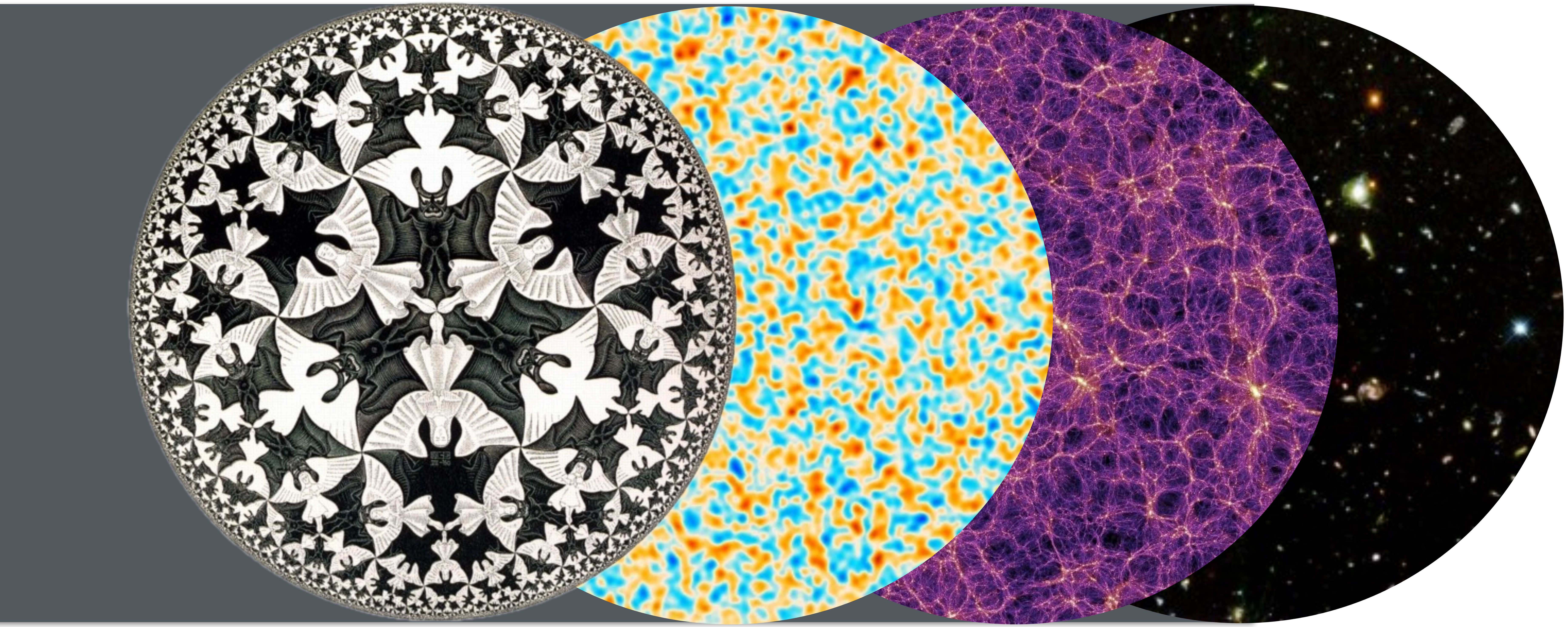
Cosmological correlators:

