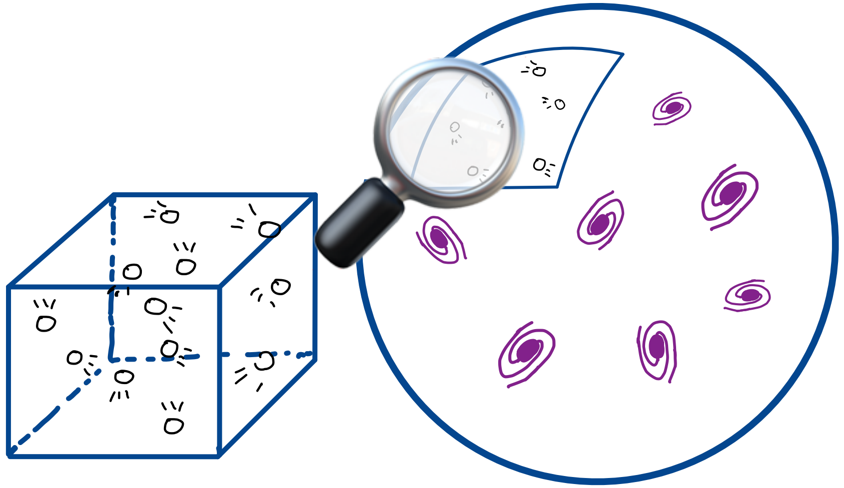
de Sitter holography from quantum information

Shira Chapman, Ben-Gurion University

Eurostrings, Southampton 04-09-2024

Based on [arXiv:2407.09604] – **SC**, Saskia Demulder, Damian Galante, Sam Sheorey, Osher Shoval



Towards de Sitter holography from ~~quantum information~~ SYK flows and their chaotic behavior

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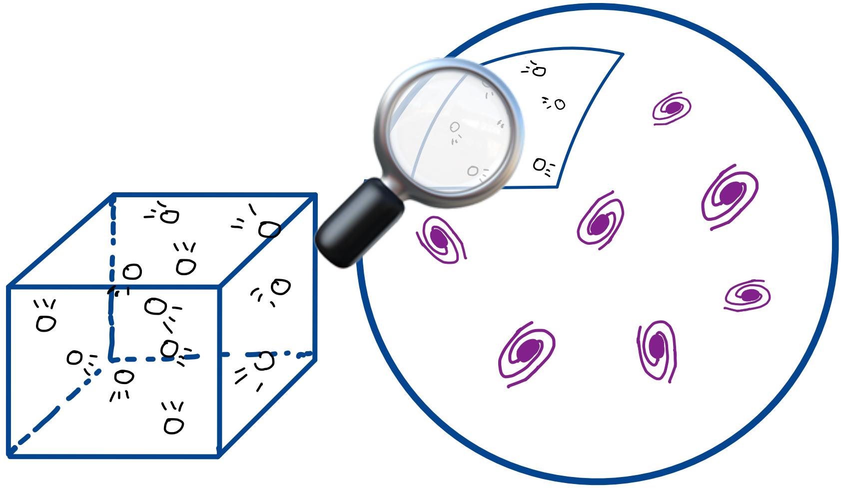
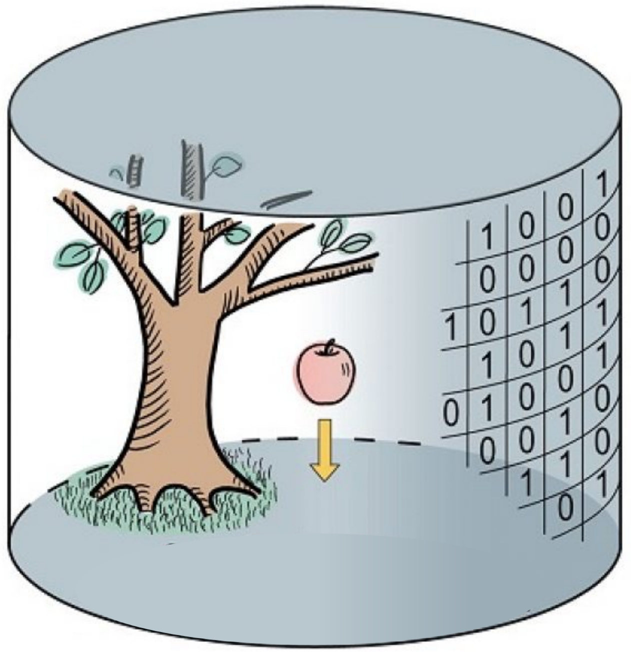


Motivation - Quantum gravity



- What are the microscopic constituents of quantum gravity?
- For gravity in AdS, holography is very useful!
- For example: in low dimensions – SYK.
- Quantum information & Chaos important!
- Can we say anything about dS?

Why would we think that dS is holographic?

- Observers in de Sitter space are surrounded by a cosmological horizon.
- This horizon has an entropy consistent with an area formula [Gibbons-Hawking (1977)]

