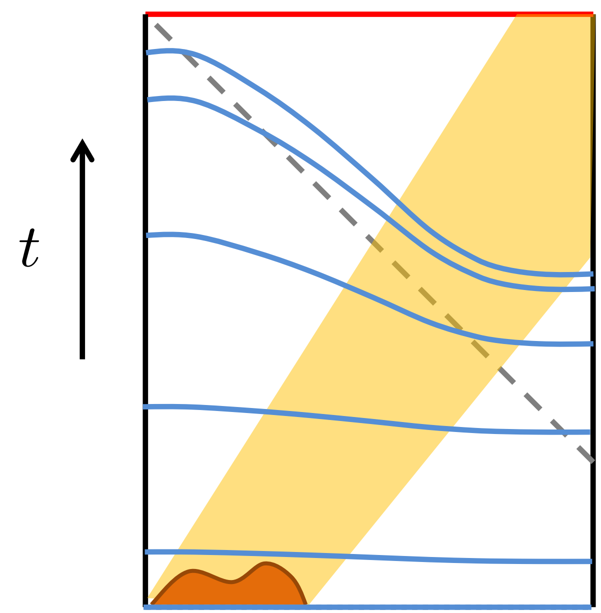
On the quantum mechanics of old black holes



Márk Mezei (Oxford)

Eurostrings 2024 meets FPUK
Southampton, 5/9/2024

Renaissance of Euclidean gravity

Euclidean gravitational path integral

- (Mesoscopic) derivation of black hole entropy [Gibbons-Hawking, …]
- No boundary wavefunction of the Universe [Hartle-Hawking, …]
- Problems: relation to Lorentzian approaches, wormholes, conformal mode problem, …

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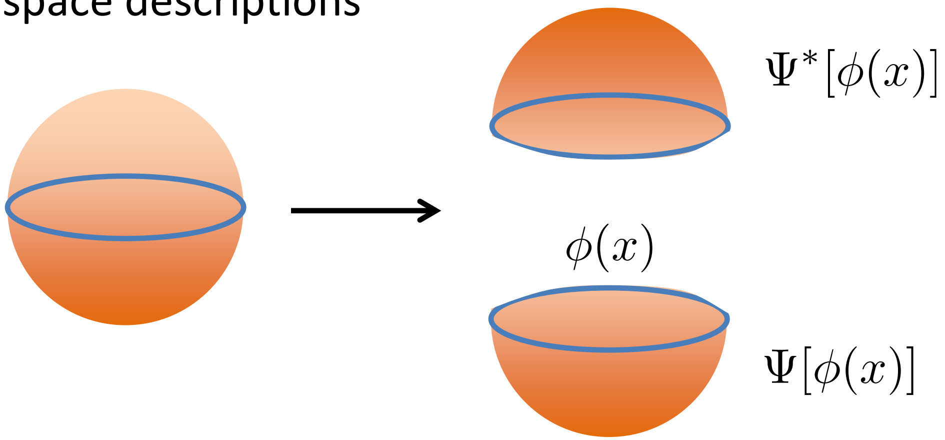
Renaissance in holographic context

- Derivation of holographic entanglement entropy [Ryu-Takayanagi, Lewkowycz-Maldacena, …]
- Wormholes encode spectral statistic of dual quantum system [Saad-Shenker-Stanford, …]
- (Mesoscopic) resolution of the black hole information paradox
  [Penington, Almheiri et al., East Coast & West Coast papers, …, Kumar’s talk]

Cutting open the Euclidean path integral

Relating path integral and Hilbert space descriptions

- Cut open QFT on a sphere



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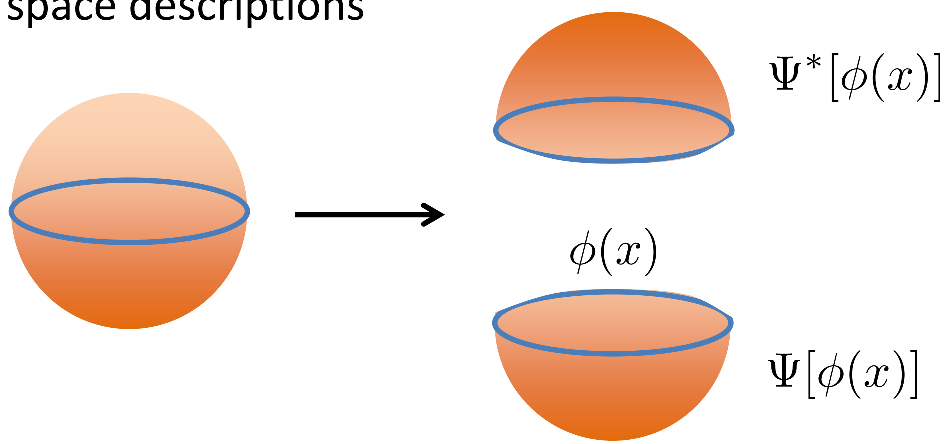

- Can insert a defect in QFT $L = \exp\left(-h \int dx\, \phi(x)\right)$

- Two ways of computing

$$\langle L \rangle = \int D\Phi \; L[\phi]\, e^{-S[\Phi]} \qquad \text{Euclidean}$$

$$= \int D\phi \; L[\phi]\, |\Psi[\phi]|^2 \qquad \text{Hilbert space}$$

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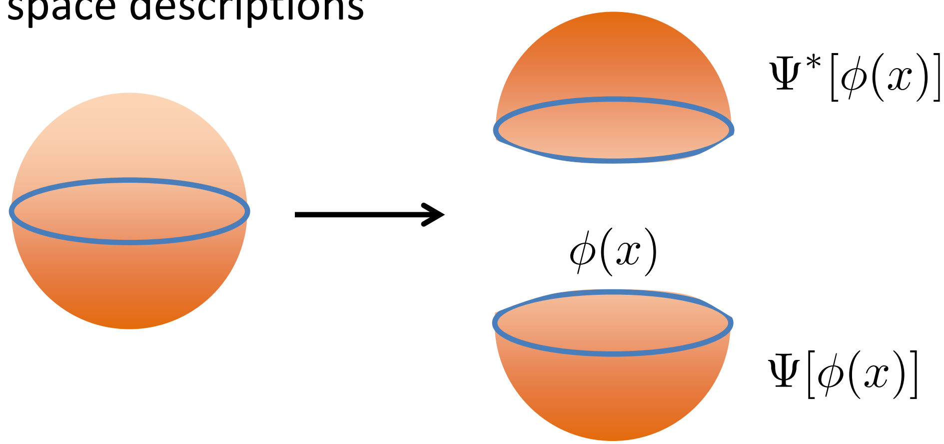

