A SCAN OF SMALL QUIVER MODELS WITH BIFUNDAMENTAL MATTER

Tom Kephart
Vanderbilt

FLASY 10

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Work with Robert Feger, Robert Saskowski and Elijah Sheridan

Use LieART to do systematically search of String Inspired nonSUSY Models with:

Small Product Gauge Groups
Three Families

Either Zero or a Few Extra Chiral Fermions

New Phenomenology

Class of bifundamental models inspired by orbifolding

$AdS_5 \times S^5$ with a discrete group Γ

generates 4D theories with gauge groups

Kachru and Silverstein 1998

$SU(n_1N)^{q_1} \times SU(n_2N)^{q_2} \times \ldots$ where the n_is are the dimensions

of the irreducible representations (irreps) of Γ

Lawrence, Nekrasov and Vafa 1998

After spontaneous symmetry breaking (to diag subgroups) we can arrive at

bifundamental models with gauge groups of the form

SU(a)XSU(b)XSU(c)

Breaking a symmetry

$$SU(3)^n = SU(3)^p \times SU(3)^q \times SU(3)^r$$

to

$$SU(3)_1 \times SU(3)_2 \times SU(3)_3$$

the gauge coupling constants adjust to

$$g_1 = \frac{g}{\sqrt{p}}; \quad g_2 = \frac{g}{\sqrt{q}}; \quad g_3 = \frac{g}{\sqrt{r}}$$

Use LieART to search space of models

LIEART 1.0: A MATHEMATICA APPLICATION FOR LIE ALGEBRAS AND REPRESENTATION THEORY ROBERT FEGER AND TWK, **COMPUT. PHYS. COMMUN., 192, 166 (2015).**

- Classical and Exceptional Lie Groups
- Properties of irreps
- Irrep decompositions
- Tensor products of irreps
- Load <<LieART` package into Mathematica
- LieART commands can be called from any Mathematica program.

- LieART 2.0 – Upgrade with many new features and tables.
  Robert Feger, TWK and Robert Saskowski,

  Communications in Computational Physics, 257, 107490 (2020)

- Code freely available at HEPFORGE

We are interested in the nonSUSY ($\mathcal{N} = 0$) case.

Fermions are in quivers (bifundamentals).

E.g., for $SU(N)^3$

$$(N, \bar{N}, 1) + (1, N, \bar{N}) + (\bar{N}, 1, N)$$

Scalars are in adjoints, etc.

E. Sheridan and TWK, Nuclear Physics B**987**, 116108 (2023)

Well studied quiver models:

Pati-Salam Model 1974

$$SU(4) \times SU(2) \times SU(2)$$

Families in

$$(4,\bar{2},1) + (\bar{4},1,2) + (1,2,\bar{2})$$

Trinification Model de Rújula, Georgi and Glashow 1984

$$SU(3) \times SU(3) \times SU(3)$$

Families in

$$(3,\bar{3},1) + (1,3,\bar{3}) + (\bar{3},1,3)$$

Quiver models with 3 automatic families

334 Model

TWK, Shafi 2001; Lee, TWK, Shafi 2006

$$SU(4) \times SU(3) \times SU(3)$$

$$3(4,\bar{3},1) + 3(\bar{4},1,3) + 4(1,3,\bar{3})$$

Coefficients are required to cancel gauge anomalies

Contains 3 family PS model, three family trinification model and several other possibilities

Grand Unification

$$E_6 \to SO(10) \to SU(5)$$

Fermion families in:

$$27 \to 16 + 10 + 1 \to (\bar{5} + 10 + 1) + (5 + \bar{5}) + 1$$

i.e., ($27 \to 16 \to (\bar{5} + 10 + 1)$ for the chiral part up to flipping)

Number of gauge generators

78, 45, 24

Three Family GUTs

SU(11) Georgi

SU(9) Frampton, Nandi

$$84 + 9(\bar{9})$$

Others

$SU(a) \times SU(b) \times SU(c)$ has $n = a^2 + b^2 + c^2 - 3$ gauge generators

Search for 3 family models with $n \leq 78$

If a, b and c are relatively prime

$$c(a, \bar{b}, 1) + b(\bar{a}, 1, c) + a(1, b, \bar{c})$$

If not remove greatest common divisor.

Building Models

Require no massless charged particle after SSB to the SM

three family models fall into subclasses:

``pristine models'' only the three families are chiral at EW scale

``chiral extensions'' three families plus extra chiral fermions—leptonic, hadronic or both at EW scale.

Recall:

A family $\mathbf{F}$ of $SU(3)_C \times SU(2)_L \times U(1)_Y$ contains

$$\mathbf{F} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} + (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} + (\mathbf{1}, \mathbf{1})_1$$

plus a right-handed neutrino.

Spontaneous Symmetry Breaking

Sequential nonAbelian breaking (maximal and regular)

$$SU(N) \rightarrow SU(N-1) \times U(1)$$

to get to

$$SU(3)_C \times SU(2)_L \times U(1)^m$$

then break Abelian symmetry group to get

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Consider all possible embeddings of $SU(3)_C$ and $SU(2)_L$

and all possible $U(1)_Y$ charge assignments in

$$SU(a) \times SU(b) \times SU(c)$$

Do Systematic Search

Select family members

$$\begin{pmatrix} q_{\kappa 1} & q_{\kappa 2} & \cdots & q_{\kappa m} \\ q_{\lambda 1} & q_{\lambda 2} & \cdots & q_{\lambda m} \\ q_{\mu 1} & q_{\mu 2} & \cdots & q_{\mu m} \\ q_{\nu 1} & q_{\nu 2} & \cdots & q_{\nu m} \\ q_{\xi 1} & q_{\xi 2} & \cdots & q_{\xi m} \\ q_{\rho 1} & q_{\rho 2} & \cdots & q_{\rho m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} \frac{1}{6} \\ \frac{1}{3} \\ -\frac{2}{3} \\ -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$$

Constrain models via SSB: only one unbroken $U(1)$ allowed

Restrict to models to where all charges are fixed

We handle this ambiguity by picking out the solution x with the smallest norm: that is, the element of the solution space whose Euclidean distance from the origin is least.

All charges are rational fractions

All potential models

No massless charged fermions

No conjugate SM fields

Possible Hypercharge assignments

Three families, plus extra chiral fermions

Three families, nothing extra

Gauge Group ($G \in \mathcal{G}'$)	$\dim G$	$N_P(G)$	NAB ($E \in \mathcal{E}$)	$N_{AB}(E)$	$N_{noMCF}(E)$	$N_{no\overline{SM}}$	$N_Y(E)$	$N_U(E)$	$N_{VL}(E)$
$SU(4) \times SU(3) \times SU(3)$	31	108	$(4,3,3)$	84	84	48	9	2	1
$SU(4) \times SU(4) \times SU(3)$	38	144	$(3,4,4)$	696	444	252	24	1	0
$SU(5) \times SU(3) \times SU(3)$	40	135	$(5,3,3)$	1086	1086	516	57	7	1
$SU(5) \times SU(4) \times SU(4)$	54	240	$(5,4,4)$	20880	9148	5124	440	17	0
$SU(5) \times SU(5) \times SU(3)$	56	225	$(3,5,5)$	16020	1280	1074	120	3	0
$SU(6) \times SU(4) \times SU(3)$	58	216	$(4,3,6)$	20520	2496	2304	67	2	0
			$(4,6,3)$	4572	252	252	48	1	1
$SU(5) \times SU(5) \times SU(4)$	63	300	$(4,5,5)$	48400	4910	4360	353	13	0
$SU(7) \times SU(3) \times SU(3)$	64	189	$(7,3,3)$	9870	9870	4920	537	55	5
$SU(6) \times SU(5) \times SU(3)$	67	270	$(5,3,6)$	74370	5024	3264	93	4	0
			$(5,6,3)$	14352	468	336	60	1	1
$SU(7) \times SU(4) \times SU(4)$	78	336	$(7,4,4)$	78696	35222	13940	1008	45	0

