Dirac neutrinos and parity solution to the strong CP problem

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FLASY 2024

UCI - June 26, 2024

Outline

- Motivation
- Lightning review on Majorana neutrino mass models and test through lepton flavor violation
- Dirac neutrinos from left-right symmetric theory and its embedding into GUT
  - Parity solves the strong CP problem
  - Dirac neutrinos can be tested through N_{eff}
  - GUT embedding predicts δ_{CP} and the lightest neutrino mass
- Summary

Strong CP Problem

- QCD Lagrangian allows term that violates Parity P and Time Reversal T symmetries, thus CP symmetry:

$$\mathscr{L}_{\text{QCD}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \theta\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \bar{q}\left(i\gamma^\mu D_\mu - m_q e^{i\theta_q\gamma_5}\right)q$$

g_s = strong coupling constant

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$$

- Transformation properties under discrete symmetries: analogous to electrodynamics

$$-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} = \frac{1}{2}\left(\vec{\boldsymbol{E}}^2 - \vec{\boldsymbol{B}}^2\right) \xRightarrow{\text{P, T}} \text{P-even, T-even}$$

$$F_{\mu\nu}\tilde{F}_{\mu\nu} = -4\,\vec{\boldsymbol{E}}\cdot\vec{\boldsymbol{B}} \xRightarrow{\text{P, T}} \text{P-odd, T-odd}$$

$\vec{\boldsymbol{E}}$ is P-odd, T-even
$\vec{\boldsymbol{B}}$ is P-even, T-odd

- Any chiral rotation of the quark field, $q \to e^{i\alpha\gamma_5}q$ would lead to the redefinition of the new parameter $\theta \to \theta + \alpha$ due to the anomalous nature of this rotation,

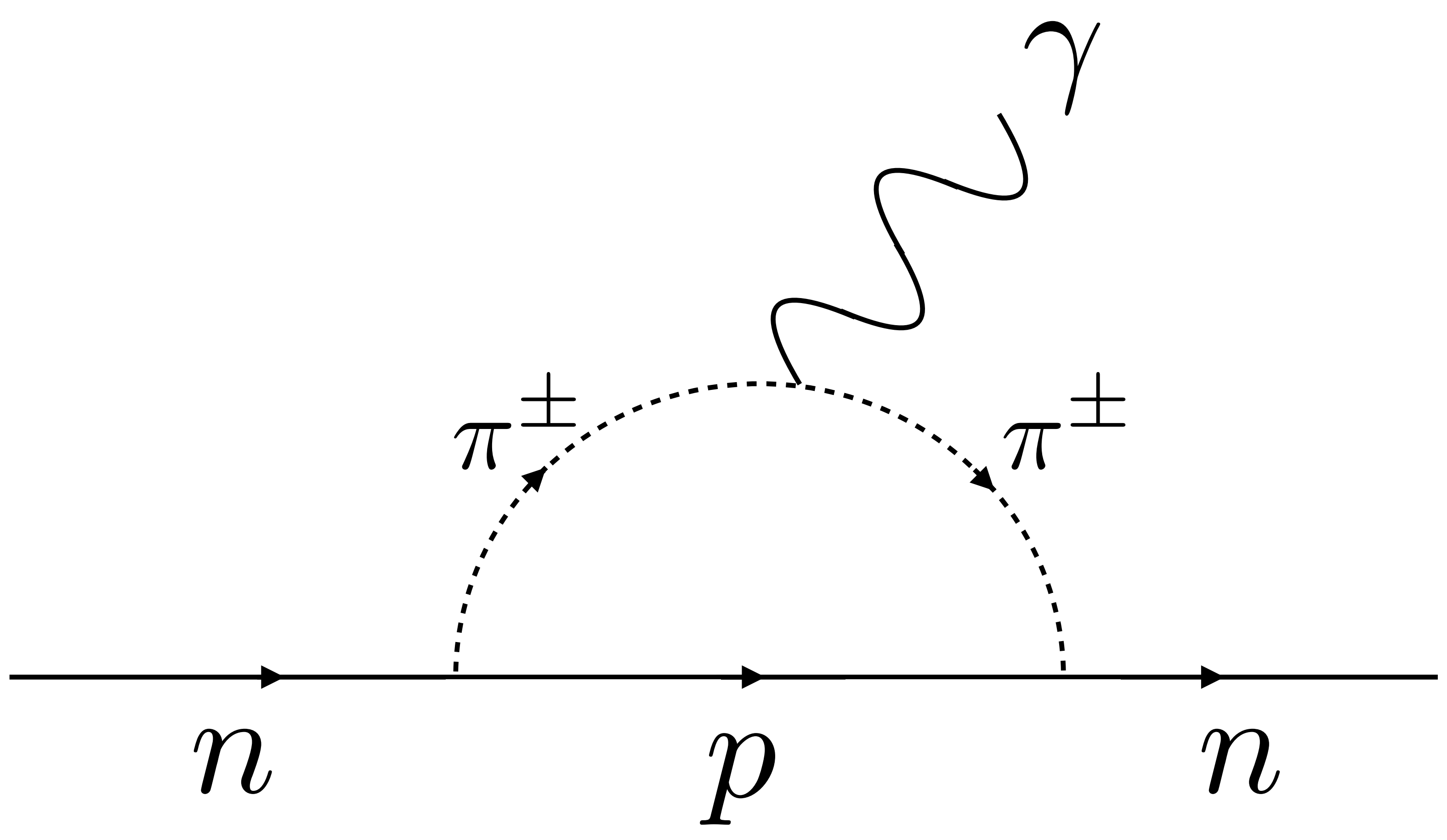
$\implies$ only invariant physical quantity is $\bar{\theta} = \theta + \theta_q$

or $\bar{\theta} = \boxed{\theta + \text{ArgDet}[M_Q]}$ with multiple flavors of the quark

No reason for them to cancel

Strong CP Problem

- $\bar{\theta}$ induces neutron electric dipole moment (neutron EDM)



$$d_n = \frac{e\bar{\theta} g_A c_+ \mu}{8\pi^2 f_\pi^2} \log \frac{\Lambda^2}{m_\pi^2} \sim 3 \times 10^{-16}\ \bar{\theta}\ \text{e cm}$$

$$\mu = \frac{m_u m_d}{m_u + m_d}\ , \quad g_A \simeq 1.27,\ c_+ \simeq 1.6,\ \Lambda = 4\pi f_\pi$$

- Current bound on neutron is $d_n < 3 \times 10^{-26}$ e cm $\implies \bar{\theta} < 10^{-10}$

The mass parameters can in principle have arbitrary phases, and one would expect $\bar{\theta} \sim \mathcal{O}(1)$

Why is $\bar{\theta}$ so small?

Strong CP Problem

Solutions to the Strong CP Problem

Massless up quark

$\bar{\theta} = \theta + \mathrm{ArgDet}[M_Q]$

- chiral rotations,
  $u \to e^{i\alpha\gamma_5} u \Longrightarrow \theta \to \theta + \alpha$
  can remove it.
- $m_u = 0$ is inconsistent with experimental data as well as lattice calculations.

H. Georgi and I. Mc Arthur'81
K. Choi, C.W. Kim and W.K. Sze'88

The Axion

Make $\bar{\theta}$ a dynamical filed.

A global chiral U(1) symmetry is introduced that is spontaneously broken. Effective interaction of axion:

$$\mathscr{L} \supset \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2} G\tilde{G}$$

Axion effective potential is such that the vacuum solution relaxes to $\bar{\theta} = 0$

R.D. Peccei and H.R. Quinn'77
F. Wilczek'78, S. Weinberg'78

P or CP

Make P or CP exact symmetry broken spontaneously in such a way that the determinant of the quark mass matrix is real.

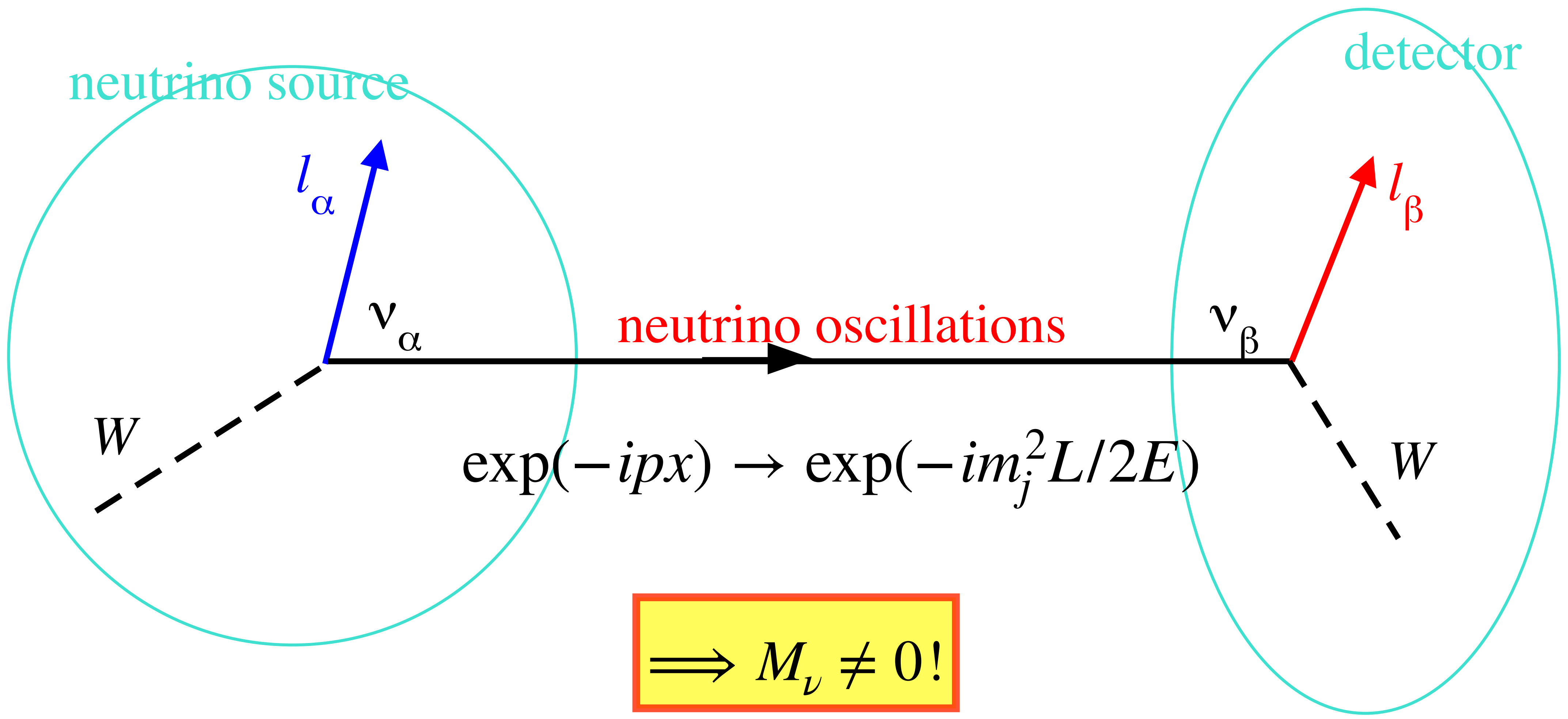
$\bar{\theta} = 0$

A. Nelson'84 and S.M. Barr'84
Babu and Mohapatra, '90

More on it later

Shortcomings of the Standard Model

Neutrino masses are predicted to be zero in the SM, but neutrino oscillates!



$\boxed{\Longrightarrow M_\nu \neq 0!}$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U parameterization:

$\theta_{12} \sim [31^\circ - 36^\circ]$

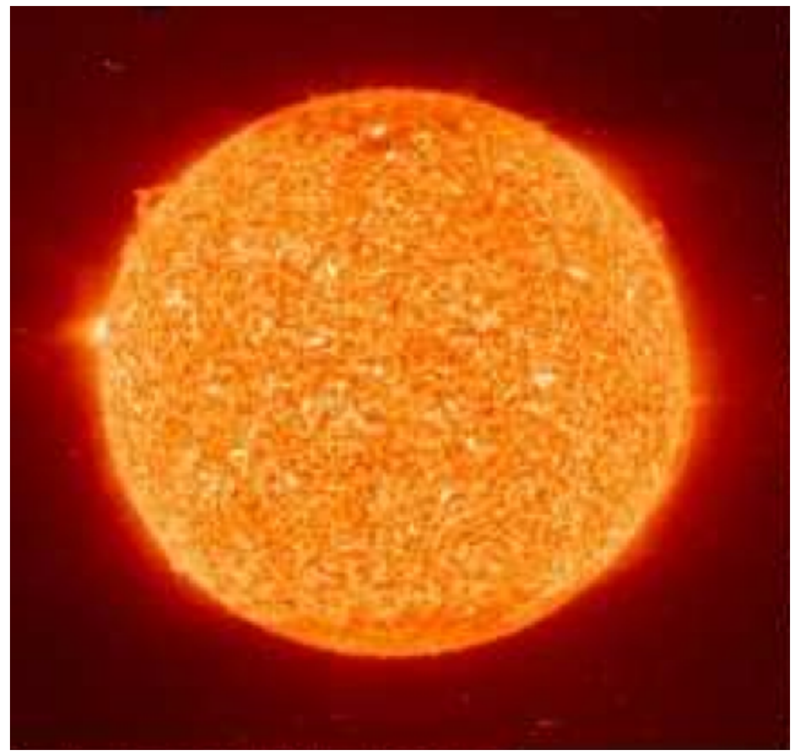
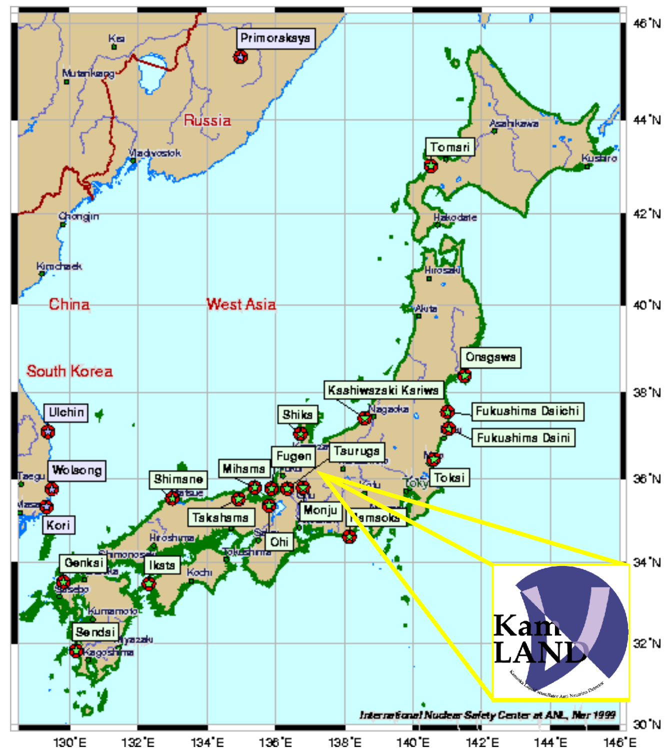
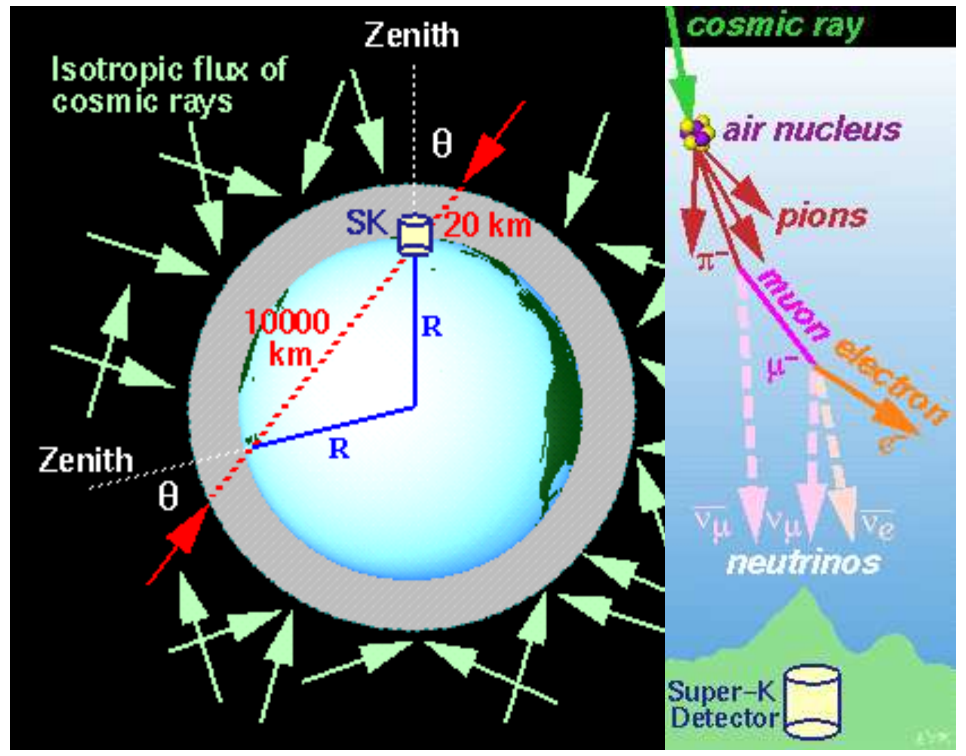
$\theta_{23} \sim [39^\circ - 52^\circ]$

$\theta_{13} \sim [8^\circ - 9^\circ]$

$\delta_{\text{CP}} \sim ?$

Kajita & McDonald, 2015

$$\mathrm{P}(\nu_\tau \to \nu_\alpha) = \left| \sum_j U^*_{\tau j} U_{\alpha j} \exp\left(-i \frac{m_j^2 L}{2E} \right) \right|^2$$

sun	reactors	atmosphere	accelerators
			
Homestake,SAGE,GALLEX SuperK, SNO, Borexino	KamLAND, D-CHOOZ DayaBay, RENO	SuperKamiokande	K2K, MINOS, T2K OPERA

Open questions

> Octant of θ_{23} ?