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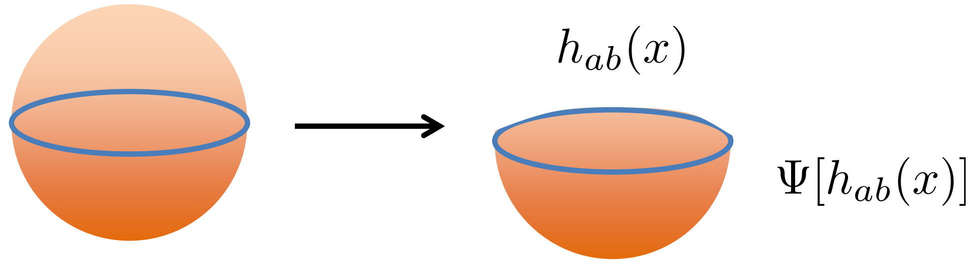
- **Complementary advantages:**
  Euclidean: saddle points, analytic continuation to Lorentzian
  Hilbert space: unitarity, probabilistic interpretation, integer degeneracies

Cutting open the Euclidean path integral

Can repeat in gravity

- Cut open sphere gravity path integral along a geodesic (minimal volume slice)
  [Wheeler-DeWitt, …, Witten]



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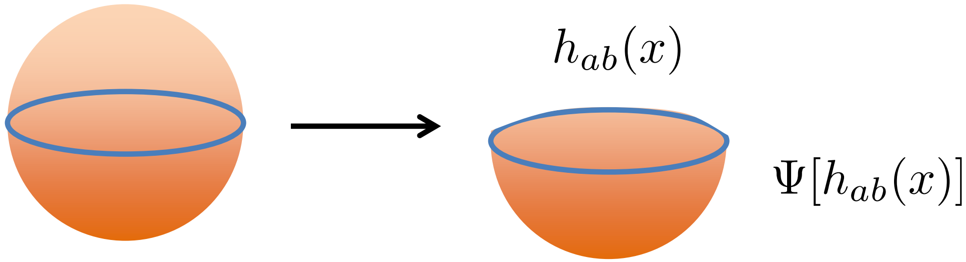
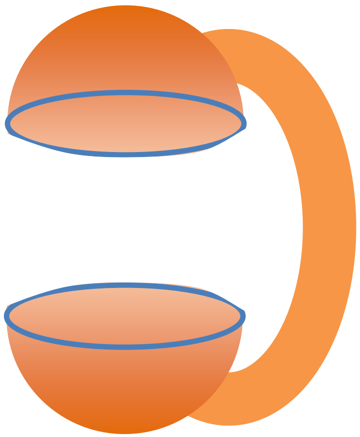


- Need to take care of diffeomorphism invariance
- Can insert (diff. invariant) operators, can compute both Euclidean and Hilbert space way
- What to do with (bra-ket) wormholes?
  Should we cut them open too?
  [Marolf-Maxfield, Gorbenko's talk]



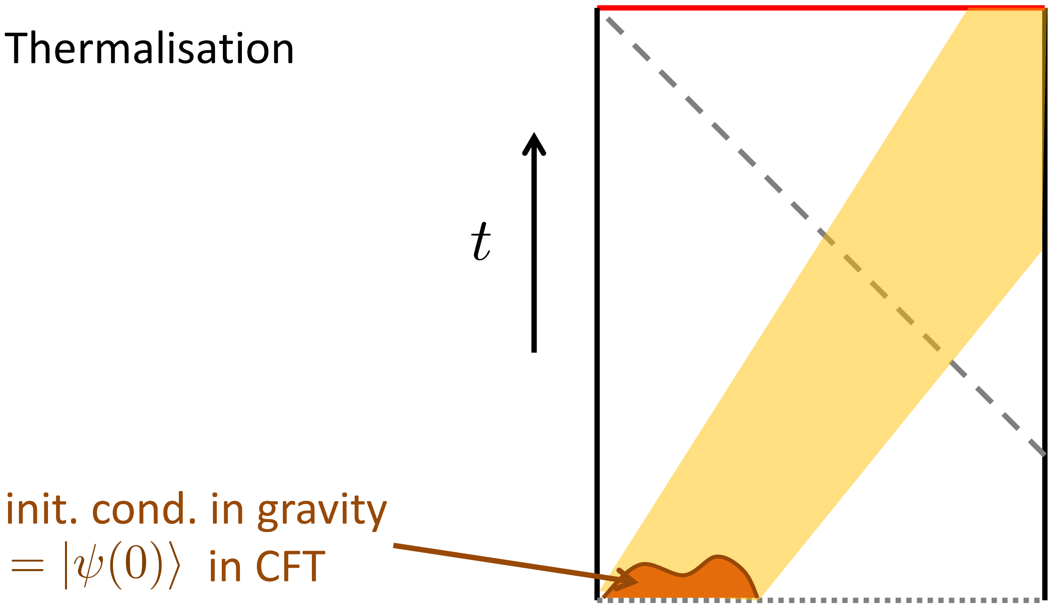
- Geometry now fluctuating

Case study: volume of the BH interior in 2d JT gravity
[Iliesiu-MM-Sárosi, Iliesiu-Levine-Lin-Maxfield-MM]

Black hole interior volume and quantum chaos

Cross-fertilisation between black holes and chaotic many-body quantum systems [Sonner's talk]

- Black hole formation ⟷ Thermalisation



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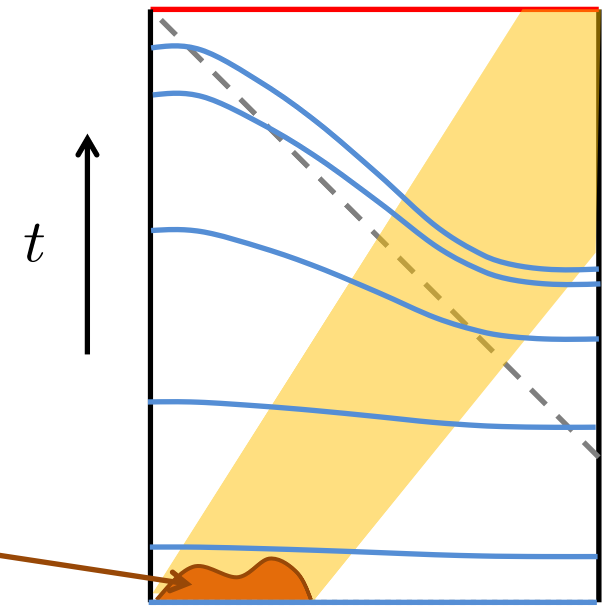

