

Modularity and Resurgence for $\mathcal{N} = 4$ Integrated Correlators

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Joint work w/
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MEETS
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$\mathcal{N} = 4$ SUSY YANG-MILLS THEORY

Beautiful “Lab” to extract exact results, e.g. correlation fcts

Uniquely specified by:

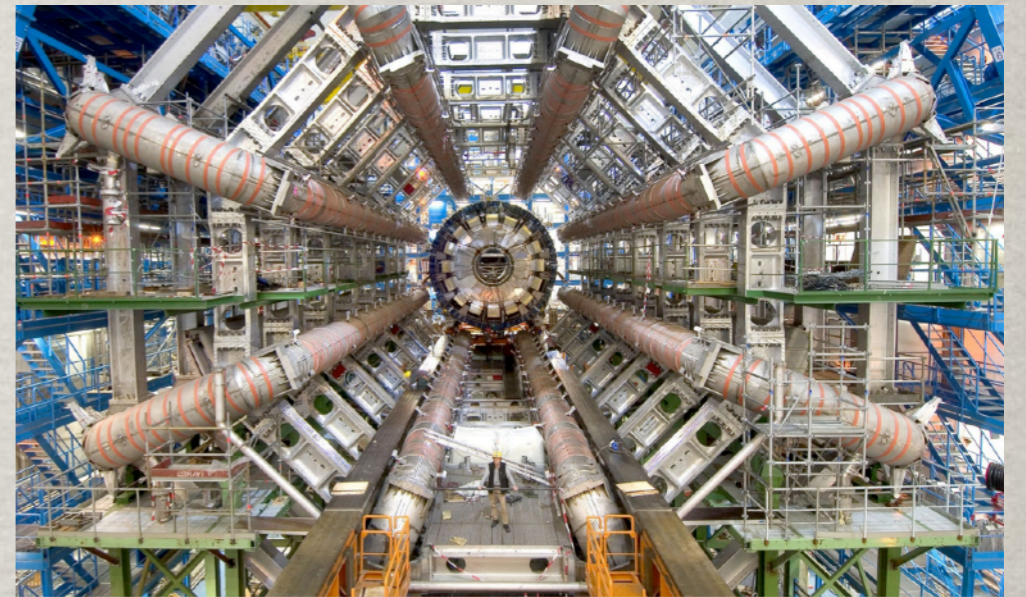
$\mathcal{N} = 4$ SYM Data:

- 4d Gauge theory: group G

In this talk: $G = \text{SU}(N)$

- single marginal deformation, coupling constant τ

$$\tau = \tau_1 + i\tau_2 = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\text{YM}}^2} \in \mathbb{H}$$



WHAT TYPE OF CORRELATORS

$\mathcal{N} = 4$ CORRELATORS

Amongst the many fields of $\mathcal{N} = 4$ we have:

Φ_I $I=1,\dots,6$ Adjoint scalar

$$\mathcal{O}_2(x, Y) = \text{Tr}(\Phi_I \Phi_J) Y^I Y^J \quad \Delta = 2$$

Null polarisation vectors for

1/2 BPS operators, super-conformal primaries in the stress-energy tensor super-multiplet ($[0,2,0]$)

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1/2 BPS operators, super-conformal primaries in the stress-energy tensor super-multiplet

We consider the (NON-PROTECTED!) 4pt function

$$\langle \mathcal{O}_2(x_1, Y_1) \dots \mathcal{O}_2(x_4, Y_4) \rangle = \frac{1}{|x_{12}|^4 |x_{34}|^4} \mathcal{I}(Y) \mathcal{T}_{G_N}(u, v)$$

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Fixed by super-conformal symmetry

$\mathcal{N} = 4$ CORRELATORS

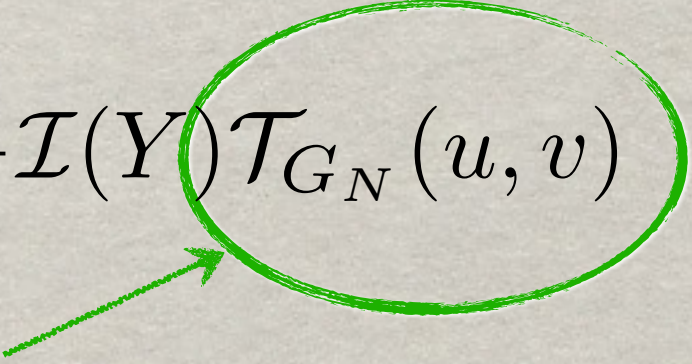
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Non-trivial function of u, v , coupling, gauge group G_N

