The flavor of a string

Saúl Ramos-Sánchez

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Reviews in collaboration with

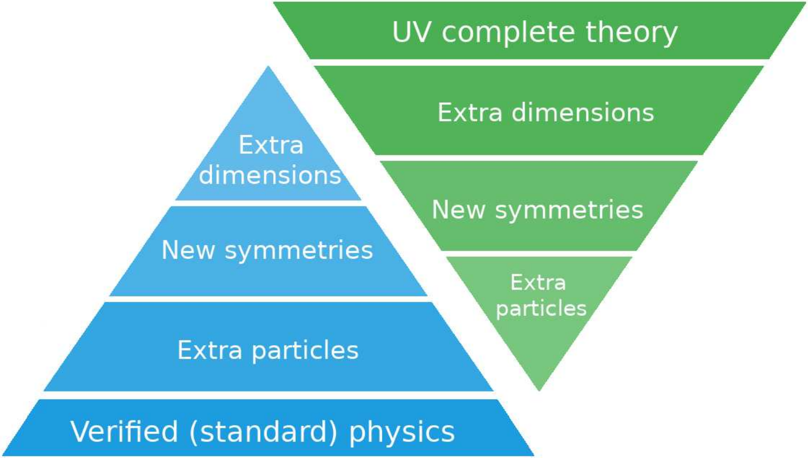
H.P. Nilles 2404.16933 (*Flavor's delight*) & M. Ratz 2401.03125 (*Heterotic orbifold models*)

based on the original works 2405.20378, 2311.10136, 2304.14437, 2207.10677, 2112.06940, 2006.03059 with

C. Arriaga, A. Baur, M-C. Chen, V. Knapp-Pérez, X-G. Liu, H.P. Nilles, M. Ratz, A. Trautner & P. Vaudrevange

Top-down meets Bottom-up: two paths, the same goal

String pheno



Particle physics & cosmology

Some flavored puzzles...

Despite the great success of the SM

- Need to explain
  - three flavors of SM particles
  - observed mass hierarchies
  - observed quark and lepton mixing *textures*
  - CP violation in CKM and PMNS
  - neutrino physics — talks by Zahra, Sudip, Anjan, Alakabha, Anil,...
  - anomalies in particle physics
  - dark matter — talks by Mauro, Monika, Xiao-Gang, Gopi, Jin,...
  - ...

$$\begin{pmatrix} 0.974 & 0.224 & 0.0039 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.039 & 1.019 \end{pmatrix}_{\text{CKM}}, \qquad \begin{pmatrix} 0.829 & 0.539 & 0.147 \\ 0.493 & 0.584 & 0.645 \\ 0.262 & 0.607 & 0.75 \end{pmatrix}_{\text{PMNS}}$$

$m_{u_i} \sim 2.16, 1270, 172900$ MeV

$m_{d_i} \sim 4.67, 93, 4180$ MeV

$\Delta m^2_{21} = 7.4 \cdot 10^{-5}, \Delta m^2_{31(23)} \approx 2.5 \cdot 10^{-3}$ eV^2

$m_{e_i} \sim 0.511, 105.7, 1776.9$ MeV

normal ordering

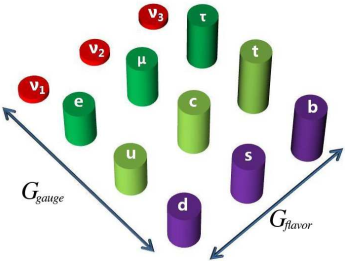
Bottom-up approaches to solve the flavor puzzle

Traditional: discrete non-Abelian flavor symmetries $G_{traditional}$ lead to models for quarks and leptons with great fits, $\theta_{13} \neq 0$,... requiring careful choice of flavon sector and flavon vevs

all Yukawa couplings: $Y \in \mathbb{C}$ $free$

[reviews by Ishimori, Kobayashi, Ohki, Okada, Shimizu, Tanimoto (2010); Feruglio, Romanino (2019)]

talks by Monika, We-Fu, Tom, Pierre, Ernest



Matter fields transform as $\phi \to \underbrace{\rho_\phi(g)}_{\text{rep of } g}\, \phi, \quad g \in G_{traditional} = S_3, A_4, \ldots$

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– some challenges include flavon vev $alignment$

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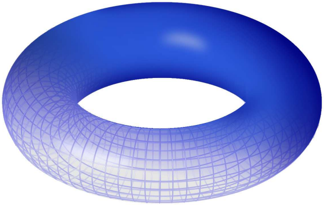
Modular: Yukawa couplings are modular forms $Y = Y(T)$ Feruglio (2017)

$$Y(T) \to Y(\gamma T) = (cT + d)^{n_Y} \rho_Y(\gamma) Y(T), \quad \gamma \in \Gamma = \mathrm{SL}(2, \mathbb{Z}), \rho_Y \in \Gamma_N$$



talks by Omar, Rukmani, Xue-Qi, Xiang-Gan

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$\Rightarrow$ finite modular groups Γ_N = modular flavor symmetry $G_{modular}$

