Electrical conductivity of warm neutron star crust in magnetic fields: Neutron-drip regime

Arus Harutyunyan [1,2] Armen Sedrakian [3,4] Narine Gevorgyan [1,5] Mekhak Hayrapetyan [2]

[1]Byurakan Astrophysical Observatory, Byurakan 0213, Armenia
[2]Department of Physics, Yerevan State University, Yerevan 0025, Armenia
[3]Frankfurt Institute for Advanced Studies, D-60438 Frankfurt am Main, Germany
[4]Institute of Theoretical Physics, University of Wroclaw, 50-204 Wroclaw, Poland
[5]A.I. Alikhanyan National Science Laboratory, Yerevan, Armenia

Abstract

We compute the anisotropic electrical conductivity tensor of the inner crust of a compact star at non-zero temperature by extending a previous work on the conductivity of the outer crust. The physical scenarios, where such crust is formed, involve proto-neutron stars born in supernova explosions, binary neutron star mergers and accreting neutron stars. The temperature-density range studied covers the transition from a semi-degenerate to a highly degenerate electron gas and assumes that the nuclei form a liquid, i.e., the temperature is above the melting temperature of the lattice of nuclei. The electronic transition probabilities include (a) the screening of electron-ion interaction in the hard-thermal-loop approximation for the QED plasma, (b) the correlations of the ionic component in a one-component plasma, and (c) finite nuclear size effects. The conductivity tensor is obtained from the Boltzmann kinetic equation in relaxation time approximation accounting for the anisotropy introduced by a magnetic field. The sensitivity of the results towards the matter composition of the inner crust is explored by using several compositions of the inner crust which were obtained using different nuclear interactions and methods of solving the many-body problem. The standard deviations of relaxation time and components of the conductivity tensor from the average are below $\leq 25\%$ except close to crust-core transition, where non-spherical nuclear structures are expected. Our results can be used in dissipative magneto-hydrodynamics (MHD) simulations of warm compact stars.

Introduction

Transport properties in hot baryonic matter are important for large-scale magnetohydrodynamic (MHD) descriotions of astrophysical phenomena associated with compact stars. One such setting offer the binary neutron star (BNS) mergers, such as the GW170817 event [1], see Fig. 1, in the premerger and postmerger phases. Here we consider the warm matter regime at subnuclear densities, where $T_m \leq T \leq T_{tr}$, with $T_m \simeq 1$ MeV being the melting temperature and $T_{tr} \simeq 5$ MeV the neutrino trapping temperature [2]. Recently, the electrical conductivity of the warm outer crust was computed in Ref. [3] in the context of BNS mergers. It was used to show the breakdown of the ideal MHD limit and the importance of the Hall conductivity in [4]. This poster considers the extension to the inner crust phase where along with the crustal lattice, there is an unbound neutron component [5].

