Implications of A_4 modular symmetry on neutrino masses, mixing and leptogenesis

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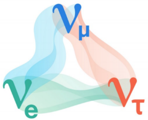
Neutrinos: What we know



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Mixing Matrix

Flavor Eigenstates

Mass Eigenstates

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$
$s_{ij} = \sin\theta_{ij}$

Measured from the following neutrino sources

Solar

Reactor

Accelerator

Atmospheric

Current values of Neutrino Oscillation Parameters

NuFIT 5.2 (2022)

without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \to 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$

Neutrino Mass Generation: Quick Review

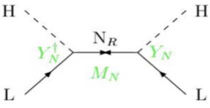
- Neutrino oscillation experiments provided compelling evidences for $m_\nu \neq 0$
- Massive neutrinos cannot be implemented in the SM through the Yukawa int, as there are no RH neutrinos.
- However, neutrino mass can arise at higher order: (dim-5 Weinberg operator)

$$\mathcal{O}_5 = \frac{Y^\nu_{ij}}{\Lambda}(\bar{L}_{L_i}\tilde{H})(\tilde{H}^T\bar{L}^C_{L_i}) + h.c. \Longrightarrow (m_\nu)_{ij} = \frac{Y^\nu_{ij}}{2\Lambda}v^2$$

- Shortcoming of Weinberg operator is that it violates L by two units, hence leading to Majorana mass term, but neutrinos could also be Dirac type
- One of the most viable theoretical frameworks used to yield neutrino masses is the see-saw mechanism.

Seesaw Mechanisms

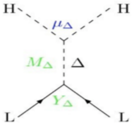
Right-handed singlet:
(type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

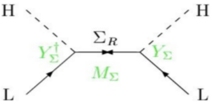
Scalar triplet:
(type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

Fermion triplet:
(type-III seesaw)



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

Courtesy: T. Hambye

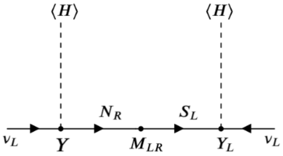
Modified Type-I Seesaw : Linear Seesaw

- The SM is extended by three RH (N_{R_i}) and three sterile (S_{Li}) singlet neutrinos
- The Yukawa interaction becomes

$$-\mathcal{L}_{linear} = Y \bar{N}_R \tilde{H} L + M_{LR} \bar{N}_R S_L + Y_L \bar{L}^C \tilde{H} S_L + h.c.$$

- The mass matrix for linear seesaw in the basis (ν_L, N_R, S_L^c)

$$M_{linear} = \begin{pmatrix} 0 & m_D & M_{LS} \\ m_D^T & 0 & M_{LR} \\ M_{LS}^T & M_{LR}^T & 0 \end{pmatrix}$$



- The mass formula for light neutrinos evolves from the above mass matrix for $M_{LR} \gg M_D, M_{LS}$

$$m_{linear} = m_D M_{LR}^{-1} M_{LS}^T + \text{transpose}$$

Radiative Neutrino mass generation (Scotogenic Model)

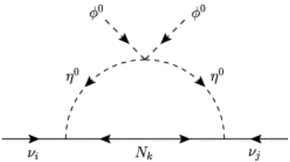
- SM is extended by: (a) 3 RHN fields N_k (b) Inert scalar doublet $\eta = (\eta^+, \eta^0)^T$ E. Ma, PRD 73, 077301 (2006)
- The structure of light neutrino mass would be

$$m_\nu \sim \frac{1}{16\pi^2}(\text{favour structure}) \times (\text{mass scale}) \times (\text{loop function})$$

- The active neutrino mass matrix obtained from the diagram

$$(\mathcal{M}_\nu)_{\alpha\beta} \simeq \sum_{k=1}^{3} \frac{\lambda_5 h_{k\alpha} h_{k\beta} v^2}{16\pi^2} M_k \left[1 - \frac{M_k^2}{(m_0^2 - M_k^2)} \ln\left(\frac{m_0^2}{M_k^2}\right)\right]$$

where $m_{\eta_R}^2 - m_{\eta_I}^2 = \lambda_5 v^2 \ll m_0^2 = (m_{\eta_R}^2 + m_{\eta_I}^2)/2$.



A_4 Discrete Flavor Symmetry

- Since indication of CP violation in lepton sector has been observed at T2K and NOvA, we are in the era to develop Flavor theory of leptons
- A_4 models are attractive, because A_4 is the minimal group which has a triplet as irreducible reps: $(\mathbf{3}, \mathbf{1}, \mathbf{1}', \mathbf{1}'')$.
- A_4 product rule: $3 \times 3 = 1 + 1' + 1'' + 3 + 3$
- It enables us to explain the flavour symmetry:, e.g., $y_\alpha \bar{L}_\alpha H \alpha_R$
  $\mathbf{3} : (L_e, L_\mu, L_\tau), \quad \mathbf{1} : e_R, \quad \mathbf{1}' : \mu_R, \quad \mathbf{1}'' : \tau_R$
- However, it yields a vanishing reactor mixing angle θ_{13}
- Inclusion of simple perturbation by introducing the flavon fields (SM singlet scalars, but transform nontrivially under flavor group) can generate nonzero θ_{13}
- The flavons play a crucial role in determining the flavour structure due to their particular vacuum alignment.

