Aspects of gauge-strings duality

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Outline

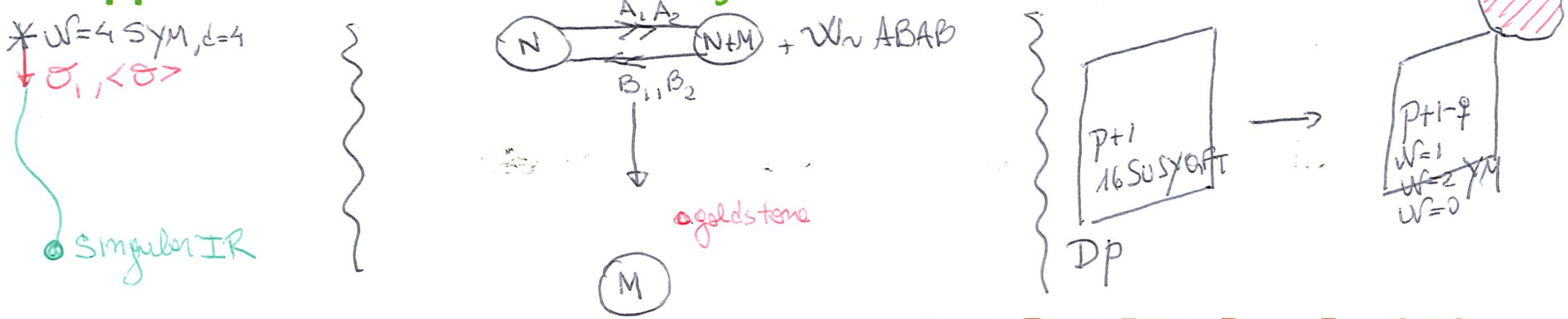
1. This is a talk about AdS/CFT. The aim is to discuss conformal and confining-screening field theories, using holography.
2. The plan is to discuss infinite families of $\mathcal{N} = 2$ SCFTs in four dimensions. Break conformality leading to gapped systems preserving less SUSY. Focus on one example and trying to get Physics of it.
3. There are many technical aspects to this. I will try to focus on the ideas, showing few technical details.
4. The talk is based on different works in the last seven months with: **Dimitris Chatzis, Ali Fatemiabhari, Federico Castellani, Mauro Giliberti, Ricardo Stuardo, Marcelo Oyarzo, Peter Weck, Leonardo Santilli and Kostya Zarembo.**

Remind some old things

This slide is schematic and very deserving references not mentioned: early 2000's saw the construction of holographic duals to Yang-Mills, QCD-like and its SUSY cousins.

- Starting from $\mathcal{N} = 4$ SYM and relevant-deforming it. The issue was that the IR ($r \to$ region of the geometry) was singular, invalidating some interesting calculations.
- Consider a SUSY two gauge nodes quiver and the holographic realisation of the dynamics towards the IR.
- Study Dp branes ($p = 4, 5, 6$) and compactified it to obtain duals to theories in $(3 + 1)$, $(2 + 1)$, $(1 + 1)$ with different SUSY preserved.

These approaches were afflicted by some issues.