init. cond. in gravity
$= |\psi(0)\rangle$ in CFT

Black hole interior volume and quantum chaos

Black holes and chaotic many-body quantum systems

- Black hole formation ⟷ Thermalisation
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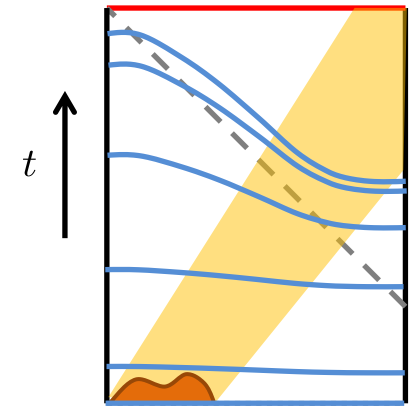
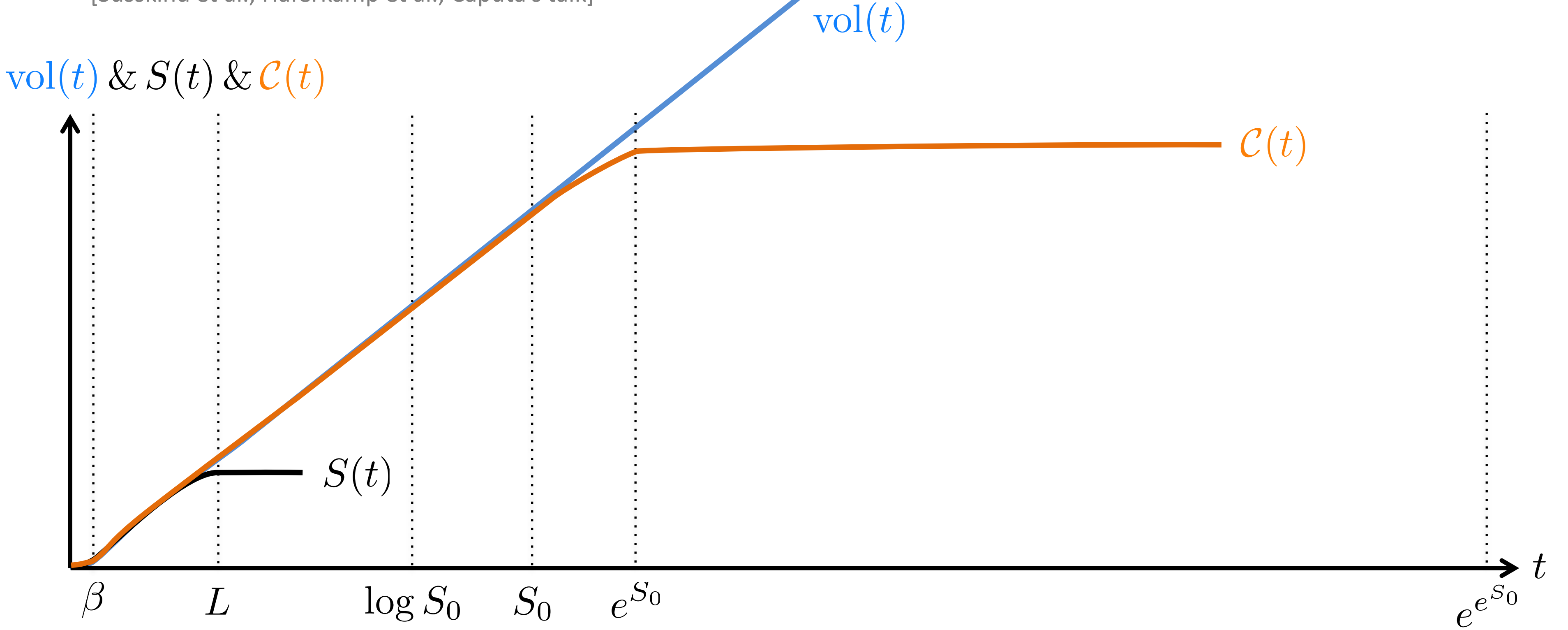
- Candidate: circuit complexity of $|\psi(t)\rangle$
  [Susskind et al., Haferkamp et al., Caputa's talk]





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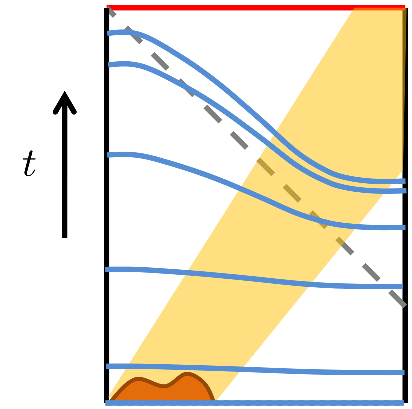

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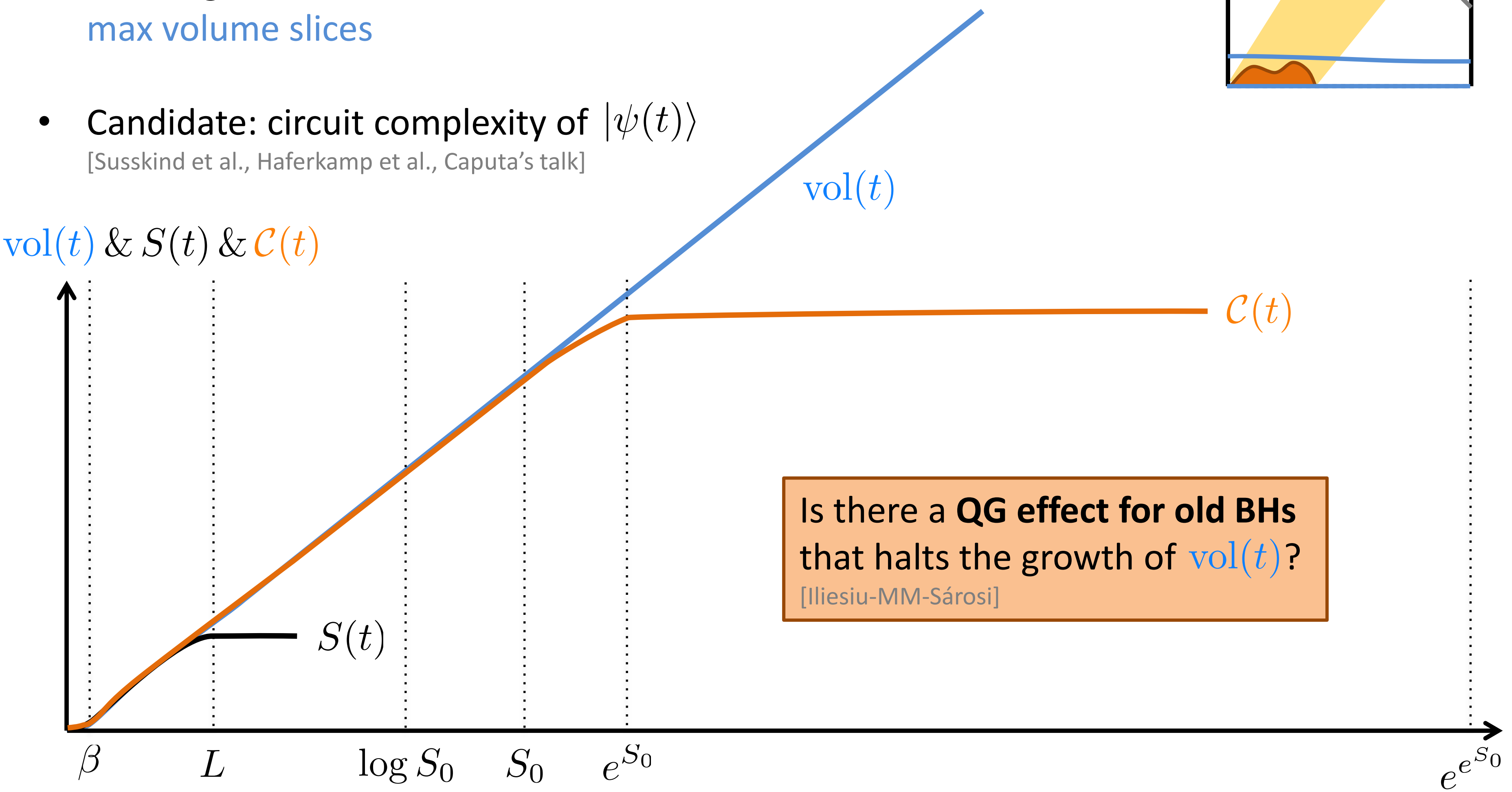

