

# Antiproton–Nucleus reactions with the Liège IntraNuclear Cascade (INCL) code

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PhD Thesis of Demid Zharenov



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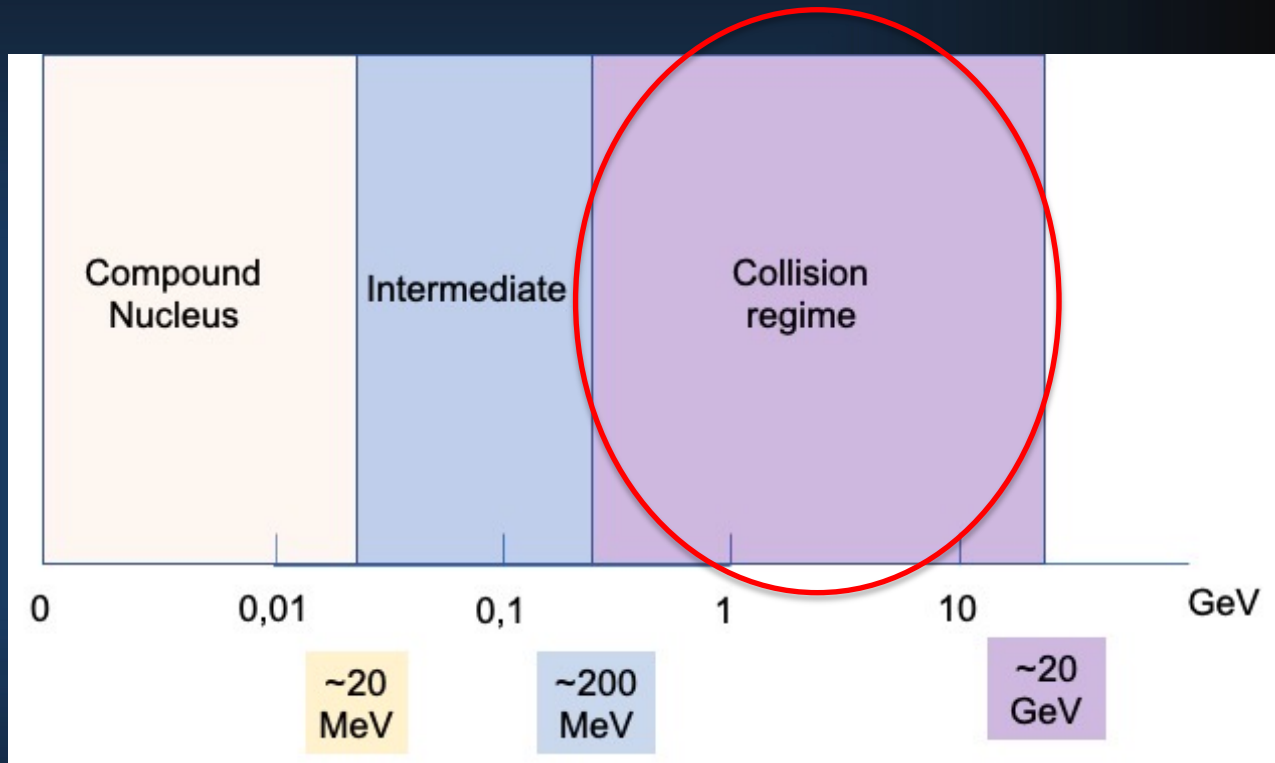
# Plan

- What is INCL?
  - Generalities
  - Capabilities
- Antiprotons in INCL
  - 90s...
  - Two mechanisms
  - Hypotheses – ingredients
  - Results



# What is INCL? Generalities

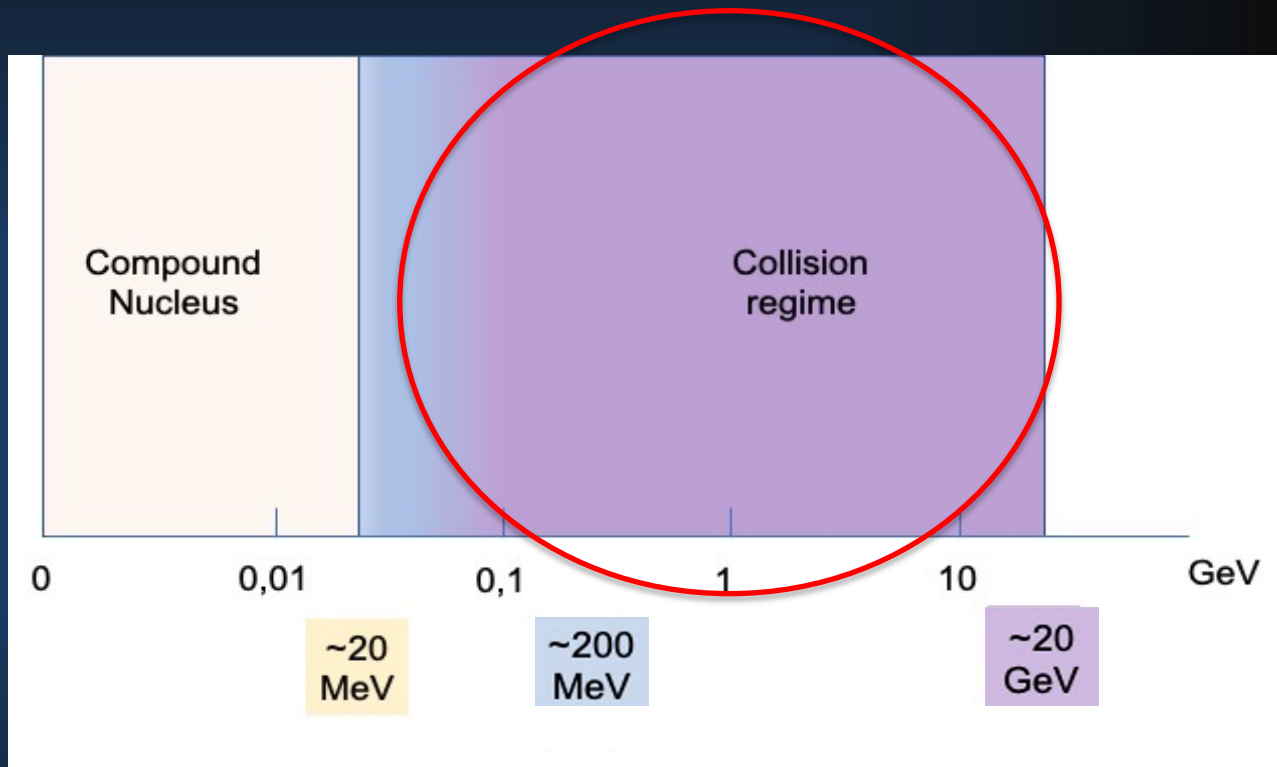
- Nuclear reaction between a light particle and a nucleus
- Energetic domain: Collision regime
- Basic assumptions
  - Intranuclear cascade
  - Binary collisions
  - Asymptotic states reached before next collision
  - Classical trajectories
  - Some quantum effects accounted for (ex. Pauli)
- Monte Carlo method
- Ingredients
  - Fermi gas model (nucleus)
  - Potential
  - Elementary cross sections
  - Final states
  - Pauli implementations
  - Cluster production
  - ...



# What is INCL? Generalities

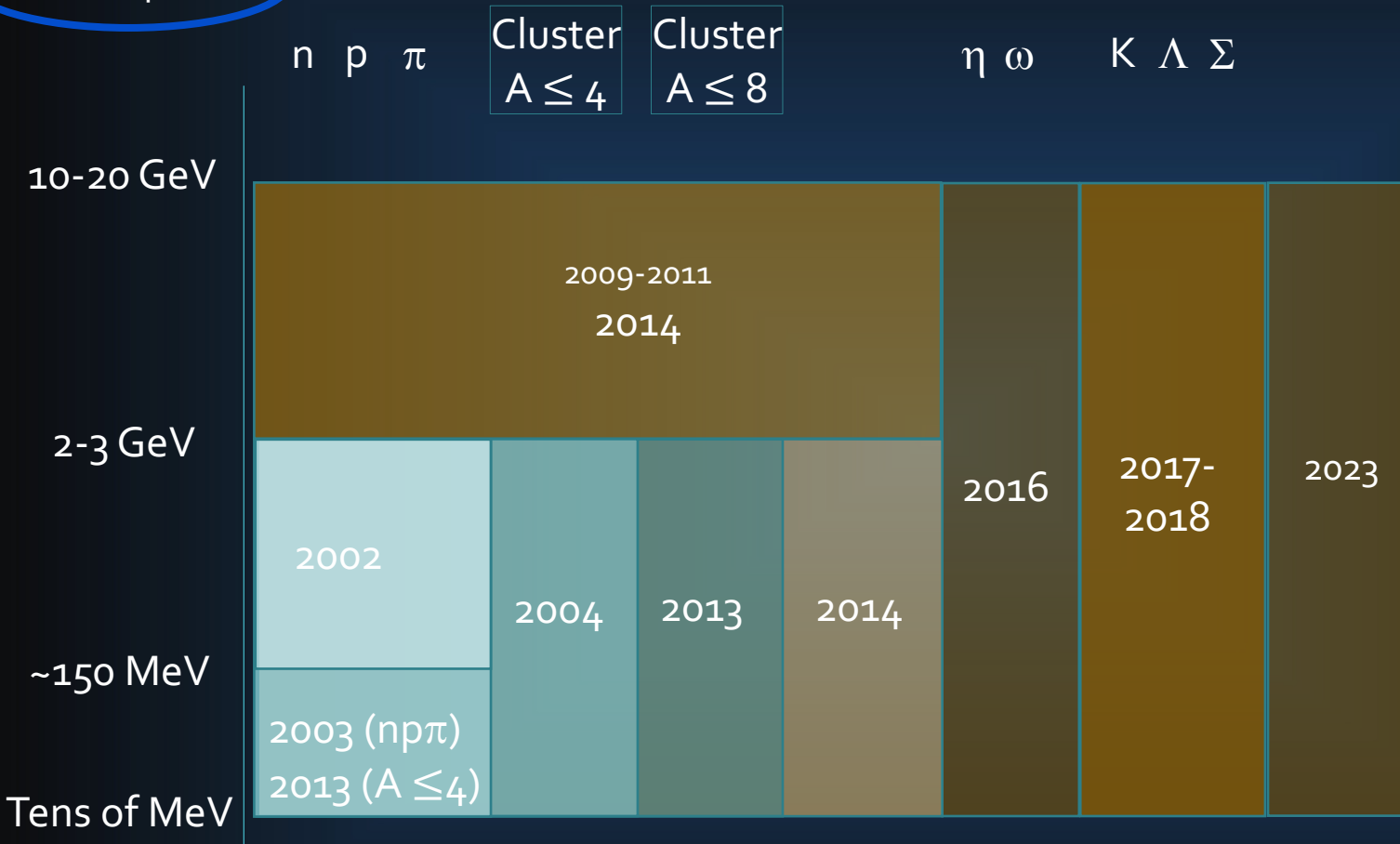
Other aspects...