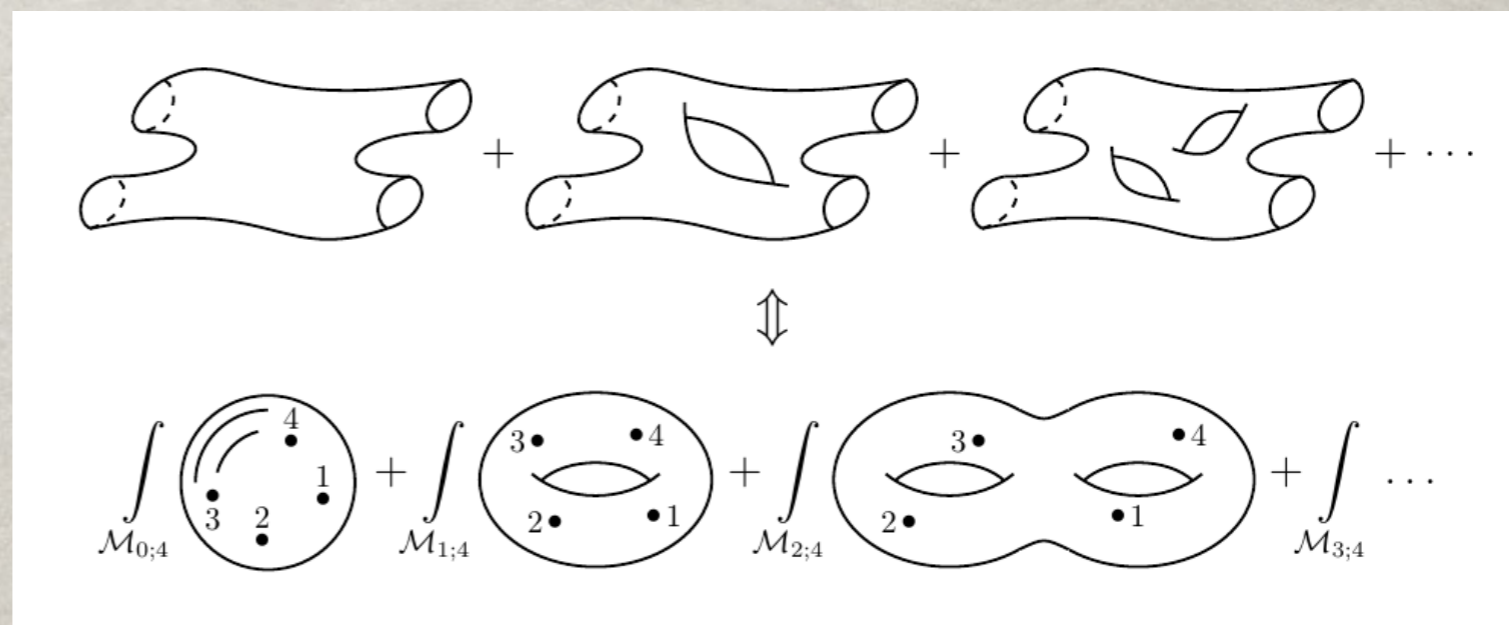
WHY DO WE CARE?

stress-energy tensor \simeq gravitons

$$\langle \mathcal{O}_2(x_1, Y_1) \dots \mathcal{O}_2(x_4, Y_4) \rangle = \frac{1}{|x_{12}|^4 |x_{34}|^4} \mathcal{I}(Y) \mathcal{T}_{G_N}(u, v)$$

Fun part = hard part

$$\langle T_{\mu_1 \nu_1}(x_1) \dots T_{\mu_4 \nu_4}(x_4) \rangle$$




4-graviton amplitude in IIB when $G_N = SU(N)$ @large-N

WHY DO WE CARE?

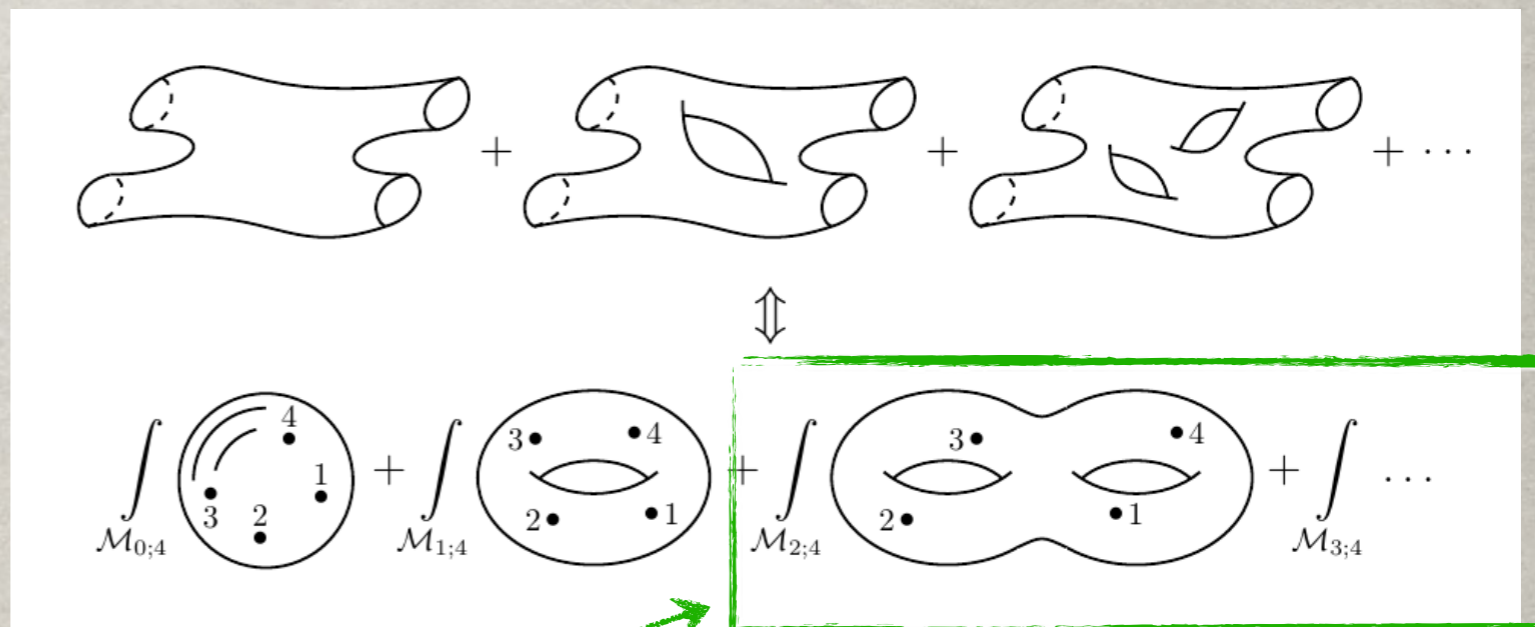
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Fun part = hard part



