On the Resurgence of D-Brane Negativity

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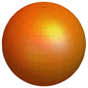
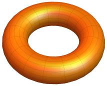
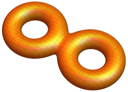
Based on work in collaboration with:

- J. Kager, J. Rodrigues, RS, N. Tamarin
  arXiv: Upcoming...
- RS, M. Schwick
  arXiv: Upcoming...
- J. Kager, J. Rodrigues, RS, M. Schwick, N. Tamarin
  arXiv: Upcoming...
- RS, M. Schwick, N. Tamarin
  arXiv: 2301.05214 [hep-th]
- M. Mariño, RS, M. Schwick
  arXiv: 2210.13479 [hep-th]
- S. Baldino, RS, M. Schwick, R. Vega
  arXiv: 2203.13726 [hep-th]

Motivation: Nonperturbative String Theory?

- String theory generically constructed *perturbatively*:

$$F = \log Z \simeq \sum_{g=0}^{+\infty} F_g(t)\, g_s^{2g-2} =$$

$= \frac{1}{g_s^2}$  $+$  $+\ g_s^2$  $+ \cdots$

- Perturbative genus expansion is *asymptotic*! $\Rightarrow F_g \sim (2g)!$

  [Gross-Periwal,Shenker]

- Obtain nonperturbative definition/construction of string theory?
  - Beyond perturbative $\sim g_s^{\bullet} \Rightarrow$ nonperturbative $\sim \exp\left(-\frac{\bullet}{g_s}\right) \cdots$
- Need perturbative expansion under control...When is this possible?

Motivation: From Perturbative String Theory!

- String theoretic perturbative expansion under control: asymptotic solutions to random matrix models ✓
  - Multicritical strings [Douglas-Shenker,Brézin-Kazakov,Gross-Migdal]
  - Minimal string theories [Seiberg-Shih]
  - Hermitian matrix models (topological strings) [Dijkgraaf-Vafa]
  - JT gravity [Saad-Shenker-Stanford]

- ~~CFT world-sheet~~ $\Rightarrow$ string equations, topological recursion [Eynard-Orantin, EGGLS] (holomorphic anomaly equations [BCOV,CESV,Gu-Mariño]) ...

- Obtain nonperturbative definition/construction of string theory?
  - From resurgence of (deep) perturbative series... [Écalle]
  - ...find full nonperturbative resurgent transseries!

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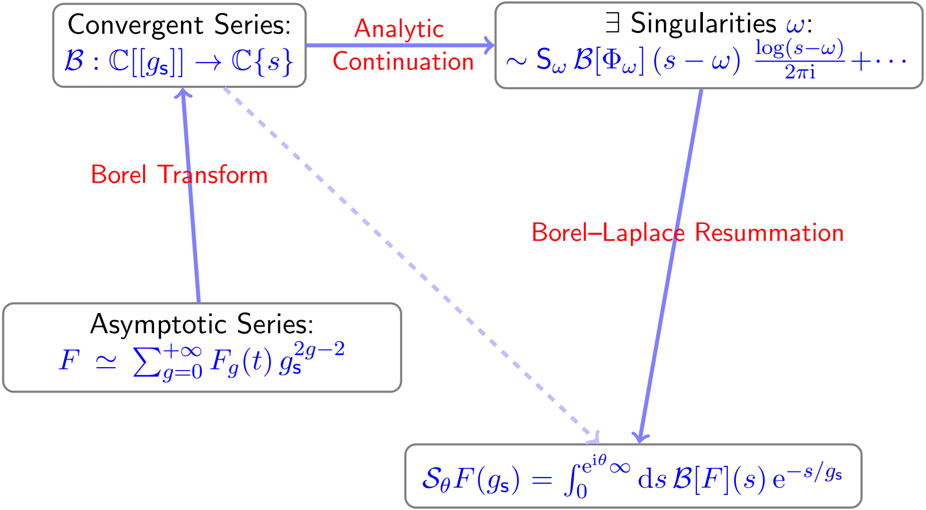
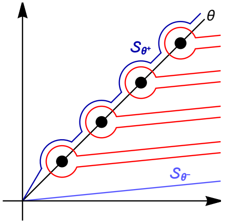
From Asymptotic Series to Resurgent Transseries



Discontinuity Upon Crossing a Stokes Line



$$\mathcal{S}_{\theta^+} F - \mathcal{S}_{\theta^-} F = - \sum_{\{\omega_n\}} \mathsf{S}_{\omega_n} \, \mathrm{e}^{-\frac{\omega_n}{g_s}} \, \mathcal{S}_{\theta^-} \Phi_{\omega_n} \equiv - \mathcal{S}_{\theta^-} \circ \operatorname{Disc}_{\theta} F$$