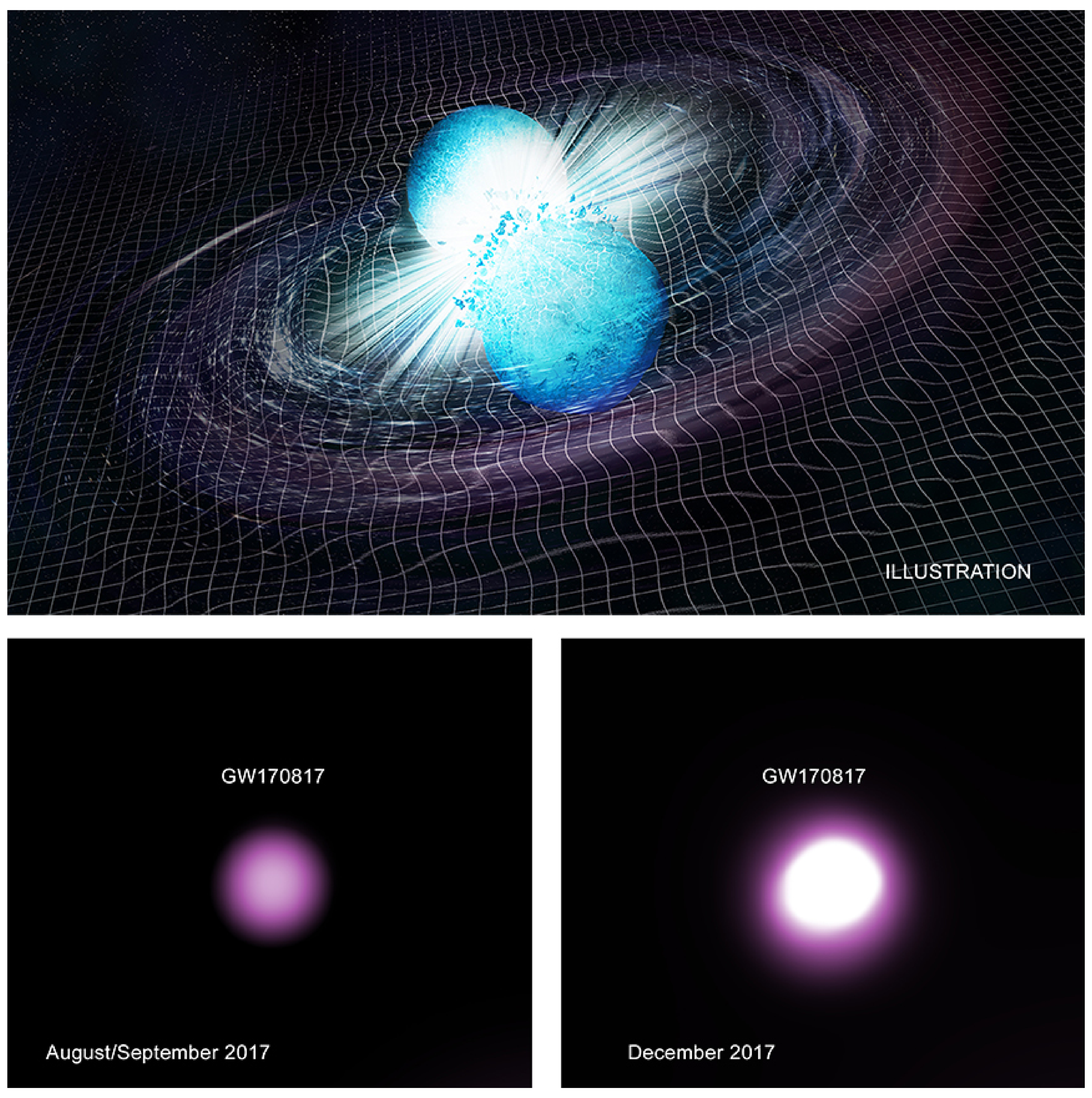

Figure 1. Artistic view of the binary neutron star merger GW170817 in the merger stage (upper panel) and optical images of the postmerger (kilonova) stage at two observation times (lower panels).

Equation of state & composition of inner crust

Above the neutron drip density $\rho_{\rm drip} = 4.3 \times 10^{11}$ gcm^{-3} the phase of fully ionized nuclei (nucleon number A and proton number Z) and relativistic electrons of the outer crust is replaced by a phase which in addition has also unbound neutrons. The total baryon density n_B is given by $\mathrm{n}_B = An_i + n'_n, where \mathrm{n}_i$ is the number density of the ions (nuclei) and $n'_n = (1 - V_N n_i)n_n$, where n_n is the number density of unbound neutrons, $V_N n_i$ is the excluded volume correction [6]. The ion-electron sub-system, viewed as Coulomb plasma, is characterized by the parameters

$$\Gamma = \frac{T_C}{T}, \qquad T_C = \frac{e^2 Z^2}{a_i},$$

where e is the elementary charge, T is the temperature, $a_i = (4\pi n_i/3)^{-1/3}$ is the radius of the spherical volume per ion, , that of the Wigner-Seitz cell. For $\Gamma \ll 1$ ($T \gg T_{\rm C}$) ions are weakly coupled and because of their large mass they form a classical Boltzmann gas. When $\Gamma \geq 1$ ions are strongly coupled and form a solid phase with nuclei arranged in a regular lattice for $\Gamma > \Gamma_m \simeq 160$. In the opposite case $\Gamma < \Gamma_m$ the liquid phase is energetically preferred. The temperature of melting of the crustal lattice is given by $T_m = T_C/\Gamma_m$. The lattice plasma temperature is defined as

$$T_p = \left(\frac{4\pi Z^2 e^2 n_i}{M}\right)^{1/2},$$

where M is the ion mass.

