In[1]:=
```
Get["e:/MB/MB.m"];
Get["e:/MB/MBresolve.m"];
Get["e:/MB/barnesroutines.m"];
Get["e:/MB/MBasymptotics.m"];
```

```
MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09

MBresolve 1.0

by Alexander Smirnov

more info in arXiv:0901.0386

last modified 4 Jan 09

Barnes Routines, v 1.1.0 of June 5, 2009
```

```
(*   The end of the derivation of the MB
  representation for the planar massless double box diagram at
  p1^2=p2^2=p3^2=p4^2=0.

     Notation:   S=-s=-(p1+p2)^2, T=-t=-(p1+p3)^2 *)

(*  This is the MB representation for the box   *)
```

In[5]:=
```
Box1[a1_, a2_, a3_, a4_] :=
  (S^(2-a1-a2-a3-a4-ep-z1) T^z1 Gamma[a1 + a2 + a3 + a4 - 2 + ep + z1] Gamma[a2 + z1] Gamma[a4 + z1]
     Gamma[2 - a1 - a2 - a4 - ep - z1] Gamma[2 - a2 - a3 - a4 - ep - z1] Gamma[-z1] ) /
   (Gamma[a1] Gamma[a2] Gamma[a3] Gamma[a4] Gamma[4 - a1 - a2 - a3 - a4 - 2 ep]);
```

In[6]:=
```
( 1 / S^(a4 + a5 + a6 + a7 + ep - 2 + z2 + z3 + z4) /
     (Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep])
   Gamma[a4 + a5 + a6 + a7 - 2 + ep + z2 + z3 + z4] Gamma[a7 + z2 + z3 + z4] Gamma[a5 + z4]
   Gamma[2 - a4 - a5 - a7 - ep - z3 - z4] Gamma[2 - a5 - a6 - a7 - ep - z2 - z4]
   Gamma[-z2] Gamma[-z3] Gamma[-z4] ) Box1[a1 - z2, a2, a3 - z3, a8 - z4];
```

In[7]:=
```
Simplify[%]
```

Out[7]=
```
(S^(4-a1-a2-a3-a4-a5-a6-a7-a8-2 ep-z1) T^z1 Gamma[-z1] Gamma[a2 + z1] Gamma[-z2] Gamma[-z3]
   Gamma[a8 + z1 - z4] Gamma[2 - a5 - a6 - a7 - ep - z2 - z4] Gamma[2 - a4 - a5 - a7 - ep - z3 - z4]
   Gamma[-2 + a1 + a2 + a3 + a8 + ep + z1 - z2 - z3 - z4] Gamma[-z4] Gamma[a5 + z4]
   Gamma[2 - a1 - a2 - a8 - ep - z1 + z2 + z4] Gamma[2 - a2 - a3 - a8 - ep - z1 + z3 + z4]
   Gamma[a7 + z2 + z3 + z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z2 + z3 + z4]) /
 (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
   Gamma[a1 - z2] Gamma[a3 - z3] Gamma[a8 - z4] Gamma[4 - a1 - a2 - a3 - a8 - 2 ep + z2 + z3 + z4])
```

```
(*   Changing variables    *)
```

In[8]:=
```
% /.  {z2 → z2 - z4, z3 → z3 - z4};
```

In[9]:=
```
% /. {z3 → z3 + z1};
```

In[10]:=
```
% /. z4 → z4 + z1;
```

In[11]:=
```
% /. z2 → z2 + z1;
```

In[12]:=
```
Simplify[% /. z4 → z4 + z2 + z3]
```

Out[12]=
$$\left(S^{4-a1-a2-a3-a4-a5-a6-a7-a8-2\,ep-z1}\,T^{z1}\,\Gamma[-z1]\,\Gamma[a2+z1]\,\Gamma[2-a5-a6-a7-ep-z1-z2]\right.$$
$$\Gamma[2-a1-a2-a8-ep+z2]\,\Gamma[2-a4-a5-a7-ep-z1-z3]$$
$$\Gamma[2-a2-a3-a8-ep+z3]\,\Gamma[a7+z1-z4]\,\Gamma[-2+a4+a5+a6+a7+ep+z1-z4]$$
$$\Gamma[a8-z2-z3-z4]\,\Gamma[-z1-z2-z3-z4]\,\Gamma[-2+a1+a2+a3+a8+ep+z4]$$
$$\left.\Gamma[z2+z4]\,\Gamma[z3+z4]\,\Gamma[a5+z1+z2+z3+z4]\right)\Big/$$
$$(\Gamma[a2]\,\Gamma[a4]\,\Gamma[a5]\,\Gamma[a6]\,\Gamma[a7]$$
$$\Gamma[4-a4-a5-a6-a7-2\,ep]\,\Gamma[4-a1-a2-a3-a8-2\,ep+z1-z4]$$
$$\Gamma[a8-z1-z2-z3-z4]\,\Gamma[a3+z2+z4]\,\Gamma[a1+z3+z4])$$

In[13]:=
```
B2[a1_, a2_, a3_, a4_, a5_, a6_, a7_, a8_] :=
 (S^(4-a1-a2-a3-a4-a5-a6-a7-a8-2 ep-z1) T^z1 Gamma[-z1] Gamma[a2+z1] Gamma[2-a5-a6-a7-ep-z1-z2]
    Gamma[2-a1-a2-a8-ep+z2] Gamma[2-a4-a5-a7-ep-z1-z3]
    Gamma[2-a2-a3-a8-ep+z3] Gamma[a7+z1-z4] Gamma[-2+a4+a5+a6+a7+ep+z1-z4]
    Gamma[a8-z2-z3-z4] Gamma[-z1-z2-z3-z4] Gamma[-2+a1+a2+a3+a8+ep+z4]
    Gamma[z2+z4] Gamma[z3+z4] Gamma[a5+z1+z2+z3+z4]) /
  (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4-a4-a5-a6-a7-2 ep]
    Gamma[4-a1-a2-a3-a8-2 ep+z1-z4]
    Gamma[a8-z1-z2-z3-z4] Gamma[a3+z2+z4] Gamma[a1+z3+z4])
```

```
(*   the function in the one-loop integration formula  *)
```

In[14]:=
```
G[a1_, a2_, s_] := Gamma[a1+a2+ep-2] Gamma[2-ep-a1]
   Gamma[2-ep-a2] / Gamma[a1] / Gamma[a2] / Gamma[4-2 ep-a1-a2] / s^(a1+a2+ep-2);
```

```
(*     a vertical check: shrink vertical lines,
a2,a5,a7→0
                                                  *)
```

