Zubarev Meets Bayes: Non-Equilibrium Pion Distribution Function in Heavy-Ion Collisions

Oleksandr Vitiuk[1], David Blaschke[1,2,3], Benjamin Dönigus[4], Gerd Röpke[5]

[1]University of Wroclaw [2]Helmholtz-Zentrum Dresden-Rossendorf [3]Center for Advanced Systems Understanding

[4]Goethe University Frankfurt [5]University of Rostock

Introduction

The transverse momentum spectra of identified particles measured by the ALICE Collaboration [1] show an interesting feature, namely low-p_T pion spectra show enhancement with respect to the predictions of various models. We apply the Zubarev approach of the non-equilibrium statistical operator [2] to develop a thermal particle generator that can account for the enhancement of soft pions by introducing an effective pion chemical potential. This is an alternative to the explanation of the low-p_T enhancement by resonance decays. Bayesian inference methods are used to find the most probable sets of thermodynamic parameters at the freeze-out hypersurface.

Zubarev Approach: Non-equilibrium Pion Distribution

The non-equilibrium state of the system is characterized by relevant observables $\{B_n\}$ in addition to the standard set of conserved ones. We look for the distribution which maximizes the information entropy $S_{\rm inf} = -\mathrm{Tr}\{\rho_{rel}(t)\ln\rho_{rel}(t)\}$:

$$\rho_{rel}(t) = \frac{1}{Z_{rel}(t)} e^{-\sum_n F_n(t) B_n}, \qquad Z_{rel}(t) = \mathrm{Tr}\{e^{-\sum_n F_n(t)B_n}\},$$

where Lagrange multipliers $F_n(t)$ are determined by the self-consistency conditions

$$\langle B_n\rangle^t = \langle B_n\rangle^t_{rel} = \mathrm{Tr}\{\rho_{rel}(t) B_n\}.$$

In this work we assume the following:

- A state overpopulated by soft pions is formed at $\tau < \tau_\pi^{CFO}$.
- For $\tau_\pi^{CFO} < \tau < \tau_\pi^{FO}$ the collisions conserve the particle number but evolve the distribution function to a thermal equilibrium distribution.

Under these assumptions, the pion number is quasi-conserved and can be chosen as a relevant observable. Then, the new self-consistency condition is:

$$\langle N_\pi\rangle^t_{rel} = \langle N_\pi\rangle^t.$$

This leads to the appearance of a non-equilibrium pion chemical potential [2]

$$f_\pi = \left(\exp\left[\frac{E}{T}\right] - 1\right)^{-1} \Rightarrow f_\pi = \left(\exp\left[\frac{E-\mu_\pi}{T}\right] - 1\right)^{-1}$$

Blast-Wave Model of Particle Freeze-out

Here, we consider chemical freeze-out on the cylindrical boost-invariant hypersurface of radius R, with constant temperature T at constant freeze-out proper time $\tau = \sqrt{t^2 - z^2}$

$$\Sigma^\mu = (\tau\cosh\eta, r\cos\varphi, r\sin\varphi, \tau\sinh\eta), \text{ where } \eta = \frac{1}{2}\ln\frac{t+z}{t-z}.$$

The longitudinal expansion is assumed to be boost invariant, and the radial flow is parametrized as $\beta_T = v(r/R)^n$ with the surface velocity v and profile exponent n

$$u^\mu = (\cosh\rho\cosh\eta, \sinh\rho\cos\varphi, \sinh\rho\sin\varphi, \cosh\rho\sinh\eta), \text{ where } \rho = \mathrm{atanh}\,\beta_T.$$

Then, for the particle with four-momentum

$$p^\mu = (m_T\cosh y, p_T\cos\psi, p_T\sin\psi, m_T\sinh y),$$

using the Cooper-Frye formula $E\frac{d^3N}{d^3\vec{p}} = \int_{\Sigma_{FO}} p^\mu d\Sigma_\mu f(x^\mu, p^\mu u_\mu)$ one finds

$$\frac{d^6 N_i}{dp_T dy d\psi dr d\eta d\varphi} \propto \tau r p_T m_T \cosh(y-\eta) \times$$
$$\times\left(\exp\left[\frac{m_T\cosh\rho\cosh(y-\eta) - p_T\sinh\rho\cos(\varphi-\psi) - \mu_i}{T}\right] \pm 1\right)^{-1}.$$

Here $\tau, R, T, \{\mu_i\}, v$ and n are free model parameters.