$$\langle T_{\mu_1 \nu_1}(x_1) \dots T_{\mu_4 \nu_4}(x_4) \rangle$$



For donughts with many holes see Oli's talk Wednesday!

HOW TO GET A HANDLE ON

$\mathcal{N} = 4$ INTEGRATED CORRELATORS

Pestun' Susy localisation for $\mathcal{N} = 2^*$ partition function on S^4
 $Z_G(m; \tau)$; massive deformation, m , of $\mathcal{N} = 4$

Path-Integral \implies Matrix Model

$$Z_G(m, \tau) := \int V_G(a) e^{-2\pi\tau_2 \langle a, a \rangle} \hat{Z}_G^{\text{pert}}(m; a) |\hat{Z}_G^{\text{inst}}(m, \tau; a)|^2 d^r a$$

1-Loop Nekrasov

$$\tau = \tau_1 + i\tau_2 = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{YM}^2}$$

G Gauge group

Perturbative
1-Loop determinant

Instanton
partition function

$\mathcal{N} = 4$ INTEGRATED CORRELATORS

Pestun' Susy localisation for $\mathcal{N} = 2^*$ partition function on S^4
 $Z_G(m; \tau)$; massive deformation, m , of $\mathcal{N} = 4$

$$\mathcal{C}_G(\tau) = \frac{1}{4} \Delta_\tau \partial_m^2 \log Z_G(m; \tau) \Big|_{m=0} \quad [\text{Binder, Chester, Pufu, Wang}]$$

$$= \int d\mu(\{x_i\}) \langle \mathcal{O}_2(x_1) \cdots \mathcal{O}_2(x_4) \rangle$$

$$\mathcal{H}_G(\tau) = \partial_m^4 \log Z_G(m; \tau) \Big|_{m=0}$$

$$= \int d\tilde{\mu}(\{x_i\}) \langle \mathcal{O}_2(x_1) \cdots \mathcal{O}_2(x_4) \rangle \quad [\text{Chester, Pufu}]$$

Very specific measures fixed by susy!

LINE-DEFECT INTEGRATED CORRELATORS

Integrated Correlator with line-defects: [Pufu, Rodriguez, Wang]

[Billo', Galvagno, Frau, Lerda]

$$\begin{aligned} I_{\mathbb{W}}(N; \tau) &= \partial_m^2 \log W_{SU(N)}(m, \tau) \Big|_{m=0} \\ &= \int d\nu(\{x_i\}) \langle \mathbb{W} \mathcal{O}_2(x_1) \mathcal{O}_2(x_2) \rangle \end{aligned}$$

1/2-BPS fundamental Wilson loop
along great circle of S^4
equivalently straight-line Wilson
loop on \mathbb{R}^4



$\mathcal{N} = 4$ INTEGRATED CORRELATORS

For the rest of the talk $G = \text{SU}(N)$:

We will discuss the integrated correlators

$\mathcal{C}(N; \tau) \longrightarrow$ **First** integrated Correlator

$\mathcal{H}(N; \tau) \longrightarrow$ **Second** integrated Correlator

$I_{\mathbb{W}}(N; \tau) \longrightarrow$ **Defect** integrated Correlator

Key Message(s):

Electro-magnetic (Olive-Montonen/GNO) duality strongly constraints $\mathcal{N} = 4$ SYM observables!