In[15]:=
```
B2[a1, a2, a3, a4, a5, a6, a7, 0]
```

Out[15]=
$$\left(S^{4-a1-a2-a3-a4-a5-a6-a7-2\,ep-z1}\,T^{z1}\,\Gamma[-z1]\,\Gamma[a2+z1]\,\Gamma[2-a5-a6-a7-ep-z1-z2]\right.$$
$$\Gamma[2-a1-a2-ep+z2]\,\Gamma[2-a4-a5-a7-ep-z1-z3]\,\Gamma[2-a2-a3-ep+z3]$$
$$\Gamma[a7+z1-z4]\,\Gamma[-2+a4+a5+a6+a7+ep+z1-z4]\,\Gamma[-z2-z3-z4]$$
$$\left.\Gamma[-2+a1+a2+a3+ep+z4]\,\Gamma[z2+z4]\,\Gamma[z3+z4]\,\Gamma[a5+z1+z2+z3+z4]\right)\Big/$$
$$(\Gamma[a2]\,\Gamma[a4]\,\Gamma[a5]\,\Gamma[a6]\,\Gamma[a7]\,\Gamma[4-a4-a5-a6-a7-2\,ep]$$
$$\Gamma[4-a1-a2-a3-2\,ep+z1-z4]\,\Gamma[a3+z2+z4]\,\Gamma[a1+z3+z4])$$

```
(*    a2→0; Gamma[-z1] Gamma[a2+z1]   *)
```

In[25]:=
```
-Residue[B2[a1, a2, a3, a4, a5, a6, a7, 0], {z1, 0}]
```

Out[25]=
$$\left(S^{4-a1-a2-a3-a4-a5-a6-a7-2\,ep}\,\Gamma[2-a5-a6-a7-ep-z2]\right.$$
$$\Gamma[2-a1-a2-ep+z2]\,\Gamma[2-a4-a5-a7-ep-z3]\,\Gamma[2-a2-a3-ep+z3]$$
$$\Gamma[a7-z4]\,\Gamma[-2+a4+a5+a6+a7+ep-z4]\,\Gamma[-z2-z3-z4]$$
$$\left.\Gamma[-2+a1+a2+a3+ep+z4]\,\Gamma[z2+z4]\,\Gamma[z3+z4]\,\Gamma[a5+z2+z3+z4]\right)\Big/$$
$$(\Gamma[a4]\,\Gamma[a5]\,\Gamma[a6]\,\Gamma[a7]\,\Gamma[4-a4-a5-a6-a7-2\,ep]$$
$$\Gamma[4-a1-a2-a3-2\,ep-z4]\,\Gamma[a3+z2+z4]\,\Gamma[a1+z3+z4])$$

In[26]:=
```
% /. a2 → 0
```

Out[26]=
$$\left(S^{4-a1-a3-a4-a5-a6-a7-2\,ep}\,\Gamma[2-a5-a6-a7-ep-z2]\right.$$
$$\Gamma[2-a1-ep+z2]\,\Gamma[2-a4-a5-a7-ep-z3]\,\Gamma[2-a3-ep+z3]$$
$$\Gamma[a7-z4]\,\Gamma[-2+a4+a5+a6+a7+ep-z4]\,\Gamma[-z2-z3-z4]$$
$$\left.\Gamma[-2+a1+a3+ep+z4]\,\Gamma[z2+z4]\,\Gamma[z3+z4]\,\Gamma[a5+z2+z3+z4]\right)\Big/$$
$$(\Gamma[a4]\,\Gamma[a5]\,\Gamma[a6]\,\Gamma[a7]\,\Gamma[4-a4-a5-a6-a7-2\,ep]$$
$$\Gamma[4-a1-a3-2\,ep-z4]\,\Gamma[a3+z2+z4]\,\Gamma[a1+z3+z4])$$

```
(*    a5→0; Gamma[a5+z2+z3+z4] Gamma[-z2-z3-z4]   *)
```

In[27]:=
```
-Residue[%, {z4, -z2 - z3}]
```

$$\left(S^{4-a1-a3-a4-a5-a6-a7-2\,ep}\,\Gamma(2-a5-a6-a7-ep-z2)\,\Gamma(-z2)\,\Gamma(2-a1-ep+z2)\,\Gamma(2-a4-a5-a7-ep-z3)\,\Gamma(-2+a1+a3+ep-z2-z3)\,\Gamma(-z3)\,\Gamma(2-a3-ep+z3)\,\Gamma(a7+z2+z3)\,\Gamma(-2+a4+a5+a6+a7+ep+z2+z3)\right)/\left(\Gamma(a4)\,\Gamma(a6)\,\Gamma(a7)\,\Gamma(4-a4-a5-a6-a7-2\,ep)\,\Gamma(a1-z2)\,\Gamma(a3-z3)\,\Gamma(4-a1-a3-2\,ep+z2+z3)\right)$$

In[28]:=
```
% /. a5 → 0
```

Out[28]=
$$\left(S^{4-a1-a3-a4-a6-a7-2\,ep}\,\Gamma(2-a6-a7-ep-z2)\,\Gamma(-z2)\,\Gamma(2-a1-ep+z2)\,\Gamma(2-a4-a7-ep-z3)\,\Gamma(-2+a1+a3+ep-z2-z3)\,\Gamma(-z3)\,\Gamma(2-a3-ep+z3)\,\Gamma(a7+z2+z3)\,\Gamma(-2+a4+a6+a7+ep+z2+z3)\right)/\left(\Gamma(a4)\,\Gamma(a6)\,\Gamma(a7)\,\Gamma(4-a4-a6-a7-2\,ep)\,\Gamma(a1-z2)\,\Gamma(a3-z3)\,\Gamma(4-a1-a3-2\,ep+z2+z3)\right)$$

```
(*    a7→0; Gamma[a7+z2+z3] Gamma[-z2]Gamma[-z3]   *)
```

In[29]:=
```
-Residue[%, {z3, 0}]
```

Out[29]=
$$\left(S^{4-a1-a3-a4-a6-a7-2\,ep}\,\Gamma(2-a3-ep)\,\Gamma(2-a4-a7-ep)\,\Gamma(2-a6-a7-ep-z2)\,\Gamma(-2+a1+a3+ep-z2)\,\Gamma(-z2)\,\Gamma(a7+z2)\,\Gamma(2-a1-ep+z2)\,\Gamma(-2+a4+a6+a7+ep+z2)\right)/\left(\Gamma(a3)\,\Gamma(a4)\,\Gamma(a6)\,\Gamma(a7)\,\Gamma(4-a4-a6-a7-2\,ep)\,\Gamma(a1-z2)\,\Gamma(4-a1-a3-2\,ep+z2)\right)$$

In[30]:=
```
-Residue[%, {z2, 0}]
```

Out[30]=
$$\left(S^{4-a1-a3-a4-a6-a7-2\,ep}\,\Gamma(2-a1-ep)\,\Gamma(2-a3-ep)\,\Gamma(2-a4-a7-ep)\,\Gamma(2-a6-a7-ep)\,\Gamma(-2+a1+a3+ep)\,\Gamma(-2+a4+a6+a7+ep)\right)/\left(\Gamma(a1)\,\Gamma(a3)\,\Gamma(a4)\,\Gamma(a6)\,\Gamma(4-a1-a3-2\,ep)\,\Gamma(4-a4-a6-a7-2\,ep)\right)$$