Modular Symmetry

- The full modular group $\Gamma \equiv SL(2,Z)$ is the group of 2×2 matrices, with integer entries and determinant 1.
- The *Principal congruence subgroup of level N*, (normal subgroup of Γ)

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

- Action of the modular group on τ, varying in the upper half complex plane $\mathcal{H} = \{\tau \in \mathrm{Im}\tau > 0\}$, has the form:

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}, \quad \{a, b, c, d \text{ integers}, \quad ad - bc = 1\}$$

- It can be generated by S and T, satisfying the relations $S^2 = (ST)^3 = \mathbb{1}$

$$S: \quad \tau \to -\frac{1}{\tau}, \quad \text{(duality)} \qquad T: \quad \tau \to \tau + 1, \quad \text{(discrete shift symmetry)}$$

Important features of Modular Symmetry

- Holomorphic functions which transform as

$$f(\tau) \to (c\tau + d)^k f(\tau)$$

  under the modular transformation are called modular forms of weight k.
- The superpotential should be invariant under modular symmetry, i.e., it should have vanishing modular weight
- The modular symmetry is broken by the vacuum expectation value of τ
- The Yukawa couplings are functions of modulus τ, and transform nontrivially under the modular symmetry
- They can be expressed in terms of Dedekind eta function $\eta(\tau)$

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \qquad q = e^{2\pi i \tau}.$$

- Γ_1 is the trivial group, $\Gamma_2 \cong S_3$, $\Gamma_3 \cong A_4$, $\Gamma_4 \cong S_4$, and $\Gamma_5 \cong A_5$.

Linear Seesaw with A_4 Modular symmetry

- Particle spectrum enriched by six extra heavy fermion fields (N_{R_i} and S_{L_i}) and one weighton (ρ)
- A global $U(1)_X$ symmetry is imposed to avoid certain unwanted terms in the superpotential

Fields	e_R^c	μ_R^c	τ_R^c	L_L	N_R	S_L^c	$H_{u,d}$	ρ
$SU(2)_L$	1	1	1	2	1	1	2	1
$U(1)_Y$	1	1	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}, -\frac{1}{2}$	0
$U(1)_X$	1	1	1	-1	1	-2	0	1
A_4	1	$1'$	$1''$	$1, 1'', 1'$	3	3	1	1
k_I	1	1	1	-1	-1	-1	0	0

MKB, SM, SS, RM, Phys. Dark Univ. 36, 101027 (2022)

Model Framework

- The importance of A_4 modular symmetry is that less no. of flavon fields are required, as Yukawa couplings have the non-trivial group transformation

Yukawa coupling	A_4	k_I
Y	**3**	**2**

- The Yukawa coupling $\mathbf{Y} = (y_1, y_2, y_3)$ with weight 2, which transforms as a triplet of A_4 can be expressed in terms $\eta(\tau)$ and its derivative

$$\begin{aligned} y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right), \\ y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\ y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right). \end{aligned}$$

Masses for the Charged leptons

- The relevant superpotential term for charged leptons is given by

$$\mathcal{W}_{M_\ell} = y_\ell^{ee} L_{e_L} H_d \; e_R^c + y_\ell^{\mu\mu} L_{\mu_L} H_d \; \mu_R^c + y_\ell^{\tau\tau} L_{\tau_L} H_d \; \tau_R^c \; .$$

- The charged lepton mass matrix takes the form:

$$M_\ell = \begin{pmatrix} y_\ell^{ee} v_d/\sqrt{2} & 0 & 0 \\ 0 & y_\ell^{\mu\mu} v_d/\sqrt{2} & 0 \\ 0 & 0 & y_\ell^{\tau\tau} v_d/\sqrt{2} \end{pmatrix} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} .$$

Dirac and pseudo Dirac mass terms for Neutral fermions

- The RH N_{R_i}'s transform as triplets under A_4 modular group with $U(1)_X$ charge of 1 and modular weight -1.
- The Yukawa couplings to transform as triplets under the A_4 with modular weight 2, so one can write invariant Dirac superpotential as

$$\mathcal{W}_D = \alpha_D L_{e_L} H_u \ (Y N_R)_1 + \beta_D L_{\mu_L} H_u \ (Y N_R)_{1'} + \gamma_D L_{\tau_L} H_u \ (Y N_R)_{1''}.$$