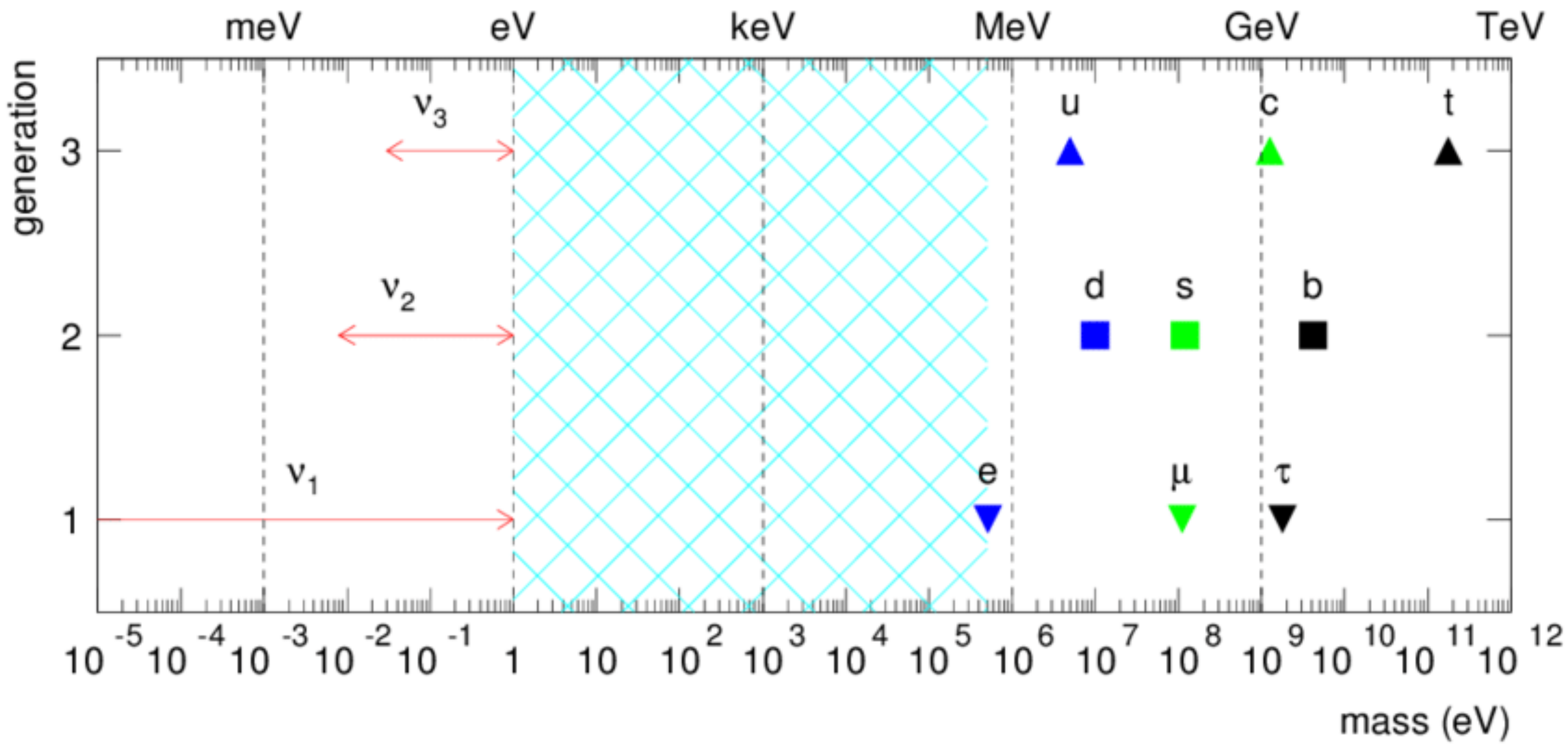
> Absolute mass scale and mass hierarchy?

> Are neutrinos their own antiparticle? Dirac vs Majorana

> Is there CP Violation in lepton sector, $P(\nu_\mu \rightarrow \nu_e) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$?

> Why is neutrino mass so tiny?

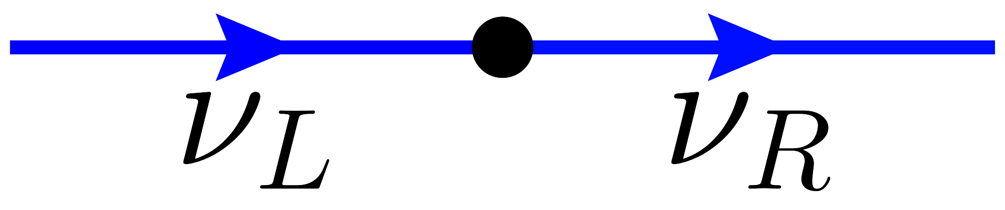
Nonzero neutrino masses
$\Longrightarrow$ existence of new fundamental fields



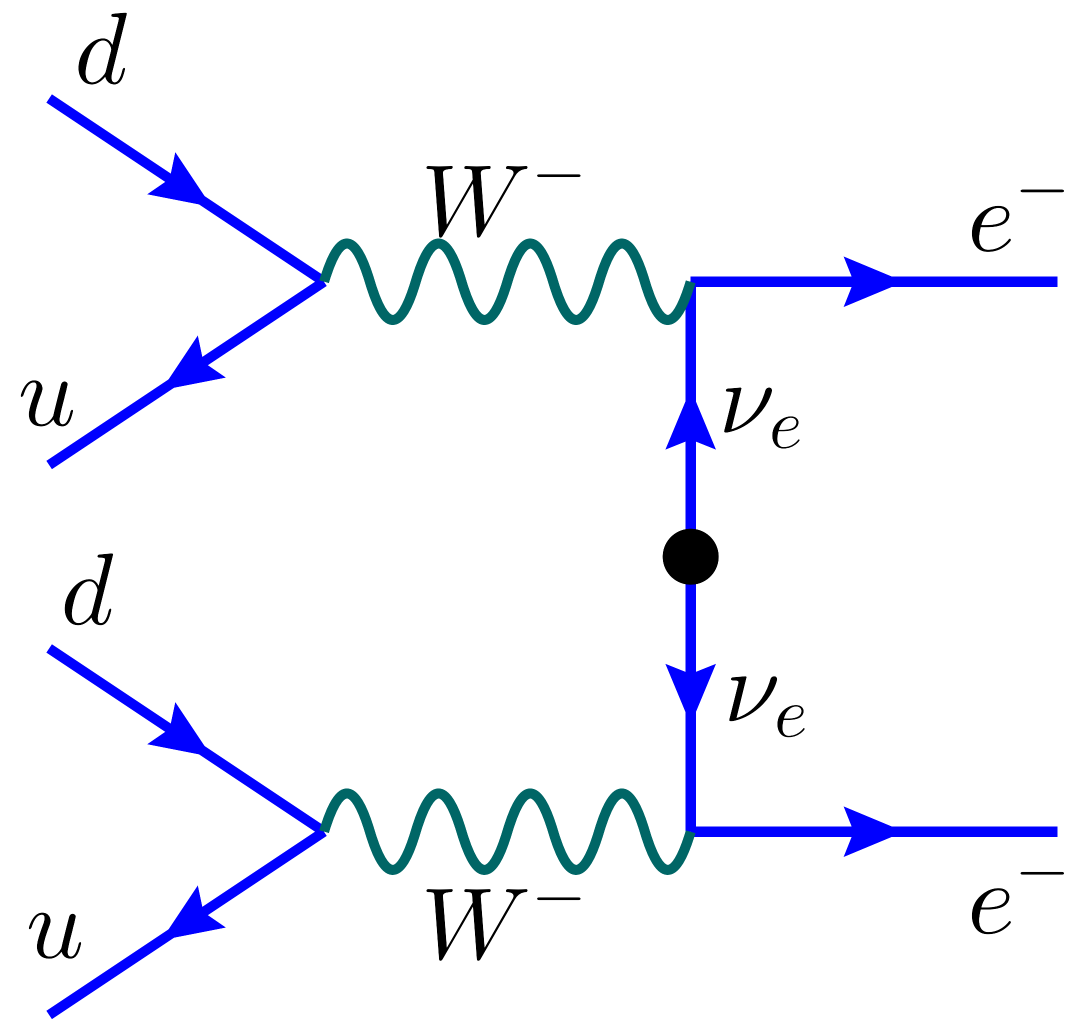
Dirac vs Majorana

- Dirac neutrinos:

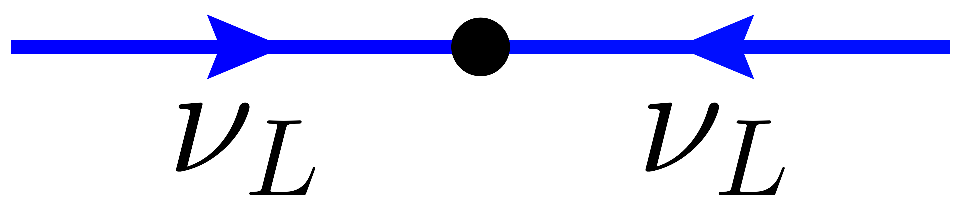
  Introduce ν_R to the SM $(SU(3)_C \times SU(2)_L \times U(1)_Y)$ allowing $\mathscr{L}_Y : y_\nu \bar{L} H \nu_R + h.c.$

  - $\nu = \nu_L + \nu_R \neq \bar{\nu}$
  - $U(1)_L$ conserved
  - $m_\nu = y_\nu \langle H \rangle \approx 0.1\text{eV},$ this means Yukawa coupling $y_\nu \sim 10^{-12}$!!
  - ν_R only couples to Higgs $\Longrightarrow$ difficult to measure



- Majorana neutrinos:

  - $\nu = \nu_L + \nu_L^c = \bar{\nu}$
  - $U(1)_L$ broken $\Longrightarrow$ neutrinoless double beta decay $0\nu\beta\beta$
  - Weinberg operator $LLHH$ generates Majorana masses

Recipe for Neutrino masses

- ν is Majorana: lowest non-renormalizable SM effective operator is the Weinberg operator

L = lepton doublet
H = Higgs doublet
y = dimensionless coupling
Λ = new $\Delta L = 2$ physics scale

$$\boxed{\frac{y}{\Lambda} LLHH} \implies \text{Majorana neutrinos} \qquad \boxed{m_\nu \sim y\frac{v^2}{\Lambda}}$$

seesaw formula

$$m_\nu \sim 0.1\text{eV}, \quad v \sim 10^2\text{GeV} \implies \Lambda \sim 10^{14}\text{GeV}$$

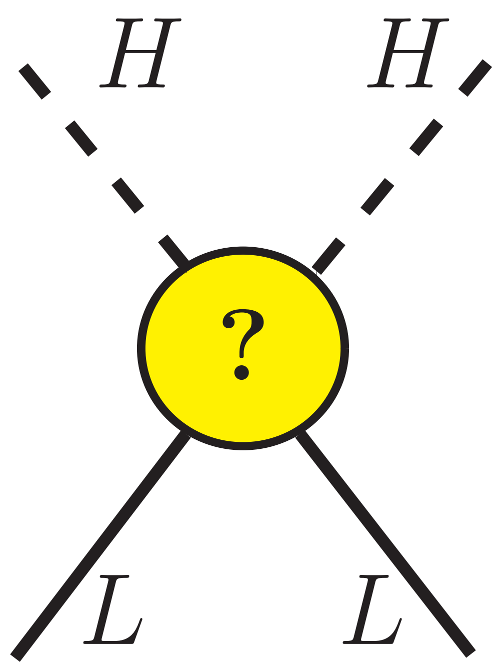
In its simplest form, seesaw scale is very high
Testability is very low

- Generalized Weinberg operator

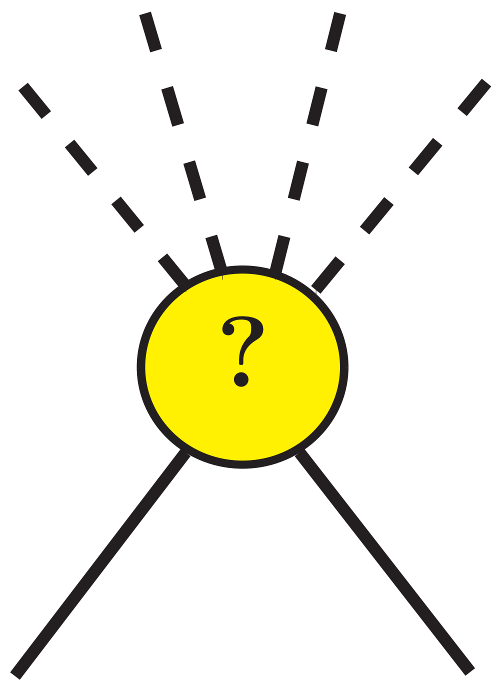
$$\frac{y}{\Lambda^{1+2n}} LLHH(H^\dagger H)^n$$

Higher the dimension lower the new physics scale

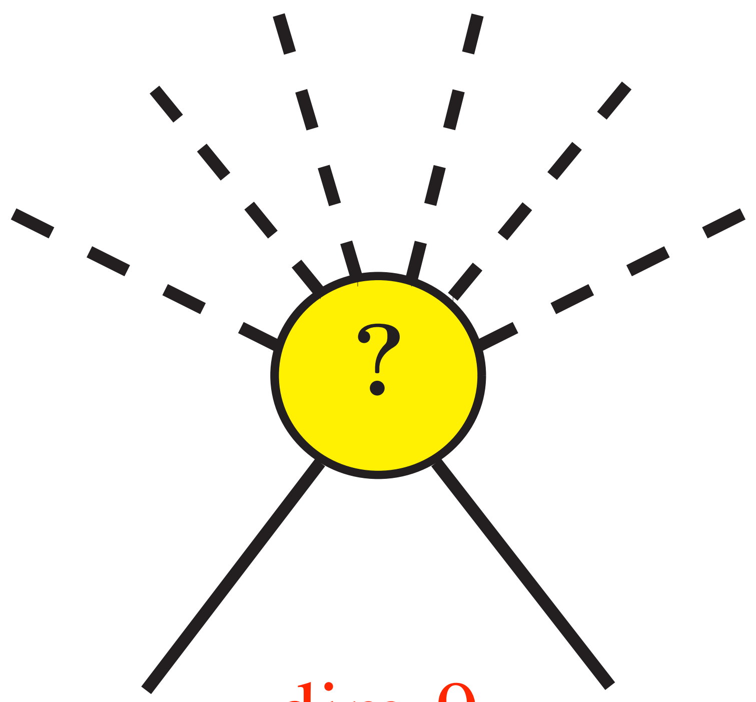
- "Open up" all such operators (UV complete) $\implies$ neutrino mass