$$S_{\mathrm{dS}} = \frac{A}{4G_N}$$

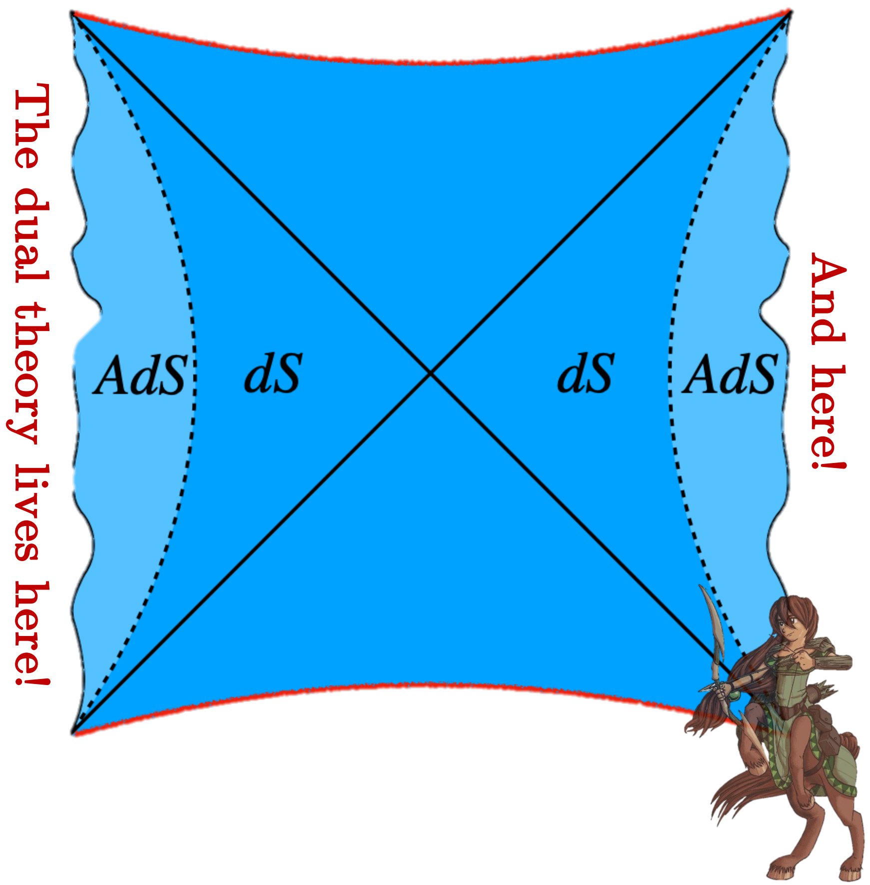
  what is it counting?

- In AdS – timelike boundary permitted progress.
- In dS – no timelike boundary. [But see Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang (2021); Svesko, Verheijden, Verlinde, Visser (2022); Anninos, Galante, Maneerat (2024)]

A hint towards dS holography

Interpolating geometries

- How to proceed without a timelike boundary?
- In low dimesions - hint - can embed a portion of dS inside AdS Space – **interpolating geometries** [Anninos, Hofman (2017)]
- No knowledge of field theory 😐
- But expect IR modification [de Boer – Verlinde^2 1999]
- Which properties are expected from the dual?
- Chaotic behavior very different than AdS [Anninos, Galante, Hofman (2019); **SC**, Galante, Kramer (2022), …]
- For instance, the OTOC has an oscillatory behavior rather than exponential



QM duals of interpolating geometries

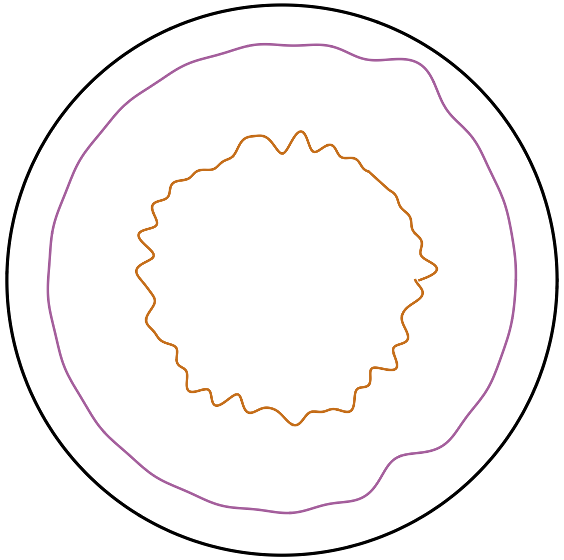
- Towards duals of interpolating geometries:

  IR modifications of AdS $\implies$ relevant deformations of SYK

- For most of my talk, I will consider IR modifications that are dual to interpolating geometries between AdS and AdS.

- Study the chaotic properties of SYK RG flows!
  [Anninos, Galante, Sheorey (2022)]

- Back to dS in the outlook

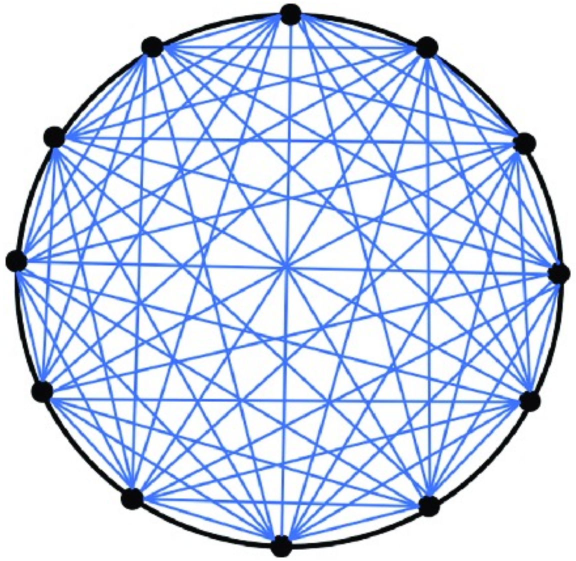


Outline

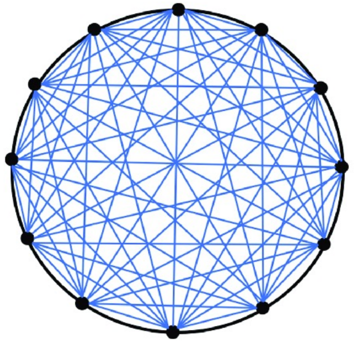
- Motivation [V]
- Background: SYK model and its deformations
- Signatures of Chaos
- Summary and outlook