Vector-Like Extensions

Pheno starts at high scale

N33 Classes (N relatively prime to 3)

$$\mathbf{R}^{N33,i} = \mathbf{R}_{\text{SM}} + \mathbf{R}^{N33,i}_{\text{universal}} + \mathbf{R}^{N33,i}_{\text{unique}}$$

where i = 1,2,…5

$$\begin{aligned}\mathbf{R}^{N33,i}_{\text{universal}} = {} & 3(\mathbf{3},\mathbf{1})_{\frac{1}{6}} + 3(\overline{\mathbf{3}},\mathbf{1})_{-\frac{1}{6}} \\ & + N(\mathbf{1},\mathbf{2})_{\pm\frac{1}{2}} + (4N-18)(\mathbf{1},\mathbf{2})_0 \\ & + (4N-15)(\mathbf{1},\mathbf{1})_{\pm\frac{1}{2}} + (7N-36)(\mathbf{1},\mathbf{1})_0\end{aligned}$$

and where $\mathbf{R}^{N33,i}_{\text{unique}}$ takes one of the 5 following forms

$$\mathbf{R}^{N33,1}_{\text{unique}}(N) = 6(\mathbf{1},\mathbf{2})_0 + 6(\mathbf{1},\mathbf{1})_{\pm\frac{1}{2}} + 12(\mathbf{1},\mathbf{1})_0$$

$$\mathbf{R}^{N33,2}_{\text{unique}}(N) = 3(\mathbf{1},\mathbf{2})_{\pm\frac{1}{2}} + 3(\mathbf{1},\mathbf{1})_{\pm 1} + 6(\mathbf{1},\mathbf{1})_{\pm\frac{1}{2}} + 6(\mathbf{1},\mathbf{1})_0$$

$$\mathbf{R}^{N33,3}_{\text{unique}}(N) = 3(\mathbf{1},\mathbf{2})_{\pm 1} + 3(\mathbf{1},\mathbf{1})_{\pm\frac{3}{2}} + 6(\mathbf{1},\mathbf{1})_{\pm 1} + 3(\mathbf{1},\mathbf{1})_{\pm\frac{1}{2}}$$

$$\mathbf{R}^{N33,4}_{\text{unique}}(N) = 3(\mathbf{1},\mathbf{2})_{\pm\frac{3}{2}} + 3(\mathbf{1},\mathbf{1})_{\pm 2} + 6(\mathbf{1},\mathbf{1})_{\pm\frac{3}{2}} + 3(\mathbf{1},\mathbf{1})_{\pm 1} + 3(\mathbf{1},\mathbf{1})_{\pm\frac{1}{2}}$$

$$\mathbf{R}^{N33,5}_{\text{unique}}(N) = 3(\mathbf{1},\mathbf{2})_{\pm\frac{1}{10}} + 3(\mathbf{1},\mathbf{1})_{\pm\frac{3}{5}} + 3(\mathbf{1},\mathbf{1})_{\pm\frac{2}{5}} + 6(\mathbf{1},\mathbf{1})_{\pm\frac{1}{10}}$$

When $SU(3)_C$ is in $SU(a)$

and $SU(2)_L$ is in $SU(b)$

we write the $U(1)$ charges as

$$A_1, A_2, \ldots, A_{a-3}; B_1, B_2, \ldots, B_{b-2}; C_1, C_2, \ldots, C_{c-1} \,.$$

Hypercharge choice examples:

$$Y^{N33,1}(N) = \frac{1}{6}A_{N-3} + \frac{1}{4}C_1 + \frac{1}{4}C_2$$
$$Y^{N33,2}(N) = \frac{1}{10}A_{N-5} - \frac{1}{10}A_{N-4} + \frac{1}{6}A_{N-3} + \frac{1}{2}C_2$$
$$Y^{N33,3}(N) = \frac{1}{5}A_{N-5} - \frac{1}{5}A_{N-4} + \frac{1}{6}A_{N-3} + \frac{1}{2}C_2$$
$$Y^{N33,4}(N) = \frac{3}{10}A_{N-5} - \frac{3}{10}A_{N-4} + \frac{1}{6}A_{N-3} + \frac{1}{2}C_2$$
$$Y^{N33,5}(N) = \frac{1}{10}A_{N-5} + \frac{1}{15}A_{N-3} + \frac{1}{2}C_2$$

N63 Classes (N relatively prime to 3 and 6)

$$\mathbf{R}^{N63,i} = \mathbf{R}_{\text{SM}} + \mathbf{R}^{N63,i}_{\text{universal}} + \mathbf{R}^{N63,i}_{\text{unique}}$$

where $i = 1{,}2$

$$\begin{aligned}\mathbf{R}^{N63,i}_{\text{universal}} = {} & 3(\mathbf{3},\mathbf{1})_{\frac{2}{3}} + 3(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + 3(\mathbf{3},\mathbf{1})_{-\frac{1}{3}} + 3(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + 6(\mathbf{3},\mathbf{1})_{\frac{1}{6}} + 6(\overline{\mathbf{3}},\mathbf{1})_{-\frac{1}{6}} \\ & + N(\mathbf{1},\mathbf{2})_{\pm\frac{1}{2}} + (4N-18)(\mathbf{1},\mathbf{2})_0 \\ & + (N+3)(\mathbf{1},\mathbf{1})_{\pm 1} + (12N-48)(\mathbf{1},\mathbf{1})_{\pm\frac{1}{2}} + (16N-42)(\mathbf{1},\mathbf{1})_0\end{aligned}$$

and

$$\mathbf{R}^{N63,1}_{\text{unique}}(N) = 6(\mathbf{1},\mathbf{2})_0 + 18(\mathbf{1},\mathbf{1})_{\pm\frac{1}{2}}$$

$$\mathbf{R}^{N63,2}_{\text{unique}}(N) = 3(\mathbf{1},\mathbf{2})_{\pm\frac{1}{10}} + 9(\mathbf{1},\mathbf{1})_{\pm\frac{3}{5}} + 9(\mathbf{1},\mathbf{1})_{\pm\frac{2}{5}} + 12(\mathbf{1},\mathbf{1})_{\pm\frac{1}{10}}$$

Hypercharge choice examples:

$$Y^{N63,1}(N) = \frac{1}{6}A_{N-3} + \frac{1}{6}B_4 - \frac{1}{6}B_3 + \frac{1}{2}C_2$$

$$Y^{N63,2}(N) = \frac{1}{10}A_{N-5} + \frac{1}{15}A_{N-3} + \frac{1}{6}B_4 - \frac{1}{6}B_3 + \frac{1}{2}C_2$$

