Tuning Ramond-Ramond flux for AdS3 strings

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Maximally supersymmetric AdS3 backgrounds

These backgrounds have 16 susys (half of $AdS_5 \times S^5$):

$$AdS_3 \times S^3 \times T^4, \qquad AdS_3 \times S^3 \times K3, \qquad AdS_3 \times S^3 \times S^3 \times S^1$$

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In this talk I will focus on the **planar spectrum** of

$$AdS_3 \times S^3 \times T^4$$

which is actually a **family of backgrounds**.

Isometries and spectrum

The AdS_3 isometries are $\mathfrak{so}(2,2) \cong \mathfrak{su}(1,1)^{\oplus 2} \cong \mathfrak{sl}(2,\mathbb{R})^{\oplus 2}$ with generators

$$\mathbf{L}_m\,, \quad m = 0, \pm 1\,, \qquad \widetilde{\mathbf{L}}_{\dot{m}}\,, \quad \dot{m} = 0, \pm 1.$$

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It will also be convient to consider the **lightcone energy**

$$\mathbf{H}_{tot} = \mathbf{E}_{tot} - \mathbf{J}_{tot}\,, \qquad \mathbf{J}_{tot} = \mathbf{J}^{+-} + \widetilde{\mathbf{J}}^{+-}$$

“Simplest” AdS3 backgrounds: pure NSNS backgrounds

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In this case, there is only the metric $G^{\mu\nu}$ and Kalb-Ramond field $B^{\mu\nu}$ so that

$$H = \mathrm{d}B = \mathrm{vol}(AdS_3) + \mathrm{vol}(S^3)$$

The only (interesting) parameter is $k = 1, 2, 3, 4, \ldots$, which becomes the string tension.

In units where $R_{AdS_3} = R_{S^3} = 1$, the tension is

$$T = \frac{k}{2\pi}$$

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The classical string action is that of a WZW model. [Giveon, Kutasov, Seiberg '98] [...]

Pure NSNS backgrouds as WZW models $(k > 1)$

$AdS_3 \times S^3 \times T^4$ can be realised as a WZW model in the RNS formalism, based on

$$\left(\mathfrak{sl}(2,\mathbb{R})_{k+2} \oplus \mathfrak{su}(2)_{k-2}\right)^{\oplus 2} \quad \text{Kač-Moody representations, plus free fermions}$$

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The short-string spectrum is generated by the modes of the Kač-Moody currents (and of the free fermions and free T^4 bosons) on a reference state $|\ell_0, j_0\rangle$, $\ell_0 \in \mathbb{R}$, $j_0 \in \mathbb{N}$. Schematically

$$\left|\Psi_{\{n_j,\tilde{n}_j\}}\right\rangle = \Big(\alpha_{-n_1}\cdots\alpha_{-n_r}|\ell_0, j_0\rangle\Big) \otimes \Big(\tilde{\alpha}_{-\tilde{n}_1}\cdots\tilde{\alpha}_{-\tilde{n}_s}|\tilde{\ell}_0, j_0\rangle\Big)$$

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The Virasoro constraint gives a quadratic equation for $\ell_0 = \tilde{\ell}_0$, so that [Maldacena, Ooguri '00] [...]

$$\mathbf{E}_{\text{tot}}\left|\Psi_{\{n_j,\tilde{n}_j\}}\right\rangle = \sqrt{\left(j_0 + \tfrac{1}{2}\right)^2 + 2k\left(n_1 + \cdots + n_r + \tilde{n}_1 + \cdots + \tilde{n}_s\right)}\,\left|\Psi_{\{n_j,\tilde{n}_j\}}\right\rangle$$

Spectrum for pure-NSNS backgrouds

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This is **highly degenerate** as it does not depend on the individual n_j.

It is also much simpler than what happens for RR backgrounds like AdS_5 and AdS_4.

(There is also a “long string” continuum spectrum.)