The Sachdev-Ye-Kitaev (SYK) Model

(and its deformations)



The SYK model



- N Majorana fermions:

$$\{\psi_i, \psi_j\} = \delta_{ij} \ , \qquad i, j = 1, \ldots, N \ .$$

- Hamiltonian: random q-body interactions:

$$H_q = i^{\frac{q}{2}} \sum_{1 \le i_1 < i_2 < \cdots < i_q \le N} J_{i_1 i_2 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q} \ , \qquad q \in 2\mathbb{Z}^+$$

$$\langle J_{i_1 i_2 \cdots i_q} \rangle = 0 \ , \qquad \langle J^2_{i_1 i_2 \cdots i_q} \rangle = \frac{2^{q-1}}{q} \frac{\mathcal{J}^2 (q-1)!}{N^{q-1}}$$

- Special case: $q = 2$ is integrable.
- Used to model strange metals (many-body quantum states without quasiparticles).

[Sachdev and Ye (1993); Maldacena and Stanford (2016); Kitaev and Suh (2018)]
[Review: Chowdhury, Georges, Parcollet, Sachdev]

The SYK model continued

- The partition function can be written in terms of the self energy Σ and fermion bilinear:

$$G(\tau) = \frac{1}{N}\sum_{i=1}^{N}\langle T\psi_i(\tau)\psi_i(0)\rangle$$

- The large N saddle is given by Schwinger-Dyson equations

$$G = (\partial_\tau - \Sigma)^{-1} \qquad , \qquad \Sigma = \mathcal{J}^2 G^{q-1}$$

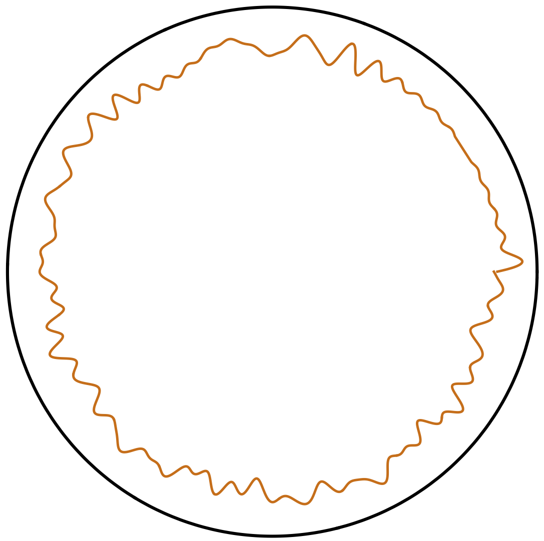
- At large q becomes ODE

- **Finite *N*,** large matrices - hard
- **Large *N***, integro-differential equations
- **Large *N*, large *q*** – ODE – analytic solution

SYK at low energies

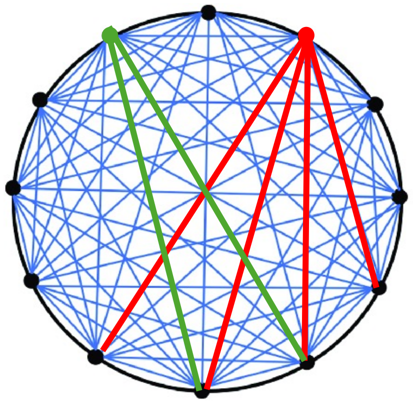
- At low energies $\beta J \gg 1$, the theory has nearly conformal behavior with Δ_ψ =1/q:

$$G_c(\tau) = \frac{b}{|\tau|^{2\Delta_\psi}} \mathrm{sgn}(\tau)$$



- Maximally chaotic (and yet tractable!)
- Dual to nearly AdS_2 solutions of JT gravity
  [Maldacena, Stanford, Yang (2016); Saad, Shenker, Stanford (2018, 2019); Stanford Witten (2019)]
- IR deformations of SYK could teach us about interpolating geometries

Deformations of the SYK model



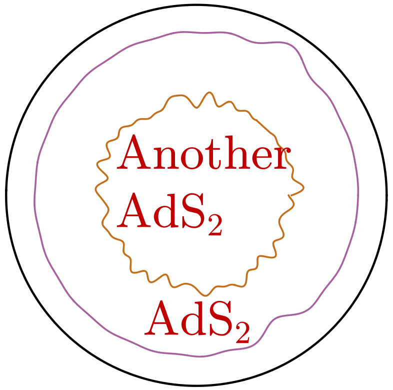
- More general gravity backgrounds? with different IR?
- Relevant deformation of SYK.
- Consider disordered operators, like the Hamiltonian itself

$$\boxed{H_{\text{def}} = {\color{red}H_q} + s{\color{teal}H_{\tilde{q}}} \;, \qquad s > 0 \;, \quad q \geq \tilde{q}}$$