Is there a **QG effect for old BHs** that halts the growth of $\mathrm{vol}(t)$?
[Iliesiu-MM-Sárosi]

Outline

Volume in Euclidean gravity

Hilbert space view

Summary

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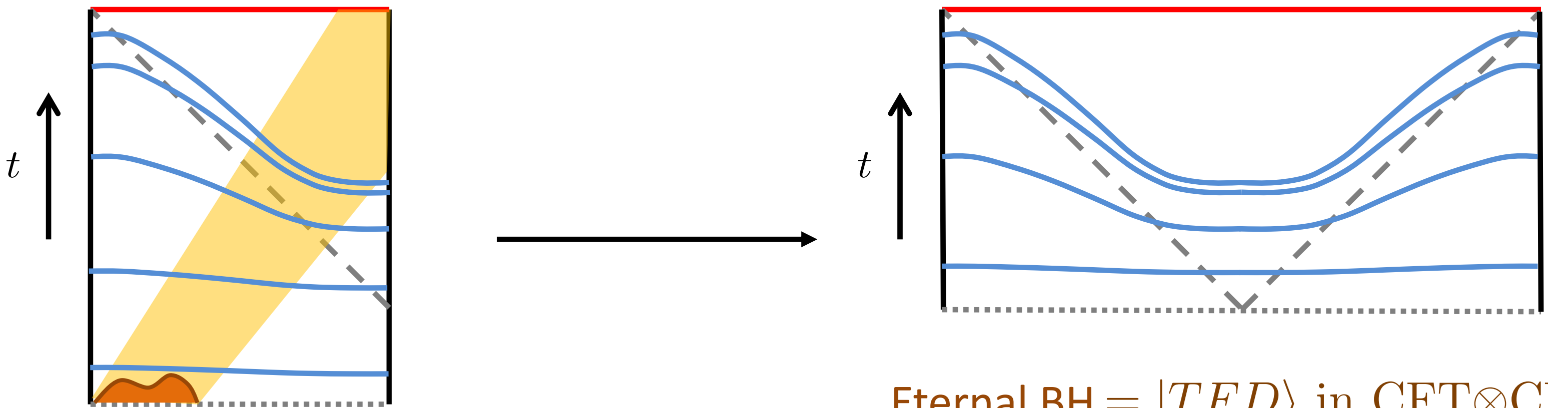
In search for the simplest model

- Analogue setup in pure gravity [Hartman-Maldacena]



Eternal BH $= |TFD\rangle$ in $\mathrm{CFT}\otimes\mathrm{CFT}$

Volume in Euclidean JT gravity

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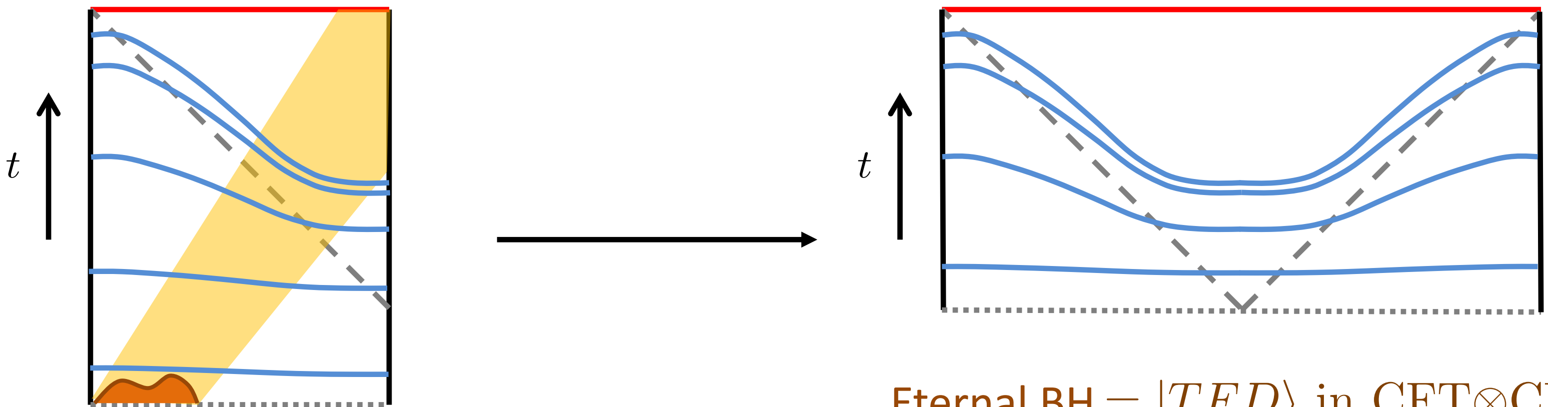


Eternal BH $= |TFD\rangle$ in CFT$\otimes$CFT

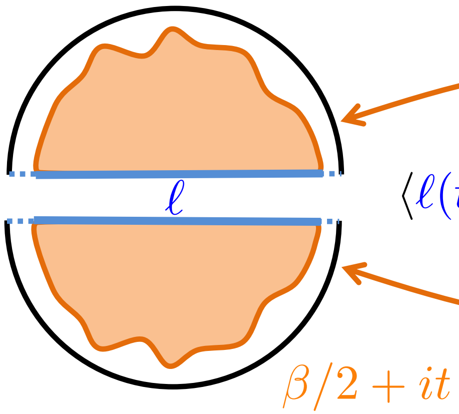
- Lowest dimension: 2d JT (dilaton) gravity
  Also effective theory for near-extremal BHs, UV complete on its own
  [Teitelboim, Jackiw, Almheiri-Polchinski, Jensen, Maldacena-Stanford-Yang]

$$Z(\beta) = \int DgD\phi \; e^{-S_0\chi - I_{\rm JT}}$$

$$I_{\rm JT} = \int \phi\,(R+2) + \text{bdy term}$$

Volume in Euclidean JT gravity

- Genus 0 [Yang]



$$\langle \ell(t) \rangle_0 = \int d\ell \; \ell \, |\psi_{\text{disk}}(\beta/2 + it, \ell)|^2$$