- The resulting Dirac neutrino mass matrix is found to be

$$M_D = \frac{v_u}{\sqrt{2}} \begin{bmatrix} \alpha_D & 0 & 0 \\ 0 & \beta_D & 0 \\ 0 & 0 & \gamma_D \end{bmatrix} \begin{bmatrix} y_1 & y_3 & y_2 \\ y_2 & y_1 & y_3 \\ y_3 & y_2 & y_1 \end{bmatrix}_{LR} .$$

Dirac and pseudo Dirac mass terms for Neutral fermions

- As we also have the extra sterile fermions S_{Li}, the pseudo-Dirac term is allowed, and the corresponding superpotential is given as

$$\mathcal{W}_{LS} = \left[\alpha'_D L_{e_L} H_u \, (Y S_L^c)_1 + \beta'_D L_{\mu_L} H_u \, (Y S_L^c)_{1'} + \gamma'_D L_{\tau_L} H_u \, (Y S_L^c)_{1''}\right] \frac{\rho^3}{\Lambda^3} \, ,$$

- The flavor structure for the pseudo-Dirac neutrino mass matrix takes the form,

$$M_{LS} = \frac{v_u}{\sqrt{2}} \left(\frac{v_\rho}{\sqrt{2}\Lambda} \right)^3 \begin{bmatrix} \alpha'_D & 0 & 0 \\ 0 & \beta'_D & 0 \\ 0 & 0 & \gamma'_D \end{bmatrix} \begin{bmatrix} y_1 & y_3 & y_2 \\ y_2 & y_1 & y_3 \\ y_3 & y_2 & y_1 \end{bmatrix}_{LR} .$$

Mixing between heavy Neutral fermions N_R and S_L^c

- One can have the interactions leading to the mixing between these additional fermion field as

$$\mathcal{W}_{M_{RS}} = [\alpha_{NS} Y(S_L^c N_R)_{\rm sym} + \beta_{NS} Y(S_L^c N_R)_{\rm Anti-sym}]\rho$$

- Using $\langle\rho\rangle = v_\rho/\sqrt{2}$, the resulting mass matrix is

$$M_{RS} = \frac{v_\rho}{\sqrt{2}} \left(\frac{\alpha_{NS}}{3} \begin{bmatrix} 2y_1 & -y_3 & -y_2 \\ -y_3 & 2y_2 & -y_1 \\ -y_2 & -y_1 & 2y_3 \end{bmatrix} + \beta_{NS} \begin{bmatrix} 0 & y_3 & -y_2 \\ -y_3 & 0 & y_1 \\ y_2 & -y_1 & 0 \end{bmatrix} \right).$$

- $\alpha_{NS}/3 \neq \beta_{NS}$, otherwise M_{RS} becomes singular
- The masses for the heavy fermions in the basis $(N_R, S_L^c)^T$, can be written as

$$M_{Hf} = \begin{pmatrix} 0 & M_{RS} \\ M_{RS}^T & 0 \end{pmatrix}.$$

- Therefore, one can have six doubly degenerate mass eigenstates for the heavy superfields upon diagonalization.

Linear Seesaw mechanism for light neutrino Masses

- Within the present model invoked with A_4 modular symmetry, the complete 9×9 mass matrix in the flavor basis of $(\nu_L, N_R, S_L^c)^T$ is

$$\mathbb{M} = \left(\begin{array}{c|ccc} & \nu_L & N_R & S_L^c \\ \hline \nu_L & 0 & M_D & M_{LS} \\ N_R & M_D^T & 0 & M_{RS} \\ S_L^c & M_{LS}^T & M_{RS}^T & 0 \end{array} \right).$$

- The linear seesaw mass formula for light neutrinos is given with the assumption $M_{RS} \gg M_D, M_{LS}$ as,

$$m_\nu \quad = \quad M_D M_{RS}^{-1} M_{LS}^T + \mathrm{transpose}.$$

- We numerically diagonalize the neutrino mass matrix through the relation $U^\dagger \mathcal{M} U = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$, where $\mathcal{M} = m_\nu m_\nu^\dagger$ and U is an unitary matrix

$$\sin^2\theta_{13} = |U_{13}|^2, \qquad \sin^2\theta_{12} = \frac{|U_{12}|^2}{1 - |U_{13}|^2}, \qquad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1 - |U_{13}|^2}.$$

Numerical Analysis

- To fit to the current neutrino oscillation data, we chose the following ranges for the model parameters:

$$\mathrm{Re}[\tau] \in [-0.5, 0.5], \quad \mathrm{Im}[\tau] \in [1, 2], \quad \{\alpha_D, \beta_D, \gamma_D\} \in 10^{-5}\ [0.1, 1],$$
$$\{\alpha'_D, \beta'_D, \gamma'_D\} \in 10^{-2}\ [0.1, 1], \qquad \alpha_{NS} \in [0, 0.5], \quad \beta_{NS} \in [0, 0.0001],$$
$$v_\rho \in [10, 100]\ \mathrm{TeV}, \quad \Lambda \in [100, 1000]\ \mathrm{TeV}.$$