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- $\Gamma_N \cong S_3, A_4, S_4, A_5$ for $N = 2,3,4,5$
  $\Rightarrow$ less parameters (despite some Kähler issues [Chen, Ratz, SRS (2019)])

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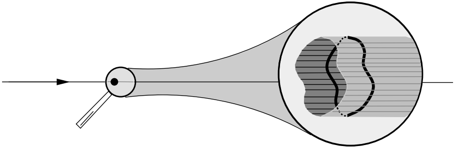
- double cover $\Gamma'_N \cong S_3, T', \mathrm{SL}(2,4), \mathrm{SL}(2,5)$ for $N = 2,3,4,5$
- 4-fold cover $\widetilde{\Gamma}_4 \cong [96,67], \widetilde{\Gamma}_8 \cong [768,1085324], \widetilde{\Gamma}_{12} \cong [2304,\ldots]$
- multiple moduli, e.g. Siegel modular groups $\Gamma_{g,N} \cong \mathrm{Sp}(2g,\mathbb{Z})/K_N$
- $\Gamma/\ker(\varrho)$ with vector-valued modular forms

Liu,Ding(2019); Liu,Yau,Qu,Ding(2020); Ding,Feruglio,Liu(2020); Ding,Liu(2021); Chen,King,Li,Medina,Petcov,Ratz,Titov,...


String
Theory

Stringy ingredients

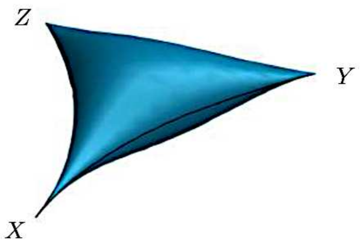
particles $\longleftrightarrow$ strings



Focus on heterotic strings

- closed strings with SUGRA & 10D space-time
  $\rightarrow$ compactify 6D on spaces with shapes and sizes set by moduli
- matter fields get all their properties from string features
  $\rightarrow$ all field charges (reps, weights,...) are computable
- field couplings arise from string interactions
  $\rightarrow$ coupling strengths are determined by CFT
  $\rightarrow$ couplings are modular forms with fixed properties
- gauge group: either $\mathrm{E}_8\times\mathrm{E}_8$ or $\mathrm{SO}(32)$ $\rightarrow$ SM by compactification

Heterotic Orbifolds



Dixon, Harvey, Vafa, Witten (1985-86)

Ibáñez, Nilles, Quevedo (1987)

Font, Ibáñez, Quevedo, Sierra (1990)

Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1990)

Kobayashi, Raby, Zhang (2004)

Buchmüller, Hamaguchi, Lebedev, Ratz (2004-06)

Kobayashi, Nilles, Plöger, Raby, Ratz (2006)

Lebedev, Nilles, Ratz, SRS, Vaudrevange, Wingerter (2006-08)

⋮

Mütter, Parr, Vaudrevange + Biermann, Ratz (2018-19)

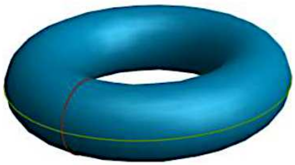
Baur, Nilles, Trautner, Vaudrevange (2018-19)

...

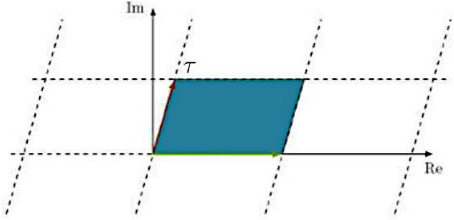
SRS, Ratz (2024)

Warm-up: 2D $\mathbb{T}^2/\mathbb{Z}_3$ orbifolds

* Start with a $\mathbb{T}^2$



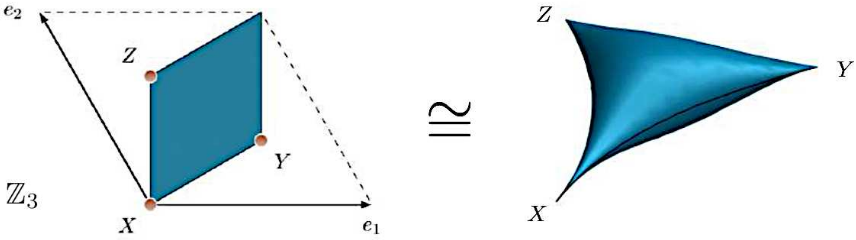
$\cong$



Two moduli: complex structure $U \longleftrightarrow$ shape $\rightarrow \Gamma_U = \mathrm{SL}(2,\mathbb{Z})_U$

Kähler modulus $T \longleftrightarrow$ size $\rightarrow \Gamma_T = \mathrm{SL}(2,\mathbb{Z})_T$

* Mod out a discrete $\mathbb{Z}_3$ symmetry generated by twist $\vartheta_{\mathbb{Z}_3} = \mathrm{e}^{2\pi\mathrm{i}/3}$



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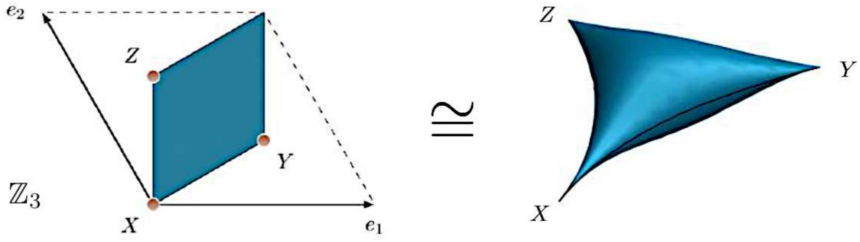

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Only one $\mathrm{SL}(2,\mathbb{Z})$ left unbroken!