- Low-energy limit lower than expected
- After the cascade follows a deexcitation (usually we use ABLA)
- INCL (and ABLA) implemented in Geant4



# What is INCL? Capabilities

Produced particles



Projectile particles



Antiproton-Nucleus reactions with the INCL code

# Antiprotons in INCL 90s...

Once upon a time...

At the turn of the 90s (LEAR era) several studies of pbar simulation **with INCL** were done  
@ Liège: Cugnon - Vandermeulen – Deneve  
at rest – in-flight; strange particle; collision on two nucleons; ...  
With rather good results.

But

this version of the code has been lost  
some assumptions are no more needed now  
INCL has been improved since then (and Fortran → C++)  
and a renew interest of pbar physics (AD...)

So

a new implementation of pbar in an up-to-date INCL based on the previous one  
has been done by D. Zharenov (PhD student) in 2023



# Antiprotons in INCL

## Two mechanisms

### In-flight (usual interaction)

- Impact parameter
- Coulomb(or not)
- Enter the nucleus

### At rest

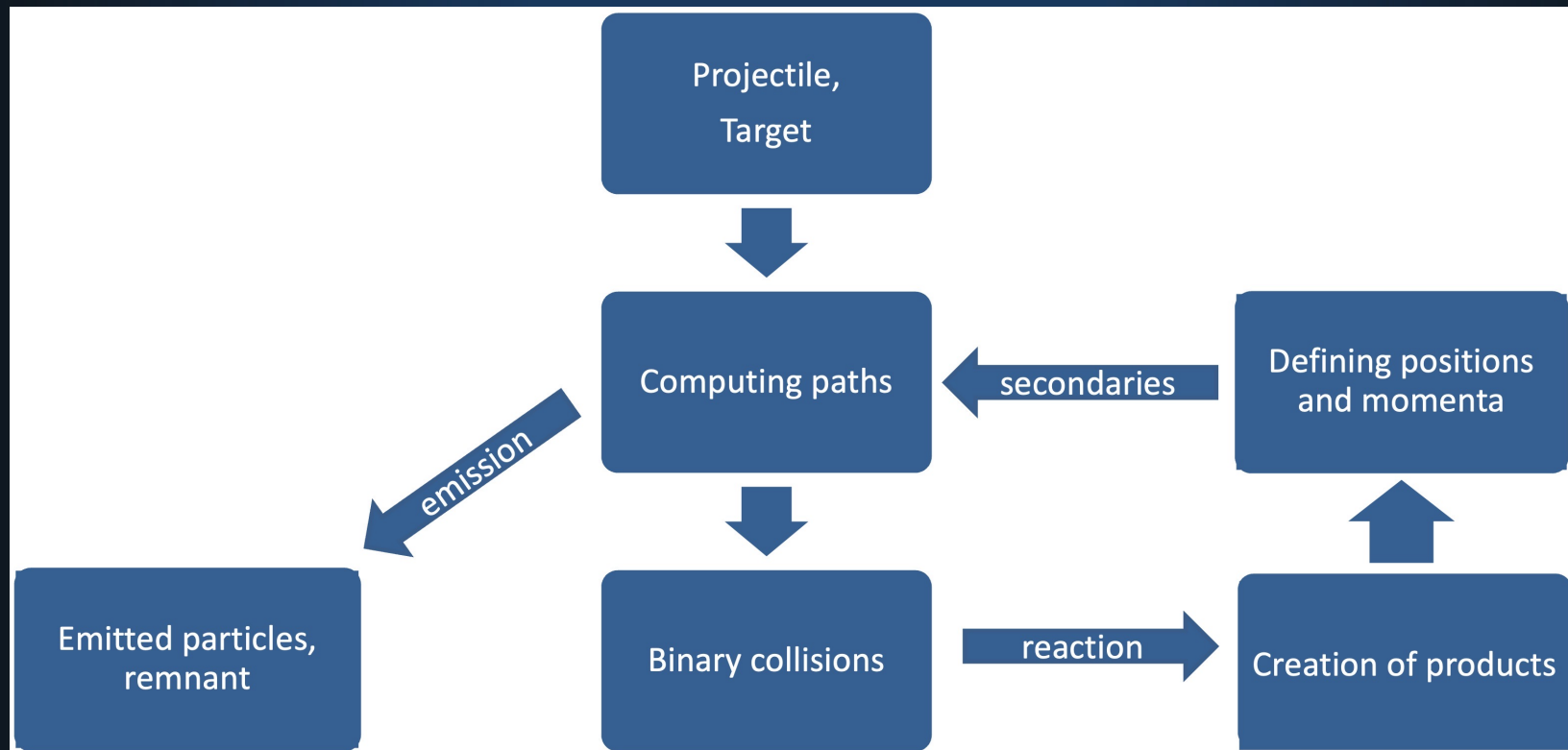
- Very low energy
- Captured in an electronic orbit
- Moved down from a high to the last orbit (annihilation)
- Cascade initiated by the annihilation products



# Antiprotons in INCL

## Two mechanisms

In-flight (usual interaction)

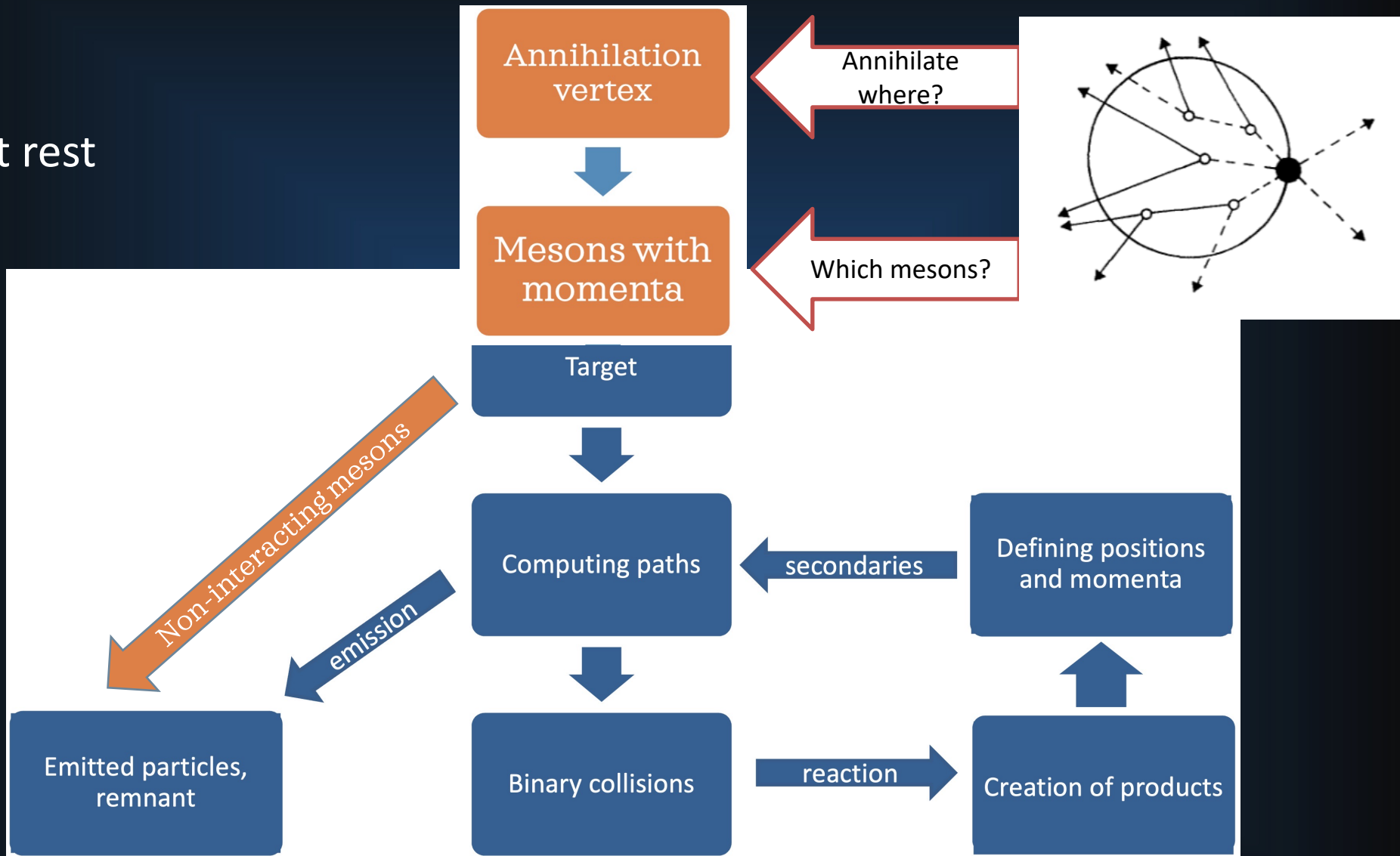




# Antiprotons in INCL

## Two mechanisms

At rest



# Antiprotons in INCL

## Hypotheses – ingredients

# Antiprotons in INCL Hypotheses – ingredients

At rest - Choice of nucleon to annihilate

$$S_p/S_n(Z, A) = S_p/S_n(D_2) \frac{Z}{A-Z}$$

Model assumption

$$S_p/S_n(D_2) = 1.331$$

R. Bizzarri

Il Nuovo Cimento A (1965-1970) 53.4 (Feb. 1968), pp. 956–968

From other values

Group	$S_p/S_n$
Rome-Syracuse[Bar+64]	$1.31 \pm 0.03$
Berkeley[CK66]	$1.33 \pm 0.07$
Padova-Pisa[Bet+67]	$1.45 \pm 0.07$