dim-5



dim-7

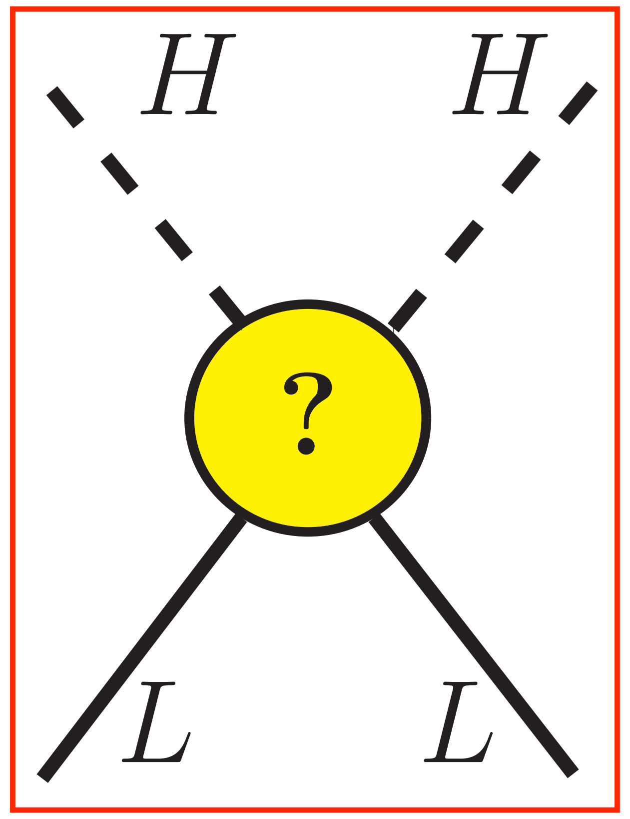


dim-9

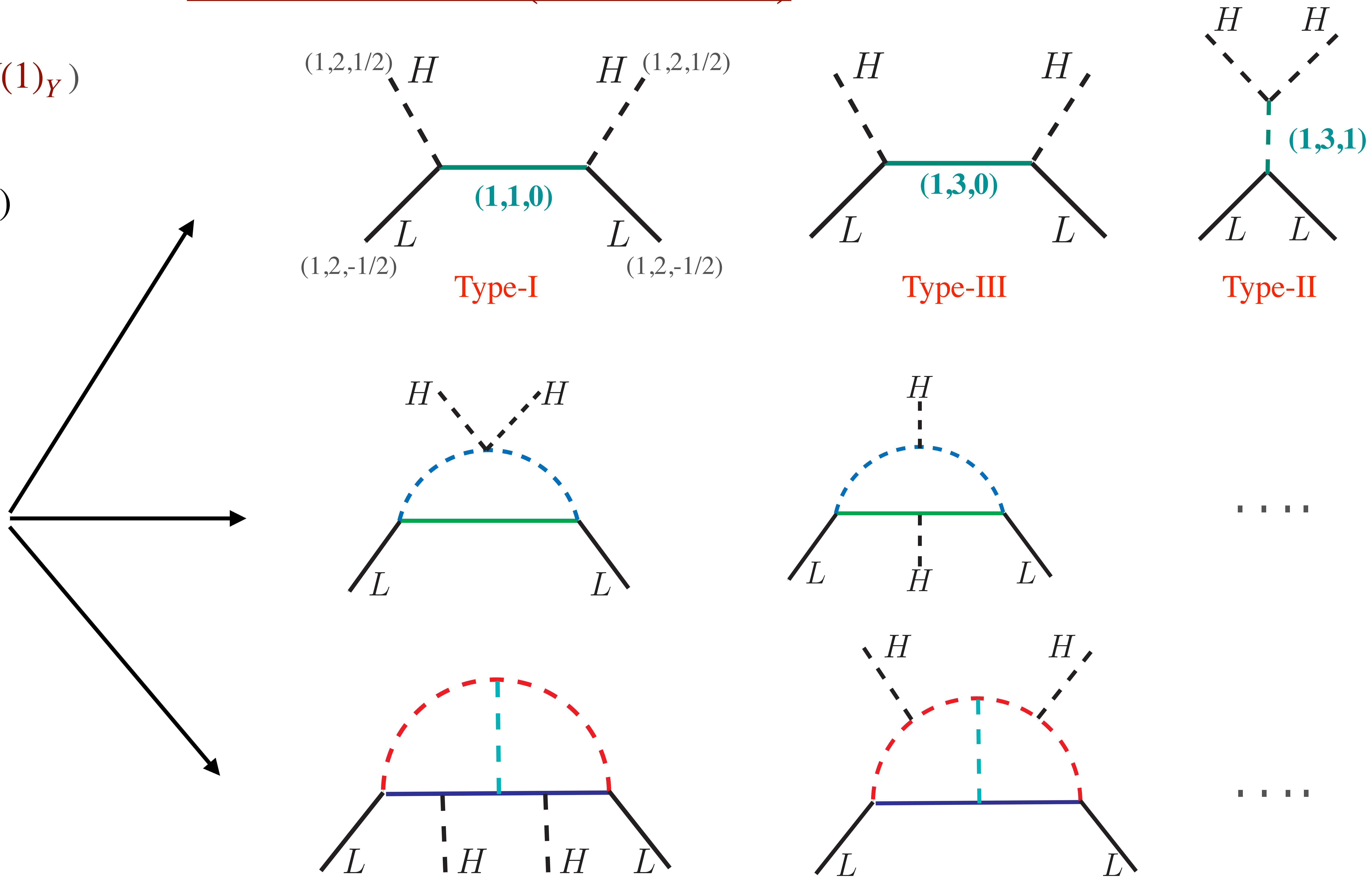
Neutrino Masses (dimension-5)

$(\,SU(3)_C\,,\,SU(2)_L\,,\,U(1)_Y\,)$

$H \sim (\,1\,,\,2\,,\,1/2)$
$L \sim (\,1\,,\,2\,,\,-1/2)$



$$\frac{y}{\Lambda} LLHH$$

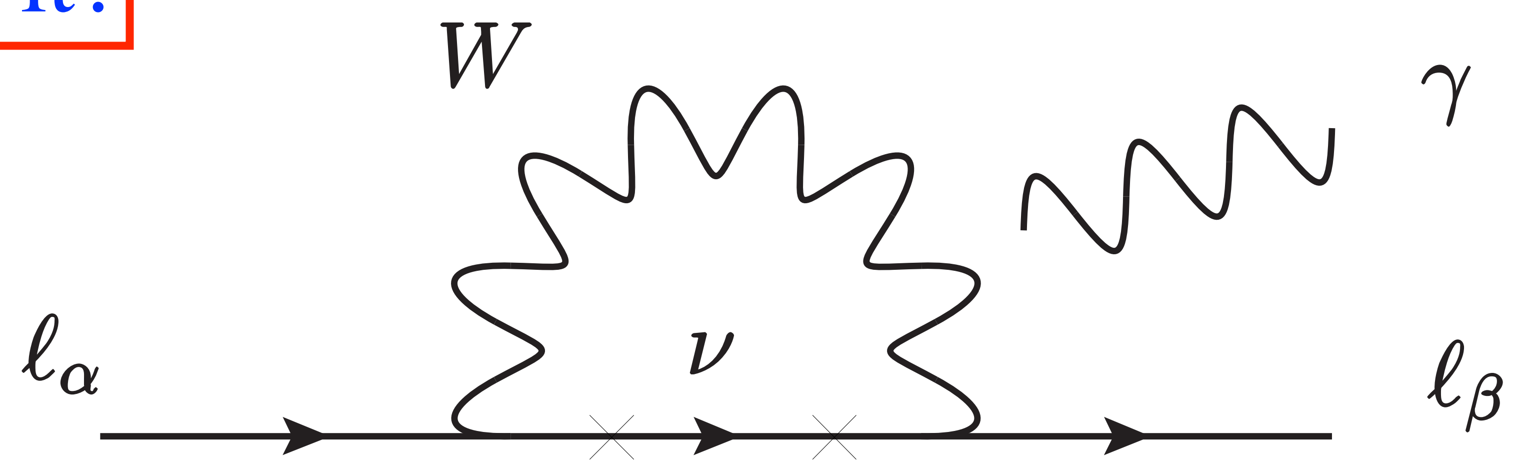


How to test such models?

$\nu_\alpha \leftrightarrow \nu_\beta$ prove that SM global symmetry $U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \Rightarrow U(1)_{L_\mu - L_\tau} \times U(1)_{L_\mu + L_\tau - 2L_e}$ is broken!

$\boxed{\text{Lepton Flavor is definitely violated, so where is it?}}$

- Dirac neutrinos: $\mathscr{L}_Y : y_\nu \bar{L} H \nu_R + h.c.$
  - $m_\nu = y_\nu \langle H \rangle \approx 0.1$ eV
  - Additional symmetry required to forbid ν_R Majorana mass
  - LFV suppressed by Dirac mass, m_ν



$$\boxed{A(\ell_\alpha \to \ell_\beta \gamma) \propto \frac{m_\nu^2}{m_W^2} < 10^{-24}!!}$$

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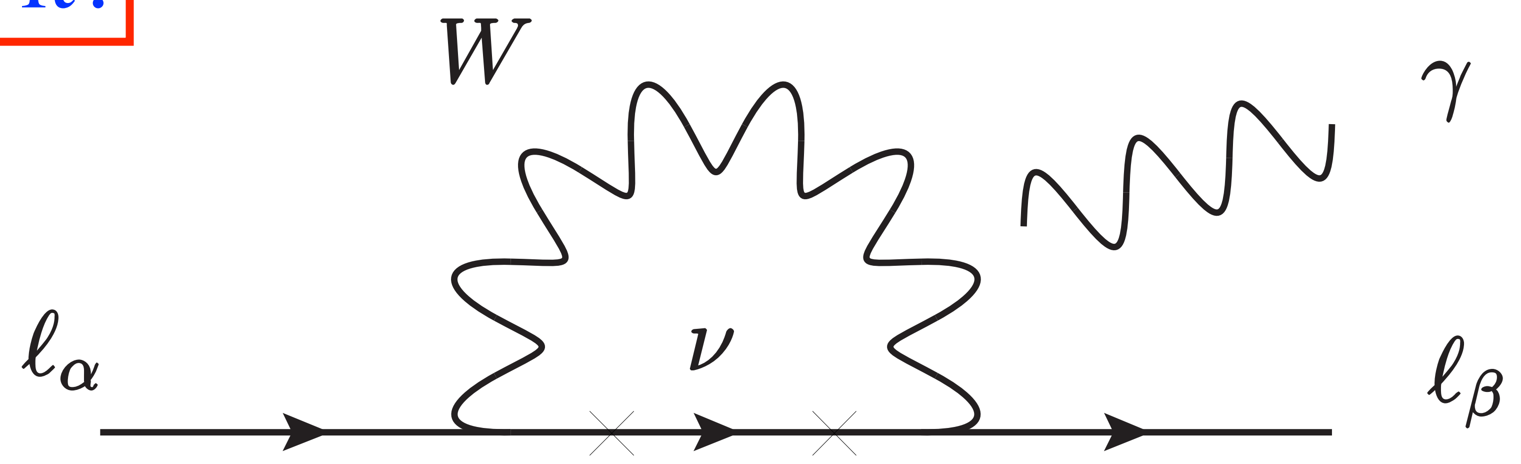

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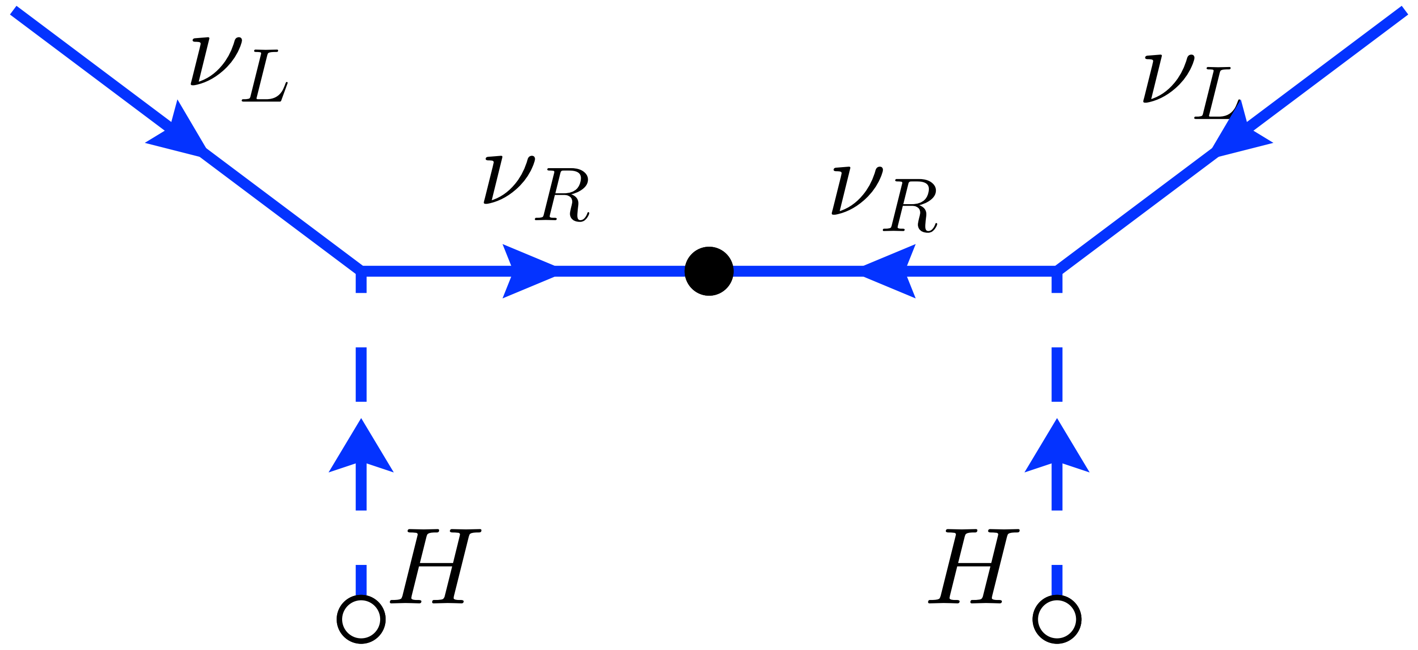
- Majorana neutrino (Seesaw mass): ν-mass is induced via Weinberg's dim-5 operator

  $\mathscr{L}_Y : 1/2\ M_R \bar{\nu}^c_R N_R + m_D \bar{\nu}_L \nu_R + h.c.$



$$\begin{array}{cc} & \overbrace{\qquad\qquad}^{m_D} \\ M = & \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & c \\ \hline 0 & 0 & 0 & 0 & \mu \\ a & b & c & \mu & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} M_R \end{array} \end{array}$$

$$\left\{0, 0, 0, \lambda_\pm \equiv \pm\sqrt{\vec{b}^2 + \mu^2}\right\}$$

$$\vec{b} \equiv \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Type I / Type III : $m_\nu \sim m_D^2 / M_R$

$A(\ell_\alpha \to \ell_\beta \gamma) \propto (m_D M_R^{-2} m_D^\dagger)_{\alpha\beta} \simeq m_\nu / M_R$

Structure in m_D can give large effect

[Coy, Frigerio, '18; Fan, **Thapa**, '23]

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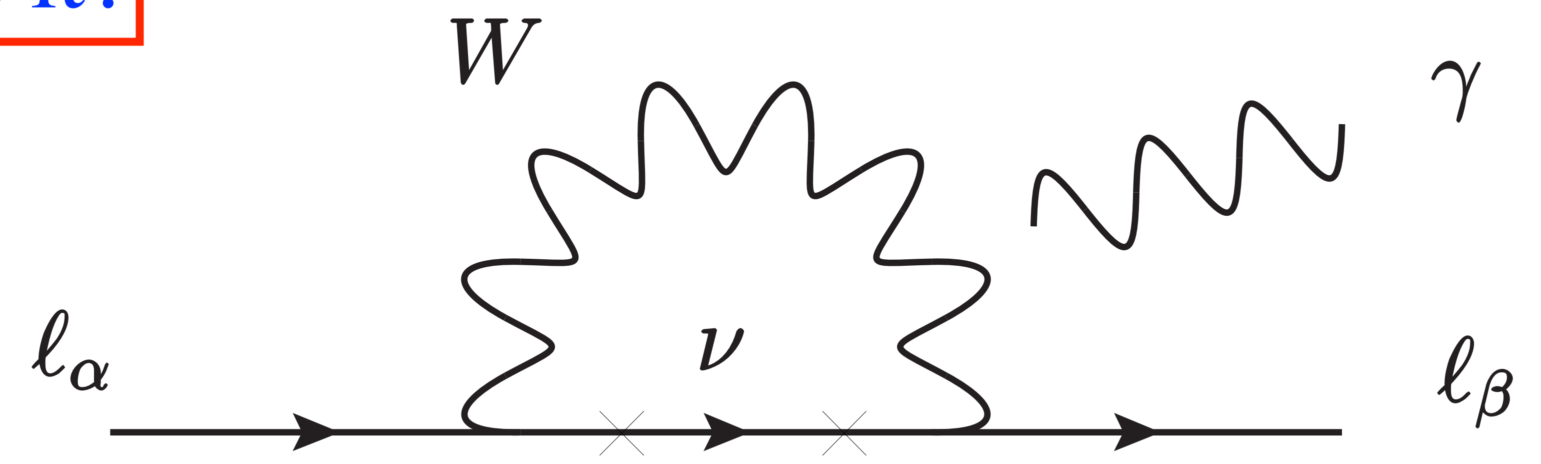

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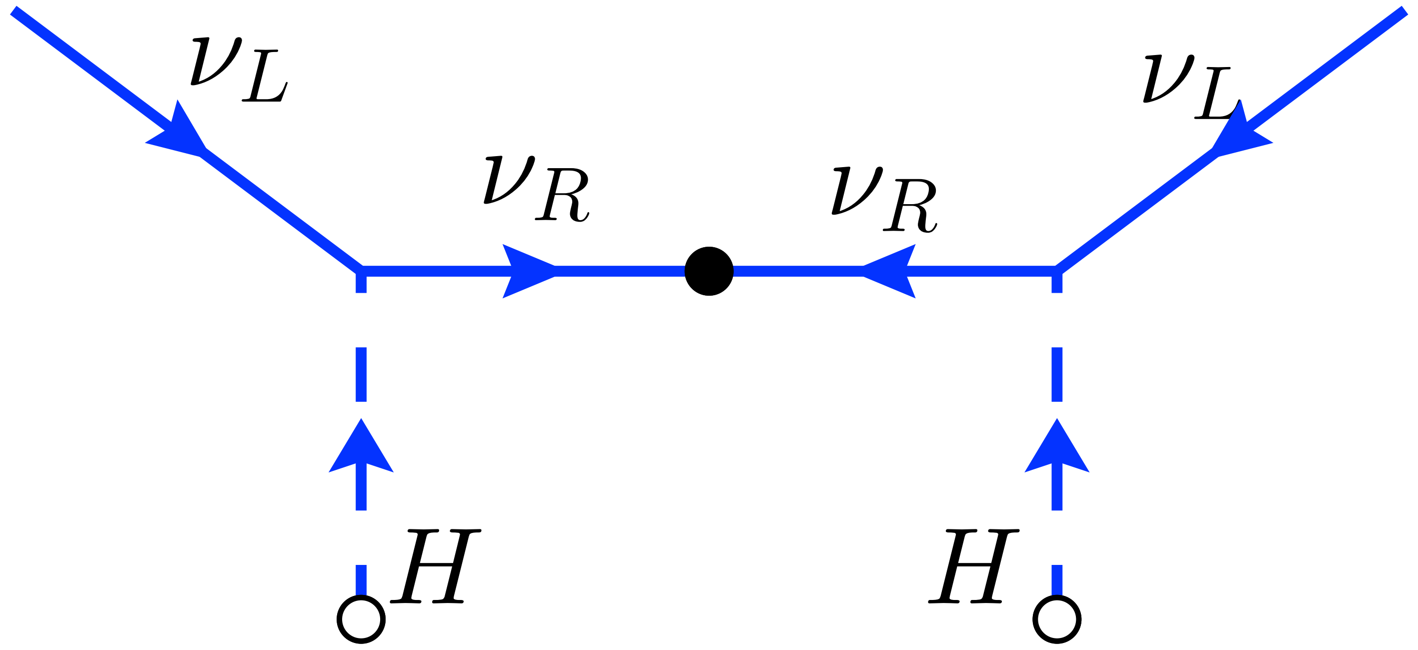