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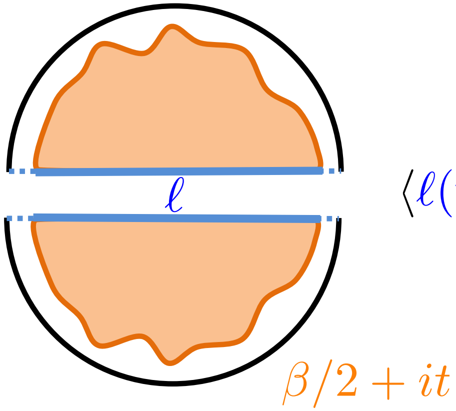


$$\langle \ell(t) \rangle_0 = \int d\ell \; \ell \, |\psi_{\text{disk}}(\beta/2 + it, \ell)|^2$$
$$= e^{S_0} C(\beta) \, t + \ldots \qquad (t \gg \beta)$$

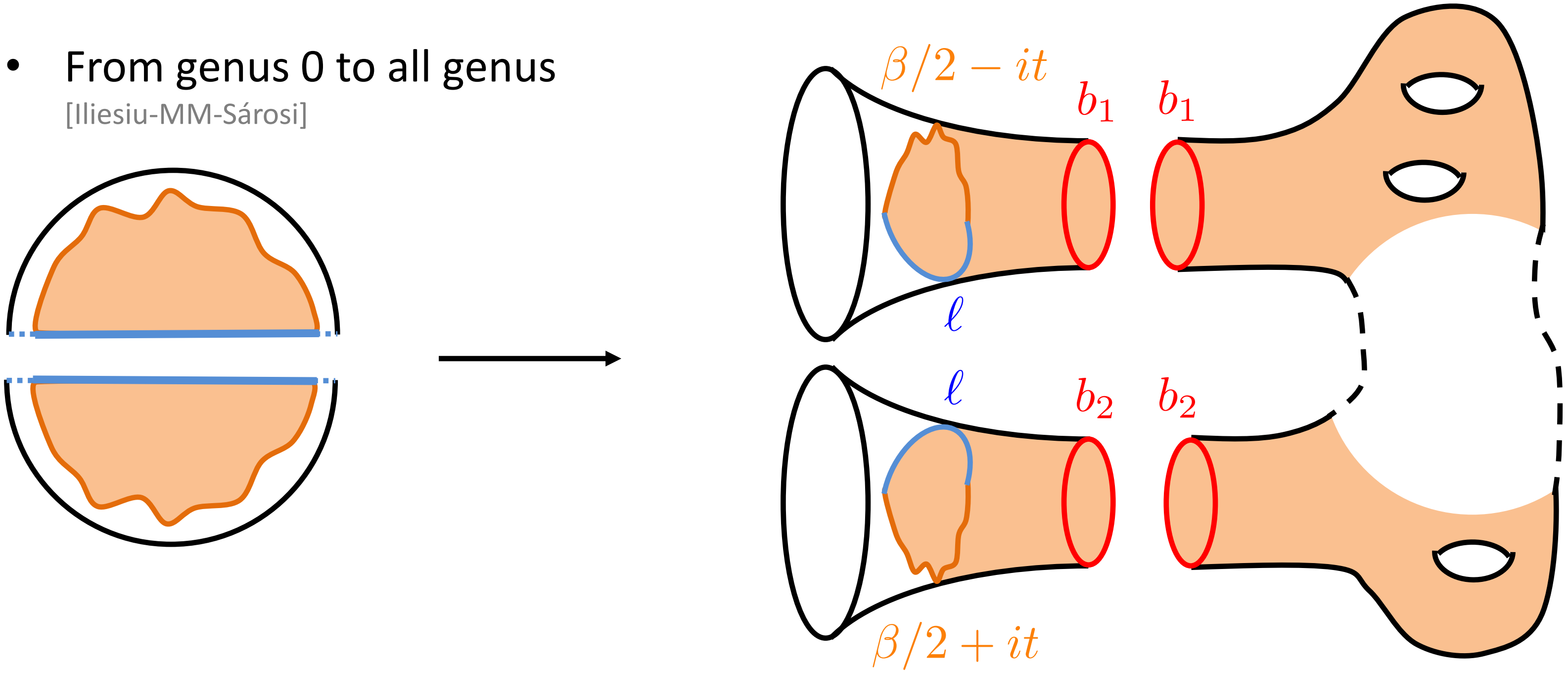
- Organise path integral computation in way suggestive of Hilbert space view

- Perturbative gravity insensitive to the Heisenberg time e^{S_0}

  Need nonperturbative effects for old BHs

Volume in Euclidean JT gravity

- From genus 0 to all genus
  [Iliesiu-MM-Sárosi]



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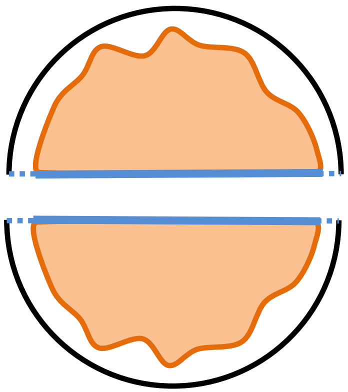

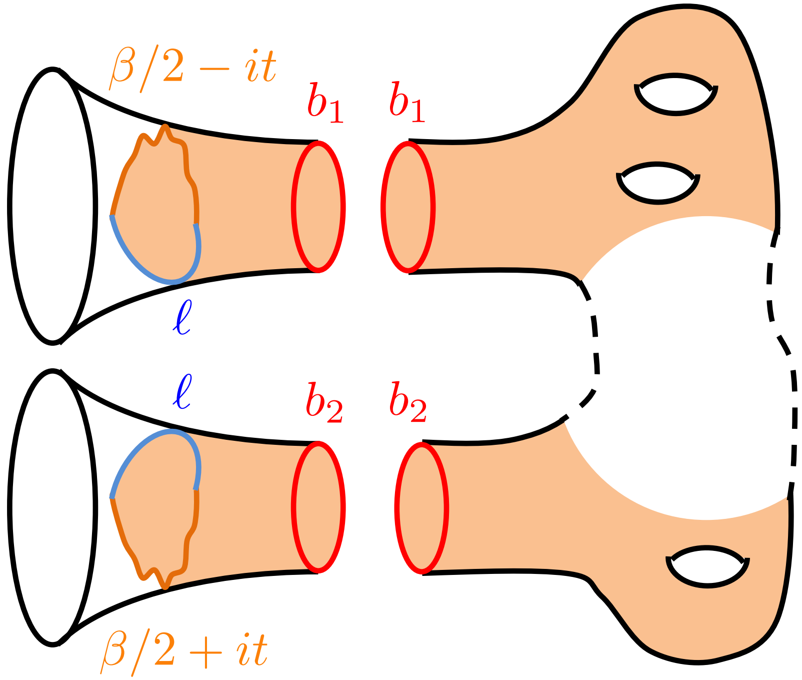

$$\langle \ell(t) \rangle_g = e^{-(1-2g)S_0} \int d\ell \, \ell \, \rho_g(\ell, \ell)$$

$$\stackrel{?}{=} e^{-(1-2g)S_0} \, \mathrm{Tr}(\ell \, \rho_g)$$

- Puzzle: $\int d\ell \, \rho_g(\ell, \ell) = \infty$

Unclear how to cut open QG path integral

Evaluation

- **Genus expansion resummed through RMT, JT is dual to an ensemble of QMs**
  [Saad-Shenker-Stanford, Mirzakhani, Eynard-Orantin, …, de Boer's talk]

$$\langle \ell(t) \rangle \approx \frac{1}{Z(\beta)} \int dE\, d\omega\, \frac{\langle \rho(E+\omega/2)\rho(E-\omega/2) \rangle}{\omega^2 \langle \rho(E) \rangle}\, e^{-\beta E} \left[1 - \cos(\omega t)\right] \qquad (t \gg \beta)$$