And, for a same experiment

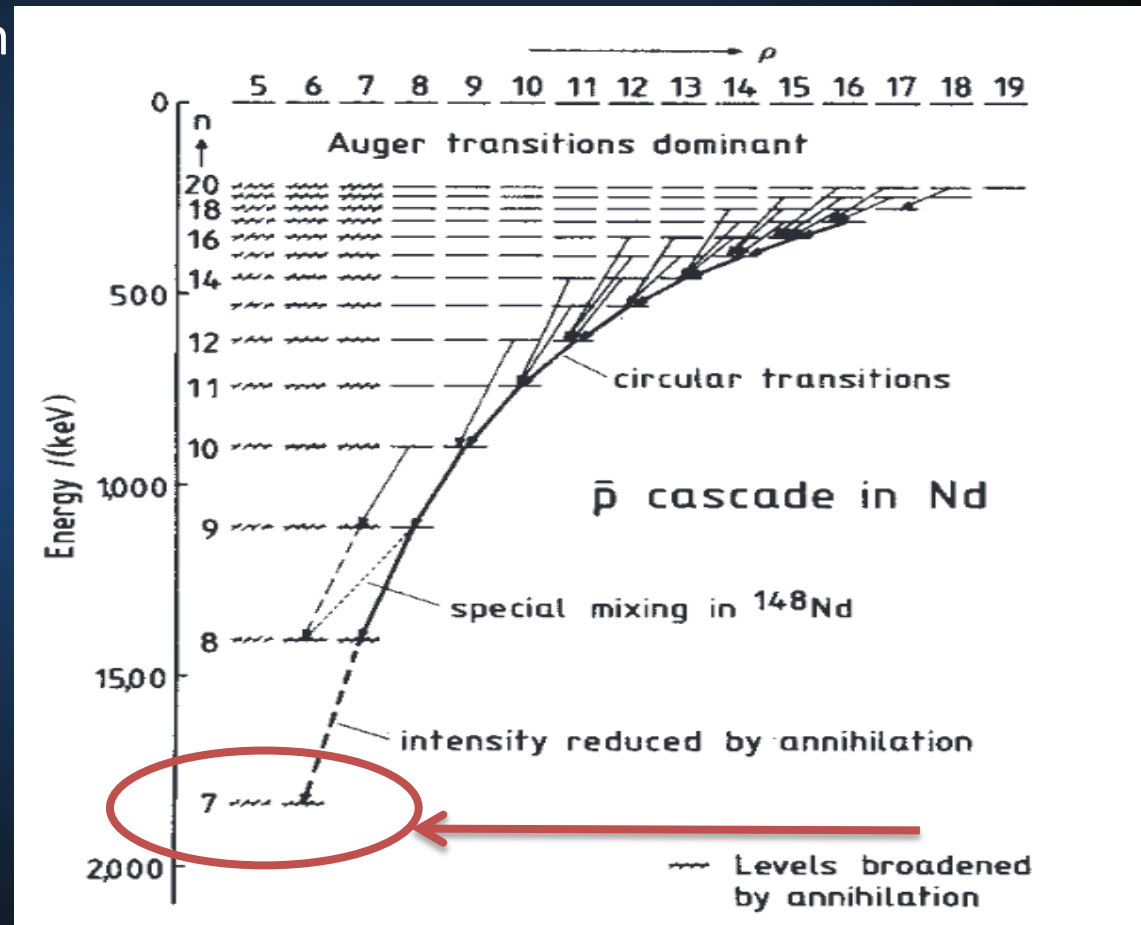
$S_p/S_n(D_2)$  between 57 and 170 MeV can range between 1.113 and 1.369



# Antiprotons in INCL Hypotheses – ingredients

## At rest - Position of annihilation

- $\bar{p}$   
Captured in a high Bohr orbit  
Cascades toward the nucleus  
Stops/annihilates at a given « n »

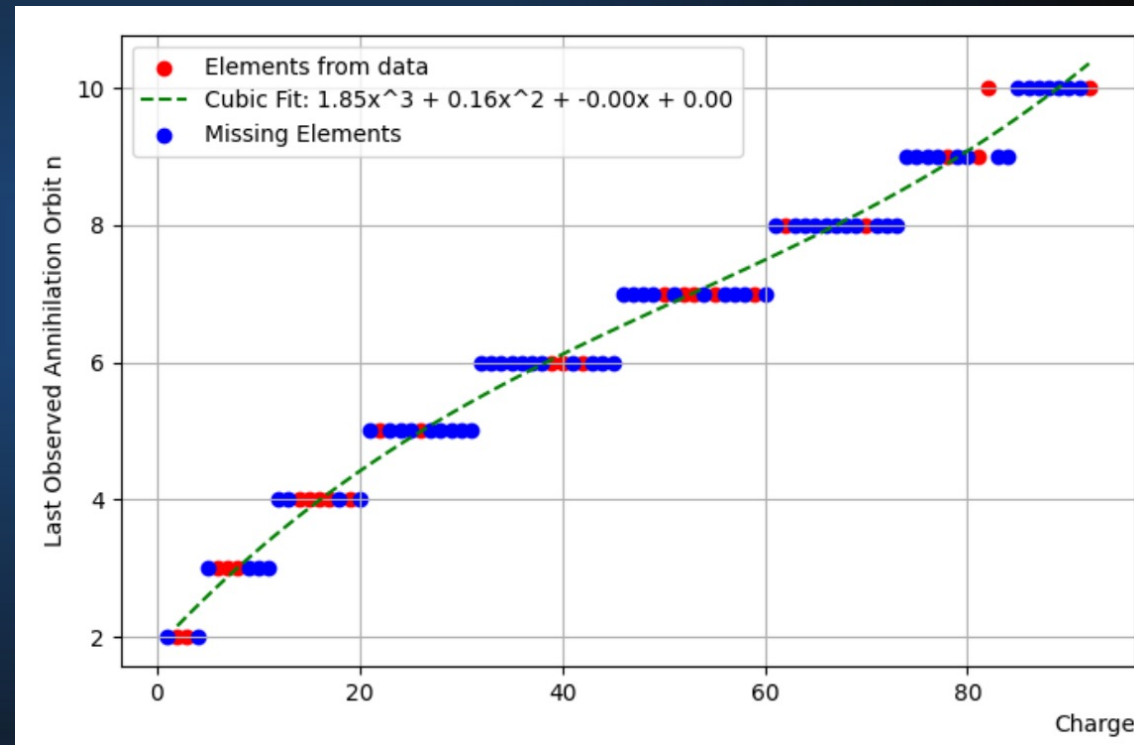


# Antiprotons in INCL

## Hypotheses – ingredients

### At rest - Position of annihilation

- $\bar{p}$   
Captured in a high Bohr orbit  
Cascades toward the nucleus  
Stops/annihilates at a given « n »
- Determination of « n »  
(fits from exp. Data)

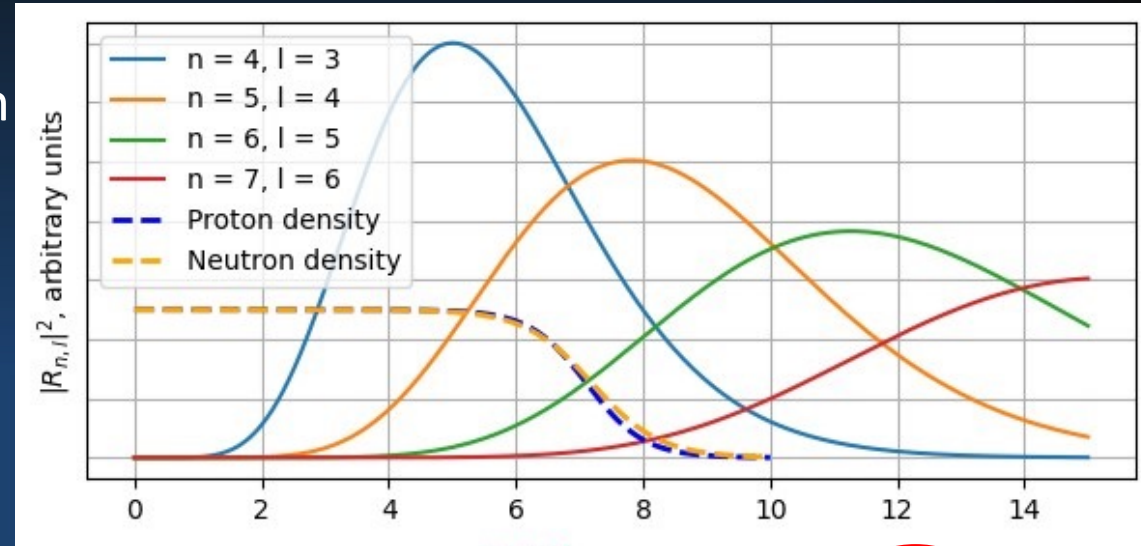


# Antiprotons in INCL

## Hypotheses - ingredients

### At rest - Position of annihilation

- $\bar{p}$   
Captured in a high Bohr orbit  
Cascades toward the nucleus  
Stops/annihilates at a given « n »
- Determination of « n »  
(fits from exp. Data)
- Position of annihilation  
→ When overlap of nuclear density and antiprotonic radial density



$$P_{neutronic}(r) = N_{nl} \times \rho_n \times r^2 \times |R_{n,l}|^2$$

$$P_{protonic}(r) = N_{nl} \times \rho_p \times r^2 \times |R_{n,l}|^2$$

$$|R_{n,l=n-1}| = ((2n)!)^{\frac{1}{2}} \left(\frac{2}{na}\right)^{\frac{3}{2}} \left(\frac{2r}{na}\right)^{(n-1)} \exp\left(\frac{-r}{na}\right)$$

$$N_{nl} = \frac{2}{n^2} \sqrt{\frac{(n-l-1)!}{((n+l)!)^3}} \quad (\text{a is the Bohr radius})$$



# Antiprotons in INCL

## Hypotheses – ingredients

### At rest - Final states

- In INCL we consider only  $\pi$ ,  $\eta$ ,  $\omega$  and K ( $\rho$  goes directly to decay products)
- Kaon frequency is put at 5%
  - 2 old values 6.82 +/- 0.25 % and 4.74 +/- 0.22 %
  - « Recent » one 5.4 +/- 1.7 %
- Final states with  $\pi$ ,  $\eta$ ,  $\omega$  taken from
  - Eberhard Klempt et al.  
Physics Reports 413.4-5 (July 2005), pp. 197–317.
  - E.S. Golubeva et al.  
Nuclear Physics A 537.3 (1992), pp. 393–417.

Channel	Probability (%)
$\eta\eta$	0.01 (1)
$\eta\omega$	0.34 (1)
$\omega\omega$	1.57 (1)
$\pi^+\pi^-$	0.40 (1)
$\pi^0\pi^0$	0.02 (1)
$\pi^+\rho^-$	1.52 (1)
$\pi^-\rho^+$	1.52 (1)
$\pi^0\rho^0$	1.57 (1)
$\rho^-\rho^+$	3.37 (2)
$\rho^0\rho^0$	0.67 (1)

Example  
Of  
Final States



# Antiprotons in INCL Results

- Multiplicities
  - $\pi$
  - n, p, d, t,  $^3\text{He}$ ,  $\alpha$
  - K
- Spectra
  - $\pi^+$
  - p
- Residues

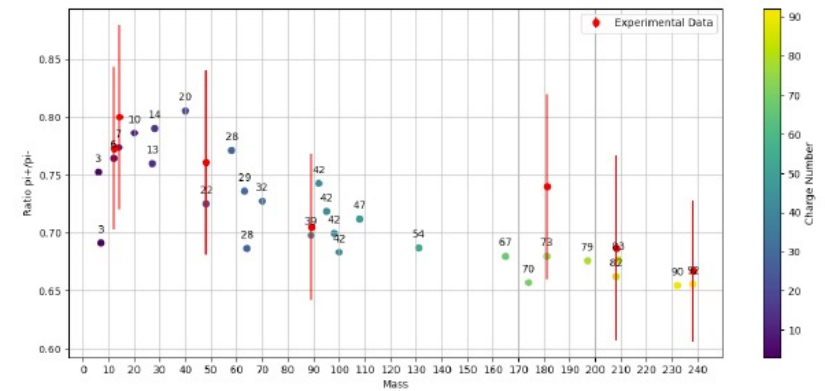
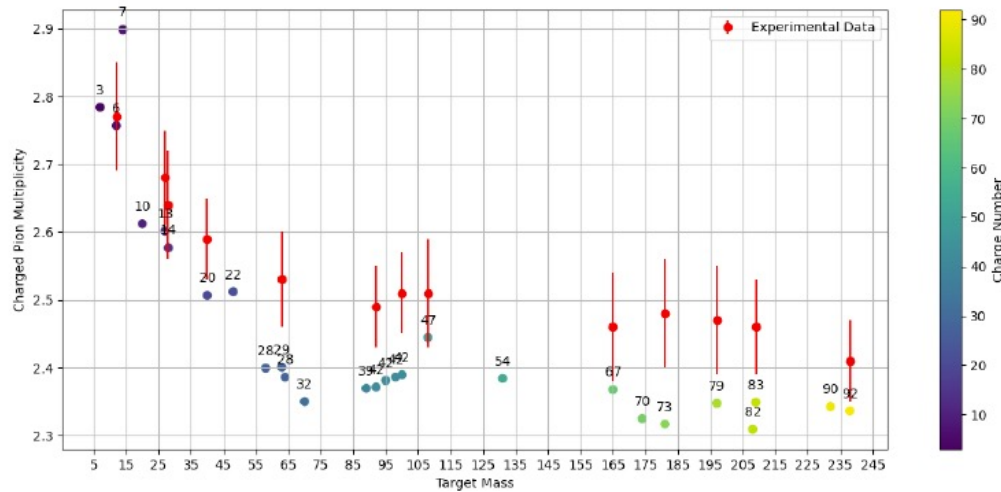