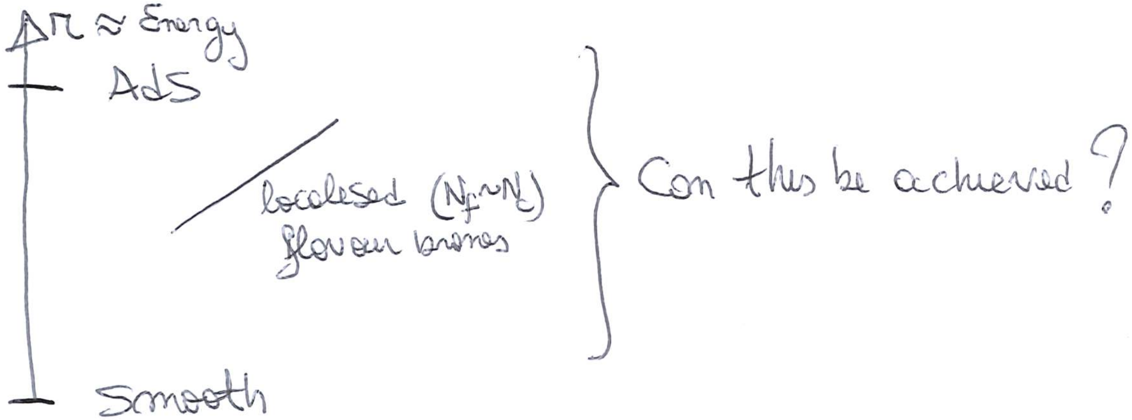
Features of these holographic duals

If the UV was well defined, the IR was singular. If the IR was smooth, the UV was not (strictly) Asymptotically AdS. In some cases, the UV was non-field theoretical.

The addition of a large amount of matter transforming in the fundamental of the gauge group (quarks), was technically challenging. It had effects on the UV and IR of the geometries. $N_f \sim N_c$ (unquenched!)

Aside from this, there were numerous things that were calculated with these holographic duals. A lot of important lessons were learn!

Would it be possible to keep the virtues (like the smooth IR), repair the UV (making it AAdS) and also add (localised) sources?



How to fix the issues, keeping the virtues?

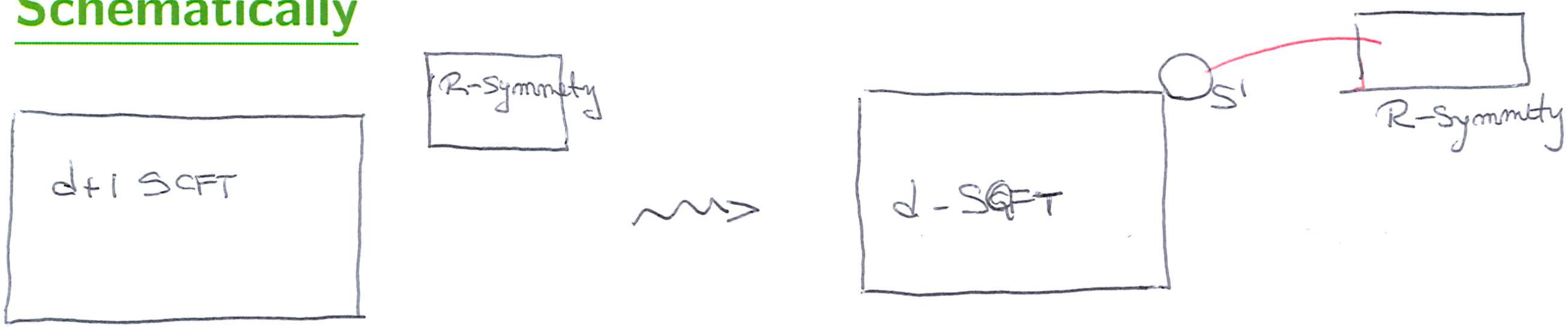
Start with a CFT that contains fields in the adjoint, bifundamental and fundamental representations.

Using these SCFTs (an infinite family of them) as UV fixed point, deform by a VEV or relevant operator, such that the IR is smooth.

To preserve SUSY, a mixing (twisting) between the R-symmetry and the Lorentz group is needed.

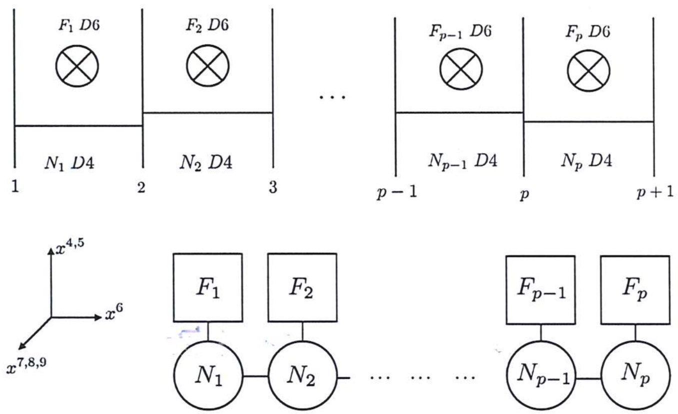
Let me present just one example (a family of examples). Various others are in recent papers written together with D. Chatzis, A. Fatemiabhari and Peter Weck.