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Key Message(s):

- non-holomorphic functions of coupling τ : Lattice sums!
- “nice” automorphic properties under

$$\tau \rightarrow \gamma \cdot \tau = \frac{a\tau + b}{c\tau + d} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z})$$

FIRST INTEGRATED CORRELATOR

FIRST INTEGRATED CORRELATOR

From matrix model localised partition function on S^4

$$\mathcal{C}_G(\tau) = \frac{1}{4} \Delta_\tau \partial_m^2 \log Z_G(m; \tau) \Big|_{m=0}$$

[Binder, Chester, Pufu, Wang]

$$= \int d\mu(\{x_i\}) \langle \mathcal{O}_2(x_1) \dots \mathcal{O}_2(x_4) \rangle$$

We found an exact formula valid

for all simple gauge groups G

and all values of the coupling τ and G NO co/in-variant

[DD, Green, Wen]

[DD, Vallarino]

Using key-results of [Chester, Green, Pufu, Wang, Wen]

and [Alday, Chester, Hansen]

FIRST INTEGRATED CORRELATOR SU(N)

Lattice sum representation for $G=SU(N)$,

$$\mathcal{C}(N; \tau) = \sum_{(m,n) \in \mathbb{Z}^2} \int_0^\infty e^{-t \frac{\pi |m+n\tau|^2}{\tau_2}} B(N; t) dt$$

- Very unexpected (from matrix model) lattice sum!
- Modular invariant (or Fricke groups) function of τ
- $B(N; t)$ rational fcts expressed in terms of Jacobi Poly

$$\text{e.g. } B(2; t) = \frac{3(3t - 10t^2 + 3t^3)}{2(1+t)^5}$$

Properties:

- At weak coupling i.e. $\tau_2 \gg 1$, only single odd-zeta values

Checked up to 4 loops from
UN-integrated correlator!
[Wen,Zhang]-[Zhang]

$$C(N; \tau) \sim \sum_{s=2}^{\infty} \mathbb{Q}\zeta(2s-1)(\pi\tau_2)^{1-s}$$

- Spectral decomposition & holo ensemble avg
[Collier,Perlmutter]



Shun-qing's talk
on Wednesday

$$\begin{aligned} C(N; \tau) &= \frac{N(N-1)}{8} + \sum_{s=2}^{\infty} c(N; s) E^*(s; \tau) \\ &= \frac{N(N-1)}{4} + \int_{\text{Res}=\frac{1}{2}} \frac{\pi}{\sin(\pi s)} c(N; s) E^*(s; \tau) \frac{ds}{2\pi i} \end{aligned}$$

NOTE: No Maass cuspforms!

e.g. $c(2; s) = s(1-s)(2s-1)^2$

NON-HOLO (REAL ANALYTIC) EISENSTEINS SERIES

$$\begin{aligned} E^*(s; \tau) &= \frac{\Gamma(s)}{2} \sum_{(m,n) \neq (0,0)} \frac{(\tau_2/\pi)^s}{|m + n\tau|^{2s}} = E^*(1-s; \tau) \\ &= \xi(2s)\tau_2^s + \xi(2s-1)\tau_2^{1-s} \\ &\quad + \sum_{k \neq 0} e^{2\pi i k \tau_1} 2\sqrt{\tau_2} |k|^{s-\frac{1}{2}} \sigma_{1-2s}(k) K_{s-\frac{1}{2}}(2\pi |k| \tau_2) \end{aligned}$$

Modular invariant functions:

$$E^*(s; \gamma \cdot \tau) = E^*(s; \tau), \quad \forall \gamma \in SL(2, \mathbb{Z})$$

Laplace eigenfunctions:

$$(\Delta_\tau - s(s-1))E^*(s; \tau) = 0$$



RESURGENT LARGE-N EXPANSION
OF THE FIRST INTEGRATED CORRELATOR

SU(N) @ LARGE N

[DD,Green,Wen,Xie]-[DD,Treilis]

$$\mathcal{C}_N(\tau) = \sum_{(m,n) \in \mathbb{Z}^2} \int_0^\infty e^{-t\pi \frac{|m+n\tau|^2}{\tau_2}} B_{SU(N)}(t) dt$$

@Large-N Fixed- τ : split into P and NP contributions (in N)

$$\mathcal{C}_N(\tau) = \mathcal{C}_N^P(\tau) + \mathcal{C}_N^{NP}(\tau)$$

$$\mathcal{C}_N^P(\tau) = \frac{N^2}{4} + \sum_{r=0}^{\infty} N^{\frac{1}{2}-r} f_r(\tau)$$

$$\mathcal{C}_N^{NP}(\tau) = O(N^2 e^{-\sqrt{N}})$$

LARGE-N PERTURBATIVE

[DD,Green,Wen,Xie]-[DD,Treilis]

$$\mathcal{C}_N(\tau) = \mathcal{C}_N^P(\tau) + \mathcal{C}_N^{NP}(\tau)$$

The large-N expansion changes dramatically only half-integer index Eisensteins appear:

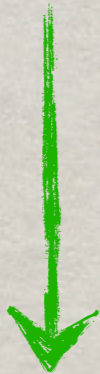


$$\begin{aligned} \mathcal{C}_N(\tau) \sim \mathcal{C}_N^P(\tau) &= \frac{N^2}{4} - \frac{3N^{\frac{1}{2}}}{2^4} E\left(\frac{3}{2}; \tau\right) + \frac{45}{2^8 N^{\frac{1}{2}}} E\left(\frac{5}{2}; \tau\right) \\ &+ \frac{3}{N^{\frac{3}{2}}} \left[\frac{1575}{2^{15}} E\left(\frac{7}{2}; \tau\right) - \frac{13}{2^{13}} E\left(\frac{3}{2}; \tau\right) \right] + O\left(N^{-\frac{5}{2}}\right) \end{aligned}$$

