Quark-Lepton Mass Relations from Modular Flavor Symmetry

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Outline

🔥 Motivation (Flavor Puzzle)

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- Modular Flavor Symmetries

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- Quark-Lepton Mass Relations
  —(model independent derivation)

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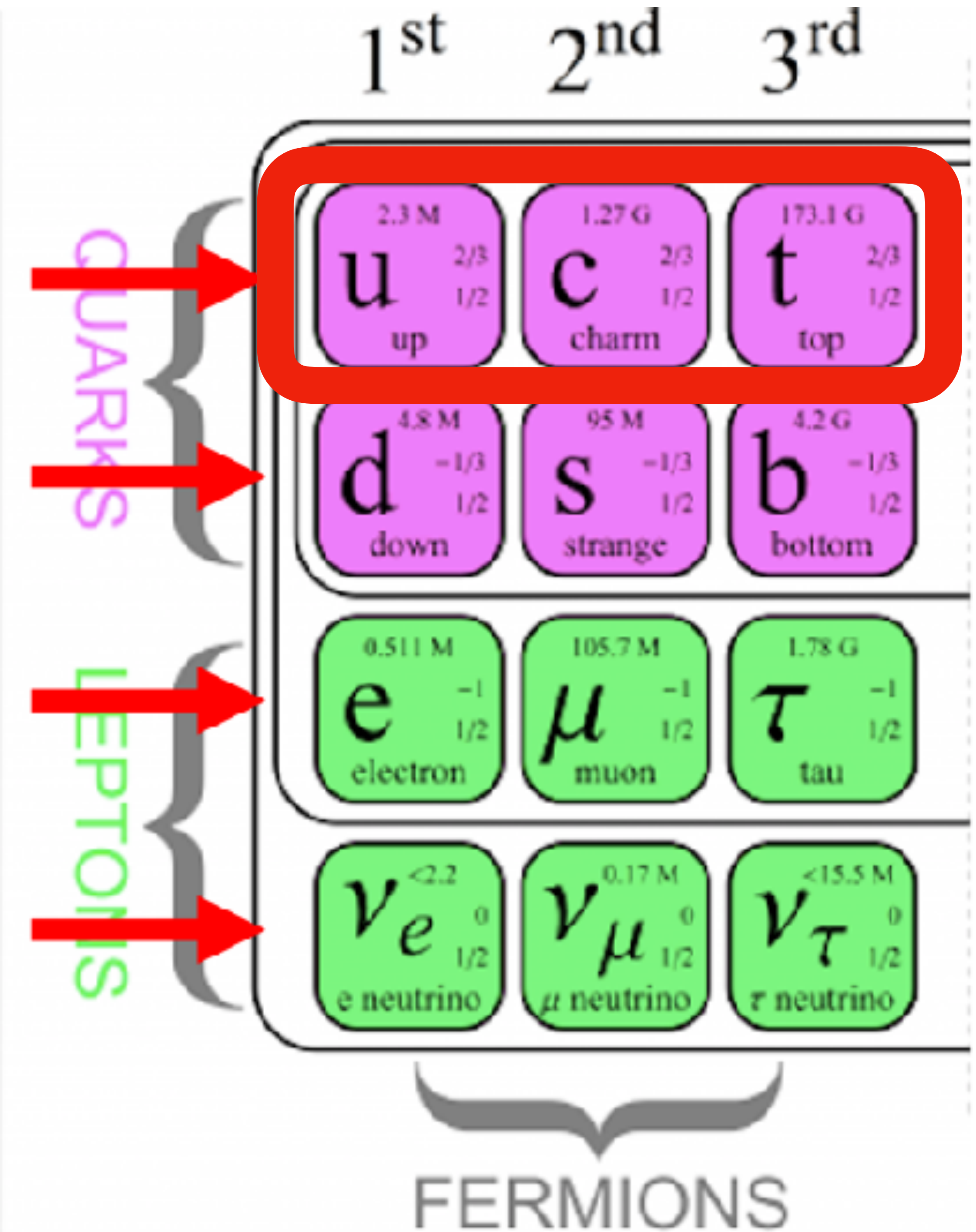
- An example and a claim:
  Experimental **hints** of a quark-lepton mass relation

Motivation (The Flavor Puzzle)

1925-GelbRotBlau-Kandinsky

The Flavor Puzzle

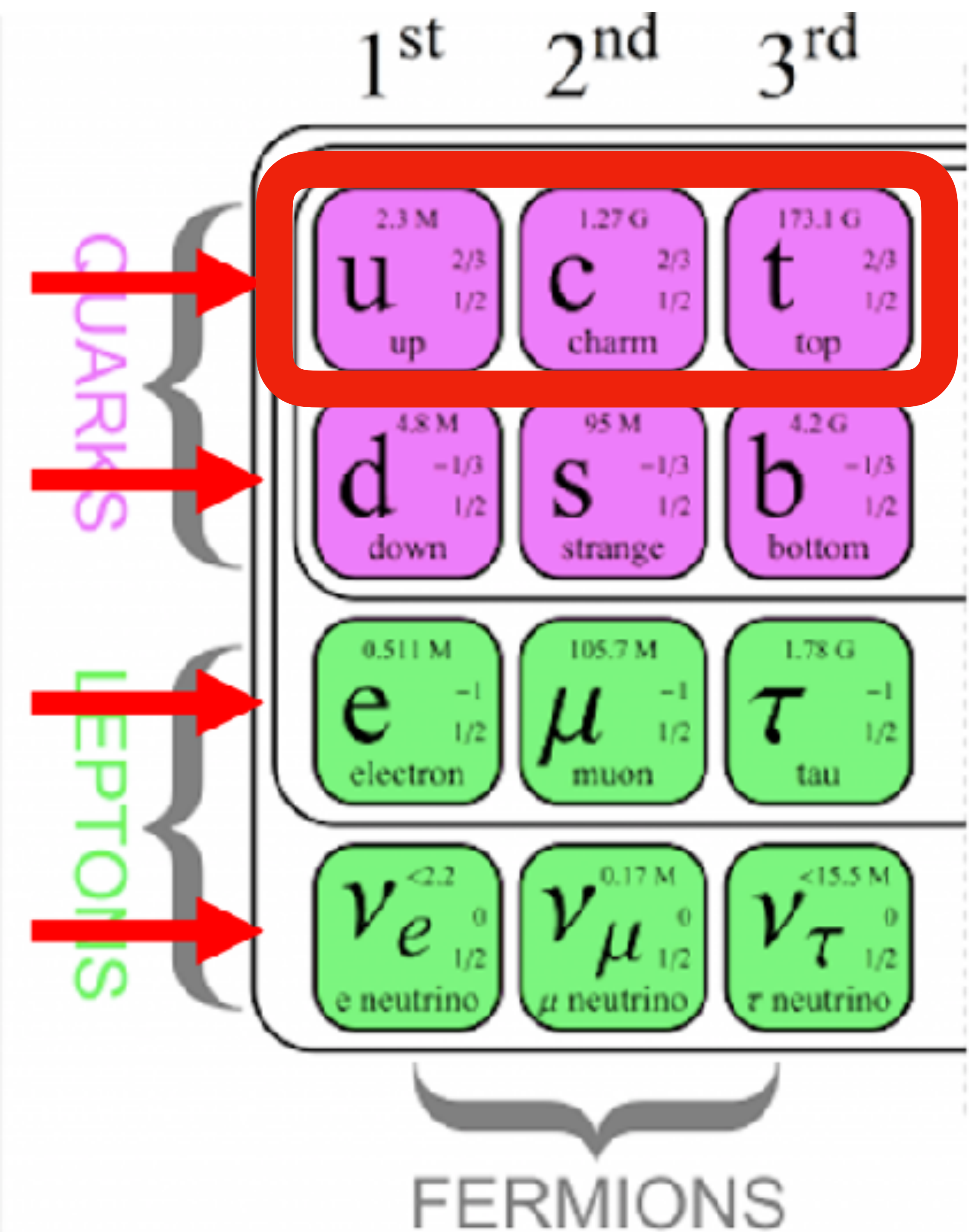


$$SU(3)_c \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{Electroweak Sector}}.$$

- The SM gauge group is **generation blind**, or in other words **flavor universal**

The Flavor Puzzle

They look like the same particle



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$$Y_e^{ij}\overline{L_i}\Phi e_{R_j} \qquad Y_e = \begin{pmatrix} y_{ee} & y_{e\mu} & y_{e\tau} \\ y_{\mu e} & y_{\mu\mu} & y_{\mu\tau} \\ y_{\tau e} & y_{\tau\mu} & y_{\tau\tau} \end{pmatrix}$$

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- The **flavor symmetry breaking** manifests in

$$m_{fermions}, \quad U_{CKM}, \quad V_{LMM}$$

1st Piece of the Flavor Puzzle

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- Large number of input parameters related to flavor

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$$\{g_e,\ \theta_W,\ g_s,\ v_h,\ m_h\}$$

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- 22 parameters come from the Yukawa sector

$$\{m_e,\ m_\mu,\ m_\tau,\ m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3},\ m_d,\ m_s,\ m_b,\ m_u,\ m_c,\ m_b\}$$

$$\{\theta^l_{12},\ \theta^l_{13},\ \theta^l_{23},\ \delta^l,\ \phi_{12},\ \phi_{13},\ \theta^q_{12},\ \theta^q_{13},\ \theta^q_{23},\ \delta^q\}$$

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Huge hint for a more fundamental theory of flavor

2nd Piece of the Flavor Puzzle

- Large hierarchies among fermion masses

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- Large hierarchies among fermion masses

$$M_e = v_\Phi \begin{pmatrix} y_{ee} & y_{e\mu} & y_{e\tau} \\ y_{\mu e} & y_{\mu\mu} & y_{\mu\tau} \\ y_{\tau e} & y_{\tau\mu} & y_{\tau\tau} \end{pmatrix} \xrightarrow{\text{Singular Values}} m_e\,, \quad m_\mu\,, \quad m_\tau$$

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- Even among same class of fermions:

Example

$$\frac{m_\tau}{m_\tau} = 1 \qquad \frac{m_\mu}{m_\tau} \approx \frac{1}{17} \qquad \frac{m_e}{m_\tau} \approx \frac{1}{3400}$$

The Flavor Puzzle

$$\{m_e,\ m_\mu,\ m_\tau,\ m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3},\ m_d,\ m_s,\ m_b,\ m_u,\ m_c,\ m_b\}$$