Mixed-flux backgrouds

It is possible to **continuously turn on a RR flux for fixed k**, e.g. by switching on an axion in the F1-NS5 system. [O-Sax, Stefanski '18]

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This gives a IIB background with the same metric but fluxes

$$H = \mathrm{d}B = q\left(\mathrm{vol}(AdS_3) + \mathrm{vol}(S^3)\right)\,, \qquad F_3 = \sqrt{1-q^2}\left(\mathrm{vol}(AdS_3) + \mathrm{vol}(S^3)\right)\,.$$

where $0 < q \leq 1$ is a new parameter. The tension is now larger

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It is convenient to use **two parameters**, $h \geq 0$ and $k \in \mathbb{N}$

$$h = \sqrt{1-q^2}\,T, \qquad \frac{k}{2\pi} = qT, \qquad T = \sqrt{h^2 + \frac{k^2}{4\pi^2}}.$$

Mixed and pure-RR background

We expect that turning on $h > 0$ will lift the degeneracies of the spectrum, and give rather intricate expression for the energies (like for $AdS_5 \times S^5$).

It is very difficult to compute the spectrum in the RNS formalism. [Cho, Collier, Yin '20]

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There is also a family of **pure-RR backgrounds**, for which

$$k = 0 \quad \Rightarrow \quad B_{\mu\nu} = 0, \qquad h > 0$$

They arise from the **D1-D5 system**, and are even harder to study in the RNS formalism.

Integrability: an alternative way to quantise the string

The classical Green-Schwarz action for $AdS_3 \times S^3 \times T^4$ is **integrable for any** h, k.

[Cagnazzo, Zarembo '12]

This provides a scheme to quantise the model in lightcone gauge, like for $AdS_5 \times S^5$.

[Arutyunov, Frolov, Zamaklar '06] [...]

Using this approach, one may compute the spectrum for any k, h, as we shall see.

Lightcone gauge for the GS string

Schematically we take $\phi \in S^3$ and $t \in AdS_3$ to make ligthcone coordinates $X^{\pm}$ and set:

$$X^+ = \tau \quad \& \quad \mathcal{P}_- = 1 \quad \text{where} \qquad \mathcal{P}_\mu = \frac{\delta \mathcal{S}_{GS}}{\delta(\partial_\tau X^\mu)}$$

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The **worldsheet Hamiltonian $\mathbf{H}_{w.s.}$ is precisely $\mathbf{H}_{tot}$**

$$\mathbf{H}_{w.s.} = -\int_0^L \mathrm{d}\sigma \mathcal{P}_+ = \mathbf{E}_{tot} - \mathbf{J}_{tot} = \mathbf{H}_{tot}$$

For convenience we split $\mathbf{H}_{tot} = \mathbf{H} + \widetilde{\mathbf{H}} = \left(\mathbf{L}_0 - \mathbf{J}^{+-}\right) + \left(\widetilde{\mathbf{L}}_0 - \widetilde{\mathbf{J}}^{+-}\right).$

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The **worldsheet size** L is also fixed

$$L = \int_0^L \mathrm{d}\sigma \mathcal{P}_- = \mathbf{J}^{+-} + \widetilde{\mathbf{J}}^{+-} = \mathbf{J}_{tot}$$

Symmetries

The full symmetry algebra is $\mathfrak{psu}(1,1|2)^{\oplus 2}$, with BPS bound

$$\mathbf{H} \equiv \mathbf{L}_0 - \mathbf{J}^{+-} \geq 0\,, \qquad \widetilde{\mathbf{H}} \equiv \widetilde{\mathbf{L}}_0 - \widetilde{\mathbf{J}}^{+-} \geq 0 \qquad \Rightarrow \qquad \mathbf{H}_{tot} \geq 0$$

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The $AdS_3 \times S^3 \times T^4$ Killing spinors fit in $\mathfrak{psu}(1,1|2)^{\oplus 2}$ as

$$\mathbf{G}^{\alpha A}_{m}, \quad m = \pm\tfrac{1}{2}, \quad \alpha = \pm, \qquad \widetilde{\mathbf{G}}^{\dot{\alpha} A}_{\dot{m}}, \quad \dot{m} = \pm\tfrac{1}{2}, \quad \dot{\alpha} = \pm,$$

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In lightcone gauge, only half of the supercharges survive.

