Ab-Initio Study of Antiproton-Nucleus System at Low Energies

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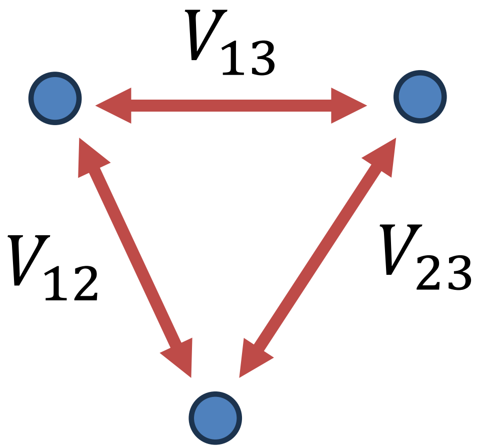
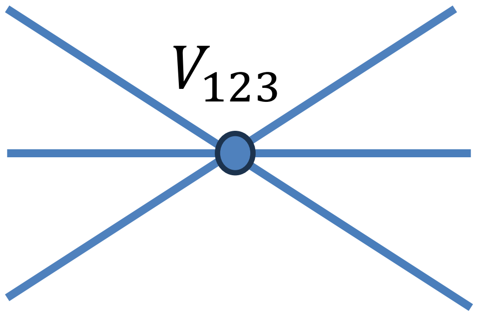
- Goal: Solving the Schrodinger equation (SE) for an A-body system:

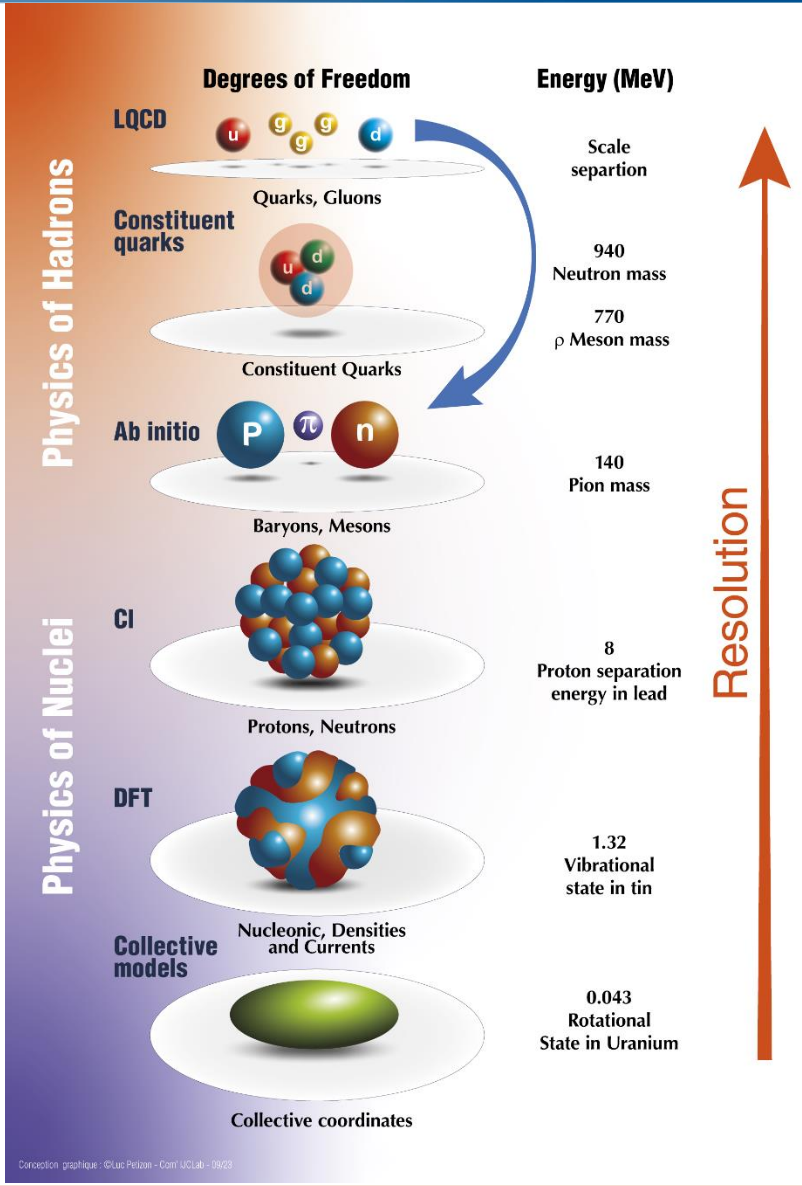
$$H|\psi^{J^\pi T}> = E|\psi^{J^\pi T}>$$

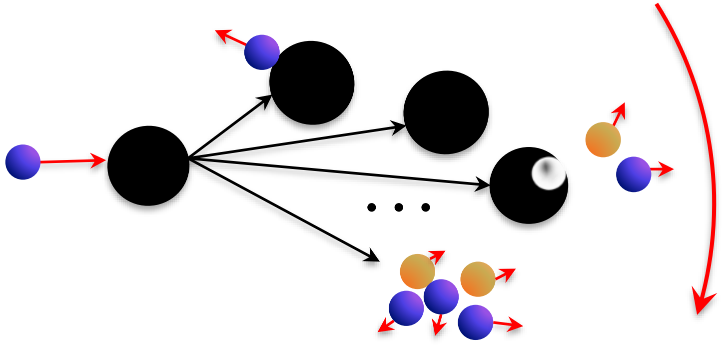
- Nucleons are considered as point-like particles.

- The SE is solved by considering two and many-body interactions between nucleons



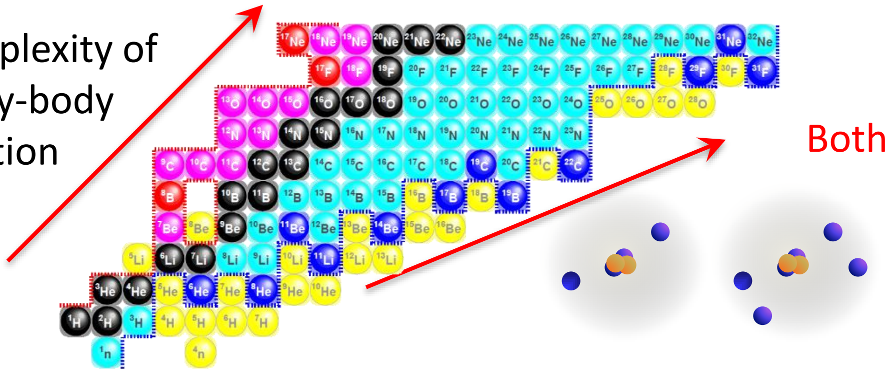




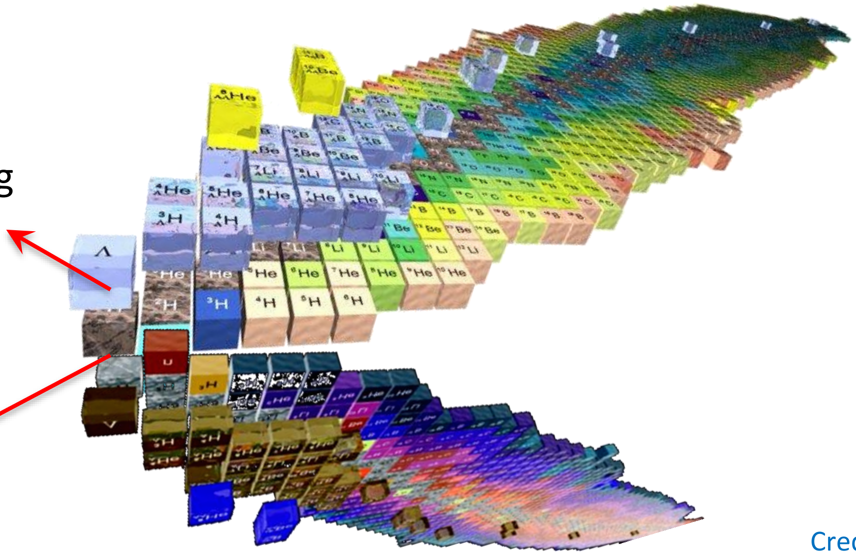


Complexity of scattering problem

Complexity of many-body solution



$\neq n, p$ particles interacting with strong force ($M_h \gg M_{n,p}$)