Schematically



Consider 4d $\mathcal{N} = 2$ SCFTs.

For its historical and conceptual value, consider these 4d SCFTs. A quiver-like description and a Hanany-Witten set-up.



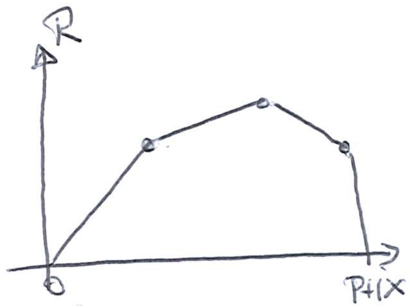
The global symmetries are $SO(2,4) \times SU(2)_R \times U(1)_r$. This will be reflected in the string dual containing $AdS_5 \times S^2 \times S^1$.

The beta function of these theories is $\beta \sim (2N_c - N_f)$. This implies

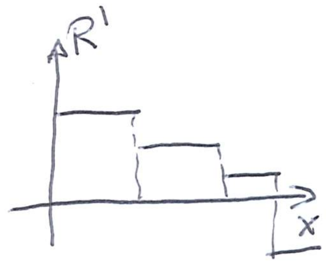
$$2N_1 = N_2 + F_1, \quad 2N_2 = N_1 + N_3 + F_2, ..., 2N_P = F_P + N_{P-1}.$$

One can define a 'Rank-function' $R(x)$, that is convex polygonal. For the quiver pictured above, we have

$$R(x) = \begin{cases} N_1 x & 0 \leq x \leq 1 \\ N_1 + (N_2 - N_1)(x-1) & 1 \leq x \leq 2. \\ .... & \\ N_k + (N_{k+1} - N_k)(x-k) & k \leq x \leq (k+1) \\ N_P - N_P(x-P) & P \leq x \leq P+1. \end{cases}$$



$$R'(x) = \begin{cases} N_1 & 0 \leq x \leq 1 \\ (N_2 - N_1) & 1 \leq x \leq 2. \\ .... & \\ (N_{k+1} - N_k) & k \leq x \leq (k+1) \\ -N_P & P \leq x \leq P+1. \end{cases}$$



$$R'' = \underbrace{(2N_1 - N_2)}_{F_1}\delta(x-1) + \underbrace{(2N_2 - N_1 - N_3)}_{F_2}\delta(x-2) + ... + \underbrace{(2N_P - N_{P-1})}_{F_{P-1}}\delta(x-P).$$

Lin, Lunin and Maldacena wrote (2005) the Type IIA backgrounds ($\alpha' = g_s = 1$). Gaiotto-Maldacena (2010).

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi, \xi) + f_4 d\beta^2.$$
$$B_2 = f_5 d\Omega_2(\chi, \xi), \;\; C_1 = f_6 d\beta, \; A_3 = f_7 d\beta \wedge d\Omega_2, \; e^{2\phi} = f_8.$$

The functions $f_i(\sigma, \eta)$ can be all written in terms of a function $V(\sigma, \eta)$ and its derivatives, $f_i \sim f_i(V, \partial_\sigma V, \partial_\eta V)$. For example

$$f_1(\sigma, \eta)^2 = \frac{2\dot{V} - \ddot{V}}{V''}, \quad f_2 = 2f_1 \frac{V''}{\dot{V}}, \ldots.$$

The function $V(\sigma,\eta)$ satisfies a Laplace-like equation with certain given boundary conditions to avoid nasty singularities,

$$\sigma^2\partial_\sigma^2 V + \sigma\partial_\sigma V + \sigma^2\partial_\eta^2 V = 0,$$
$$V(\sigma\to\infty,\eta)\to 0,\quad V(\sigma,\eta=0) = V(\sigma,\eta=P+1) = 0.$$
$$R(\eta) = \sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0},\qquad R(0) = R(P+1) = 0.$$

Importantly, the rank function $R(\eta)$ appears as an initial condition for the Laplace problem.
Given $R(\eta)$, one can write the solution for $V(\sigma,\eta)$ as a Fourier series

$$V(\sigma,\eta) = -\sum_{n=1}^{\infty}\frac{c_n}{w_n}K_0(w_n\sigma)\sin(w_n\eta),\qquad w_n = \frac{n\pi}{P+1}.$$
$$c_n = \frac{n\pi}{(P+1)^2}\int_{-(P+1)}^{P+1} R(\eta)\sin(w_n\eta)d\eta.$$

The backgrounds are trustable if the numbers P, N_k are large.