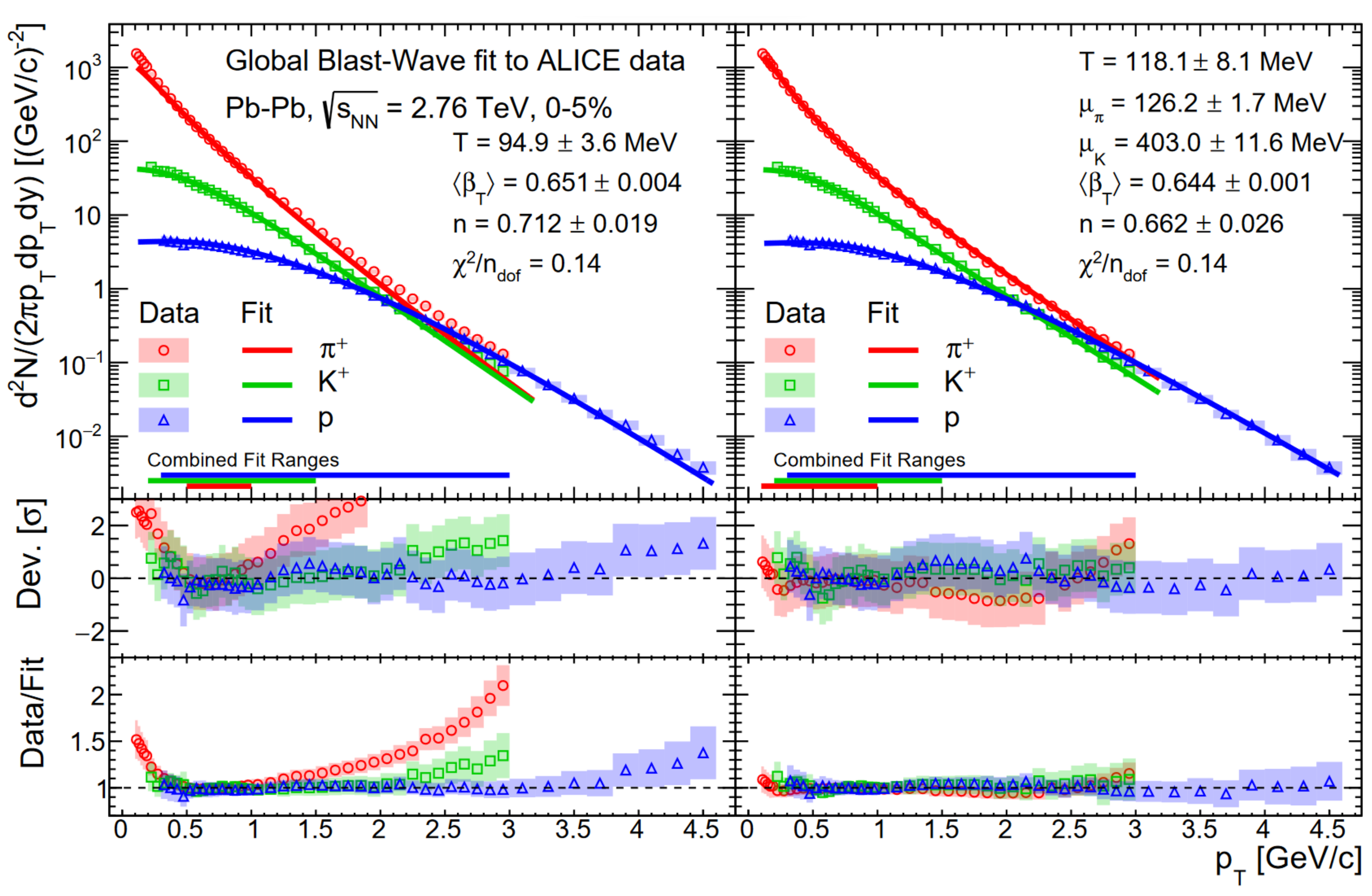


Figure 1. Left panel: The traditional blast-wave fit of ALICE data with Boltzmann statistics without chemical potentials. Right panel: Fit involving the Bose-Einstein statistics and chemical potentials of pions and kaons.

✓ Using two additional free parameters μ_π and μ_K one can achieve much better agreement between model and experimental data

× Feed-down and final state interactions are not taken into account explicitly

❖ Improved model: Thermal particle generator + Hadronic transport afterburner

Bayesian Inference

For a model, which for an input parameter vector $\vec{x} = (x_1, \ldots, x_n)$ gives an output $\vec{y} = \vec{y}(\vec{x}) = (y_1, \ldots, y_m)$, we want to find the "optimal" value of $\vec{x}$ to describe the experimental data $\vec{y}^{obs}$

$$\underbrace{P(\vec{x}|\vec{y}^{obs})}_{\text{Posterior}} = \frac{\mathcal{L}(\vec{x};\vec{y}^{obs})P(\vec{x})}{P(\vec{y}^{obs})} \propto \underbrace{\mathcal{L}(\vec{x};\vec{y}^{obs})}_{\text{Likelihood}} \times \underbrace{P(\vec{x})}_{\text{Prior}}.$$

The goal is to find the Maximum a Posteriori (MAP) parameter, i.e. the point in parameter space which maximizes the posterior distribution.