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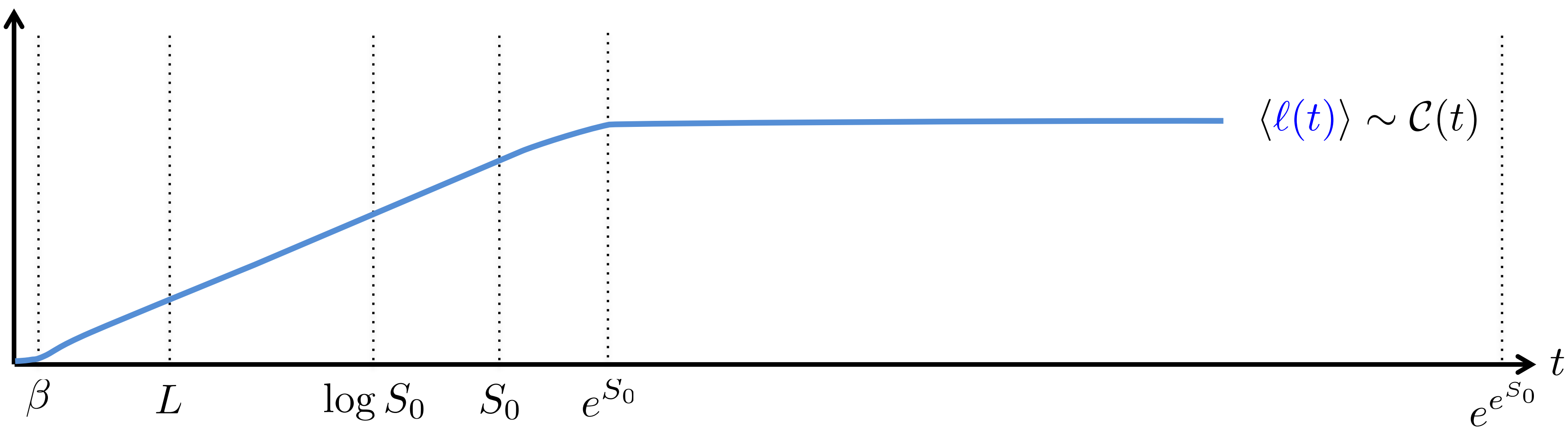
- Linear growth and saturation [Iliesiu-MM-Sárosi]



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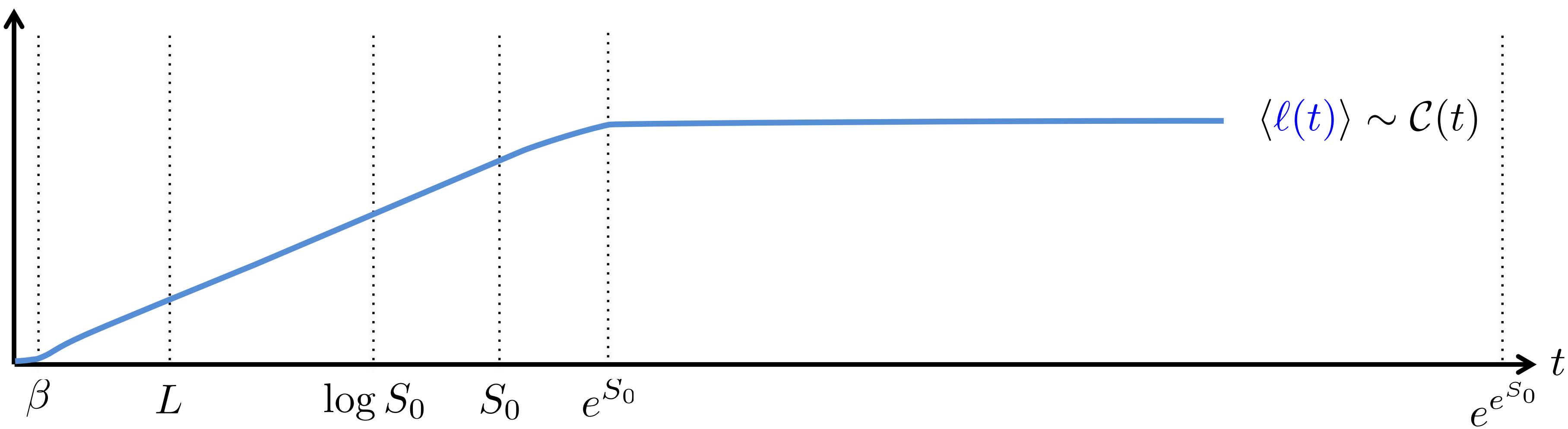

- Motivates spectral complexity for an individual Hamiltonian

$$\mathcal{C}(t) = \sum_{E_1 \neq E_2} \frac{e^{-\beta(E_1+E_2)/2}}{(E_1-E_2)^2} \left[1-\cos\left((E_1-E_2)t\right)\right]$$

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Goal: understand and go beyond Euclidean result using Hilbert space perspective

[Iliesiu-Levine-Lin-Maxfield-MM]

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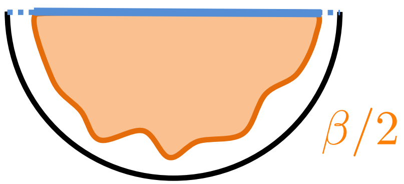
Idea: semiclassical space of states with modified inner product

- Label states with length $|\ell\rangle$
  Energy eigenstate wavefunctions unchanged $\langle \ell | E \rangle = \phi_E(\ell)$



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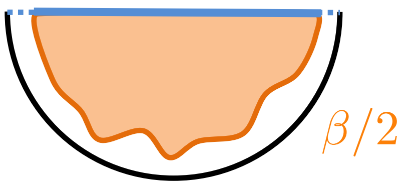

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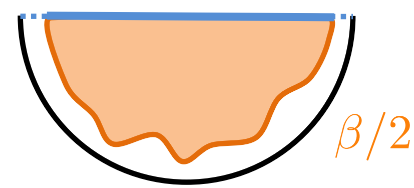

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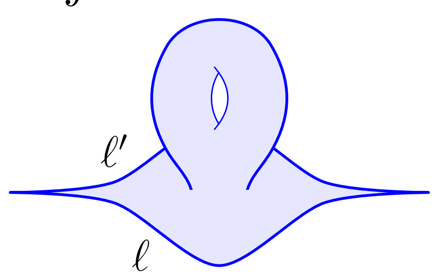
- Inner product $$\langle \ell' | \ell \rangle = \sum_n \langle \ell' | E_n \rangle \langle E_n | \ell \rangle = \sum_n \phi_{E_n}(\ell') \phi_{E_n}(\ell)$$
- Ensemble average $$\overline{\langle \ell' | \ell \rangle} = \int dE \, \overline{\rho(E)} \, \phi_E(\ell') \phi_E(\ell)$$