Chiral Extensions

New pheno near the EW scale

Minimal Chiral Extensions

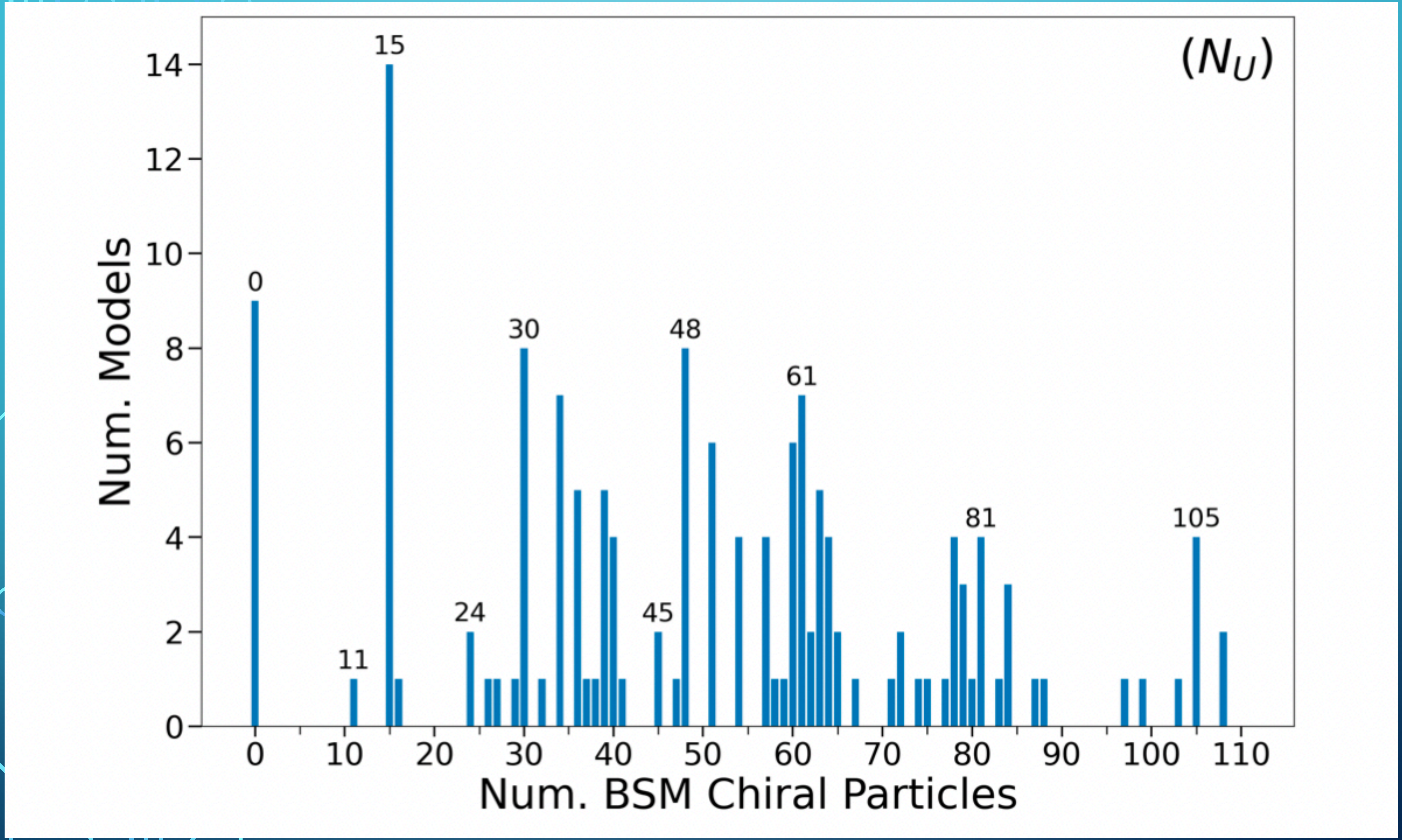
$$3(\mathbf{F}+\mathbf{1})+\mathbf{R_C}$$

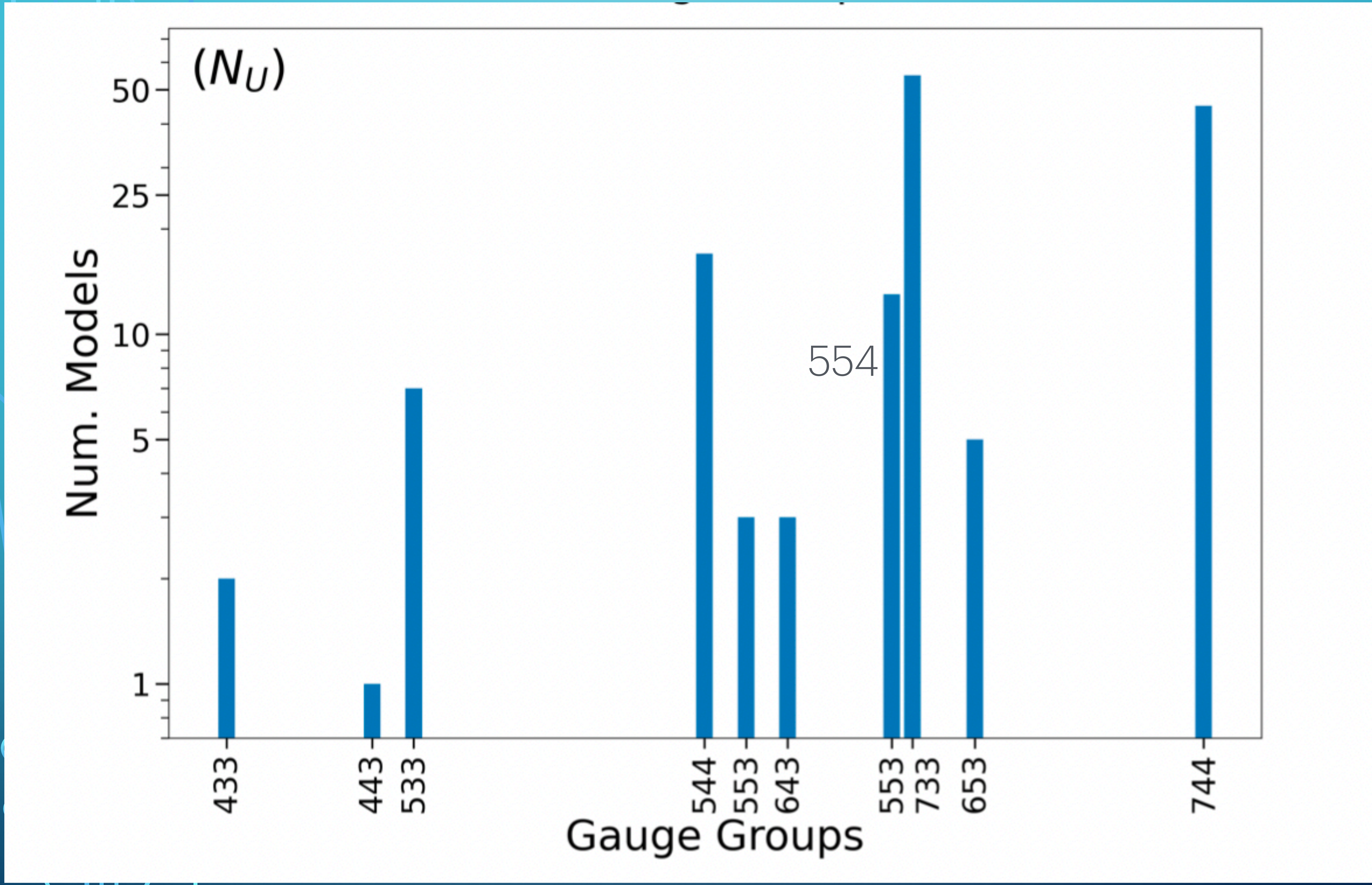
Smallest example, 11 extra leptons:

$$\mathbf{R}_C^{11}=(\mathbf{1},\mathbf{2})_{\frac{3}{2}}+3(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}+(\mathbf{1},\mathbf{1})_{-2}+2(\mathbf{1},\mathbf{1})_{1}$$

where

$$Y^{11}=\frac{1}{4}A_3+\frac{5}{12}A_4+\frac{1}{2}B_1+C_2.$$


(N_U)
Num. Models
Num. BSM Chiral Particles
0
11
15
24
30
45
48
61
81
105
0 2 4 6 8 10 12 14
0 10 20 30 40 50 60 70 80 90 100 110



(N_U)
Num. Models
50
25
10
5
1
554
433
443
533
544
553
643
553
733
653
744
Gauge Groups

FUTURE WORK: SU(A) x SU(B) x SU(C) PHENO

Leptoquarks

Bileptons

Biquarks

Z's

Gearing up for pheno with G. Corcella, R. Feger, P. Frampton, etc.

FUTURE WORK: OTHER BIFUNDAMENTAL MODELS

Quantification

R Foot, H Lew and R Volkas (1990)

Flipped Quantification

J Dent, H Pas, TWK and T Weiler (2024)

Both based on SU(3) x SU(3) x SU(3) x SU(3)

Explore SU(a) x SU(b) x SU(c) x SU(d) models

for models with up to 78 gauge generators

Extend to SUSY models

with SU(a) x SU(b) x SU(c)

and SU(a) x SU(b) x SU(c) x SU(d)

E. Ma, et al.

Conclusions

Landscape of unexplored models with small gauge group

Can have new particles at or near the EW scale

Some with fractional charge

Hence multi-charged magnetic monopoles

Possible diquarks, dileptons, leptoquarks, Z’s, …

Thank you!!!