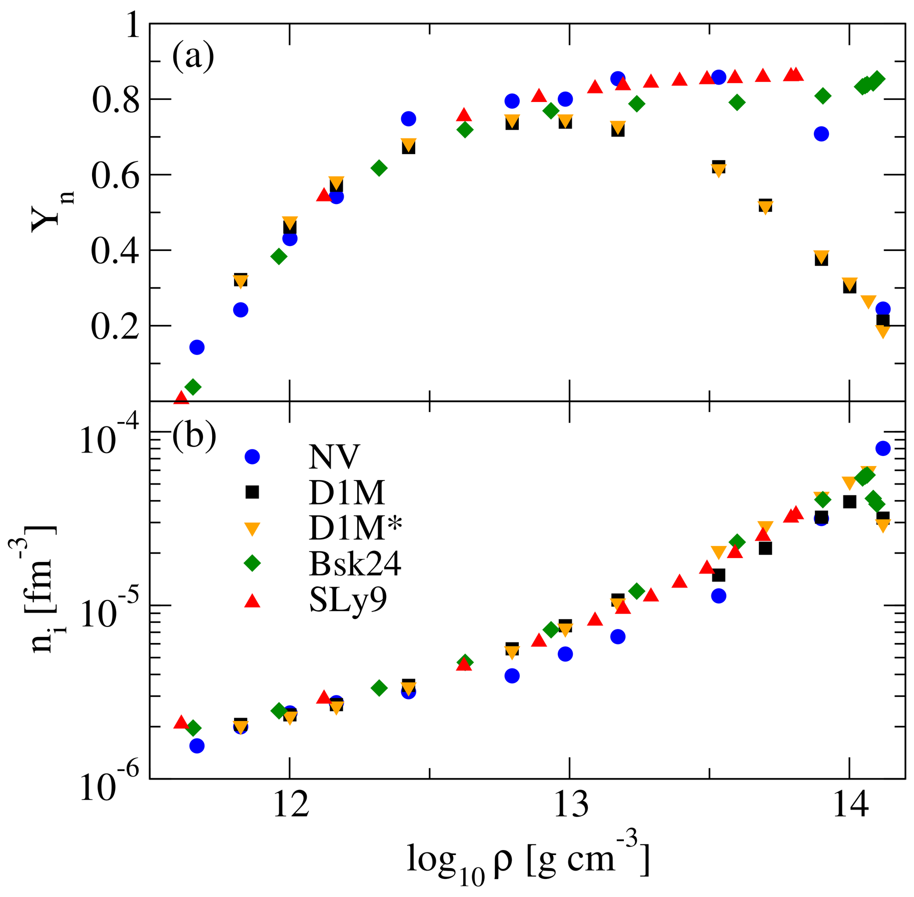

Figure 2. The fraction of free neutrons $Y_n = n'_n/n_B$ (a) and the number density of ions (b) as functions of the mass density for five compositions of stellar matter.