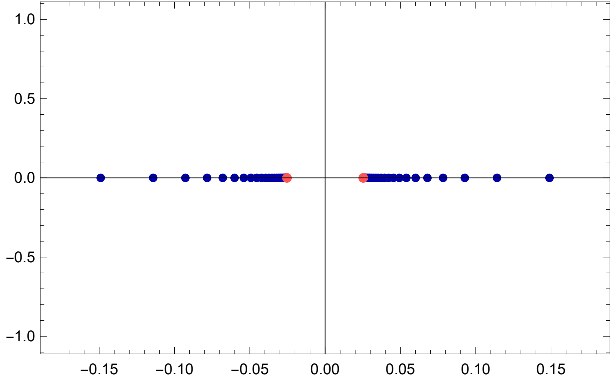
$$\Rightarrow \qquad \operatorname{Disc}_{\theta} F = \sum_{\{\omega_n\}} \mathsf{S}_{\omega_n} \, \mathrm{e}^{-\omega_n / g_s} \, \Phi_{\omega_n}$$

- All sectors Φ_{ω_n} must be included in full solution, as perturbative series not enough! $\Rightarrow$ Leads to *transseries* and to *resurgence*...

Borel Analysis of String-Theoretic Perturbative Expansion

- Borel transform of asymptotic series in $\sim g_s^2$ yields parity-fixed

$$\mathcal{B}\left[F\right](s) = -\mathcal{B}\left[F\right](-s)$$



- Singularities on Borel plane are symmetric $\Rightarrow$ Resonance!

String Equations

String Equation for Painlevé I $= (2,3)$ Minimal String

- String equation in DSL $\Rightarrow$ specific-heat $u(z)$, with $F''(z) = -\frac{1}{2}u(z)$.
- $k=2$ or $(2,3)$ or $c=0$ multicritical theory string equation:

$$u^2 - \frac{1}{3}u'' = z, \qquad g_s = z^{-5/4}$$

- Perturbative expansion:

[Douglas-Shenker,Brézin-Kazakov,Gross-Migdal,Ginsparg-ZinnJustin,Seiberg-Shih]

$$F_{(2,3)} \simeq -\frac{4}{15}z^{\frac{5}{2}} - \frac{1}{24}\log z + \frac{7}{1440}z^{-\frac{5}{2}} + \frac{245}{41472}z^{-5} + \frac{259553}{9953280}z^{-\frac{15}{2}} + \cdots$$

- Instanton action and characteristic exponent [GIKM] ...**Resonant**!

$$A_{(2,3)} = \pm\frac{4}{5}\sqrt{6}, \qquad \beta_{(2,3)} = \frac{1}{2}$$

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Two-Parameter Transseries Solution: Nonperturbative

- General two-parameter transseries solution is resonant:

  [GIKM,Aniceto-RS-Vonk]

$$u(g_{\mathsf{s}},\sigma_1,\sigma_2) = \sum_{n=0}^{+\infty}\sum_{m=0}^{+\infty} {\sigma_1}^n {\sigma_2}^m \, \mathrm{e}^{-(n-m)\frac{A}{g_{\mathsf{s}}}} \left(\sum_{k=0}^{\min(n,m)} \log^k(g_{\mathsf{s}}) \cdot \Phi_{(n|m)}^{[k]}(g_{\mathsf{s}}) \right)$$

  When $n=m \Rightarrow$ all diagonal *nonperturbative* sectors of same weight!

- Asymptotic sectors have different starting orders β_{nm},

$$\Phi_{(n|m)} \simeq \sum_{g=0}^{+\infty} u_g^{(n|m)} \, g_{\mathsf{s}}^{g+\beta_{nm}}$$

  Log-sectors not independent: $\Phi_{(n|m)}^{[k]} = \frac{1}{k!}\left(\frac{8(m-n)}{\sqrt{6}}\right)^k \Phi_{(n-k|m-k)}^{[0]}$.

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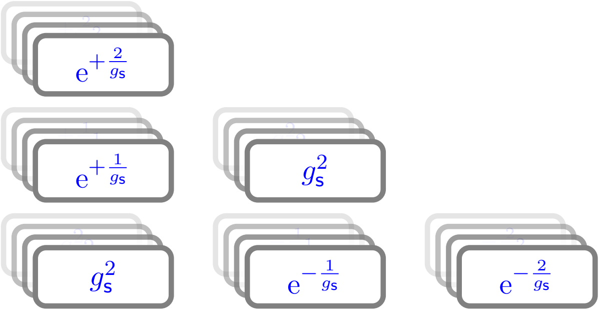
This Resonant Structure is Generic ✓

- Many many examples worked out: Multicritical models ... Minimal string theories ... Hermitian matrix models ... JT gravity ...