# Antiprotons in INCL Results

Multiplicities  $\pi^{+/-}$

Ratio  $\pi^{+}/\pi^{-}$



**Figure 5.9:** Ratio  $\pi^{+}/\pi^{-}$  for a group of target nuclei from INCL, experimental points are taken from [Bug+73; McG+86a; Rie+89; WL76]. No nuclei with the same mass number are present on this plot; the charge numbers are color-coded. Note, that incident beam energy was slightly different in the references, but in all cases assumed to annihilate through the capture. Also, sometimes materials of natural isotope composition were used as targets, in these cases the mass value was taken as for most abundant isotope.

- Ratio: Good!
- Multiplicities: quite good, except a little too low multiplicities (4% too low)

Could come from

Lack of information on annihilation with (very) high meson multiplicity

*Not exact* annihilation position

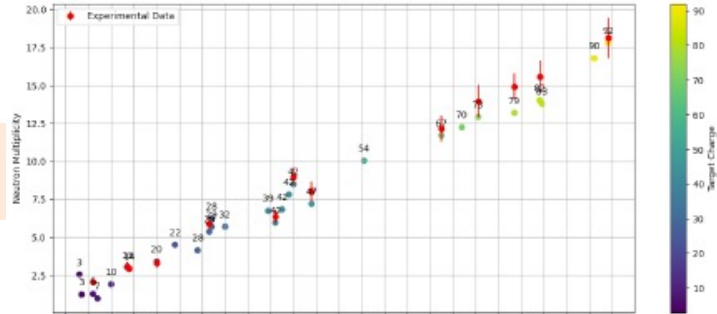


# Antiprotons in INCL Results

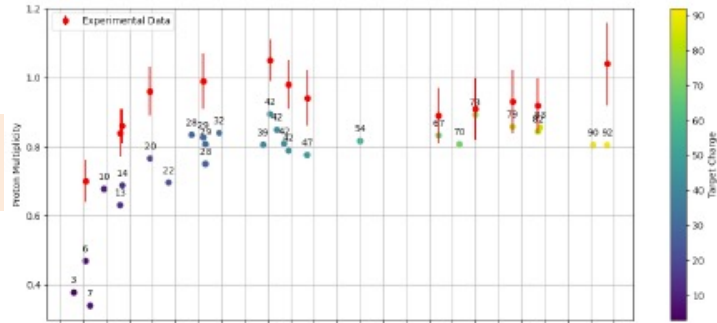
## Multiplicities n & p

## Multiplicities d & t

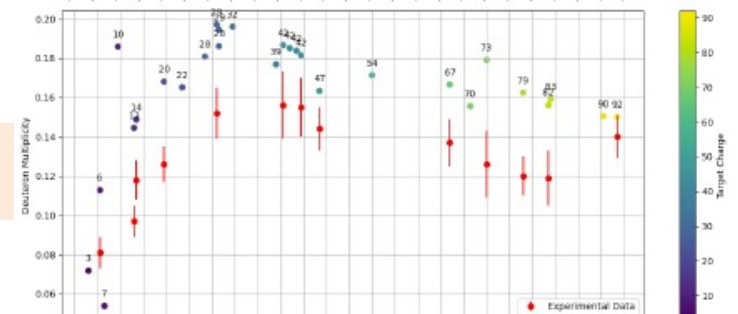
n



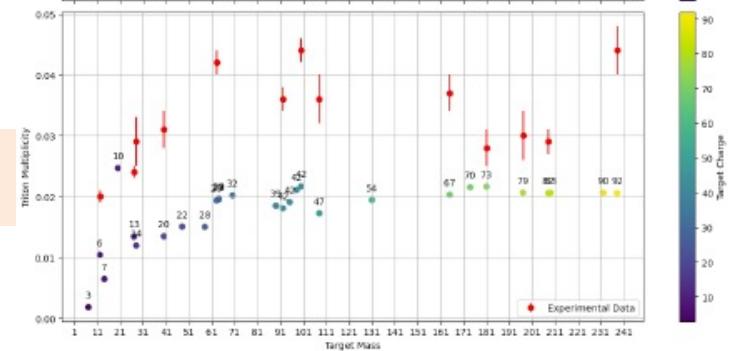
p



d



t



- n: perfect
- p: little underestimation (< 20%)
- d: overestimation (< 25%)
- t: underestimation (< x2)

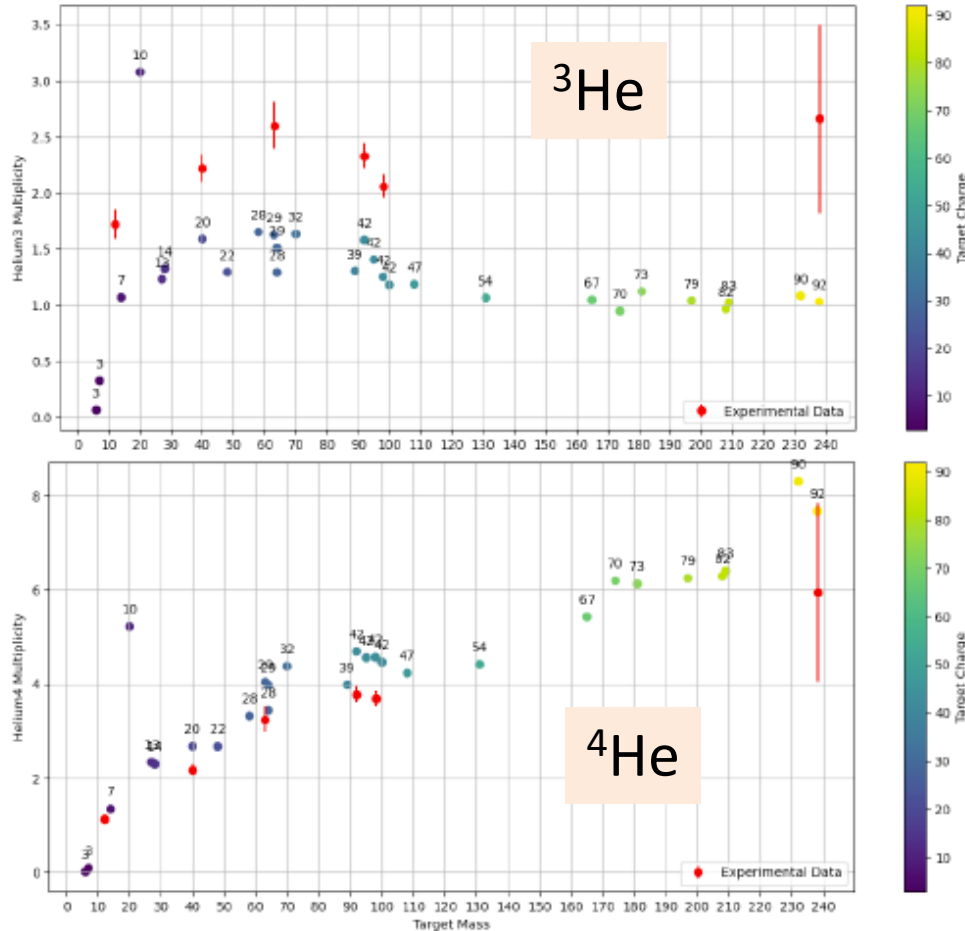
**Figure 5.11:** Total neutron/proton/deuteron/triton multiplicities per 100  $\bar{p}$  for kinetic energy range (0-300)/(35-200)/(50-160)/(60-150) MeV for a group of target nuclei, experimental values with errors are taken from [Pol+95]. No nuclei with the same mass number are present on this plot; the charge numbers are color-coded.



# Antiprotons in INCL

## Results

### Multiplicities ${}^3\text{He}$ & ${}^4\text{He}$



- ${}^3\text{He}$ : underestimation ( $< \times 2.5$ )
- ${}^4\text{He}$ : rather good

So...

Here for given kinetic ranges...  
Coalescence model?

Figure 5.12: Total  ${}^3\text{He}$  and  ${}^4\text{He}$  multiplicities per 100  $\bar{p}$  for kinetic energy range 30-70 MeV for a group of target nuclei, experimental values with errors are taken from [Mar+88]. No nuclei with the same mass number are present on this plot; the charge numbers are color-coded.



# Antiprotons in INCL Results

Multiplicities p to  $^4\text{He}$ , even beyond  
(comparisons to FLUKA, FTF)

**Table 5.4:** Particle multiplicities for a given energy range after antiproton annihilation. The top value in each cell is taken from [Mar+88], statistical error in superscript, while systematic is subscript, error values are given with respect to the last digit (e.g.  $74.2^{+3}_{-38} \equiv 74.2^{+0.3}_{-3.8}$ ). The second value is the INCL, the red is FTF and the blue is FLUKA. The FLUKA and FTF results were kindly provided by Angela Gligorova (Stefan Meyer Institute).