In[31]:=
```
% /. a7 → 0
```

Out[31]=
$$\left(S^{4-a1-a3-a4-a6-2\,ep}\,\Gamma(2-a1-ep)\,\Gamma(2-a3-ep)\,\Gamma(2-a4-ep)\,\Gamma(2-a6-ep)\,\Gamma(-2+a1+a3+ep)\,\Gamma(-2+a4+a6+ep)\right)/\left(\Gamma(a1)\,\Gamma(a3)\,\Gamma(a4)\,\Gamma(a6)\,\Gamma(4-a1-a3-2\,ep)\,\Gamma(4-a4-a6-2\,ep)\right)$$

In[32]:=
```
G[a1, a3, S] G[a4, a6, S] / %
```

Out[32]=
$$1$$

```
(*     a horizontal check: shrink horizontal lines,
a1,a3,a4,a6→0
                                                  *)
```

In[33]:=
```
B2[a1, a2, a3, a4, a5, a6, a7, 0]
```

Out[33]=
$$\left(S^{4-a1-a2-a3-a4-a5-a6-a7-2\,ep-z1}\,T^{z1}\,\Gamma(-z1)\,\Gamma(a2+z1)\,\Gamma(2-a5-a6-a7-ep-z1-z2)\,\Gamma(2-a1-a2-ep+z2)\,\Gamma(2-a4-a5-a7-ep-z1-z3)\,\Gamma(2-a2-a3-ep+z3)\,\Gamma(a7+z1-z4)\,\Gamma(-2+a4+a5+a6+a7+ep+z1-z4)\,\Gamma(-z2-z3-z4)\,\Gamma(-2+a1+a2+a3+ep+z4)\,\Gamma(z2+z4)\,\Gamma(z3+z4)\,\Gamma(a5+z1+z2+z3+z4)\right)/\left(\Gamma(a2)\,\Gamma(a4)\,\Gamma(a5)\,\Gamma(a6)\,\Gamma(a7)\,\Gamma(4-a4-a5-a6-a7-2\,ep)\,\Gamma(4-a1-a2-a3-2\,ep+z1-z4)\,\Gamma(a3+z2+z4)\,\Gamma(a1+z3+z4)\right)$$

```
Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4] Gamma[2 - a4 - a5 - a7 - ep - z1 - z3] Gamma[z3 + z4]

-2 + a4 + a5 + a6 + a7 + ep + z1 - z4 + 2 - a4 - a5 - a7 - ep - z1 - z3 + z3 + z4

a6
```

In[47]:=
```
Residue[B2[a1, a2, a3, a4, a5, a6, a7, 0], {z3, -z4}]
```

Out[47]=
```
(S^(4-a1-a2-a3-a4-a5-a6-a7-2 ep-z1) T^z1 Gamma[-z1] Gamma[a2 + z1]
   Gamma[2 - a5 - a6 - a7 - ep - z1 - z2] Gamma[-z2] Gamma[2 - a1 - a2 - ep + z2] Gamma[a5 + z1 + z2]
   Gamma[2 - a2 - a3 - ep - z4] Gamma[a7 + z1 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4]
   Gamma[-2 + a1 + a2 + a3 + ep + z4] Gamma[2 - a4 - a5 - a7 - ep - z1 + z4] Gamma[z2 + z4]) /
 (Gamma[a1] Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7]
   Gamma[4 - a4 - a5 - a6 - a7 - 2 ep] Gamma[4 - a1 - a2 - a3 - 2 ep + z1 - z4] Gamma[a3 + z2 + z4])
```

In[48]:=
```
-Residue[%, {z4, -2 + a4 + a5 + a6 + a7 + ep + z1}]
```

Out[48]=
```
(S^(4-a1-a2-a3-a4-a5-a6-a7-2 ep-z1) T^z1 Gamma[2 - a4 - a5 - a6 - ep]
   Gamma[4 - a2 - a3 - a4 - a5 - a6 - a7 - 2 ep - z1] Gamma[-z1]
   Gamma[a2 + z1] Gamma[-4 + a1 + a2 + a3 + a4 + a5 + a6 + a7 + 2 ep + z1]
   Gamma[2 - a5 - a6 - a7 - ep - z1 - z2] Gamma[-z2] Gamma[2 - a1 - a2 - ep + z2]
   Gamma[a5 + z1 + z2] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 + z2]) /
 (Gamma[a1] Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a7]
   Gamma[6 - a1 - a2 - a3 - a4 - a5 - a6 - a7 - 3 ep] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
   Gamma[-2 + a3 + a4 + a5 + a6 + a7 + ep + z1 + z2])
```

In[49]:=
```
% /. a6 → 0
```

Out[49]=
```
(S^(4-a1-a2-a3-a4-a5-a7-2 ep-z1) T^z1 Gamma[2 - a4 - a5 - ep]
   Gamma[4 - a2 - a3 - a4 - a5 - a7 - 2 ep - z1] Gamma[-z1] Gamma[a2 + z1]
   Gamma[-4 + a1 + a2 + a3 + a4 + a5 + a7 + 2 ep + z1] Gamma[2 - a5 - a7 - ep - z1 - z2] Gamma[-z2]
   Gamma[2 - a1 - a2 - ep + z2] Gamma[a5 + z1 + z2] Gamma[-2 + a4 + a5 + a7 + ep + z1 + z2]) /
 (Gamma[a1] Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a7] Gamma[6 - a1 - a2 - a3 - a4 - a5 - a7 - 3 ep]
   Gamma[4 - a4 - a5 - a7 - 2 ep] Gamma[-2 + a3 + a4 + a5 + a7 + ep + z1 + z2])
```

```
(* a1,a3,a4→0  *)

(*    let a4→0    *)

Gamma[-2 + a4 + a5 + a7 + ep + z1 + z2] Gamma[2 - a5 - a7 - ep - z1 - z2]

-2 + a4 + a5 + a7 + ep + z1 + z2 + 2 - a5 - a7 - ep - z1 - z2

a4
```

In[50]:=
```
-Residue[ %, {z2, 2 - a5 - a7 - ep - z1}]
```

Out[50]=
```
(S^(4-a1-a2-a3-a4-a5-a7-2 ep-z1) T^z1 Gamma[2 - a4 - a5 - ep] Gamma[2 - a7 - ep]
   Gamma[4 - a1 - a2 - a5 - a7 - 2 ep - z1] Gamma[4 - a2 - a3 - a4 - a5 - a7 - 2 ep - z1] Gamma[-z1]
   Gamma[a2 + z1] Gamma[-2 + a5 + a7 + ep + z1] Gamma[-4 + a1 + a2 + a3 + a4 + a5 + a7 + 2 ep + z1]) /
 (Gamma[a1] Gamma[a2] Gamma[a3 + a4] Gamma[a5] Gamma[a7]
   Gamma[6 - a1 - a2 - a3 - a4 - a5 - a7 - 3 ep] Gamma[4 - a4 - a5 - a7 - 2 ep])
```