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$$M = \begin{array}{cc} \begin{array}{c} \overbrace{}^{m_D} \end{array} & \\ \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & c \\ \hline 0 & 0 & 0 & 0 & \mu \\ a & b & c & \mu & 0 \end{array}\right] & \begin{array}{c} \\ \\ \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} M_R \end{array} \end{array}$$

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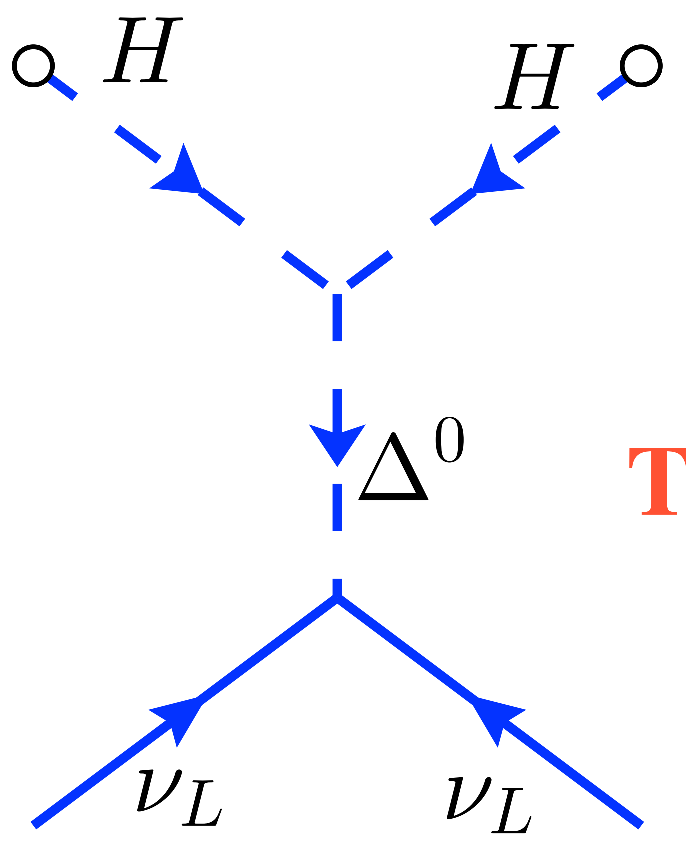
$$\vec{b} \equiv \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

[Coy, Frigerio, '18; Fan, **Thapa**, '23]

$\mathscr{L} : y \bar{L}^c \Delta L + \mu H \Delta H + h.c.$

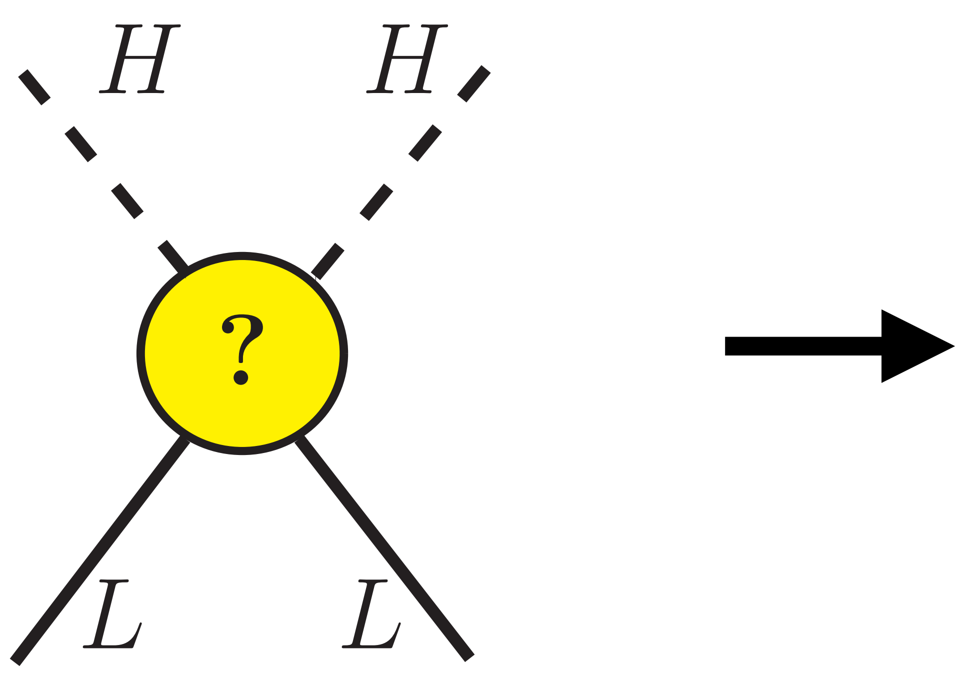


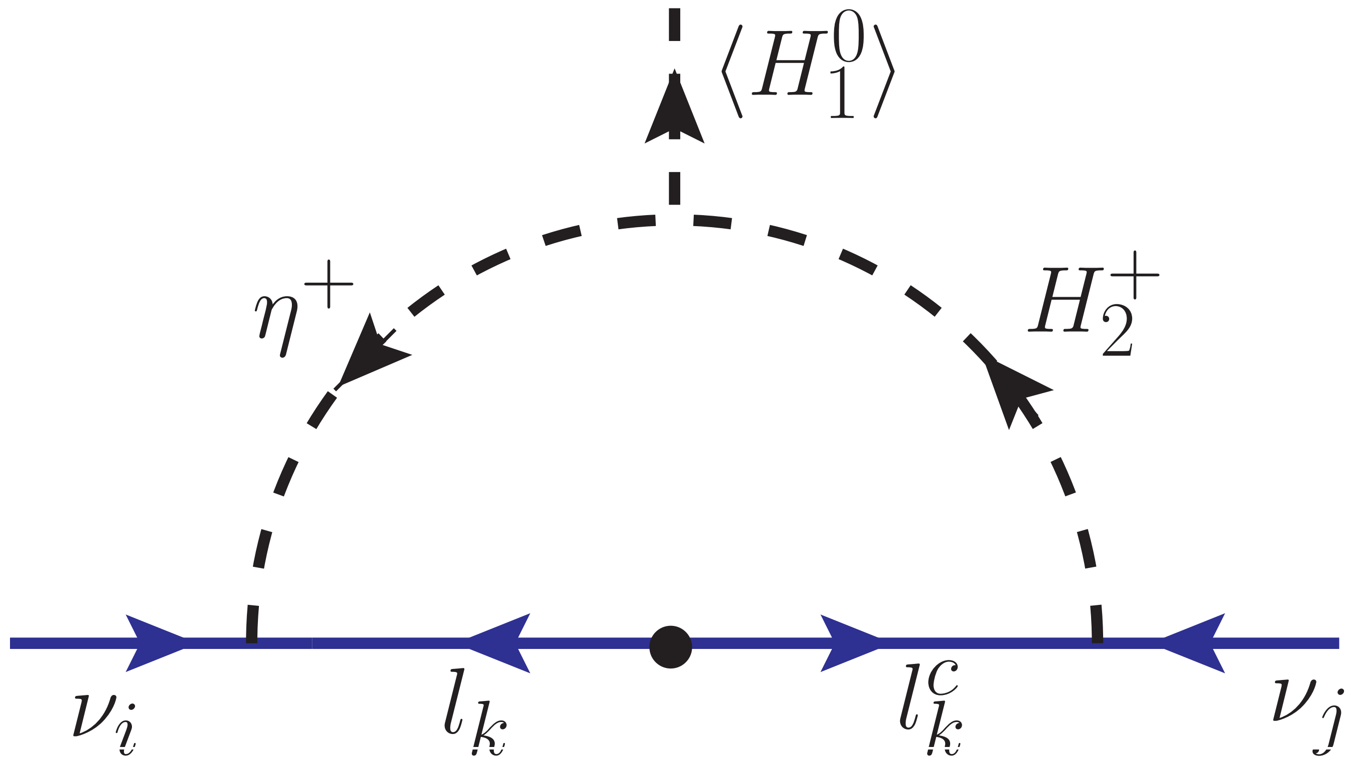
Type II : $m_\nu \simeq y \langle \Delta \rangle$

$$\boxed{\begin{array}{c} \mathrm{BR}(\tau \to \mu\gamma) \simeq 23\mathrm{BR}(\tau \to e\gamma) \simeq 3.5\mathrm{BR}(\mu \to e\gamma) \\ \text{Prediction of LFV ratios via } m_\nu \end{array}}$$

[Chakrabortty++, '12]

Example : Zee Model





$$H_1(1,2,1/2) = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG^0\right) \end{pmatrix}$$

$$H_2(1,2,1/2) = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$$

$$\eta^+(1,1,1)$$

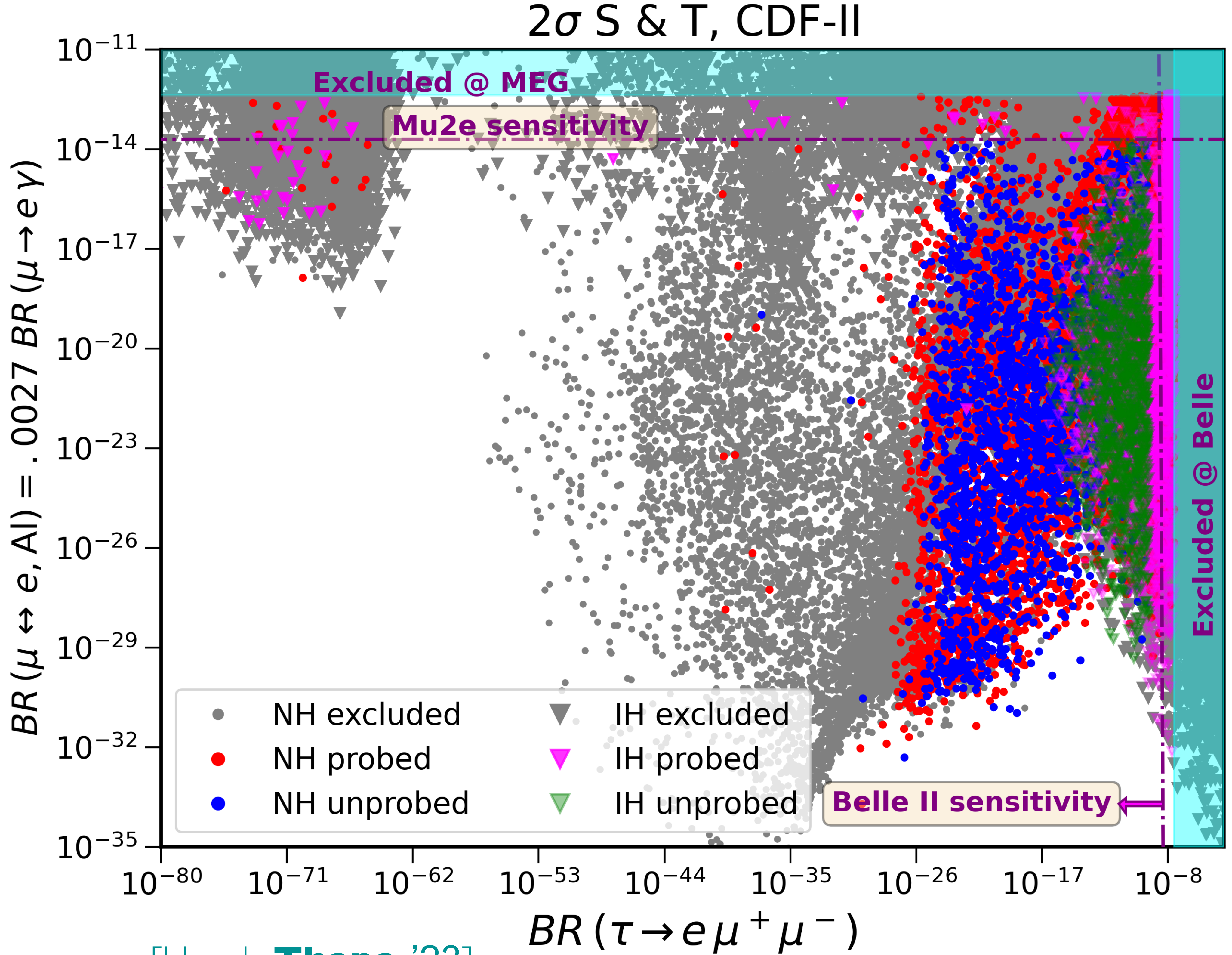
$$M_\nu = \kappa \left(f M_\ell Y + Y^T M_\ell f^T \right)$$

$$\kappa = \frac{1}{16\pi^2} \sin 2\phi \log\left(\frac{m_{h^+}^2}{m_{H^+}^2}\right)$$

- Neutrino masses are zero at tree level: ν_R is absent
- Involves exchange of two scalars leading to lepton number violation $\Longrightarrow$ Majorana Masses
- Smallness of neutrino mass is explained via loop and chiral suppression
- New physics in this framework may lie at the TeV scale if connected to $(g-2)_\mu$ $\Longrightarrow$ Prediction for LFV

Blue and Green Points:

texture zero	ordering	$\sum_j m_j$/meV	$\langle m_{\beta\beta}\rangle$/meV
$M_{ee} = 0$	normal	$\in [60, 65]$	0
$M_{ee} = 0$	inverted	–	–
$M_{\mu\mu} = 0$	normal	> 150	> 41
$M_{\mu\mu} = 0$	inverted	> 98	> 15



[Heeck, **Thapa**, '23]

- Neutrinos may well be Dirac particles just as the electron $\Longrightarrow$ $\Delta L = 0$ (Lepton number is conserved)

- Oscillation experiments cannot distinguish Dirac neutrinos from Majorana neutrinos

- If Dirac nature $\Longrightarrow$ important to understand the smallness of their masses

- Dirac leptogenesis to explain observed baryon asymmetry is an attractive feature
  [Dick, Lindner, Ratz, Wrig, '99]

- Dirac seesaw can be achieved in Mirror Models
  [Lee, Yang '56; Foot, Volkas '95; Berezhiani, Mohapatra '95, Silagadze '97]

- $\boxed{\text{Dirac neutrinos from left-right symmetric theory and GUT}}$

Dirac Neutrinos from Left-Right Symmetry

- Gauge symmetry is extended to:

$$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_X$$

- Fermion representation:

$$Q_L\ (3,2,1,1/3) = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad Q_R\ (3,1,2,1/3) = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$L_L\ (1,2,1,-1) = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad L_R\ (1,1,2,-1) = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

- Higgs sector for symmetry breaking is very simple:

$$H_L\ (1,2,1,1) = \begin{pmatrix} H_L^+ \\ H_L^0 \end{pmatrix}_L \qquad H_R\ (1,1,2,1) = \begin{pmatrix} H_R^+ \\ H_R^0 \end{pmatrix}_R$$

$$SU(2)_L \times SU(2)_R \times U(1)_X \xrightarrow{\langle H_R^0 \rangle} SU(2)_L \times U(1)_Y \xrightarrow{\langle H_L^0 \rangle} U(1)_{\text{EM}}$$

- Parity symmetry is spontaneously broken

Standard LR Higgs fileds

$$\boxed{\begin{array}{l} \Phi(1,2,2,0) \\ \Delta_L(1,3,1,0) \\ \Delta_R(1,1,3,0) \end{array}}$$

Dirac Neutrinos from Left-Right Symmetry

- Vector-like fermion introduced to realize "seesaw" for charged fermion masses

$$U\ (3,1,1,4/3), \quad D\ (3,1,1,-2/3), \quad E\ (1,1,1,-2)$$

[Davidson, Wali '87]

$$\boxed{M_F = \begin{pmatrix} 0 & y\,\kappa_L \\ y^\dagger\,\kappa_R & M \end{pmatrix} \quad \Longrightarrow \quad m_f \approx \frac{y^2 \kappa_L \kappa_R}{M}}$$

Seesaw for charged fermion masses (no seesaw for neutrinos)