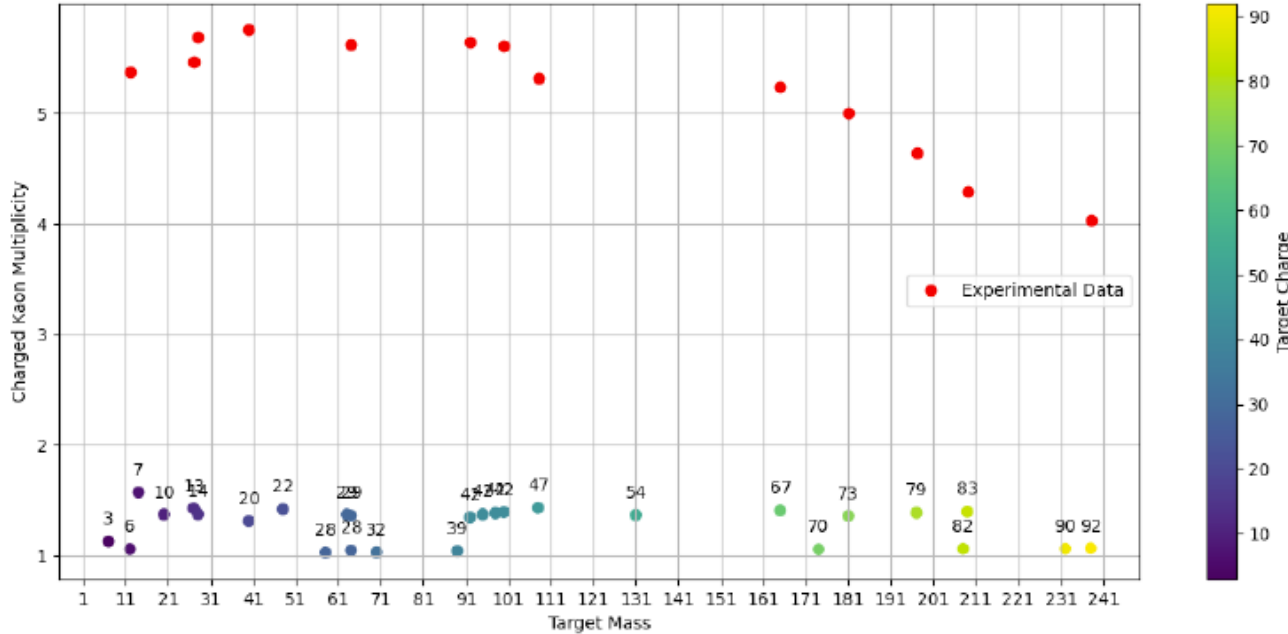
range(MeV)	C12	Ca40	Cu63	Mo92	Mo98	U238
p (6-18)	$23.3^{+2}_{-18}$	$74.2^{+3}_{-38}$				
	21.2	122.2	$94.5^{+4}_{-78}$	$127.2^{+4}_{-58}$	$124.3^{+3}_{-64}$	$76.6^{+3}_{-240}$
	3.0	6.7	115.3	155.6	98.5	34.9
	18.3	30.2				
d (8-24)	$9.3^{+1}_{-7}$	$18.1^{+2}_{-9}$				
	19.9	25.6	$28.0^{+2}_{-23}$	$29.0^{+2}_{-13}$	$30.4^{+2}_{-15}$	$31.3^{+2}_{-99}$
	0.0	0.0	31.0	34.1	29.9	14.9
	13.1	19.1				
t (11-29)	$4.5^{+1}_{-3}$	$5.7^{+1}_{-3}$				
	5.4	5.0	$9.9^{+1}_{-8}$	$11.8^{+1}_{-5}$	$12.7^{+1}_{-7}$	$18.8^{+2}_{-59}$
	0.0	0.0	8.4	8.7	10.6	12.1
	5.0	8.1				
$^3\text{He}$ (30-70)	$1.72^{+4}_{-13}$	$2.22^{+5}_{-12}$				
	1.74	1.59	$2.60^{+6}_{-21}$	$2.33^{+5}_{-11}$	$2.06^{+4}_{-10}$	$2.66^{+6}_{-84}$
	0.0	0.1	1.62	1.58	1.25	1.03
	2.0	0.2				
$^4\text{He}$ (30-70)	$1.14^{+3}_{-9}$	$2.18^{+5}_{-11}$				
	1.32	2.67	$3.25^{+7}_{-26}$	$3.78^{+6}_{-17}$	$3.69^{+6}_{-17}$	$5.94^{+9}_{-190}$
	12.0	4.0	4.04	4.69	4.57	7.66
	2.5	1.6				
$^6\text{He}$ (39-89)	$0.025^{+5}_{-2}$	$0.045^{+7}_{-3}$	$0.048^{+8}_{-4}$	$0.061^{+8}_{-3}$	$0.060^{+8}_{-3}$	$0.150^{+20}_{-50}$
	0.022	0.046	0.083	0.077	0.111	0.194
$^8\text{He}$ (44-90)	$0.0041^{+18}_{-3}$	$0.014^{+4}_{-1}$	$0.0094^{+36}_{-8}$	$0.011^{+3}_{-1}$	$0.013^{+4}_{-1}$	$0.041^{+8}_{-13}$
	0.0	0.004	0.017	0.021	0.036	0.088
Li (61-96)	$0.017^{+4}_{-2}$	$0.075^{+9}_{-4}$	$0.058^{+9}_{-5}$	$0.086^{+9}_{-4}$	$0.083^{+9}_{-4}$	$0.180^{+16}_{-60}$
	0.003	0.022	0.051	0.054	0.067	0.120

INCL is clearly competitive



# Antiprotons in INCL Results

Multiplicities  $K^{+/-}$



**Figure 5.34:** Charged kaon multiplicity in the kinetic energy range 60-200 MeV for targets from  $^{12}\text{C}$  to  $^{238}\text{U}$ . Values were digitized manually from Ref. [Pol+95].

... Some domains needs more efforts !

Kaon are too much underestimated

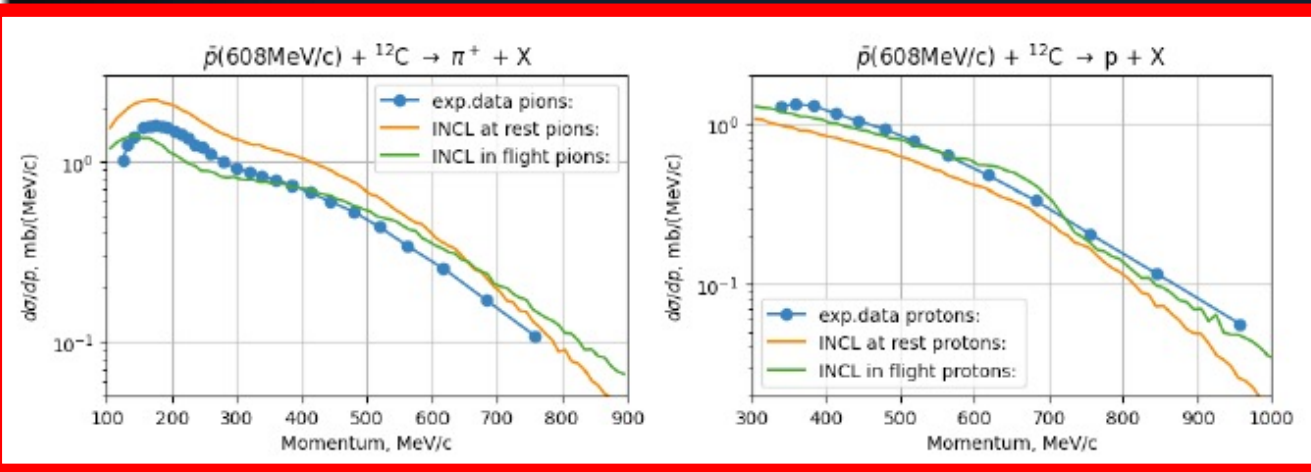
Why?

- More than 5%?
- Wrong relative contributions? (because here kinetic range 60-200 MeV)  
 low multiplicity  $\rightarrow$  high energies  
 high multiplicity  $\rightarrow$  low energies

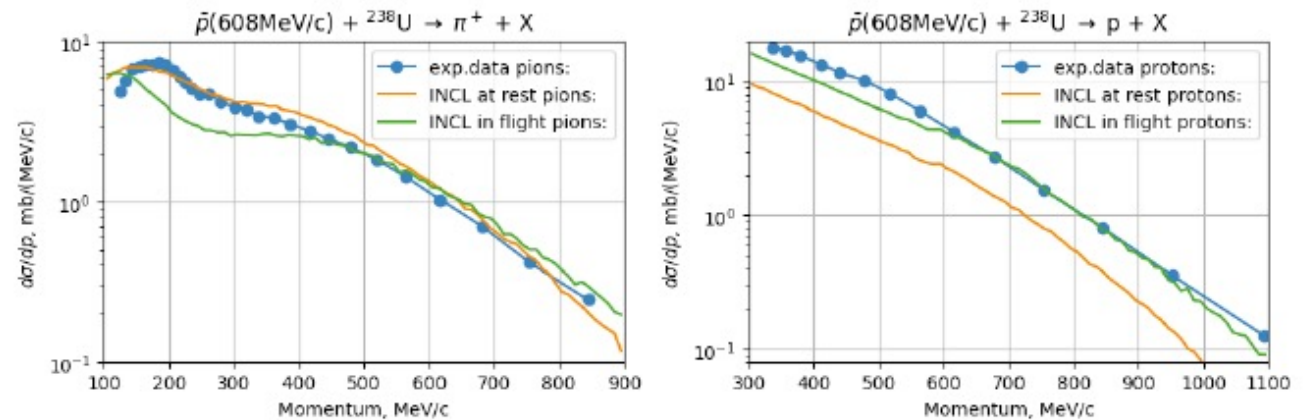


# Antiprotons in INCL Results

Spectra  $\pi^+$  & p



Carbon  
 $\pi^+$  over and p under...!?  
Not really...

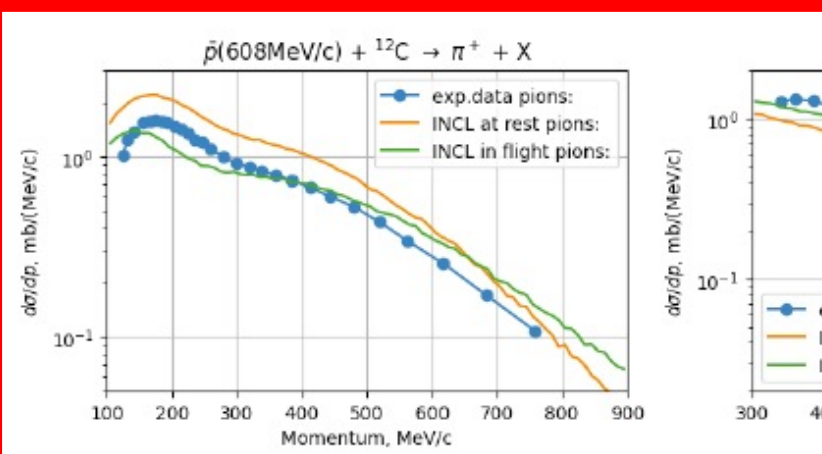


**Figure 5.16:**  $\pi^+$  and proton spectra at  $P_{lab} = 608\text{MeV}/c \approx 180\text{ MeV}$   $\bar{p}$  incident energy. Data are taken from Ref. [McG+86a] and Ref. [McG+86b].