Symmetries in lightcone gauge

There are four “left” and four “right” superchages

$$\mathbf{G}^{-A}_{+\frac{1}{2}},\ \mathbf{G}^{+A}_{-\frac{1}{2}},\quad A=1,2, \qquad\qquad \widetilde{\mathbf{G}}^{\dot{-}A}_{\dot{+}\frac{1}{2}},\ \widetilde{\mathbf{G}}^{\dot{+}A}_{\dot{-}\frac{1}{2}},\quad A=1,2$$

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Their algebra is quite simple when $\mathbf{J}_{tot}\sim L\to\infty$

$$\left\{\mathbf{G}^{+A}_{-\frac{1}{2}},\mathbf{G}^{-B}_{+\frac{1}{2}}\right\}=\varepsilon^{AB}\,\mathbf{H}\,, \qquad\qquad \left\{\widetilde{\mathbf{G}}^{\dot{+}A}_{\dot{-}\frac{1}{2}},\widetilde{\mathbf{G}}^{\dot{-}B}_{\dot{+}\frac{1}{2}}\right\}=\varepsilon^{AB}\,\widetilde{\mathbf{H}}\,,$$

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$$\{\mathbf{G}^{+A}_{-\frac{1}{2}},\widetilde{\mathbf{G}}^{\dot{+}B}_{\dot{-}\frac{1}{2}}\}=\varepsilon^{AB}\,\mathbf{C}^{+\dot{+}}_{-\frac{1}{2},\dot{-}\frac{1}{2}}\,,\qquad \{\widetilde{\mathbf{G}}^{\dot{+}A}_{\dot{-}\frac{1}{2}},\mathbf{G}^{-B}_{+\frac{1}{2}}\}=\varepsilon^{AB}\,\mathbf{C}^{-\dot{-}}_{+\frac{1}{2},\dot{+}\frac{1}{2}}\,,$$

[Borsato, O-Sax, AS '12] [Lloyd, O-Sax, AS, Stefanski '14]

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The charges $\mathbf{C}\equiv\mathbf{C}^{+\dot{+}}_{-\frac{1}{2},\dot{-}\frac{1}{2}}$ and $\mathbf{C}^{\dagger}\equiv\mathbf{C}^{-\dot{-}}_{+\frac{1}{2},\dot{+}\frac{1}{2}}$ are central extensions due to the gauge fixing.

On the central extensions

If we consider an asymptotic state on the worldsheet (at $L \to \infty$)

$$|p_1, \dots p_n\rangle_L \equiv A^\dagger(p_1) \cdots A^\dagger(p_n)|0\rangle_L$$

we find

$$\mathbf{C}|p_1, \dots p_n\rangle_L = \frac{i h}{2} \left(e^{i(p_1+\cdots+p_n)} - 1 \right) |p_1, \dots p_n\rangle_L$$

Level-matching: $p_1 + \cdots + p_n = 0 \ \mathrm{mod} 2\pi$.

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Fundamental excitations of the model (the modes of the transverse $AdS_3 \times S^3 \times T^4$) obey

$$\mathbf{H}\,\widetilde{\mathbf{H}} = \mathbf{C}^\dagger\mathbf{C}$$

This allows to derive an exact dispersion relation $\omega(p)$

$$\mathbf{H}_{tot}\,|p_1, \dots p_n\rangle_L = \sum_{j=1}^{n} \omega(p_j)\,|p_1, \dots p_n\rangle_L\,.$$

The dispersion relation

Exact dispersion relation ($L = \infty$): [Hoare, Stepanchuk, Tseytlin '13] [Lloyd, O-Sax, AS, Stefanski '14]

$$\omega(p) = \sqrt{\left(\frac{k}{2\pi} p + \mu\right)^2 + 4h^2 \sin^2\left(\frac{p}{2}\right)},$$

where $\mu = 0, 1, \ldots k - 1$ labels the representations.