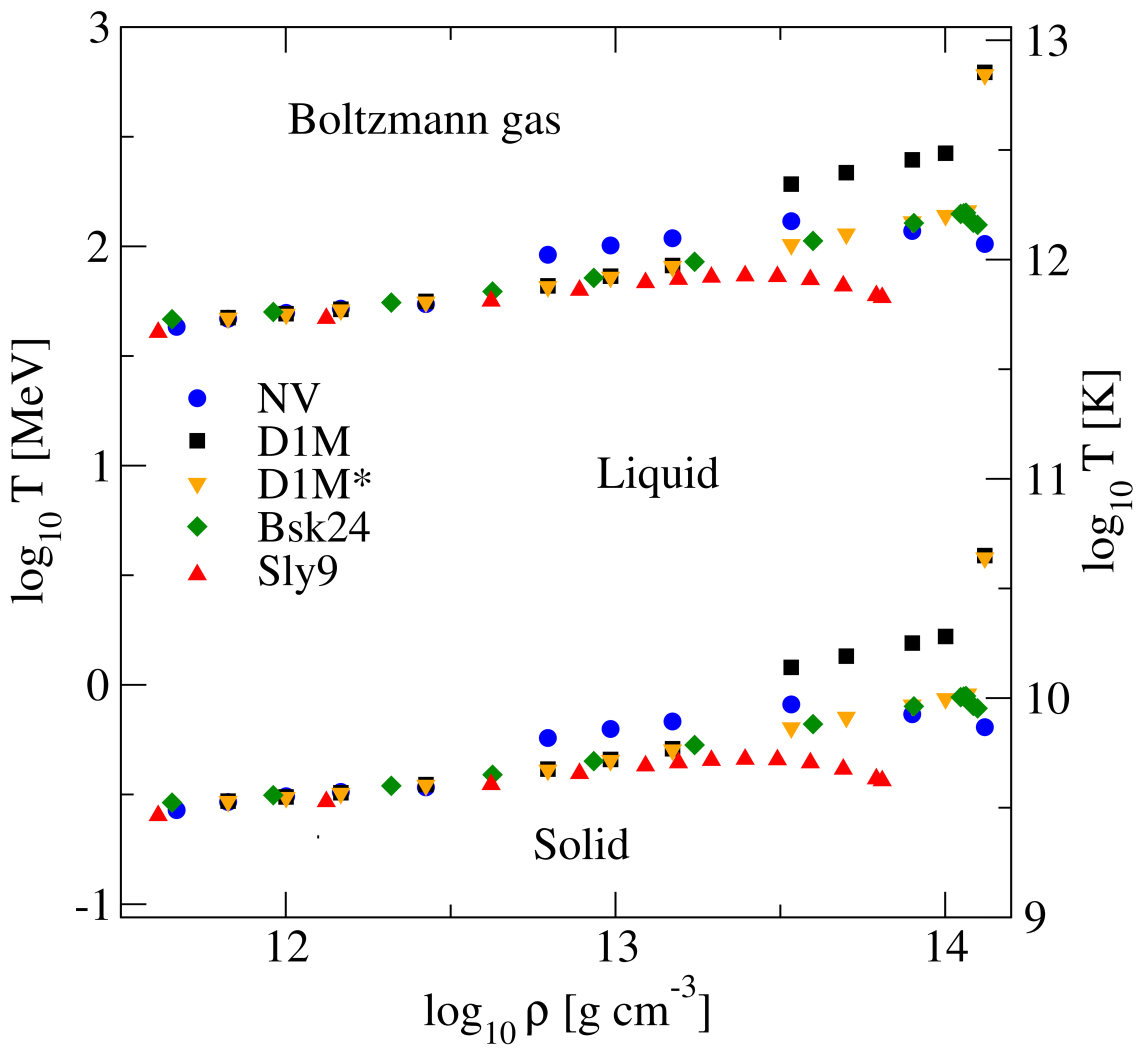

Figure 3. The phase diagram of dense plasma in the inner crust of the neutron star in the temperature-density plane for five different compositions. The lower curves show the melting temperature T_m below which the ionic component solidifies. Upper curves show $T_{\rm C}$ above which the ionic component forms a Boltzmann gas. The present study covers the liquid portion of the phase diagram.