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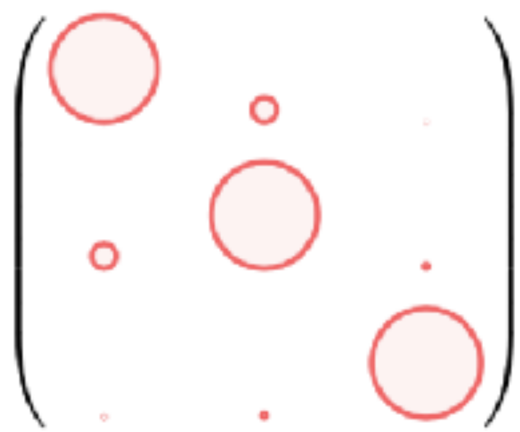
CKM

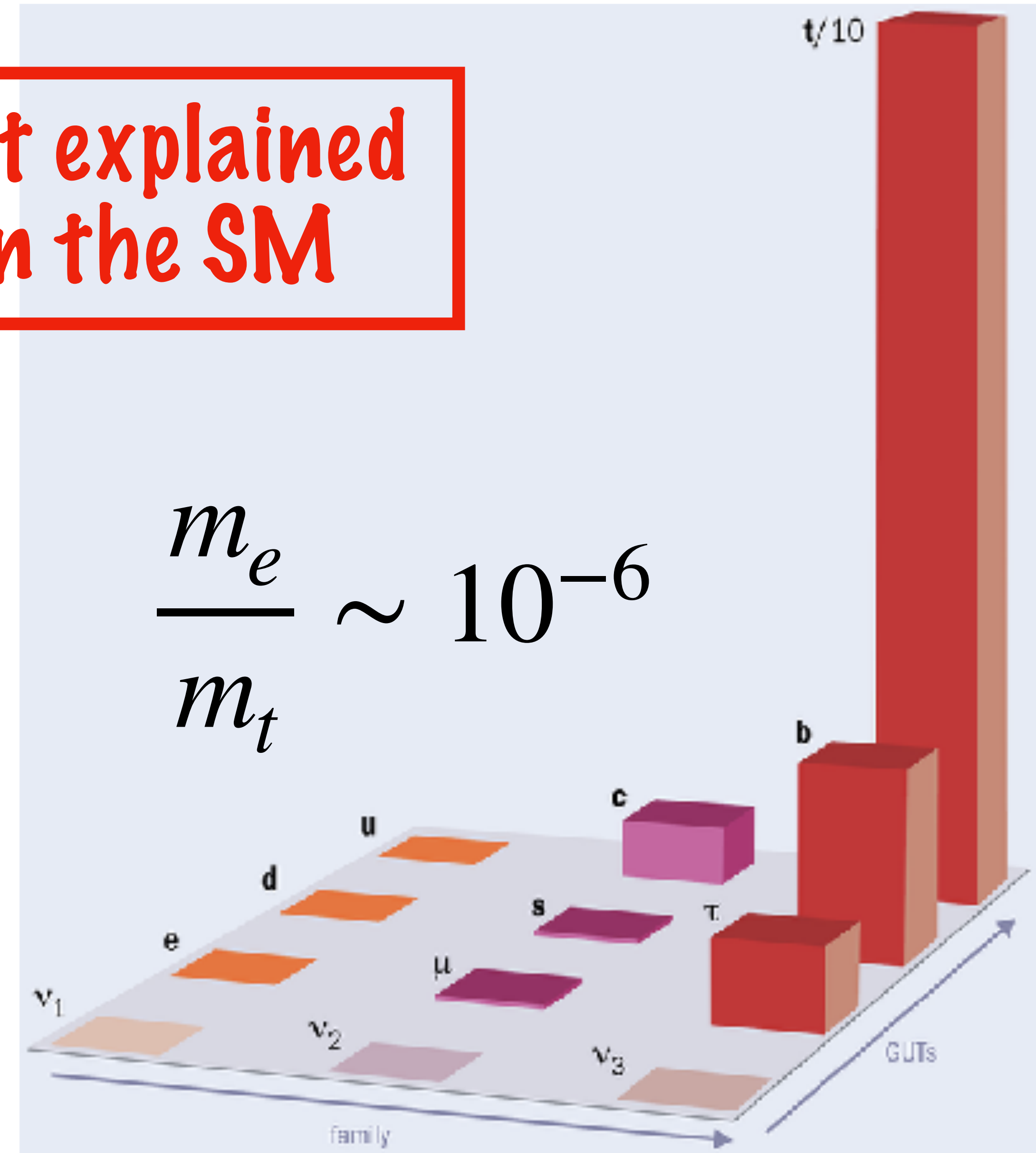
$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \sim$$



Not explained in the SM

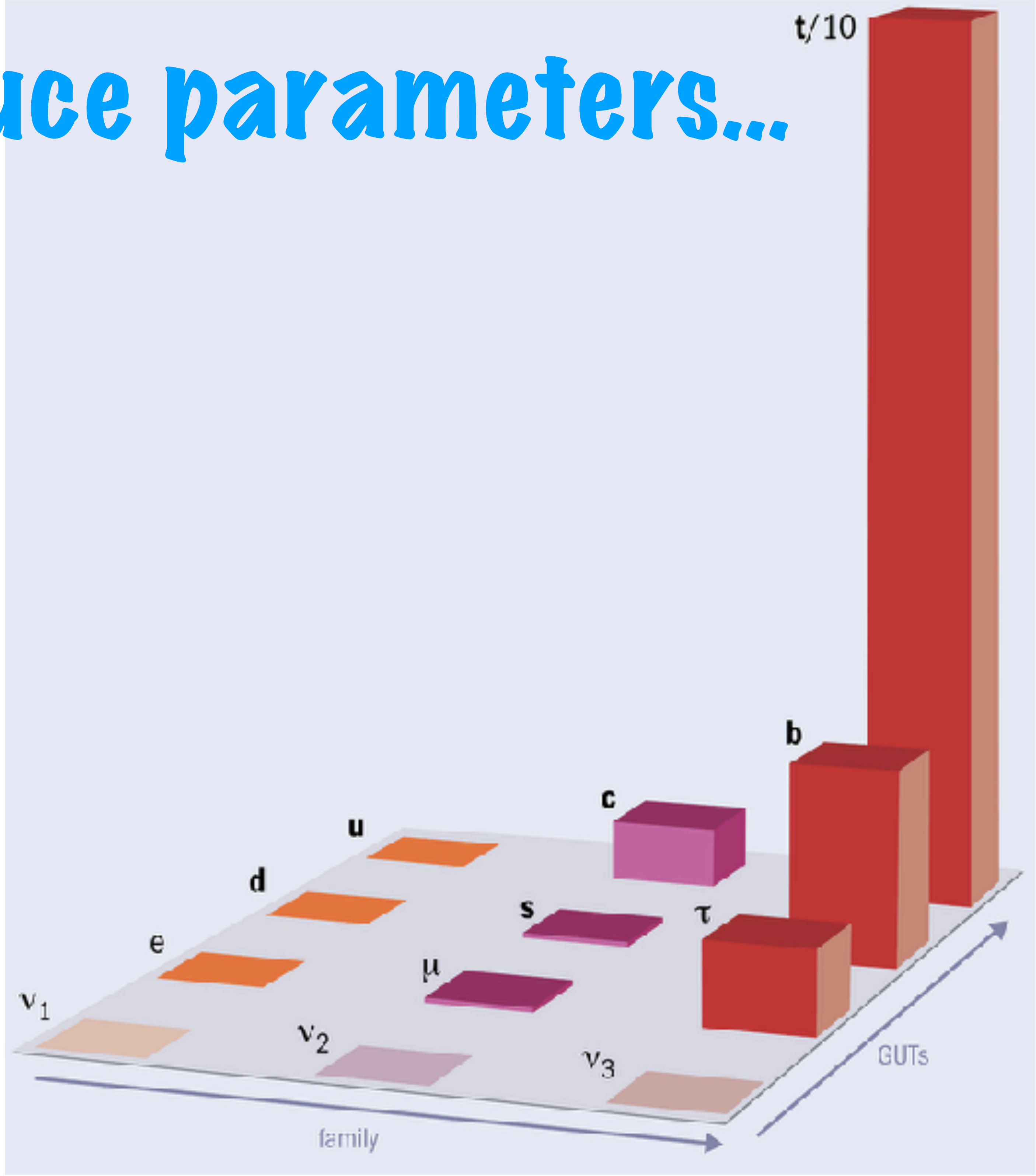
PMNS (Normal Ordering)

$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} \sim$$



$$\frac{m_e}{m_t} \sim 10^{-6}$$

To reduce parameters...