There are numerous checks that these supergravity backgrounds capture aspects of the SCFTs in question

These checks range from the calculation of free energies, central charges (number of degrees of freedom), Wilson loops, VEVs of certain operators, etc.

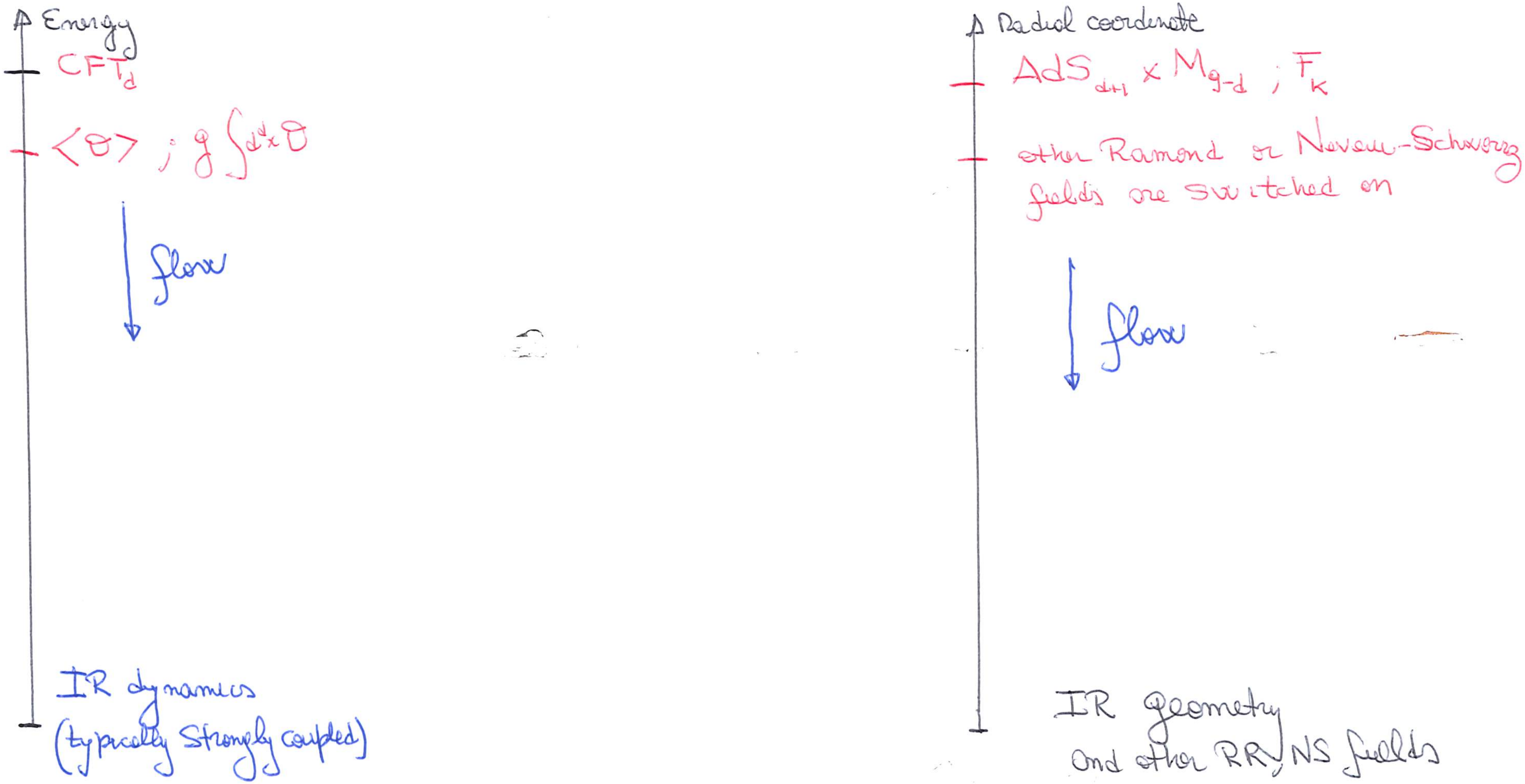
These calculations in String Theory are very precisely matched by localisation+matrix model computations. This works also for SCFTs in diverse dimensions.