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If $h = 0$, it is chiral (WZW model) [Baggio, AS '17] [Dei, AS '18]

$$\omega(p) = \left|\frac{k}{2\pi}p + \mu\right|, \qquad \mu = 0, 1, \dots k-1.$$

If $k = 0$, periodic (like for $AdS_5 \times S^5$) [Borsato, O-Sax, AS '12]

$$\omega(p) = \sqrt{\mu^2 + 4h^2 \sin^2\left(\frac{p}{2}\right)}, \qquad \mu \in \mathbb{Z}$$

S-matrix and spectrum

The symmetries allow to fix an S matrix for worldsheet excitations

$$\mathbf{S}\ A^\dagger_a(p_1)A^\dagger_b(p_2)\,|0\rangle_\infty = e^{i\Phi(p_1,p_2)}\,S^{cd}_{ab}(p_1,p_2)\ A^\dagger_c(p_2)A^\dagger_d(p_4)\,|0\rangle_\infty$$

The dressing factor $\Phi(p_1, p_2)$ is hardest to fix.

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The energy spectrum at fixed L is then fixed by taking

$$e^{ip_i L}\prod_{j=1}^{n} S(p_i,p_j) = 1\,, \qquad i = 1,\dots n \quad \Rightarrow \quad p_i = \frac{2\pi\nu_i}{L} + \frac{F_i(\{\nu_j\})}{L^2} + \mathcal{O}(L^{-3}), \quad \nu_i \in \mathbb{Z}$$

and plugging the solutions in

$$H_{tot} = \sum_{i=1}^{n} \omega(p_i(\nu)) \qquad \text{up to ``wrapping'' corrections}$$

Results

Spectrum at $h = 0$ and any $k \in \mathbb{N}$, which agrees with the WZW construction. [Dei, AS '18]

Spectrum at $k = 0$ and any $h > 0$, which displays new intriguing features.

[Ekhammar, Volin '21] [Cavaglià, Gromov, Stefanski, Torrielli '21] [Frolov, AS '21] [Brollo, le Plat, AS, Suzuki '23]

S-matrix and dressing factors when $k > 0$ and $h > 0$.

[Lloyd, O-Sax, AS, Stefasnki '14] [Frolov, Polvara, AS '23] [O-Sax, Riabchenko, Stefanski '23] [Frolov, Polvara, AS '24]

Weak-tension limit

Recall that the string tension is

$$T = \sqrt{\frac{k^2}{4\pi^2} + h^2}$$

Tensionless limit(s):

- $k = 0$ and $h \ll 1$, related to the D1-D5 system of branes
- $k = 1$ and $h \ll 1$, related to the symmetric-orbifold CFT

Weak tension at $k = 1$

At $k = 1$ and $h = 0$ the CFT dual is the **symmetric-orbifold CFT of T^4**: N-fold tensor product of a free theory, symmetrised under S_N.

[Giribet, Hull, Kleban, Porrati, Rabinovici '18] [Gaberdiel, Gopakumar '18] [Eberhardt, Gaberdiel, Gopakumar '19]

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States are labeled by conjugacy classes of S_N (cycles). If $N \to \infty$ consider single-cycle states.

Weak tension at $k = 1$

At $k = 1$ and $h = 0$ the CFT dual is the **symmetric-orbifold CFT of T^4**: N-fold tensor product of a free theory, symmetrised under S_N.

[Giribet, Hull, Kleban, Porrati, Rabinovici '18] [Gaberdiel, Gopakumar '18] [Eberhardt, Gaberdiel, Gopakumar '19]

States are labeled by conjugacy classes of S_N (cycles). If $N \to \infty$ consider single-cycle states.