Unification

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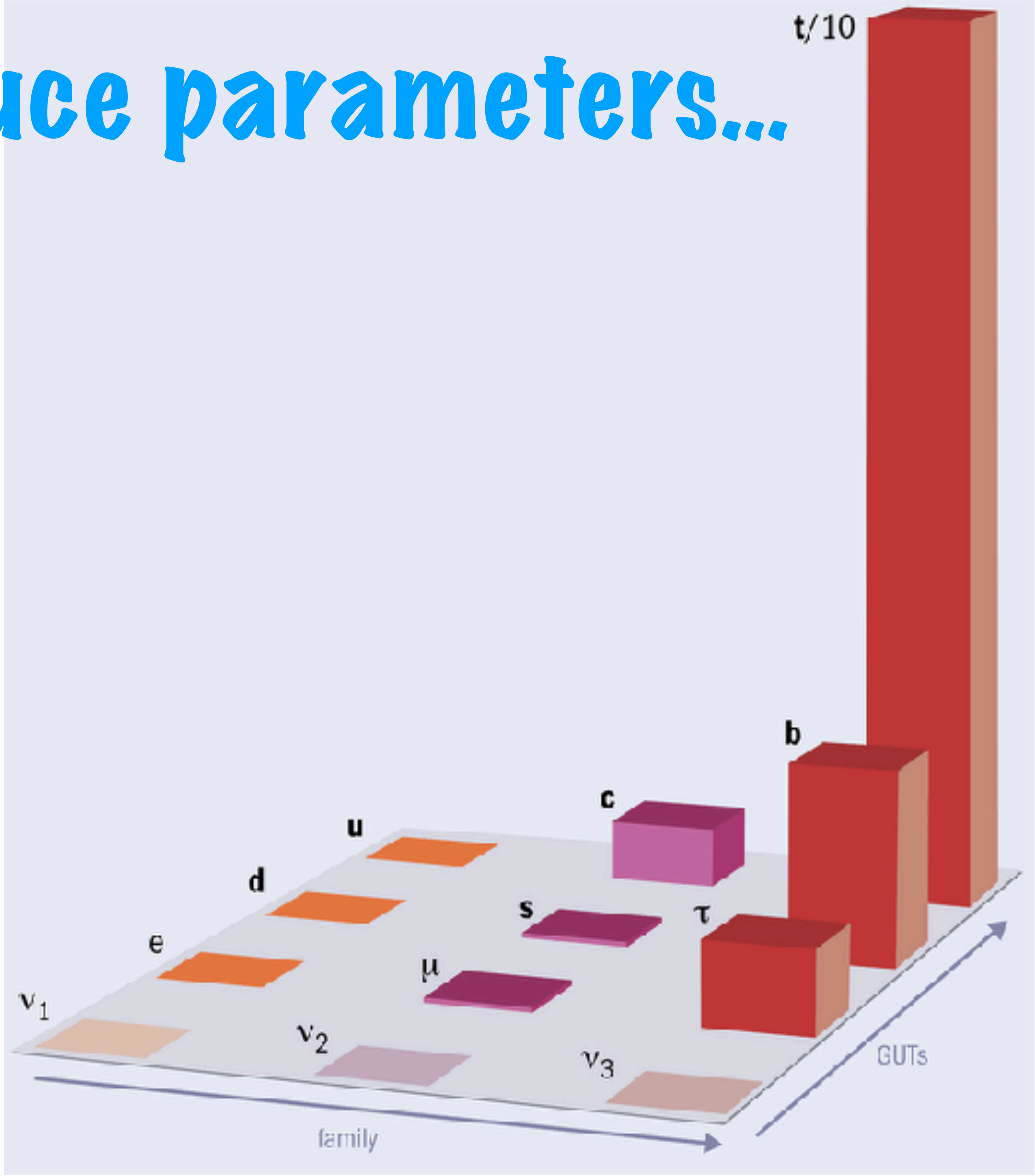

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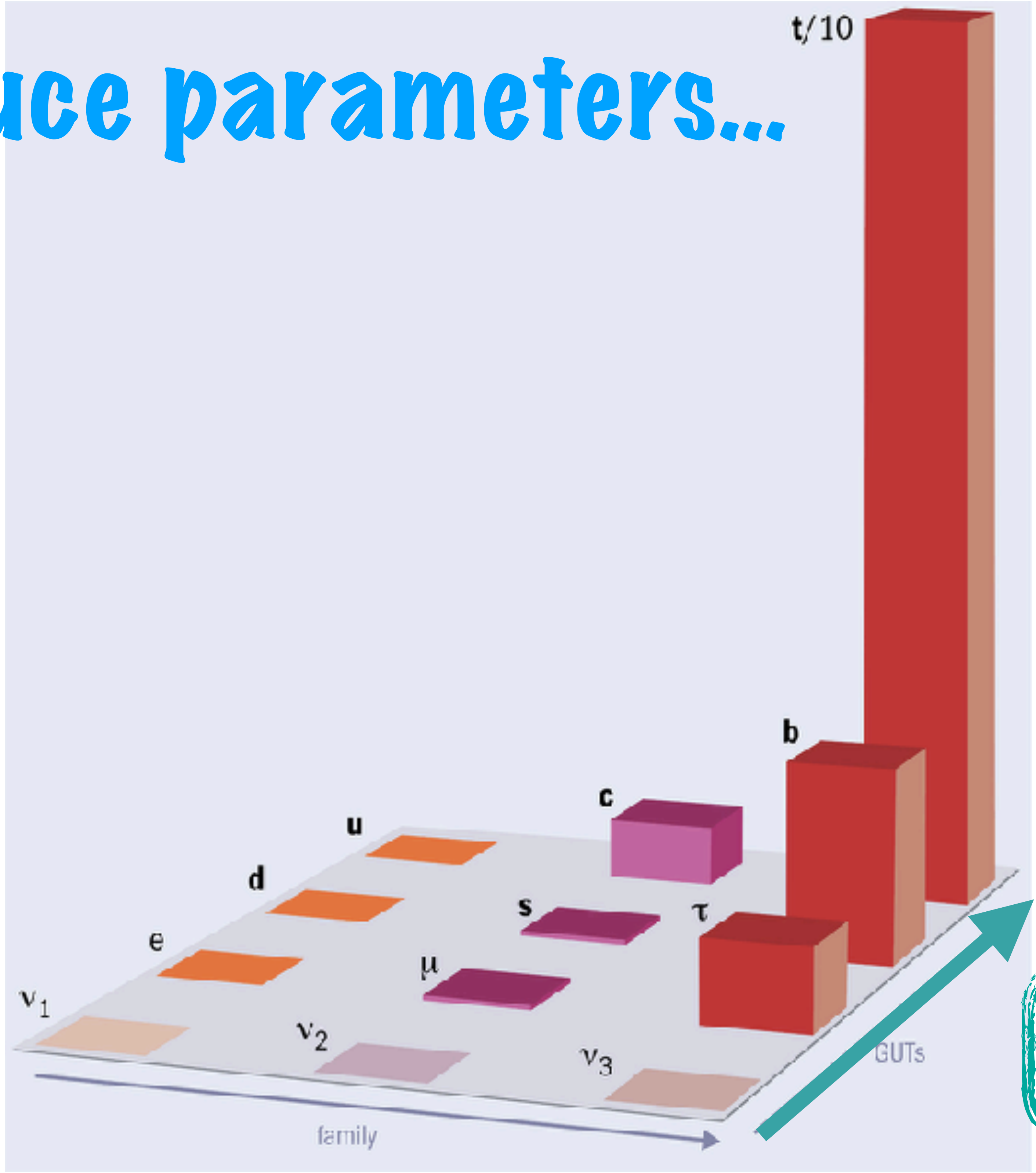

Gauge Unification

Unification

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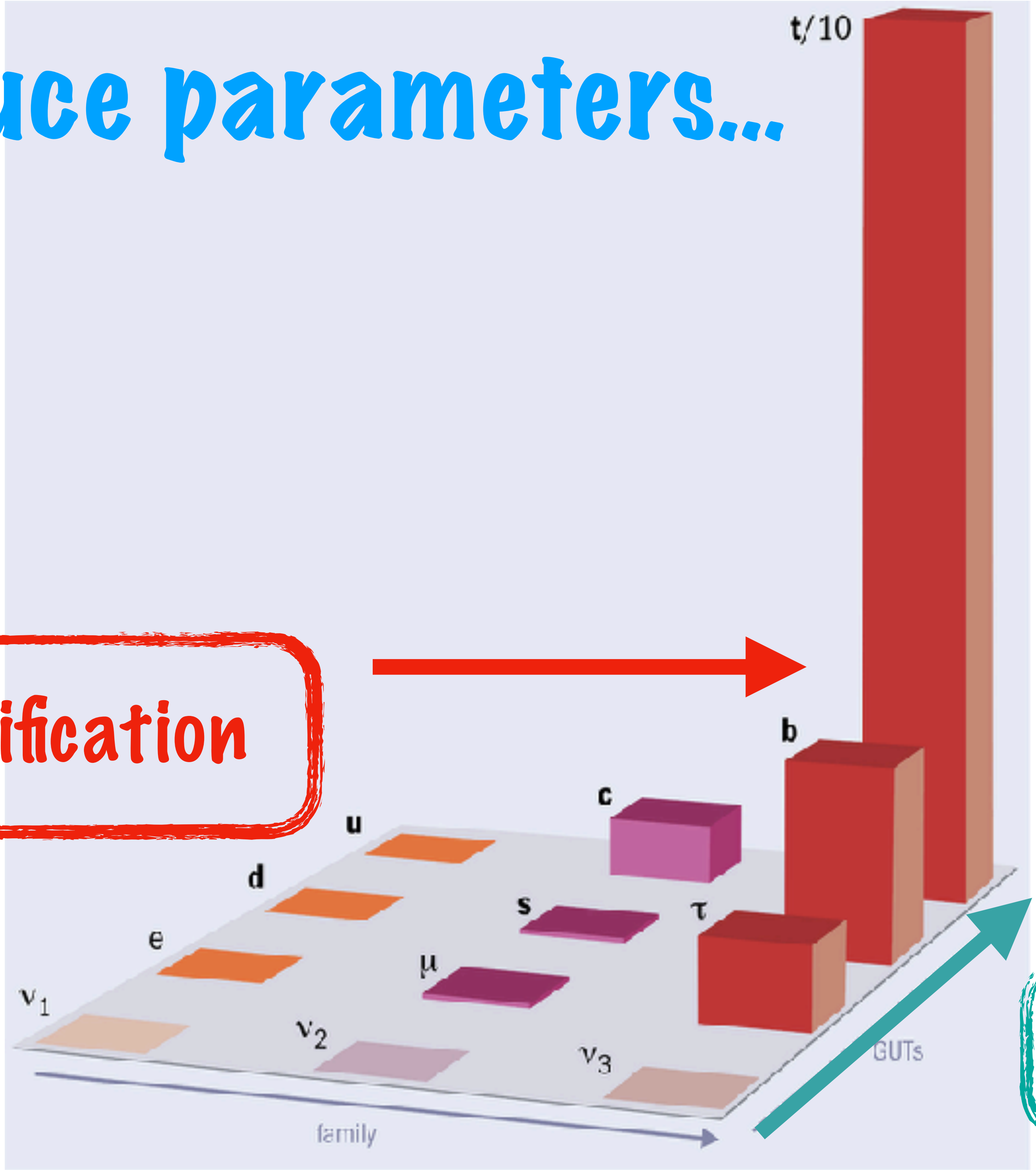

Flavor Unification

Gauge Unification

Flavor Symmetry

- A **new symmetry** at a higher energy scale

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes \underbrace{G_{flavor}}_{\text{Flavor Symmetry}}$$

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$$G_{Flavor} \implies m_{fermions}, V_{CKM}, U_{PMNS}$$

- Potentially explains the masses and mixings of quark and leptons **by few parameters** **(through correlations)**

$$\{m_e,\ m_\mu,\ m_\tau,\ m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3},\ m_d,\ m_s,\ m_b,\ m_u,\ m_c,\ m_b\}$$

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Example

Discrete Flavor Symmetries

Example: The S_4 group has five irreps

$$\mathbf{1},\ \mathbf{1}',\ \mathbf{2},\ \mathbf{3},\ \mathbf{3}'$$

Putting different fields in packages!