All the formalism is easily extended to infinite families of SCFTs/String backgrounds.

Let us now depart from the 'conformal world'.

I shall tell you about how to use these systems to learn about RG flows away from the fixed point. Less symmetries and less constrained dynamics are the features.

The main idea can be expressed with a drawing

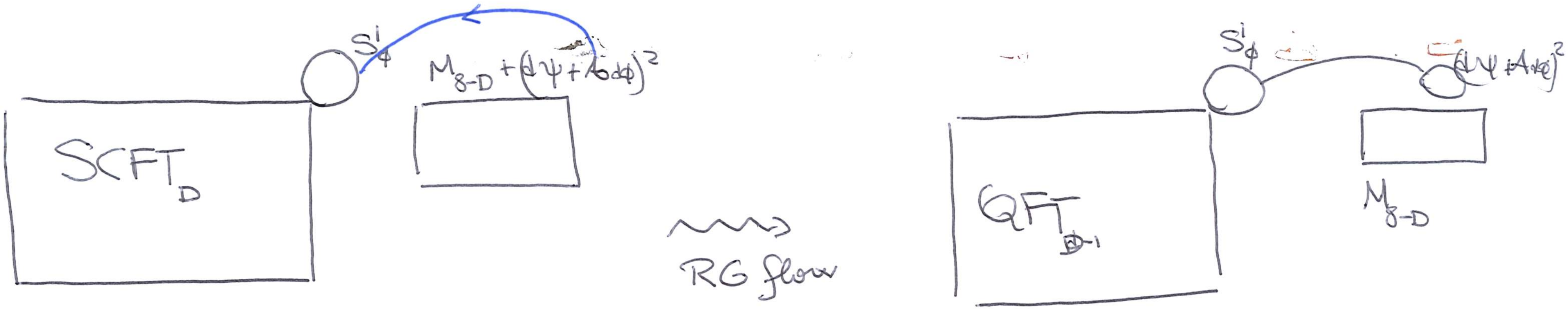


The idea is to use metrics of the form $\mathrm{AdS}_{d+1} \times M_{9-d}$,

$$ds^2 \sim r^2\left(-dt^2 + d\vec{x}^2_{d-2} + f(r)d\phi^2\right) + \frac{dr^2}{r^2 f(r)} + M_{8-d} + (d\psi + A_\phi d\phi)^2,$$

$$f(r) = 1 - \frac{\mu}{r^d} - \frac{q}{r^{2d-2}}, \quad A_\phi = q\left(\frac{1}{r^2} - \frac{1}{r_+^2}\right)d\phi.$$

This is inspired on work by Anabalón and Ross (2021). Studied for $\mathcal{N} = 4$ SYM by Kumar and Stuardo (2024).



What is happening in the QFT?

A UV CFT has an R-symmetry. Pick a $U(1)_R$ and mix it with the circle direction on which we compactify the CFT. The background is smooth.

Thanks to this 'twisting', one can preserve some amount of SUSY. The gravity indicates the transition from a CFT_d to a $\mathrm{QFT}_{(d-1)}$ that is gapped (smooth IR).

One can play these trick in various scenarios. For example in the CFTs in $d = 4$ with eight SUSYs discussed before.

I will show the metric (there are accompanying RR and NS fields that I will not display). These are the holographic description of SCFTs deformed by VEVs that lead to a gapped system

We show the resulting metric in the case for which the UV is $\mathrm{AdS}_5 \times S^2 \times S^1 \times \Sigma_2$. Other examples are available.