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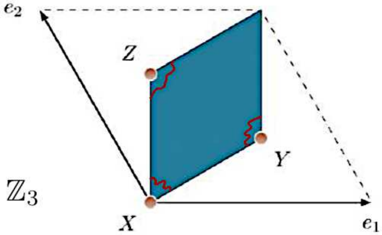
* Consider twisted string states localized at singularities of $\vartheta^k_{\mathbb{Z}_3}$ sector



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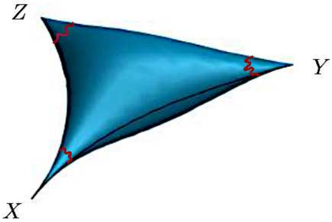

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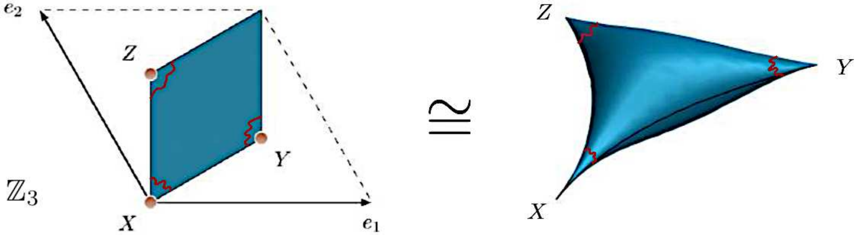

Strings perceive two kinds of trafos:

- $G_{modular}$ = associated with $\mathrm{SL}(2,\mathbb{Z}) \to T$-dependent [Lauer,Mas,Nilles (1989)]
- $G_{traditional}$ = associated with localization [Kobayashi,Nilles,Plöger,Raby,Ratz (2006)]

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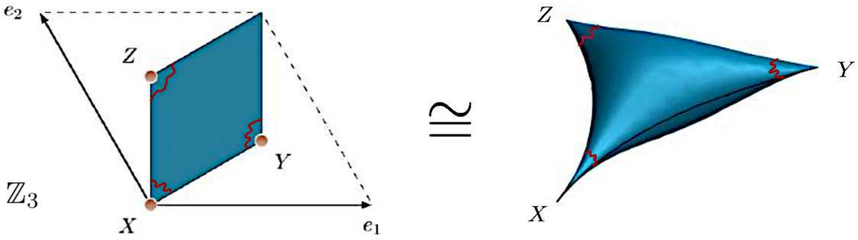

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Eclectic flavor scheme: $G_{modular} \cup G_{traditional}$ ☺

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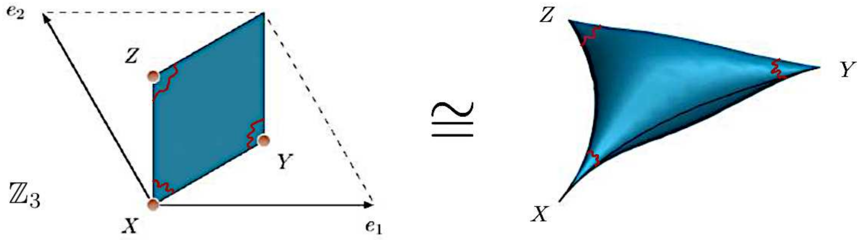

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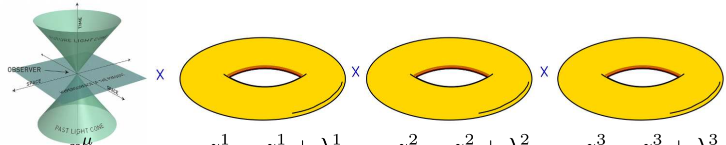
In $\mathbb{T}^2/\mathbb{Z}_3$: $T' \cup \Delta(54) \cong [658, 533]$

[Baur, Nilles, Trautner, Vaudrevange (2019); Nilles, SRS, Vaudrevange (2020)]

Toroidal heterotic orbifolds

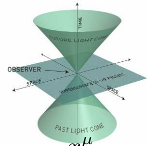
$\mathbb{M}^4 \times \mathbb{T}^6$



x^μ $z^1 \sim z^1 + \lambda^1$ $z^2 \sim z^2 + \lambda^2$ $z^3 \sim z^3 + \lambda^3$

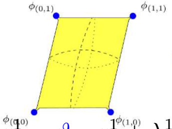
Toroidal heterotic orbifolds

$\mathbb{M}^4 \times \mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_3$ $\longleftarrow$ Abelian heterotic orbifold