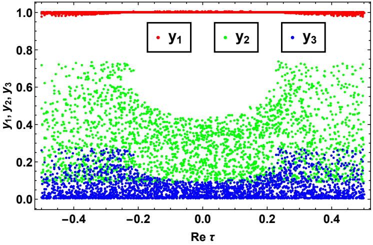
- The input parameters are randomly scanned over to obtain the allowed parameter space satisfying the neutrino oscillation data and $\sum m_i < 0.12$ eV
- The typical range of τ is found to be:

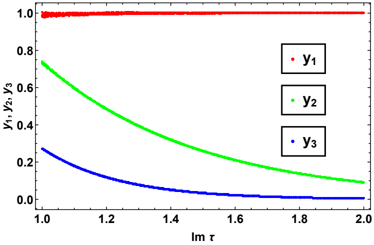
$$-0.5 \lesssim Re[\tau] \lesssim 0.5 \ \text{ and } \ 1 \lesssim Im[\tau] \lesssim 2 \ \text{ for } \ \mathrm{NO}.$$

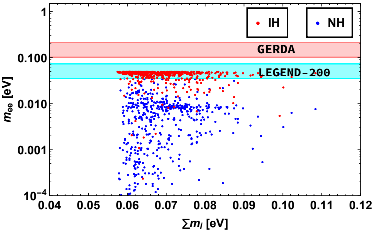
- Yukawa couplings as function of τ are found to vary in the region:

$$0.99 \lesssim y_1(\tau) \lesssim 1, \qquad 0.1 \lesssim y_2(\tau) \lesssim 0.8 \ \text{ and } \ 0.01 \lesssim y_3(\tau) \lesssim 0.3.$$

Some Results







Baryon Asymmetry of the Universe

- Understanding the origin of neutrino mass and BAU are two major challenges in Particle Physics
- Leptogenesis plays a vital role in relating these two issues
- The BAU is parametrized in terms of the following quantity:

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.04 \pm 0.08) \times 10^{-10}$$

- An alternative way to express matter-antimatter asymmetry is to use the ratio

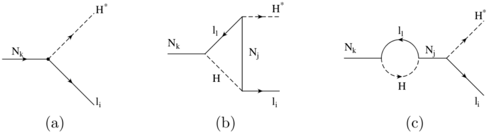
$$Y_B = \frac{n_b - n_{\bar{b}}}{s}$$

- The two formulations in terms of Y_B and η, at the present time, are easily related:

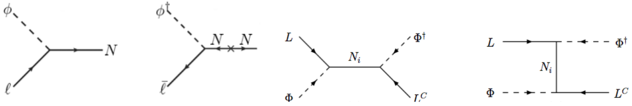
$$Y_B = \frac{n_\gamma^0}{s^0}\eta = 0.142\eta = (8.77 \pm 0.24) \times 10^{-11}.$$

Leptogenesis in 3 Basic Steps

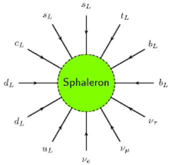
1. Generation of lepton asymmetry by the decay of heavy Majorana neutrino



2. Partial washout of the asymmetry due to inverse decays and scattering

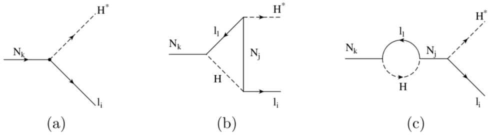


3. Conversion of the residual lepton asymmetry to baryon asymmetry



Resonant Leptogenesis

- For quasi-degenerate heavy Majorana neutrinos, i.e., $M_{N_i} - M_{N_j} \ll M_{N_i}$, the self energy ($\varepsilon$) contribution to the CP asymmetry becomes dominant



- Resonant leptogenesis occurs when $M_{N_i} - M_{N_j} \sim \Gamma_{N_i}$, in this case CP asymmetry can become very large (even order 1)
- The ε-type CP asymmetry,

$$\varepsilon_{N_i} = \frac{\mathrm{Im}(h^{\nu\dagger}h^{\nu})^2_{ij}}{(h^{\nu\dagger}h^{\nu})_{ii}(h^{\nu\dagger}h^{\nu})_{jj}} \frac{(m^2_{N_i} - m^2_{N_j})m_{N_i}\Gamma^{(0)}_{N_j}}{(m^2_{N_i} - m^2_{N_j})^2 + m^2_{N_i}\Gamma^{(0)\ 2}_{N_j}}$$