More on LieART

MAIN NEW FEATURES IN LIEART 2.0

- Even more user friendly
- Extended tables of properties of irreps, tensor products and branching rules
- Branching rules to special maximal subalgebras for all classical and exceptional Lie algebras through rank 15.

FEATURES IN COMPANION PAPER

- Download instructions
- Automatic Installation
- Manual Installation
- Documentation
- LaTeX package

QUICK START GUIDE

- Entering Irreps
- Decomposing Tensor Products
- Decomposition to Subalgebras
- Tables of all LieART Commands

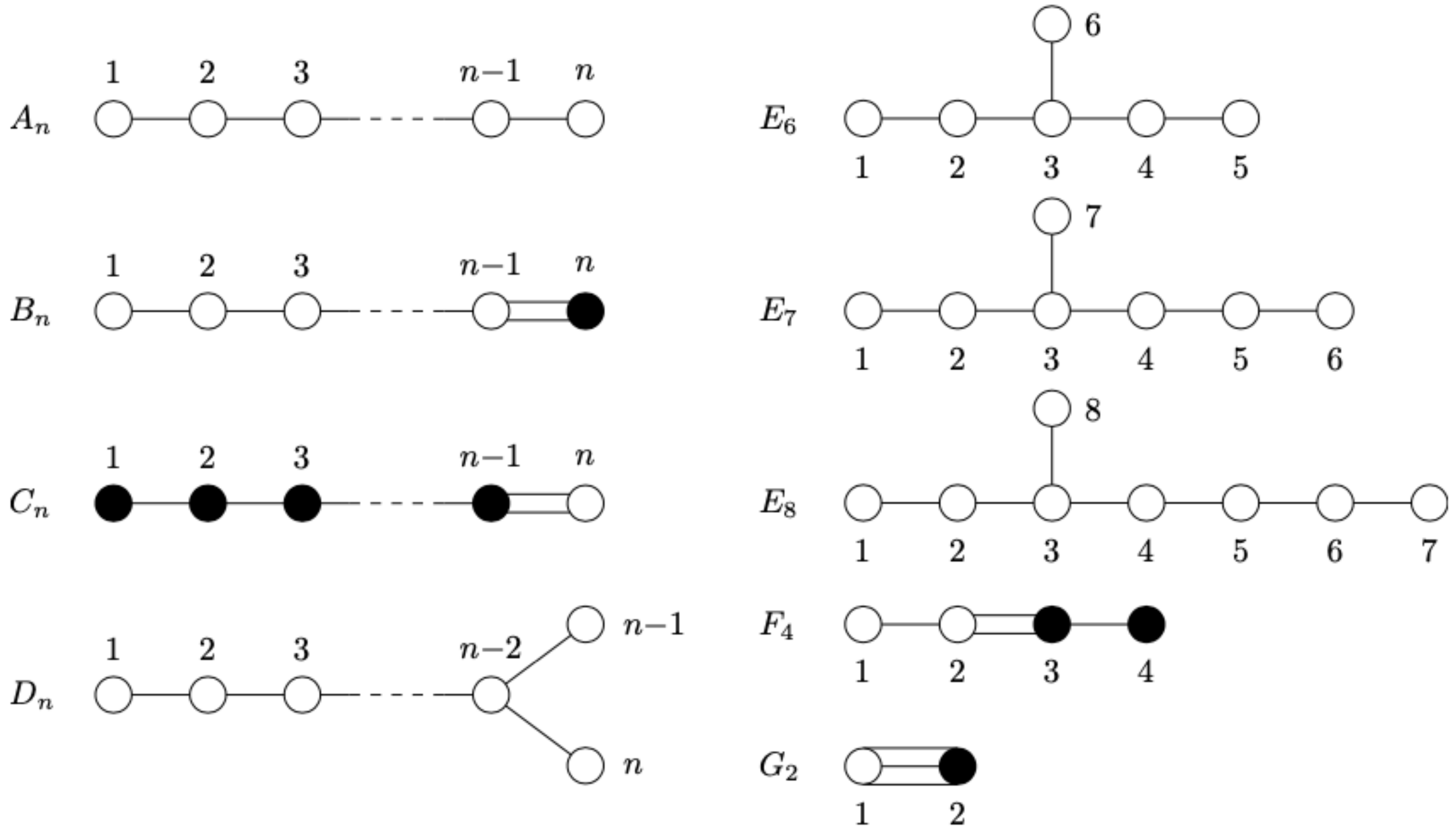


Figure 1: Dynkin Diagrams of classical and exceptional simple Lie algebras.

Type	Cartan	Name	Rank	Description
classical	A_n	SU(n+1)	$n \geq 1$	Special unitary algebras of n+1 complex dimension
	B_n	SO(2n+1)	$n \geq 3$	Special orthogonal algebras of odd (2n+1) real dimension
	C_n	Sp(2n)	$n \geq 2$	Symplectic algebras of even (2n) complex dimension
	D_n	SO(2n)	$n \geq 4$	Special orthogonal algebras of even (2n) real dimension
exceptional	E_6	E_6	6	Exceptional algebra of rank 6
	E_7	E_7	7	Exceptional algebra of rank 7
	E_8	E_8	8	Exceptional algebra of rank 8
	F_4	F_4	4	Exceptional algebra of rank 4
	G_2	G_2	2	Exceptional algebra of rank 2

Table 5.1: Classification of simple Lie algebras.

All Maximal Subalgebras through rank 15

Table 6.3: Maximal Subalgebras

Rank	Algebra		Maximal subalgebras	Type
1	SU(2)	⊃	U(1)	(*R*)
	(SU(2), SO(3), and Sp(2) are all isomorphic.)			
2	SU(3)	⊃	SU(2)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(4)	⊃	SU(2)⊗SU(2); SU(2)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	(SO(5) is isomorphic to Sp(4), and SO(4) is isomorphic to SU(2)⊗SU(2).)			
	G_2	⊃	SU(3); SU(2)⊗SU(2)	(*R*)
		⊃	SU(2)	(*S*)
3	SU(4)	⊃	SU(3)⊗U(1); SU(2)⊗SU(2)⊗U(1)	(*R*)
		⊃	Sp(4); SU(2)⊗SU(2)	(*S*)
	SO(7)	⊃	SU(4); SU(2)⊗SU(2)⊗SU(2); Sp(4)⊗U(1)	(*R*)
		⊃	G_2	(*S*)
	Sp(6)	⊃	SU(3)⊗U(1); SU(2)⊗Sp(4)	(*R*)
		⊃	SU(2); SU(2)⊗SU(2)	(*S*)
	(SO(6) is isomorphic to SU(4).)			
4	SU(5)	⊃	SU(4)⊗U(1); SU(3)⊗SU(2)⊗U(1)	(*R*)
		⊃	Sp(4)	(*S*)
	SO(9)	⊃	SO(8); SU(2) ⊗SU(2)⊗Sp(4); SU(4)⊗SU(2); SO(7)⊗U(1)	(*R*)
		⊃	SU(2); SU(2)⊗SU(2)	(*S*)
	Sp(8)	⊃	SU(4)⊗U(1); SU(2)⊗Sp(6); Sp(4)⊗Sp(4)	(*R*)
		⊃	SU(2); SU(2)⊗SU(2)⊗SU(2)	(*S*)
	SO(8)	⊃	SU(2)⊗SU(2)⊗SU(2)⊗SU(2); SU(4)⊗U(1)	(*R*)
		⊃	SU(3); SO(7); SU(2)⊗Sp(4)	(*S*)
	F_4	⊃	SO(9); SU(3)⊗SU(3); SU(2)⊗Sp(6)	(*R*)
		⊃	SU(2); SU(2)⊗G_2	(*S*)
5	SU(6)	⊃	SU(5)⊗U(1); SU(4)⊗SU(2)⊗U(1); SU(3)⊗SU(3)⊗U(1)	(*R*)
		⊃	SU(3); SU(4); Sp(6); SU(3)⊗SU(2)	(*S*)
	SO(11)	⊃	SO(10); SO(8)⊗SU(2); SU(4)⊗Sp(4); SU(2)⊗SU(2)⊗SO(7); SO(9)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(10)	⊃	SU(5)⊗U(1); SU(2)⊗Sp(8); Sp(4)⊗Sp(6)	(*R*)
		⊃	SU(2); SU(2)⊗Sp(4)	(*S*)
	SO(10)	⊃	SU(5)⊗U(1); SU(2)⊗SU(2)⊗SU(4); SO(8)⊗U(1)	(*R*)
		⊃	Sp(4); SO(9); SU(2)⊗SO(7); Sp(4)⊗Sp(4)	(*S*)
6	SU(7)	⊃	SU(6)⊗U(1); SU(5)⊗SU(2)⊗U(1); SU(4)⊗SU(3)⊗U(1)	(*R*)
		⊃	SO(7)	(*S*)
	SO(13)	⊃	SO(12); SO(10)⊗SU(2); SO(8)⊗Sp(4); SU(4)⊗SO(7); SU(2)⊗SU(2)⊗SO(9); SO(11)⊗U(1)	(*R*)