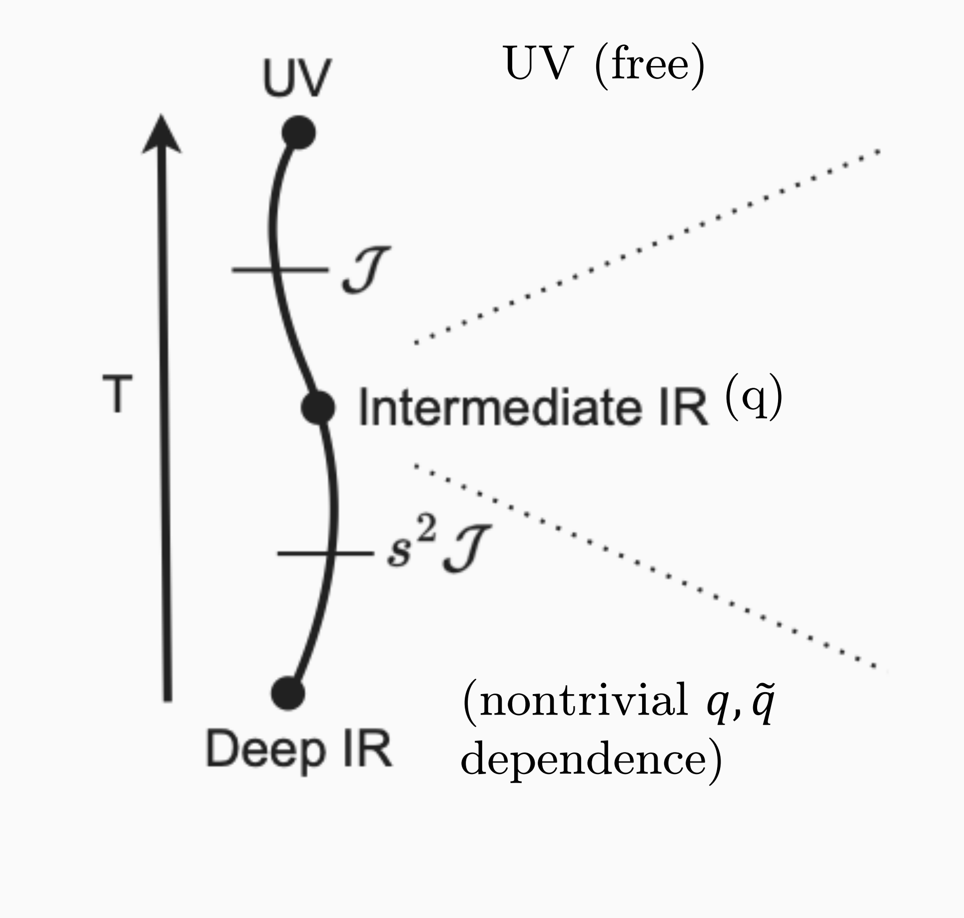
→ Becomes important at very low energies

[Jiang and Yang (2019); Anninos, Galante (2020), Anninos, Galante, Sheorey (2022)]
[Garcia Garcia et al. (2018); Lunkin et al. (2020); Nandy et al. (2022)]

- All the techniques at large N and large q that we used before can be generalized to this case.
- Dual understood (at least the thermodynamics) – near AdS deformations.



Regimes along the RG flows



*If not $s \ll 1$ go directly to deep IR

$$H_{\text{def}} = H_q + sH_{\tilde{q}} \; , \qquad s > 0 \; , \quad q \geq \tilde{q}$$

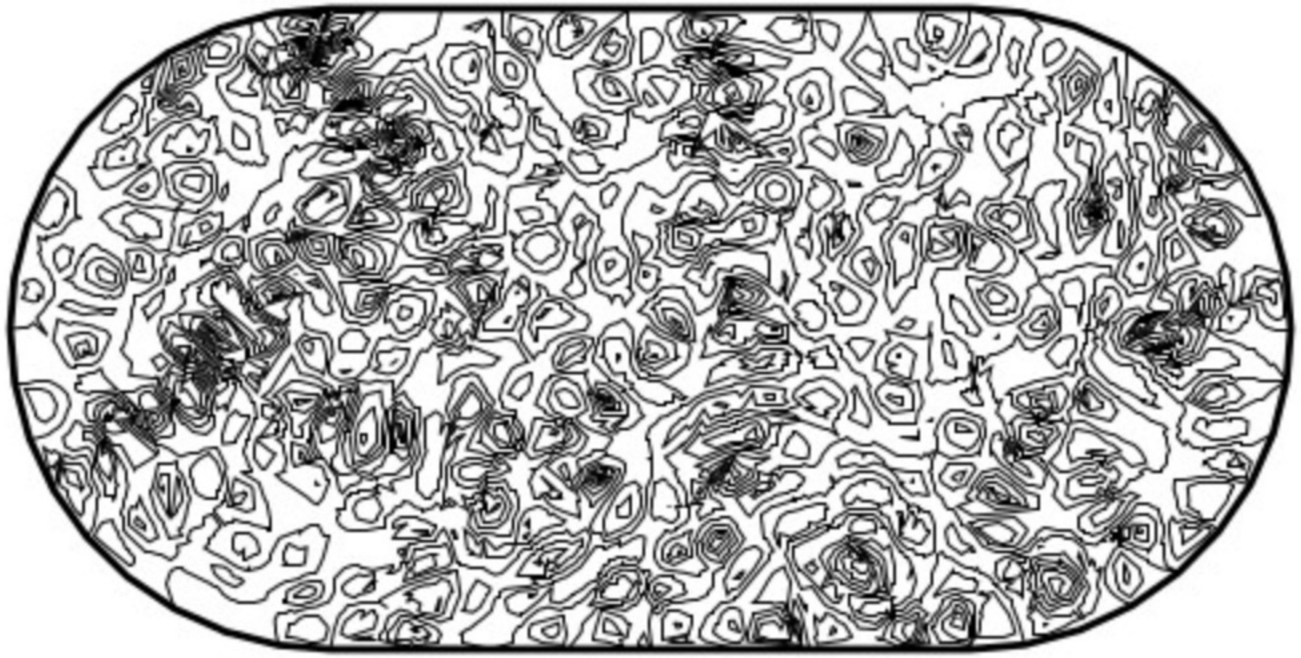
- Renormalization group flows away from the near-fixed point of the Sachdev-Ye-Kitaev (SYK) model

- **Expect two different regimes of nearly maximal chaos, can we see them?**

[Jiang, Yang]
[Anninos, Galante, Sheorey, ….]