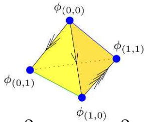
x^μ

×



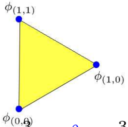
$z^1 \sim \vartheta_{\mathbb{Z}_2} z^1 + \lambda^1$

×



$z^2 \sim \vartheta_{\mathbb{Z}_2} z^2 + \lambda^2$

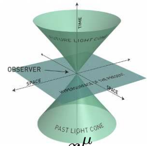
×



$z^3 \sim \vartheta_{\mathbb{Z}_3} z^3 + \lambda^3$

Toroidal heterotic orbifolds

$\mathbb{M}^4 \times \mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_3 = \mathbb{M}^4 \times \mathbb{R}^6/S$ with S: space group



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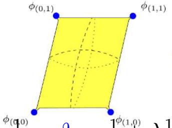

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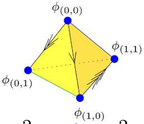

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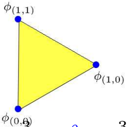

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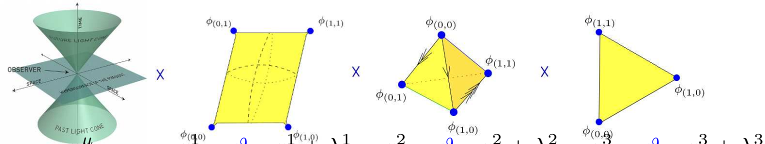

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Advantages:

- Flat everywhere, but at fixed points
- Matter states: free in $\mathbb{M}^4$, but localized at fixed points
- Fundamental forces from gauge symmetries

  $$\mathcal{G}_{10D} \to \mathcal{G}_{4D} \quad \text{with} \quad \mathcal{G}_{4D} \cong \mathcal{G}_{SM} \text{ possible}$$

  In heterotic strings: $\mathcal{G}_{10D} = \mathrm{E}_8 \times \mathrm{E}_8, \mathrm{SO}(32), \mathrm{SO}(16) \times \mathrm{SO}(16)$
- Allows for grand unification schemes at $M_{GUT} \lesssim 10^{17}$ GeV
- Moduli of compact space controls Yukawa couplings
- (Scalar fields/moduli can serve as inflaton or dark matter,...)

What is an acceptable heterotic orbifold model?

Must inspect all $\mathbb{R}^6/S$ with $S \supset \mathbb{Z}_N$ or $\mathbb{Z}_N \times \mathbb{Z}_M$, such that

- Gauge group $\mathcal{G}_{4D} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathcal{G}_{hidden}$
- 3 families of quarks and leptons
- Some (at least two) Higgs fields (with SUSY)
- Anomaly free $\mathrm{U}(1)_Y$
- Largest top mass: $m_t > m_{q_i,\ell_i,h}$
- Possible exotics are vectorlike, i.e. develop stringy masses
- SUSY broken due to hidden-sector dynamics
- Many right-handed neutrinos [Buchmüller, Hamaguchi, Lebedev, SRS, Ratz (2007)]

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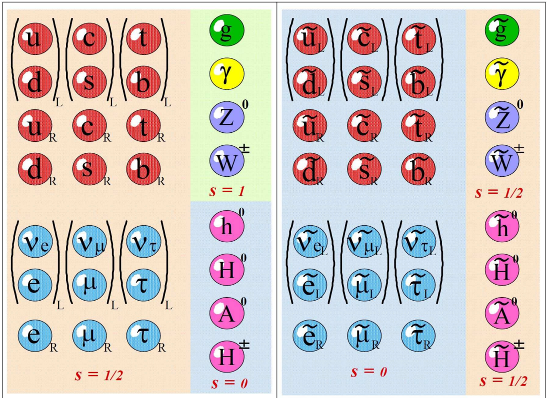
One can systematically find them! ☺

[Lebedev, Nilles, Raby, SRS, Ratz, Vaudrevanger, Wingerter (2006); Olguín-Trejo, Pérez-Martínez, SRS (2018)]

Orbifolder App [Nilles, SR-S, Vaudrevange, Wingerter (1110.5229)]

Viable heterotic orbifolds

121,246 MSSM-like models: (also $> 170,000$ *explicit* non-SUSY SM-like models, with no chiral exotics)



Observed particles

Superpartners

Viable heterotic orbifolds

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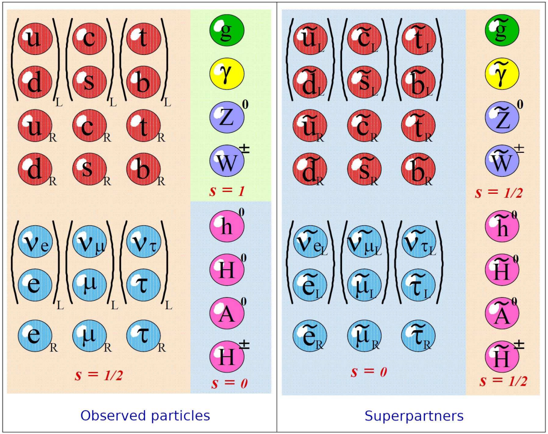