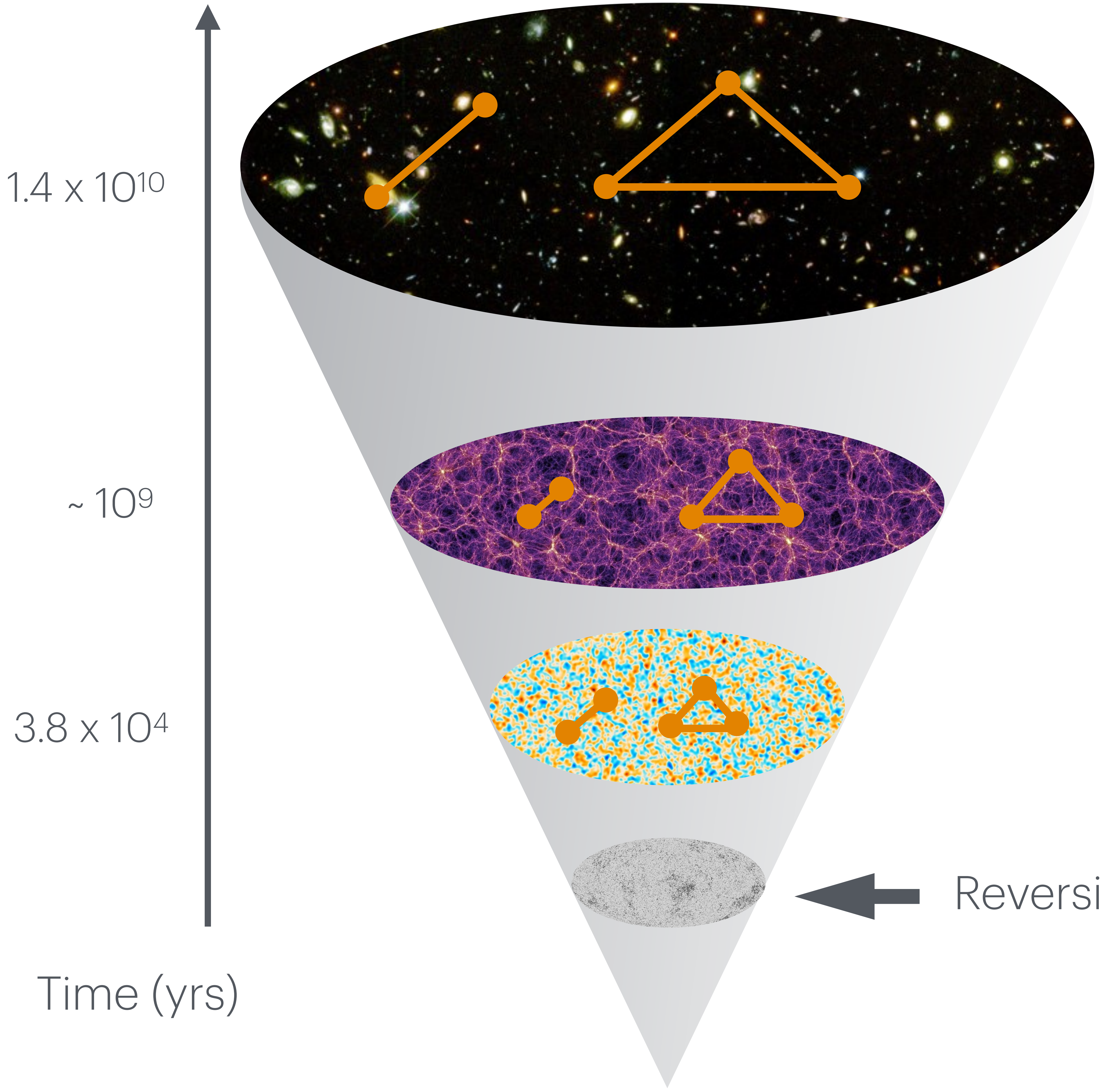
Symmetries, singularities, synergies.



Paul McFadden

Newcastle University

Primordial perturbations



All structure in the universe today grew from quantum vacuum fluctuations!