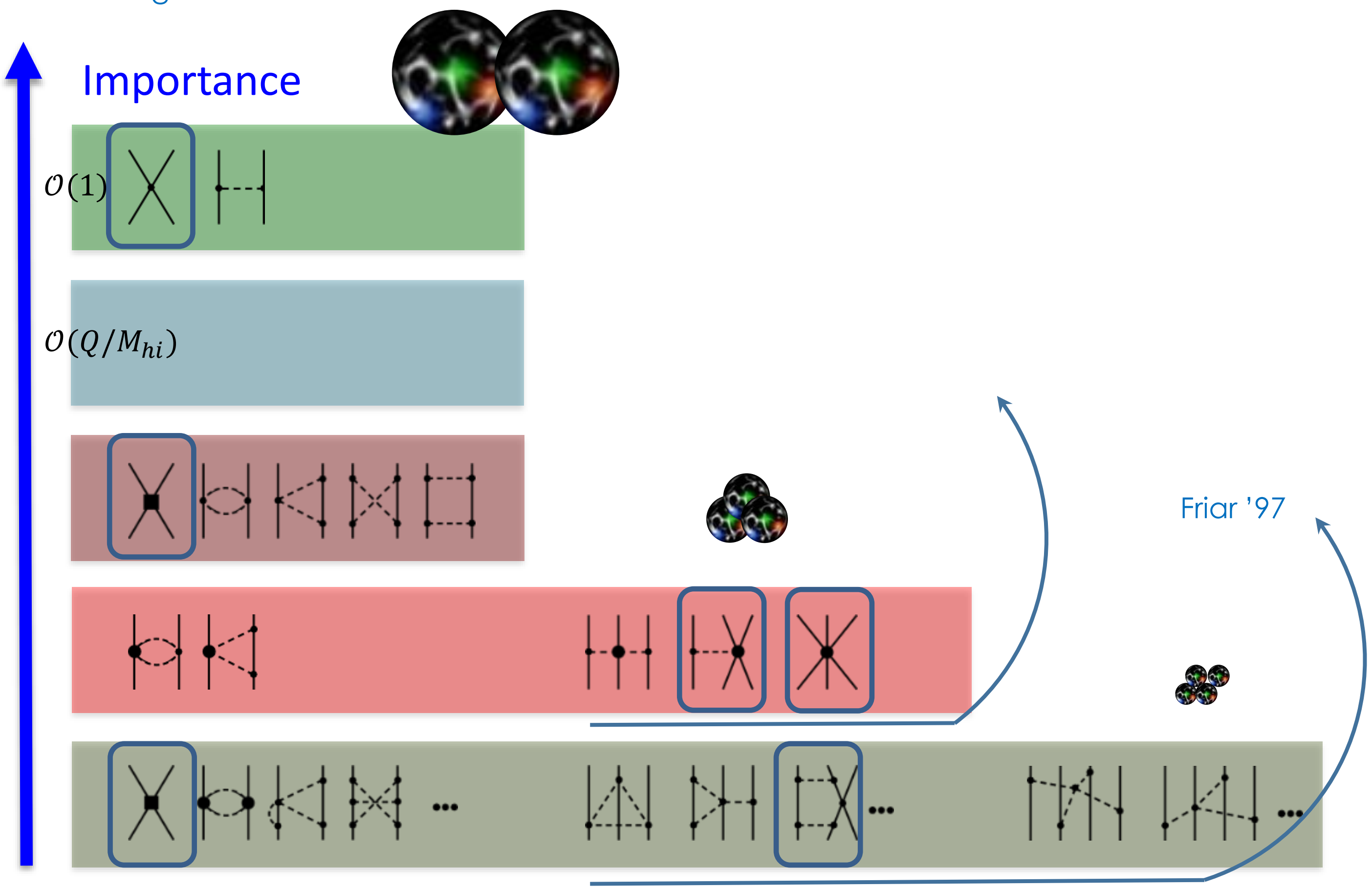
$M_h \leq M_{n,p}$

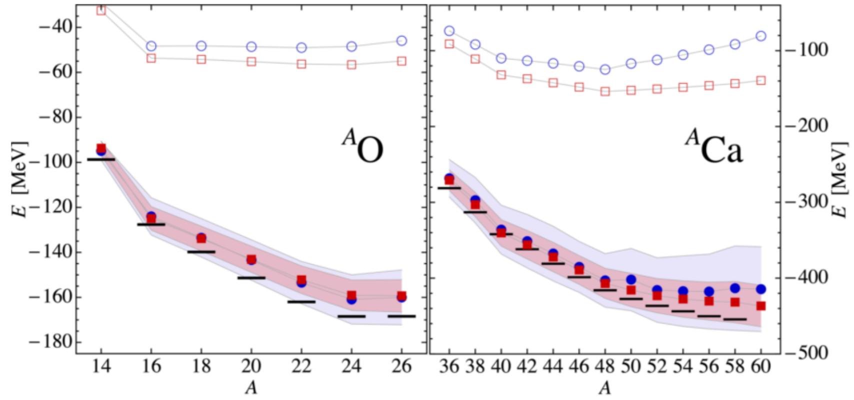
Credits H. Lenske

- ☺ Nuclear theory is **data driven**.
- ☹ Global optical models (NN or $N\bar{p}$ or $N\pi$ scattering) are not applicable to exotic systems.

Building block - Chiral EFTs

Weinberg '90'91'92 Ordóñez and vK '92 etc…



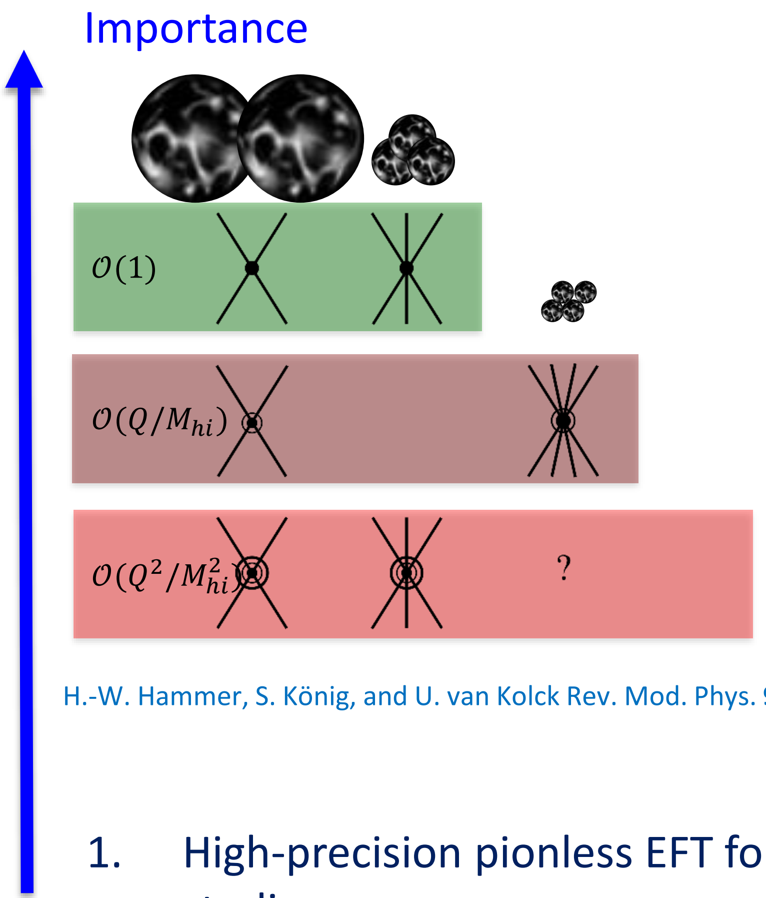


A. Tichai, *et. al.* Front. Phys. **8**, 164 (2020)

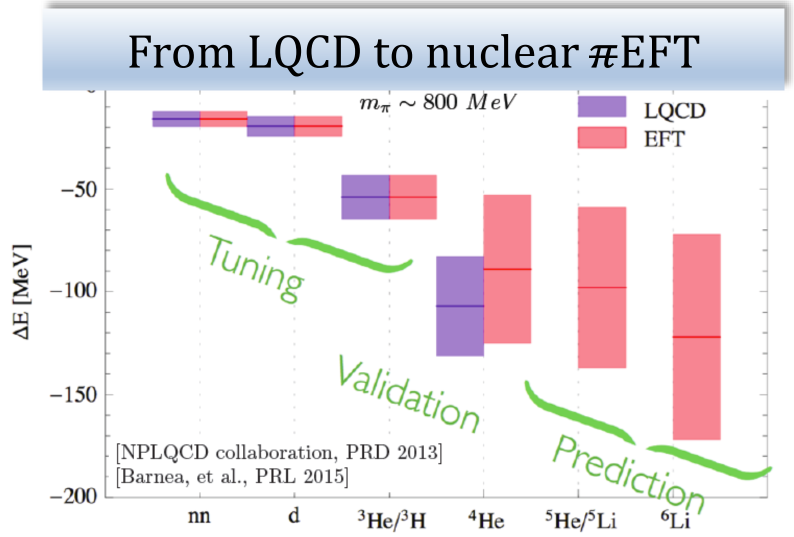
- ☺ **High quality** nuclear interactions (at N^{3}LO).
- ☺ **Various** fits and successes.
- ☹ Weinberg PC wrong: no **renormalizability**.
- ☺ Correct power counting: active research

1. Development of improved power counting ⟶ high-precision/high-orders.
2. Stability of p-shell nuclei.
3. How does chiral expansion perform under forthcoming LQCD data?