HOLOGRAPHIC PICTURE

stress-energy tensor \simeq gravitons

$$\langle \mathcal{O}_2(x_1, Y_1) \dots \mathcal{O}_2(x_4, Y_4) \rangle = \frac{1}{|x_{12}|^4 |x_{34}|^4} \mathcal{I}(Y) \mathcal{T}_{G_N}(u, v)$$



$$\langle T_{\mu_1 \nu_1}(x_1) \dots T_{\mu_4 \nu_4}(x_4) \rangle$$



4-graviton amplitudes in IIB

$$\tau = \chi + i/g_s$$
$$\frac{(\alpha')^2}{L^4} = \frac{1}{Ng_{YM}^2}$$

Axio-dilaton

LARGE-N PERTURBATIVE

-Fixed g_{YM} large-N (modularity is preserved):

[Chester, Green, Pufu, Wang, Wen]

$$\mathcal{C}_{SU(N)}(\tau, \bar{\tau}) \sim \frac{N^2}{4} - \frac{3N^{\frac{1}{2}}}{2^4} E\left(\frac{3}{2}; \tau, \bar{\tau}\right) + \frac{45}{2^8 N^{\frac{1}{2}}} E\left(\frac{5}{2}; \tau, \bar{\tau}\right) + \frac{3}{N^{\frac{3}{2}}} \left[\frac{1575}{2^{15}} E\left(\frac{7}{2}; \tau, \bar{\tau}\right) - \frac{13}{2^{13}} E\left(\frac{3}{2}; \tau, \bar{\tau}\right) \right] + O(N^{-\frac{5}{2}})$$


4-graviton effective action in type IIB low-energy expansion


[Green, Gutperle - Green, Vanhove- Green, Miller, Vanhove]


$$\tau = \chi + i/g_s$$


$$\frac{(\alpha')^2}{L^4} = \frac{1}{Ng_{YM}^2}$$

$$\mathcal{L}_{eff} = (\alpha')^{-4} g_s^{-2} R + f_1(\tau, \bar{\tau}) (\alpha')^{-1} g_s^{-1/2} R^4 + f_2(\tau, \bar{\tau}) \alpha' g_s^{1/2} d^4 R^4 + f_3(\tau, \bar{\tau}) (\alpha')^2 g_s d^6 R^4 + \dots$$


 N^2


 $N^{1/2}$


 $N^{-1/2}$


 N^{-1}

MODULAR RESURGENCE @ LARGE-N

@Large-N Perturbative expansion diverges factorially!

[DD,Green,Wen,Xie]- [DD,Treilis]

$$C_P(N; \tau) = \frac{N^2}{4} + \sum_{g=0}^{\infty} N^{2-2g} \mathcal{C}_P^{(g)}(N; \tau)$$

Resurgence analysis of “genus”-expansion
at large-N and fixed- τ

$$\mathcal{C}_P^{(g)}(N; \tau) = \sum_{k=0}^{\infty} b_{g,k} N^{-\frac{3}{2}-k} E^*\left(\frac{3}{2} + k; \tau\right)$$


e.g. $g=0$

$$b_{0,k} = \frac{(k+1)\Gamma(k-\frac{1}{2})\Gamma(k+\frac{5}{2})}{2^{2k+1}\pi^{3/2}\Gamma(k+1)}$$

LARGE-N NON-PERTURBATIVE

Modular Invariant Borel-Ecalle resummation:

[DD, Treillis]

$$\mathcal{C}^{(g)}(N; \tau) = \mathcal{C}_P^{(g)}(N; \tau) + \mathcal{C}_{NP}^{(g)}(N; \tau)$$


Infinite tower of modular invariant non-perturbative corrections at large-N recovered from Perturbative data!

$$\mathcal{C}_{NP}^{(g)}(N; \tau) = \sum_{k=-3g-1}^{\infty} d_{g,k} N^{-\frac{k+1}{2}} D_N\left(\frac{k+1}{2}; \tau\right)$$

$$D_N(s; \tau) = \sum_{(m,n) \neq (0,0)} \exp\left(-4\sqrt{\frac{N|m+n\tau|^2}{\tau_2}}\right) \frac{\tau_2^s}{|m+n\tau|^{2s}}$$

LARGE-N NON-PERTURBATIVE

Novel NP modular invariant functions!