Rank	Algebra		Maximal subalgebras	Type
		⊃	SU(2)	(*S*)
	Sp(12)	⊃	SU(6)⊗U(1); SU(2)⊗Sp(10); Sp(4)⊗Sp(8); Sp(6)⊗Sp(6)	(*R*)
		⊃	SU(2); SU(2)⊗SU(4); SU(2)⊗Sp(4)	(*S*)
	SO(12)	⊃	SU(6)⊗U(1); SU(2)⊗SU(2)⊗SO(8); SU(4)⊗SU(4); SO(10)⊗U(1)	(*R*)
		⊃	SU(2)⊗Sp(6); SU(2)⊗SU(2)⊗SU(2); SO(11); SU(2)⊗SO(9); Sp(4)⊗SO(7)	(*S*)
	E_6	⊃	SO(10)⊗U(1); SU(6)⊗SU(2); SU(3)⊗SU(3)⊗SU(3)	(*R*)
		⊃	F_4; SU(3)⊗G_2; Sp(8); G_2; SU(3)	(*S*)
7	SU(8)	⊃	SU(7)⊗U(1); SU(6)⊗SU(2)⊗U(1); SU(5)⊗SU(3)⊗U(1); SU(4)⊗SU(4)⊗U(1)	(*R*)
		⊃	SO(8); Sp(8); SU(4)⊗SU(2)	(*S*)
	SO(15)	⊃	SO(14); SO(12)⊗SU(2); SO(10)⊗Sp(4); SO(8)⊗SO(7); SU(4)⊗SO(9); SU(2)⊗SU(2)⊗SO(11); SO(13)⊗U(1)	(*R*)
		⊃	SU(2); SU(4); SU(2)⊗Sp(4)	(*S*)
	Sp(14)	⊃	SU(7)⊗U(1); SU(2)⊗Sp(12); Sp(4)⊗Sp(10); Sp(6)⊗Sp(8)	(*R*)
		⊃	SU(2); SU(2)⊗SO(7)	(*S*)
	SO(14)	⊃	SU(7)⊗U(1); SU(2)⊗SU(2)⊗SO(10); SU(4)⊗SO(8); SO(12)⊗U(1)	(*R*)
		⊃	Sp(4); Sp(6); G_2; SO(13); SU(2)⊗SO(11); Sp(4)⊗SO(9); SO(7)⊗SO(7)	(*S*)
	E_7	⊃	E_6⊗U(1); SU(8); SO(12)⊗SU(2); SU(6)⊗SU(3)	(*R*)
		⊃	SU(2)⊗F_4; G_2⊗Sp(6); SU(2)⊗G_2; SU(3); SU(2)⊗SU(2); SU(2); SU(2)	(*S*)
8	SU(9)	⊃	SU(8)⊗U(1); SU(7)⊗SU(2)⊗U(1); SU(6)⊗SU(3)⊗U(1); SU(5)⊗SU(4)⊗U(1)	(*R*)
		⊃	SO(9); SU(3)⊗SU(3)	(*S*)
	SO(17)	⊃	SO(16); SO(14)⊗SU(2); SO(12)⊗Sp(4); SO(10)⊗SO(7); SO(8)⊗SO(9); SU(4)⊗SO(11); SU(2)⊗SU(2)⊗SO(13); SO(15)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(16)	⊃	SU(8)⊗U(1); SU(2)⊗Sp(14); Sp(4)⊗Sp(12); Sp(6)⊗Sp(10); Sp(8)⊗Sp(8)	(*R*)
		⊃	SU(2); Sp(4); SU(2)⊗SO(8)	(*S*)
	SO(16)	⊃	SU(8)⊗U(1); SU(2)⊗SU(2)⊗SO(12); SU(4)⊗SO(10); SO(8)⊗SO(8); SO(14)⊗U(1)	(*R*)
		⊃	SO(9); SU(2)⊗Sp(8); Sp(4)⊗Sp(4); SO(15); SU(2)⊗SO(13); Sp(4)⊗SO(11); SO(7)⊗SO(9)	(*S*)
	E_8	⊃	SO(16); SU(5)⊗SU(5); E_6⊗SU(3); E_7⊗SU(2); SU(9)	(*R*)
		⊃	G_2⊗F_4; SU(2)⊗SU(3); Sp(4); SU(2); SU(2); SU(2)	(*S*)

Rank	Algebra		Maximal subalgebras	Type
9	SU(10)	⊃	SU(9)⊗U(1); SU(8)⊗SU(2)⊗U(1); SU(7)⊗SU(3)⊗U(1); SU(6)⊗SU(4)⊗U(1); SU(5)⊗SU(5)⊗U(1)	(*R*)
		⊃	SU(3); SU(4); SU(5); Sp(4); SO(10); Sp(10); SU(5)⊗SU(2)	(*S*)
	SO(19)	⊃	SO(18); SO(16)⊗SU(2); SO(14)⊗Sp(4); SO(12)⊗SO(7); SO(10)⊗SO(9); SO(8)⊗SO(11); SU(4)⊗SO(13); SU(2)⊗SU(2)⊗SO(15); SO(17)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(18)	⊃	SU(9)⊗U(1); SU(2)⊗Sp(16); Sp(4)⊗Sp(14); Sp(6)⊗Sp(12); Sp(8)⊗Sp(10)	(*R*)
		⊃	SU(2); SU(2)⊗SO(9); SU(2)⊗Sp(6)	(*S*)
	SO(18)	⊃	SU(9)⊗U(1); SU(2)⊗SU(2)⊗SO(14); SU(4)⊗SO(12); SO(8)⊗SO(10); SO(16)⊗U(1)	(*R*)
		⊃	SU(2)⊗SU(4); SO(17); SU(2)⊗SO(15); Sp(4)⊗SO(13); SO(7)⊗SO(11); SO(9)⊗SO(9)	(*S*)
10	SU(11)	⊃	SU(10)⊗U(1); SU(9)⊗SU(2)⊗U(1); SU(8)⊗SU(3)⊗U(1); SU(7)⊗SU(4)⊗U(1); SU(6)⊗SU(5)⊗U(1)	(*R*)
		⊃	SO(11)	(*S*)
	SO(21)	⊃	SO(20); SO(18)⊗SU(2); SO(16)⊗Sp(4); SO(14)⊗SO(7); SO(12)⊗SO(9); SO(10)⊗SO(11); SO(8)⊗SO(13); SU(4)⊗SO(15); SU(2)⊗SU(2)⊗SO(17); SO(19)⊗U(1)	(*R*)
		⊃	SU(2); SU(2)⊗SO(7); SO(7); Sp(6)	(*S*)
	Sp(20)	⊃	SU(10)⊗U(1); SU(2)⊗Sp(18); Sp(4)⊗Sp(16); Sp(6)⊗Sp(14); Sp(8)⊗Sp(12); Sp(10)⊗Sp(10)	(*R*)
		⊃	SU(2); Sp(4)⊗Sp(4); SU(2)⊗SO(10); SU(6)	(*S*)
	SO(20)	⊃	SU(10)⊗U(1); SU(2)⊗SU(2)⊗SO(16); SU(4)⊗SO(14); SO(8)⊗SO(12); SO(10)⊗SO(10); SO(18)⊗U(1)	(*R*)
		⊃	SU(2)⊗Sp(10); SO(19); SU(2)⊗SO(17); Sp(4)⊗SO(15); SO(7)⊗SO(13); SO(9)⊗SO(11); SU(2)⊗SU(2)⊗Sp(4); SU(4)	(*S*)
11	SU(12)	⊃	SU(11)⊗U(1); SU(10)⊗SU(2)⊗U(1); SU(9)⊗SU(3)⊗U(1); SU(8)⊗SU(4)⊗U(1); SU(7)⊗SU(5)⊗U(1); SU(6)⊗SU(6)⊗U(1)	(*R*)
		⊃	SO(12); Sp(12); SU(6)⊗SU(2); SU(4)⊗SU(3)	(*S*)
	SO(23)	⊃	SO(22); SO(20)⊗SU(2); SO(18)⊗Sp(4); SO(16)⊗SO(7); SO(14)⊗SO(9); SO(12)⊗SO(11); SO(10)⊗SO(13); SO(8)⊗SO(15); SU(4)⊗SO(17); SU(2)⊗SU(2)⊗SO(19); SO(21)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(22)	⊃	SU(11)⊗U(1); SU(2)⊗Sp(20); Sp(4)⊗Sp(18); Sp(6)⊗Sp(16); Sp(8)⊗Sp(14); Sp(10)⊗Sp(12)	(*R*)
		⊃	SU(2)	(*S*)
	SO(22)	⊃	SU(11)⊗U(1); SU(2)⊗SU(2)⊗SO(18); SU(4)⊗SO(16); SO(8)⊗SO(14); SO(10)⊗SO(12); SO(20)⊗U(1)	(*R*)
		⊃	SO(21); SU(2)⊗SO(19); Sp(4)⊗SO(17); SO(7)⊗SO(15); SO(9)⊗SO(13); SO(11)⊗SO(11)	(*S*)