Quantum Chaos

(and how to recognize it)



Quantum Chaos

- Quantum version of hypersensitivity to initial conditions.

Lyapunov exponent

$$\mathrm{OTOC}(t) = \frac{1}{N^2}\sum_{i,j=1}^{N} \mathrm{Tr}\left(\rho^{\frac{1}{4}}\psi_i(t)\rho^{\frac{1}{4}}\psi_j(0)\rho^{\frac{1}{4}}\psi_i(t)\rho^{\frac{1}{4}}\psi_j(0)\right) , \qquad \rho = \frac{1}{Z(\beta)}e^{-\beta H}$$

$$\boxed{\mathrm{OTOC} = f_0 - \frac{f_1}{N}\exp\lambda_L t + \cdots}$$

- A bound on chaos [Maldacena, Shenker, Stanford (2016)] $\lambda_L \leq \dfrac{2\pi}{\beta}$

- Need separation between the dissipation and scrambling time.

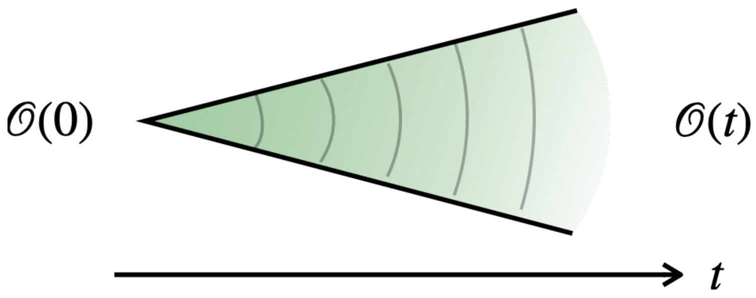
Krylov complexity

Measure for operator spreading under Hamiltonian evolution:

$$\mathcal{H}_{\mathcal{O}} = \text{span}\{\mathcal{O}, [H,\mathcal{O}], [H,[H,\mathcal{O}]], \ldots\} \quad \longrightarrow \quad \mathcal{H}_{\text{Krylov}} = \text{span}\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3 \ldots\}$$

How many operators are necessary to expand $\mathcal{O}(t) = e^{iHt}\mathcal{O}e^{-iHt}$

$$\langle n \rangle_t = \exp(\lambda_K t)$$



- Can be computed from the thermal 2pt function.
- Much easier than Lyapunov.
- No need for large N

[Pawel Caputa's talk!
Parker, Cao, Avdoshkin, Scaffidi, Altman (2019)]

Relation between the two exponents

- Bounds on Krylov complexity [Parker, Cao, Avdoshkin, Scaffidi, Altman (2019); Avdoshkin, Dymarsky (2020); Gu, Kitaev, Zhang (2022)]

$$\boxed{\lambda_L \leq \lambda_K \leq \frac{2\pi}{\beta}}$$

- Conjectured: tight bound $\lambda_L = \lambda_K$ – easy way to calculate Lyapunov?
- Tested for SYK at large q.
- But at the next order in 1/q already get more corrections!

$$\frac{\beta}{2\pi}(\lambda_K - \lambda_L) = \frac{4\pi^2}{3q\beta\mathcal{J}} + \cdots .$$

[**SC**, Demulder, Galante, Sheorey, Shoval (2024)]

Numerical techniques at finite q

Outline of the numerical procedure:

- Solve the Schwinger-Dyson equations using an iterative procedure in Fourier space to obtain the fermion 2pt function and various analytic continuations in Lorentzian time.
- From the 2pt function can already obtain the Krylov exponent.
- Solve 4pt function from Kernel equation: $\mathrm{OTOC}(t_1, t_2) = F_0(t_1, t_2) + \frac{1}{N} F(t_1, t_2) + \cdots$.

$$F\left(t_1, t_2\right) = \int dt_3 dt_4 \; K\left(t_1, t_2, t_3, t_4\right) F\left(t_3, t_4\right)$$