Building block - Pionless EFT



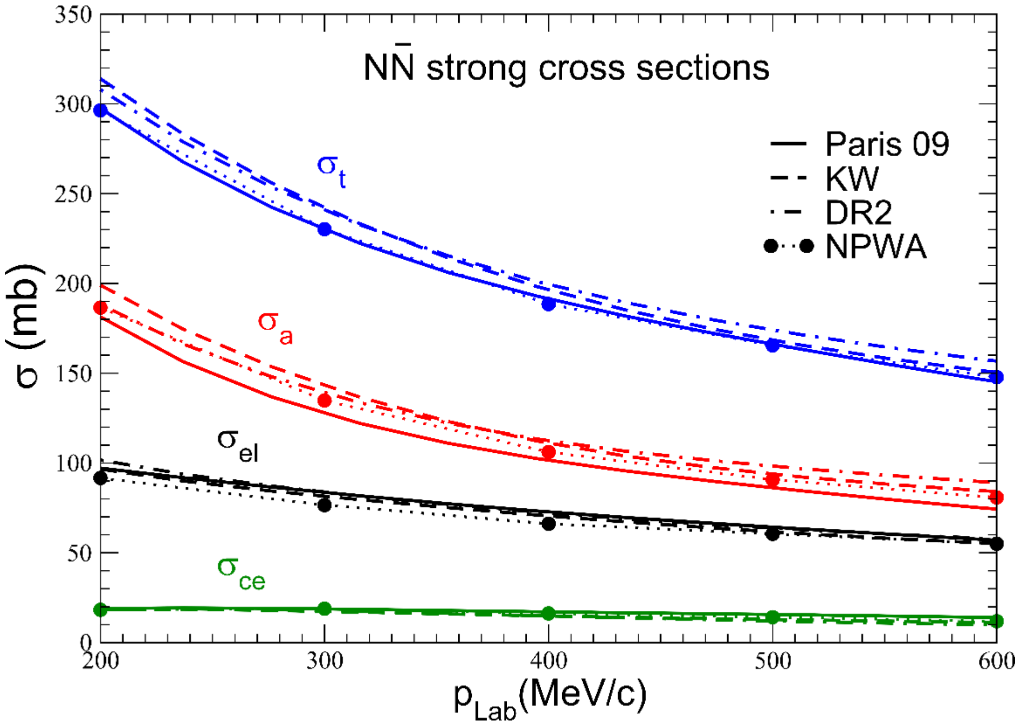
H.-W. Hammer, S. König, and U. van Kolck Rev. Mod. Phys. **92**, 025004 (2020)



☺ **Renormalizable**, connection to LQCD.

1. High-precision pionless EFT for *ab initio* nuclear studies.
2. Stability of p-shell nuclei (NLO).
3. Pionless EFT in the many-body sector: mass reach.
4. Pionless EFT for antinucleon systems /hypernuclei ?

$N\bar{N}$: agreement to data



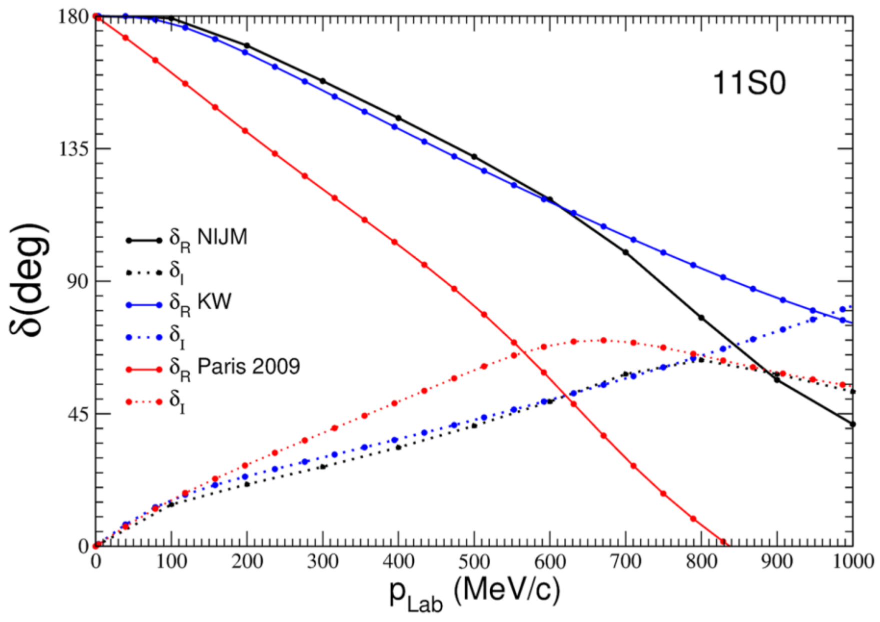
$N\bar{N}$ strong integral cross sections for DR2 (dashed dotted line), KW (dashed line) and Paris 2009 (solid line) optical models, and the Nijmegen Partial Wave analysis.

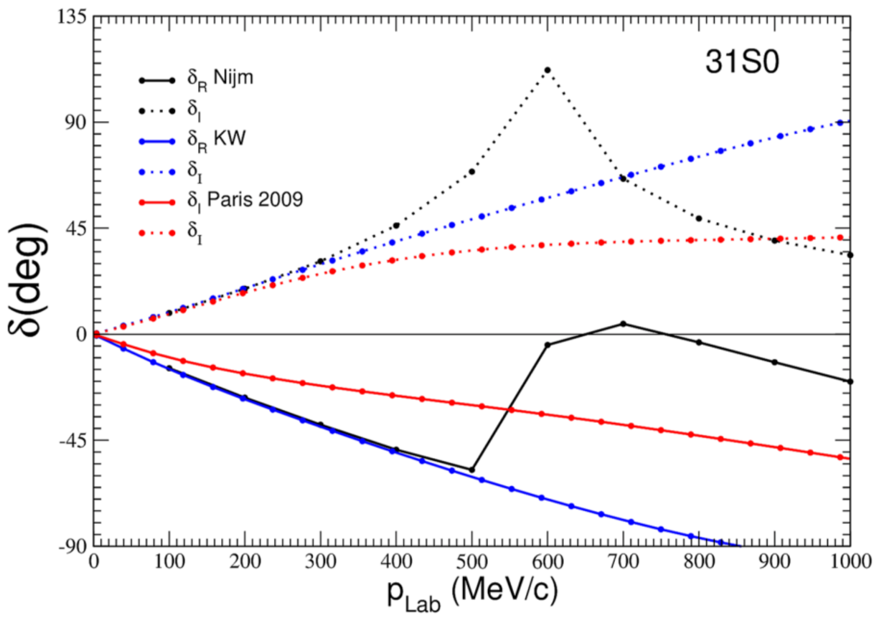
Despite significant disparities, old models, evaluation (PWA), and Chiral EFT parametrization yield **similar agreements** in **integrated cross-sections**.

J. Carbonell *et al.* Eur. Phys. Jour. A **59** (2023)

$N\bar{N}$: phase-shift analysis

- Old models and evaluations, which were primarily fitted to intermediate energy data, may display significant discrepancies in partial wave content.
- Need for low-energy measurements ($^{d\sigma}/_{d\Omega}$, A_y etc …)





$N\bar{N}$ phase-shifts comparison between two models (Paris 2009 and Kohno-Weise) and Partial Wave Analysis (PWA) of Nijmegen (data evaluation).