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$$L = \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, -1, \mathbf{3})$$

$$\mathscr{L}'_{S_4} = \mathscr{L}_{s_4}$$

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$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathscr{L}'_{S_4} = \mathscr{L}_{S_4}$$

Finite set of transformations

Discrete Flavor Symmetries

- **Flavour symmetry** at high-energy regime.

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \underbrace{\otimes\, G}_{\text{Flavour}} .$$

Abelian

$$\mathbb{Z}_n \quad (n \geq 2)$$

Deletes couplings

$$Y_e^{ij} \overline{L_i} \Phi e_{R_j}$$

$$Y_e = \begin{pmatrix} y_{ee} & \cancel{y_{e\mu}} & y_{e\tau} \\ \cancel{y_{\mu e}} & y_{\mu\mu} & \cancel{y_{\mu\tau}} \\ y_{\tau e} & \cancel{y_{\tau\mu}} & y_{\tau\tau} \end{pmatrix}$$

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Deletes couplings

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Non-Abelian

$$A_4, S_3, T', \ldots$$

Deletes and Relates couplings

$$Y_e = \begin{pmatrix} y_{ee} & y_{e\mu} & y_{e\tau} \\ y_{\mu e} & y_{\mu\mu} & y_{\mu\tau} \\ y_{\tau e} & y_{\tau\mu} & y_{\tau\tau} \end{pmatrix}$$

Residual Symmetries

- Residual $\mathbb{Z}_2$ symmetry from S_4

Flavon

$$\langle \phi \rangle = \begin{pmatrix} v_\phi \\ 0 \\ 0 \end{pmatrix} \sim \mathbf{3}$$

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S_4	$\mathbb{Z}_2$
$\mathbf{1}$	$\mathbf{1}_+$
$\mathbf{1}'$	$\mathbf{1}_+$
$\mathbf{2}$	$\mathbf{1}_+ \oplus \mathbf{1}_+$
$\mathbf{3}$	$\mathbf{1}_+ \oplus \mathbf{1}_- \oplus \mathbf{1}_-$
$\mathbf{3}'$	$\mathbf{1}_+ \oplus \mathbf{1}_- \oplus \mathbf{1}_-$

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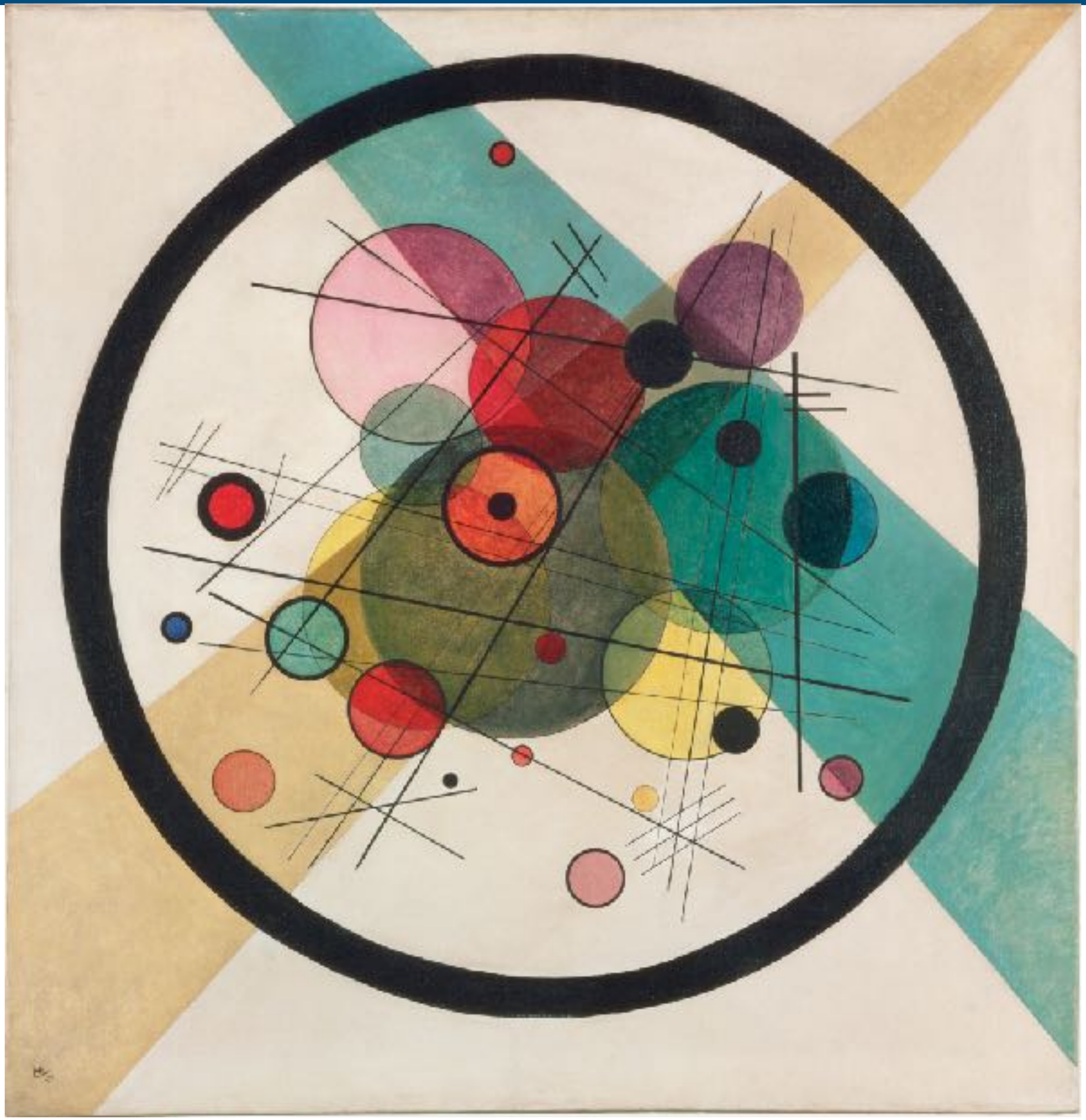
Modular Flavor Symmetries



1923-CirclesInACircle-Kandinsky

Modular Flavor Symmetries

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes SL(2,\mathbb{Z})$$

- The **modular group** becomes the Flavor Symmetry

$$SL(2,\mathbb{Z}) \equiv \Gamma$$

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- It is the infinite discrete group

$$\gamma \in \Gamma \qquad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad ad - cb = 1$$

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- With generators

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

Modular Flavor Symmetries

- Modular symmetries have rich mathematical structure and fundamental origin:

Extra-Dimensional field theories

e.g. Almumin et al. (2021)

String Theory

e.g. Nilles and Ramos-Sanchez (2021)

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- But the central idea is very simple

Yukawa Couplings
are modular forms

$$Y(\tau)$$

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Yukawa Couplings are modular forms $Y(\tau)$ $\tau \in \mathbb{C}$ **Modulus "spurion"**

- For example:

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \longrightarrow m_\nu = \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix} \frac{{v_u}^2}{\Lambda}$$

Building a Modular Flavor Model

- **Effective** (SUSY $\mathcal{N} = 1$) action

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta}\, \underbrace{\mathcal{K}(\bar{\tau}, \bar{\psi}, \tau, \psi)}_{\text{Kähler Potential}} + \int d^4x d^2\theta \underbrace{\mathcal{W}(\tau, \psi) + h.c}_{\text{Superpotential}},$$

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Two kinds of super fields

Modulus τ

Matter ψ

Field Transformations

- The transformation of the fields under $SL(2,\mathbb{Z})$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Modulus

$$\tau \xrightarrow{\gamma} \frac{a\tau + b}{c\tau + d}$$

Field Transformations

- The transformation of the fields under $SL(2,\mathbb{Z})$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Example

$$L \equiv \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix}$$

Matter fields

weight

$$L \xrightarrow{\gamma} (c\tau + d)^{-k} \rho_{\mathbf{r}}(\gamma)\, L$$

Automorphic Factor

Matrix Representation of a discrete flavor symmetry

Field Transformations

- The transformation of the fields under $SL(2,\mathbb{Z})$

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Example

Matter fields

$$L \equiv \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix}$$

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The authomorphic factor perceives $SL(2,\mathbb{Z})$

$S_3,\ A_4,\ S_4,\ A_5,\ \ldots$

So it contains a "tradicional" flavor symmetry

Modular Invariance

- Under these $SL(2,\mathbb{Z})$ transformations the **superpotential** must be invariant

$$\mathscr{W} \supset \sum_{i,k,\beta} \alpha_i \left(\psi \, \Phi_{u,d} \, \psi^c Y^{(k)}_{\mathbf{r}_\beta}(\tau) \right)_{\mathbf{1}} \; ,$$

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- Thus, Yukawa couplings are **modular form multiplets of the discrete flavor symmetry**

Yukawa Couplings

$$Y^{(k)}(\tau) \xrightarrow{\gamma} (c\tau + d)^k \, \rho(\gamma) Y^{(k)}(\tau)$$

$$S_3, \; A_4, \; S_4, \; A_5, \; \ldots$$

Modular Forms

- Example from an A_4 model

Feruglio (2017)