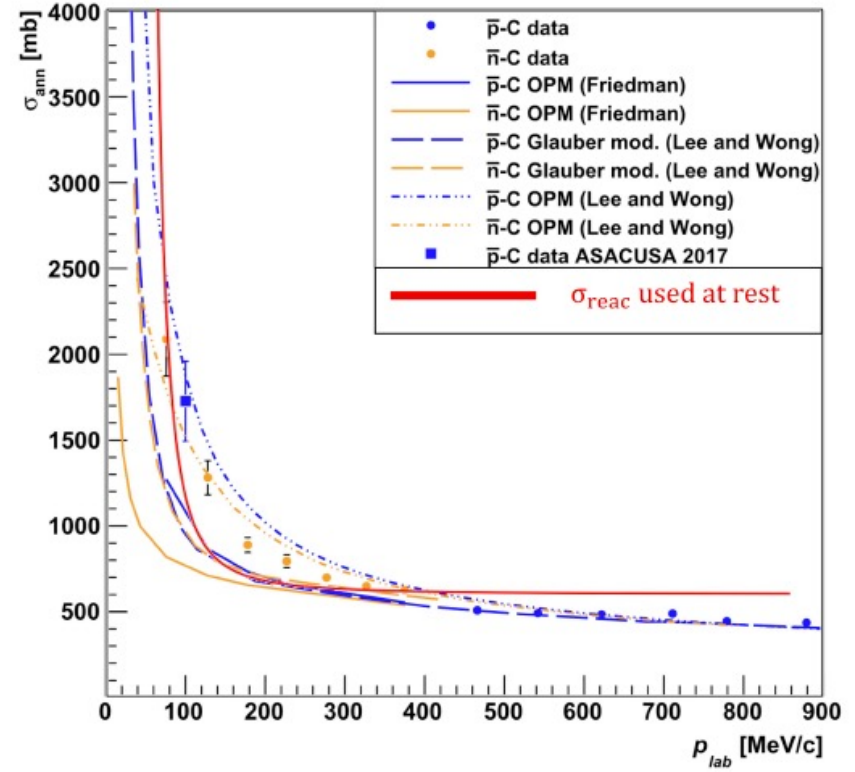
# Antiprotons in INCL Results

Spectra  $\pi^+$  & p



$\sigma_{\text{reac}}$

pbar+C

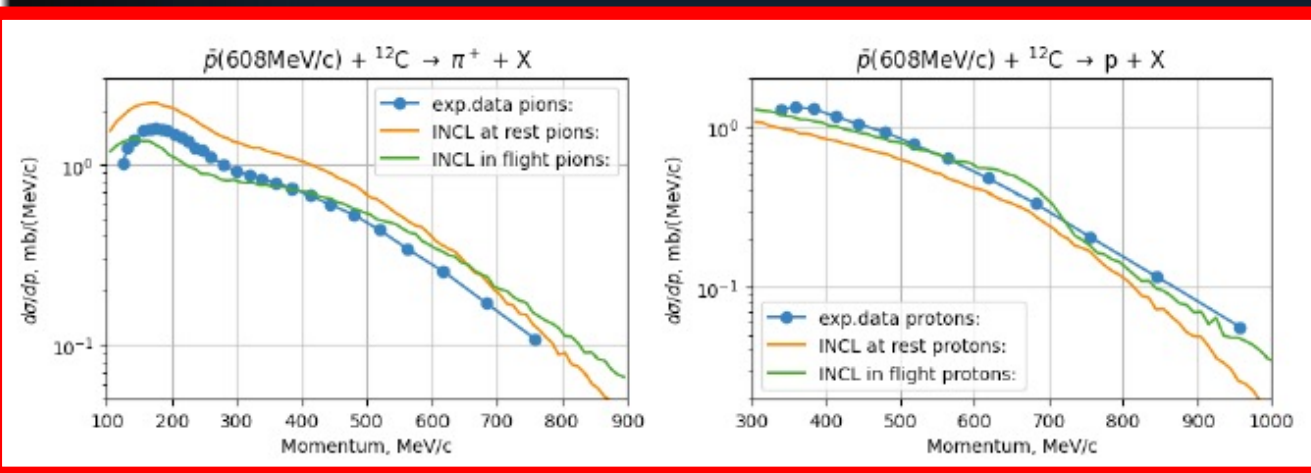


**Figure 5.6:** Antinucleon  $\sigma_{\text{reac}}$  at low energies on carbon. In orange the antineutron values, in blue those for antiproton. The points are the experimental data. The continuous lines represent the calculations with the optical potential model. The dashed lines are from the calculations with the extended Glauber model. The dotted-dashed lines are preliminary calculations obtained by means of a phenomenological optical model whose parameters are tuned to reproduce the N-nucleus annihilation data. Red line is the formula used in INCL from Ref.[Bia+11]. The original plot is taken from Ref.[Agh+18].

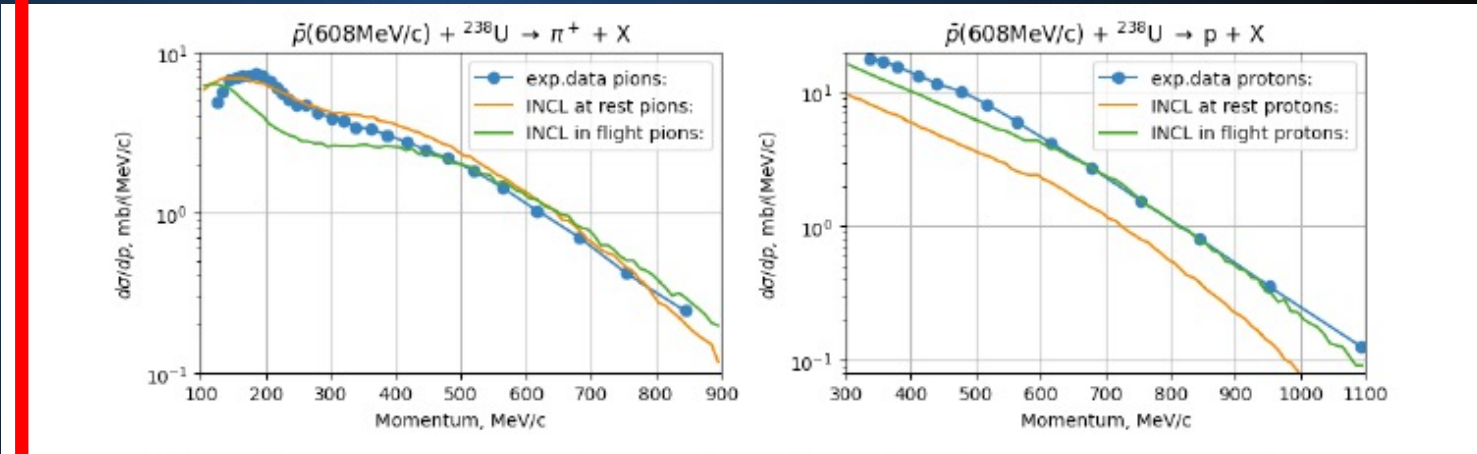
Carbon  
 $\pi^+$  over and p under...!?  
 Not really...

# Antiprotons in INCL Results

Spectra  $\pi^+$  & p



Carbon  
 $\pi^+$  over and p under...!?  
 Not really...  
 Too low normalization ( $\sigma_{\text{reac}}$ )  
 $\pi^+$  might be fine and  
 p underestimated  
 Good shapes!

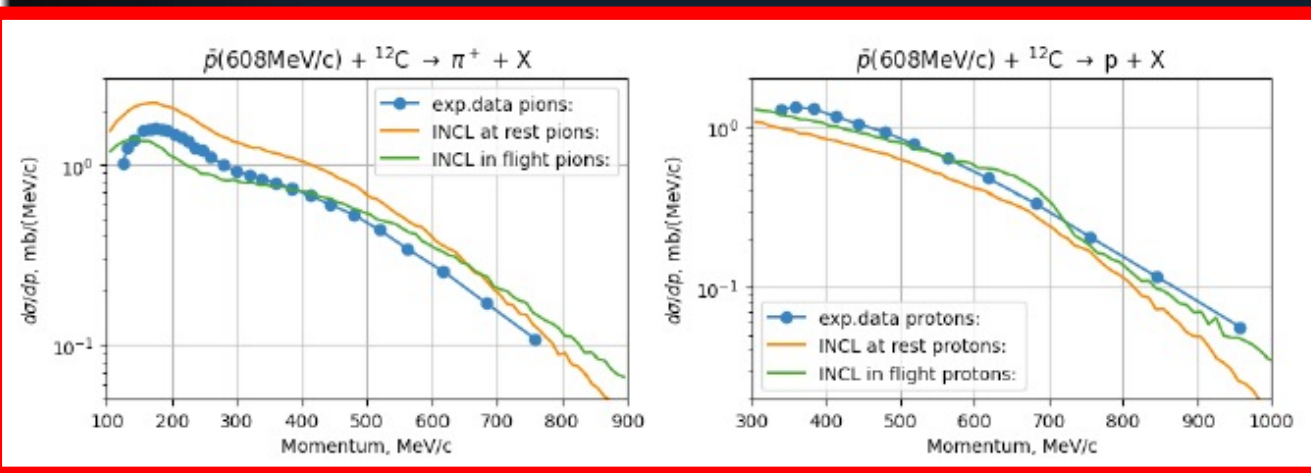


**Figure 5.16:**  $\pi^+$  and proton spectra at  $P_{\text{lab}} = 608 \text{ MeV}/c \approx 180 \text{ MeV}$   $\bar{p}$  incident energy. Data are taken from Ref. [McG+86a] and Ref. [McG+86b].



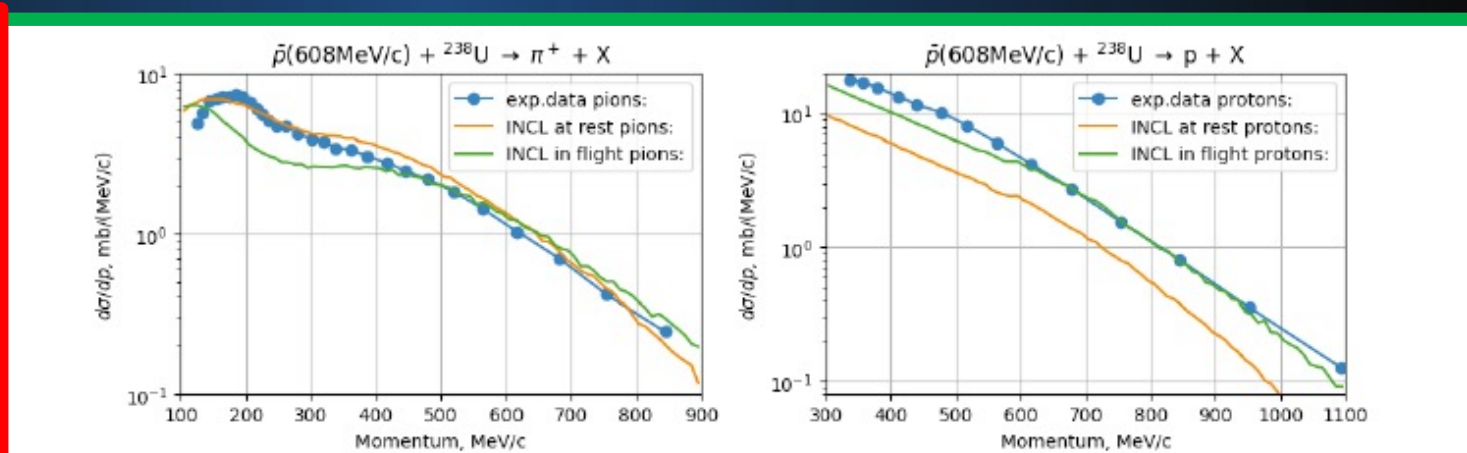
# Antiprotons in INCL Results

Spectra  $\pi^+$  & p



Uranium  
 $\pi^+$  OK  
 p underestimated!  
 What about the  $\sigma_{\text{reac}}$ ?  
 Good shapes!