These *primordial perturbations* seeded the formation of:

Large-scale structure (LSS): *distribution of galaxies*

Cosmic microwave background (CMB): *fluctuations in temperature & polarisation*

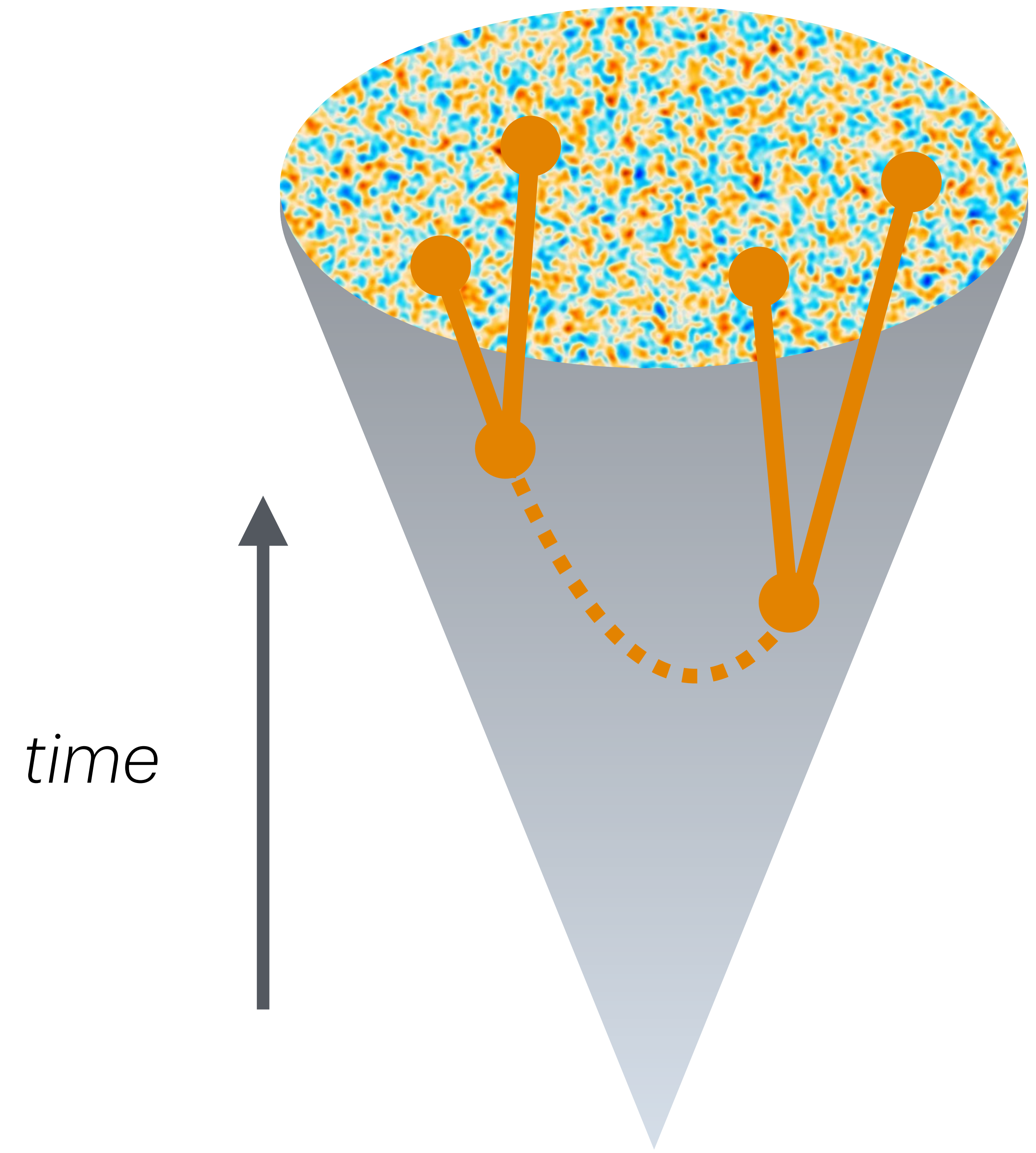
Reversing this, we can reconstruct the primordial perturbations.