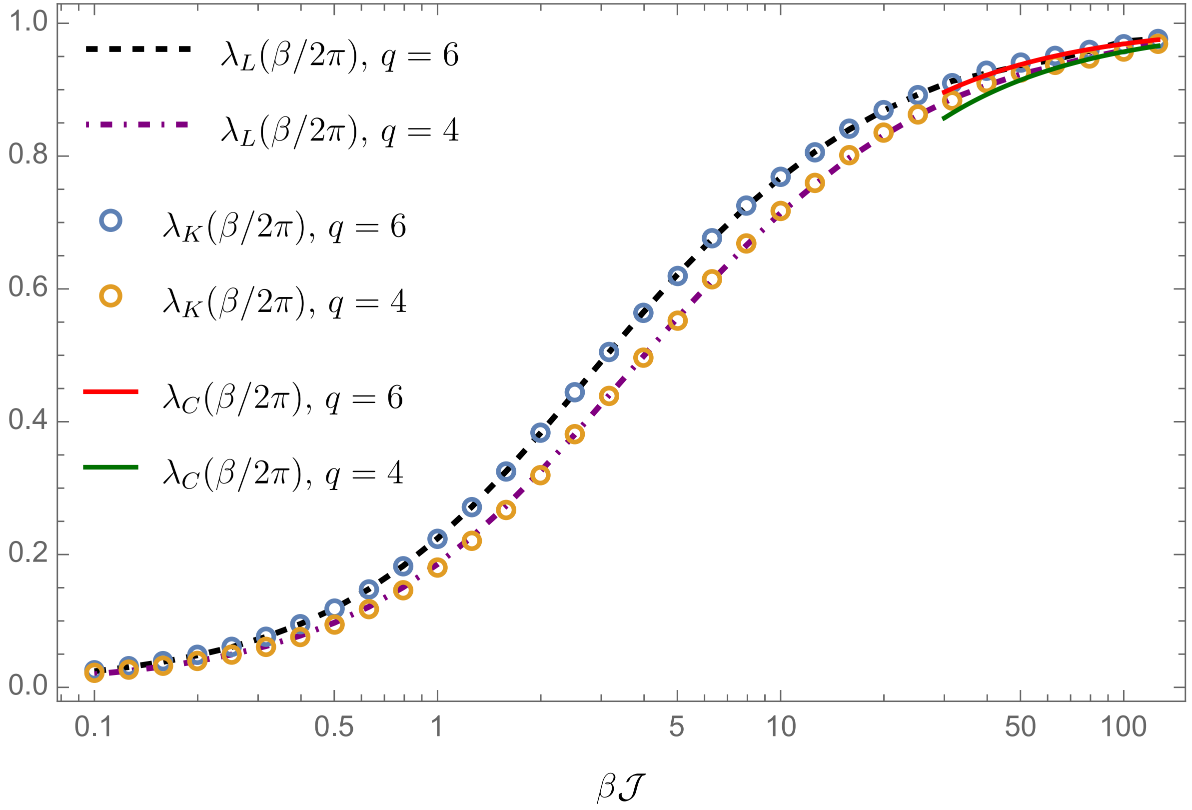
$$K\left(t_1, t_2, t_3, t_4\right) = G^R(t_{13}) G^R(t_{24}) \mathcal{J}^2 \left(\frac{2^{q-1}}{q} (q-1) G^W(t_{34})^{q-2} + s^2 \frac{2^{\tilde{q}-1}}{\tilde{q}} (\tilde{q}-1) G^W(t_{34})^{\tilde{q}-2} \right)$$

- Numerical difficulties associated with low temperatures (many coding hours of our talented students & postdocs)

Chaos diagnostics in a single SYK

q = 4/6 of fermions in each interaction term

- Maximal chaos at low temperatures.
- Krylov – good approximation to the Lyapunov exponent.
- But not exactly equal (differences fall within numerical accuracy)

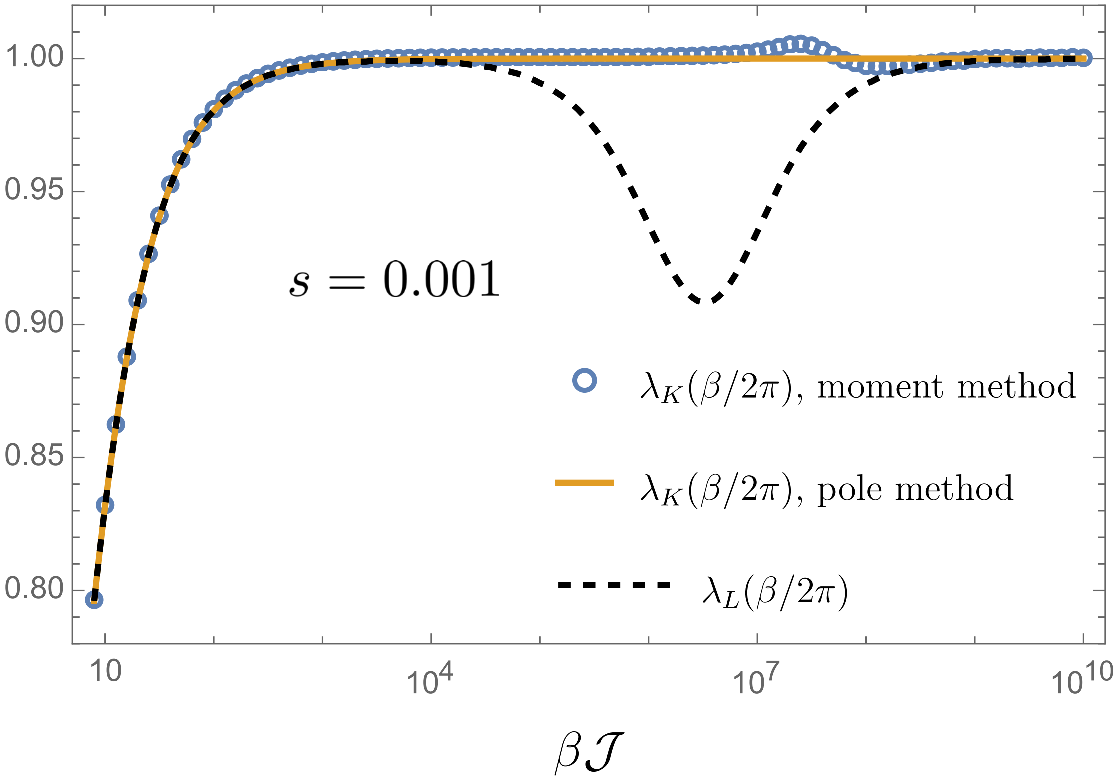


[**SC,** Demulder, Galante, Sheorey, Shoval (2024)]

Chaos to Chaos flows

$$H_{\text{def}} = H_q + sH_{q/2}, \qquad q \to \infty$$

- **Two regimes of nearly maximal chaos.**
- Chaos transition at $\beta\mathcal{J} \sim 1/s^2$
- Krylov complexity correctly (but poorly!) bounds (above) the Lyapunov exponent.
- Krylov does **not** diagnose chaos transitions!
- Analytic expansions near maximally chaotic regimes available.