Rank	Algebra		Maximal subalgebras	Type
12	SU(13)	⊃	SU(12)⊗U(1); SU(11)⊗SU(2)⊗U(1); SU(10)⊗SU(3)⊗U(1); SU(9)⊗SU(4)⊗U(1); SU(8)⊗SU(5)⊗U(1); SU(7)⊗SU(6)⊗U(1)	(*R*)
		⊃	SO(13)	(*S*)
	SO(25)	⊃	SO(24); SO(22)⊗SU(2); SO(20)⊗Sp(4); SO(18)⊗SO(7); SO(16)⊗SO(9); SO(14)⊗SO(11); SO(12)⊗SO(13); SO(10)⊗SO(15); SO(8)⊗SO(17); SU(4)⊗SO(19); SU(2)⊗SU(2)⊗SO(21); SO(23)⊗U(1)	(*R*)
		⊃	SU(2); Sp(4)⊗Sp(4)	(*S*)
	Sp(24)	⊃	SU(12)⊗U(1); SU(2)⊗Sp(22); Sp(4)⊗Sp(20); Sp(6)⊗Sp(18); Sp(8)⊗Sp(16); Sp(10)⊗Sp(14); Sp(12)⊗Sp(12)	(*R*)
		⊃	SU(2); SU(2)⊗SU(2)⊗Sp(6); SU(2)⊗Sp(8); SU(4)⊗Sp(4)	(*S*)
	SO(24)	⊃	SU(12)⊗U(1); SU(2)⊗SU(2)⊗SO(20); SU(4)⊗SO(18); SO(8)⊗SO(16); SO(10)⊗SO(14); SO(12)⊗SO(12); SO(22)⊗U(1)	(*R*)
		⊃	SO(23); SU(2)⊗SO(21); Sp(4)⊗SO(19); SO(7)⊗SO(17); SO(9)⊗SO(15); SO(11)⊗SO(13); Sp(6)⊗Sp(4); SU(2)⊗SO(8); SU(5)	(*S*)
13	SU(14)	⊃	SU(13)⊗U(1); SU(12)⊗SU(2)⊗U(1); SU(11)⊗SU(3)⊗U(1); SU(10)⊗SU(4)⊗U(1); SU(9)⊗SU(5)⊗U(1); SU(8)⊗SU(6)⊗U(1); SU(7)⊗SU(7)⊗U(1)	(*R*)
		⊃	SO(14); Sp(14); SU(7)⊗SU(2)	(*S*)
	SO(27)	⊃	SO(26); SO(24)⊗SU(2); SO(22)⊗Sp(4); SO(20)⊗SO(7); SO(18)⊗SO(9); SO(16)⊗SO(11); SO(14)⊗SO(13); SO(12)⊗SO(15); SO(10)⊗SO(17); SO(8)⊗SO(19); SU(4)⊗SO(21); SU(2)⊗SU(2)⊗SO(23); SO(25)⊗U(1)	(*R*)
		⊃	SU(2); SU(3); SO(7); SU(2)⊗SO(9)	(*S*)
	Sp(26)	⊃	SU(13)⊗U(1); SU(2)⊗Sp(24); Sp(4)⊗Sp(22); Sp(6)⊗Sp(20); Sp(8)⊗Sp(18); Sp(10)⊗Sp(16); Sp(12)⊗Sp(14)	(*R*)
		⊃	SU(2)	(*S*)
	SO(26)	⊃	SU(13)⊗U(1); SU(2)⊗SU(2)⊗SO(22); SU(4)⊗SO(20); SO(8)⊗SO(18); SO(10)⊗SO(16); SO(12)⊗SO(14);SO(24)⊗U(1)	(*R*)
		⊃	SO(25); SU(2)⊗SO(23); Sp(4)⊗SO(21); SO(7)⊗SO(19); SO(9)⊗SO(17); SO(11)⊗SO(15); SO(13)⊗SO(13); F_4	(*S*)
14	SU(15)	⊃	SU(14)⊗U(1); SU(13)⊗SU(2)⊗U(1); SU(12)⊗SU(3)⊗U(1); SU(11)⊗SU(4)⊗U(1); SU(10)⊗SU(5)⊗U(1); SU(9)⊗SU(6)⊗U(1); SU(8)⊗SU(7)⊗U(1)	(*R*)
		⊃	SO(15); SU(5)⊗SU(3); SU(3); SU(3); SU(5); SU(6)	(*S*)
	SO(29)	⊃	SO(28); SO(26)⊗SU(2); SO(24)⊗Sp(4); SO(22)⊗SO(7); SO(20)⊗SO(9); SO(18)⊗SO(11); SO(16)⊗SO(13); SO(14)⊗SO(15); SO(12)⊗SO(17); SO(10)⊗SO(19); SO(8)⊗SO(21); SU(4)⊗SO(23); SU(2)⊗SU(2)⊗SO(25); SO(27)⊗U(1)	(*R*)
		⊃	SU(2)	(*S*)
	Sp(28)	⊃	SU(14)⊗U(1); SU(2)⊗Sp(26); Sp(4)⊗Sp(24); Sp(6)⊗Sp(22); Sp(8)⊗Sp(20); Sp(10)⊗Sp(18); Sp(12)⊗Sp(16); Sp(14)⊗Sp(14)	(*R*)