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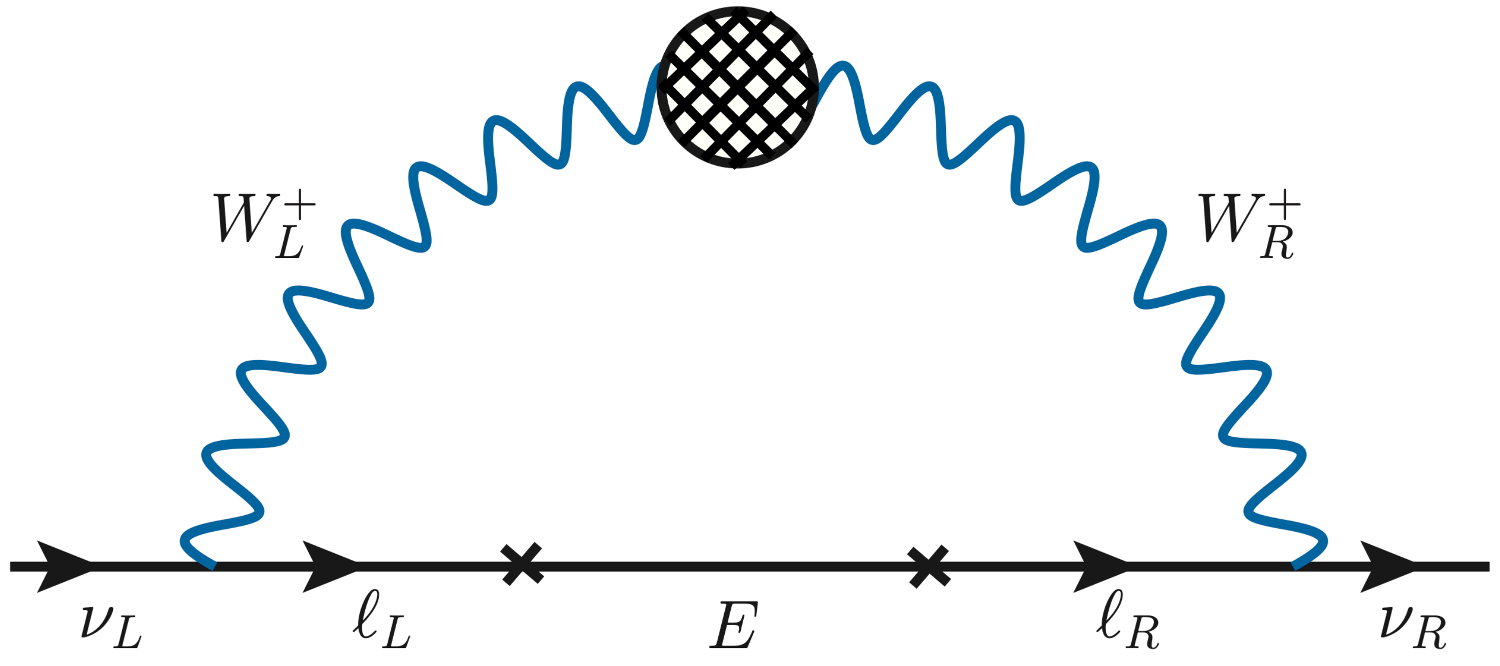
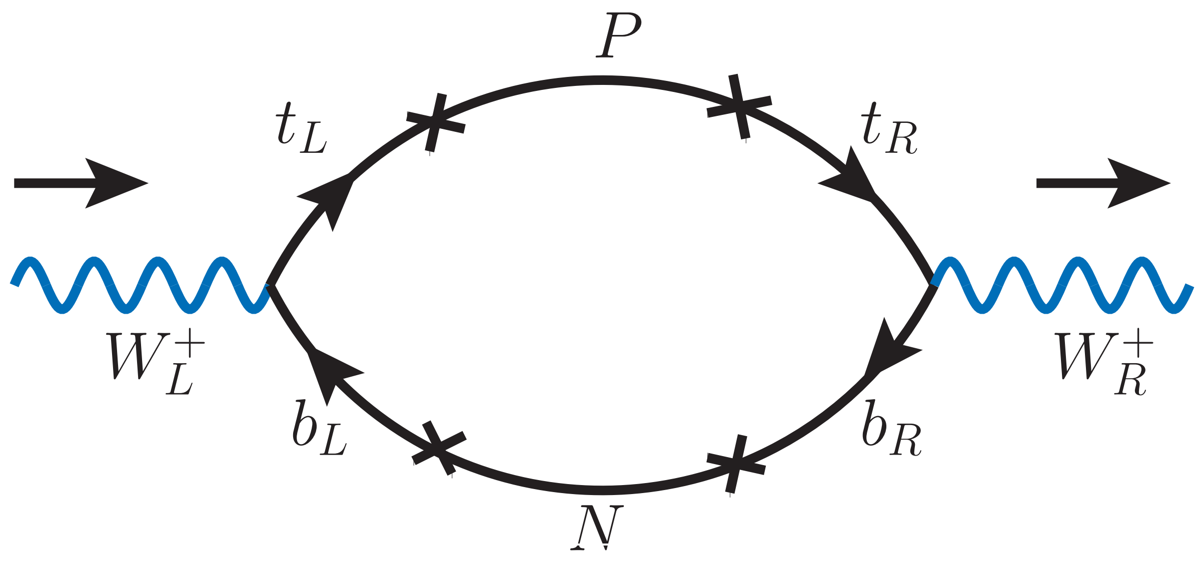
Seesaw for charged fermion masses (no seesaw for neutrinos)

- $W_L^+ \leftrightarrow W_R^+$ mixing is absent at the tree level
- $W_L^+ \leftrightarrow W_R^+$ mixing is induced at the loop level, which in turn induces two-loop Dirac masses for neutrino

[Babu, He '89]





$$\boxed{\xi \approx \frac{\alpha}{4\pi \sin^2\theta_W} \frac{m_b m_t}{M_{W_R}^2}}$$

$$M_{\nu^D} = \frac{-g^4}{2} y_t^2 y_b^2 y_\ell^2 \kappa_L^3 \kappa_R^3 \frac{r\ M_P M_N M_{E_\ell}}{M_{W_L}^2 M_{W_R}^2}\ I_{E_\ell}$$

- Flavor structure of the the two loop need to be studied to check its consistency with oscillation data

[Babu, He, Su, **Thapa** '22]

Fit to Oscillation Data

$$\boxed{M_{\nu^D} = y_\ell M_E I_E y_\ell^\dagger}$$

- Enough parameters to fit oscillation data
- Both normal and inverted ordering allowed
- Dirac CP phase is not constrained
- No neutrinoless double beta decay
- Left-right symmetry breaking is not constrained

Oscillation parameters	3σ range NuFit5.1	Model prediction			
		BP I (NH)	BP II (NH)	BP III (IH)	BP IV (IH)
$\Delta m^2_{21}(10^{-5}\ \text{eV}^2)$	6.82 - 8.04	7.42	7.38	7.35	7.35
$\Delta m^2_{23}(10^{-3}\ \text{eV}^2)$(IH)	2.410 - 2.574	-	-	2.48	2.52
$\Delta m^2_{31}(10^{-3}\ \text{eV}^2)$(NH)	2.43 - 2.593	2.49	2.51	-	-
$\sin^2\theta_{12}$	0.269 - 0.343	0.324	0.301	0.306	0.310
$\sin^2\theta_{23}$ (IH)	0.410 - 0.613	-	-	0.510	0.550
$\sin^2\theta_{23}$ (NH)	0.408 - 0.603	0.491	0.533	-	-
$\sin^2\theta_{13}$ (IH)	0.02055 - 0.02457	-	-	0.0219	0.0213
$\sin^2\theta_{13}$(NH)	0.02060 - 0.02435	0.0234	0.0213	-	-
δ_{CP} (IH)	192 - 361	-	-	236°	279°
δ_{CP} (NH)	105 - 405	199°	280°	-	-
m_{light} (10^{-3}) eV		0.66	2.04	14.1	8.50
M_{E_1}/M_{W_R}		917	45.5	1936	1990
M_{E_2}/M_{W_R}		0.650	0.43	0.12	0.11
M_{E_3}/M_{W_R}		0.019	0.029	0.015	0.012

[Babu, He, Su, **Thapa** 2205.09127]

- Universal left-right symmetric theory can solve strong CP problem without the need for an axion
- Model can be tested through N_{eff}

Testing Dirac Neutrinos with N_{eff}

- CMB is sensitive to extra radiation density arising from new extra degrees of freedom that were in thermal equilibrium with the SM plasma
- ν_R (ultra-light new particles, new degrees of freedom) couples to other particles and contributes to additional radiation density in early universe.
- The effect of such light particles is parameterized as ΔN_{eff} and is measured in units of extra neutrino degrees of freedom
- Dirac neutrino modes of this type will modify N_{eff} by about 0.14

$$\Delta N_{\text{eff}} \simeq 0.027 \left(\frac{106.75}{g_\star (T_{\text{dec}})} \right)^{4/3} g_{\text{eff}}$$

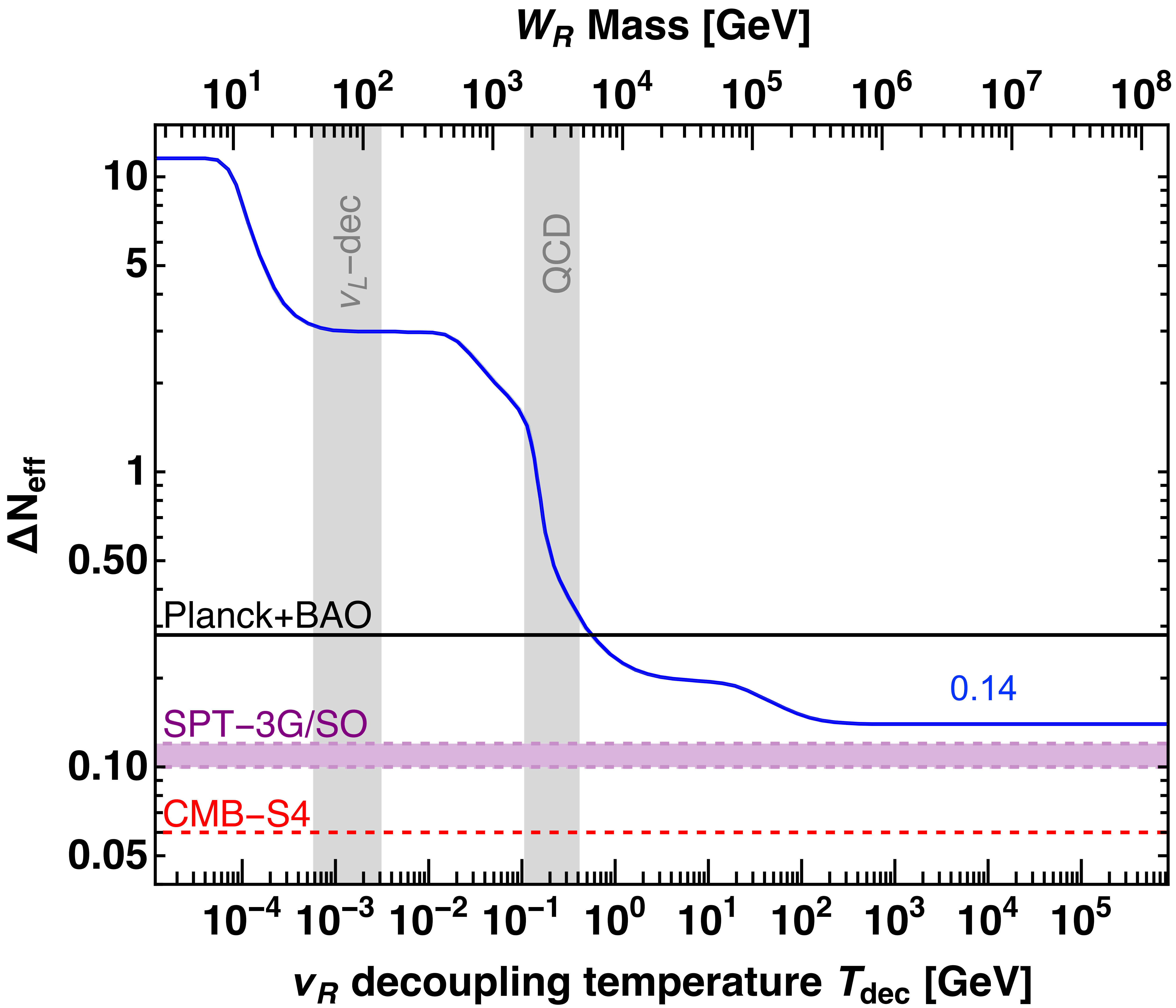
$$g_{\text{eff}} = (7/8) \times (2) \times (3) = 21/4$$

$$G_F^2 \left(\frac{M_{W_L}}{M_{W_R}} \right)^4 T_{\text{dec}}^5 \approx \sqrt{g_\star (T_{\text{dec}})} \frac{T_{\text{dec}}^2}{M_{\text{Pl}}}$$

$$T_{\text{dec}} \simeq 400\text{MeV} \left(\frac{g_* (T_{\text{dec}})}{70} \right)^{1/6} \left(\frac{M_{W_R}}{5\text{TeV}} \right)^{4/3}$$

Dirac Neutrino in Cosmology

- In SM $N_{\rm eff} \simeq 3$
- Improvement on $\Delta N_{\rm eff}$ in CMB-S4
- Valid for 3 ν_R were in thermal equilibrium with SM
- This gives strong constraints for any (e.g., LR model) Dirac neutrino mass model
- Planck+BAO sets a lower limit of 7 TeV on W_R mass



[Heeck, Abazajian '19; Babu, He, Su, **Thapa** '22]

- Can we embed this version of the LR model into GUT while preserving the Dirac nature for neutrinos?
- Any predictions in neutrino oscillation (normal vs inverted, Dirac CP, …)?
- Can we still solve the strong CP problem?
- What else can the model do?

Embedding in $SU(5)_L \times SU(5)_R$

- The fermion spectrum of the model has a natural embedding in $SU(5)_L \times SU(5)_R$ unification

- All left-handed (right-handed) fermions of the SM fit into $\mathbf{10} + \bar{\mathbf{5}}$ of $SU(5)_L$ $(SU(5)_R)$

- The remaining vector-like quarks and leptons fill rest of the multiples

$$\psi_{L,R} = \left(\begin{array}{c} \left.\begin{matrix} D_1^c \\ D_2^c \\ D_3^c \end{matrix}\right\} D \\ \left.\begin{matrix} e \\ -\nu \end{matrix}\right\} L_{L,R} \end{array}\right)_{L,R} \qquad \chi_{L,R} = \frac{1}{\sqrt{2}} \begin{pmatrix} \overbrace{\begin{matrix} 0 & U_3^c & -U_2^c \\ -U_3^c & 0 & U_1^c \\ U_2^c & -U_1^c & 0 \end{matrix}}^{U} & \overbrace{\begin{matrix} u_1 & d_1 \\ u_2 & d_2 \\ u_3 & d_3 \end{matrix}}^{Q} \\ \begin{matrix} -u_1 & -u_2 & -u_3 \\ -d_1 & -d_2 & -d_3 \end{matrix} & \underbrace{\begin{matrix} 0 & E^c \\ -E^c & 0 \end{matrix}}_{E} \end{pmatrix}_{L,R}$$

- Parity can be imposed under which $\psi_L \leftrightarrow \psi_R$ and $\chi_L \leftrightarrow \chi_R$

GUT Symmetry Breaking and Gauge Coupling Unification

- With the SM particles, we obtain the following beta function coefficients with properly normalized gauge couplings: $b_1 = \frac{41}{26}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -\frac{7}{2}$

- If $SU(5) \times SU(5)$ directly break to the SM group, where g_i meet at a single value. $\alpha_{\text{GUT}} = 2\,\alpha_3 = \alpha_2 = \frac{13}{3}\alpha_1$

  $\Longrightarrow$ $\sin^2\theta_W = 3/16$

  $\Longrightarrow$ Cannot reconcile value measured at EW scale

  $\Longrightarrow$ An intermediate symmetry is needed

$$\sin^2\theta_W(m_t) = \frac{3}{16}\left[1 + \frac{\alpha}{6\pi}\left\{-\frac{185}{3}\log\frac{M_G}{m_t}\right\}\right]$$

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To break $SU(5)_L \times SU(5)_R$ spontaneously to $SU(3)_c \times U(1)_{\text{em}}$ we choose the following Higgs multiplets

$\{\Sigma_L(\mathbf{75},1) + \Sigma_R(1,\mathbf{75})\},$

Why not $(\mathbf{24},1)$+$(1,\mathbf{24})$?

> allows $(\mathbf{24},1)H_R^\dagger \Phi H_L$ and $(\mathbf{24},1)\eta^\dagger\Phi\Phi$ that spoils strong CP solution

$\{H_L(\mathbf{5},1) + H_R(1,\mathbf{5})\}, \quad \Phi(\bar{\mathbf{5}},\mathbf{5}),$

Required for fermion mass generation

Required for symmetry breaking

$\eta(\overline{\mathbf{15}},\mathbf{15})$

Required for gauge coupling unification

Why not $(\overline{\mathbf{10}},\mathbf{10})$?