  [GIKM,Aniceto-RS-Vonk,RS-Vaz,Gregori-RS,Mariño-RS-Schwick,RS-Schwick-Tamarin,KRSST,KRST]

- Generic transseries solutions $\Rightarrow$ more sectors than expected!

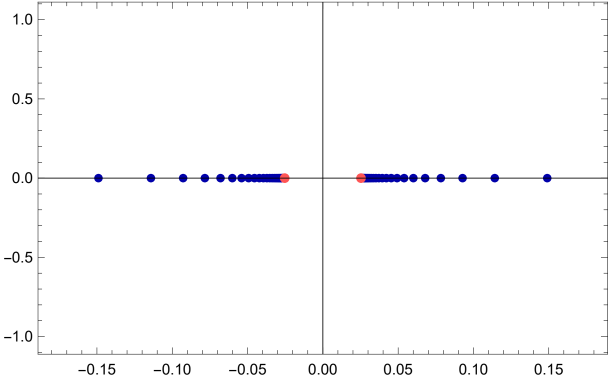


- What are these new resonant sectors?

Borel Analysis of String-Theoretic Perturbative Expansion

- Borel transform of asymptotic series in $\sim g_s^2$ yields parity-fixed

$$\mathcal{B}[F](s) = -\mathcal{B}[F](-s)$$

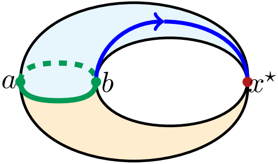


- Singularities on Borel plane are symmetric $\Rightarrow$ Resonance!

From Familiar Nonperturbative Corrections...

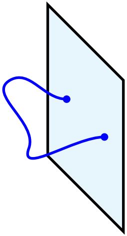
- Random matrix model *leading* nonperturbative corrections

$\sim \mathrm{e}^{-N}$ $\Rightarrow$ Eigenvalue Tunneling **[David]**



- String theoretic *leading* nonperturbative corrections

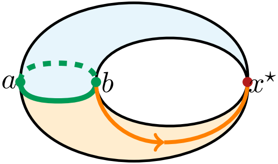
$\sim \mathrm{e}^{-\frac{1}{g_s}}$ $\Rightarrow$ D-Branes **[Polchinski]**



...To Resonant Nonperturbative Corrections!

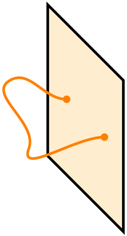
- Random matrix model [Mariño-RS-Schwick]

$\sim \mathrm{e}^{+N}$ Anti-Eigenvalue Tunneling



- String theory [RS-Schwick-Tamarin]

$\sim \mathrm{e}^{+\frac{1}{g_s}}$ Negative-Tension D-Branes



- Negative D-branes not new [Vafa,Okuda-Takayanagi,Dijkgraaf-Heidenreich-Jefferson-Vafa] ...
  ... but herein a requirement of resurgence!

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Matrix Integrals

How is a Matrix Integral a Transseries Sector?

- Transseries from string equations *versus* matrix integrals?
  - String-equation transseries with (n,m) resurgent sectors,

$$Z = \sum_{n=0}^{+\infty} \sum_{m=0}^{+\infty} {\sigma_1}^n {\sigma_2}^m \, \mathrm{e}^{-(n-m)\frac{A}{g_s}} Z_{(n|m)}(g_s)$$

  - Matrix-integral with eigenvalue integrations contours $\mathscr{C}_i$,

$$\mathcal{Z}_N = \frac{1}{\mathrm{vol}\,(\mathsf{U}(N))} \int \mathrm{d}M \, \mathrm{e}^{-\frac{1}{g_s}\mathbb{T}\mathrm{r} V(M)} = \frac{1}{N!} \int_{\{\mathscr{C}_i\}} \prod_{i=1}^{N} \frac{\mathrm{d}\lambda_i}{2\pi} \, \Delta(\lambda)^2 \, \mathrm{e}^{-\frac{1}{g_s}\sum_{i=1}^{N} V(\lambda_i)}$$

- Write *which* transseries resurgent-sectors as matrix integrals?
- Choose *which* integration contours via saddle-point/steepest-descent?

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Computing Matrix Integrals with Anti-Eigenvalues

- Involution map flips sheets:

$$\zeta \mapsto \sigma(\zeta) = \frac{1}{\zeta} \qquad \Rightarrow \qquad \begin{cases} x(\sigma(\zeta)) = x(\zeta), \\ y(\sigma(\zeta)) = -y(\zeta) \end{cases}$$

- Need to flip sheets at level of determinant correlators...