This is a $\mathcal{N} = (4,4)$ theory (4 bosons and fermions). In the sector with a cycle of length L there is **a susy BPS state** $|0\rangle_L$ with $\mathbf{J}_{tot}|0\rangle_L = L\,|0\rangle_L$. We have

$$\left|\Psi_{\{n_i,\tilde{n}_i\}}\right\rangle = \alpha^{A_1\dot{A}_1}_{-\frac{n_1}{L}} \cdots \chi^{-\dot{A}_r}_{+\frac{1}{2}-\frac{n_r}{L}} \cdots \chi^{+\dot{A}_s}_{-\frac{1}{2}-\frac{n_s}{L}} \cdots (\text{anti-chiral})\,|0\rangle_L$$

subject to the physical state condition [Lunin, Mathur '01] [. . .]

$$\sum n_i - \sum \tilde{n}_i = 0 \bmod L$$

Weak tension at $k = 1$: energy at $h = 0$

$$|\Psi_{\{n_i,\tilde{n}_i\}}\rangle = \alpha^{A_1\dot{A}_1}_{-\frac{n_1}{L}} \cdots \chi^{-\dot{A}_r}_{+\frac{1}{2}-\frac{n_r}{L}} \cdots \chi^{+\dot{A}_s}_{-\frac{1}{2}-\frac{n_s}{L}} \cdots (\text{anti-chiral})\,|0\rangle_L$$

the lighcone energy from the symmetric orbifold CFT is

$$H_{tot} = \sum_j \frac{n_i}{L} + \sum_j \frac{\tilde{n}_j}{L}$$

We can reproduce this spectrum by setting $k = 1$ and $h = 0$ in $\omega(p)$

$$\omega(p) = \frac{1}{2\pi}|p|\,, \qquad H_{tot} = \sum_j \omega(p_j)$$

with

$$p_j = \frac{2\pi\nu_j}{L} \qquad \sum p_j = 0 \mod 2\pi$$

Weak tension at $k = 1$: turning on $h > 0$

We want $\left\{ G^{+A}(z),\, \widetilde{G}^{\dot{+}B}(\bar{z}) \right\} \neq 0$ in the deformed theory

Weak tension at $k = 1$: turning on $h > 0$

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The marginal operator that correponds to turning RR flux comes from the $L = 2$ sector:

$$S \to S + \lambda \int \mathrm{d}z\mathrm{d}\bar{z}\, \mathcal{D}_{(2)}(z, \bar{z})$$

where $\lambda \ll 1$ is a deformation parameter such that $h(\lambda) = c_0\lambda + \mathcal{O}(\lambda^2)$ for $h \ll 1$.

Weak tension at $k = 1$: representations at $h > 0$

In the deformed theory we expect e.g.

$$\widetilde{\mathbf{G}}^{\dot{+}B}_{-1/2}\,\alpha^{A\dot{A}}_{-\frac{n}{L}}|0\rangle_L = c(n,L)\,\varepsilon^{BA}\psi^{-\dot{A}}_{+\frac{1}{2}-\frac{n}{L+1}}|0\rangle_{L+1}\,, \qquad c(n,L) = \mathcal{O}(\lambda),$$

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which only makes sense as $L \to \infty$.

The coefficients $c(n,L)$ should give the representations of [Lloyd, O-Sax, AS, Stefanski '14] in the limit

$$L \to \infty, \qquad n \to \infty, \qquad p = \frac{2\pi n}{L} \quad \text{fixed}$$

Weak tension at $k = 1$: computing the representation coefficients

$$c(n, L) = \oint \mathrm{d}\bar{\zeta}\ \lambda \int \mathrm{d}z\mathrm{d}\bar{z}\ \left\langle \mathcal{V}^{(\chi_{-n})}_{(L+1)}(\infty)\ \widetilde{\mathbf{G}}(\bar{\zeta})\ \mathcal{D}_{(2)}(z, \bar{z})\ \mathcal{V}^{(\alpha_{-n})}_{(L)}(0) \right\rangle$$

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- [AS, Frolov '24] corrected the match, reproduces [Lloyd, O-Sax, Stefanski, Sfondrini '14] at

$$h(\lambda) = \lambda + \mathcal{O}(\lambda^2)$$

Weak tension at $k = 0$ (the D1-D5 system)

For worldsheet integrability we know

$$H_{tot} = \sum_{j=1}^{n} \sqrt{\mu_j^2 + 4h^2 \sin^2(p_j/2)} + \text{corrections}$$

with momentum $p_j = 2\pi\nu_j/L$ + corrections.

Leading-order anomalous terms come from $\mu = 0$ modes, ie from T^4.