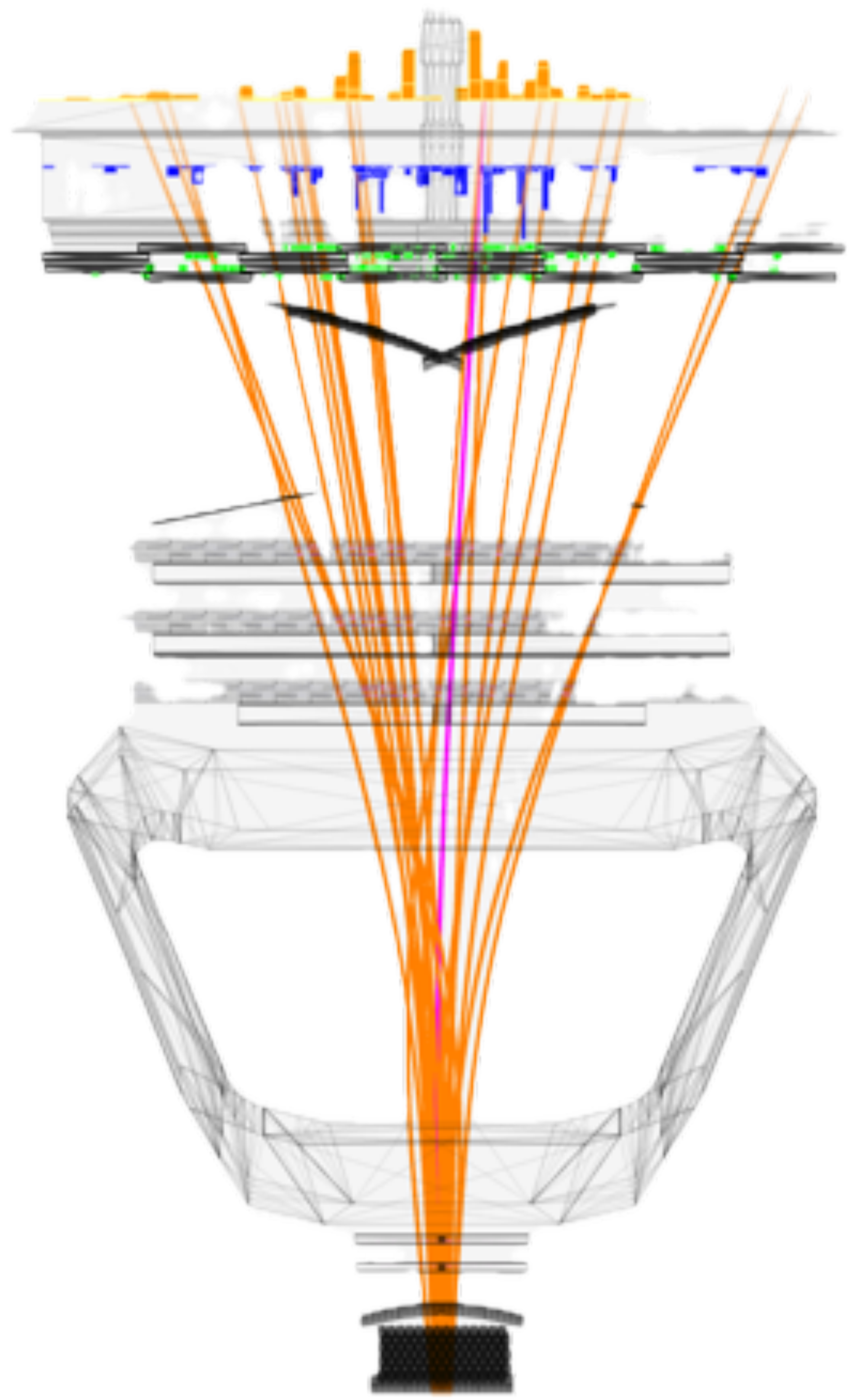
Correlations

Fluctuations at different locations, but the same time, are not independent, but *correlated*.

These correlations exist even on *super-horizon scales*, requiring an era prior to the hot big-bang.

They encode physics at the earliest of times & the highest accessible energies.