Rank	Algebra		Maximal subalgebras	Type
		⊃	SU(2); SO(7)⊗Sp(4)	(S)
	SO(28)	⊃	SU(14)⊗U(1); SU(2)⊗SU(2)⊗SO(24); SU(4)⊗SO(22); SO(8)⊗SO(20); SO(10)⊗SO(18); SO(12)⊗SO(16); SO(14)⊗SO(14); SO(26)⊗U(1)	(R)
		⊃	SO(27); SU(2)⊗SO(25); Sp(4)⊗SO(23); SO(7)⊗SO(21); SO(9)⊗SO(19); SO(11)⊗SO(17); SO(13)⊗SO(15); SU(2)⊗SU(2)⊗SO(7)	(S)
15	SU(16)	⊃	SU(15)⊗U(1); SU(14)⊗SU(2)⊗U(1); SU(13)⊗SU(3)⊗U(1); SU(12)⊗SU(4)⊗U(1); SU(11)⊗SU(5)⊗U(1); SU(10)⊗SU(6)⊗U(1); SU(9)⊗SU(7)⊗U(1); SU(8)⊗SU(8)⊗U(1)	(R)
		⊃	SO(16); Sp(16); SO(10); SU(8)⊗SU(2); SU(4)⊗SU(4)	(S)
	SO(31)	⊃	SO(30); SO(28)⊗SU(2); SO(26)⊗Sp(4); SO(24)⊗SO(7); SO(22)⊗SO(9); SO(20)⊗SO(11); SO(18)⊗SO(13); SO(16)⊗SO(15); SO(14)⊗SO(17); SO(12)⊗SO(19); SO(10)⊗SO(21); SO(8)⊗SO(23); SU(4)⊗SO(25); SU(2)⊗SU(2)⊗SO(27); SO(29)⊗U(1)	(R)
		⊃	SU(2)	(S)
	Sp(30)	⊃	SU(15)⊗U(1); SU(2)⊗Sp(28); Sp(4)⊗Sp(26); Sp(6)⊗Sp(24); Sp(8)⊗Sp(22); Sp(10)⊗Sp(20); Sp(12)⊗Sp(18); Sp(14)⊗Sp(16)	(R)
		⊃	SU(2); SU(2)⊗Sp(10); Sp(4)⊗Sp(6)	(S)
	SO(30)	⊃	SU(15)⊗U(1); SU(2)⊗SU(2)⊗SO(26); SU(4)⊗SO(24); SO(8)⊗SO(22); SO(10)⊗SO(20); SO(12)⊗SO(18); SO(14)⊗SO(16); SO(28)⊗U(1)	(R)
		⊃	SO(29); SU(2)⊗SO(27); Sp(4)⊗SO(25); SO(7)⊗SO(23); SO(9)⊗SO(21); SO(11)⊗SO(19); SO(13)⊗SO(17); SO(15)⊗SO(15); SU(2)⊗SO(10); Sp(4)⊗SU(4)	(S)

TABLE OF TABLES

SO(21)	A.31	71	A.90	119	A.149	271
SO(22)	A.32	71	A.91	120	A.150	275
SO(23)	A.33	72	A.92	120	A.151	278
SO(24)	A.34	72	A.93	120	A.152	281
SO(25)	A.35	73	A.94	121	A.153	285
SO(26)	A.36	73	A.95	121	A.154	289
SO(27)	A.37	74	A.96	121	A.155	293
SO(28)	A.38	74	A.97	121	A.156	298
SO(29)	A.39	75	A.98	121	A.157	302
SO(30)	A.40	75	A.99	122	A.158	306
SO(31)	A.41	75	A.100	122	A.159	310
Sp(4)	A.42	76	A.101	123	A.160	314
Sp(6)	A.43	78	A.102	124	A.161	316
Sp(8)	A.44	80	A.103	125	A.162	320
Sp(10)	A.45	81	A.104	126	A.163	324
Sp(12)	A.46	82	A.105	127	A.164	327
Sp(14)	A.47	83	A.106	127	A.165	332
Sp(16)	A.48	83	A.107	128	A.166	335
Sp(18)	A.49	84	A.108	128	A.167	339
Sp(20)	A.50	84	A.109	129	A.168	342
Sp(22)	A.51	84	A.110	129	A.169	344
Sp(24)	A.52	85	A.111	129	A.170	346
Sp(26)	A.53	85	A.112	129	A.171	349
Sp(28)	A.54	85	A.113	130	A.172	351
Sp(30)	A.55	85	A.114	130	A.173	354
E_6	A.56	86	A.115	131	A.174	358
E_7	A.57	87	A.116	132	A.175	363
E_8	A.58	87	A.117	132	A.176	368
F_4	A.59	88	A.118	133	A.177	371
G_2	A.60	90	A.119	135	A.178	376

Table A.1: Table of tables

```
In[38]:= << LieART`Tables`

In[39]:= BranchingRulesTable[E8, {SU2}, 1, MaxDim -> 147250]
```

Out[38]:=

E_8	→	SU(2)
248	=	3 + 11 + 15 + 19 + 23 + 27 + 29 + 35 + 39 + 47
3875	=	2(1) + 3(5) + 7 + 4(9) + 2(11) + 6(13) + 3(15) + 6(17) + 4(19) + 7(21) + 4(23) + 7(25) + 5(27) + 7(29) + 5(31) + 6(33) + 4(35) + 7(37) + 4(39) + 5(41) + 3(43) + 5(45) + 3(47) + 4(49) + 2(51) + 3(53) + 2(55) + 2(57) + 59 + 2(61) + 63 + 65 + 69 + 73
27000	=	7(1)+3+13(5)+7(7)+19(9)+14(11)+25(13)+19(15)+29(17)+23(19)+33(21)+26(23)+35(25)+28(27)+36(29)+28(31)+35(33)+28(35)+34(37)+27(39)+31(41)+24(43)+28(45)+22(47)+25(49)+18(51)+21(53)+15(55)+18(57)+12(59)+14(61)+9(63)+11(65)+7(67)+8(69)+5(71)+6(73)+3(75)+4(77)+2(79)+3(81)+83+2(85)+89+93
30380	=	10(3)+6(5)+17(7)+14(9)+24(11)+22(13)+30(15)+26(17)+35(19)+31(21)+37(23)+34(25)+40(27)+34(29)+40(31)+34(33)+38(35)+34(37)+36(39)+30(41)+33(43)+27(45)+29(47)+24(49)+25(51)+19(53)+21(55)+16(57)+16(59)+13(61)+13(63)+9(65)+10(67)+6(69)+7(71)+5(73)+5(75)+2(77)+3(79)+2(81)+2(83)+85+87+91
147250	=	8(1) + 22(3) + 41(5) + 49(7) + 69(9) + 80(11) + 93(13) + 102(15) + 118(17) + 121(19) + 133(21)+138(23)+144(25)+147(27)+153(29)+149(31)+153(33)+151(35)+149(37)+144(39)+144(41)+134(43)+132(45)+124(47)+118(49)+110(51)+105(53)+94(55)+89(57)+81(59)+73(61)+66(63)+61(65)+51(67)+47(69)+41(71)+36(73)+30(75)+27(77) + 21(79) + 19(81) + 16(83) + 12(85) + 10(87) + 9(89) + 6(91) + 5(93) + 4(95) + 3(97) + 2(99) + 2(101) + 105 + 107

Click on A.174 to find:

$E_6 \to F_4$ (S)		
$\mathbf{27}$	=	$\mathbf{1} + \mathbf{26}$
$\mathbf{78}$	=	$\mathbf{26} + \mathbf{52}$
$\mathbf{351}$	=	$\mathbf{26} + \mathbf{52} + \mathbf{273}$
$\mathbf{351'}$	=	$\mathbf{1} + \mathbf{26} + \mathbf{324}$
$\mathbf{650}$	=	$\mathbf{1} + 2(\mathbf{26}) + \mathbf{273} + \mathbf{324}$
$\mathbf{1728}$	=	$\mathbf{26} + \mathbf{52} + \mathbf{273} + \mathbf{324} + \mathbf{1053}$
$\mathbf{2430}$	=	$\mathbf{324} + \mathbf{1053} + \mathbf{1053'}$
$\mathbf{2925}$	=	$\mathbf{52} + 2(\mathbf{273}) + \mathbf{1053} + \mathbf{1274}$
$\mathbf{3003}$	=	$\mathbf{1} + \mathbf{26} + \mathbf{324} + \mathbf{2652}$
$\mathbf{5824}$	=	$\mathbf{26} + \mathbf{52} + \mathbf{273} + \mathbf{324} + \mathbf{1053} + \mathbf{4096}$
$\mathbf{7371}$	=	$\mathbf{26} + \mathbf{52} + 2(\mathbf{273}) + \mathbf{324} + \mathbf{1053} + \mathbf{1274} + \mathbf{4096}$
$\mathbf{7722}$	=	$\mathbf{1} + 2(\mathbf{26}) + \mathbf{273} + 2(\mathbf{324}) + \mathbf{2652} + \mathbf{4096}$
$\mathbf{17550}$	=	$\mathbf{273} + \mathbf{324} + 2(\mathbf{1053}) + \mathbf{1053'} + \mathbf{1274} + \mathbf{4096} + \mathbf{8424}$
$\mathbf{19305}$	=	$\mathbf{26} + \mathbf{52} + \mathbf{273} + \mathbf{324} + \mathbf{1053} + \mathbf{2652} + \mathbf{4096} + \mathbf{10829}$

Tables from appendix are in supplementary material of paper.