In[51]:=
```
% /. a4 → 0
```

Out[51]=
```
(S^(4-a1-a2-a3-a5-a7-2 ep-z1) T^z1 Gamma[2 - a5 - ep] Gamma[2 - a7 - ep]
   Gamma[4 - a1 - a2 - a5 - a7 - 2 ep - z1] Gamma[4 - a2 - a3 - a5 - a7 - 2 ep - z1] Gamma[-z1]
   Gamma[a2 + z1] Gamma[-2 + a5 + a7 + ep + z1] Gamma[-4 + a1 + a2 + a3 + a5 + a7 + 2 ep + z1]) /
 (Gamma[a1] Gamma[a2] Gamma[a3] Gamma[a5] Gamma[a7]
   Gamma[6 - a1 - a2 - a3 - a5 - a7 - 3 ep] Gamma[4 - a5 - a7 - 2 ep])
```

```
(* a1,a3→0  *)

(*    let a1→0    *)

Gamma[-4 + a1 + a2 + a3 + a5 + a7 + 2 ep + z1] Gamma[4 - a2 - a3 - a5 - a7 - 2 ep - z1]
```

```
-4 + a1 + a2 + a3 + a5 + a7 + 2 ep + z1 + 4 - a2 - a3 - a5 - a7 - 2 ep - z1

a1
```

In[52]:=
```
-Residue[ %, {z1, 4 - a2 - a3 - a5 - a7 - 2 ep}]
```

Out[52]=
```
(S^-a1 T^(4-a2-a3-a5-a7-2 ep) Gamma[-a1 + a3] Gamma[4 - a3 - a5 - a7 - 2 ep] Gamma[2 - a2 - a3 - ep]
   Gamma[2 - a5 - ep] Gamma[2 - a7 - ep] Gamma[-4 + a2 + a3 + a5 + a7 + 2 ep]) / (Gamma[a2]
   Gamma[a3] Gamma[a5] Gamma[a7] Gamma[6 - a1 - a2 - a3 - a5 - a7 - 3 ep] Gamma[4 - a5 - a7 - 2 ep])
```

In[53]:=
```
% /. a1 → 0
```

Out[53]=
```
(T^(4-a2-a3-a5-a7-2 ep) Gamma[4 - a3 - a5 - a7 - 2 ep] Gamma[2 - a2 - a3 - ep]
   Gamma[2 - a5 - ep] Gamma[2 - a7 - ep] Gamma[-4 + a2 + a3 + a5 + a7 + 2 ep]) /
 (Gamma[a2] Gamma[a5] Gamma[a7] Gamma[6 - a2 - a3 - a5 - a7 - 3 ep] Gamma[4 - a5 - a7 - 2 ep])
```

In[54]:=
```
% /. a3 → 0
```

Out[54]=
```
(T^(4-a2-a5-a7-2 ep) Gamma[2 - a2 - ep]
   Gamma[2 - a5 - ep] Gamma[2 - a7 - ep] Gamma[-4 + a2 + a5 + a7 + 2 ep]) /
 (Gamma[a2] Gamma[a5] Gamma[a7] Gamma[6 - a2 - a5 - a7 - 3 ep])
```

In[55]:=
```
G[a2, a7, 1] G[a2 + a7 + ep - 2, a5, T] / %
```

Out[55]=
```
1
```

```
(*  In addition to the usual factor  (I Pi^(d/2))^2,
let us pull out the factor

  S^(4-a1-a2-a3-a4-a5-a6-a7-a8-2 ep).

   Let us turn to the variable x=T/S = t/s      *)
```

In[56]:=
```
K2[a1_, a2_, a3_, a4_, a5_, a6_, a7_, a8_] :=
 ( x^z1 Gamma[-z1] Gamma[a2 + z1] Gamma[2 - a5 - a6 - a7 - ep - z1 - z2]
    Gamma[2 - a1 - a2 - a8 - ep + z2] Gamma[2 - a4 - a5 - a7 - ep - z1 - z3]
    Gamma[2 - a2 - a3 - a8 - ep + z3] Gamma[a7 + z1 - z4] Gamma[-2 + a4 + a5 + a6 + a7 + ep + z1 - z4]
    Gamma[a8 - z2 - z3 - z4] Gamma[-z1 - z2 - z3 - z4] Gamma[-2 + a1 + a2 + a3 + a8 + ep + z4]
    Gamma[z2 + z4] Gamma[z3 + z4] Gamma[a5 + z1 + z2 + z3 + z4]) /
  (Gamma[a2] Gamma[a4] Gamma[a5] Gamma[a6] Gamma[a7] Gamma[4 - a4 - a5 - a6 - a7 - 2 ep]
    Gamma[4 - a1 - a2 - a3 - a8 - 2 ep + z1 - z4]
    Gamma[a8 - z1 - z2 - z3 - z4] Gamma[a3 + z2 + z4] Gamma[a1 + z3 + z4])
```

```
(*  The double box with the powers of the propagators equal to one *)
```

In[57]:=
```
B2 = x K2[1, 1, 1, 1, 1, 1, 1, 0]
```

Out[57]=
```
(x^(1+z1) Gamma[-z1] Gamma[1 + z1] Gamma[-1 - ep - z1 - z2] Gamma[-ep + z2] Gamma[-1 - ep - z1 - z3]
   Gamma[-ep + z3] Gamma[1 + z1 - z4] Gamma[2 + ep + z1 - z4] Gamma[-z2 - z3 - z4]
   Gamma[1 + ep + z4] Gamma[z2 + z4] Gamma[z3 + z4] Gamma[1 + z1 + z2 + z3 + z4]) /
 (Gamma[-2 ep] Gamma[1 - 2 ep + z1 - z4] Gamma[1 + z2 + z4] Gamma[1 + z3 + z4])
```

In[58]:= **B2rules = MBoptimizedRules[B2, ep → 0, {}, {ep}]**

MBrules::norules : no rules could be found to regulate this integral

MBrules::norules : no rules could be found to regulate this integral

MBrules::norules : no rules could be found to regulate this integral

General::stop : Further output of MBrules::norules will be suppressed during this calculation. ≫

Out[58]= $\left\{\left\{\text{ep} \to -\frac{9}{16}\right\}, \left\{\text{z1} \to -\frac{1}{2}, \text{z2} \to -\frac{5}{16}, \text{z3} \to -\frac{3}{8}, \text{z4} \to \frac{7}{16}\right\}\right\}$