Conductivity in magnetic field from Boltzmann equation

Boltzmann equation for the electron distribution function

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial r} - e(+[\times])\frac{\partial f}{\partial} = I[f],$$

where the collision integral for electron-ion collisions has the form

$$I = -(2\pi)^4 \sum_{234} |\mathcal{M}_{12\to 34}|^2 \delta^{(4)}(p + p_2 - p_3 - p_4) \times [f(1-f_3)g_2 - f_3(1-f)g_4].$$

The electrical conductivity is obtained by computing the electrical current

$$j_i = -2\int \frac{dp}{(2\pi)^3} ev_i\delta f = \sigma_{ij}E_j\ , \ \sigma_{ij} = \delta_{ij}\sigma_0 - \epsilon_{ijm}b_m\sigma_1 + b_ib_j\sigma_2,$$

where

$$\sigma_n = \frac{e^2}{3\pi^2 T}\int_m^\infty d\varepsilon \frac{p^3}{\varepsilon}\frac{\tau(\omega_c\tau)^n}{1+(\omega_c\tau)^2} f^0(1-f^0), \quad n = 0, 1, 2.$$

Figure 4 shows the relaxation time τ and the Hall parameter $\omega_c\tau$ for five compositions as functions of the mass density for the temperature $T = 5$ MeV. To assess the variations in the relaxation time and Hall parameter $\omega_c\tau$ with the composition we use

$$\tau^{-1} = \frac{4Ze^4\varepsilon_F}{3\pi}\int_0^{2p_F}\frac{dq}{q}\left(1 - \frac{q^2}{4\varepsilon_F^2}\right) S(q)F^2(q),$$

where ion-ion correlatios are taken into account by the structure factor $S(q)$ and the nuclear formfactor $F(q)$ accounts for the finite nuclear size.