> allows rapid proton decay

> spoils strong CP

> makes g_{5R} nonperturbative

[Babu, Mohapatra, **Thapa**, '24]

GUT Symmetry Breaking and Gauge Coupling Unification

$$
\begin{array}{c}
SU(5)_L \times SU(5)_R \\
\downarrow M_G \sim \langle \Sigma_L \rangle \\
SU(3)_{CL} \times SU(2)_L \times U(1)_L \times SU(5)_R \\
\downarrow M_I \sim \langle \Phi \rangle, \langle H_R \rangle \\
SU(3)_C \times SU(2)_L \times U(1)_Y \\
\downarrow M_W \sim \langle H_L \rangle \\
SU(3)_C \times U(1)_{\rm em}
\end{array}
$$

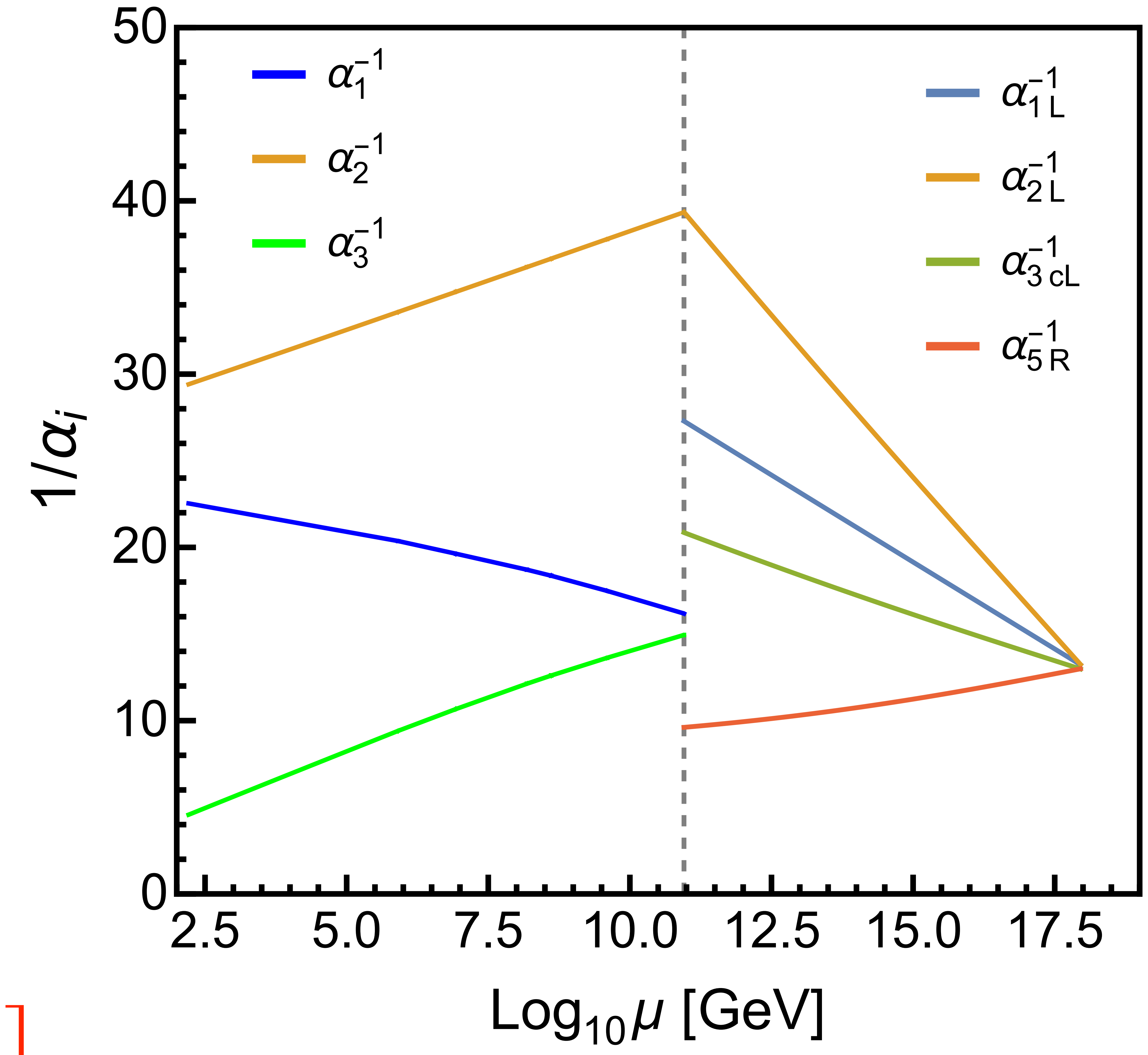
- The evolution of the gauge couplings are governed by the following RGEs

$$\boxed{16\pi^2 \frac{dg_i}{dt} = g_i^3 b_i + \frac{g_i^3}{16\pi^2}\left[\sum_j b_{ij} g_j^2 - \sum_k C_{ik} \mathrm{Tr}\left(Y_k^\dagger Y_k\right)\right]}$$

- $\sin^2\theta_W$ at one-loop accuracy (ignoring threshold effect from VLF)

$$\sin^2\theta_W(m_t) = \frac{3}{16}\left[1 + \frac{\alpha}{6\pi}\left\{-\frac{185}{3}\log\frac{M_I}{m_t} + (46 + \boxed{39)}\log\frac{M_G}{M_I}\right\}\right]$$

$(\bar{\mathbf{3}}, \mathbf{2}, -1/6, \mathbf{15}) \supset (\overline{\mathbf{15}}, \mathbf{15})$



$$\boxed{\begin{array}{c} M_I = 9.02 \times 10^{10}\ \mathrm{GeV} \\ M_G = 8.0 \times 10^{17}\ \mathrm{GeV} \\ \alpha_G^{-1} = 13.18 \end{array}}$$

Fermion Mass Generation

$$-\mathscr{L}_{\text{Yuk}} = \frac{(Y_u^\star)_{ij}}{4}\epsilon_{\alpha\beta\gamma\delta\rho}\left\{\chi_{Li}^{\alpha\beta}\chi_{Lj}^{\gamma\delta}H_L^\rho + \chi_{Ri}^{\alpha\beta}\chi_{Rj}^{\gamma\delta}H_R^\rho\right\} + \sqrt{2}\,(Y_\ell^\star)_{ij}\left\{\psi_{Li\alpha}\chi_{Lj}^{\alpha\beta}H_{L\beta}^\star + \psi_{Ri\alpha}\chi_{Rj}^{\alpha\beta}H_{R\beta}^\star\right\} + (Y_D^\star)_{ij}\,\overline{\psi}_{Li}^{\alpha}\,\Phi_\alpha^\beta\,\psi_{Rj\beta}$$

- After spontaneous symmetry breaking, the masses of fermions read as

$$M_u = \begin{pmatrix} 0 & Y_u\,\kappa_L \\ Y_u^\dagger\,\kappa_R & \boxed{0} \end{pmatrix}, \qquad M_\ell = \begin{pmatrix} 0 & Y_\ell\,\kappa_L \\ Y_\ell^\dagger\,\kappa_R & \boxed{0} \end{pmatrix}, \qquad M_d = \begin{pmatrix} 0 & Y_\ell^T\,\kappa_L \\ Y_\ell^\star\,\kappa_R & Y_D v_\phi \end{pmatrix}$$

Crucial for the model to be compatible with proton decay with $SU(5)_R$ intermediate symmetry.

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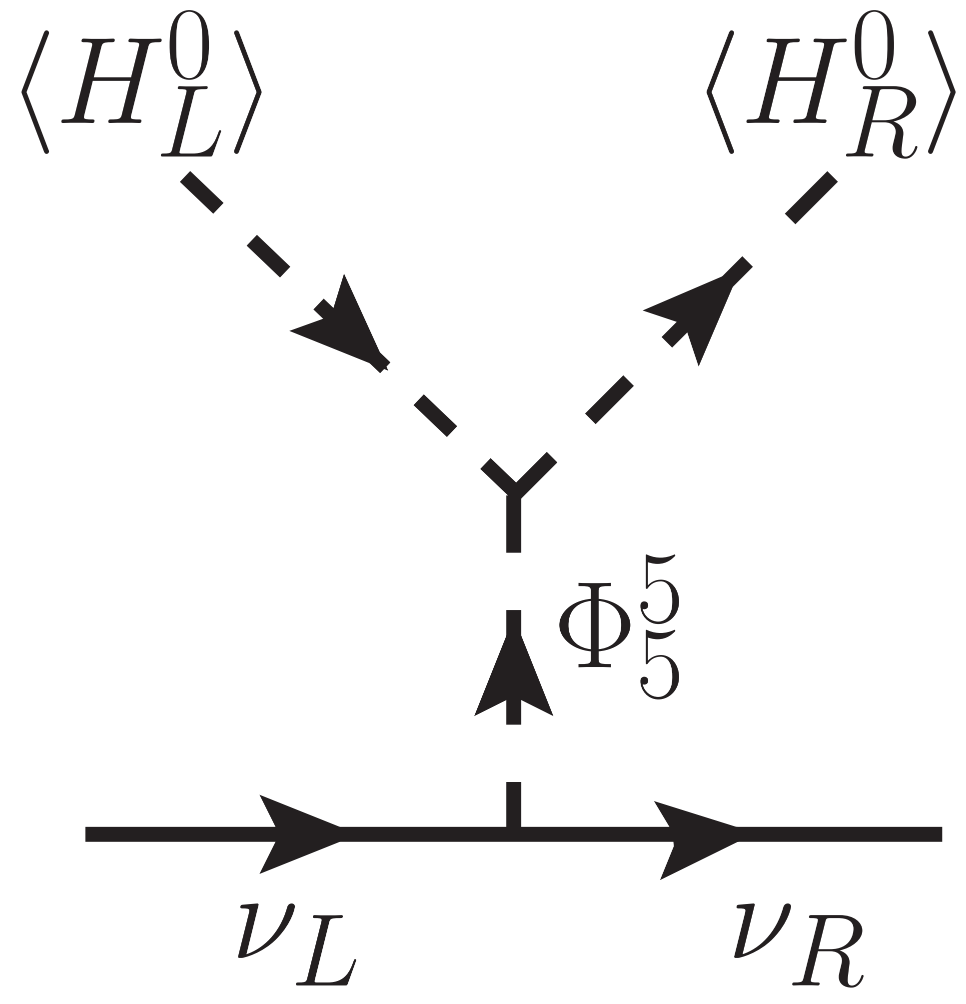
Crucial for the model to be compatible with proton decay with $SU(5)_R$ intermediate symmetry.

- Small Dirac neutrinos masses are induced naturally at the tree level via type-II Dirac seesaw



$$\boxed{\mathscr{L}_{\nu-\rm mass}^{\rm Dirac} = \frac{\overline{\nu}_L\nu_R\langle H_L^0\rangle\langle H_R^0\rangle}{M_G} \implies Y_\nu^{\rm Dirac} \sim \frac{M_I}{M_G} \approx 10^{-7}}$$

Majorana mass for ν_R is forbidden by unbroken $B-L$ symmetry

Preditions for Neutrino Oscillations

- In the basis where Y_u and Y_ℓ are diagonal, down-type quark mass matrix M_d read as

$$M_u = \begin{pmatrix} 0 & \hat{M}_u \kappa_L \\ \hat{M}_u \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix}, \qquad M_\ell = \begin{pmatrix} 0 & \hat{M}_\ell \kappa_L \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix},$$

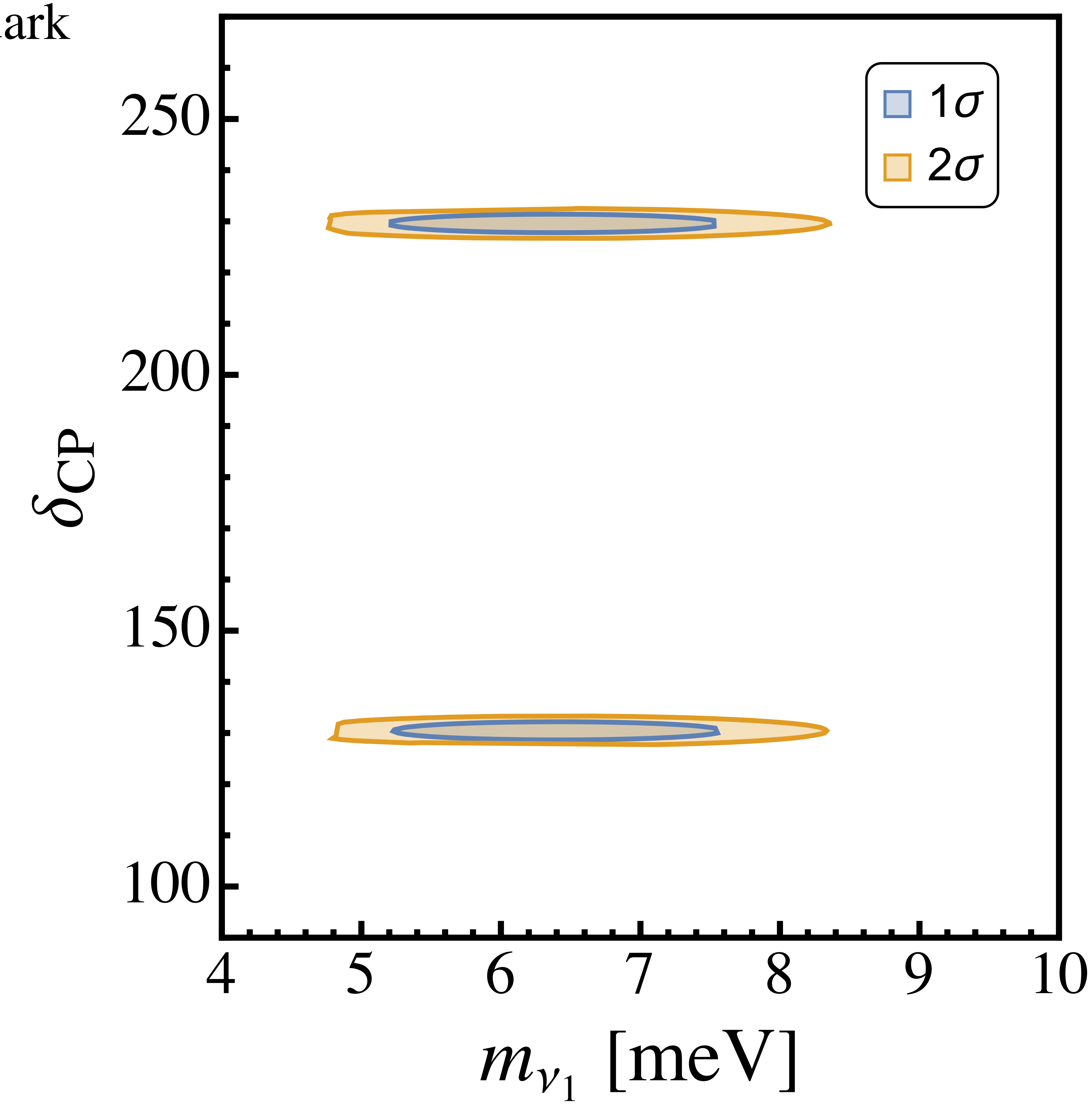
$$M_d = \begin{pmatrix} 0 & \hat{M}_\ell \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & \frac{v_\phi}{v_\nu} U^*_{\mathrm{PMNS}} \hat{M}_\nu U^T_{\mathrm{PMNS}} \end{pmatrix}$$

- Only one parameter in M_d to fit three light down-quark masses

$\Longrightarrow$ Predicts δ_{CP} and lightest neutrino mass m_{ν_1}

$$\boxed{\begin{array}{c} \delta_{CP} = (130.4 \pm 1.2)^\circ \text{ or } (229.6 \pm 1.2)^\circ \\ m_{\nu_1} = (4.8 - 8.4)\ \mathrm{meV} \end{array}}$$