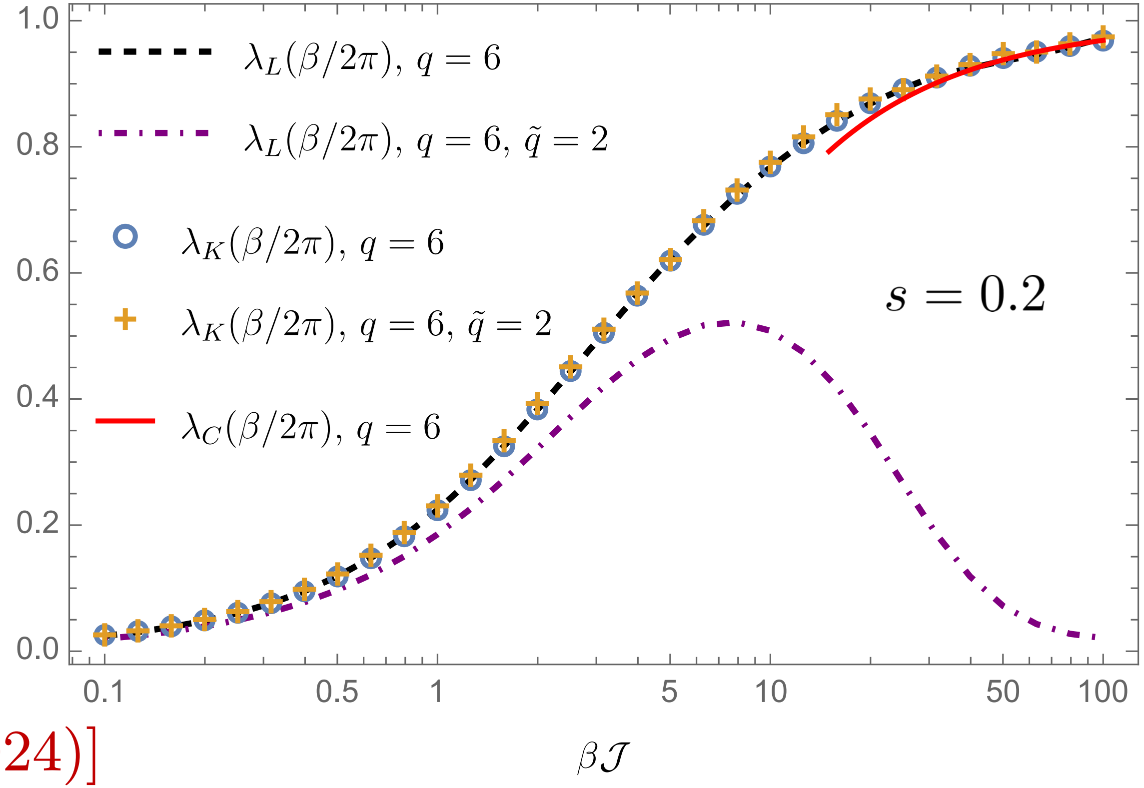


[**SC**, Demulder, Galante, Sheorey, Shoval (2024)]

Chaos to integrable flows

$$H_{\text{def}} = H_6 + sH_2$$

- Lyapunov exponent sees the transition
- Krylov seems unable to diagnose the chaotic properties of the flows