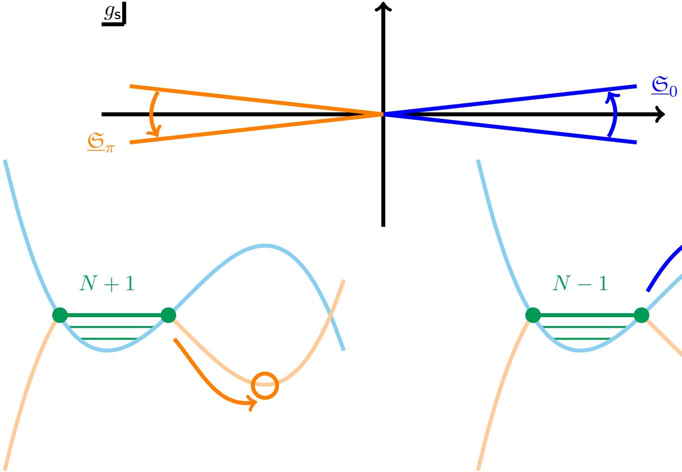
$$\mathcal{Z}^{(0,0)}_{N-1} \left\langle \det \left(x(\sigma(\zeta)) - M \right)^2 \right\rangle_{N-1} \mathrm{e}^{-\frac{1}{g_s} V(x(\sigma(\zeta)))} =$$

$$= \mathcal{Z}^{(0,0)}_{N+1} \left\langle \frac{1}{\det \left(x(\zeta) - M \right)^2} \right\rangle_{N+1} \mathrm{e}^{+\frac{1}{g_s} V(x(\zeta))}$$

  - Flipping sheets flips determinants...
  - Involuted eigenvalues behave like holes on physical sheet (reminiscent of "Dirac sea" picture $\Rightarrow$ "anti-eigenvalues")... [Mariño-RS-Schwick]

- Denote involuted-sheet eigenvalues by $\bar{x} \equiv$ anti-eigenvalues...

The Mechanics of (Anti) Eigenvalue Tunneling



- Eigenvalues ↔ Forward Stokes automorphism ✓
- Anti-eigenvalues ↔ Backward Stokes automorphism ✓
- Anti-eigenvalues behave like holes on physical sheet! [Klemm-Mariño-Rauch]

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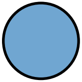
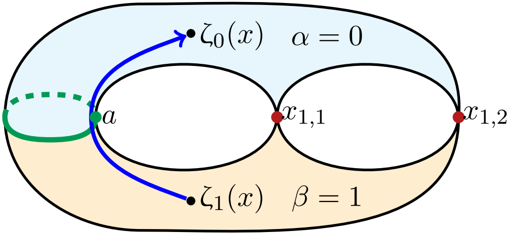
Boundary CFT

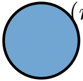
Minimal Strings and Liouville BCFT $(p,q) = (2, 2k-1)$

- Minimal strings = minimal model CFT + Liouville theory + ghosts:

$$c_{p,q} = 1 - 6\,\frac{(p-q)^2}{pq}, \quad c_{\mathsf{L}} = 1 + 6\left(b + \frac{1}{b}\right)^2, \quad c_{\mathsf{gh}} = -26, \quad b^2 = \frac{p}{q}$$

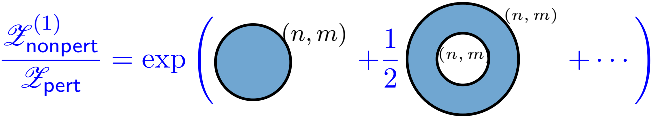
- Liouville FZZT-branes: $\mathsf{A}_{\mathsf{D}}(\zeta) =$  $^{\zeta} = \mu^{\frac{p+q}{2p}} \int^{x(\zeta)} \mathrm{d}x\, y(x).$



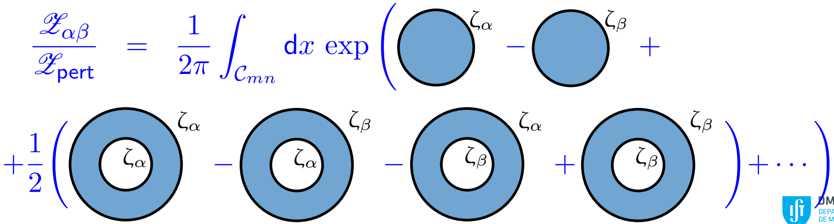
- Liouville ZZ-branes: $\mathsf{A}_{\mathsf{D}}(m,n) =$  $^{(n,m)} = \mu^{\frac{p+q}{2p}} \oint_{B_{mn}} \mathrm{d}x\, y(x).$