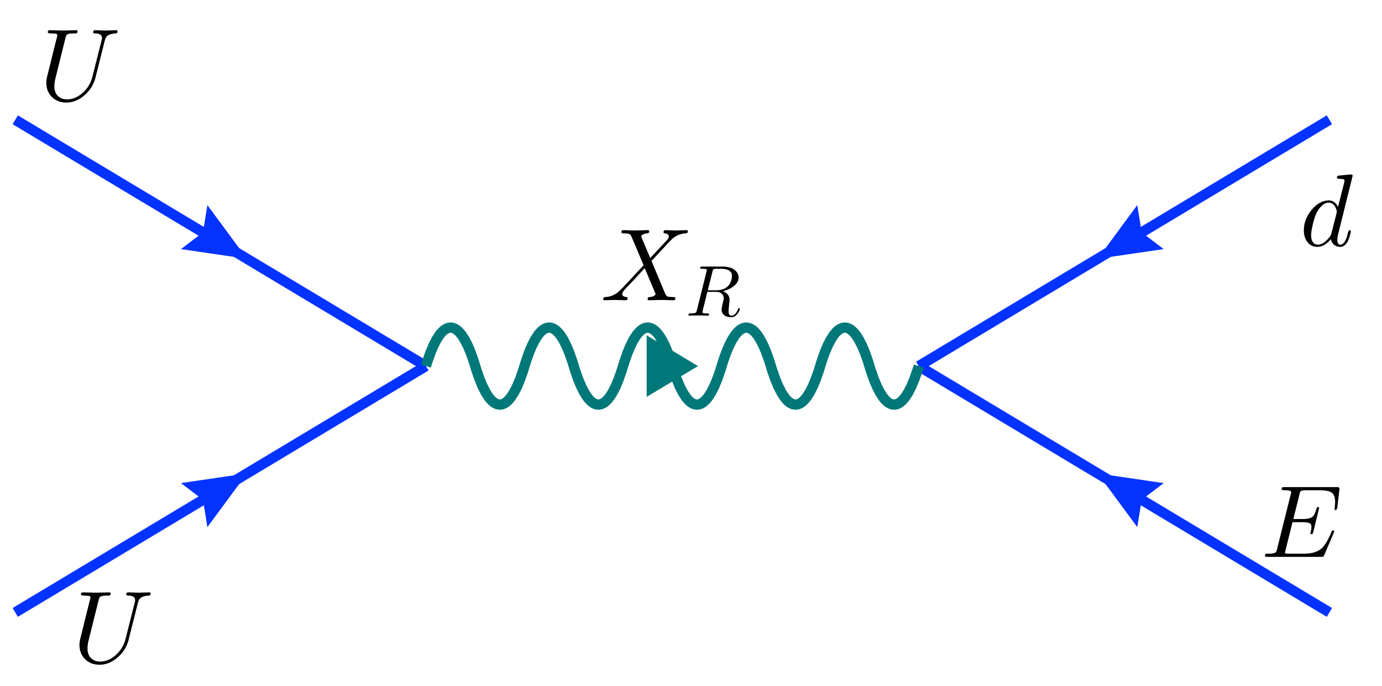
$\Longrightarrow$ Only normal hierarchy

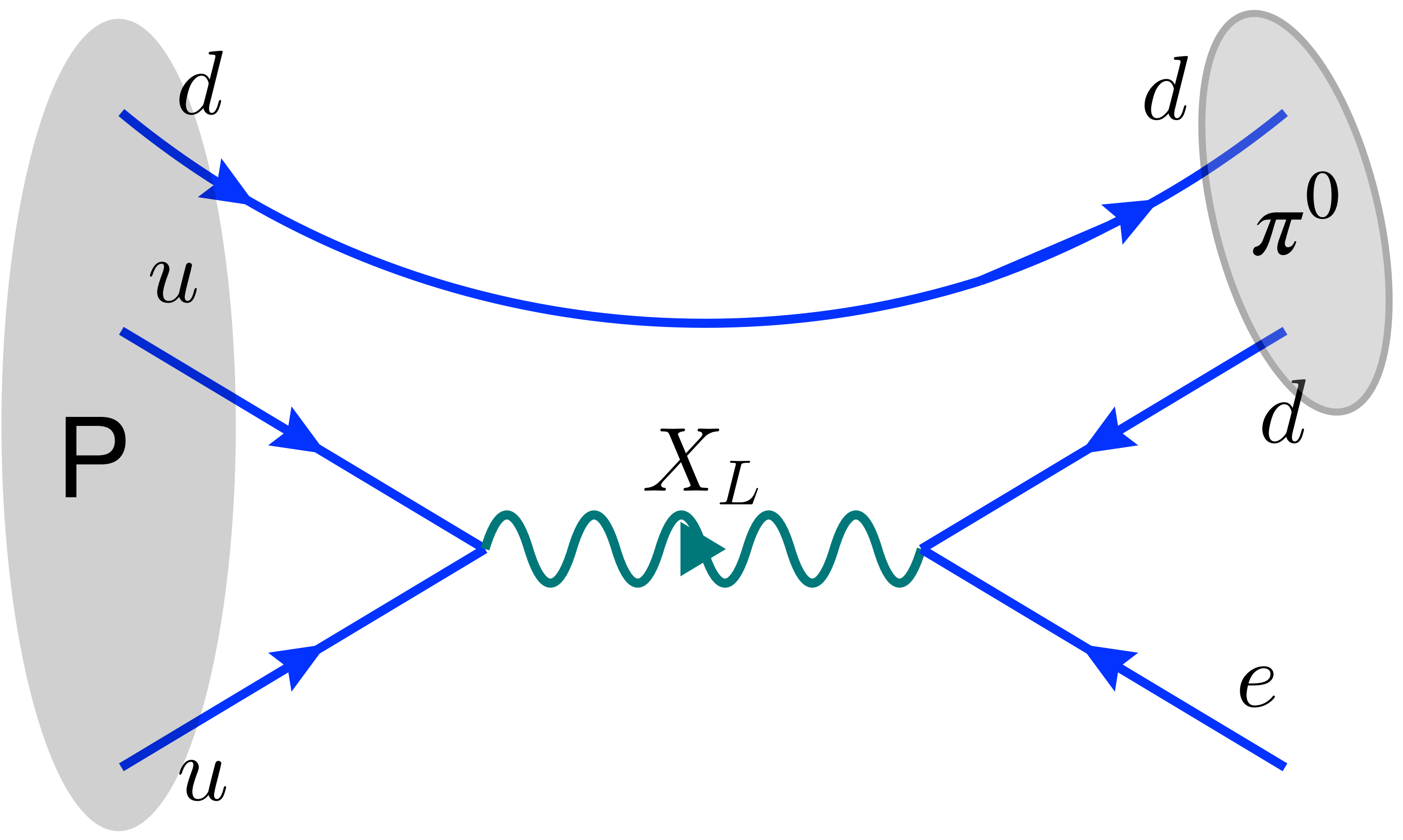


[Babu, Mohapatra, **Thapa**, '24]

Proton Decay

- Gauge bosons of $SU(5)_R$ with masses $M_{X_R,Y_R} \simeq M_I \sim 10^{11}$ GeV do not lead to proton decay owing to the structure of the zeros in (2,2) blocks of M_u and M_ℓ
- These couplings involve at least one heavy field
- Same is true with $H_R(1,\mathbf{5})$ Higgs field which has mass of order M_I





- B-violating interactions of X_L and Y_L gauge bosons of $SU(5)_L$ with masses of order $M_G = (7 \times 10^{16} - 8 \times 10^{17})$ GeV mediate proton decay.
- The leading decay mode of proton is $p \to e^+\pi^0$ with lifetime $\tau_p \approx (10^{38} - 10^{42})$ years. (Well beyond the reach of forthcoming experiments like JUNO, Hyperkamiokande, and DUNE)

Parity Solves the Strong CP Problem

quark mass matrix

$$\bar{\theta} = \boxed{\theta} + \text{Arg Det}\,\boxed{[M_Q]}$$

$M_Q \propto$ parity breaking VEVs, need to make sure the determinant is real.

$G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \propto \vec{E}_{\text{color}} \cdot \vec{B}_{\text{color}}$

θ is odd under parity, therefore in parity symmetric theory it would vanish.

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- $SU(5)_L \times SU(5)_R$ with parity has the following quark mass matrices

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$\Longrightarrow$ Det $[M_Q]$ = Det $[M_u M_d] \equiv$ Real

$\Longrightarrow$ $\bar{\theta} = 0$ at tree level

All the Higgs potential parameters with the fields [$\{\Sigma_L(\mathbf{75},1) + \Sigma_R(1,\mathbf{75})\}$, $\{H_L(\mathbf{5},1) + H_R(1,\mathbf{5})\}$, $\Phi(\bar{\mathbf{5}},\mathbf{5})$, $\eta(\overline{\mathbf{15}},\mathbf{15})$] are real with parity. Thus CP conserving vacuum is admitted, where all the VEVs are real.

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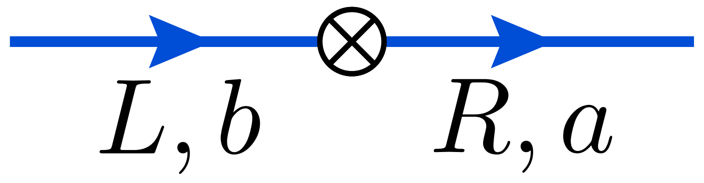
- Quantum corrections would in general induce $\bar{\theta} \neq 0$, but this may be within experimentally allowed range $\bar{\theta} \leq 1.19 \times 10^{-10}$ arising from neutron EDM limits.

Vanishing of one loop $\bar{\theta}$ contributions

- Convenient to work in the flavor basis, where the mass matrices M_u and M_d are treated as part of the interaction Lagrangian.

$\Longrightarrow$ need to sum all possible chirality flip in the propagator



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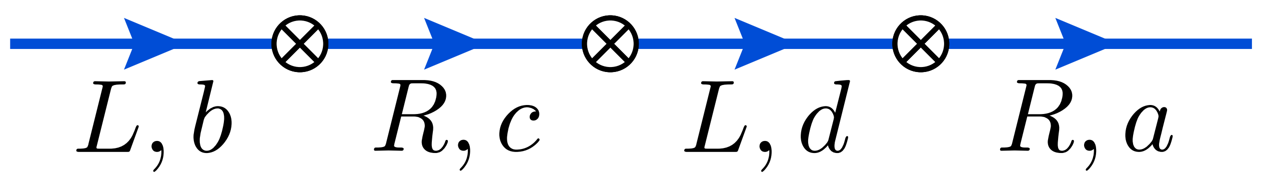

$$+ \quad \ldots\ldots = \bar{f}_R \left(M_d^\dagger \frac{k^2}{k^2 - M_d M_d^\dagger} \right) f_L \qquad f_{L,R} = \begin{pmatrix} d \\ D \end{pmatrix}_{L,R}$$

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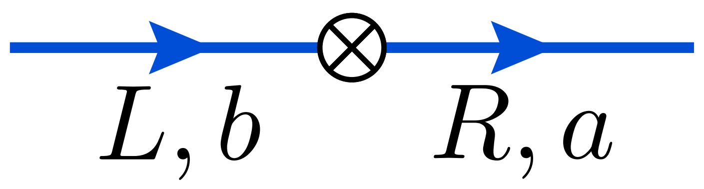

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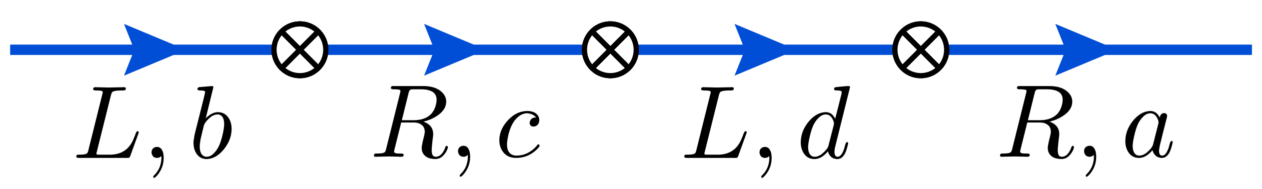

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- Loop-corrected quark mass matrix

tree level quark mass for $q = u, d$
where Arg Det $[M_q^{(0)}] = 0$

$C = C_1 + C_2 + \ldots.$ contribution from 1-loop, 2-loop, ..

$$M_q = M_q^{(0)} + \delta M_q = M_q^{(0)}(1 + C)$$

L : light sector
H : heavy sector

$$\delta M_q = \begin{pmatrix} \delta M_{LL}^q & \delta M_{LH}^q \\ \delta M_{HL}^q & \delta M_{HH}^q \end{pmatrix}$$

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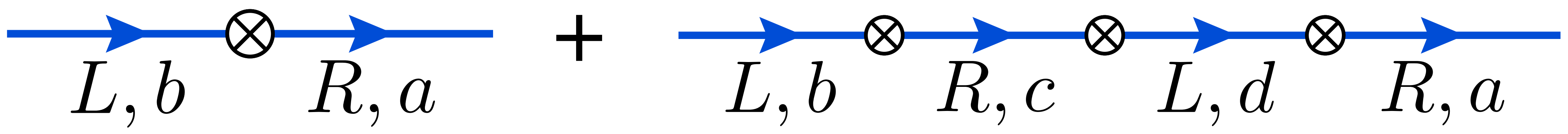

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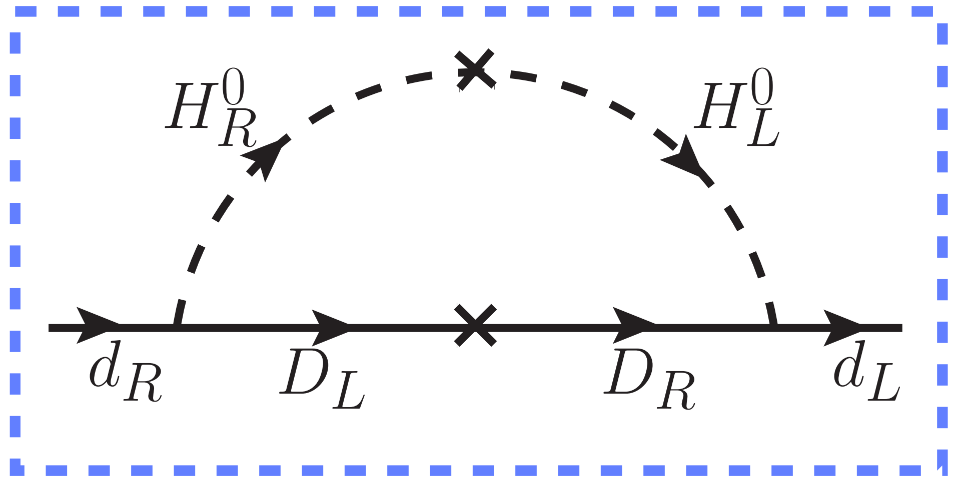
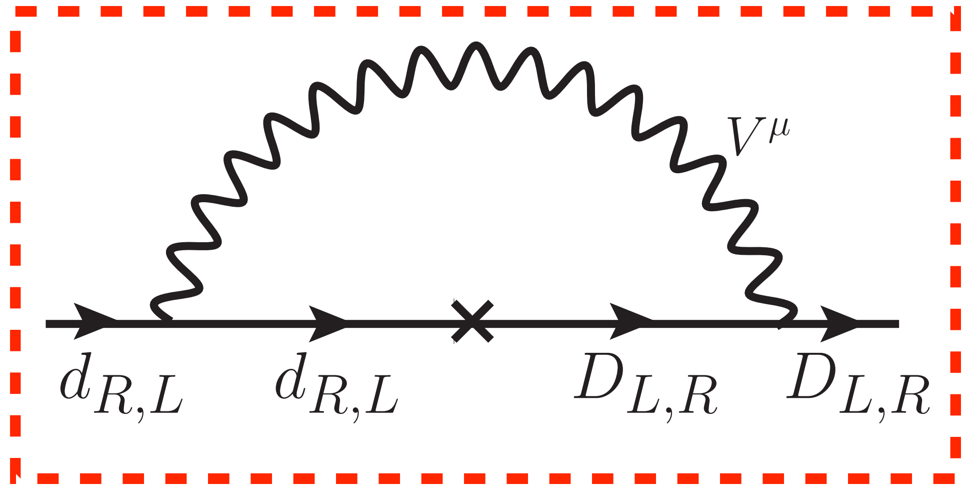