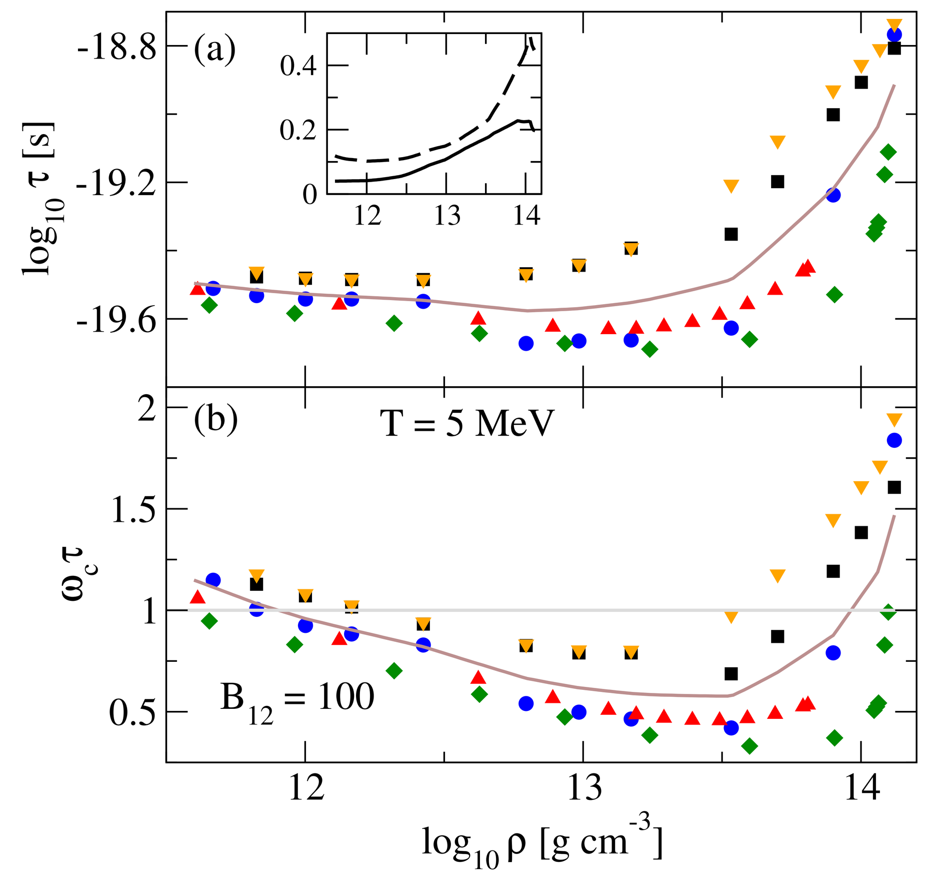

Figure 4. The relaxation time τ and the Hall parameter $\omega_c\tau$ at the Fermi energy as functions of the mass density for five compositions as labeled in Fig. **??**. The temperature is fixed at $T = 5$ MeV, and the magnetic field is fixed at $B_{12} = 100$ for panel (b). The solid lines show the values of these quantities averaged over the five compositions. The solid and dashed curves in the inset show the standard deviations of $\log\tau$ and $\omega_c\tau$, respectively.

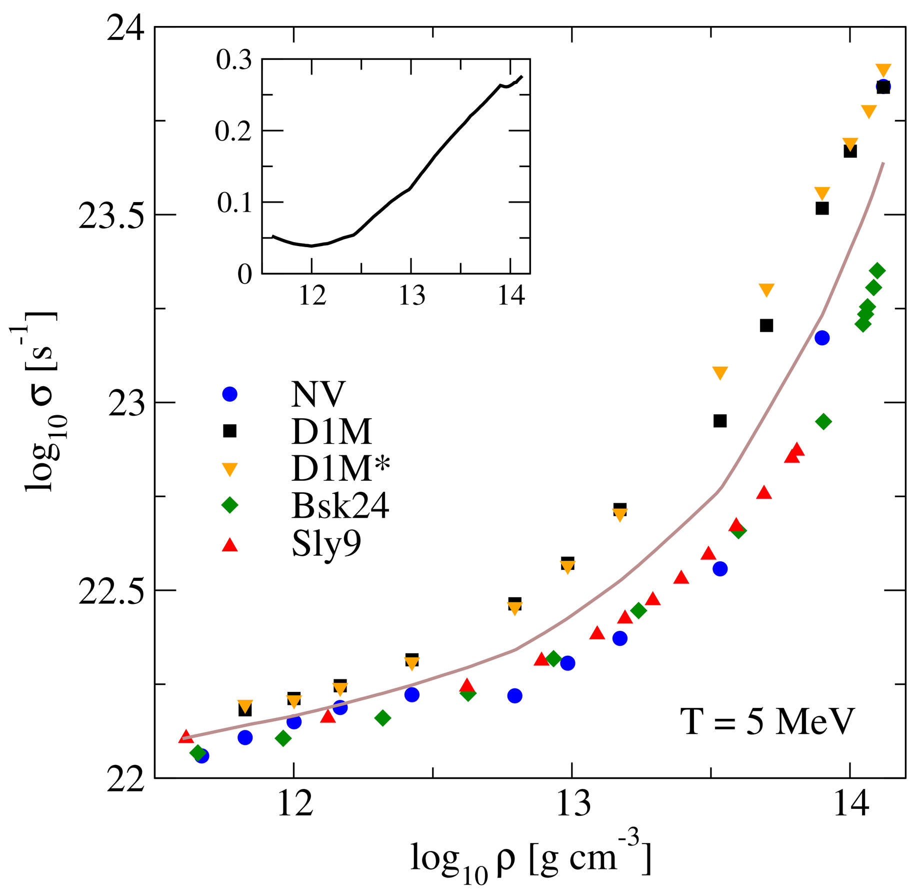

Figure 5. Dependence of the scalar conductivity on density for five compositions. The temperature is fixed at $T = 5$ MeV. The solid line shows the logarithm of the conductivity averaged over the five compositions, and the inset shows the standard derivation for $\log\sigma$.

Conclusions

We have quantified the conductivity of the warm inner crust of a compact star within the Boltzmann quasiparticle transport of electrons in the liquid phase of inner crust matter taking into account the screening of electron-ion interaction, finite nuclear size, ion structure factor, and magnetic fields. In the future, this study can be extended to higher temperatures by accounting for the multi-ion composition of matter, which includes α-particles and other light clusters [7].

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