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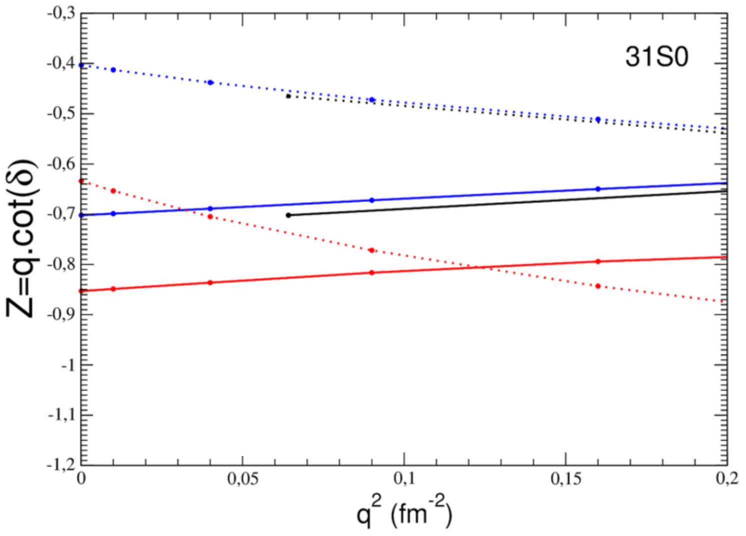
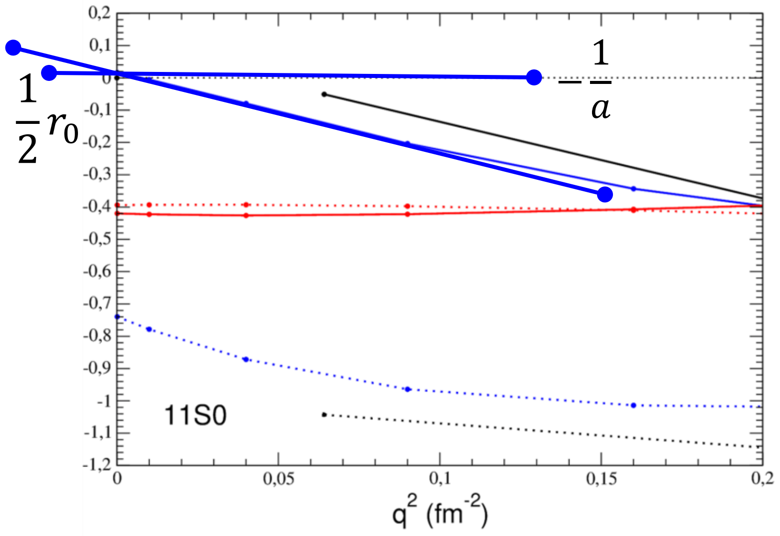
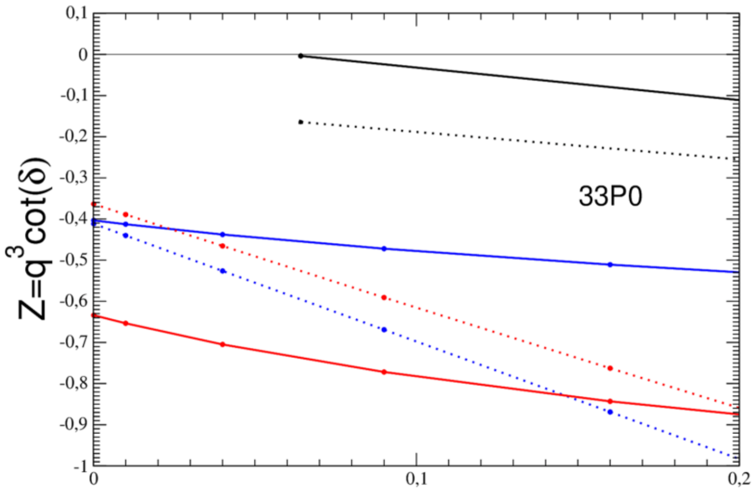
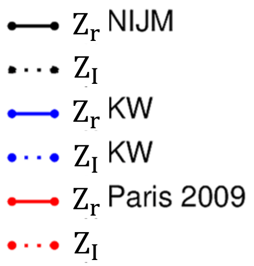
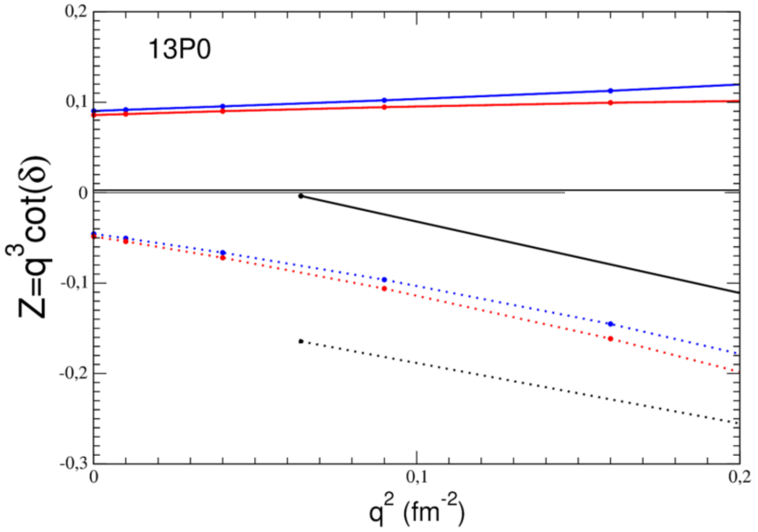
$N\bar{N}$: low-energy limit











Problems:

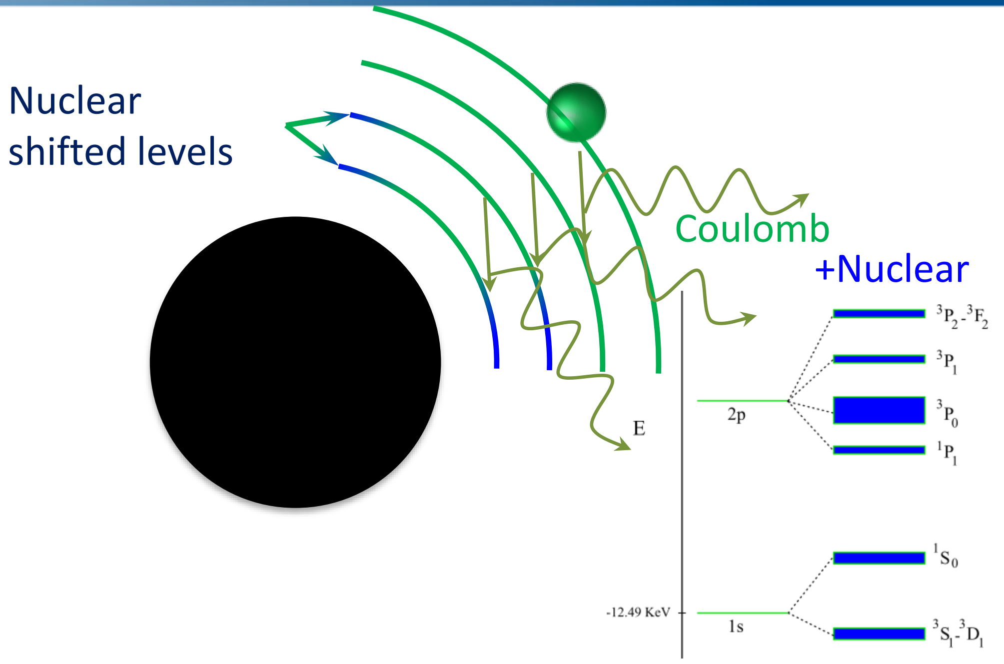
- $N\bar{N}$ low-energy parameters (scattering length a and effective range r_0) are weakly constrained by the present data.
- Nijmegen evaluation do not extrapolate to potential models [unlike in NN case].

$N\bar{N}$: Effective Range Expansion parameters and protonium data

- Qualitative agreement for some waves but accuracy do not exceed 10%
- Low-energy theories need high accuracy at and below threshold

T=0	$^{11}P_1$		$^{13}P_0$		$^{13}P_1$		$^{3}PF_2$	
Nijm*	$-3.34-1.22i$	$9.3-1.2i$	$-3.06-7.23i$	$-1.7-1.5i$	$4.36-0.00i$	$-3.5-0.0i$	--	--
Jülich	$-2.87-0.36i$	--	$-2.83-7.82i$	--	$4.61-0.05i$	--	$-0.74-1.13i$	--
Paris 09	$-3.62-0.34i$	$3.8-0.8i$	$-8.78-4.99i$	$0.23-1.1i$	$5.12-0.02i$	$-3.4-0.02$	$-0.49-0.87i$	--
KW	$-3.36-0.62i$	$3.7-1.6i$	$-8.83-4.45i$	$0.25-0.97i$	$4.73-0.08i$	$-3.5-0.1i$	$-0.46-1.09i$	--
DR2	$-3.28-0.78i$	$4.2-2.3i$	$-8.53-3.50i$	$0.63-1.0i$	$5.14-0.09i$	$-3.4-0.1i$	$-0.59-0.85i$	--

Scattering length and effective range over the years, same data.