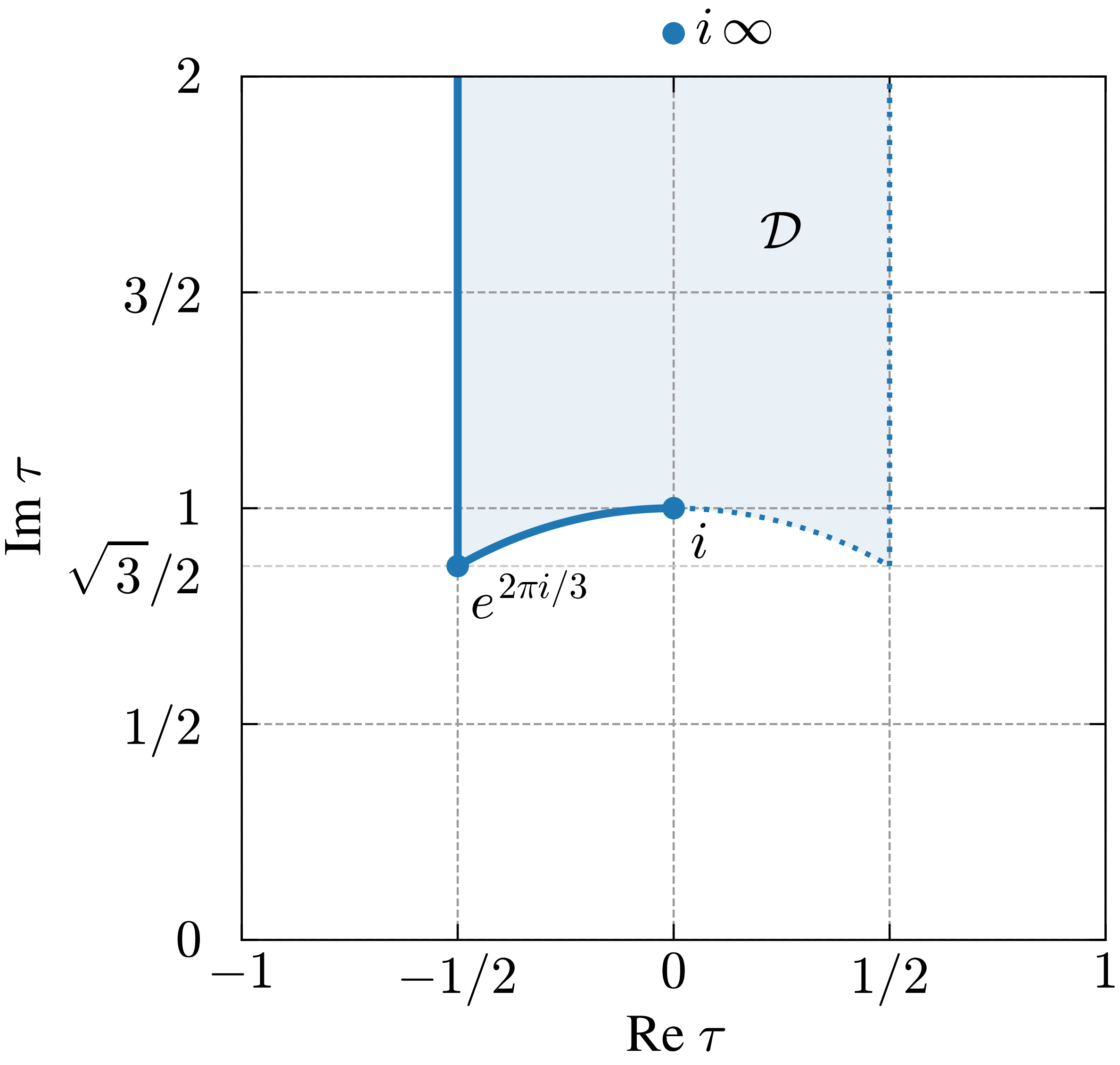
$$L \equiv \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \sim (\mathbf{3}, -1)$$

weighted representation

$$Y_{\mathbf{3}}^{(2)}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \longrightarrow m_\nu = \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix} \frac{v_u{}^2}{\Lambda}$$

Symmetry points

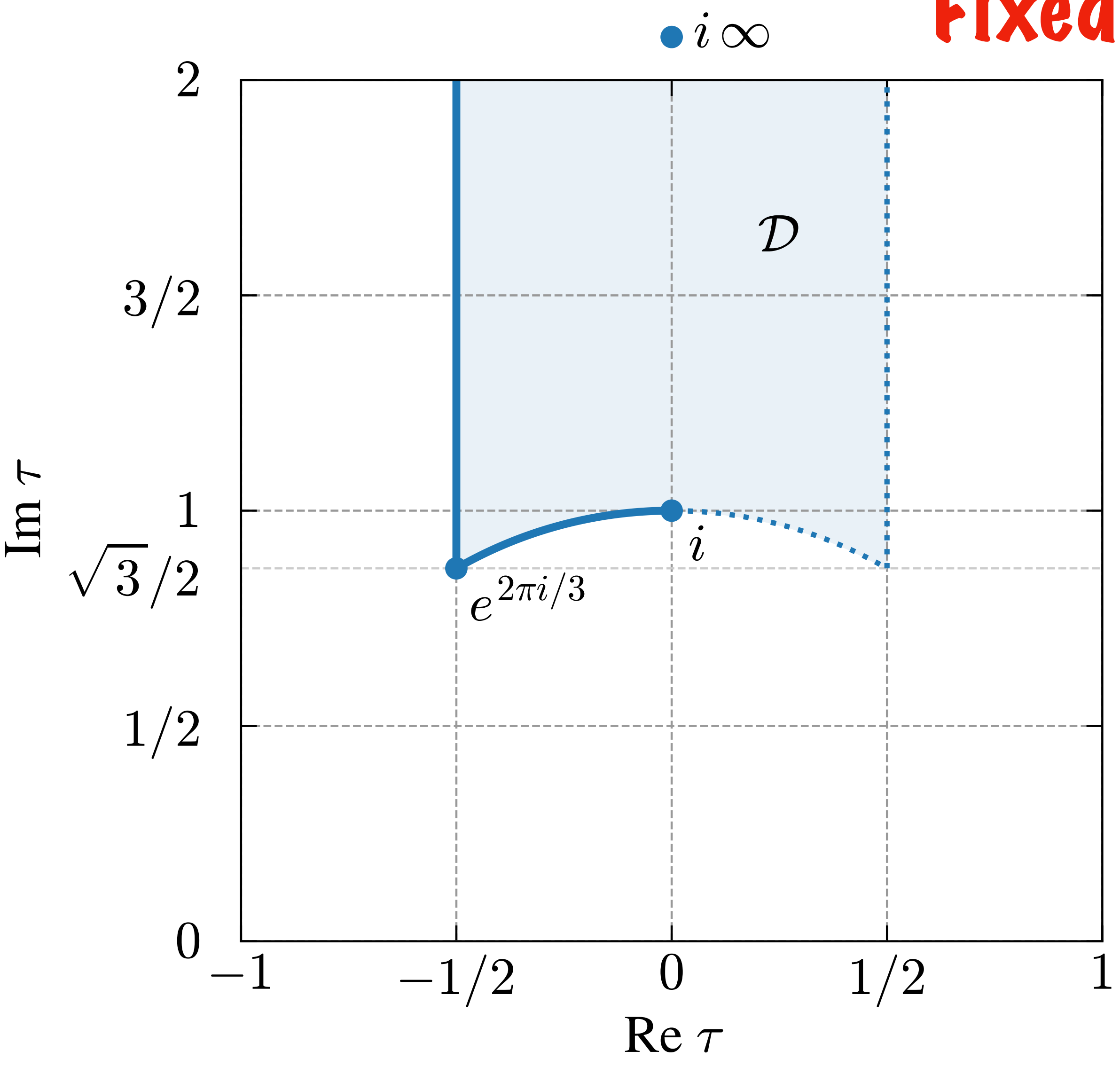
τ



Symmetry points

- Some values of τ lead to residual symmetries

Fixed Points



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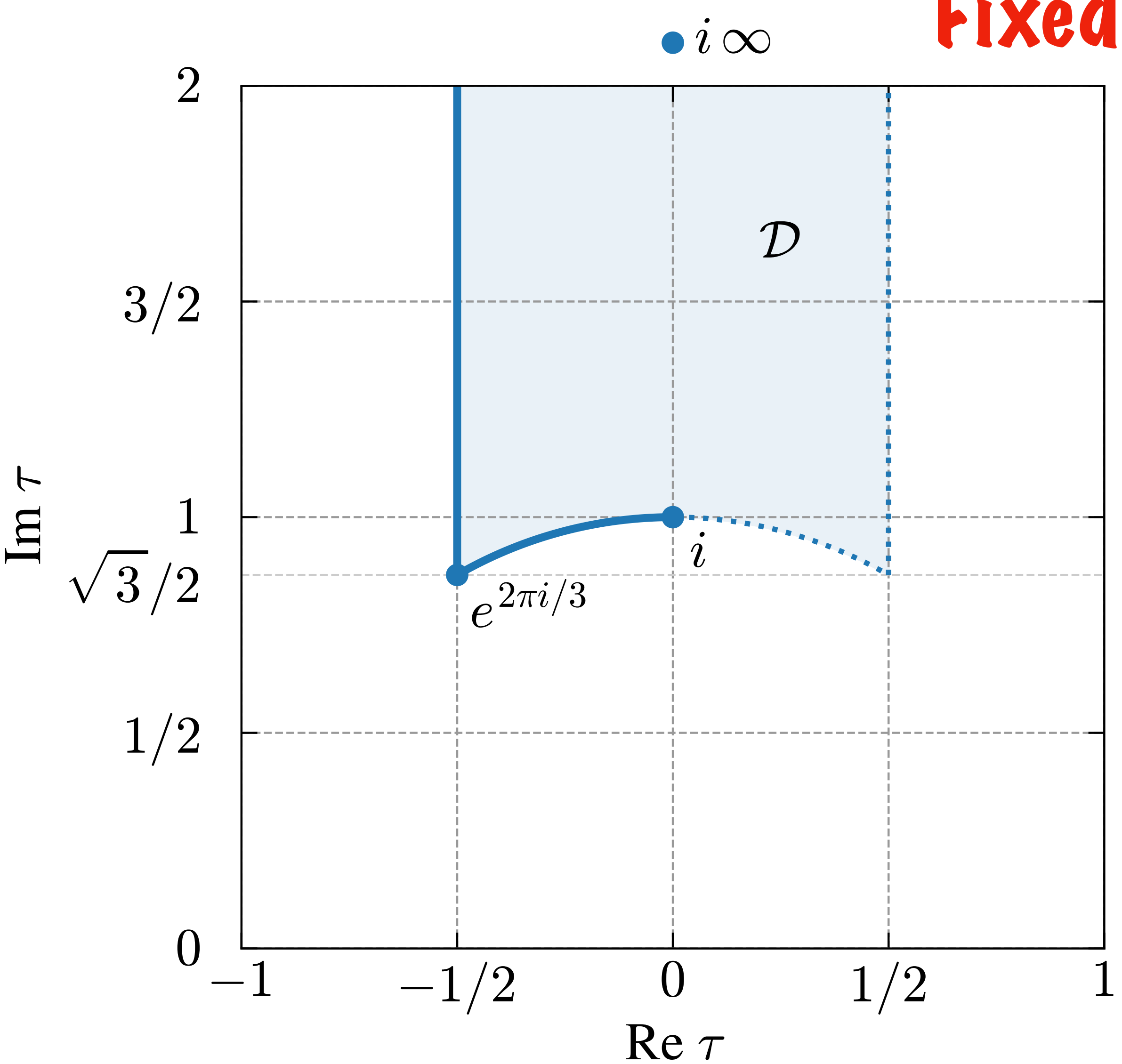

Fixed Points

$$SL(2,\mathbb{Z})$$

$$\tau_T = i\infty \quad\rightarrow\quad \mathbb{Z}_N \quad\rightarrow\quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\tau_S = i \quad\rightarrow\quad \mathbb{Z}_2 \quad\rightarrow\quad \begin{pmatrix} 1 \\ 1-\sqrt{3} \\ -2+\sqrt{3} \end{pmatrix}$$

$$\tau_{ST} = e^{i\frac{2\pi}{3}} \quad\rightarrow\quad \mathbb{Z}_3 \quad\rightarrow\quad \begin{pmatrix} 1 \\ \omega \\ \frac{\omega^2}{2} \end{pmatrix}$$

Modular Forms align at the symmetry points!