- $\mathcal{O}(1)$ CP asymmetries are possible when,

$$m_{N_2} - m_{N_1} \sim \frac{1}{2}\Gamma^{(0)}_{N_{1,2}}, \qquad \frac{\mathrm{Im}(h^{\nu\dagger}h^{\nu})^2_{ij}}{(h^{\nu\dagger}h^{\nu})_{ii}(h^{\nu\dagger}h^{\nu})_{jj}} \sim 1$$

Resonant Leptogenesis in the present framework

- Six heavy states with doubly degenerate masses for each pair, obtained by diagonalization of the heavy fermion mass matrix

$$M_{Hf} = \begin{pmatrix} 0 & M_{RS} \\ M_{RS}^T & 0 \end{pmatrix}$$

- But one can introduce a higher dimensional mass term for the heavy RH neutrinos (N_R) as

$$L_M = -\alpha_R Y N_R^c N_R^c \frac{\rho^2}{\Lambda}$$

- The lightest pair, assumed to be in the TeV scale, dominantly contribute to the CP asymmetry, i.e., the contribution from one loop self energy dominates over the vertex diagram.

One Flavor Approximation

- The evolution of lepton asymmetry can be deduced from the Boltzmann equations.
- Sakharov criteria demand the decay of parent fermion to be out of equilibrium to generate the lepton asymmetry.
- To impose this condition, one has to compare the Hubble rate with the decay rate

$$K = \frac{\Gamma_{N_1^-}}{H(T = M_1^-)} \,.$$

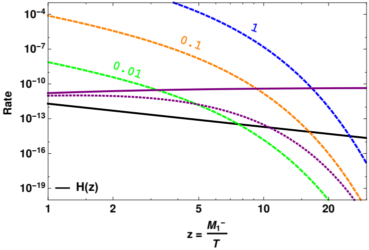
- The Boltzmann equations for the evolution of the number densities of RH fermions, in terms of yield parameter

$$\frac{dY_{N-}}{dz} = -\frac{z}{sH(M_1^-)}\left[\left(\frac{Y_{N-}}{Y_{N-}^{\rm eq}} - 1\right)\gamma_D + \left(\left(\frac{Y_{N-}}{Y_{N-}^{\rm eq}}\right)^2 - 1\right)\gamma_S\right],$$

$$\frac{dY_{B-L}}{dz} = -\frac{z}{sH(M_1^-)}\left[\epsilon_{N-}\left(\frac{Y_{N-}}{Y_{N-}^{\rm eq}} - 1\right)\gamma_D - \frac{Y_{B-L}}{Y_\ell^{\rm eq}}\frac{\gamma_D}{2}\right]$$

One Flavor Approximation

Figure: Interaction rates with Hubble expansion.



- Decay (Γ_D) in Purple solid line and inverse decay $\left(\Gamma_D \frac{Y^{eq}_{N^-}}{Y^{eq}_{\ell}}\right)$ dotted purple line with the coupling strength $\sim 10^{-6}$.
- The scattering rate $\left(\frac{\gamma_S}{s Y^{eq}_{N^-}}\right)$ for $N_1^- N_1^- \to \rho\rho$ is projected for various set of values for coupling, consistent with neutrino oscillation study.

One Flavor Approximation

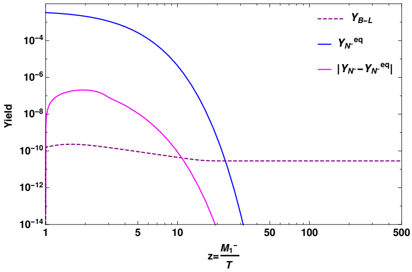


Figure: Evolution of Y_{B-L} (dashed) as a function of $z = M_1^- / T$.

- The obtained lepton asymmetry gets converted to the observed baryon asymmetry through sphaleron transition

$$Y_B = \left(\frac{8N_f + 4N_H}{22N_f + 13N_H} \right) Y_{B-L} \sim \mathcal{O}(10^{-10}).$$

Flavor Consideration

- One flavor approximation is reasonable at high scale ($T > 10^{12}$ GeV), where all the Yukawa interactions are out of equilibrium.
- But for temperatures below 10^{12} GeV, various Yukawa couplings come into equilibrium $\Longrightarrow$ flavor effects play a crucial role in generating the final lepton asymmetry.
- For temperatures below 10^5 GeV, all the Yukawa interactions are in equilibrium and the asymmetry is stored in the individual lepton sector.
- The Boltzmann equation for generating lepton asymmetry in each flavor is

$$\frac{dY^{\alpha}_{B-L_\alpha}}{dz} = -\frac{z}{sH(M_1^-)}\left[\epsilon^{\alpha}_{N-}\left(\frac{Y_{N-}}{Y^{eq}_{N-}} - 1\right)\gamma_D - \left(\frac{\gamma^{\alpha}_D}{2}\right)\frac{A_{\alpha\alpha}Y^{\alpha}_{\mathrm{B-L}_\alpha}}{Y^{eq}_{\ell}}\right],$$

where

$$\gamma^{\alpha}_D = sY^{eq}_{N-}\Gamma^{\alpha}_{N-}\frac{K_1(z)}{K_2(z)}, \quad \gamma_D = \sum_{\alpha}\gamma^{\alpha}_D$$

