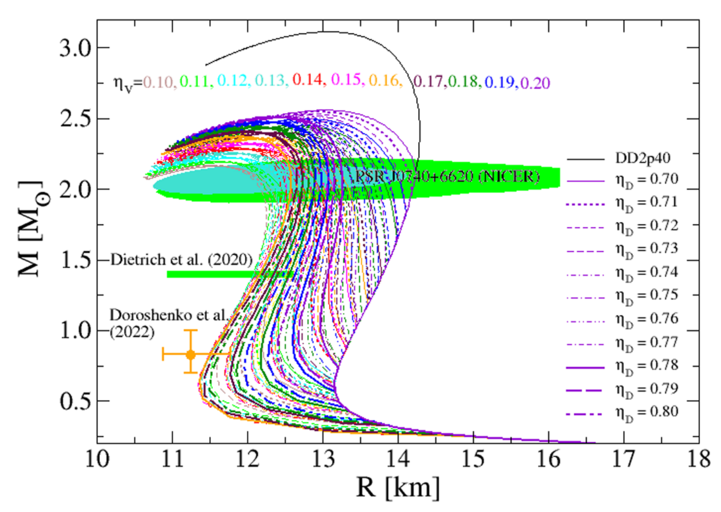


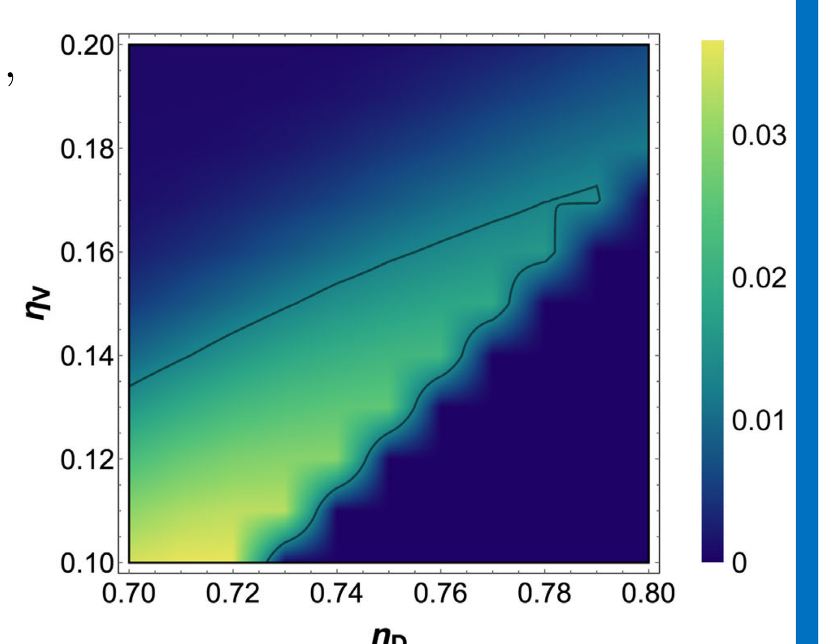
$$\mathcal{L} = \bar\psi\left(-i\partial\!\!\!/ + m_c\right)\psi - \frac{G_S}{2} j_S^f j_S^f - \frac{G_D}{2}[j_D^a]^\dagger j_D^a + \frac{G_V}{2} j_V^\mu j_V^\mu$$

$$j_S^f(x) = \int d^4z\, g(z) \bar\psi(x + \frac{z}{2}) \Gamma^f \psi(x - \frac{z}{2}),$$