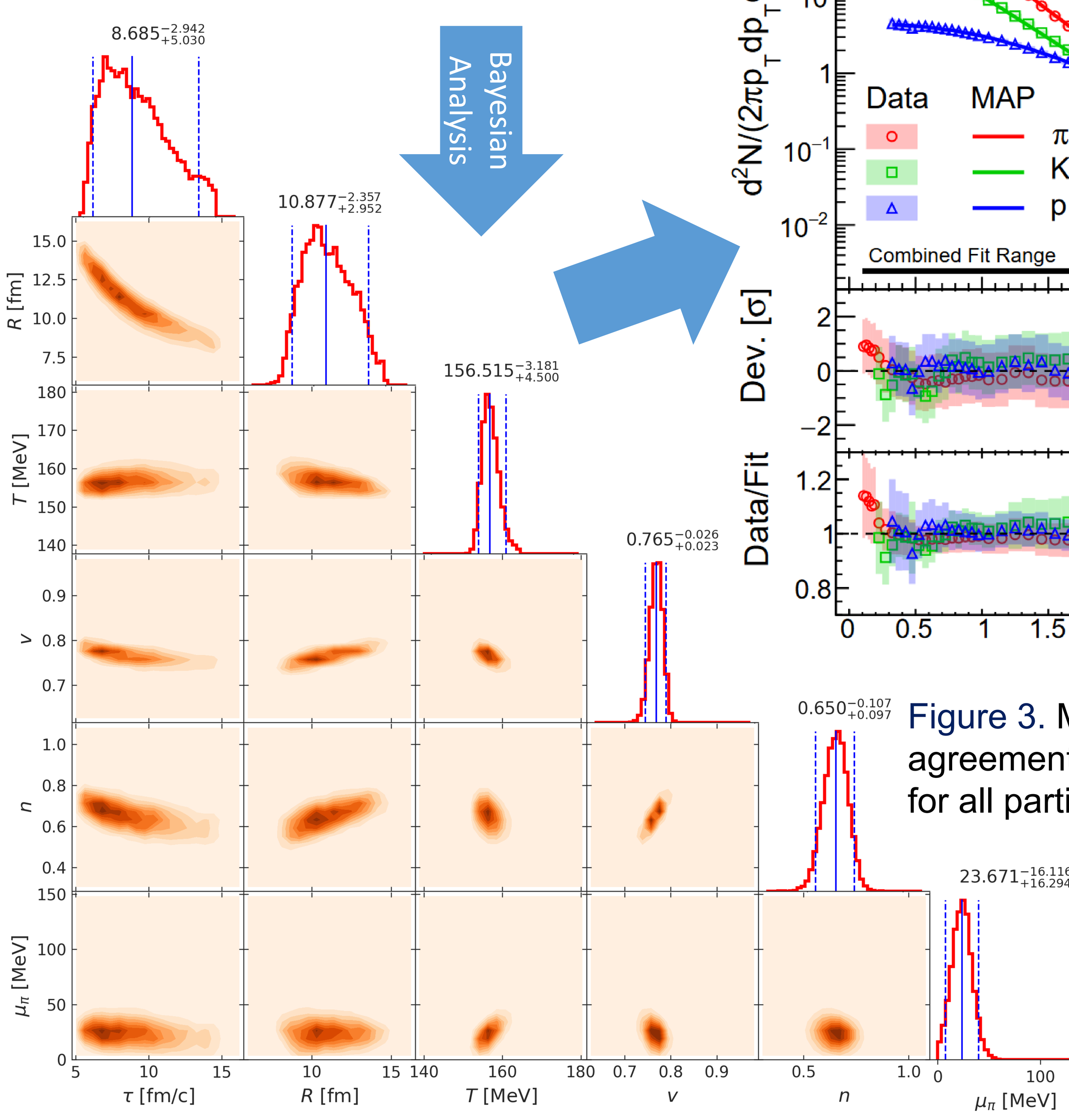
Model setup:

- Blast-Wave thermal particle generator model with SMASH [3] afterburner
- Observables: $p, \bar{p}, \pi^+, \pi^-, K^+, K^-$ spectra in 0-5% Pb-Pb@2.76 TeV collisions
- 200 training and 50 validation data sets
- Principal Component Analysis + Gaussian Processes emulator
- Markov Chain MC to recover posterior distribution
- Uniform prior