In[59]:= **B2cont = MBcontinue[B2, ep → 0, B2rules];**

```
Level 1
Taking -residue in z2 = -1 - ep - z1
Taking +residue in z2 = ep
Taking -residue in z3 = -1 - ep - z1
Taking +residue in z3 = ep
Level 2
Integral {1}
Taking -residue in z3 = -1 - ep - z1
Taking +residue in z4 = 1 + ep + z1
Integral {2}
Taking -residue in z1 = -1 - 2 ep
Taking -residue in z3 = -1 - ep - z1
Taking -residue in z4 = -ep - z3
Integral {3}
Taking +residue in z4 = 1 + ep + z1
Integral {4}
Taking -residue in z1 = -1 - 2 ep
Taking -residue in z2 = -1 - ep - z1
Taking +residue in z2 = ep
Taking -residue in z4 = -ep - z2
Level 3
Integral {1, 1}
Taking +residue in z4 = 1 + 2 ep + z1
Taking +residue in z4 = 1 + ep + z1
Integral {1, 2}
Integral {2, 1}
Taking -residue in z4 = -2 ep
Taking -residue in z4 = -ep - z3
Integral {2, 2}
```

Taking +residue in z4 = 1 + ep + z1

Integral {2, 3}

Integral {3, 1}

Integral {4, 1}

Taking −residue in z4 = −2 ep

Taking −residue in z4 = −ep − z2

Integral {4, 2}

Taking +residue in z4 = 1 + ep + z1

Integral {4, 3}

Taking −residue in z1 = −1 − 2 ep

Taking −residue in z4 = −2 ep

Integral {4, 4}

Level 4

Integral {1, 1, 1}

Integral {1, 1, 2}

Integral {2, 1, 1}

Taking +residue in z3 = 3 ep

Taking +residue in z3 = 2 ep

Integral {2, 1, 2}

Integral {2, 2, 1}

Integral {4, 1, 1}

Taking +residue in z2 = 3 ep

Taking +residue in z2 = 2 ep

Integral {4, 1, 2}

Integral {4, 2, 1}

Integral {4, 3, 1}

Taking −residue in z4 = −4 ep

...no contribution

Taking −residue in z4 = −2 ep

Integral {4, 3, 2}

Level 5

Integral {2, 1, 1, 1}

Integral {2, 1, 1, 2}

Integral {4, 1, 1, 1}

Integral {4, 1, 1, 2}

Integral {4, 3, 1, 1}

30 integral(s) found

In[60]:=
```
B2select = MBpreselect[B2cont, {ep, 0, 0}];
```

In[61]:=
```
B2exp = Simplify[MBexpand[B2select, E^(2 EulerGamma ep), {ep, 0, 0}]]
```

Out[61]=

$$\left\{\text{MBint}\left[\frac{1}{\text{ep}} 4\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[2+z1]\right.\right.$$
$$(-1+\text{ep EulerGamma}+4\,\text{ep PolyGamma}[0,-1-z1]-3\,\text{ep PolyGamma}[0,2+z1]),$$
$$\left.\left\{\{\text{ep}\to 0\},\left\{z1\to -\frac{1}{2}\right\}\right\}\right],\ \text{MBint}\left[-\frac{1}{\text{ep}} 2\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\right.$$
$$\text{Gamma}[1+z1]^2\, \text{Gamma}[2+z1]\,(-1+\text{ep EulerGamma}+4\,\text{ep PolyGamma}[0,-1-z1]+$$
$$\left.2\,\text{ep PolyGamma}[0,1+z1]-5\,\text{ep PolyGamma}[0,2+z1]),\ \left\{\{\text{ep}\to 0\},\left\{z1\to -\frac{1}{2}\right\}\right\}\right],$$
$$\text{MBint}\left[\frac{1}{288\,\text{ep}^4}\left(72+84\,\text{ep}^2\,\pi^2+59\,\text{ep}^4\,\pi^4+24\,\text{ep}^2\left(6+7\,\text{ep}^2\,\pi^2\right)\text{Log}[x]^2-96\,\text{ep}^3\,\text{Log}[x]^3+\right.\right.$$
$$\left.\left.48\,\text{ep}^4\,\text{Log}[x]^4+96\,\text{ep}^3\,\text{PolyGamma}[2,1]-24\,\text{ep Log}[x]\left(6+7\,\text{ep}^2\,\pi^2+8\,\text{ep}^3\,\text{PolyGamma}[2,1]\right)\right),\right.$$
$$\left.\{\{\text{ep}\to 0\},\{\}\}\right],\ \text{MBint}\left[-\frac{2}{\text{ep}^4}-\frac{\pi^2}{\text{ep}^2}-\frac{17\,\pi^4}{60}+\left(-\frac{4}{\text{ep}^2}-2\,\pi^2\right)\text{Log}[x]^2+\frac{8\,\text{Log}[x]^3}{3\,\text{ep}}-\right.$$
$$\left.\frac{4\,\text{Log}[x]^4}{3}-\frac{14\,\text{PolyGamma}[2,1]}{3\,\text{ep}}+\text{Log}[x]\left(\frac{4}{\text{ep}^3}+\frac{2\,\pi^2}{\text{ep}}+\frac{28}{3}\,\text{PolyGamma}[2,1]\right),\ \{\{\text{ep}\to 0\},\{\}\}\right],$$
$$\text{MBint}\left[\frac{2\,\text{Gamma}[-z3]^2\,\text{Gamma}[z3]^3}{\text{Gamma}[1+z3]},\ \left\{\{\text{ep}\to 0\},\left\{z3\to -\frac{3}{8}\right\}\right\}\right],$$
$$\text{MBint}\left[\frac{1}{12\,\text{ep}^2}\right.$$
$$\text{Gamma}[-z3]^2\,\text{Gamma}[z3]\,\text{Gamma}[1+z3]\left(6+12\,\text{ep EulerGamma}+12\,\text{ep}^2\,\text{EulerGamma}^2+4\,\text{ep}^2\,\pi^2-\right.$$
$$12\,\text{ep Log}[x]-24\,\text{ep}^2\,\text{EulerGamma Log}[x]+12\,\text{ep}^2\,\text{Log}[x]^2+3\,\text{ep}^2\,\text{PolyGamma}[0,-z3]^2+$$
$$6\,\text{ep}\,(1+2\,\text{ep EulerGamma}-2\,\text{ep Log}[x])\,\text{PolyGamma}[0,z3]+3\,\text{ep}^2\,\text{PolyGamma}[0,z3]^2+$$
$$\left.6\,\text{ep PolyGamma}[0,-z3]\,(1+2\,\text{ep EulerGamma}-2\,\text{ep Log}[x]+\text{ep PolyGamma}[0,z3])+\right.$$
$$\left.3\,\text{ep}^2\,\text{PolyGamma}[1,-z3]-21\,\text{ep}^2\,\text{PolyGamma}[1,z3]\right),\ \left\{\{\text{ep}\to 0\},\left\{z3\to -\frac{3}{8}\right\}\right\}\Bigg],$$
$$\text{MBint}\left[\frac{\text{Gamma}[-z3]\,\text{Gamma}[z3]\,\text{Gamma}[1-z4]\,\text{Gamma}[-z3-z4]\,\text{Gamma}[z4]\,\text{Gamma}[z3+z4]^2}{\text{Gamma}[1+z3+z4]},\right.$$
$$\left.\left\{\{\text{ep}\to 0\},\left\{z3\to -\frac{3}{8},\ z4\to \frac{7}{16}\right\}\right\}\right],$$
$$\text{MBint}\left[6\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^3,\ \left\{\{\text{ep}\to 0\},\left\{z1\to -\frac{1}{2}\right\}\right\}\right],$$
$$\text{MBint}\left[2\,x^{1+z1}\,\text{Gamma}[-1-z1]\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\,\text{Gamma}[-1-z1-z3]\right.$$
$$\left.\text{Gamma}[-z3]\,\text{Gamma}[z3]\,\text{Gamma}[2+z1+z3],\ \left\{\{\text{ep}\to 0\},\left\{z1\to -\frac{1}{2},\ z3\to -\frac{3}{8}\right\}\right\}\right],$$