+ hidden sector: extra gauge symmetries, SUSY breakdown, vectorlike/massive exotics

[Lebedev, Nilles, Raby, SR-S, Ratz (2006); Olguín-Trejo, Pérez-Figueroa, Pérez-Martínez, SR-S (2019)]

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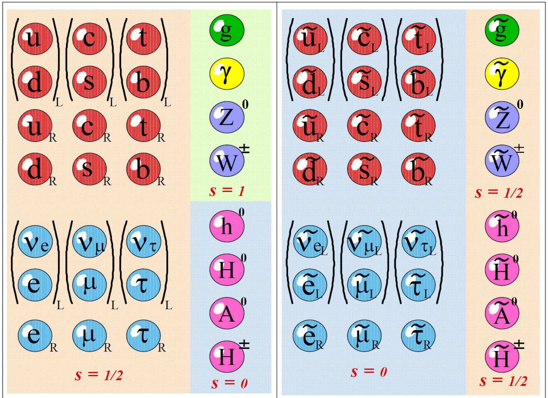


Observed particles | Superpartners

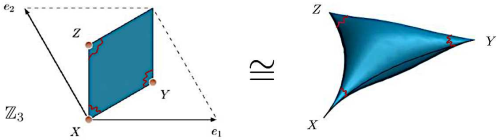
$+$ flavor symmetries ☺

Large number of MSSM-like models exhibit $\mathbb{T}^2/\mathbb{Z}_3$ ☺

with $G_{traditional} \cup G_{modular} = \Delta(54) \cup T'$

[Olguín-Trejo, Pérez-Martínez, SRS (2018)]

More on matter fields in $\mathbb{T}^2/\mathbb{Z}_3$



Define $\Phi_n = (X, Y, Z)$ with n: a computable modular weight

[Ibáñez,Lüst(1992), Olguín-Trejo,SRS(2017)]

n depends on: twisted sector, winding, momentum in compact space.

- In $\mathbb{T}^2/\mathbb{Z}_3$: $n = -{}^2\!/_3$ for $\vartheta^{k=1}_{\mathbb{Z}_3}$, $n = -{}^1\!/_3$ for $\vartheta^{k=2}_{\mathbb{Z}_3}$
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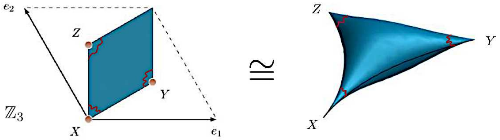

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  Strings that are (untwisted) not localized, get only $n = 0, -1$
- The transformations under $G_{modular}$ and $G_{traditional}$ can be computed

[Lauer,Mas,Nilles (1989)]

	Φ_0	Φ_{-1}	$\Phi_{-2/3}$	$\Phi_{-1/3}$
$\Delta(54)$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}_2$	$\bar{\mathbf{3}}_2$
T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$

[Baur, Nilles, Trautner, Vaudrevange (2019); Nilles, SRS, Vaudrevange (2020)]

More symmetries in $\mathbb{T}^2/\mathbb{Z}_3$?

- Always also a $\mathbb{Z}_2^{\mathcal{CP}}$ $\mathcal{CP}$-like trafo: $T \to -\bar{T}$ and $X, Y, Z \to \bar{X}, \bar{Y}, \bar{Z}$

  [Dent(2001); Baur,Nilles,Trautner,Vaudrevange (2019)] also in bottom-up:[Novichkov,Penedo,Petcov,Titov (2019)]

- For the torus to be consistent with $\vartheta_{\mathbb{Z}_3}$, $\langle U \rangle = \omega := \mathrm{e}^{2\pi \mathrm{i}/3}$

  $\to$ some discrete modular symmetry is unbroken: $\mathbb{Z}_9^R$ in $\mathbb{T}^2/\mathbb{Z}_3$

$$q_R = (-\langle U \rangle - 1)^n = \omega^{2n}, \qquad n = 0, -1, -2/3, -1/3$$

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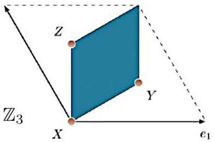
In summary



$\mathbb{T}^2/\mathbb{Z}_3$	bulk matter		θ sector		θ^2 sector		
	Φ_0	Φ_{-1}	$\Phi_{-2/3}$	$\Phi_{-5/3}$	$\Phi_{-1/3}$	$\Phi_{+2/3}$	$\mathcal{W}$
traditional $\Delta(54)$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}_2$	$\mathbf{3}_1$	$\bar{\mathbf{3}}_2$	$\bar{\mathbf{3}}_1$	$\mathbf{1}'$
modular T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{1}$
modular weight n_T	0	-1	$-2/3$	$-5/3$	$-1/3$	$+2/3$	-1
R-charge of $\mathbb{Z}_9^R$	0	3	1	-2	2	5	3