Flavor Consideration

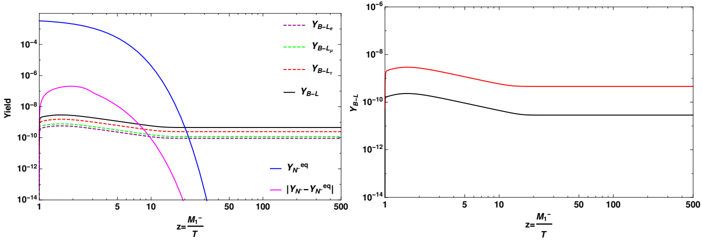


Figure: Left panel displays yield with inclusion of flavor effects. Right panel shows the enhancement in the yield due to three-flavor case over one-flavor approximation.

- The enhancement is because, in one flavor approximation the decay of heavy fermion to a specific lepton flavor final state ($N \to \ell_\alpha H$) can get washed out by the inverse decays of any flavor ($\ell_\beta + H \to N$) unlike the flavored case

Probing the Model in upcoming LBL Expts

- Due to the predictive features of the model, it can be probed in the forthcoming neutrino oscillation experiments: DUNE and T2HK
- T2HK and DUNE are expected to precisely measure $\delta_{\rm CP}$ as well as the octant of θ_{23}. Additionally, they will also determine the true nature of neutrino mass ordering
- It should be mentioned here that the current measurements of θ_{23} and $\delta_{\rm CP}$ are very weak.
- Hence, a large number of models are currently allowed, which predict a wide range of values regarding these two parameters.
- However, with the future measurements of these parameters by T2HK and DUNE, we expect to rule out many such models.

RM et al, JHEP 09, 144 (2023)

Numerical Analysis

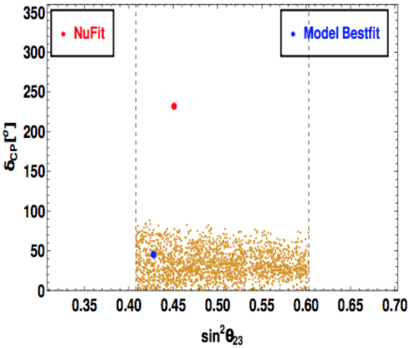
- The input parameters are randomly scanned over and the parameter space for the allowed regions is initially filtered by:
  - the observed 3σ limit of Δm^2_{21} & Δm^2_{31} , the mixing angles and the observed sum of active neutrino masses $0.058 \leq \Sigma m_{\nu_i} \leq 0.12$ eV
- The best-fit values of the input parameters are obtained by utilizing the chi-square minimization technique,

$$\chi^2 = \sum_i \left(\frac{T_i(z) - E_i}{\sigma_i} \right)^2 ,$$

  where E_i is the experimental best-fit values of oscillation parameters from NuFIT and $T_i(z)$ is the theoretical predictions for the corresponding oscillation parameter as a function of z (z indicates input parameters in the model)
- These calculations yield a cumulative $\chi^2|_{\min}$, which yields the best-fit values of model parameters

Results

The model predicts NO and constraints the δ_{CP} value



Range obtained for $\delta_{CP} \in [0^\circ, 89^\circ]$

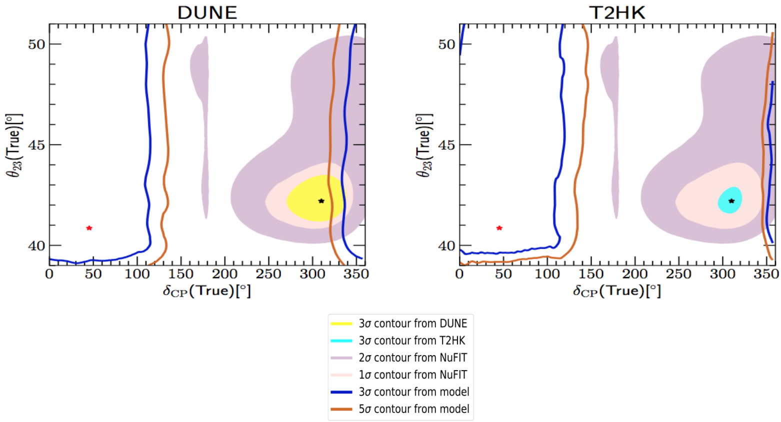
Numerical Analysis

- Next, we study the capability of T2HK and DUNE to constrain the model, using the χ^2 analysis
- Defining the Poisson χ^2 as

$$\chi^2 = 2\sum_j \left[N_j^{\rm th} - N_j^{\rm true} - N_j^{\rm true} \ln\left(\frac{N_j^{\rm th}}{N_j^{\rm true}}\right)\right],$$

  where j is the number of energy bins, $N_j^{\rm th}$ ($N_j^{\rm true}$) is the number of events in the test (true) spectrum.
- The current values of the oscillation parameters are taken as the true and the values of predicted from the models are as test.
- For each set of true parameters, we minimize the χ^2 w.r.t. all sets of predicted parameters.
- We find the $\chi^2_{\rm min}$ for all sets of true parameters and calculate $\Delta\chi^2$ as $\chi^2 - \chi^2_{\rm min}$.