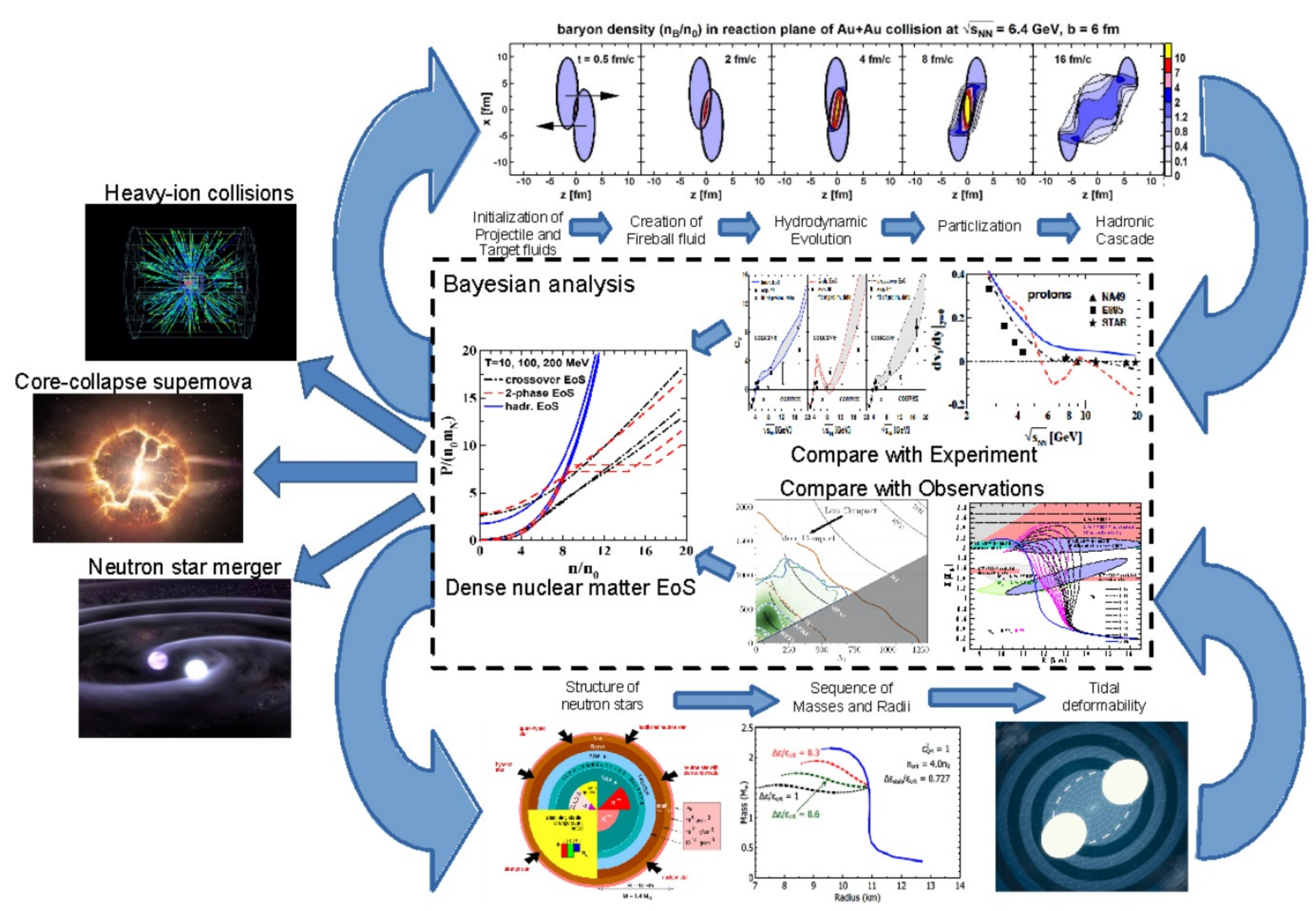
$$j_D^a(x) = \int d^4z\, g(z) \bar\psi_C(x + \frac{z}{2}) i\gamma_5 \tau_2 \lambda^a \psi(x - \frac{z}{2}), \qquad \eta_D = G_D/G_S$$

$$j_V^\mu(x) = \int d^4z\, g(z) \bar\psi(x + \frac{z}{2}) i\gamma_\mu \psi(x - \frac{z}{2}), \qquad \eta_V = G_V/G_S$$





We propose a new project to develop the Bayesian analysis (BA) approach and its implementation to simulations of HIC and NS astrophysics to answer the question: "Is a first−order phase transition to quark matter accessible in heavy−ion collisions and in the interiors of compact stars?" and "How can one identify it within already operating and planned HIC experiments and observational campaigns?"



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