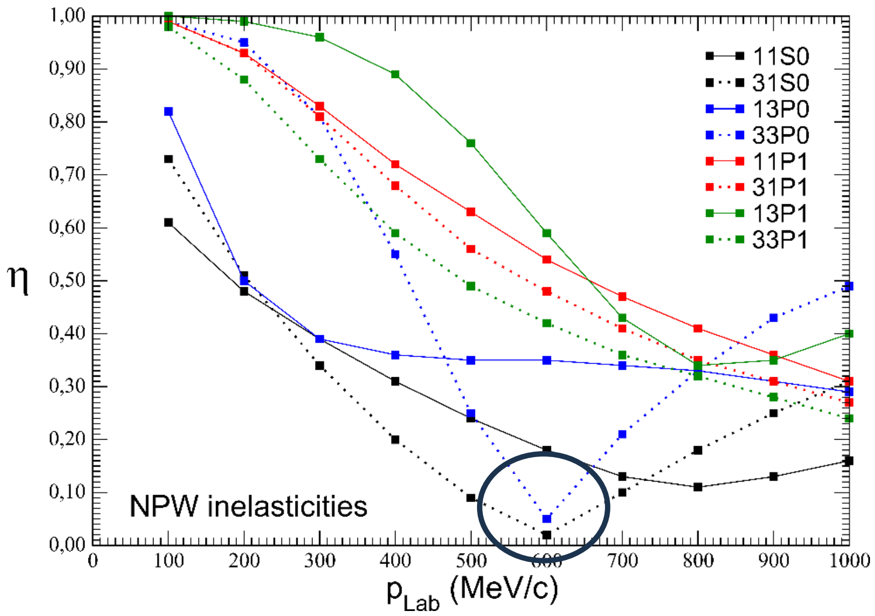


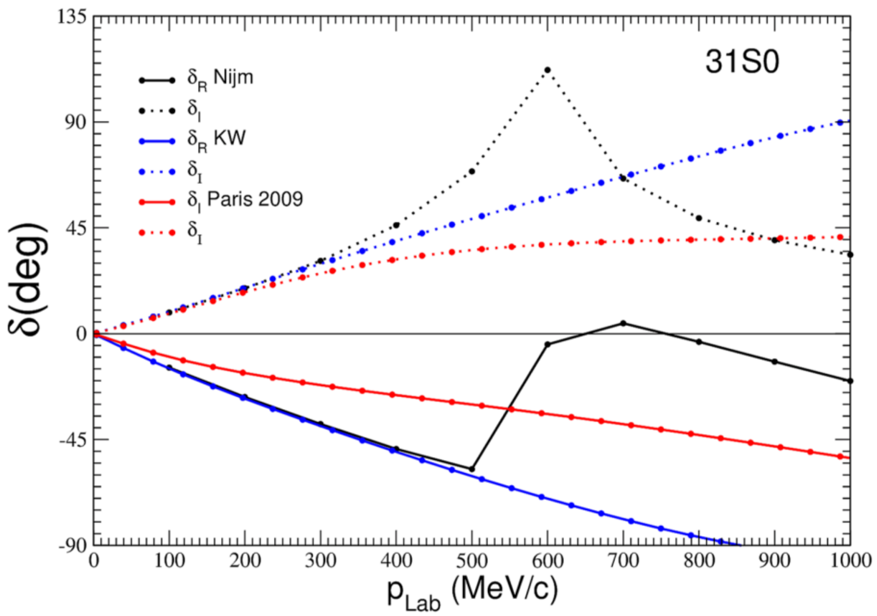
$p\bar{p}$ state	Exp	Paris 2009	Jülich	KW
$^1S_0$	$0.493(92) - i\,0.732(146)$	$0.92 - i\,0.67$	$0.50 - i\,0.71$	$0.57 - i\,0.77$
$^3P_0$	$-5.68(123) - i\,2.45\,(49)$	$-2.74 - i\,2.46$	$-0.32 - i\,3.85$	$-2.81 - i\,1.99$

No agreement between average value extracted from atomic data and $N\bar{N}$

$N\bar{N}$: an example of issues to address

Fitting S-matrix components when the number data points is low, may lead to ambiguities, e.g. limit where a wave is totally absorptive and do not contribute to the elastic/c.e. cross-section. Alternative fits reproduced equally well the data (overfitting!).





$N\bar{N}$ inelasticities extracted from data (PWA).

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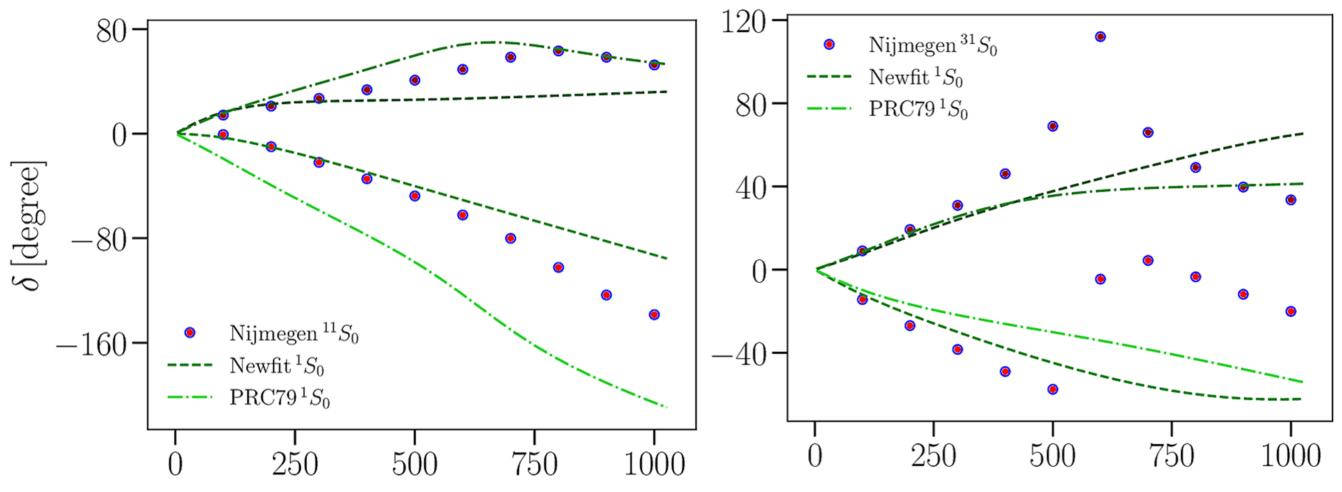
$N\bar{N}$: Needs for new fit with uncertainty quantifications from $A > 2-3$ propagation

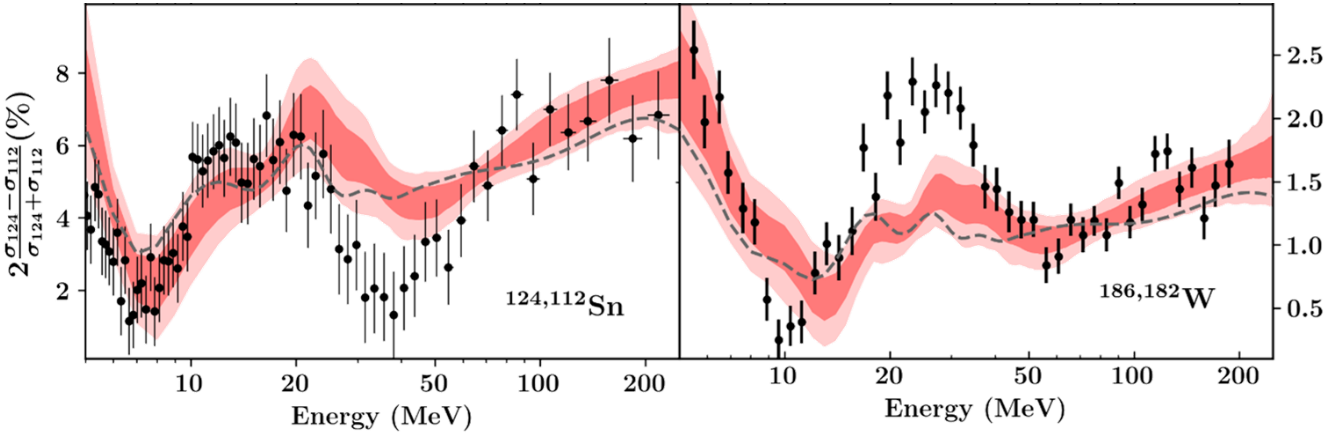
We can upgrade antediluvian $N\bar{N}$ models to match the most accurate PWA.