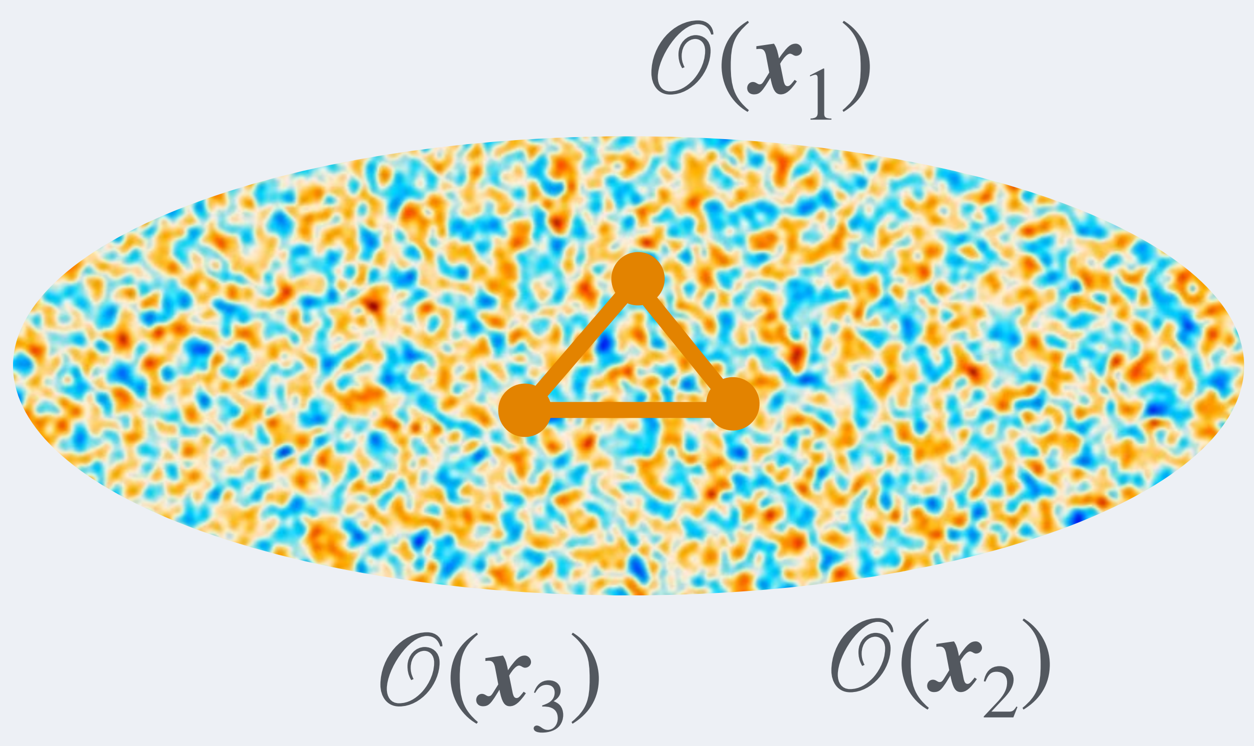




Up to 10^{14} GeV, *i.e.*, 7 *orders of magnitude* above a 100 TeV collider!

How can we decode these correlations to find the underlying symmetries, particles & interactions?

Cosmological correlators



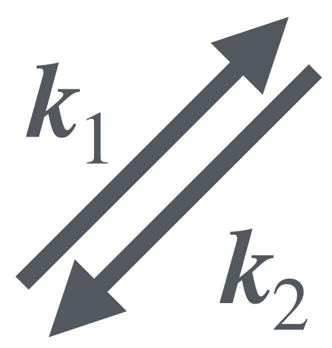
On a 3d constant-time slice: $\langle \mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\ldots\mathcal{O}(\boldsymbol{x}_n)\rangle$