$\text{AdS}_5 \times S^2 \times S^1 \times \Sigma_2$

These backgrounds represent the effect of performing the above procedure to any of the infinite backgrounds in the Gaiotto-Maldacena family of solutions. The metric reads

SU(2)$_R$

$$ds^2 = \tilde{f}_1\Big[4ds_5^2 + \tilde{f}_2(d\theta^2 + \sin^2\theta(d\varphi - B_1)^2) + \tilde{f}_4(d\sigma^2 + d\eta^2)$$

$$+\tilde{f}_3(d\chi + B_1)^2\Big], \quad f(r) = 1 - \frac{\mu}{r^4} - \frac{q}{r^6}, \quad B_1 = q\left(\frac{1}{r^2} - \frac{1}{r_+^2}\right) d\phi.$$

U(1)$_R$

$$ds_5^2 = \frac{r^2}{L^2}(-\mathrm{d}t^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + f(r)\mathrm{d}\phi^2) + \frac{L^2 \mathrm{d}r^2}{f(r)r^2}.$$

$\Phi, \quad F_2, \quad H_3, \quad F_4.$

The functions $\tilde{f}_i(\sigma, \eta)$ are those in the Gaiotto-Maldacena system, written in terms of a potential (infinite family).

We are twisted-compactifying a 4d $N = 2$ SCFT linear quiver to three dimensions, preserving 4 SUSY, leading to a "confining" QFT in the IR.

gapped

What can we calculate with these backgrounds?

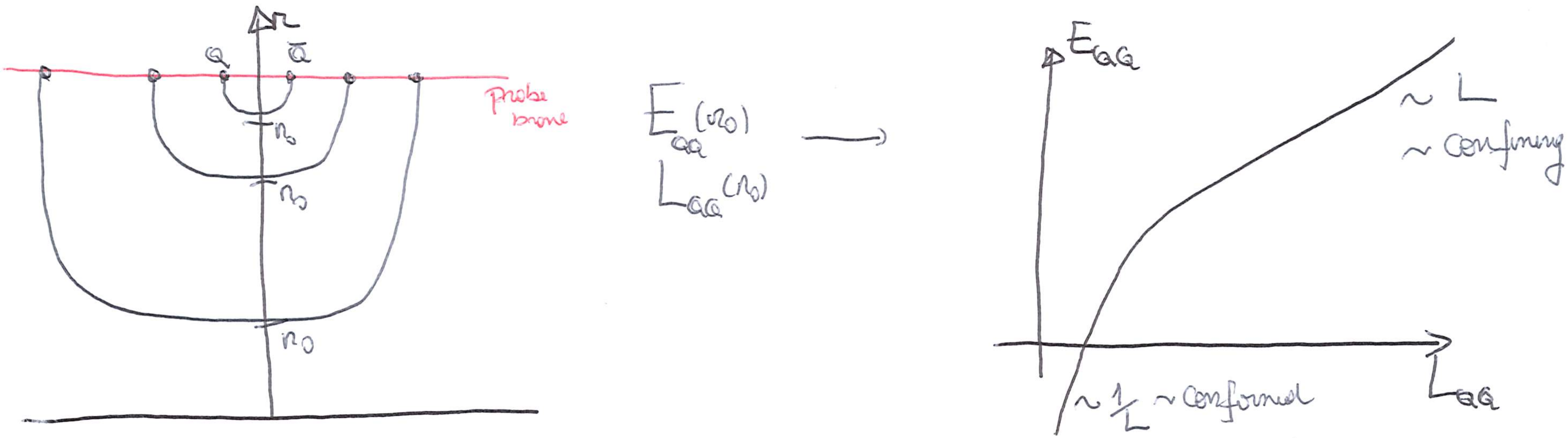
Analysing asymptotics, one finds that (in terms the parameters of the solution μ, q)

$$< J_1 >= -q, \quad - < T_{tt} >=< T_{x_i x_i} >=< \frac{T_{\phi\phi}}{4} >= \mu.$$