BENCHMARKS

As an example for subalgebra decomposition of a large irrep we decompose the **6696000** of E_8 to $G_2 \otimes F_4$:

```
In[49]:= Timing[DecomposeIrrep[Irrep[E8][6696000], ProductAlgebra[G2, F4]]]
```

Out[48]:=

{1066.14, 2(**7**, **1**) + 2(**14**, **1**) + (**1**, **26**) + (**27**, **1**) + 6(**7**, **26**) + 5(**14**, **26**) + 2(**1**, **52**) + 6(**27**, **26**) + 3(**7**, **52**) +
2(**64**, **1**) + 3(**14**, **52**) + 2(**77**, **1**) + 5(**27**, **52**) + 5(**64**, **26**) + 4(**77**, **26**) + (**77′**, **26**) + 3(**64**, **52**) + 2(**77**, **52**) +
(**77′**, **52**) + (**189**, **1**) + (**182**, **26**) + 2(**189**, **26**) + (**182**, **52**) + (**189**, **52**) + 3(**1**, **273**) + 6(**7**, **273**) + 4(**14**, **273**) +
8(**27**, **273**) + (**1**, **324**) + 6(**7**, **324**) + 5(**64**, **273**) + 5(**14**, **324**) + 3(**77**, **273**) + (**77′**, **273**) + 4(**27**, **324**) +
3(**64**, **324**) + 3(**77**, **324**) + (**182**, **273**) + (**189**, **273**) + (**189**, **324**) + 2(**1**, **1053**) + 5(**7**, **1053**) + (**7**, **1053′**) +
3(**14**, **1053**) + (**14**, **1053′**) + 5(**27**, **1053**) + 3(**64**, **1053**) + 2(**77**, **1053**) + (**77**, **1053′**) + (**189**, **1053**) +
2(**1**, **1274**) + 2(**7**, **1274**) + 2(**14**, **1274**) + 3(**27**, **1274**) + (**64**, **1274**) + (**77**, **1274**) + (**77′**, **1274**) +
2(**7**, **2652**) + (**14**, **2652**) + (**27**, **2652**) + 2(**1**, **4096**) + 5(**7**, **4096**) + 3(**14**, **4096**) + 4(**27**, **4096**) +
2(**64**, **4096**) + (**77**, **4096**) + 2(**7**, **8424**) + (**14**, **8424**) + (**27**, **8424**) + (**64**, **8424**) + (**1**, **10829**) + (**7**, **10829**) +
(**14**, **10829**) + (**27**, **10829**) + (**1**, **19278**) + (**7**, **19278**) + (**27**, **19278**) + (**7**, **19448**) + (**14**, **19448**) +
(**1**, **34749**) + (**7**, **34749**)}

FUN WITH LIEART

- SU(2) irreps have dimensions: 1, 2, 3, 4, 5,…
- Others look more random. E.g., SU(3) irreps are of dim:1, 3, 3bar, 6, 6bar, 8, 10, 10bar, 15, 15bar 15', 15'bar, 21, 21bar, 24, 27, …
- There is a sequence of SU(3) irreps (0,0), (1,1), (2,2), (3,3),…

with dim

$1, 8, 27, 64, \ldots = 1^3, 2^3, 3^3, 4^3, 5^3, \ldots$

- Similar result hold for any classical or exceptional group where [a,a,a,a,…a] has dim $(a+1)^p$
- All have series of irreps 1^p, 2^p, 3^p, 4^p, … where for A_n, p = 1, 3, 6, 10,…(binomial coeff) for n = 1,2,3,4,… for B_n and C_n p = n2, for D_n, p = 2X the binomial coeff of A_n and for the exceptionals p = 6, 24, 36, 63 and 120 for G2, F4, E6, E7 and E8.
- E.g., Irrep[E8][9,9,9,9,9,9,9,9]= 10^{120}

which is an exact result.

ONE-LINE PROOF FROM WEYL DIM FORMULA

First we rewrite the Weyl's dimension formula for a general irrep Λ in the form

$$\dim(\Lambda) = \frac{\prod_{\alpha\in\Delta^+}(\Lambda+\delta,\alpha)}{\prod_{\alpha\in\Delta^+}(\delta,\alpha)} \tag{16}$$

where Δ^+ is the set of positive roots α. Recall that in the Dynkin bases one has $\delta = (1,1,1,...,1)$. Now for the specific irrep $\Lambda = (a,a,a,...,a) = a\delta$ that means we have

$$\dim(\Lambda) = \frac{\prod_{\alpha\in\Delta^+}(a+1)(\delta,\alpha)}{\prod_{\alpha\in\Delta^+}(\delta,\alpha)} = \frac{\prod_{\alpha\in\Delta^+}(a+1)}{\prod_{\alpha\in\Delta^+}(1)} = (a+1)^p \tag{17}$$

where p is the number of positive roots, i.e., the number of elements in the set $\Delta+$, which agrees with the numbers in the examples above.

END

ENTERING IRREPS

- Entering the 10 of SU(5) by its Dynkin label and algebra class:

In[2]:= **Irrep[A][0,1,0,0]//FullForm**

Out[2]:= Irrep[A][0,1,0,0]

- In[3]:= **Irrep[A][0,1,0,0]//StandardForm**
- Out[3]:= (0100)
- TraditionalForm (default):
- In[4]:= **Irrep[A][0,1,0,0]**
- Out[4]:= 10

ENTERING IRREPS

- Entering the 10 of SU(5) by its dimensional name specifying the algebra by its Dynkin classification in A_4:
- In[6]:= **Irrep[A4][10]//InputForm**
- Out[6]:= Irrep[A][0,1,0,0]
- The traditional name of the algebra SU(5) may also be used: In[7]:= **Irrep[SU5][10]//InputForm**
- Out[7]:= Irrep[A][0,1,0,0]

DECOMPOSING TENSOR PRODUCTS

- In[14]:=**DecomposeProduct[Irrep[E6][27],Irrep[E6][Bar[27]]]**
- Out[14]:= 1+78+650

- Decompose the tensor product 3⊗3⊗3 of SU(3):
- In[15]:= **DecomposeProduct[Irrep[SU3][3],Irrep[SU3][3],Irrep[SU3][3]]**
- Out[15]:= 1+2(8)+10

DECOMPOSITION TO SUBALGEBRAS (VIA PROJECTION MATRICES)

- Decompose the 16 of SO(10) to SU(5)⊗U(1):
- In[26]:= **DecomposeIrrep[Irrep[SO10][16],ProductAlgebra[SU5,U1]]**
- Out[26]:= (1)(−5)+(5bar)(3)+(10)(−1)

- E.g., E_7 has two SU(2) special maximal subalgebras
- In[34]:= **DecomposeIrrep[Irrep[E7][56], SU2, 1]**
- Out[34]:= 10+18+28
- In[35]:= **DecomposeIrrep[Irrep[E7][56], SU2, 2]**
- Out[35]:= 6+12+16+22

String Inspired Search for
nonSUSY Models with:

Small product gauge groups

Either Zero or a Few Extra Chiral Fermions

New Phenomenology

CONCLUSIONS

- We continue to develop LieART to make it more:
- user friendly
- versatile
- comprehensive

- NEXT
- SuperLieART ?

$$
\begin{array}{ccc}
G = SU(a) \times SU(b) \times SU(c) \in \mathcal{G}' & & \mathbf{R} \\
\Big\downarrow \text{NAB } (E \in \mathcal{E}) & & \Big\downarrow \\
\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)^m & & \mathbf{R}_E \\
\Big\downarrow \text{AB } (x \in \mathbb{R}^m) & \text{plus Higgs doublet} & \Big\downarrow \\
G_{SM} & & \mathbf{R}_x
\end{array}
$$