Results



The 5σ allowed region is well separated from the 3σ allowed region for T2HK, but for DUNE, they are consistent.

Scoto-seesaw with A_4 modular symmetry PLB 853, 138635 (2024)

- To date, the only measured neutrino mass parameters are: $|\Delta m^2_{32}|$ and $|\Delta m^2_{21}|$, that might suggest that the origins of these two scales stem from separate mechanisms
- In the scoto-seesaw setup, the neutrino masses will be generated at tree level from type-I seesaw and from scoto-loop
- Two implement scoto-seesaw, new superfields added are: N_{R_1}, N_{R_2}, f, which are $SU(2)_L$ singlets and the scalar doublet η.

	Fermions					Scalars			Yukawa couplings					
Fields	L_ℓ	ℓ^c_R	N_{R_1}	N_{R_2}	f	$H_{u,d}$	η	η'	$Y_1^{(4)}$	$Y_{1'}^{(4)}$	$Y_1^{(8)}$	$Y_{1'}^{(8)}$	$Y_{1''}^{(8)}$	$Y_1^{(10)}$
$SU(2)_L$	2	1	1	1	1	2	2	2	–	–	–	–	–	–
$U(1)_Y$	$-1/2$	1	0	0	0	$\pm1/2$	$1/2$	$-1/2$	–	–	–	–	–	–
A_4	$1, 1', 1''$	$1, 1'', 1'$	1	$1'$	1	1	1	1	1	$1'$	1	$1'$	$1''$	1
k_I	0	0	4	4	5	0	3	3	4	4	8	8	8	10

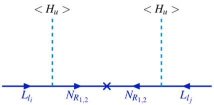
Neutrino Mass generation

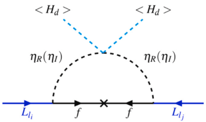
- The neutrino mass will have contributions from both tree and loop levels
- The super potential contributing to the tree level mass generation is

$$\mathcal{W}_\nu^T = \alpha_T \left(Y_1^{(4)} L_e H_u N_{R_1} + Y_{1'}^{(4)} L_\tau H_u N_{R_1} + Y_{1'}^{(4)} L_\mu H_u N_{R_2} + Y_1^{(4)} L_\tau H_u N_{R_2} \right) + \kappa_1 Y_1^{(8)} N_{R_1} N_{R_1} + \kappa_2 Y_{1''}^{(8)} N_{R_2} N_{R_2}.$$

giving rise to the Dirac and Majorana mass matrices M_D and M_R as:

$$M_D = \begin{pmatrix} Y_1^{(4)} & 0 \\ 0 & Y_{1'}^{(4)} \\ Y_{1'}^{(4)} & Y_1^{(4)} \end{pmatrix} \alpha_T v_u, \qquad M_R = \begin{pmatrix} \kappa_1 Y_1^{(8)} & 0 \\ 0 & \kappa_2 Y_{1''}^{(8)} \end{pmatrix},$$





- Considering the type-I seesaw formula, the light neutrino mass matrix is

$$(M_\nu)_{\rm tree} = -M_D M_R^{-1} M_D^T \ .$$

Using the expressions of M_D and M_R, one can have

$$(M_\nu)_{\rm tree} = -(\alpha_T v_u)^2 \begin{pmatrix} A & 0 & p\sqrt{AB} \\ 0 & B & r\sqrt{AB} \\ * & * & p^2 B + r^2 A \end{pmatrix} ,$$

- The neutrino mass term can also be generated at one loop level through the scotogenic process due to inert doublet η and the fermion f in the loop.
- The superpotential becomes

$$\mathcal{W}_\nu^L = \beta_L \left(Y_1^{(8)} L_e \eta f + Y_{1''}^{(8)} L_\mu \eta f + Y_{1'}^{(8)} L_\tau \eta f \right) + \kappa_S Y_1^{(10)} f f \ ,$$

- The neutrino masses generated effectively at the one loop level are as follows:

$$\left(M_\nu^{ij}\right)_{\text{loop}} = \mathcal{F}\left(m_{\eta_R}, m_{\eta_I}, M_f\right) M_f \ \mathbf{h^i} \ \mathbf{h^j} \ ,$$