Yet:

1. Impossible to reproduce P-waves.
2. Unreliable extension of the partial wave analysis to low-energy.
3. Not enough information on subthreshold states.



Modern Nucleon-Nucleus optical potentials come with uncertainty quantifications, which reflect the measurements uncertainties and the limitation of the functional form of optical model



C. D. Pruitt *et al.* Phys. Rev. C **107** (2023)

Ab-Initio No-Core Shell Model (NCSM)

- Suitable for studying static properties of nuclei like the energy spectrum.

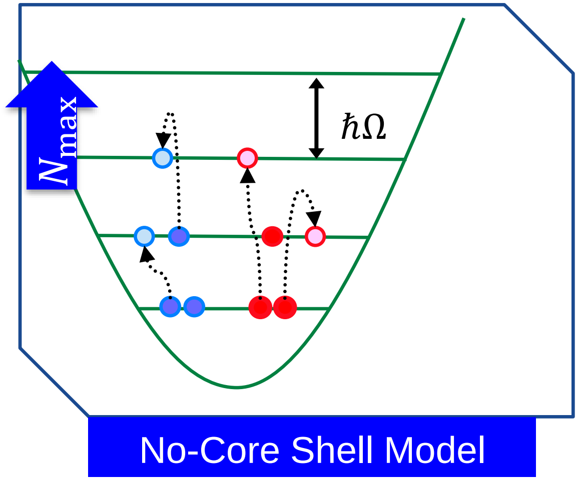
- In the NCSM, the wavefunction of the A-body system is expanded using A-body Harmonic oscillator (HO) basis, e. g., for A=2:

$$\Psi^{(A)}_{NCSM} = \sum_{n,l,s} c_{n,l,s} \, |n \, (l \, s) \, J^{\pi} T\rangle$$

Expansion coefficients (unknown)

Basis states

$$N_{max} = \max(2n + l)$$

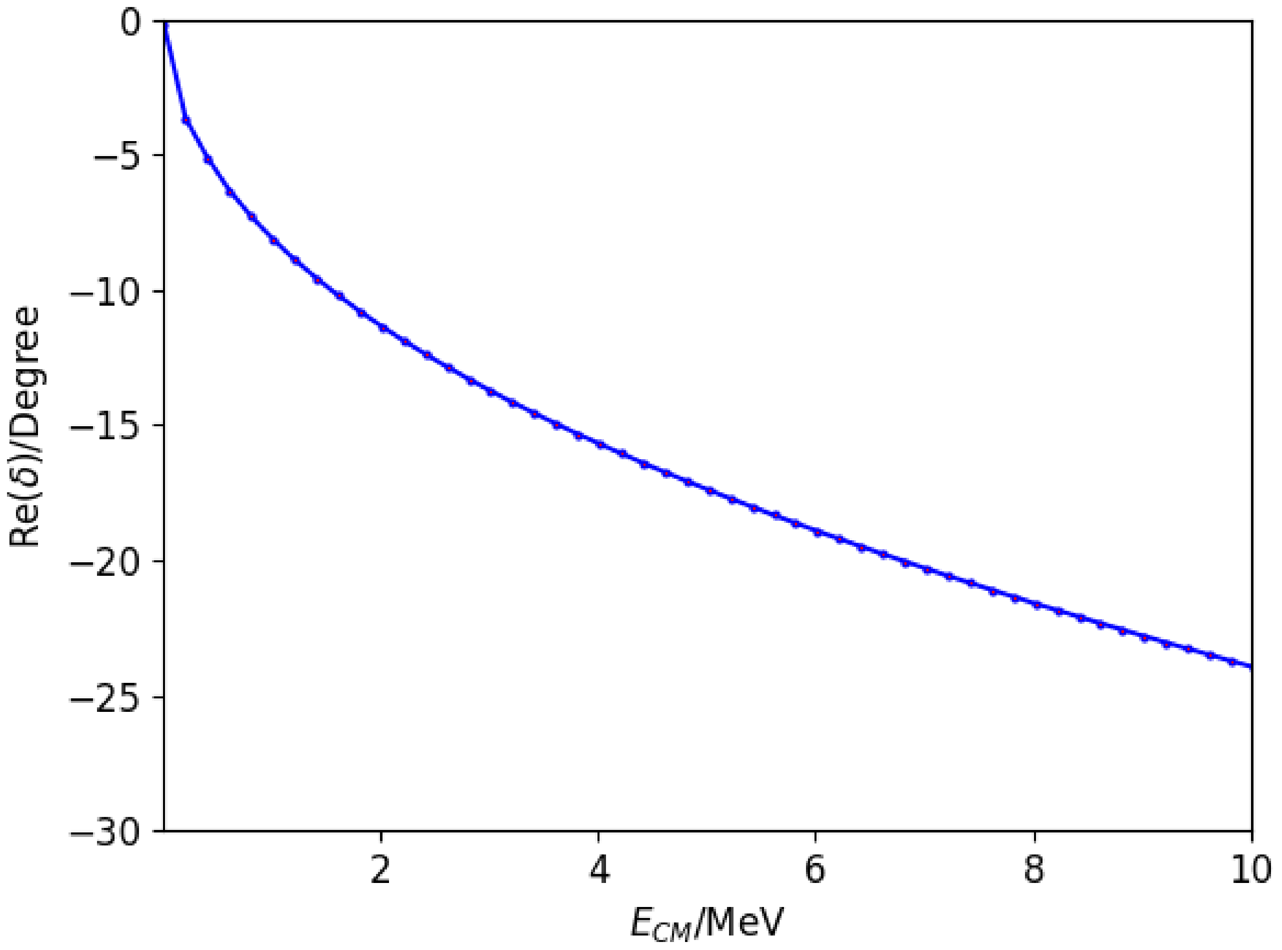


Example of No-Core Shell Model calculation: Quasi-bound states

Channel	$p\bar{n}$	$n\bar{n}$
$^1S_0$	———	———
$^3SD_1$	———	$-110-379i$
$^3P_0$	———	———
$^3PF_2$	———	$-242-345i$

Quasi-bound states (in MeV) obtained from Kohno-Weise $N\bar{N}$ potential using NCSM.
Results are obtained using $N_{\max}=200$ and $\hbar\omega=20$.

Example of No-Core Shell Model calculation: Scattering

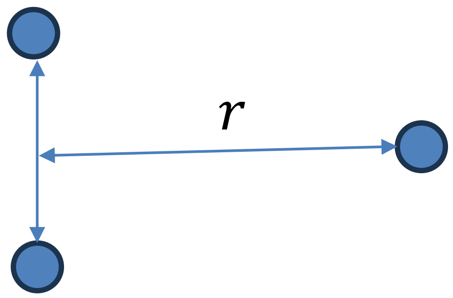


Real part of $^1S_0$ $\bar{p}n$ scattering phaseshift calculated using Complex Scaling method with $\theta = 20°$ and $n_r = 450$.

- RGM can be used for studying nuclear reaction as well as nuclear structure.

- The A-body system is considered as two clusters, i.e., projectile and target.