Vanishing Masses at the symmetry points

- At the symmetry points some masses can vanish

Example

$$(m_\tau, m_\mu, m_e) \sim (m_\tau, 0, 0)$$

Vanishing Masses at the symmetry points

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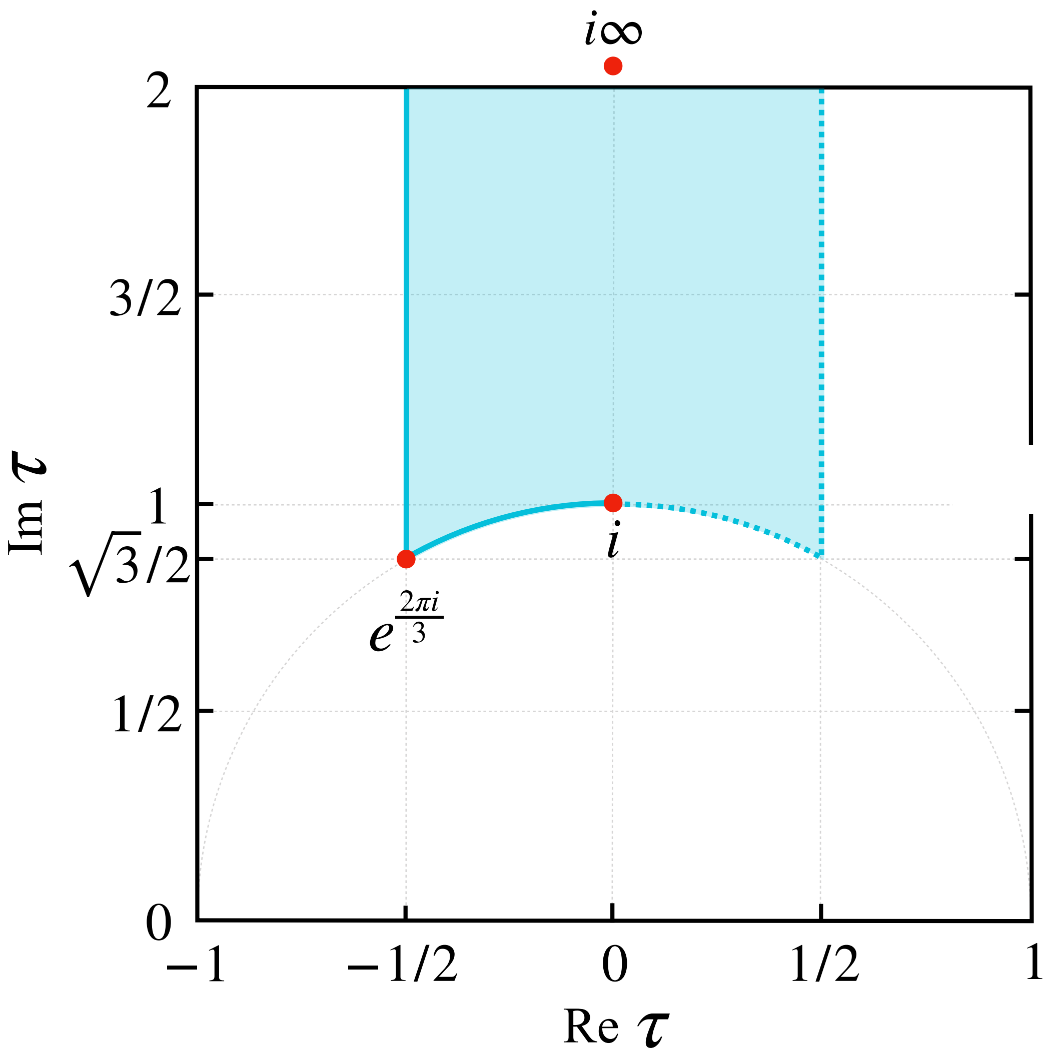
Example

$$(m_\tau, m_\mu, m_e) \sim (m_\tau, 0, 0)$$

- Thus, masses can be generated from a deviation from a symmetry point

Deviation Parameter

$$\epsilon(\tau)$$



The so-called “Near critical behavior”

- Near the symmetry points we can obtain textures of **hierarchical masses**

Example

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$\epsilon(\tau)$ **near** $\tau \sim i\infty$

Example

$$m_\nu \approx \begin{pmatrix} 2 & 18\epsilon^2 & 6\epsilon \\ 18\epsilon^2 & 12\epsilon & -1 \\ 6\epsilon & -1 & 36\epsilon^2 \end{pmatrix} \frac{v_u^2}{\Lambda}$$

The so-called "Near critical behavior"

- This might look like **Froggatt-Nielsen** but there is a crucial difference: **Fixed coefficients** (or reduced)

Can lead to actual predictions!

$\epsilon(\tau)$ **near** $\tau \sim i\infty$

Example

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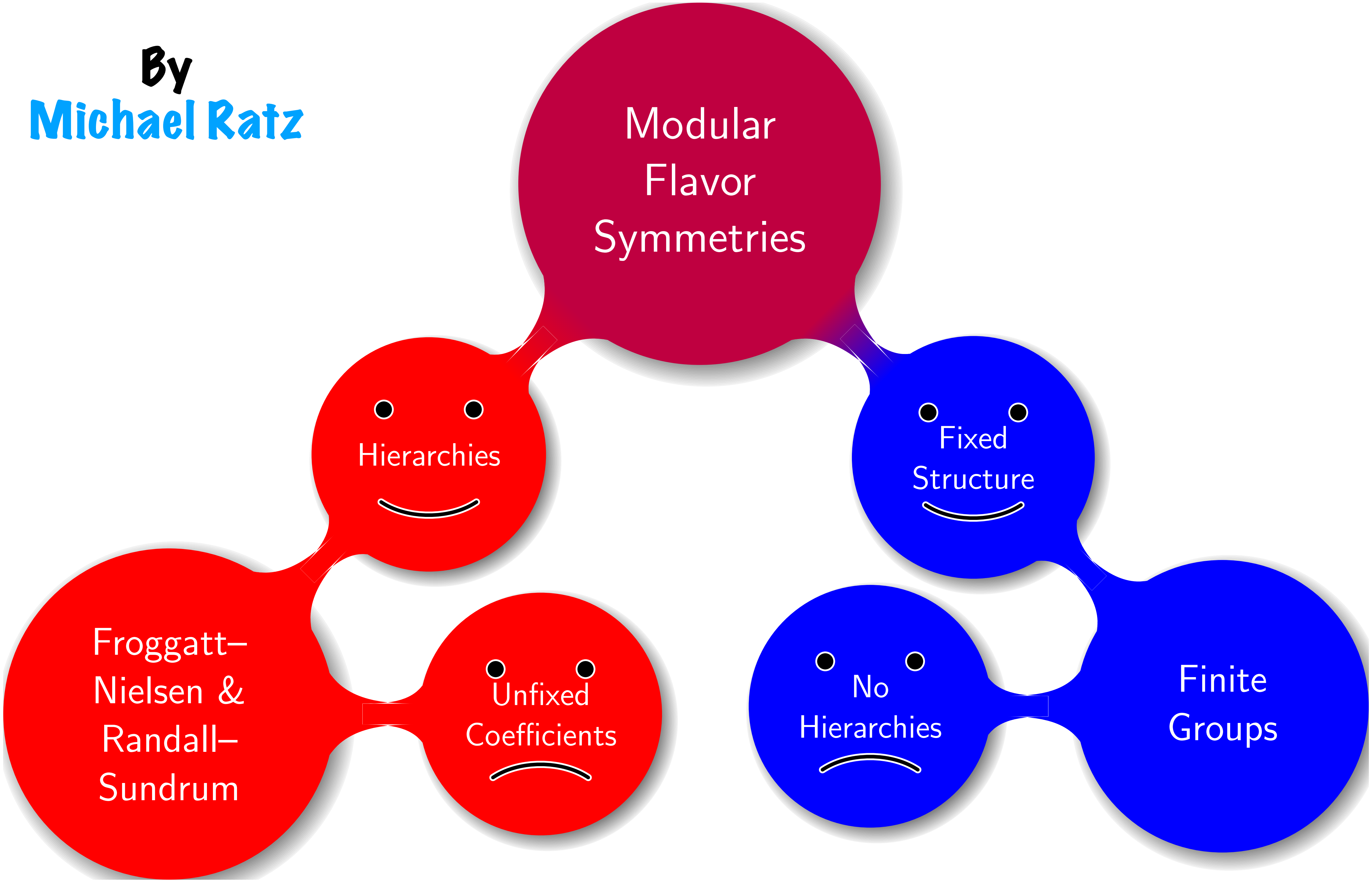
What modular flavor symmetries?

By
Michael Ratz



What modular flavor symmetries?