Carbon  
 $\pi^+$  over and p under...!?  
 Not really...  
 Too low normalization ( $\sigma_{\text{reac}}$ )  
 $\pi^+$  might be fine and  
 p underestimated  
 Good shapes!

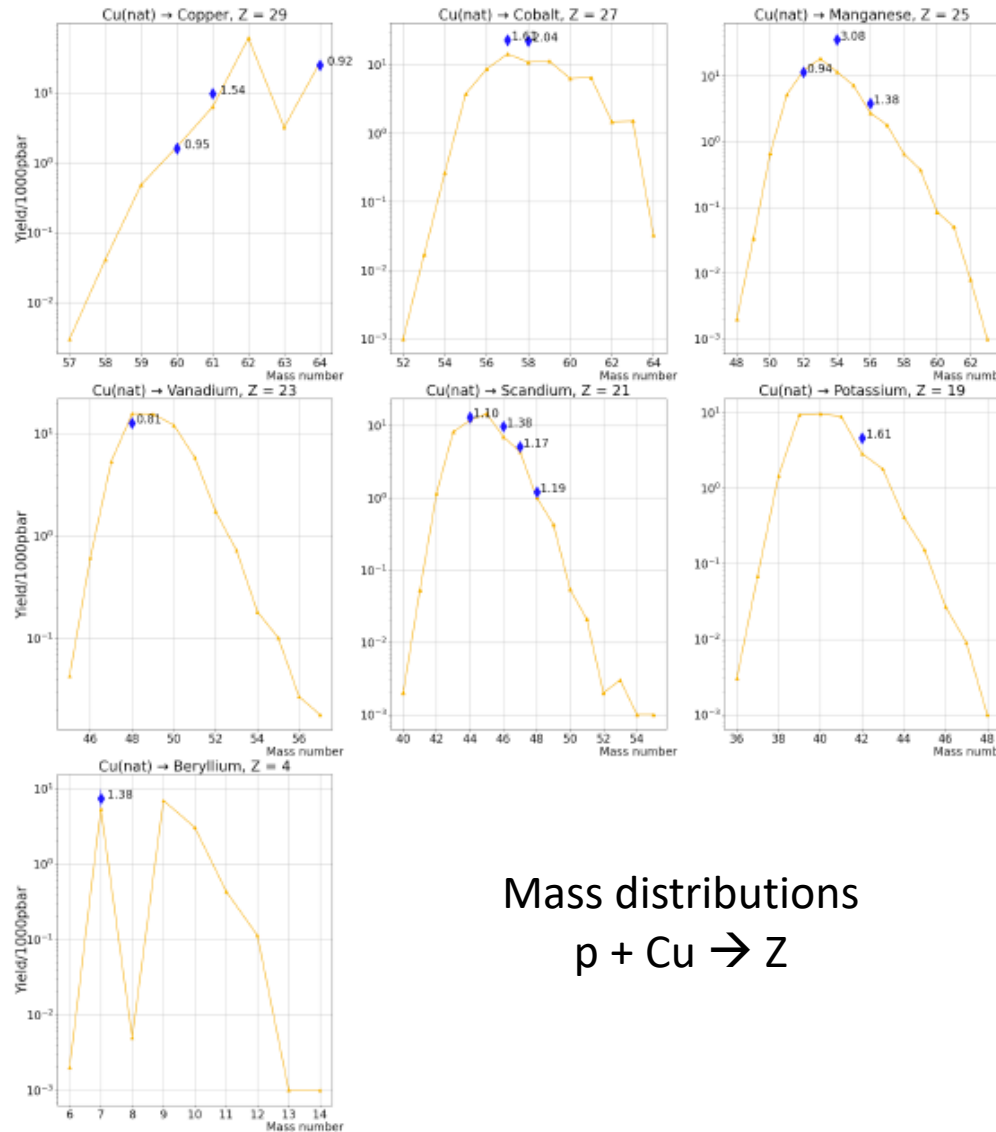


**Figure 5.16:**  $\pi^+$  and proton spectra at  $P_{\text{lab}} = 608\text{MeV}/c \approx 180\text{ MeV}$   $\bar{p}$  incident energy. Data are taken from Ref. [McG+86a] and Ref. [McG+86b].

# Antiprotons in INCL

## Results

### Residue production



Mass distributions  
 $p + Cu \rightarrow Z$

Pretty good!

As good as with usual  
projectiles (p...)

Figure 5.22: Independent isotopic distributions from the reaction  $\bar{p} + \text{nat} Cu$ . Calculated results are in orange. Data are from [Jas+93].

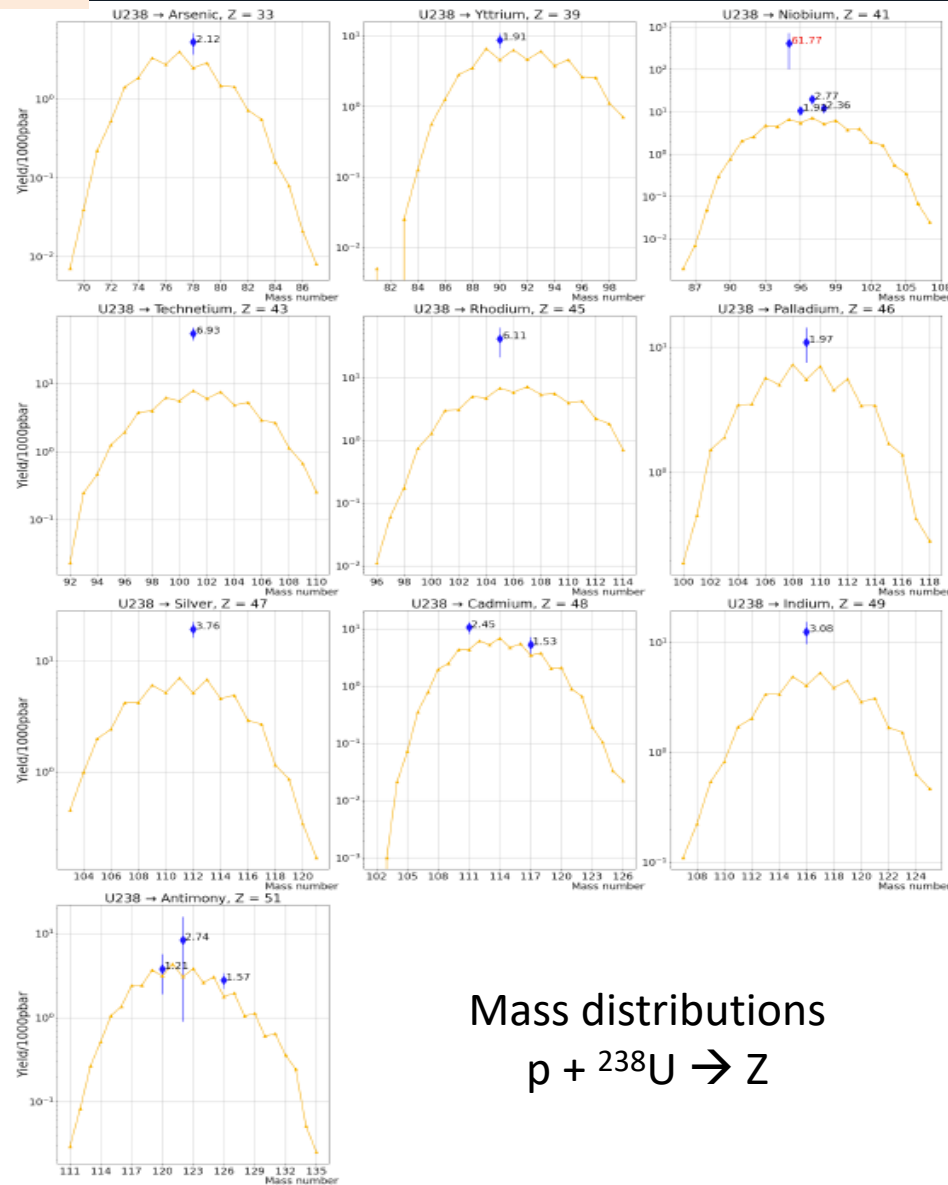
Antiproton-Nucleus reactions  
with the INCL code



# Antiprotons in INCL

## Results

### Residue production



Mass distributions  
 $p + {}^{238}\text{U} \rightarrow Z$

Figure 5.24: Independent isotopic distributions from the reaction  $\bar{p} + {}^{238}\text{U}$ . Calculated results are in orange. Data are from [Mac+92].

Experimental data...  
Consistency...

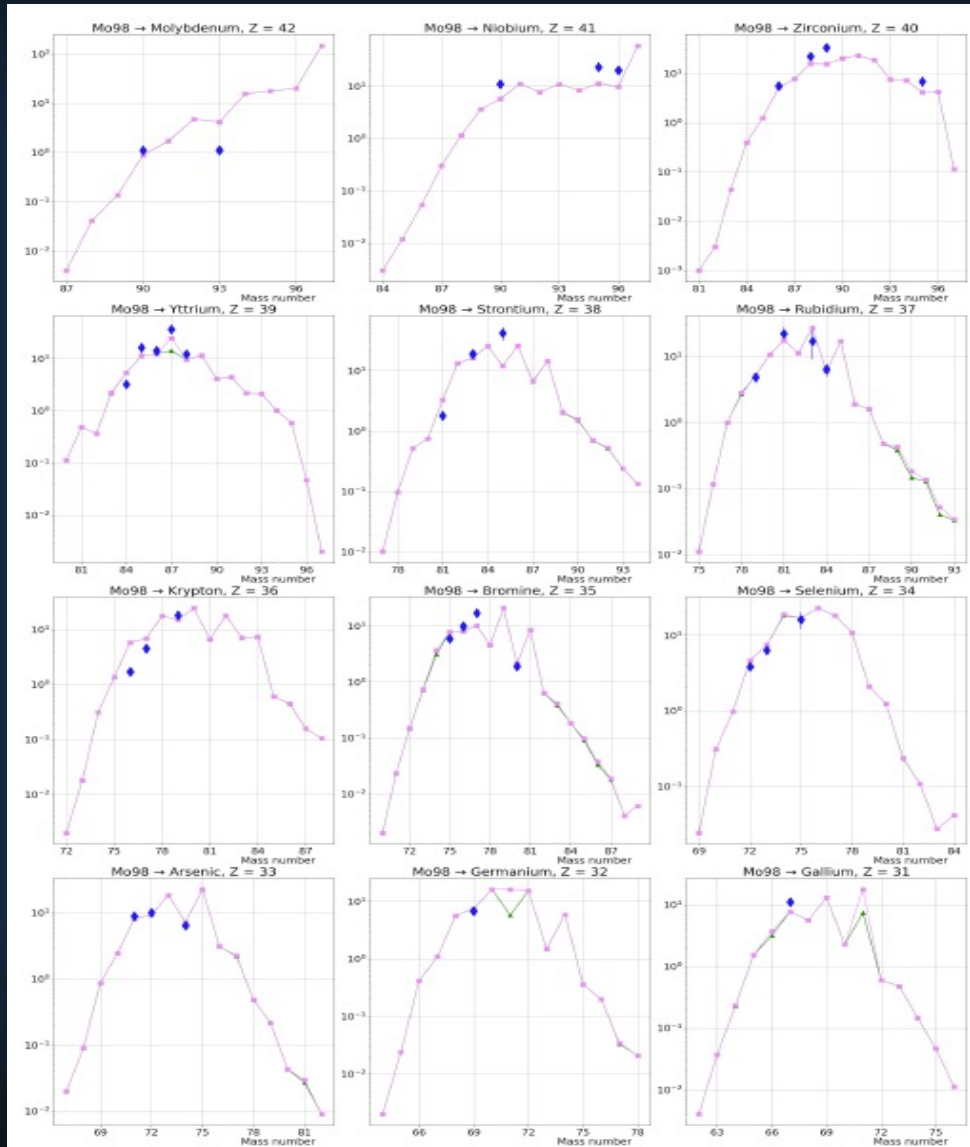
Don't know what to  
conclude.