Yukawa couplings in $\mathbb{T}^2/\mathbb{Z}_3$

Yukawa coupling coefficients $\hat{Y}$ are modular forms

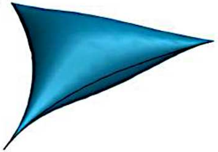
modular forms $\hat{Y}_{\boldsymbol{s}}^{(n_Y)}$	eclectic flavor group $\Omega(1)$							
	modular T' subgroup				traditional $\Delta(54)$ subgroup			
	irrep $\boldsymbol{s}$	$\rho_{\boldsymbol{s}}(\mathrm{S})$	$\rho_{\boldsymbol{s}}(\mathrm{T})$	n_Y	irrep $\boldsymbol{r}$	$\rho_{\boldsymbol{r}}(\mathrm{A})$	$\rho_{\boldsymbol{r}}(\mathrm{B})$	$\rho_{\boldsymbol{r}}(\mathrm{C})$
$\hat{Y}_{\mathbf{2}''}^{(1)}$	$\mathbf{2}''$	$\rho_{\mathbf{2}''}(\mathrm{S})$	$\rho_{\mathbf{2}''}(\mathrm{T})$	1	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1}}^{(4)}$	$\mathbf{1}$	1	1	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1}'}^{(4)}$	$\mathbf{1}'$	1	ω	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{3}}^{(4)}$	$\mathbf{3}$	$\rho_{\mathbf{3}}(\mathrm{S})$	$\rho_{\mathbf{3}}(\mathrm{T})$	4	$\mathbf{1}$	1	1	1

$$\hat{Y}_{\mathbf{2}''}^{(1)} := \begin{pmatrix} \hat{Y}_1(T) \\ \hat{Y}_2(T) \end{pmatrix} = \begin{pmatrix} -3\sqrt{2} & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} \eta(3T)^3/\eta(T) \\ \eta(T/3)^3/\eta(T) \end{pmatrix}$$

No arbitrary modular weights n_Y nor representations $\boldsymbol{s}$! ☺

Restricted superpotential

[Baur,Nilles,Trautner,SRS,Vaudrevange (2021-22)]



$$\Rightarrow \mathcal{W} \supset c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3+Y_1\,Y_2\,Y_3+Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3+X_1\,Y_3\,Z_2+X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2+X_2\,Y_3\,Z_1+X_3\,Y_2\,Z_1\big)\Big]\,,$$

with $\Phi^{i}_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c\in\mathbb{R}$

Superpotential and Kähler in $\mathbb{T}^2/\mathbb{Z}_3$

Restricted superpotential

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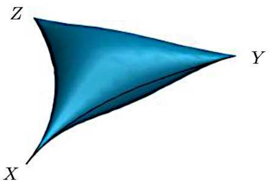

$$\Rightarrow \mathcal{W} \supset c\Big[\hat{Y}_2(T)\big(X_1\,X_2\,X_3 + Y_1\,Y_2\,Y_3 + Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3 + X_1\,Y_3\,Z_2 + X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2 + X_2\,Y_3\,Z_1 + X_3\,Y_2\,Z_1\big)\Big]\,,$$

with $\Phi^i_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

More interestingly

$$\mathcal{K} = -\log(-\mathrm{i}T + \mathrm{i}\bar{T}) + \textstyle\sum_i \Big[(-\mathrm{i}T + \mathrm{i}\bar{T})^{-2/3} + (-\mathrm{i}T + \mathrm{i}\bar{T})^{1/3}|\hat{Y}^{(1)}_{\mathbf{2''}}|^2 + \ldots\Big]\,|\Phi^i_{-2/3}|^2$$

$+$ suppressed corrections with flavon fields

Only canonical terms are allowed

$\rightarrow$ predictivity of bottom-up models with Γ'_N recovered! ☺

[Chen,SRS,Ratz (2019); Nilles,SRS,Vaudrevange (2004.05200)]

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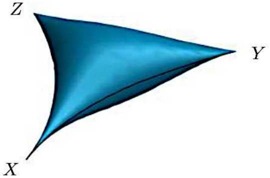

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with $\quad \Phi^i_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

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Only canonical terms are allowed (due to traditional symmetry)

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[Chen,SRS,Ratz (2019); Nilles,SRS,Vaudrevange (2004.05200)]

Lessons from $\mathbb{T}^2/\mathbb{Z}_3$

- Common origin of all kinds of flavor symmetries
- $G_{modular}$ and $G_{traditional}$ appear together $\rightarrow$ eclectic picture with

$$G_{modular} \subset \mathrm{Out}(G_{traditional})$$

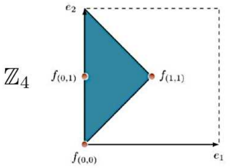
Also in bottom-up [Nilles,SRS,Vaudrevange (2020); Ding,King,Li,Liu,Lu (2023); Arriaga,Liu,SRS (2023)]

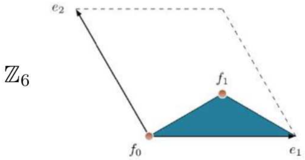
- $G_{modular}$ is a double cover (e.g. T') $\rightarrow n_Y \in \mathbb{Z}$
- Discrete $\mathbb{Z}_M^R$ symmetry
- Generalized $\mathbb{Z}_2^{\mathcal{CP}}$ $\mathcal{CP}$ trafo
- Fractional modular weights - only $0, -1$ for untwisted matter
- $\mathcal{K}$ constrained to canonical form by $G_{traditional}$
- $\mathcal{W}$ constrained by all
- Flavons are needed ☹

Generalization?