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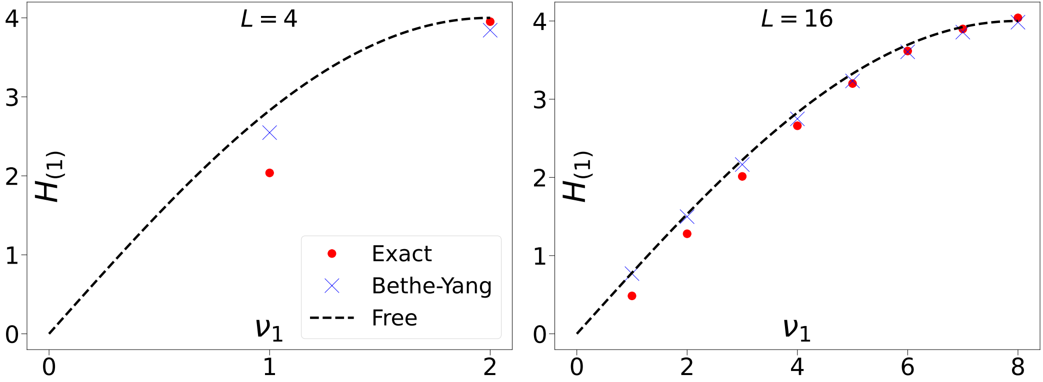
Expand the exact answer

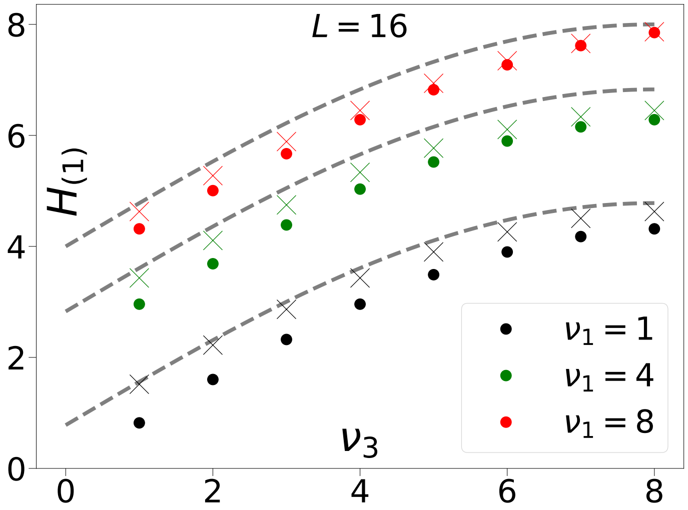
$$H_{tot}(\nu_1, \dots \nu_n) = h\, H_{(1)} + \mathcal{O}(h^2), \qquad H_{(1)} \approx \sum_{j=1}^{n} 2 \left| \sin\left(\frac{\pi\nu_j}{L}\right)\right| + \text{corrections}$$

Weak tension at $k = 0$: numerical results

Two excitations with $\mu = 0$ and $p_1 = -p_2$, ie $\nu_1 = -\nu_2$. [Brollo, le Plat, AS, Suzuki '23]



Weak tension at $k = 0$: Four excitations, $p_1 = -p_2$, $p_3 = -p_4$



Summary

Pure-NSNS (max $B_{\mu\nu}$)

- $\mathfrak{sl}(2) \oplus \mathfrak{su}(2)$ WZW model
- From F1-NS5 system
- Partially known dual
- Quantised tension $k \in \mathbb{N}$
- Simple, degenerate spectrum
- Can do integrability too

Mixed-flux case

- Hardest case
- From generic setup
- Both $h > 0$ and $k \in \mathbb{N}$
- Dual known for $k = 1$
- Nondegenerate spectrum
- "Only" S-matrix so far

Pure-RR, $B_{\mu\nu} = 0$

- Most similar to AdS5
- From D1-D5 system
- Continuous tension $h > 0$
- Nondegenerate spectrum
- Spectrum known
- Weak-tension dual?

[see Seibold, AS '24 for a review and references]

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THEORY
CHALLENGES

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