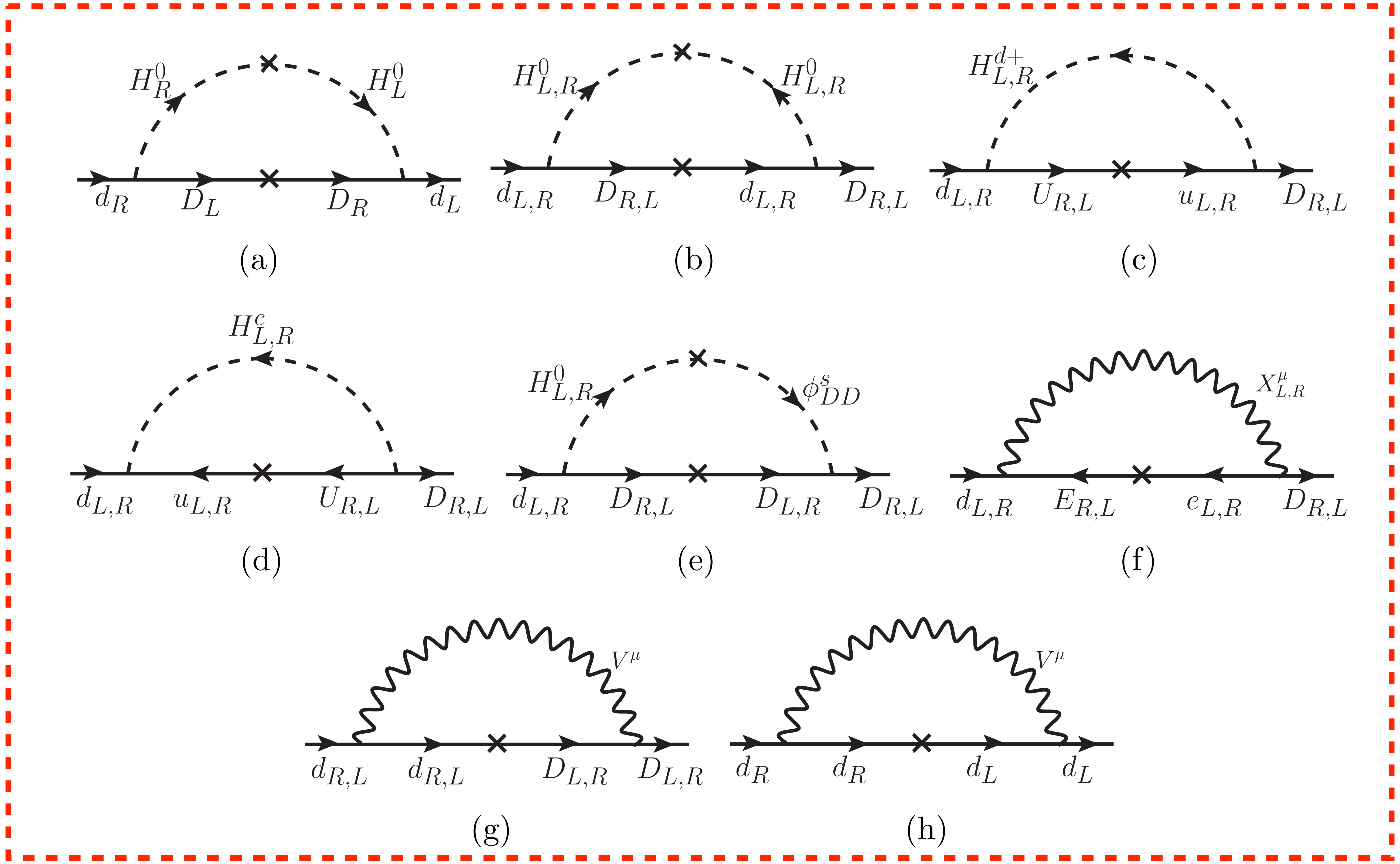
- $\bar{\theta}$ is given by

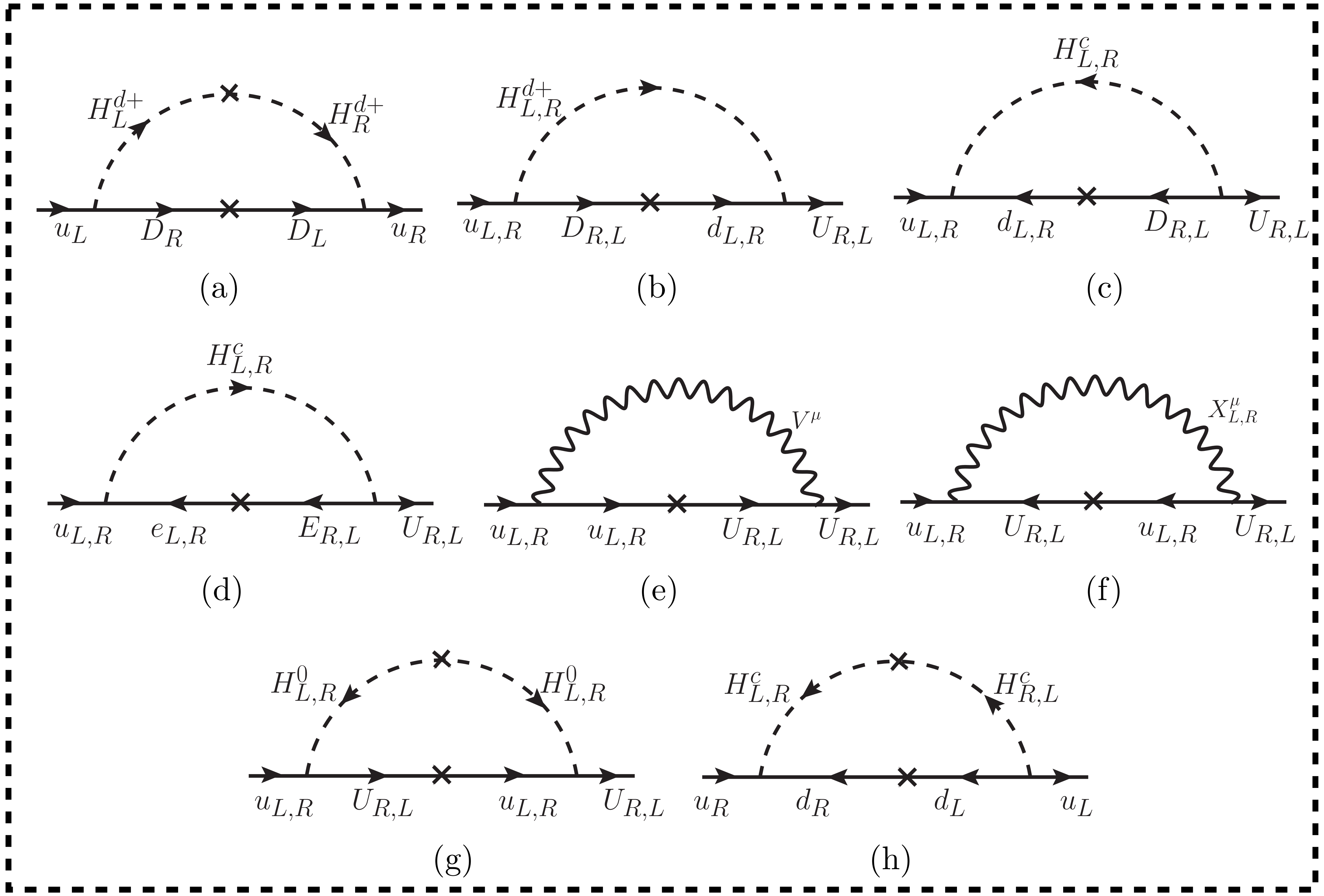
$$\bar{\theta} = \mathrm{Im}\ \mathrm{Tr} C_1 + \mathrm{Im}\ \mathrm{Tr}(C_2 - \frac{1}{2}C_1^2) + \ldots$$

$$\boxed{\bar{\theta} = \mathrm{Im}\ \mathrm{Tr}\left[-\frac{v_\phi}{\kappa_L \kappa_R}\delta M_{LL}^d (Y_d^\dagger)^{-1} Y_D Y_d^{-1} + \frac{1}{\kappa_L}\delta M_{LH}^d Y_d^{-1} + \frac{1}{\kappa_R}\delta M_{HL}^d (Y_d^\dagger)^{-1}\right]}$$

Vanishing of one loop $\bar{\theta}$

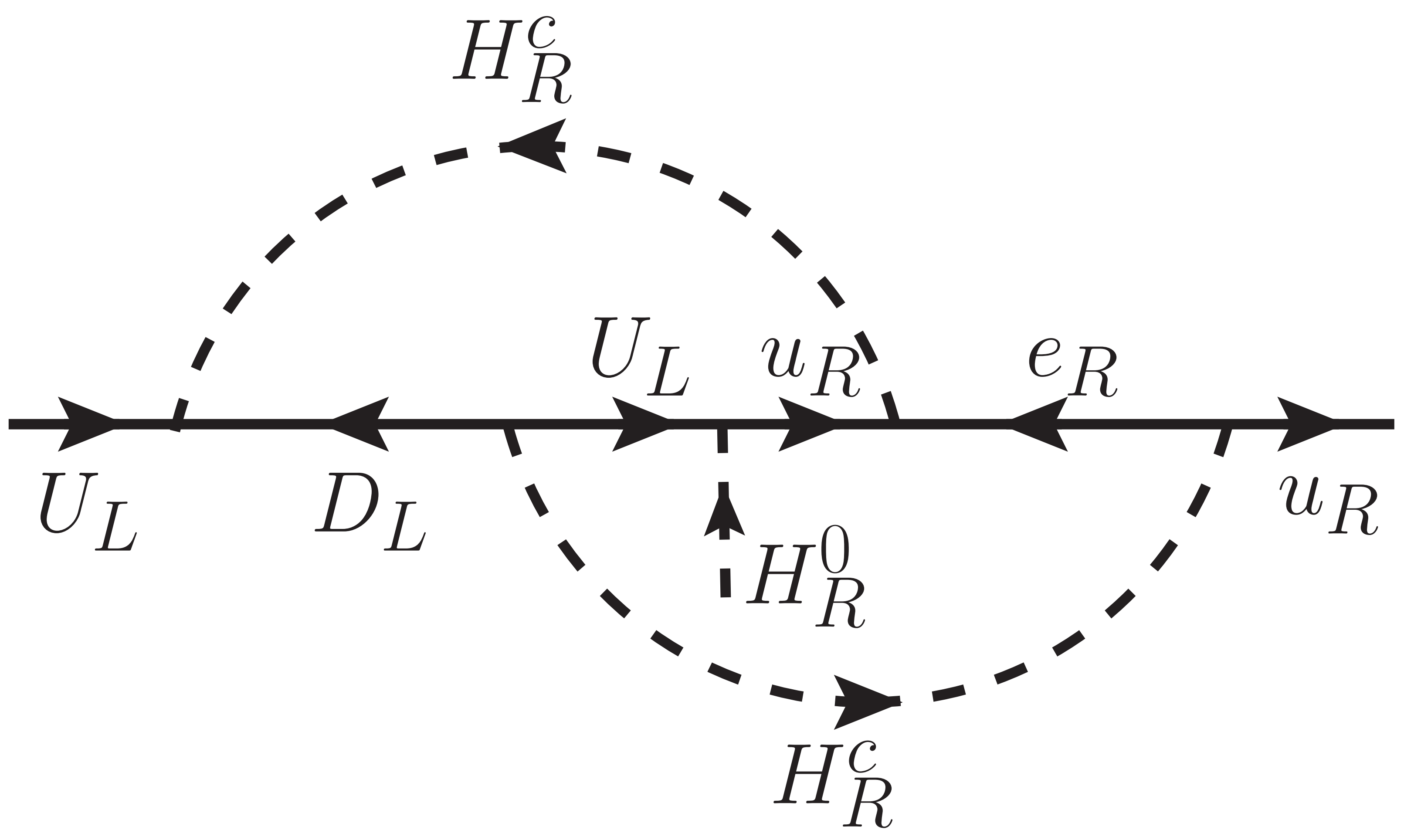
[Babu, Mohapatra, **Thapa**, '24]





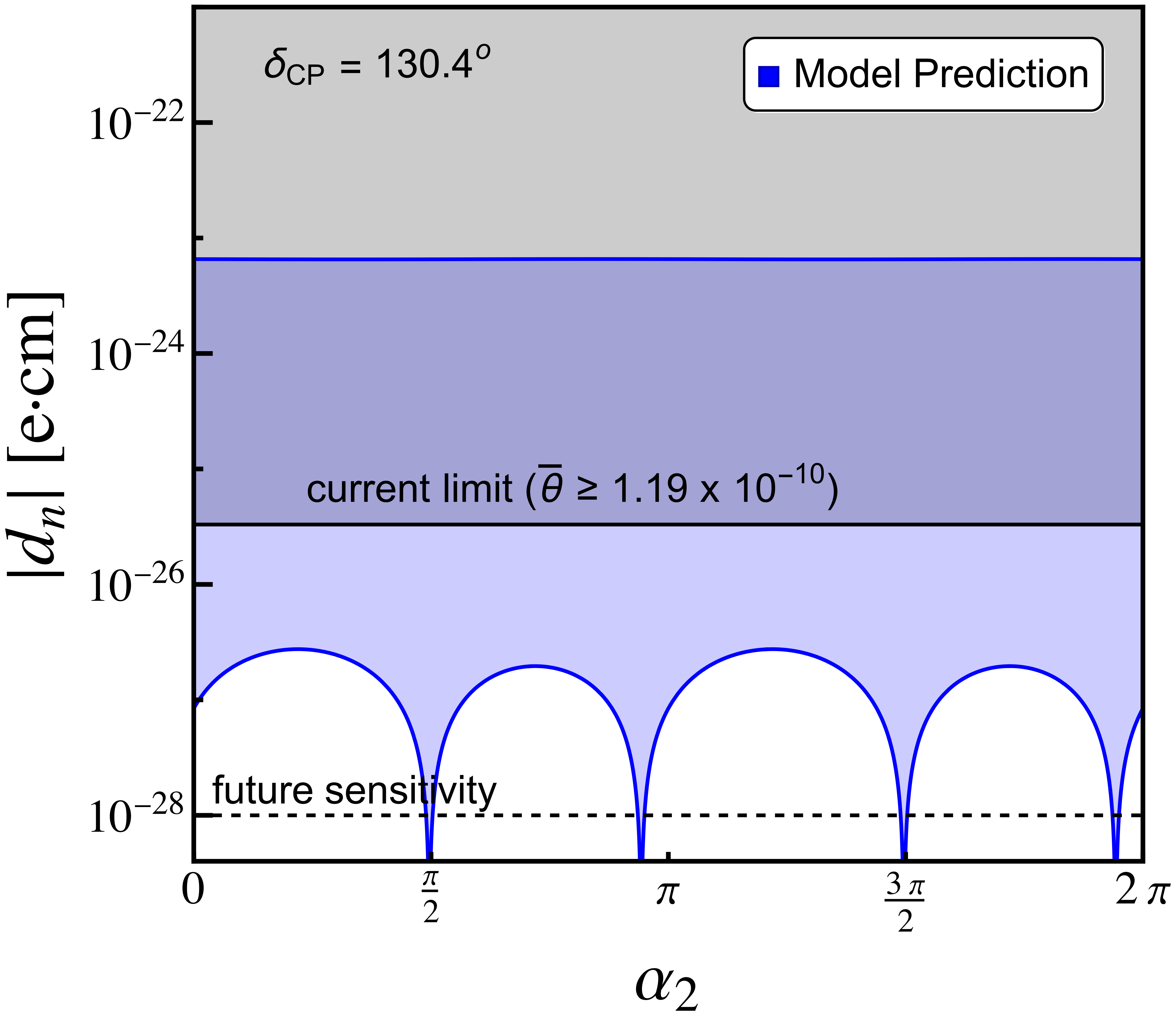
Each diagram individually gives
$\bar{\theta} = 0$

Two loop contribution to $\bar{\theta}$



$$\bar{\theta} \approx \frac{8}{(4\pi)^4} \mathrm{Im\,Tr}\left[Q^* U^\dagger_{\mathrm{PMNS}} \hat{Y}^2_\ell U_{\mathrm{PMNS}} Q^2 \right.$$
$$\left. \hat{Y}_u\, U^T_{\mathrm{PMNS}} \hat{Y}^2_\ell\, U^*_{\mathrm{PMNS}} (\hat{Y}_u)^{-1}\right] \ln\left(\frac{\mathrm{M}_{\mathrm{H}^c_\mathrm{L}}}{\mathrm{M}_{\mathrm{H}^c_\mathrm{R}}}\right)$$

All parameters are known expect for
$Q = \mathrm{diag}\,.(e^{i\alpha_1}, e^{i\alpha_2}, 1)$ with
$\alpha_1 = 0.128 + n\pi/2 \ (n = 0,1,2,..)$



Summary

- Universal LRSM has natural embedding in $SU(5)_L \times SU(5)_R$

- Open questions in neutrino oscillations

  > Absolute mass scale and mass hierarchy? ✓

  $m_{\nu_1} = (4.8 - 8.4)$ meV and Normal hierarchy

  > Are neutrinos their own antiparticle? ✓

  Dirac neutrino via type-II seesaw

  > Is there CP Violation in lepton sector? ✓

  Predicts $\delta_{CP} = (130.4 \pm 1.2)^\circ$ or $(229.6 \pm 1.2)^\circ$

  > Why is neutrino mass so tiny? ✓

  Dirac mass suppressed by $\mathcal{O}\left(M_I/M_G\right) \approx 10^{-7}$

- The model solves strong CP problem without the need for an axion

  $\bar{\theta} = 0$ at tree level and one-loop level.

- No $0\nu\beta\beta$ and suppressed proton decay

Thank you for your time

Backup Slides

Renormalization group evolution of $\bar{\theta}$

- There is the possibility that extrapolation of the Yukawa couplings by the RGE from the GUT scale to the weak scale could generate a nonzero $\bar{\theta}$

- The induced $\bar{\theta}$ via RGE from the up-quark sector read as

$$\delta(\bar{\theta}) = \mathrm{Im}\,\mathrm{Tr}\left[\frac{d}{dt}\left(Y_{uL}Y_{uR}^{\dagger}\right)\left(Y_{uL}Y_{uL}^{\dagger}\right)^{-1}\right]$$

$$\beta^{(1)}\left(Y_{uL}\right) = +\frac{3}{2}Y_{uL}Y_{uL}^{\dagger}Y_{uL} - \frac{3}{2}Y_{dL}Y_{dL}^{\dagger}Y_{uL} + 3\,\mathrm{Tr}\left(Y_{uL}^{\dagger}Y_{uL}\right)Y_{uL} + 3\,\mathrm{Tr}\left(Y_{dL}^{\dagger}Y_{dL}\right)Y_{uL} + \mathrm{Tr}\left(Y_{lL}^{\dagger}Y_{lL}\right)Y_{uL} - \frac{17}{20}g_{1L}^{2}Y_{uL} - \frac{9}{4}g_{2L}^{2}Y_{uL} - 8g_{3L}^{2}Y_{uL}$$

$$\vdots$$

- $\frac{d}{dt}\left(Y_{uL}Y_{uR}^{\dagger}\right)$ is a hermitian matrix $\Longrightarrow$ does not generate $\bar{\theta}$ if the initial $\bar{\theta}$ is zero

Fermion mass fitting

- Redefine the down-type quarks (d, D) and the charged leptons (e, E) to go from the original basis to new basis such that $\hat{M}_\ell$ and $\hat{M}_u$ are diagonal

$d_L = V_R P^* d'_L, \quad d_R = V_R P^* d'_R, \quad D_L = Q U^T_{\text{PMNS}} D'_L, \quad D_R = Q U^T_{\text{PMNS}} D'_R e_L = Q^* U^\dagger_{\text{PMNS}} e'_L, \quad e_R = Q^* U^\dagger_{\text{PMNS}}$
$e'_R, \quad \nu_L = Q^* \nu'_L, \quad \nu_R = Q^* \nu'_R E_L = V^*_R P E'_L, \quad E_R = V^*_R P E'_R .$

$$M_u = \begin{pmatrix} 0 & \hat{M}_u \kappa_L \\ \hat{M}_u \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix}, \qquad M_\ell = \begin{pmatrix} 0 & \hat{M}_\ell \kappa_L \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & 0 \end{pmatrix}, \qquad M_d = \begin{pmatrix} 0 & \hat{M}_\ell \\ \hat{M}_\ell \frac{\kappa_R}{\kappa_L} & \frac{v_\phi}{v_\nu} U^*_{\text{PMNS}} \hat{M}_\nu U^T_{\text{PMNS}} \end{pmatrix}$$

$$\xi^\dagger_L M_d \xi_R = \text{diag}\,.\left(m_d, m_s, m_b, m_{D_1}, m_{D_2}, m_{D_3}\right) \text{ where } \xi_{L,R} = \begin{pmatrix} \xi^{11} & \xi^{12} \\ \xi^{21} & \xi^{22} \end{pmatrix}_{L,R}$$

- CKM matrix is given by $V_{\text{CKM}} = P'^* V_R P^* \xi^{11}_L Q'^*$

unspecified unitary matrix V_R, thus V_{CKM} is unconstrained

$$\boxed{m_{D_1}\left(M_I\right) = 1.05 \times 10^7 \text{ GeV} \qquad m_{D_2}\left(M_I\right) = 1.62 \times 10^8 \text{ GeV} \qquad m_{D_1}\left(M_I\right) = 4.38 \times 10^9 \text{ GeV}}$$