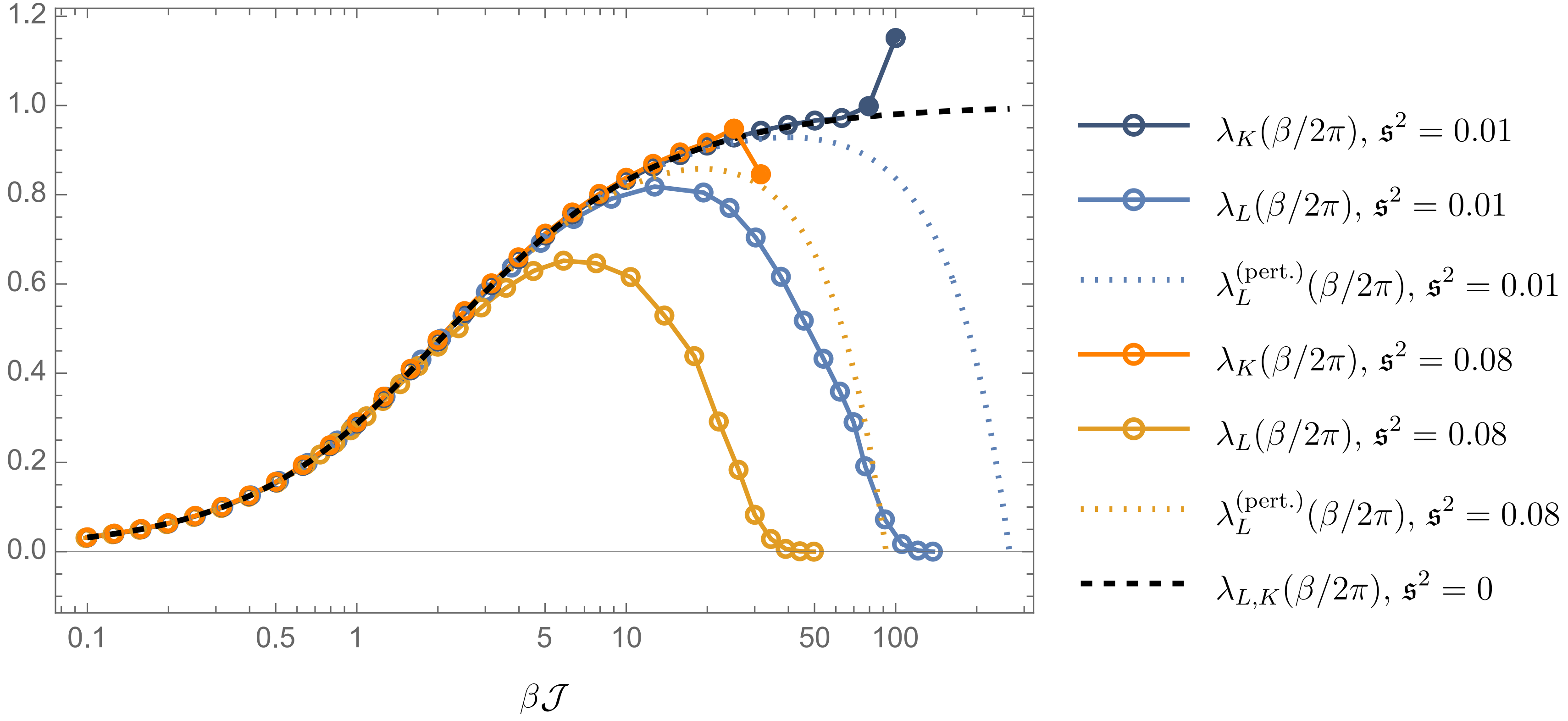


[**SC**, Demulder, Galante, Sheorey, Shoval (2024)]

Other studies of chaos-integrable transitions: [Berkooz, Brukner, Jia, Mamroud, 2024; Garcia-Garcia, Loureiro, Romero-Bermudez, Tezuka, 2018; Kim, Cao, 2021; Lunkin, Kitaev, Feigel'man, 2020; Nandy, Cadez, Dietz, Andreanov, Rosa 2022; Menzler, Jha 2024...]

Bonus plot: exclusive to Eurostrings 2024

Chaotic to integrable phase transitions?

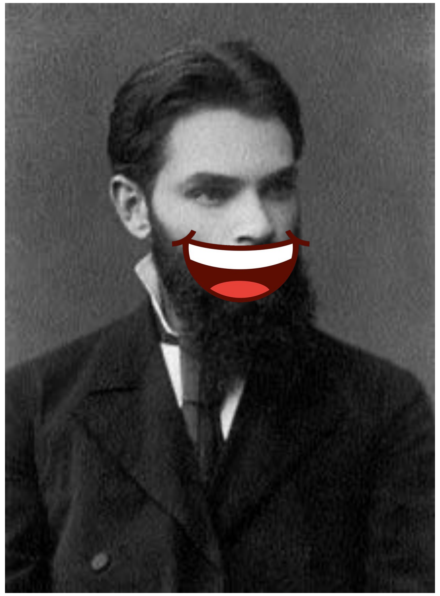


$$H_{\text{def}} = H_\infty + sH_2$$

Summary

Krylov bad!

Lyapunov good!

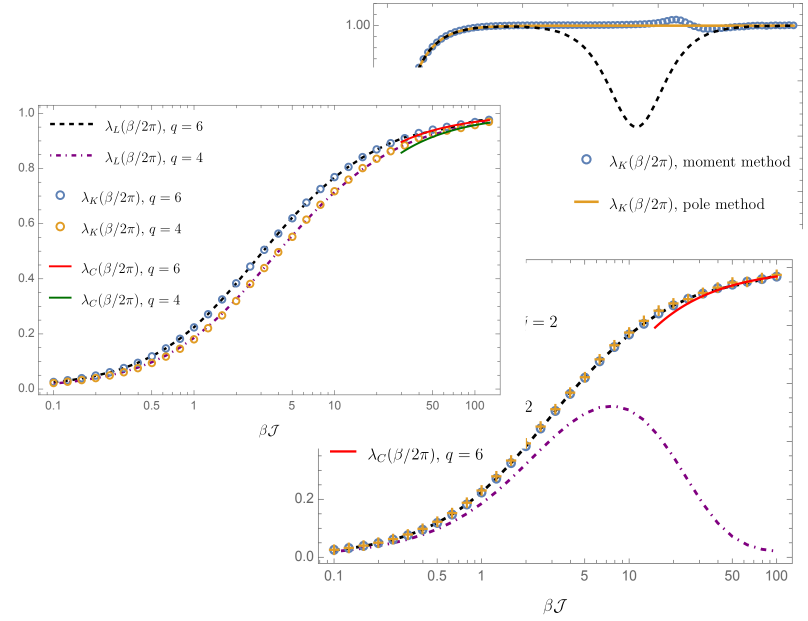


for diagnosing chaos along RG flows

*Note that the late time volume in flow geometries **also** does not seem to see the flow [Rabinovici, Sanchez-Garrido, Shir, Sonner 2023; **SC**, Galante, Kramer 2022]

Is the Krylov exponent always monotonic?
(when it can be defined & in Hermitian systems)

$$\beta\partial_\beta\left(\frac{\lambda_K\beta}{2\pi}\right)\geq 0$$



Back to de Sitter

- dS space has non-standard chaotic behavior
  [Anninos, Galante, Hofman 2019; **SC**, Galante, Kramer 2022, …]
- Which RG flow could lead to a dS interior?
- **Maybe dS is dual to an open quantum system**
  - Open boundary at future infinity
  - Perturbations make contact with a larger part of the system.
  - Thermodynamic behavior matches when $H_{\text{def}} = H_q + sH_{\tilde{q}} \Rightarrow H_q + isH_{\tilde{q}}$
    [Anninos, Galante, Sheorey (2022)]
- Non Hermitian Hamiltonian
- Signatures of chaos in open quantum systems
  [Sá, Ribeiro, Prosen 2021; Bhattacharya, Nandy, Nath and Sahu, 2022; Liu, Tang , Zhai 2023; Bhattacharjee, Cao, Nandy, Pathak 2023; Srivatsa, Keyserlingk 2024…]
- Interesting to explore connection to dS!


Thank you!
Any questions?