$\overline{\langle \ell' | \ell \rangle} = \delta(\ell - \ell') +$  $+ \quad O(e^{-4S_0})$

- Reproduces all Euclidean gravity results, resolves $\int d\ell \, \rho_g(\ell, \ell) = \infty$ puzzle

Construction of Hilbert space

- Construct null states for one member of the ensemble

$$|\psi\rangle = \int d\ell\, \psi(\ell)|\ell\rangle\,, \quad \psi(\ell) \equiv \int dE\, \rho_0(E)\chi(E)\phi_E(\ell)\,,$$

$$\langle\psi|\psi\rangle = \sum_n |\chi(E_n)|^2$$

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Reduction in dimension of Hilbert space to e^{S_0}

- Hamiltonian is unmodified [Harlow-Jafferis]

$$H|\psi\rangle = \int d\ell \left(-\frac{1}{2}\partial_\ell^2 + 2e^{-\ell} \right) \psi(\ell)|\ell\rangle$$

Length operators

A family of length operators

$$\hat{\ell}_\Delta = -\frac{1}{\Delta}\log\left(\int d\ell\, e^{-\Delta\ell}|\ell\rangle\langle\ell|\right)$$

- If $|\ell\rangle$'s were all orthogonal $\hat{\ell}_\Delta = \int d\ell\, \ell|\ell\rangle\langle\ell|$
- The exponential penalises long geodesics

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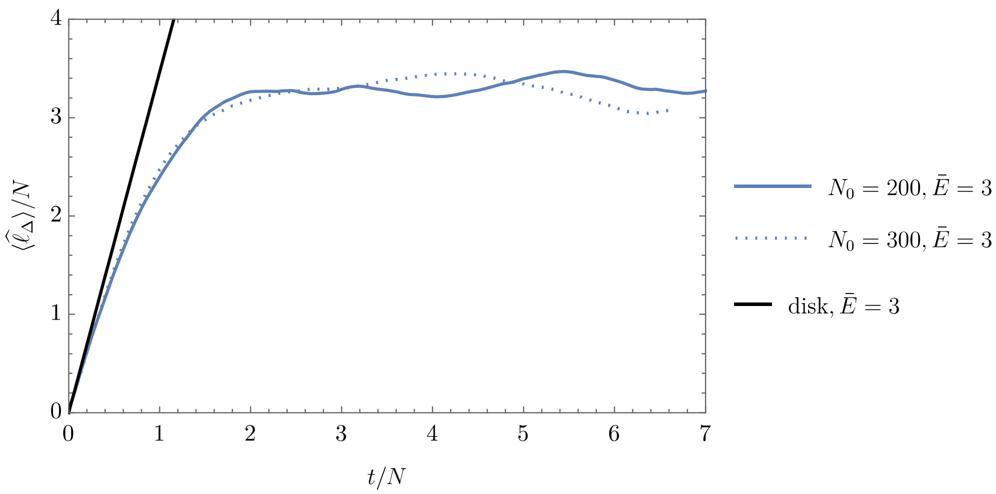
  For $\Delta \to \infty$ minimal length operator (up to subtleties)



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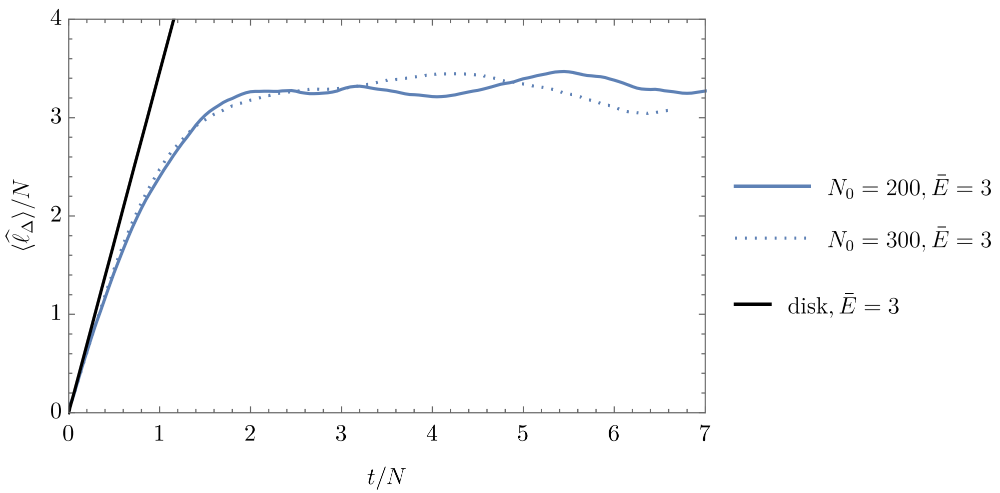
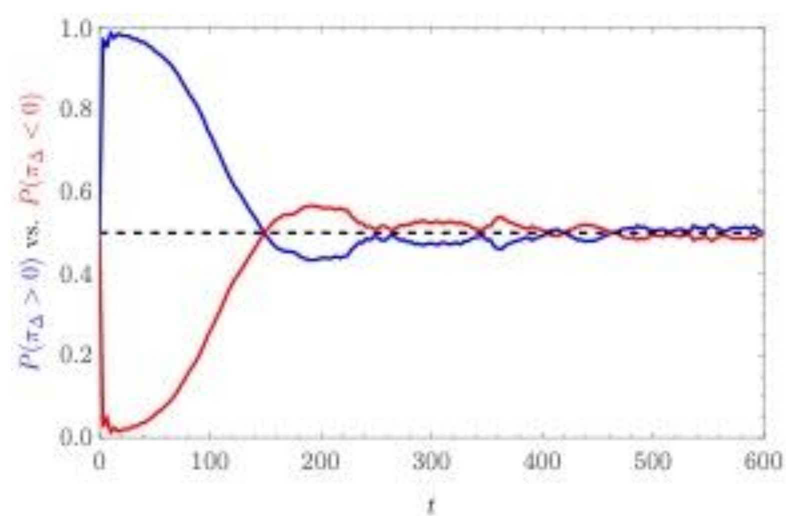


BH interior velocity: $\hat{\pi}_\Delta = i[H, \hat{\ell}_\Delta]$



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- Can absorb topology change into inner product on state space (at least in JT)
  Gives rise to plethora of null states

- Studied the volume of black hole interior from both the path integral and Hilbert space perspectives, cross-fertilisation with many-body quantum chaos

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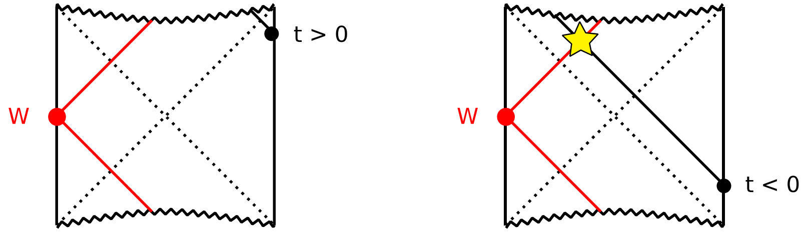
Outlook

- More analytic control in JT, extension to higher dimensions

- Experience of infalling observer? Firewall?
  [Stanford-Yang, Mathur, AMPS(S), Saad, Blommaert-Chen-Nomura]



Thank you!