D-Branes as Nonperturbative Contributions

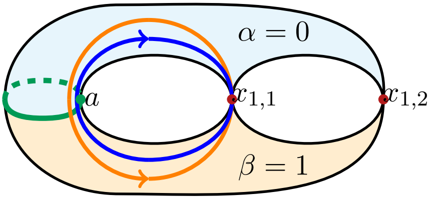
- Combinatorics of multiple, disconnected D-boundaries exponentiates [Polchinski]



- ZZ = differences FZZT on different sheets: [Martinec,Seiberg-Shih,KOPSS,MMSS, Fukuma-Irie-Matsuo]



Computing Nonperturbative Contributions from D-Branes



- One D-instanton + one negative-tension D-instanton contribution:

$$\frac{\mathcal{Z}_{\alpha\beta,\beta\alpha}}{\mathcal{Z}_{\text{pert}}} = \int \frac{\mathrm{d}x}{2\pi} \int \frac{\mathrm{d}\tilde{x}}{2\pi} \, \frac{1}{\left(x-\tilde{x}\right)^2} \times$$

$$\times \exp\left(\mathcal{A}^{[-1]}_{\alpha\beta}(x) + \mathcal{A}^{[-1]}_{\beta\alpha}(\tilde{x}) + \mathcal{A}^{[0]}_{\alpha\beta}(x) + \mathcal{A}^{[0]}_{\beta\alpha}(\tilde{x}) + \widehat{\mathcal{A}}^{[0]}_{\alpha\beta,\beta\alpha}(x,\tilde{x}) + \cdots \right)$$

- Minus sign in regularized annulus amplitude results in double pole!
- Not just "minus signs" $\Rightarrow$ leads to non-trivial transseries sectors!

[RS-Schwick-Tamarin]

EVERYONE MATCHES ✓

Exact Solutions

Exact Solutions to Matrix Models and String Theories

- Include all anti-eigenvalues/negative-brane contributions ⇒ find closed-form expression for full nonperturbative transseries ✓
- Full nonperturbative partition function as (sectorial) DFT: [KRSST]

$$\underbrace{Z_{\text{trans}}(t,\mu)}_{\textit{one}\text{-cut}} = \sum_{\ell\in\mathbb{Z}} {\rho_0}^{\ell} \exp\left\{ \underbrace{\widehat{F}_{\text{pert}}\left(t - g_{\text{s}}\left(\mu+\ell\right), g_{\text{s}}\left(\mu+\ell\right)\right)}_{\text{(regularized) } \textit{two}\text{-cut perturbative } F} \right\}$$

with (always need $N_1^{(1)}$ Stokes data!) [BSSV,KRST]

$$\rho_0 = N_1^{(1)}\, \sigma_2 \, \frac{\left(96\sqrt{3}\right)^{\frac{2}{\sqrt{3}}\,\mu}}{\Gamma\left(1+\frac{2}{\sqrt{3}}\,\mu\right)}, \qquad \mu = \sigma_1\sigma_2$$

- One-cut transseries parameters upgraded to two-cut moduli!

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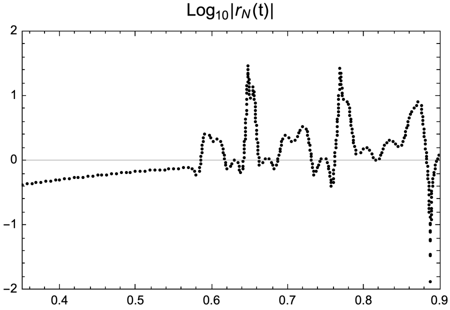
Quartic Matrix Model: Orthogonal Polynomials

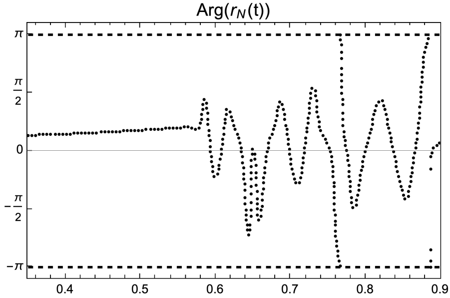
- Orthogonal polynomials solve matrix models,

$$p_{n+1}(z) = z\, p_n(z) - r_n\, p_{n-1}(z)$$

- Recursion coefficients r_n for quartic matrix model:

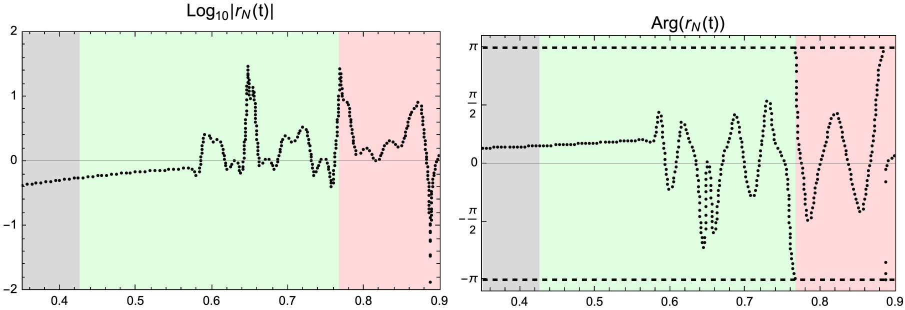
$$r_n \left(1 - \frac{\lambda}{6}\left(r_{n-1} + r_n + r_{n+1}\right)\right) = n\, g_{\mathsf{s}}, \qquad r_N = \frac{Z_{N+1}\, Z_{N-1}}{Z_N^2}$$





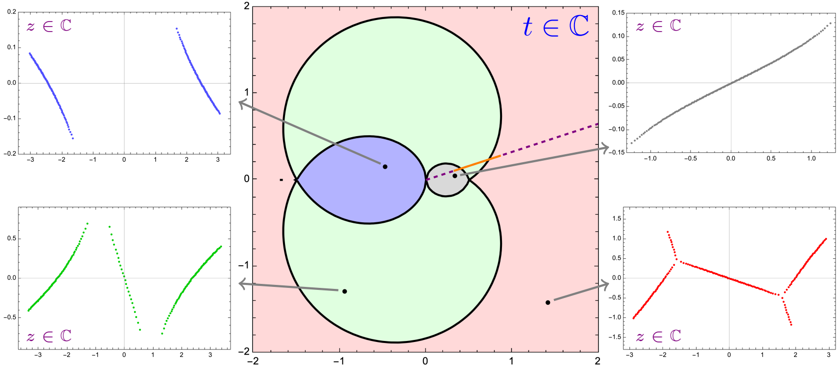
Phases of Quartic Matrix Model

Different r_n behavior $=$ different large N phases on 't Hooft plane...



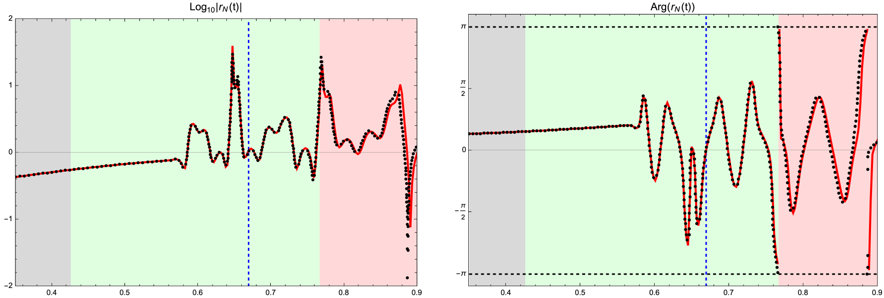
Phase Diagram of Quartic Matrix Model

Different phases on $t \in \mathbb{C}$ plane = different eigenvalue behavior...

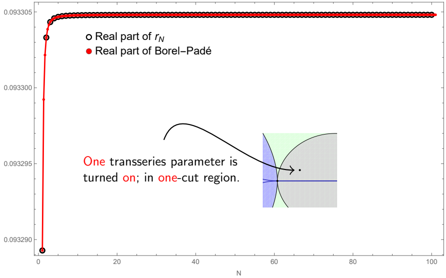


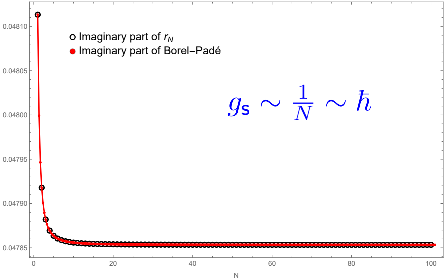
Exact Weak to Strong Coupling Interpolation ✓

- Compute recursion coefficients from our resurgent transseries...
- Implement all required Stokes transitions... [BSSV,KRST]
- Obtain exact reproduction of all phases $t \sim 0$ to $t \sim \infty$! [KRSST]