[DD,Green,Wen,Xie]

$$D_N(s; \tau) = \sum_{(m,n) \neq (0,0)} \exp\left(-4\sqrt{\frac{N|m+n\tau|^2}{\tau_2}}\right) \frac{\tau_2^s}{|m+n\tau|^{2s}}$$

- Arise also in large-charge expansion of \mathcal{O}_p integrated corr.
[Paul,Perlmutter,Raj]-[Brown,Wen Xie]
- Torodial Casimir energy in 3-dimensional CFTs
[Luo,Wang]

In 't Hooft limit: $\lambda = 4\pi N/\tau_2$ fixed / $\tilde{\lambda} = N^2/\lambda$ fixed

F-string world-sheet instantons $e^{-2\ell\sqrt{\lambda}}$ and

“dyonic” instantons $e^{-2\ell\sqrt{\tilde{\lambda}}}$

Reproducing resurgence results at large- λ , large- $\tilde{\lambda}$

[DD,Green,Wen] - [Collier,Perlmutter] - [Hatsuda,Okuyama]

HOLOGRAPHIC INTERPRETATION:

Holo. Dictionary: consider $AdS_5 \times S^5$ with scale L

$$g_{YM}^2 = \frac{4\pi}{\tau_2} = 4\pi g_s \quad \text{and} \quad \sqrt{g_{YM}^2 N} = \frac{L^2}{\alpha'}$$

$$T_F = \frac{1}{2\pi\alpha'} \quad \Rightarrow \quad T_{p,q} = T_F |p + q\tau|$$

$$\mathcal{C}_{SU(N)}^{N.P.}(\tau, \bar{\tau}) \rightarrow \sum_{\ell=1}^{\infty} \sum_{\gcd(p,q)=1} \exp\left(-4\pi L^2 \ell T_{p,q}\right)$$

NP terms are given by sum over ℓ coincident (p,q) -strings euclidean world-sheet wrapping a great S^2 inside S^5

[Some key differences for SO and USp]

Defect integrated correlator



Second integrated correlator

First integrated correlator

LARGE-N EXPANSION
OF THE SECOND INTEGRATED
CORRELATOR

SECOND INTEGRATED CORRELATOR

[Alday, Chester, DD, Green, Wen]

$$\begin{aligned}\mathcal{H}(N; \tau) &= \partial_m^4 \log Z_{SU(N)}(m; \tau) \Big|_{m=0} \\ &= \int d\tilde{\mu}(\{x_i\}) \langle \mathcal{O}_2(x_1) \dots \mathcal{O}_2(x_4) \rangle\end{aligned}$$

[Chester, Pufu]

Large-N Perturbative expansion:

$$\mathcal{H}(N; \tau) \sim 6N^2 + \mathcal{H}^h(N; \tau) + \mathcal{H}^i(N; \tau)$$

SECOND INTEGRATED CORRELATOR

[Alday, Chester, DD, Green, Wen]

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Large-N Perturbative expansion:

$$\mathcal{H}(N; \tau) \sim 6N^2 + \mathcal{H}^h(N; \tau) + \mathcal{H}^i(N; \tau)$$

Half-integer powers in $1/N \longrightarrow$ Same structure as first
Integrated Correlator

Can find NP completion with Modular/Resurgence

[DD, Treilis]

SECOND INTEGRATED CORRELATOR

[Alday, Chester, DD, Green, Wen]

$$\mathcal{H}(N; \tau) \sim 6N^2 + \mathcal{H}^h(N; \tau) + \mathcal{H}^i(N; \tau)$$

Integer powers in $1/N \rightarrow$ Generalised Eisenstein Series

Modular invariant solutions to inhomogeneous Laplace eq.

$$(\Delta_\tau - s(s-1))\mathcal{E}(s; s_1, s_2; \tau) = E(s_1; \tau)E(s_2; \tau)$$

e.g. higher derivative correction $d^6 R^4$ in IIB: $\mathcal{E}(4; \frac{3}{2}, \frac{3}{2}; \tau)$

[Green, Miller, Vanhove]

SECOND INTEGRATED CORRELATOR

[Alday, Chester, DD, Green, Wen]

Order by order in $1/N$: $\mathcal{H}_N^i(\tau)$ Lattice sum!!

$$\mathcal{E}_{i,j}^w(\tau) = \sum_{\substack{p_1, p_2, p_3 \neq 0 \\ p_1 + p_2 + p_3 = 0}} \int_0^\infty d^3t B_{i,j}^w(t_1, t_2, t_3) \exp\left(-\frac{\pi}{\tau_2} \sum_{i=1}^3 t_i |p_i|^2\right)$$

$p_i \in \mathbb{Z} + \tau\mathbb{Z}$

Modular Local Harmonic Maass-forms

[Zagier], see also [Bringmann, Kane]

Special rational linear combinations of generalised Eisenstein series for which L-values of holomorphic cusp forms drops out

[DD, Kleinschmidt, Schlotterer]-[Fedosova, Klinger-Logan, Radchenko]

LAPLACE-DIFFERENCE EQUATIONS

First Integrated Correlator: [DD, Green, Wen]

$$\begin{aligned} & [\Delta_\tau + 2(N^2 - 1)]\mathcal{C}_N(\tau) \\ &= N(N - 1)\mathcal{C}_{N+1}(\tau) + N(N + 1)\mathcal{C}_{N-1}(\tau) \end{aligned}$$

Second Integrated Correlator: [Alday, Chester, DD, Green, Wen]