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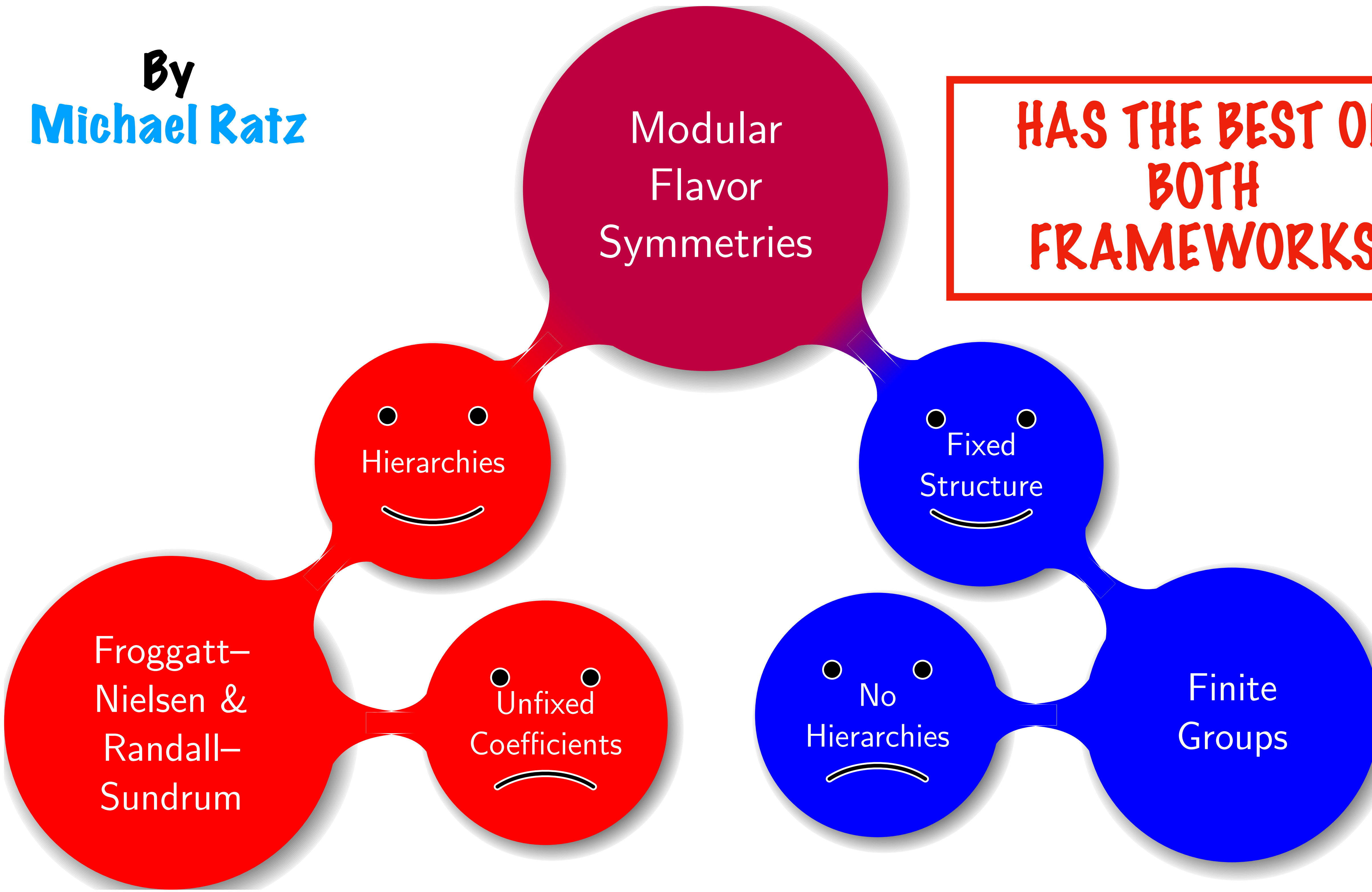
HAS THE BEST OF BOTH FRAMEWORKS



Quark-Lepton mass relations

1926- SeveralCircles-Kandinsky

Quark-Lepton mass relations

- This section follows: JHEP 02 (2024) 160

- Main result:

Viable and testable correlations among quark and lepton masses can emerge in modular symmetry models

MODEL INDEPENDENT DERIVATION

Fixing notation

- We will study the mass matrix of three generations of fermions:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \qquad \psi^c = \begin{pmatrix} \psi_1^c \\ \psi_2^c \\ \psi_3^c \end{pmatrix}$$

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- In modular flavor models we have invariance under

$$SL(2,\mathbb{Z}) \longrightarrow \Gamma_N : \; S_3 , A_4 , \ldots$$

$$\mathscr{W} \supset \sum_{i,k,\beta} \alpha_i \left(\psi \Phi_{u,d} \psi^c Y^{(k)}_{\mathbf{r}_\beta}(\tau) \right)_{\mathbf{1}} ,$$

Fixing notation

- From the superpotential we obtain the mass matrix

$$\mathscr{W} \supset \sum_{i,k,\beta} \alpha_i \left(\psi \Phi_{u,d} \psi^c Y_{\mathbf{r}_\beta}^{(k)}(\tau) \right)_{\mathbf{1}} ,$$

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$$\mathscr{W} \supset \sum_{i,k,\beta} \alpha_i \left(\psi \Phi_{u,d} \psi^c Y^{(k)}_{\mathbf{r}_\beta}(\tau) \right)_{\mathbf{1}} ,$$

- We will define the dimensionful parameters

$$a_i \equiv \frac{\alpha_i}{\sqrt{2}} v_{u,d} \quad \text{dimensionful parameters}$$

Fixing notation

- The mass matrix will be a function of a_i and τ

$$\mathscr{W} \supset \psi \, M_\psi(a_i, \tau) \, \psi^c$$

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- From the mass matrix we can compute the masses as a function of the parameters

$$M_\psi(a_i, \tau) \longrightarrow m_1 \leq m_2 \leq m_3 \, (a_i, \tau)$$

By definition

Fixing notation

- It is more convenient to work with the Hermitian matrix

$$H_\psi \equiv M_\psi M_\psi^\dagger = U_\psi D_\psi^2 U_\psi^\dagger$$

Eigenvalues

$$D_\psi^2 = \mathrm{Diag}\left(m_1^2, m_2^2, m_3^2\right)$$

Invariant Equations

- Using H_ψ we can find three (basis invariant) equations for the masses in terms of the parameters

MASTER EQUATIONS

Three eqs.
Thress masses

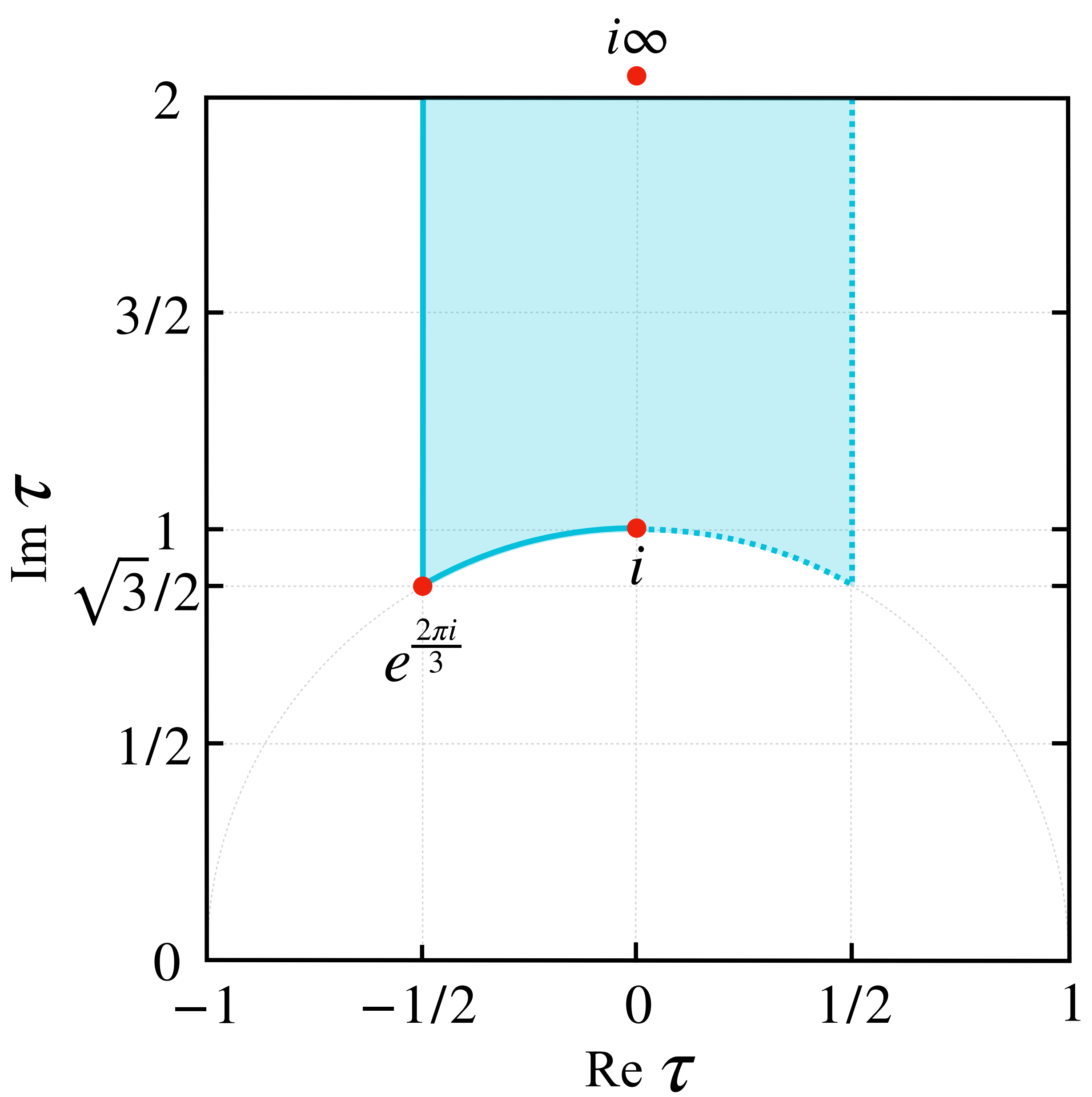
a) $$\mathrm{Det}[H_\psi] \equiv f(a_i, \tau) = m_1^2 m_2^2 m_3^2$$

b) $$\frac{1}{2}\{\mathrm{Tr}[H_\psi]^2 - \mathrm{Tr}[H_\psi^2]\} \equiv g(a_i, \tau) = m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2$$

c) $$\mathrm{Tr}[H_\psi] \equiv h(a_i, \tau) = m_1^2 + m_2^2 + m_3^2$$

Invariant Equations

- We assume closeness to a symmetry point (near critical behavior)



Deviation Parameter

$$\epsilon(\tau)$$

Instead of

$$\tau$$

Conditions for mass relation

- In **some model** the following two conditions should satisfied **to obtain a mass relation**