- The A-body wavefunction is taken to have the following form:



$$\Psi^{(A)}_{RGM} = \sum_{v} \frac{g_\nu(r)}{r} \left| \left(\left(\left| \alpha \quad I_1^{\pi_1} \, T_1 \right\rangle \left| \frac{1}{2} \quad \frac{1}{2} \right\rangle \right) s \, Y_l(\theta, \varphi) \right) J^\pi T \right\rangle$$

Relative — Target — Projectile — Relative (angular part)

The Hamiltonian (neglecting the three-body force) can be written as:

$$H = T_{rel}(r) + \bar{V}_c(r) + H_{tar} + \sum_{j=1}^{A-1} \left(V_{jA}^{s}(rjA) + V_{jA}^{c}(r_{jA})\right) - \bar{V}_c(r)$$

The RGM equation then becomes:

$$\left(T_{rel}(r) + \bar{V}_c(r) - \left(E - E_{\nu}^{\alpha_1}\right)\right)\frac{g_{\nu}(r)}{r} + \sum_{\nu'} \int dr' \, W_{\nu\nu'}(r,r') \frac{g_{\nu'}(r')}{r'} = 0$$

$\bar{V}_c(r)$: $\frac{-Ze^2}{r}$

$E_{\nu}^{\alpha_1}$: Target (binding) energy

$W_{\nu\nu'}(r,r')$: Contribution of interaction between target and projectile

For illustration, we consider a two body spinless system with a local potential:

$$(T_l + V_c + V_s - E)|u_l(E)\rangle = 0$$

We then add the Bloch operator $\mathcal{L}(B)$ to both sides:

$$\left(T_l + V_c + V_s - E + \mathcal{L}(B)\right)|u_l(E)\rangle = \mathcal{L}(B)\,|u_l(E)\rangle \quad , \quad \mathcal{L}(B) = \frac{\hbar^2}{2\mu}\delta(r-a)\left(\frac{\mathrm{d}}{\mathrm{d}r} - B\right)$$

Channel radius (a)

Arbitrary parameter (B)

For r outside the range of the potential, one can write the right hand side as:

$$\mathcal{L}(B)\,|u_l(E)\rangle = \mathcal{L}(B)\,|u_l^{ext}(E)\rangle \propto I_l(kr) + S_l(E)O_l(kr)$$

We then expand the internal wavefunction using the lagrange basis:

$$|u_l^{int} >= \sum_j A_j \left|f_j\right\rangle$$

The R-Matrix (setting $B = 0$) can be defined as:

$$R_l(E) = \frac{u_l(a)}{a\, u_l'(a)} = \frac{u_l^{int}(a)}{a\, u_l^{ext'}(a)} = \frac{\hbar^2}{2\mu a} \sum_{i,j}^{n_s} f_i(a)(C^{-1})_{ij} f_j(a)$$

$$C_{ij} = \left\langle f_i \middle| (T_l + V_c + V_s - E + \mathcal{L}(0)) \middle| f_j \right\rangle$$

Some algebra gives:

$$S_l(E) = \mathrm{e}^{2i\phi_l} \frac{1 - L_l^* R_l(E,0)}{1 - L_l R_l(E,0)}$$

Numerical Challenges

$$\left(T_{rel}(r) + \bar{V}_c(r) - (E - E_\nu^{\alpha_1})\right)\frac{g_\nu(r)}{r} + \sum_{\nu'}\int dr'\, W_{\nu\nu'}(r,r')\,\frac{g_{\nu'}(r')}{r'} = 0$$

$$W_{\nu\nu'}(r,r') = \sum_{nn'}^{N_{\max}} R_{n'l}(r')R_{nl}(r)\left\langle \phi_{\nu'n'}^{J^\pi T}\middle| V_{A,A-1} \middle| \phi_{\nu n}^{J^\pi T}\right\rangle$$

$$\left|\phi_{\nu n}^{J^\pi T}\right\rangle \equiv \left[\left[\left(|\alpha_1 I_1^{\pi_1} T_1\rangle\ \left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle\right) s\, Y_l(\theta,\varphi)\right]\right]^{J^\pi T} R_{nl}(r_{A,A-1})$$

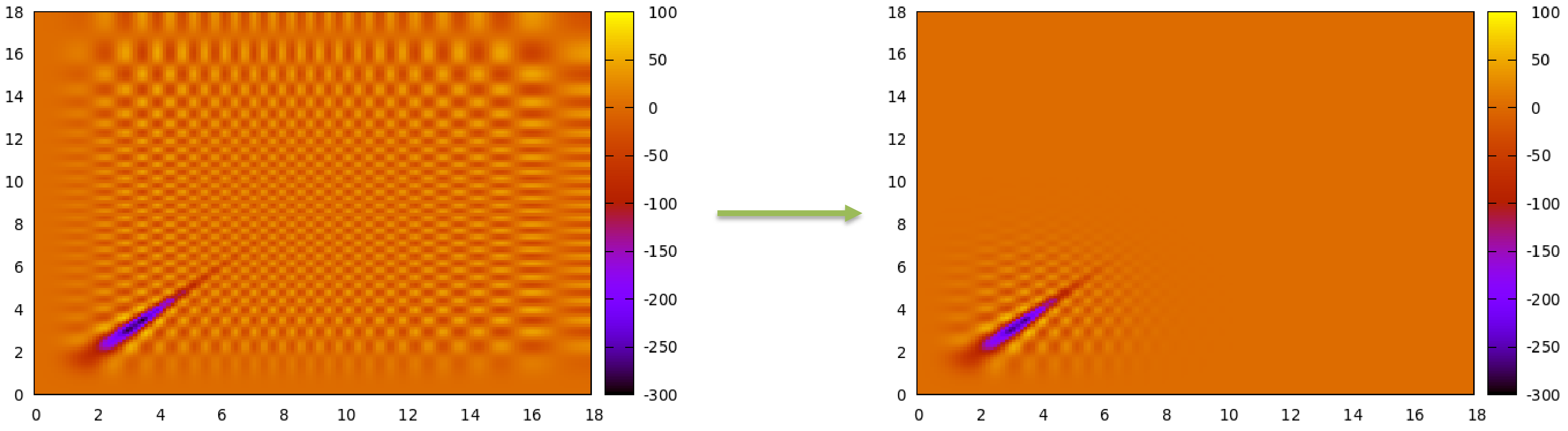
Harmonic oscillator wavefunctions

Still considerable at large n, n'

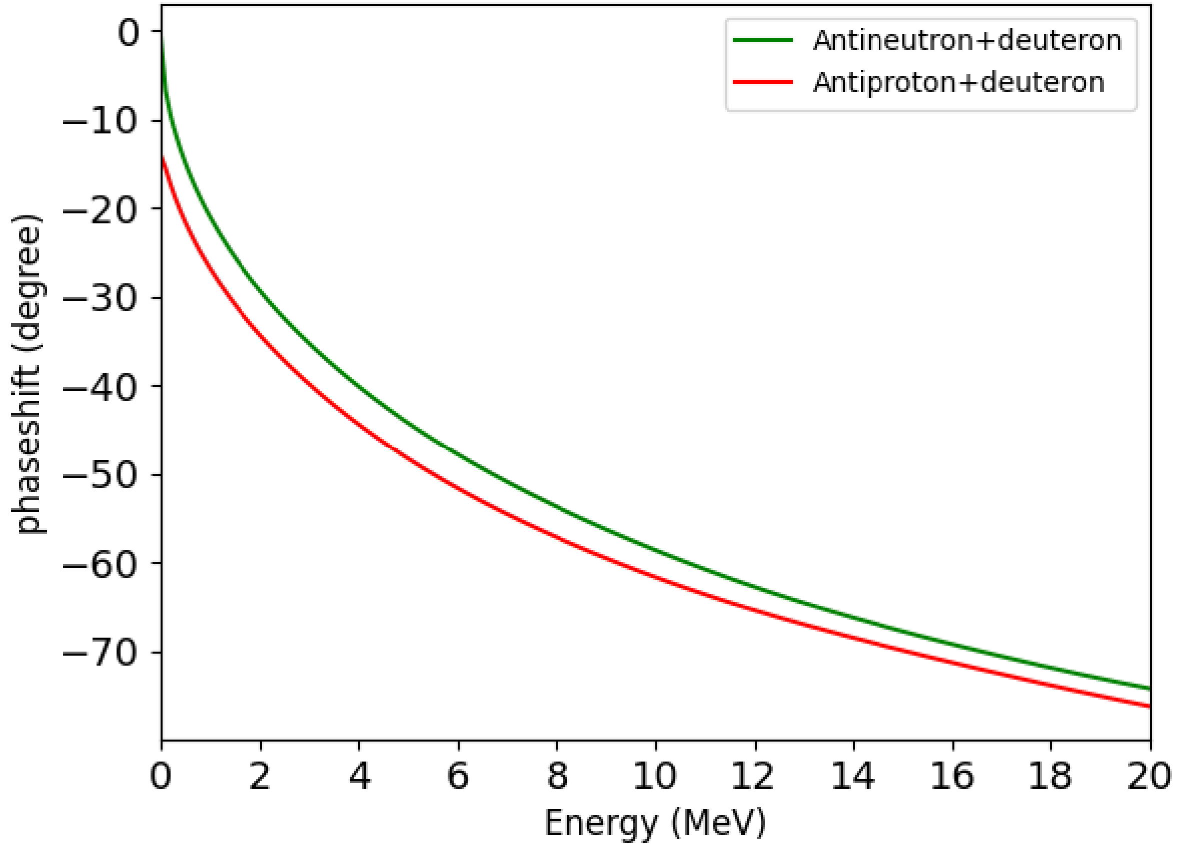
Convergence and other numerical issues

Numerical Challenges

- A very high N_{max} is needed for convergence.
- The high N_{max} introduces noise in the potential:



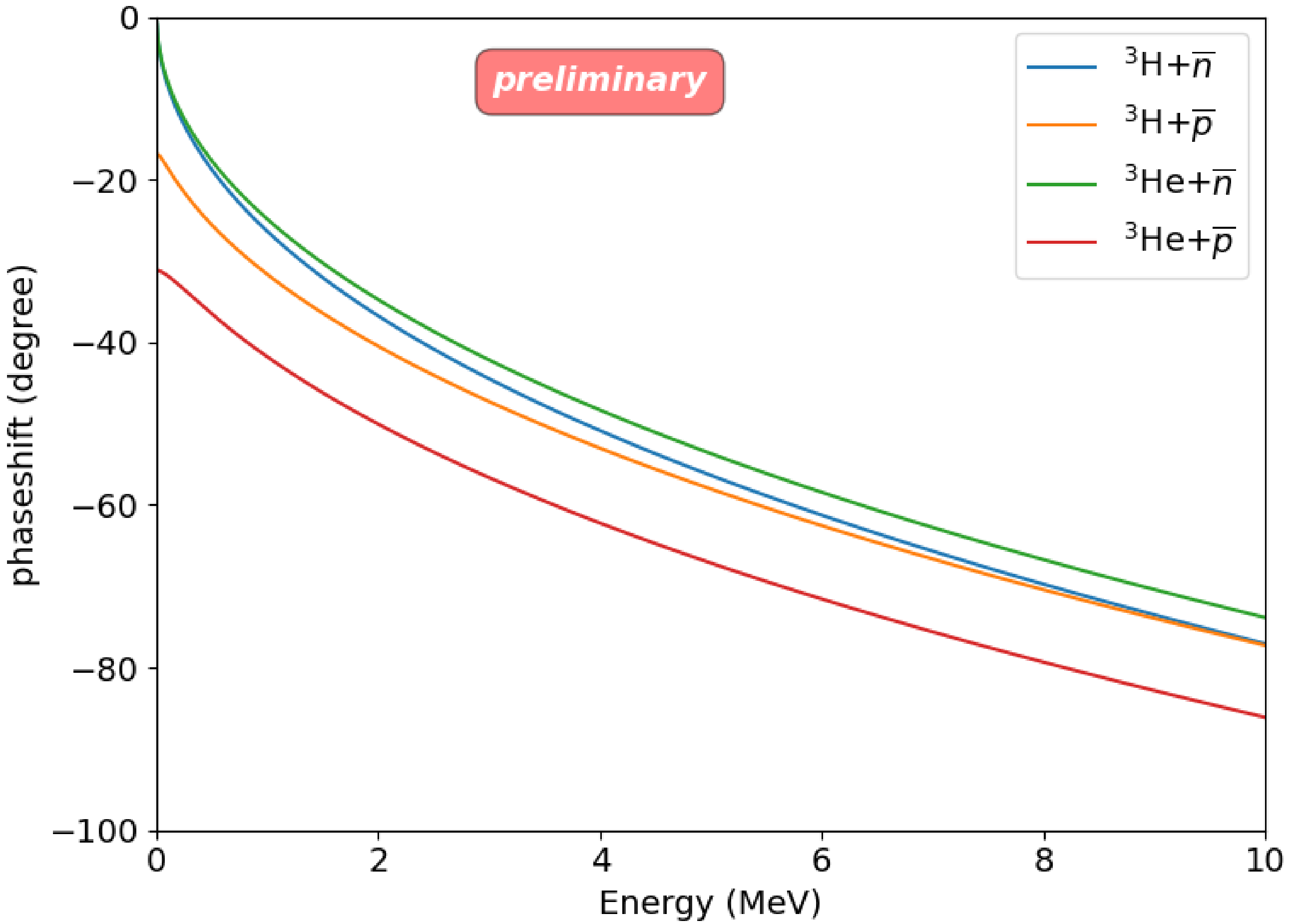
Real part of the RGM potential for antineutron-deuteron system in $^2S_{1/2}$ channel before and after regularization.



Quasi-bound state

$$E_{QB} = -21 \ - 153i \text{ MeV}$$

Real part of Antinucleon-deuteron phaseshift in $^2S_{1/2}$ channel with a = 18 fm and N_{max}= 80.



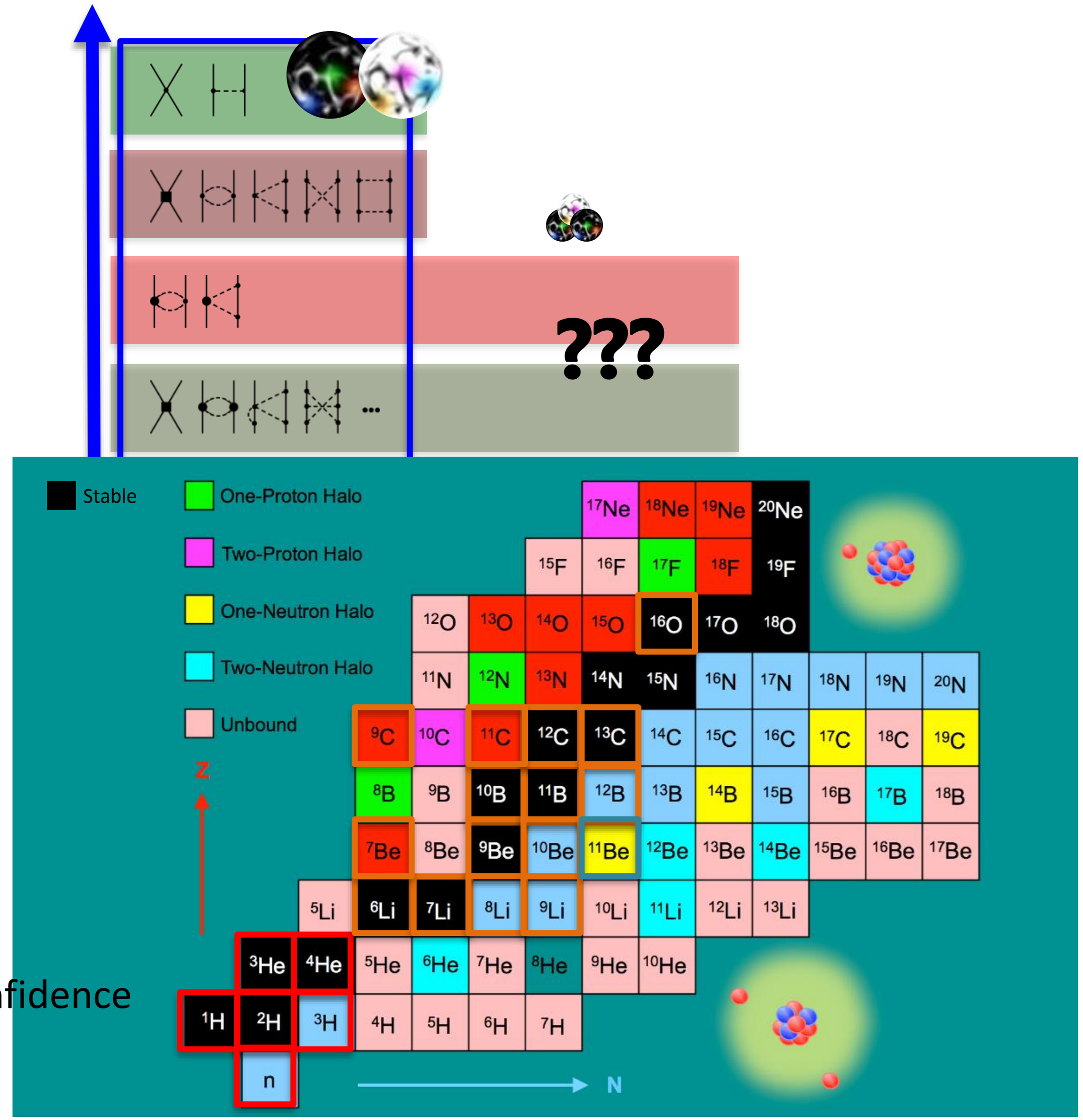
Real part of A=4 system phaseshift in $^1S_0$ channel with a = 10 fm and N_{max}= 30.

- There is no consensus among current $N\bar{N}$ potentials.

- In order to develop high-quality $N\bar{N}$ potentials, one needs low-energy data from experiments.

- Currently, there are no $N\bar{N}$ observable data available at threshold.

- The advanced *ab-initio* methods designed for NN systems can be applied to $N\bar{N}$ systems.



Halo nuclei
Within some confidence
Exact