$\tau \in [5;15]\ fm/c$ $\quad R \in [6;15]\ fm$

$T \in [145;165]$ MeV $\quad \mu_\pi \in [0;137.9]$ MeV

$v \in [0.65;0.9]$ $\quad n \in [0.3;1.0]$



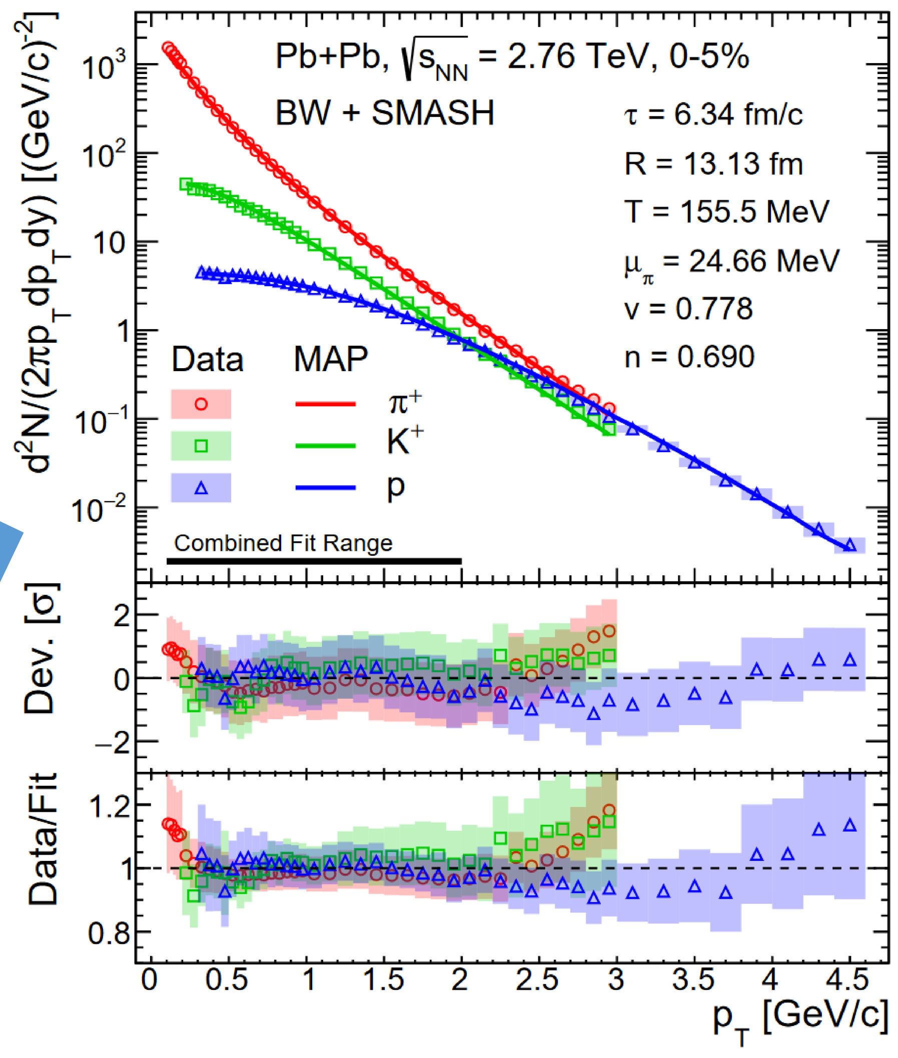


Figure 3. MAP prediction shows good agreement with the experimental data for all particles in the wide p_T range.

Figure 2. Marginal and joint marginal posterior probability distributions of model parameters. Blue solid lines indicate medians of the marginal distributions, while blue dashed lines show 90% credible intervals.

The advanced version of the model gives a non-zero value of the pion chemical potential $\mu_\pi \simeq 24$ MeV

Summary

- The non-equilibrium process of pion production within the Zubarev approach of the non-equilibrium statistical operator leads to the appearance of a non-equilibrium pion chemical potential
- A naive model gives a value of non-equilibrium chemical potential close to the pion mass and can describe data well, but it does not take into account a feed-down from resonance decays and final state interactions
- A more sophisticated model gives smaller, but non-zero value of a non-equilibrium pion chemical potential $\mu_\pi \simeq 24$ MeV

References

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Uniwersytet Wrocławski · CASUS Center for Advanced Systems Understanding · HZDR Helmholtz-Zentrum Dresden Rossendorf · Goethe Universität Frankfurt am Main · Universität Rostock · National Science Centre Poland

Grant № 2022/45/N/ST2/02391

oleksandr.vitiuk@uwr.edu.pl