Half-integer powers

$$\begin{aligned} \Delta_\tau \mathcal{H}_N^h(\tau) &= N^2(N^2 - 1) \left[\frac{\mathcal{H}_{N+1}^h(\tau)}{(N+1)^2} - 2\frac{\mathcal{H}_N^h(\tau)}{N^2} + \frac{\mathcal{H}_{N-1}^h(\tau)}{(N-1)^2} \right] \\ &\quad + \frac{16N}{N^2 - 1} \left[\frac{3N^3}{4} - (N-1)^2\mathcal{C}_{N+1}(\tau) + 3N\mathcal{C}_N(\tau) + (N+1)^2\mathcal{C}_{N-1}(\tau) \right] \end{aligned}$$

Integer powers

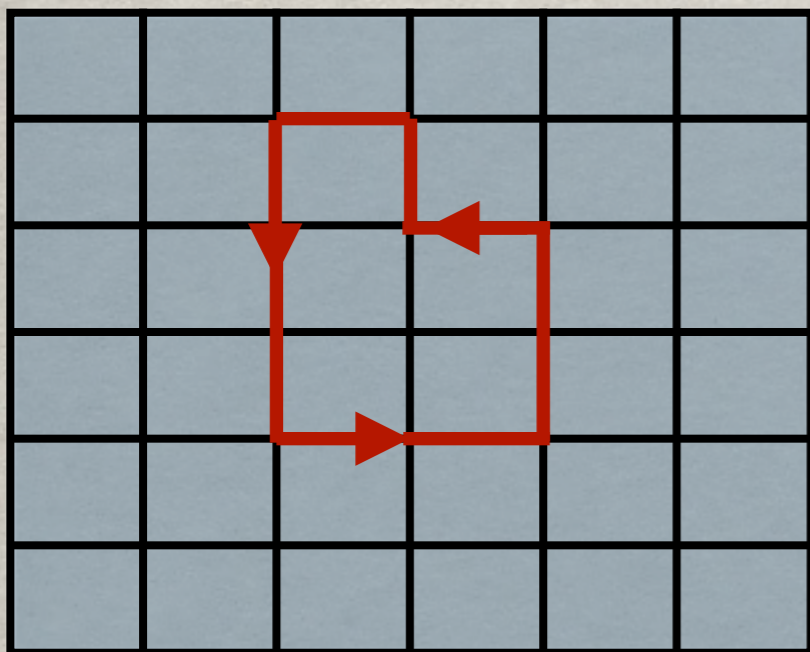
$$\Delta_\tau \mathcal{H}_N^i(\tau) \Big|_{\text{Eisenstein}} = \frac{96}{N^2 - 1} \left(\mathcal{C}_N(\tau) - \frac{N^2}{4} \right)^2$$

**LINE-DEFECT
INTEGRATED CORRELATORS**

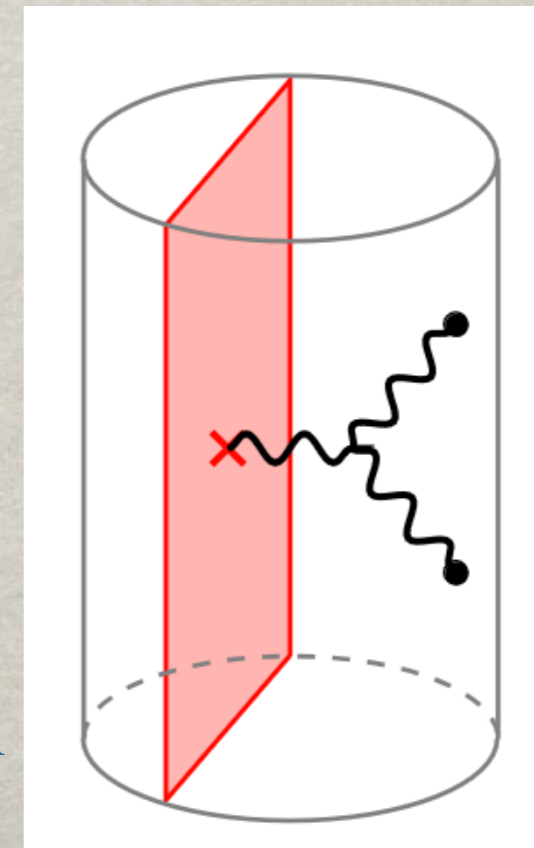
IMPORTANCE OF LINE-DEFECTS:

Important examples of non-local operators:

- QCD: confinement/deconfinement phase transition;
- important for higher form symmetries;
- class of $\mathcal{N} = 4$ SYM line defect integrated correlators teaches us about graviton scattering from branes.