To calculate Wilson loops, propose the usual string configuration

$$t = \tau, \quad x = \gamma, \quad r = r(\gamma), \quad \eta = \eta_0, \quad \sigma = \sigma_0.$$

$$S_{NG} = \frac{T}{2\pi} \int d\gamma \sqrt{F^2 + G^2 r'^2}, \; F^2 = g_{tt} g_{xx}, \; G^2 = g_{tt} g_{rr}.$$



A quantity that measures degrees of freedom

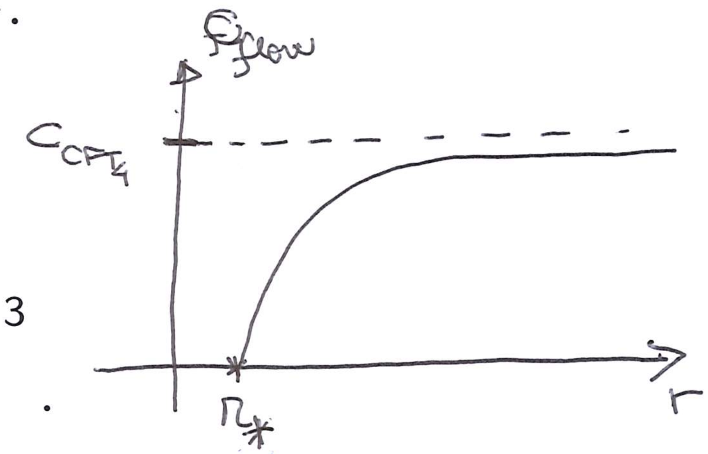
For backgrounds of the form (anisotropic like ours!)

$$ds^2 = -\alpha_0 dt^2 + \alpha_1 dy_1^2 + \alpha_2 dy_2^2 + ... + \alpha_d dy_d^2 +$$
$$\Pi_{i=1}^{d}(\alpha_1....\alpha_d)^{\frac{1}{d}} b(r) dr^2 + g_{ij}(d\theta^i - A_1^i)(d\theta^j - A_1^j), \quad \Phi(r, \vec{\theta}).$$
$$V_{\rm int} = \int_X \sqrt{\det[g_{\rm int}] e^{-4\Phi}}, \qquad \hat{H} = V_{\rm int}^2.$$

The holographic central charge along the flow–called $c_{\rm flow}$ is a generalisation of the Brown-Henneaux expression.

$$c_{\rm flow} = \frac{d^d}{G_N} b(r)^{d/2} \frac{\hat{H}^{\frac{2d+1}{d}}}{\hat{H}'^d}.$$

Using the functions in the backgrounds,



$$\frac{c_{\rm flow}}{c_{\rm AdS}} = \left(\frac{\sqrt{f(r)}}{1 + \frac{r f'(r)}{6}} \right)^3.$$

A behaviour that occurs for various other observables

The quantity is the product of two parts, one referring to the UV SCFT and a radial dependent part indicating the flow in energy. This also occurs for other quantities (Entanglement Entropy, Complexity).

These 'universal' behaviours are actually a consequence of a development by Gauntlett and Varela in 2007! This is very non trivial from the QFT viewpoint.

This c_{flow} is monotonic. It detects the UV conformal point and also shows the gapped IR. We have a flow across dimensions.

One can also calculate masses for spin-two excitations along the Minkowski directions of the QFT.

Let me briefly comment on something (in progress!) that is non-universal.

Screening

We have systems with colour and flavour groups. Why did the Wilson loop presented before give a confining result?

The string probe we are using is not allowing moves in the η-direction. If we allow this

$$t = \tau, \quad x = \gamma, \quad r = r(\gamma), \quad , \eta = \eta(\gamma).$$