# Antiprotons in INCL

## Results

### Residue production



Here, cumulative production (progenitors accounted for)

Not bad at all, is it?

Mass distributions  
 $p + {}^{98}\text{Mo} \rightarrow Z$

**Figure 5.21:** Cumulative isotopic distributions from the reaction  $\bar{p} + {}^{98}\text{Mo}$ . Calculated results are in violet (option  $\frac{\lambda_P}{\lambda_{P\bar{P}}}$  = 0.1), and in green (option  $\frac{\lambda_P}{\lambda_{P\bar{P}}}$  = 0.5). Definition of  $\frac{\lambda_P}{\lambda_{P\bar{P}}}$  in section 5.3.3.1. Data are from [Mos+89].

Antiproton-Nucleus reactions with the INCL code



# Conclusions

- Antiproton (at rest and in-flight) as projectile in INCL (and Geant4)
- Results
  - Generally good
  - But place for improvements
    - $\pi$  high multiplicities
    - d overestimated; t and  $^3\text{He}$  underestimated
    - K underestimated
    - Normalization -  $\sigma_{\text{reaction}}$  (outside of INCL)
- Improvements...
  - Position of annihilation?
  - Ratio  $S_p/S_n$ ?
  - Kaon contribution?
  - Annihilation on two nucleons?
- More exp. data needed
  - Which ones?
  - To do what with it?



*Thanks for your  
attention!*

And thanks to  
D. Zharenov who did the work  
J. Hirtz who gave him advices  
and  
Joseph Cugnon for his unfailing support



# Backup



# What is INCL? Capabilities

Produced particles

n p  $\pi$  Cluster  $A \leq 4$  Cluster  $A \leq 8$   $\eta$   $\omega$   $K$   $\Lambda$   $\Sigma$

10-20 GeV

2009-2011  
2014

2-3 GeV

2002

2004

2013

2014

2016

2017-  
2018

2023

~150 MeV

2003 (np $\pi$ )  
2013 ( $A \leq 4$ )

Tens of MeV

n p  $\pi$  Cluster  $A \leq 4$

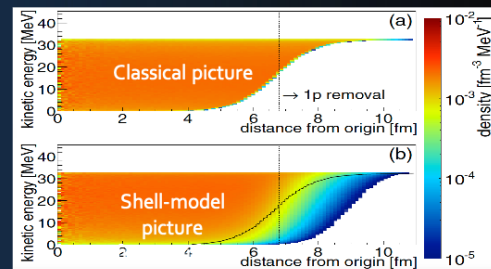
Cluster  $A \leq 18$

$K$   $\Lambda$   $\Sigma$   $\bar{p}$

Projectile particles

2015-2017

Few nucleon removal  
→  
New momentum distribution  
(quantum effects)



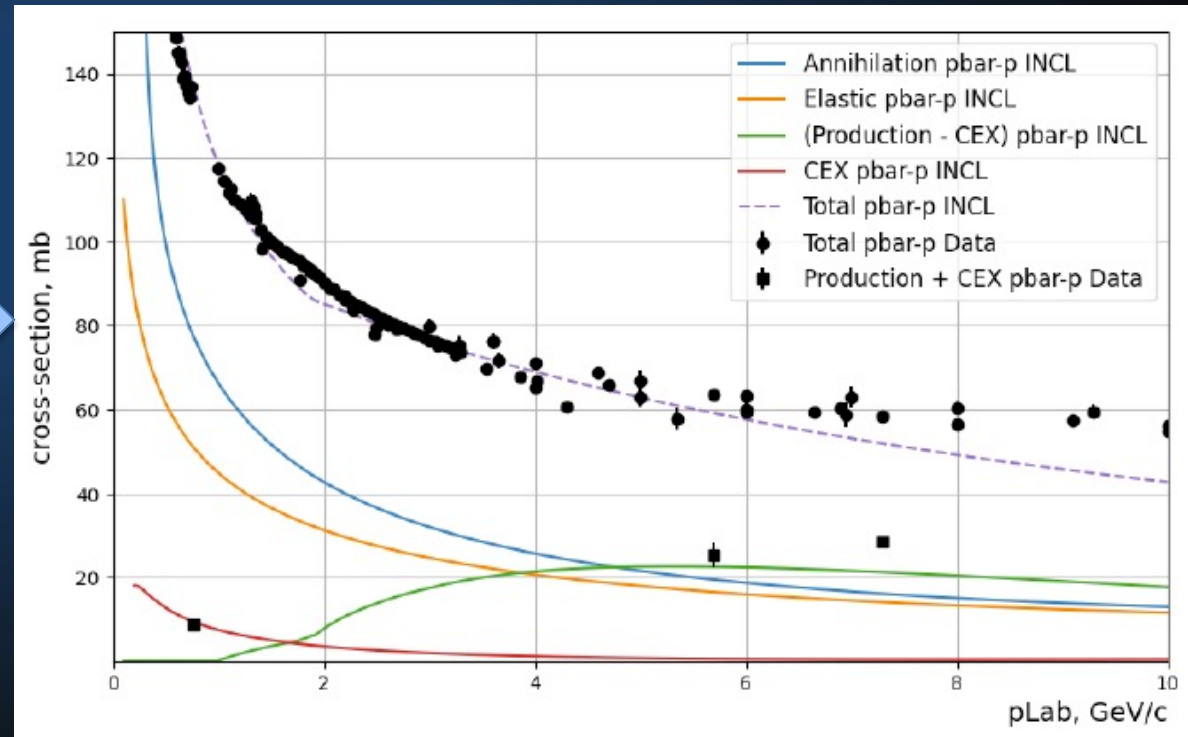
Antiproton-Nucleus reactions  
with the INCL code



# Antiprotons in INCL Hypotheses – ingredients

## In-flight (usual interaction)

- Potential of the pbar ( $V = -150$  MeV)
- Reaction cross sections  
(fits from exp. Data)
- Final states  
(fits from exp. Data)

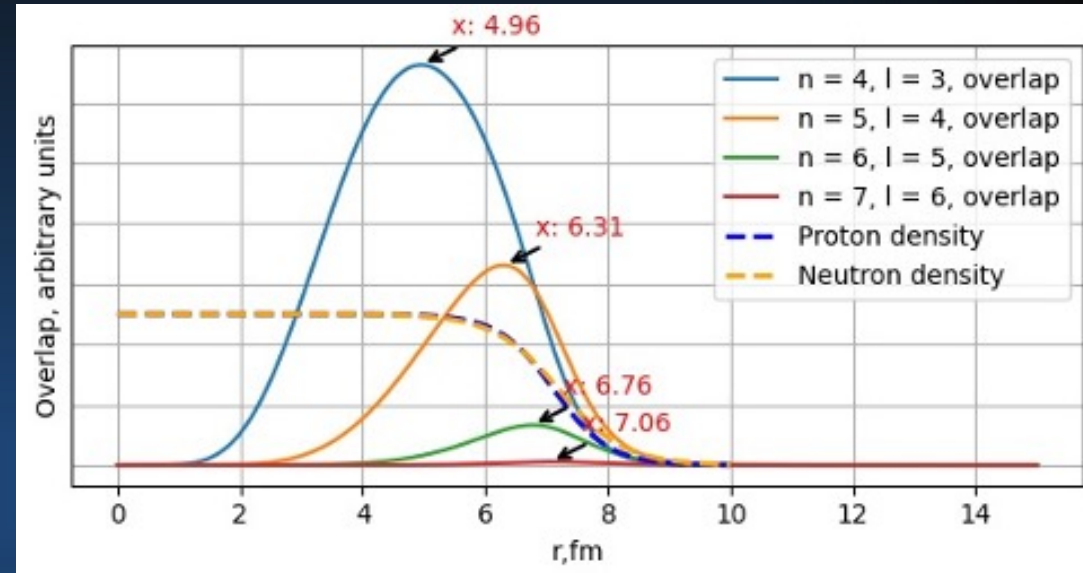


# Antiprotons in INCL

## Hypotheses – ingredients

### At rest - Position of annihilation

- pbar  
Captured in a high Bohr orbit  
Cascades toward the nucleus  
Stops/annihilates at a given « n »
- Determination of « n »  
(fits from exp. Data)
- Position of annihilation  
→ When overlap of nuclear density and antiprotonic radial density



$$P_{neutronic}(r) = N_{nl} \times \rho_n \times r^2 \times |R_{n,l}|^2$$

$$P_{protonic}(r) = N_{nl} \times \rho_p \times r^2 \times |R_{n,l}|^2$$

$$|R_{n,l=n-1}| = ((2n)!)^{\frac{1}{2}} \left(\frac{2}{na}\right)^{\frac{3}{2}} \left(\frac{2r}{na}\right)^{(n-1)} \exp\left(\frac{-r}{na}\right)$$

$$N_{nl} = \frac{2}{n^2} \sqrt{\frac{(n-l-1)!}{((n+l)!)^3}} \quad (\text{a is the Bohr radius})$$



# Antiprotons in INCL Results

values deduced from Crystal Barrel data:

$$\begin{aligned}N(\pi) &= 4.98 \pm 0.35, \\N(\pi^\pm) &= 3.14 \pm 0.28, \\N(\pi^0) &= 1.83 \pm 0.21\end{aligned}\tag{5.6}$$

while in INCL we have for  $p\bar{p}$ :

$$\begin{aligned}N(\pi) &= 4.904, \\N(\pi^\pm) &= 3.1, \\N(\pi^0) &= 1.804\end{aligned}\tag{5.7}$$

and for  $n\bar{p}$ :

$$\begin{aligned}N(\pi) &= 4.911, \\N(\pi^\pm) &= 3.195, \\N(\pi^0) &= 1.717.\end{aligned}\tag{5.8}$$