Shamelessly 'borrowed' from
[Pufu, Rodriguez, Wang]



LINE-DEFECTS & EM DUALITY:

We consider line defect \mathbb{L} in $\mathcal{N} = 4$ $SU(N)$ SYM parametrised by an electric and a magnetic charge:

$p, q \in \mathbb{Z}$ with $(p, q) = 1$

- $(p, q) = (1, 0) \longrightarrow$ Wilson loop \mathbb{W}
- $(p, q) = (0, 1) \longrightarrow$ 't Hooft loop \mathbb{T}



Under electromagnetic duality (Olive-Montonen/GNO):

$$\tau \rightarrow \tau' = \gamma \cdot \tau = \frac{a\tau + b}{c\tau + d}$$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$(p, q) \rightarrow (p', q') = (p, q) \begin{pmatrix} a & -c \\ -b & d \end{pmatrix}$$

$$\mathbb{L}_{p, q} \longrightarrow \mathbb{L}_{p', q'}$$

LINE-DEFECTS & EM DUALITY:

Line-defect integrated correlator:

[Pufu, Rodriguez, Wang]

[Billo', Galvagno, Frau, Lerda]

$$I_{\mathbb{W}}(N; \tau) = \partial_m^2 \log W_{SU(N)}(m, \tau) \Big|_{m=0}$$
$$= \int d\nu(\{x_i\}) \langle \mathbb{W} \mathcal{O}_2(x_1) \mathcal{O}_2(x_2) \rangle$$



1/2-BPS fundamental Wilson loop along great circle

$\mathcal{N} = 4$ SYM EM duality constraint:

$$I_{\mathbb{W}}(N; \gamma \cdot \tau) = I_{\mathbb{L}_{p,q}}(N; \tau)$$

$$\gamma \cdot \tau = \frac{a\tau + b}{q\tau + p}$$

e.g. $I_{\mathbb{W}}(N; -\frac{1}{\tau}) = I_{\mathbb{T}}(N; \tau)$

$$\gamma = \begin{pmatrix} a & b \\ q & p \end{pmatrix} \in SL(2, \mathbb{Z})$$

LINE-DEFECTS & EM DUALITY:

to appear soon [DD,Duan, Pavarini, Xie, Wen]

$\mathcal{N} = 4$ SYM integrated correlator of a line defect \mathbb{L} with charges (p, q)

- lattice-sum representation
- novel automorphic functions:

$$F_{\mathbb{L}_{p,q}}(s_1, s_2, s_3; \tau) = \frac{\tau_2^{s_1}}{|q\tau + p|^{2s_1}} \sum_{(n,m) \neq \mathbb{Z}(q,p)} \frac{\tau_2^{s_2}}{|n\tau + m|^{2s_2}} (np - mq)^{s_3}$$

$s_1, s_2 \in \mathbb{Z}$ or $\mathbb{Z} + \frac{1}{2}$ \longrightarrow Finite-N/Large-N transition
 $s_3 \in 2\mathbb{Z}$

LINE-DEFECTS & EM DUALITY:

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$\mathcal{N} = 4$ SYM integrated correlator of a line defect \mathbb{L} with charges (p, q)

- lattice-sum representation
- novel automorphic functions:

Importantly:

$$F_{\mathbb{L}_{p,q}}(s_1, s_2, s_3; \tau) = F_{\mathbb{L}_{p',q'}}(s_1, s_2, s_3; \gamma \cdot \tau)$$

E/M Duality:

$$\tau \rightarrow \tau' = \gamma \cdot \tau$$

$$(p, q) \rightarrow (p', q') = (p, q) \begin{pmatrix} a & -c \\ -b & d \end{pmatrix}$$

E.g. We can predict the exact 't Hooft line-defect integrated correlator say in $SU(2)$ $\mathcal{N} = 4$ SYM



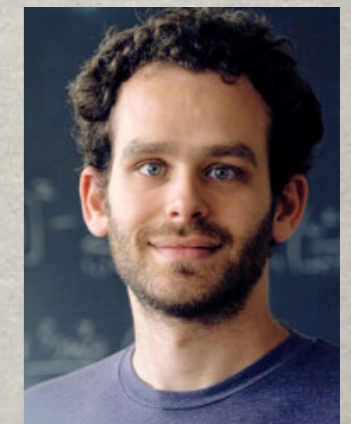


CONCLUSIONS:

- ✿ The power of modularity: Astonishingly simple & beautiful non-perturbative results for non-protected correlators in $\mathcal{N} = 4$ SYM!
- ✿ Integrated correlators and super conformal bootstrap [Chester,Dempsey,Pufu] see also [Behan,Chester,Ferrero]
- ✿ Other integrated correlators:
 - Higher point functions MUV [DD,Green,Wen]
 - Higher-charge operators [Brown,Wen,Xie]-[Paul,Perlmutter,Raj]
 - Giant-graviton operators [Brown,Galvagno,Wen]
 - $\mathcal{N} = 2$ SYM [Billò,Frau,Lerda,Pini,Vallarino]-[Pini,Vallarino]



Then come and listen to
Paolo & Francesco
this afternoon
and Shai's talk
tomorrow



OPEN QUESTIONS:

- ✱ Systematic of finite- N /large- N for second correlator & defect correlator? 't Hooft loop from susy localisation?
- ✱ String theory/QFT origin of these lattice-sum representations?
- ✱ What is the origin of these differential structures?
2d/4d correspondence?
- ✱ **Math.NT**: Why does string theory only like MZVs?
- ✱ How far can we push integrated correlators to learn about un-integrated stuff?

**THANK YOU
FOR YOUR ATTENTION!**