$$\text{MBint}\left[\frac{1}{288\,\text{ep}^4}\left(72+84\,\text{ep}^2\,\pi^2+59\,\text{ep}^4\,\pi^4+24\,\text{ep}^2\,\left(6+7\,\text{ep}^2\,\pi^2\right)\,\text{Log}[x]^2-96\,\text{ep}^3\,\text{Log}[x]^3+\right.\right.$$
$$\left.\left.48\,\text{ep}^4\,\text{Log}[x]^4+96\,\text{ep}^3\,\text{PolyGamma}[2,1]-24\,\text{ep}\,\text{Log}[x]\,\left(6+7\,\text{ep}^2\,\pi^2+8\,\text{ep}^3\,\text{PolyGamma}[2,1]\right)\right),\right.$$
$$\left.\{\{\text{ep}\to 0\},\{\}\}\right],\ \text{MBint}\left[-\frac{2}{\text{ep}^4}-\frac{\pi^2}{\text{ep}^2}-\frac{17\,\pi^4}{60}+\left(-\frac{4}{\text{ep}^2}-2\,\pi^2\right)\,\text{Log}[x]^2+\frac{8\,\text{Log}[x]^3}{3\,\text{ep}}-\right.$$
$$\left.\frac{4\,\text{Log}[x]^4}{3}-\frac{14\,\text{PolyGamma}[2,1]}{3\,\text{ep}}+\text{Log}[x]\,\left(\frac{4}{\text{ep}^3}+\frac{2\,\pi^2}{\text{ep}}+\frac{28}{3}\,\text{PolyGamma}[2,1]\right),\{\{\text{ep}\to 0\},\{\}\}\right],$$
$$\text{MBint}\left[\frac{2\,\text{Gamma}[-z2]^2\,\text{Gamma}[z2]^3}{\text{Gamma}[1+z2]},\left\{\{\text{ep}\to 0\},\left\{z2\to-\frac{5}{16}\right\}\right\}\right],\ \text{MBint}\left[\frac{1}{12\,\text{ep}^2}\right.$$
$$\text{Gamma}[-z2]^2\,\text{Gamma}[z2]\,\text{Gamma}[1+z2]\,\left(6+12\,\text{ep}\,\text{EulerGamma}+12\,\text{ep}^2\,\text{EulerGamma}^2+4\,\text{ep}^2\,\pi^2-\right.$$
$$12\,\text{ep}\,\text{Log}[x]-24\,\text{ep}^2\,\text{EulerGamma}\,\text{Log}[x]+12\,\text{ep}^2\,\text{Log}[x]^2+3\,\text{ep}^2\,\text{PolyGamma}[0,-z2]^2+$$
$$6\,\text{ep}\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x])\,\text{PolyGamma}[0,z2]+3\,\text{ep}^2\,\text{PolyGamma}[0,z2]^2+$$
$$\left.6\,\text{ep}\,\text{PolyGamma}[0,-z2]\,(1+2\,\text{ep}\,\text{EulerGamma}-2\,\text{ep}\,\text{Log}[x]+\text{ep}\,\text{PolyGamma}[0,z2])+\right.$$
$$\left.3\,\text{ep}^2\,\text{PolyGamma}[1,-z2]-21\,\text{ep}^2\,\text{PolyGamma}[1,z2]\right),\left\{\{\text{ep}\to 0\},\left\{z2\to-\frac{5}{16}\right\}\right\}\Big],$$
$$\text{MBint}\left[\frac{\text{Gamma}[-z2]\,\text{Gamma}[z2]\,\text{Gamma}[1-z4]\,\text{Gamma}[-z2-z4]\,\text{Gamma}[z4]\,\text{Gamma}[z2+z4]^2}{\text{Gamma}[1+z2+z4]},\right.$$
$$\left.\left\{\{\text{ep}\to 0\},\left\{z2\to-\frac{5}{16},\,z4\to\frac{7}{16}\right\}\right\}\right],$$
$$\text{MBint}\left[6\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^3,\left\{\{\text{ep}\to 0\},\left\{z1\to-\frac{1}{2}\right\}\right\}\right],$$
$$\text{MBint}\left[\left(\frac{9}{\text{ep}^2}-\frac{17\,\pi^2}{6}\right)\,\text{Log}[x]^2-\frac{4\,\text{Log}[x]^3}{\text{ep}}+\text{Log}[x]^4+\text{Log}[x]\,\left(-\frac{12}{\text{ep}^3}+\frac{7\,\pi^2}{3\,\text{ep}}-33\,\text{PolyGamma}[2,1]\right)+\right.$$
$$\left.\frac{1800-220\,\text{ep}^2\,\pi^2-229\,\text{ep}^4\,\pi^4+4920\,\text{ep}^3\,\text{PolyGamma}[2,1]}{240\,\text{ep}^4},\{\{\text{ep}\to 0\},\{\}\}\right],$$
$$\text{MBint}\left[-\frac{1}{2\,\text{ep}\,\text{Gamma}[1+z4]}\text{Gamma}[1-z4]\,\text{Gamma}[-z4]\,\text{Gamma}[z4]^3\,(1+4\,\text{ep}\,\text{EulerGamma}+\right.$$
$$\left.\text{ep}\,\text{PolyGamma}[0,1-z4]+4\,\text{ep}\,\text{PolyGamma}[0,z4]-\text{ep}\,\text{PolyGamma}[0,1+z4]),\right.$$
$$\left.\left\{\{\text{ep}\to 0\},\left\{z4\to\frac{7}{16}\right\}\right\}\right],\ \text{MBint}\left[-\frac{1}{\text{ep}}2\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\right.$$
$$\text{Gamma}[2+z1]\,(1+\text{ep}\,\text{EulerGamma}-4\,\text{ep}\,\text{PolyGamma}[0,-1-z1]+$$
$$\left.2\,\text{ep}\,\text{PolyGamma}[0,1+z1]+3\,\text{ep}\,\text{PolyGamma}[0,2+z1]),\left\{\{\text{ep}\to 0\},\left\{z1\to-\frac{1}{2}\right\}\right\}\right],$$
$$\text{MBint}\left[2\,x^{1+z1}\,\text{Gamma}[-1-z1]\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\,\text{Gamma}[-1-z1-z2]\right.$$
$$\left.\left.\text{Gamma}[-z2]\,\text{Gamma}[z2]\,\text{Gamma}[2+z1+z2],\left\{\{\text{ep}\to 0\},\left\{z1\to-\frac{1}{2},\,z2\to-\frac{5}{16}\right\}\right\}\right]\right\}$$