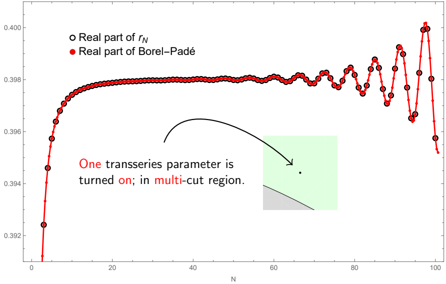


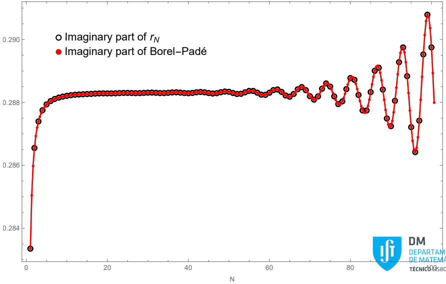
Exact Quantum Interpolation: One- and Multi-Cut Phases





$$g_s \sim \frac{1}{N} \sim \hbar$$





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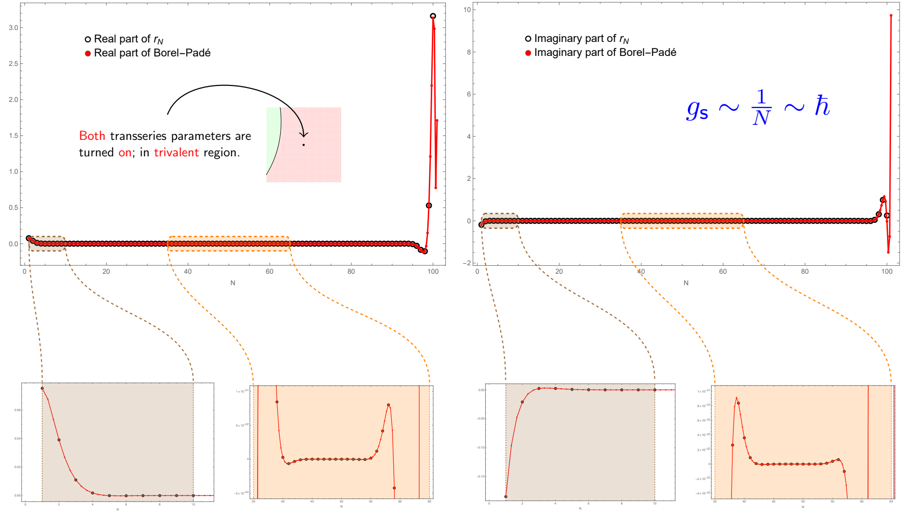
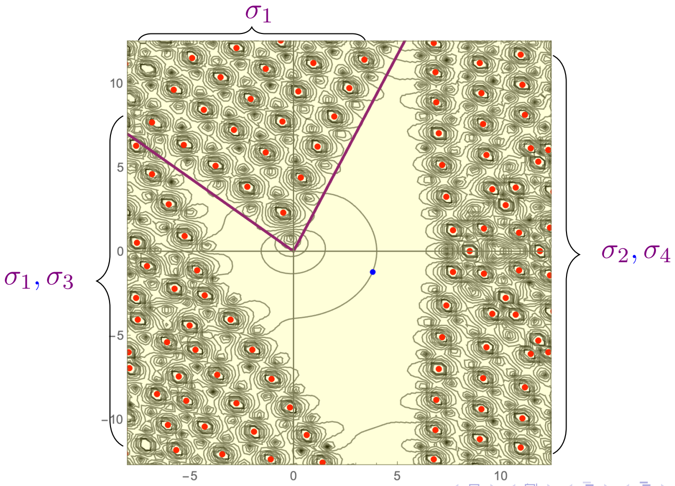
Exact Quantum Interpolation: Trivalent Phase



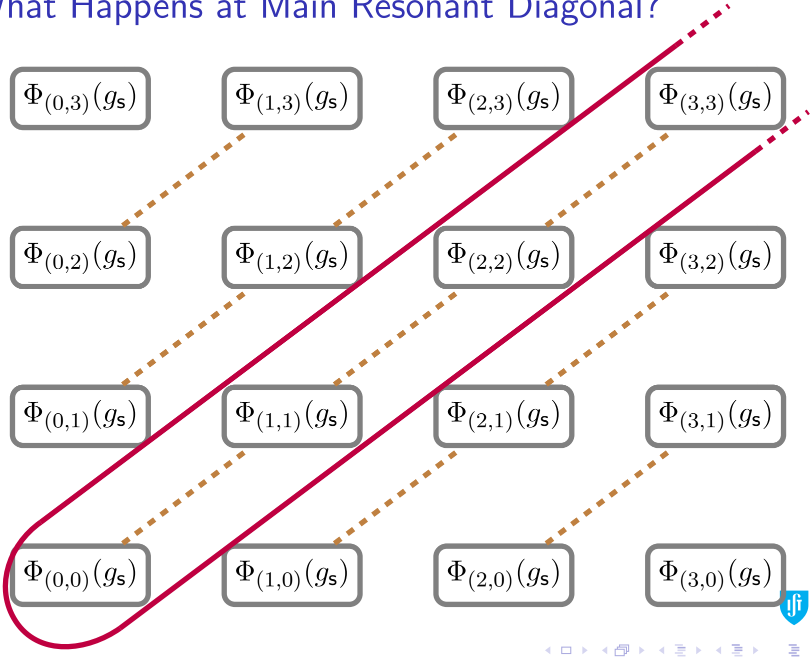
Partition Function Zeroes: Numerical *versus* Analytical

- Yang–Lee $(2, 5)$ multicritical string theory $\Rightarrow$ Four transseries parameters $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ ... Match numerical $=$ analytical ✓
- Different instantons dominate distinct regions: [KRSST]



One Lingering Question

...What Happens at Main Resonant Diagonal?