$\mathcal{O}(\boldsymbol{x}_1)$ = fluctuation at $\boldsymbol{x}_1$,

e.g., curvature perturbation ζ, graviton γ_{ij}

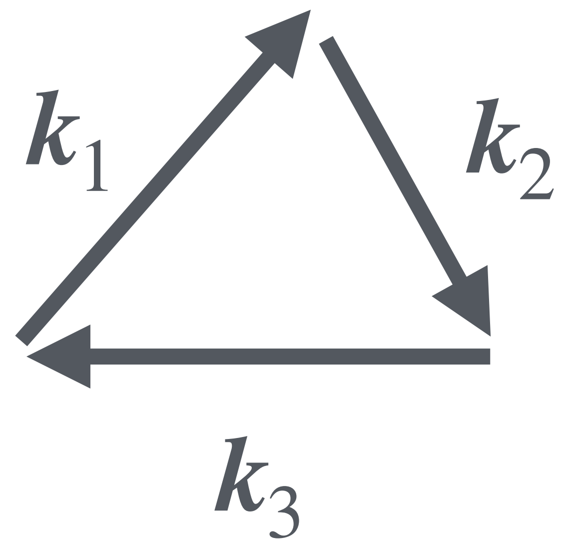
Background geometry of these spatial slices is flat $\Rightarrow$ convenient to work in ***momentum space:***

$$\langle \mathcal{O}(\boldsymbol{k}_1)\mathcal{O}(\boldsymbol{k}_2)\ldots\mathcal{O}(\boldsymbol{k}_n)\rangle = \langle\langle \mathcal{O}(\boldsymbol{k}_1)\mathcal{O}(\boldsymbol{k}_2)\ldots\mathcal{O}(\boldsymbol{k}_n)\rangle\rangle\,(2\pi)^3\delta(\boldsymbol{k}_T)$$



Power spectra (2-pt fns):

background evolution



Non-Gaussianities ($\geq$ 3-pt fns):

interactions

What we know so far...

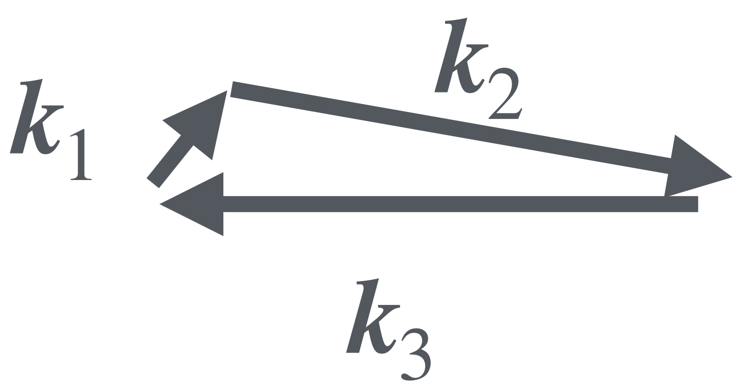
- From Planck & other experiments: $$\Delta_S(k) = \frac{k^3}{2\pi}\langle\langle \zeta(k)\zeta(-k)\rangle\rangle = A_s \left(\frac{k}{k_0}\right)^{n_s-1}$$

$$A_s \approx 2 \times 10^{-9}, \qquad n_s \approx 0.96$$

Small amplitude, nearly scale-invariant

Nearly Gaussian: Upper bounds for amplitudes f_{NL} of various phenomenological templates for $\langle\zeta\zeta\zeta\rangle$ which peak for different momentum configurations ('shapes').