In[62]:= **B2expB = DoAllBarnes[B2exp] // MergeAll**

Out[62]= $$\left\{\text{MBint}\left[8\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^3, \left\{\{\text{ep}\to 0\}, \left\{z1\to -\frac{1}{2}\right\}\right\}\right],\right.$$
$$\text{MBint}\left[-\text{Gamma}[1-z4]\, \text{Gamma}[-z4]\, \text{Gamma}[z4]^2\right.$$
$$\left(2\, \text{EulerGamma}\, \text{PolyGamma}[0, 1+z4] + 2\, \text{PolyGamma}[0, -z4]\, \text{PolyGamma}[0, 1+z4] + \right.$$
$$2\, \text{PolyGamma}[0, z4]\, \text{PolyGamma}[0, 1+z4] - \text{PolyGamma}[0, 1+z4]^2 -$$
$$\left.\left.\text{PolyGamma}[1, z4] + 3\, \text{PolyGamma}[1, 1+z4]\right), \left\{\{\text{ep}\to 0\}, \left\{z4\to \frac{7}{16}\right\}\right\}\right],$$
$$\text{MBint}\left[\frac{1}{\text{ep}} 4\, x^{1+z1}\, \text{Gamma}[-1-z1]^2\, \text{Gamma}[-z1]\, \text{Gamma}[1+z1]^2\, \text{Gamma}[2+z1]\right.$$
$$(-1 + \text{ep}\, \text{EulerGamma} + 4\, \text{ep}\, \text{PolyGamma}[0, -1-z1] -$$
$$\left.\left.2\, \text{ep}\, \text{PolyGamma}[0, 1+z1] - \text{ep}\, \text{PolyGamma}[0, 2+z1]), \left\{\{\text{ep}\to 0\}, \left\{z1\to -\frac{1}{2}\right\}\right\}\right],\right.$$
$$\text{MBint}\left[-\frac{1}{720\, \text{ep}^4}\left(-2880 + 1800\, \text{ep}^2\, \pi^2 + 60\, \text{ep}^4\, \text{EulerGamma}^2\, \pi^2 + 1061\, \text{ep}^4\, \pi^4 + 3600\, \text{ep}\, \text{Log}[x] -\right.\right.$$
$$3960\, \text{ep}^3\, \pi^2\, \text{Log}[x] - 1440\, \text{ep}^2\, \text{Log}[x]^2 + 4320\, \text{ep}^4\, \pi^2\, \text{Log}[x]^2 - 480\, \text{ep}^3\, \text{Log}[x]^3 +$$
$$960\, \text{ep}^4\, \text{Log}[x]^4 - 7800\, \text{ep}^3\, \text{PolyGamma}[2, 1] - 3600\, \text{ep}^4\, \text{EulerGamma}\, \text{PolyGamma}[2, 1] +$$
$$\left.\left.10\,560\, \text{ep}^4\, \text{Log}[x]\, \text{PolyGamma}[2, 1]\right), \{\{\text{ep}\to 0\}, \{\}\}\right],$$
$$\text{MBint}\left[-\frac{1}{2\, \text{Gamma}[1+z4]}\text{Gamma}[1-z4]\, \text{Gamma}[-z4]\, \text{Gamma}[z4]^3\right.$$
$$(\text{PolyGamma}[0, 1-z4] - 4\, \text{PolyGamma}[0, -z4] - 12\, \text{PolyGamma}[0, z4] + 7\, \text{PolyGamma}[0, 1+z4]),$$
$$\left.\left\{\{\text{ep}\to 0\}, \left\{z4\to \frac{7}{16}\right\}\right\}\right], \text{MBint}\left[\frac{1}{2}\, \text{Gamma}[-z2]^2\, \text{Gamma}[z2]\, \text{Gamma}[1+z2]\right.$$
$$\left.\left.\left(\text{PolyGamma}[0, z2]^2 + \text{PolyGamma}[1, -z2] - 7\, \text{PolyGamma}[1, z2]\right), \left\{\{\text{ep}\to 0\}, \left\{z2\to -\frac{5}{16}\right\}\right\}\right]\right\}$$

In[63]:=
```
B2exp = Simplify[MBexpand[B2select, E^(2 EulerGamma ep), {ep, 0, -2}]];
```

In[64]:=
```
B2expB = DoAllBarnes[B2exp] // MergeAll
```

Out[64]= $$\left\{\text{MBint}\left[-\frac{-8 + 5\, \text{ep}^2\, \pi^2 + 10\, \text{ep}\, \text{Log}[x] - 4\, \text{ep}^2\, \text{Log}[x]^2}{2\, \text{ep}^4}, \{\{\text{ep}\to 0\}, \{\}\}\right]\right\}$$

In[65]:=
```
Apart[%[[1]][[1]], ep]
```

Out[65]= $$\frac{4}{\text{ep}^4} - \frac{5\, \text{Log}[x]}{\text{ep}^3} + \frac{-5\, \pi^2 + 4\, \text{Log}[x]^2}{2\, \text{ep}^2}$$

In[66]:=
```
B2cont = MBresolve[B2, ep, OptimizeNow → True]
```

In[67]:=
```
B2select = MBpreselect[B2cont, {ep, 0, 0}];
```

In[68]:=
```
B2exp = Simplify[MBexpand[B2select, E^(2 EulerGamma ep), {ep, 0, 0}]];
```

In[69]:=
```
B2expB = DoAllBarnes[B2exp] // MergeAll
```

Out[69]=
$$\left\{\text{MBint}\left[-4\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^3,\ \left\{\{\text{ep}\to 0\},\ \left\{\text{z1}\to -\frac{26}{79}\right\}\right\}\right],\right.$$

$$\text{MBint}\left[\frac{1}{2}\,\text{Gamma}[1-\text{z4}]\,\text{Gamma}[-\text{z4}]\,\text{Gamma}[\text{z4}]^2\right.$$
$$\left(\text{PolyGamma}[0,\,1-\text{z4}]^2-4\,\text{EulerGamma}\,\text{PolyGamma}[0,\,1+\text{z4}]-\right.$$
$$4\,\text{PolyGamma}[0,\,-\text{z4}]\,\text{PolyGamma}[0,\,1+\text{z4}]-4\,\text{PolyGamma}[0,\,\text{z4}]\,\text{PolyGamma}[0,\,1+\text{z4}]+$$
$$2\,\text{PolyGamma}[0,\,1+\text{z4}]^2+\text{PolyGamma}[1,\,1-\text{z4}]-8\,\text{PolyGamma}[1,\,-\text{z4}]+$$
$$\left.7\,\text{PolyGamma}[1,\,\text{z4}]-6\,\text{PolyGamma}[1,\,1+\text{z4}]\right),\ \left\{\{\text{ep}\to 0\},\ \left\{\text{z4}\to\frac{1}{2}\right\}\right\}\Big],$$