More flavors for $\mathbb{T}^2/\mathbb{Z}_N$

Not every $\mathbb{Z}_N$ is possible: $N = 2, 3, 4, 6$ only!





orbifold	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$	$\mathbb{Z}_6$
$G_{modular}$	$(S_3 \times S_3) \rtimes \mathbb{Z}_4$	T'	$2D_3$	$S_3 \times T'$
$G_{traditional}$	$(D_8 \times D_8)/\mathbb{Z}_2$	$\Delta(54)$	$(D_8 \times \mathbb{Z}_4)/\mathbb{Z}_2$	$\mathbb{Z}_6$
$\mathbb{Z}_M^R$	$\mathbb{Z}_4^R$	$\mathbb{Z}_9^R$	$\mathbb{Z}_{16}^R$	$\mathbb{Z}_{36}^R$

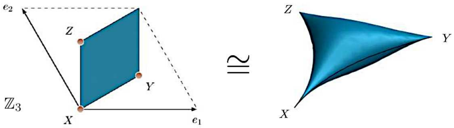
[Baur,Nilles,SRS,Trautner,Vaudrevange (2024)]

Flavor in

semi-realistic orbifold models

Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- Contains a sector $\mathbb{T}^2/\mathbb{Z}_3$



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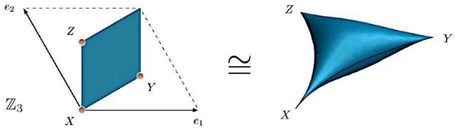

- $G_{traditional} = \Delta(54)$ & $G_{modular} = T' \cong \Gamma'_3$

[Lauer, Mas, Nilles (89-90)]

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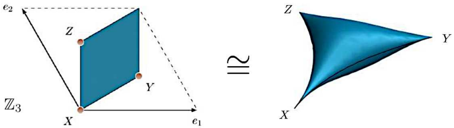

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use only the few representations for quarks and leptons:

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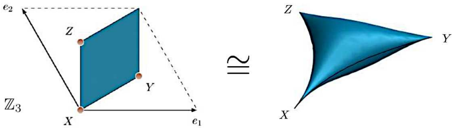

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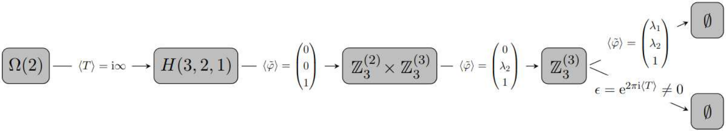
use only the few representations for quarks and leptons:

	quarks and leptons						Higgs fields		flavons							
label	q	$\bar{u}$	$\bar{d}$	ℓ	$\bar{e}$	$\bar{\nu}$	$H_{\rm u}$	$H_{\rm d}$	$\varphi_{\rm e}$	$\varphi_{\rm u}$	φ_ν	ϕ^0	$\phi^0_{\rm M}$	$\phi^0_{\rm e}$	$\phi^0_{\rm u}$	$\phi^0_{\rm d}$
$\mathrm{SU}(3)_c$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{SU}(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{U}(1)_Y$	$1/6$	$-2/3$	$1/3$	$-1/2$	1	0	$1/2$	$-1/2$	0	0	0	0	0	0	0	0
$\Delta(54)$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{3}_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
T'	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbb{Z}_9^R$	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0
n	$-2/3$	$-2/3$	$-2/3$	$-2/3$	$-2/3$	$-2/3$	0	0	$-2/3$	$-2/3$	$-2/3$	0	0	0	0	0

[Baur, Nilles, SRS, Trautner, Vaudrevange (2207.10677)]

Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- write the corresponding action,
- fit the value of the modulus ($\langle T \rangle \sim 3\mathrm{i}$), and
- compute effective particle interactions (with 20 params)
- breake the eclectic flavor symmetry



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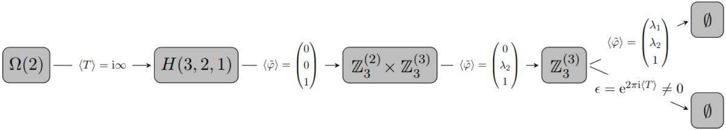
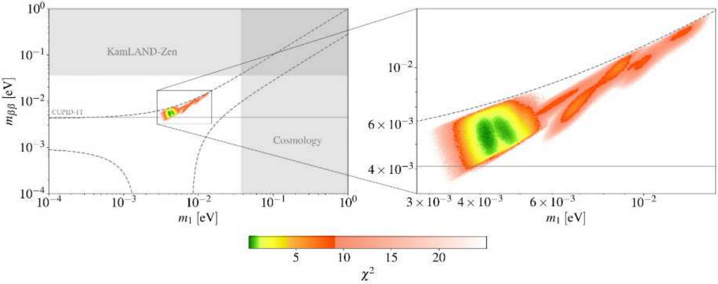