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- **C1:** The mass matrix has at most two coefficients

$$a_1\,, a_2 \Longrightarrow M_\psi(a_1, a_2, \epsilon)$$

Conditions for mass relation

- In **some model** the following two conditions should satisfied **to obtain a mass relation**

- **C1:** The mass matrix has at most two coefficients

$$a_1\,, a_2 \implies M_\psi(a_1, a_2, \epsilon)$$

- **C2:** At least one mass is generated as a deviation from a symmetry point

$$\lim_{\epsilon\to 0} Det[H_\psi] = 0$$

Conditions for mass relation

- If in **some model** these two conditions are satisfied for two species of fermions, **there will be a mass relation**

Example

	Charged Leptons	Down-quarks
C1	$M_e(a_1^e, a_2^e, \epsilon)$	$M_d(a_1^d, a_2^d, \epsilon)$
C2	$\lim_{\epsilon \to 0} Det[H_e] = 0$	$\lim_{\epsilon \to 0} Det[H_d] = 0$

Conditions for mass relation

- We can compute the general form of the mass relations

MODEL INDEPENDENT

- If C1 and C2 are satisfied, we have

Expanding

$$\mathrm{Det}\left[H_\varphi(a_1,a_2,\epsilon)\right] = \sum_{m=0}^{\infty} f_m(a_1,a_2)\,|\epsilon|^{2m}$$

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No epsilon independent term

$$f_0(a_1, a_2) = 0$$

Conditions for mass relation

- If C1 and C2 are satisfied, we can write

$$\mathrm{Det}\left[H_\omega(a_1, a_2, \epsilon) \right] = m_1^2 m_2^2 m_3^2 \equiv f_\eta(a_1, a_2)\, |\epsilon|^\eta + \mathcal{O}(|\epsilon|^{\eta+1})$$

Conditions for mass relation

- If C1 and C2 are satisfied, we can write

$$\mathrm{Det}\left[H_\psi(a_1, a_2, \epsilon) \right] = m_1^2 m_2^2 m_3^2 \equiv f_\eta(a_1, a_2)|\epsilon|^\eta + \mathcal{O}(|\epsilon|^{\eta+1})$$

$f_\eta(a_1, a_2)$ is the leading term polynomial

$$m_1^2 m_2^2 m_3^2 = f_\eta(a_1, a_2)|\epsilon|^\eta$$

Solutions at the symmetry point

- At the **symmetry point**… $\epsilon \to 0$

$$\lim_{\epsilon \to 0} \begin{pmatrix} m_3 \\ m_2 \\ m_1 \end{pmatrix} \sim \begin{pmatrix} m_3(a_1, a_2) \\ m_2(a_1, a_2) \\ 0 \end{pmatrix}$$

At least one vanishes

$$h_\varphi(a_1, a_2, 0) = m_2^2 + m_3^2$$

$$g_\varphi(a_1, a_2, 0) = m_2^2 m_3^2$$

to equations two variables.

Solutions at the symmetry point

- Solving this equation system

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$$\tilde{a}_1(m_2, m_3)\,,\ \tilde{a}_2(m_2, m_3)$$

$$m_1^2 m_2^2 m_3^2 = f_\eta(\tilde{a}_1, \tilde{a}_2)|\epsilon|^\eta \quad \Longrightarrow \quad m_1^2 m_2^2 m_3^2 \equiv F_\eta(m_2, m_3)|\epsilon|^\eta$$

General mass relation

- Using these solutions we find an approximate mass correlation

$$\frac{m_1^2 m_2^2 m_3^2}{F(m_2, m_3)} \approx |\epsilon|^\eta$$

- The prediction in a specific model is the polynomial

$$F(m_2, m_3)$$

General mass relation

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Order 6 polynomial

Homogeneous

General mass relation

- Using these solutions we find an approximate mass correlation

$$\frac{m_1^2 m_2^2 m_3^2}{F(m_2, m_3)} \approx |\epsilon|^\eta$$

- In a given model the coefficients are determined by the specific modular symmetry

$$F(m_2, m_3) = C_{3,6} m_3^6 + C_{3,5} m_3^5 m_2 \cdots$$

General mass relation

- In particular there is a class that are appealing

$$F^{leading}(m_2, m_3)$$

CG - Modular form coefficients

- $f^{leading}(m_2, m_3) = C_{3,6}\, m_3^6 + \dots$
- $f^{non\text{-}leading}(m_2, m_3) = \cancel{C_{3,6}}^{\,0}\, m_3^6 + C_{3,5}\, m_3^5 m_2 + \dots$

General mass relation

- **Interesting Fact:** Experimental data is compatible with a F-leading correlation between

$$\frac{m_e^2 m_\mu^2 m_\tau^2}{m_\tau^6 + \ldots} \approx \frac{m_d^2 m_s^2 m_b^2}{m_b^6 + \ldots}$$

Highly non-trivial

Example

- In our paper we explicitly obtain four relations in an S_4 modular symmetry model

$$\frac{m_d m_s}{m_b(m_b \pm 3m_s)} \approx |\epsilon|^4 \approx \frac{m_e m_\mu}{m_\tau(m_\tau \pm 3m_\mu)}$$

Example

- In our paper we explicitly obtain four relations in an S_4 modular symmetry model

$$\frac{m_d m_s}{m_b(m_b \pm 3m_s)} \approx |\epsilon|^4 \approx \frac{m_e m_\mu}{m_\tau(m_\tau \pm 3m_\mu)}$$

- When comparing against data, one fits better

R1

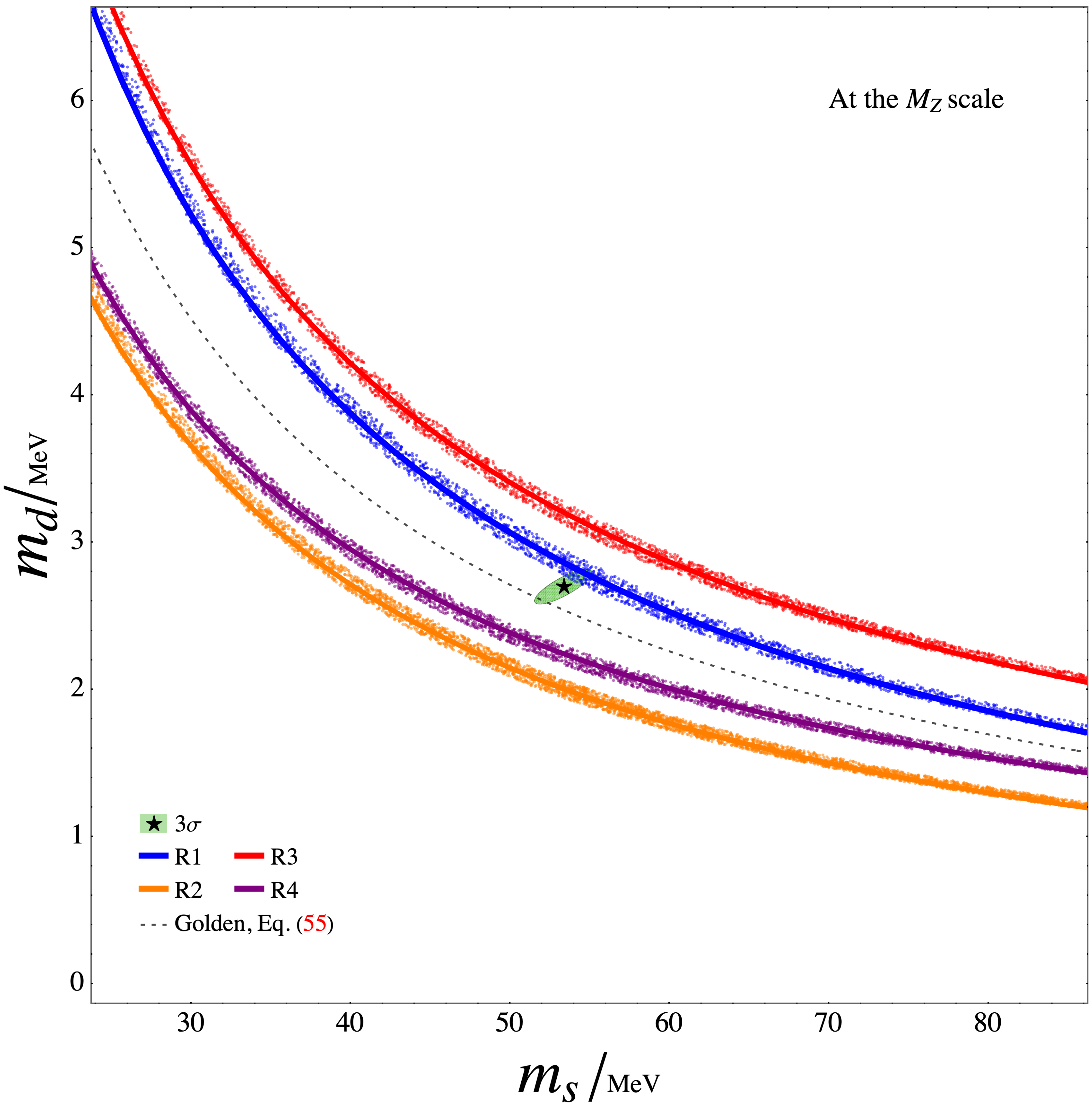
$$\frac{m_d m_s}{m_b(m_b - 3m_s)} \approx \frac{m_e m_\mu}{m_\tau(m_\tau - 3m_\mu)}$$

Testing the mass relations

R1

$$\frac{m_d m_s}{m_b(m_b - 3m_s)} \approx \frac{m_e m_\mu}{m_\tau(m_\tau - 3m_\mu)}$$

Fairly Stable under RG-Running



Some comments

This is only a **particular example**

Different mass-relations for different modular groups

mass relations can allow to **test** modular flavor symmetries

For more details...

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Summary

- We review the need for a fundamental theory of flavor

- We discusses modular flavor symmetries

$$SL(2,\mathbb{Z})$$

Minimal Parameters **Natural Hierarchies**

- One generic prediction are mass-relations

$$\frac{m_1^2 m_2^2 m_3^2}{F(m_2, m_3)} \approx |\epsilon|^\eta$$

non-trivial

$$\frac{m_e^2 m_\mu^2 m_\tau^2}{m_\tau^6 + \dots} \approx \frac{m_d^2 m_s^2 m_b^2}{m_b^6 + \dots}$$

APPROVED

Thank you!