$$\text{MBint}\left[-\frac{1}{2\,\text{Gamma}[1+\text{z4}]}\text{Gamma}[1-\text{z4}]\,\text{Gamma}[-\text{z4}]\,\text{Gamma}[\text{z4}]^2\,\left(\text{Gamma}[\text{z4}]\,\text{PolyGamma}[0,\,1-\text{z4}]-\right.\right.$$
$$4\,\text{Gamma}[\text{z4}]\,\text{PolyGamma}[0,\,-\text{z4}]-12\,\text{Gamma}[\text{z4}]\,\text{PolyGamma}[0,\,\text{z4}]+$$
$$\left.7\,\text{Gamma}[\text{z4}]\,\text{PolyGamma}[0,\,1+\text{z4}]+4\,x^{\text{z4}}\,\text{Gamma}[-\text{z4}]\,\text{Gamma}[1+\text{z4}]^2\,\text{PolyGamma}[0,\,1+\text{z4}]\right),$$
$$\left.\left\{\{\text{ep}\to 0\},\ \left\{\text{z4}\to\frac{1}{2}\right\}\right\}\right],\ \text{MBint}\left[\frac{1}{\text{ep}}2\,x^{1+z1}\,\text{Gamma}[-1-z1]^2\,\text{Gamma}[-z1]\,\text{Gamma}[1+z1]^2\right.$$
$$\text{Gamma}[2+z1]\,(-1+\text{ep}\,\text{EulerGamma}+4\,\text{ep}\,\text{PolyGamma}[0,\,-1-z1]-$$
$$\left.2\,\text{ep}\,\text{PolyGamma}[0,\,1+z1]-\text{ep}\,\text{PolyGamma}[0,\,2+z1]),\ \left\{\{\text{ep}\to 0\},\ \left\{\text{z1}\to -\frac{26}{79}\right\}\right\}\right],$$

$$\text{MBint}\left[-\frac{1}{144\,\text{ep}^4}\left(-576+360\,\text{ep}^2\,\pi^2+12\,\text{ep}^4\,\text{EulerGamma}^2\,\pi^2+115\,\text{ep}^4\,\pi^4+720\,\text{ep}\,\text{Log}[x]-\right.\right.$$
$$648\,\text{ep}^3\,\pi^2\,\text{Log}[x]+144\,\text{ep}^4\,\text{EulerGamma}\,\pi^2\,\text{Log}[x]-288\,\text{ep}^2\,\text{Log}[x]^2+576\,\text{ep}^4\,\pi^2\,\text{Log}[x]^2-$$
$$48\,\text{ep}^3\,\text{Log}[x]^3+48\,\text{ep}^4\,\text{EulerGamma}\,\text{Log}[x]^3+120\,\text{ep}^4\,\text{Log}[x]^4-1560\,\text{ep}^3\,\text{PolyGamma}[2,\,1]-$$
$$\left.\left.576\,\text{ep}^4\,\text{EulerGamma}\,\text{PolyGamma}[2,\,1]+1824\,\text{ep}^4\,\text{Log}[x]\,\text{PolyGamma}[2,\,1]\right),\ \{\{\text{ep}\to 0\},\ \{\}\}\right],$$

$$\text{MBint}\left[12\,x^{\text{z4}}\,\text{Gamma}[1-\text{z4}]\,\text{Gamma}[-\text{z4}]^2\,\text{Gamma}[\text{z4}]^3,\ \left\{\{\text{ep}\to 0\},\ \left\{\text{z4}\to\frac{1}{2}\right\}\right\}\right],$$

$$\text{MBint}\left[\frac{1}{\text{ep}}2\,x^{\text{z4}}\,\text{Gamma}[1-\text{z4}]\,\text{Gamma}[-\text{z4}]^2\,\text{Gamma}[\text{z4}]^2\,\text{Gamma}[1+\text{z4}]\right.$$
$$(-1+\text{ep}\,\text{EulerGamma}+4\,\text{ep}\,\text{PolyGamma}[0,\,-\text{z4}]-2\,\text{ep}\,\text{PolyGamma}[0,\,\text{z4}]),$$
$$\left.\left.\left\{\{\text{ep}\to 0\},\ \left\{\text{z4}\to -\frac{1}{2}\right\}\right\}\right]\right\}$$

In[70]:=
```
B2exp = Simplify[MBexpand[B2select, E^(2 EulerGamma ep), {ep, 0, -2}]]
```

In[71]:=
```
B2expB = DoAllBarnes[B2exp] // MergeAll
```

Out[71]=
$$\left\{\text{MBint}\left[-\frac{-8+5\,\text{ep}^2\,\pi^2+10\,\text{ep}\,\text{Log}[x]-4\,\text{ep}^2\,\text{Log}[x]^2}{2\,\text{ep}^4},\ \{\{\text{ep}\to 0\},\ \{\}\}\right]\right\}$$

In[72]:=
```
Apart[%[[1]][[1]], ep]
```

Out[72]=
$$\frac{4}{\text{ep}^4}-\frac{5\,\text{Log}[x]}{\text{ep}^3}+\frac{-5\,\pi^2+4\,\text{Log}[x]^2}{2\,\text{ep}^2}$$

```
(*                the result                *)
1 / x ( 4/ep^4 - (5 Log[x])/ep^3 + (-5 π^2 + 4 Log[x]^2)/(2 ep^2) +
    1/(6 ep) (33 π^2 Log[x] + 4 Log[x]^3 - 12 π^2 Log[1 + x] - 12 Log[x]^2 Log[1 + x] -
      24 Log[x] PolyLog[2, -x] + 24 PolyLog[3, -x] - 130 Zeta[3]) +
    1/30 (-29 π^4 - 180 π^2 Log[x]^2 - 40 Log[x]^4 + 100 π^2 Log[x] Log[1 + x] + 80 Log[x]^3 Log[1 + x] -
      30 π^2 Log[1 + x]^2 - 30 Log[x]^2 Log[1 + x]^2 - 200 π^2 PolyLog[2, -x] - 60 Log[x]^2
       PolyLog[2, -x] - 120 Log[x] Log[1 + x] PolyLog[2, -x] + 720 Log[x] PolyLog[3, -x] +
      120 Log[1 + x] PolyLog[3, -x] - 1320 PolyLog[4, -x] - 120 Log[x] PolyLog[1, 2, -x] +
      120 PolyLog[2, 2, -x] + 880 Log[x] Zeta[3] - 120 Log[1 + x] Zeta[3]) )
```