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Appendix

Spectral complexity

Related quantity: $\langle \ell(t) \rangle \approx \frac{1}{Z(\beta)} \int dE\, d\omega\, \frac{\langle \rho(E+\omega/2)\rho(E-\omega/2)\rangle}{\omega^2 \langle \rho(E)\rangle} e^{-\beta E} \left[1-\cos(\omega t)\right]$

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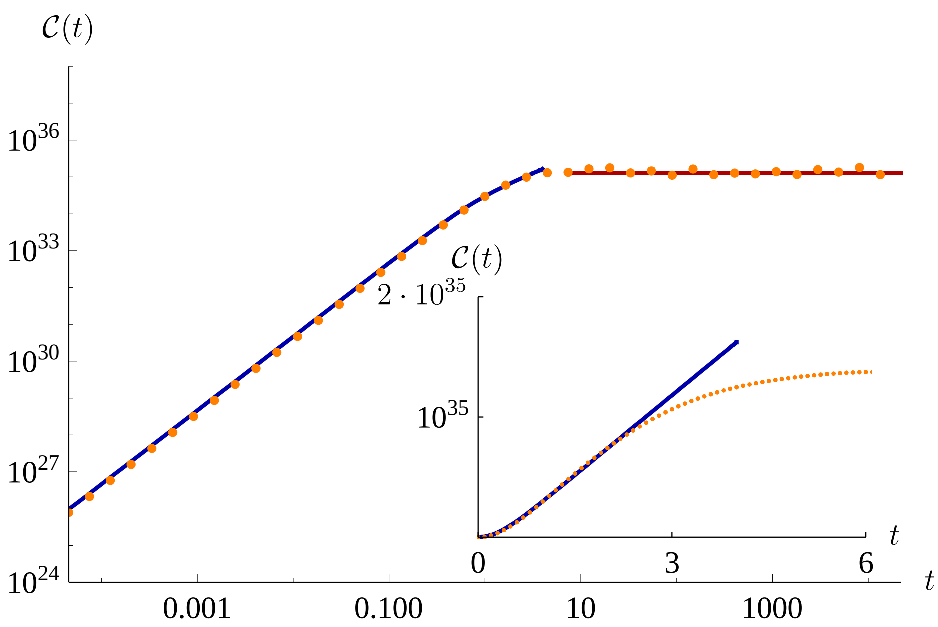
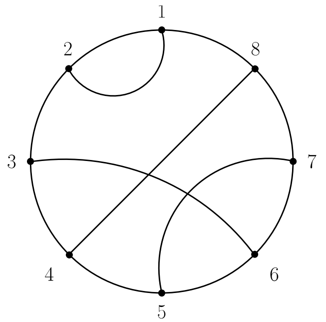


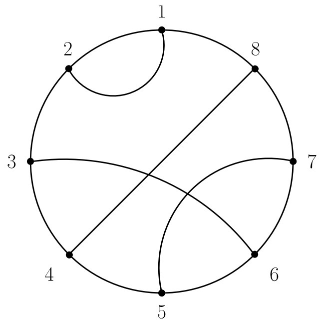
Analogy with DSSYK

Hilbert space for DSSYK

- DSSYK is solved by summing chord diagrams [Berkooz et al., Erdős-Schröder]



Analogy with DSSYK



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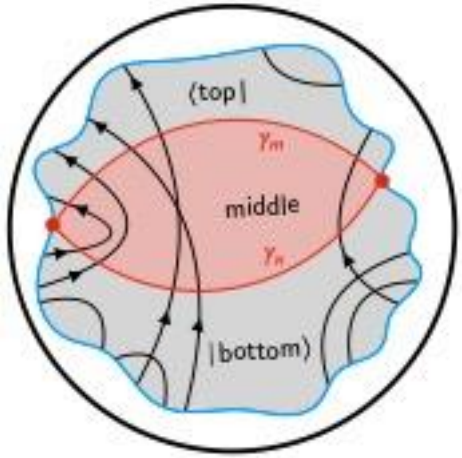

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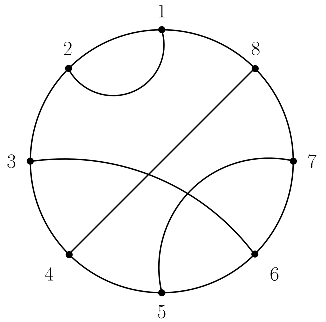
Analogy with DSSYK

Hilbert space for DSSYK



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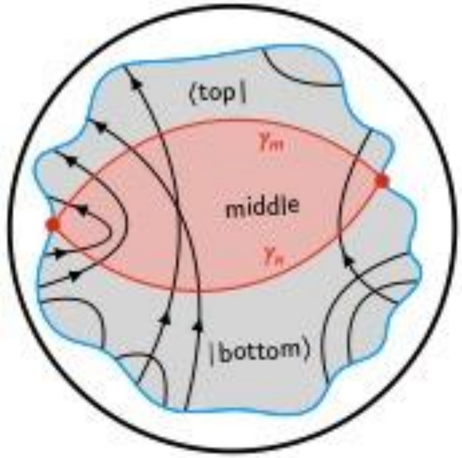

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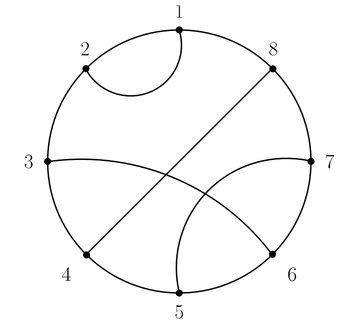

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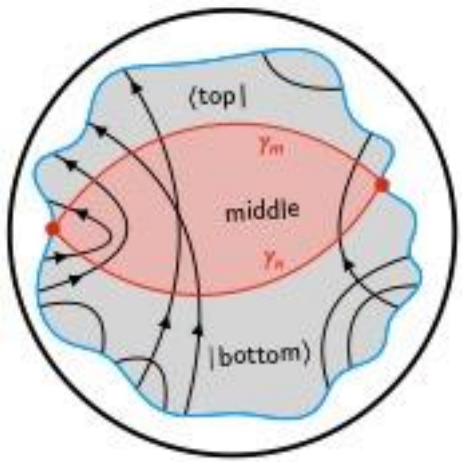

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- Partition function is a transition amplitude $Z(\beta) = \langle 0|e^{-\beta T}|0\rangle$
- Limit $\lambda = \dfrac{p^2}{N} \to 0\,, \quad \ell = \lambda m = \text{fixed}$ gives JT at genus 0, length is Krylov complexity

Is there a genus expansion for DSSYK?