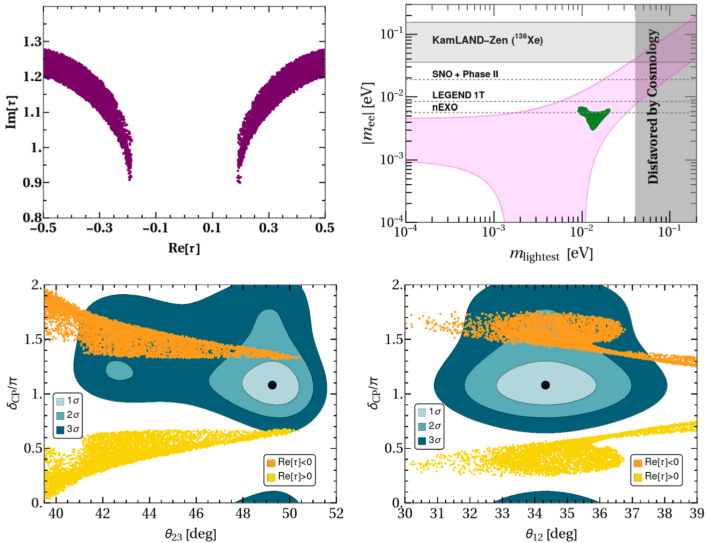
- Hence, the neutrino mass matrix at the loop level evolves as

$$(M_\nu)_{\text{loop}} = \beta_L^2 M_f \begin{pmatrix} \left(Y_1^{(8)}\right)^2 & \left(Y_1^{(8)} Y_{1''}^{(8)}\right) & \left(Y_1^{(8)} Y_{1'}^{(8)}\right) \\ * & \left(Y_{1''}^{(8)}\right)^2 & \left(Y_{1''}^{(8)} Y_{1'}^{(8)}\right) \\ * & * & \left(Y_{1'}^{(8)}\right)^2 \end{pmatrix} \mathcal{F}\left(m_{\eta_R}, m_{\eta_I}, M_f\right).$$

- The total contribution of neutrino mass matrix becomes

$$M_\nu = (M_\nu)_{\text{tree}} + (M_\nu)_{\text{loop}} \, .$$

Some Results



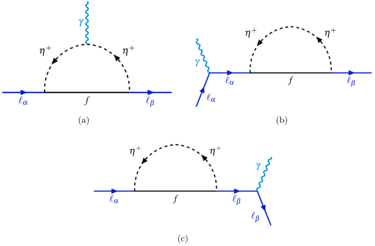
LFV decay: $\mu \to e\gamma$

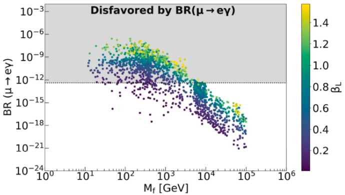
- The BR for the rare decay $\mu \to e\gamma$

$$\mathrm{BR}(\mu \to e\gamma) = \frac{3(4\pi)^3\alpha}{4G_F^2}|\mathcal{A}_1|^2\mathrm{BR}(\mu \to \mathrm{e}\overline{\nu}_\mathrm{e}\nu_\mu), \quad (1)$$

- $\mathcal{A}_1$ is the dipole contribution, expressed as

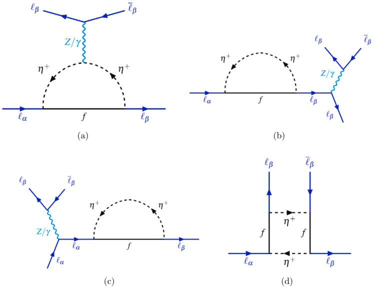
$$\mathcal{A}_1 = \beta_L^2 \frac{Y_1^{(8)*}\,Y_{1''}^{(8)}}{32\pi^2}\frac{1}{m_{\eta^+}^2}\,\mathcal{G}_1(x)\,, \quad x = M_f^2/m_{\eta^+}^2 \quad (2)$$

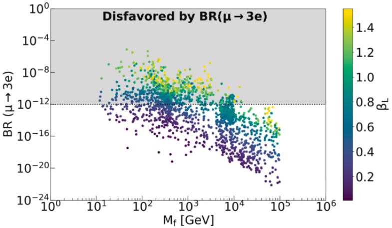




LFV decay: $\mu \to 3e$

The three body LFV decay $\ell_\alpha \to \ell_\beta \overline{\ell_\beta} \ell_\beta$ can proceed through penguin and box diagrams





Conclusion

- The modular A_4 flavor symmetry is quite successful in accommodating the observed neutrino oscillation data.
- The important aspect of modular symmetry is that the Yukawa couplings to transform non-trivially under modular A_4 group, which replaces the role of conventional flavon fields.
- Leptogenesis can be explained through the decay of lightest heavy fermion eigenstate
- Scoto-seesaw can also be implemented using modular symmetry.

Thank you for your attention !