$$S_{NG} = \frac{T}{2\pi} \int d\gamma \sqrt{F^2 + G^2 r'^2 + S^2 \eta'^2}.$$

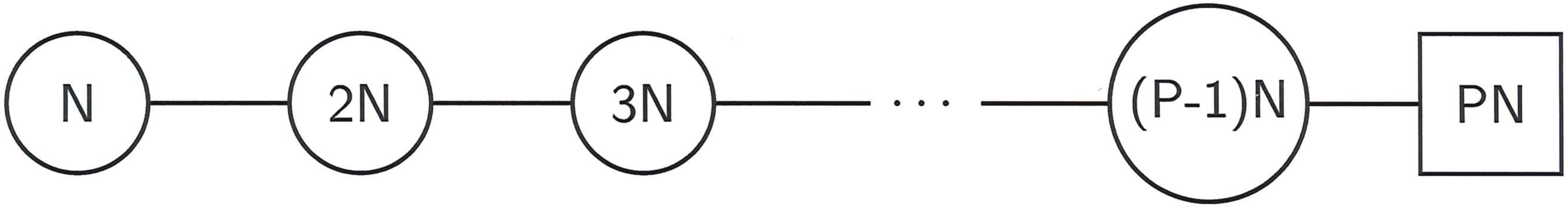
$$F^2 = g_{tt} g_{xx}, \quad G^2 = g_{tt} g_{rr}, \quad S^2 = g_{tt} g_{\eta\eta}.$$

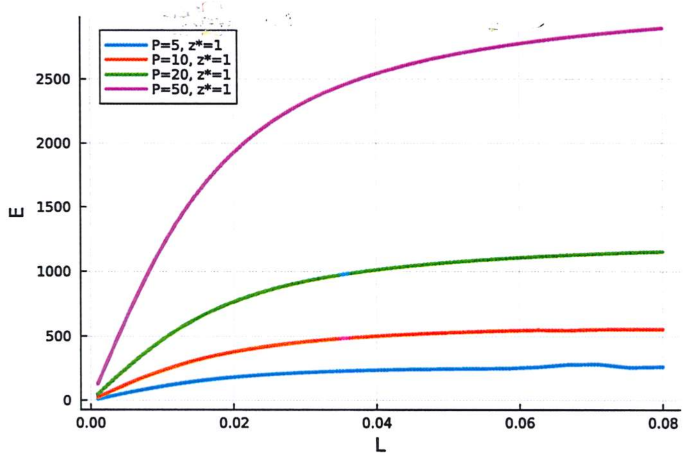
One try to numerically solve the Euler-Lagrange equations. In work with Fatemiabhari and Giliberti, a minimisation of the action is done in a system similar to the one discussed here.

We find that the probe F1 "dives" into the flavour group. There is a form of screening, which is very non-universal.

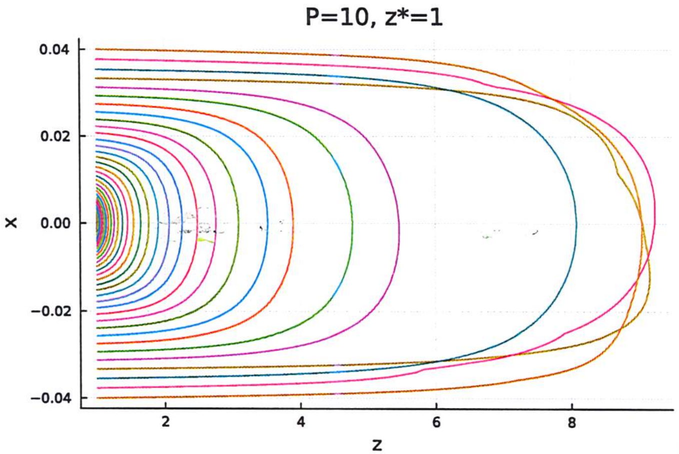
I show some plots in the next page. There are still various things to understand and explore!

For the quiver





$$E(L) = -\frac{|Q|}{L} + |B|\left(1 - e^{-|b|L}\right)$$



Some closing comments and conclusions

- I have presented one example $d = 4$ of a generic formalism to write string duals to SCFTs in d=1,2,3,4,5,6 with eight Poincare SUSYs.
- I have shown how to use these SCFTs as UV-fixed point for an RG flow, ending in a gapped field theory. Ending in another fixed point is also possible.
- Various observables calculated in the SCFT are interesting along the flow. Some are universal, some are not. It is interesting to better understand the dynamics induced by the presence of flavour degrees of freedom.
- **I am presently exploring similar solutions, with or without SUSY. Calculating correlations functions and other observables.**