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Diagonal Structure of Generic *Resonant* Solutions

- Explicit free energy in "diagonal" rather than "rectangular" framing:

$$F(g_s, \boldsymbol{\sigma}) = \underbrace{\sum_{m=0}^{+\infty} (\sigma_1\sigma_2)^m \, \Phi_{(m|m)}(g_s)} + \cdots$$

$$\sum_{m=0}^{+\infty} (\sigma_1\sigma_2)^m \, \Phi_{(m|m)}(g_s) = \sum_{g=0}^{+\infty}\sum_{h=0}^{+\infty} (\sigma_1\sigma_2)^h \, F_g^{(h|h)} g_s^{2g+h-2}$$

$$= \underbrace{\sum_{g=0}^{+\infty} F_g(\mu)\, g_s^{2g-2}}_{F_g(\mu)=\sum_{h=0}^{+\infty} F_g^{(h|h)}\mu^h}$$

- With 't Hooft-like parameter $\mu = g_s\sigma_1\sigma_2$ find closed string theory $\Rightarrow$ Transseries parameters upgraded to string-background moduli!

From Background Independence to Argyres–Douglas

- Main resonant diagonal encodes all other (multi-cut) backgrounds ⇒ Full nonperturbative partition function is background independent ✓
  [Bonnet-David-Eynard,Eynard,Eynard-Mariño,KRSST,RS-Schwick]

- Pre-potential of multi-cut backgrounds ⇒ Pre-potential of Argyres–Douglas theories:
  - (A_1, A_{2k-1}) AD theories from topological gravity...
  - (A_1, A_{2k}) AD theories from $\deg = k+1$ hermitian matrix models...
  - Generalizes Painlevé/gauge theory correspondences... **[GIL,BLMST]**
    - ⋆ Painlevé I ∼ Nekrasov–Okounkov dual partition function of $4d$ $\mathcal{N}=2$ SU(2) $N_{\mathsf{f}}=1$ gauge theory at Argyres–Douglas point...

Resurgence: Stokes Automorphisms and Wall-Crossing

- Complete nonperturbative map includes its resurgence...
  [Gu-Mariño,Iwaki-Mariño,RS-Schwick]
- Direct resurgence of "diagonal" theory ⇒ Usual string-theoretic resurgence + wall-crossing formulae of AD theory ✓ [Écalle]
- Giving rise to wall-crossing phenomena on Argyres–Douglas side:
  - Transseries birth of new cut ⇒ New Gaussian/conifold point...
  - Gaussian/conifold has own Stokes automorphism... [Pasquetti-RS]
  - Wall-crossing occurs when $\underline{\mathfrak{S}}_{\theta}^{\text{TG/MM}}$, $\underline{\mathfrak{S}}_{\theta'}^{\text{conif}}$ automorphisms cross ✓

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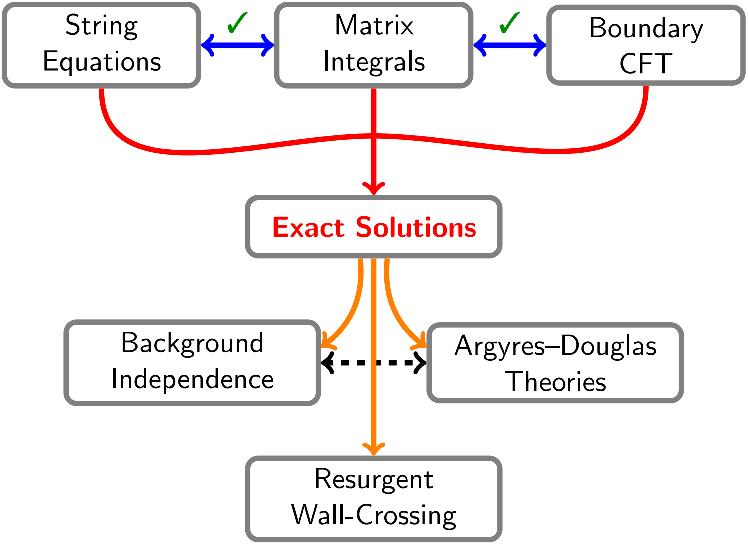

Thank You for Listening!