Results:



Explicit string model $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

Results:

	parameter	best-fit value
superpotential	$\mathrm{Im}\,\langle T\rangle$	3.195
	$\mathrm{Re}\,\langle T\rangle$	0.02279
	$\langle\tilde{\varphi}_{\mathrm{u},1}\rangle$	$2.0332 \cdot 10^{-4}$
	$\langle\vartheta_{\mathrm{u},1}\rangle$	1.6481
	$\langle\tilde{\varphi}_{\mathrm{u},2}\rangle$	$6.3011 \cdot 10^{-2}$
	$\langle\vartheta_{\mathrm{u},2}\rangle$	-1.5983
	$\langle\tilde{\varphi}_{\mathrm{e},1}\rangle$	$-4.069 \cdot 10^{-5}$
	$\langle\tilde{\varphi}_{\mathrm{e},2}\rangle$	$5.833 \cdot 10^{-2}$
	$\langle\tilde{\varphi}_{\nu,1}\rangle$	$1.224 \cdot 10^{-3}$
	$\langle\tilde{\varphi}_{\nu,2}\rangle$	-0.9857
	Λ_ν [eV]	0.05629
Kähler potential	α_1^{u}	-0.94917
	α_2^{u}	0.0016906
	α_3^{u}	0.31472
	α_1^{d}	0.95067
	α_2^{d}	0.0077533
	α_3^{d}	0.30283
	α_1^{q}	-0.96952
	α_2^{q}	-0.20501
	α_3^{q}	0.041643

(a)

	observable	model best fit	exp. best fit	exp. 1σ interval
quark sector	$m_{\mathrm{u}}/m_{\mathrm{c}}$	0.00193	0.00193	$0.00133 \to 0.00253$
	$m_{\mathrm{c}}/m_{\mathrm{t}}$	0.00280	0.00282	$0.00270 \to 0.00294$
	$m_{\mathrm{d}}/m_{\mathrm{s}}$	0.0505	0.0505	$0.0443 \to 0.0567$
	$m_{\mathrm{s}}/m_{\mathrm{b}}$	0.0182	0.0182	$0.0172 \to 0.0192$
	ϑ_{12} [deg]	13.03	13.03	$12.98 \to 13.07$
	ϑ_{13} [deg]	0.200	0.200	$0.193 \to 0.207$
	ϑ_{23} [deg]	2.30	2.30	$2.26 \to 2.34$
	$\delta_{\mathcal{CP}}^{\mathrm{q}}$ [deg]	69.2	69.2	$66.1 \to 72.3$
lepton sector	m_{e}/m_{μ}	0.00473	0.00474	$0.00470 \to 0.00478$
	m_{μ}/m_{τ}	0.0586	0.0586	$0.0581 \to 0.0590$
	$\sin^2\theta_{12}$	0.303	0.304	$0.292 \to 0.316$
	$\sin^2\theta_{13}$	0.0225	0.0225	$0.0218 \to 0.0231$
	$\sin^2\theta_{23}$	0.449	0.450	$0.434 \to 0.469$
	$\delta_{\mathcal{CP}}^{\ell}/\pi$	1.28	1.28	$1.14 \to 1.48$
	η_1/π	0.029	-	-
	η_2/π	0.994	-	-
	$J_{\mathcal{CP}}$	-0.026	-0.026	$-0.033 \to -0.016$
	$J_{\mathcal{CP}}^{\max}$	0.0335	0.0336	$0.0329 \to 0.0341$
	$\Delta m_{21}^2/10^{-5}$ [eV^2]	7.39	7.42	$7.22 \to 7.63$
	$\Delta m_{31}^2/10^{-3}$ [eV^2]	2.521	2.510	$2.483 \to 2.537$
	m_1 [eV]	0.0042	<0.037	-
	m_2 [eV]	0.0095	-	-
	m_3 [eV]	0.0504	-	-
	$\sum_i m_i$ [eV]	0.0641	<0.120	-
	$m_{\beta\beta}$ [eV]	0.0055	<0.036	-
	m_β [eV]	0.0099	<0.8	-
	χ^2	0.11		

[Baur, Nilles, SRS, Trautner, Vaudrevange (2207.10677)]

In summary

Concluding remarks

- Flavor puzzle: open questions about flavor (number and mixings of particles)

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To work on

- flavor with $\mathbb{T}^2/\mathbb{Z}_4$ & $\mathbb{T}^2/\mathbb{Z}_6$?
  Baur, Nilles, SRS, Trautner, Vaudrevange (2024)
- $\mathcal{CP}$ and $\mathcal{CP}$ violation ?
  Nilles, Ratz, Trautner, Vaudrevange (2018)
- bottom-up understanding of these fetures?
  all of us?
- dynamic moduli stabilization & de Sitter ?
  e.g. Knapp-Pérez, Liu, Nilles, SRS, Ratz (2023)
- more pheno & COSMO in these models ?
  should we join forces?
- already testable predictions ?

You wanna discuss top-down?