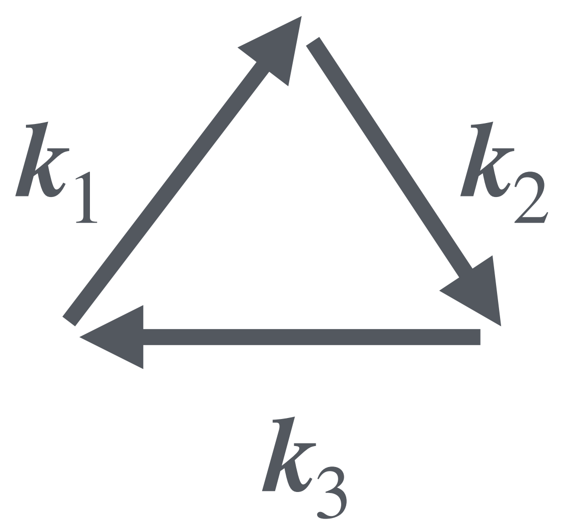
e.g.,



$\sigma(f_{NL}^{loc}) \sim 5$

'local'-type: probes single vs multi-field inflation
via *single-field consistency relation* $f_{NL}^{loc} \sim n_s - 1$

[Maldacena '03; Creminelli & Zaldarriaga '04]



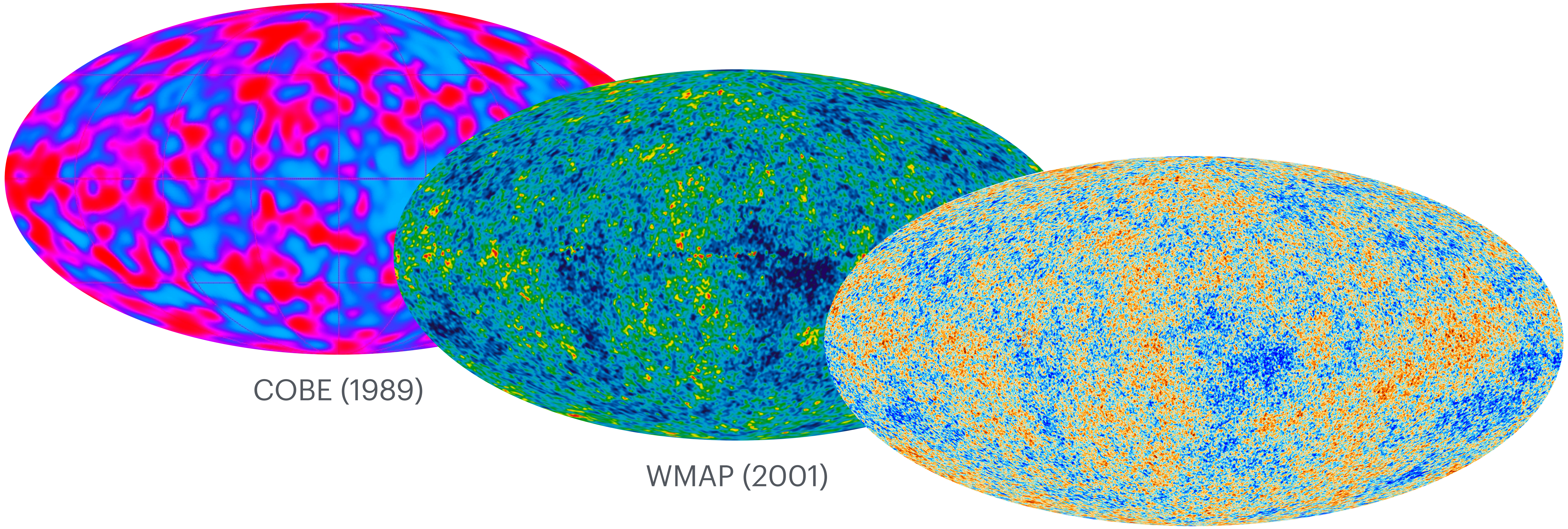
$\sigma(f_{NL}^{eq}) \sim 50$

'equilateral'-type

The decade ahead…

- Cosmic microwave background:

Temperature fluctuations well-characterised, but so far only ~10% of available information in polarisation extracted.



COBE (1989)

WMAP (2001)

Planck (2009)

ACTPol, SPTPol (on-going)

Upcoming:

Simons Observatory (2024-8)

LiteBIRD (launch ~2032)

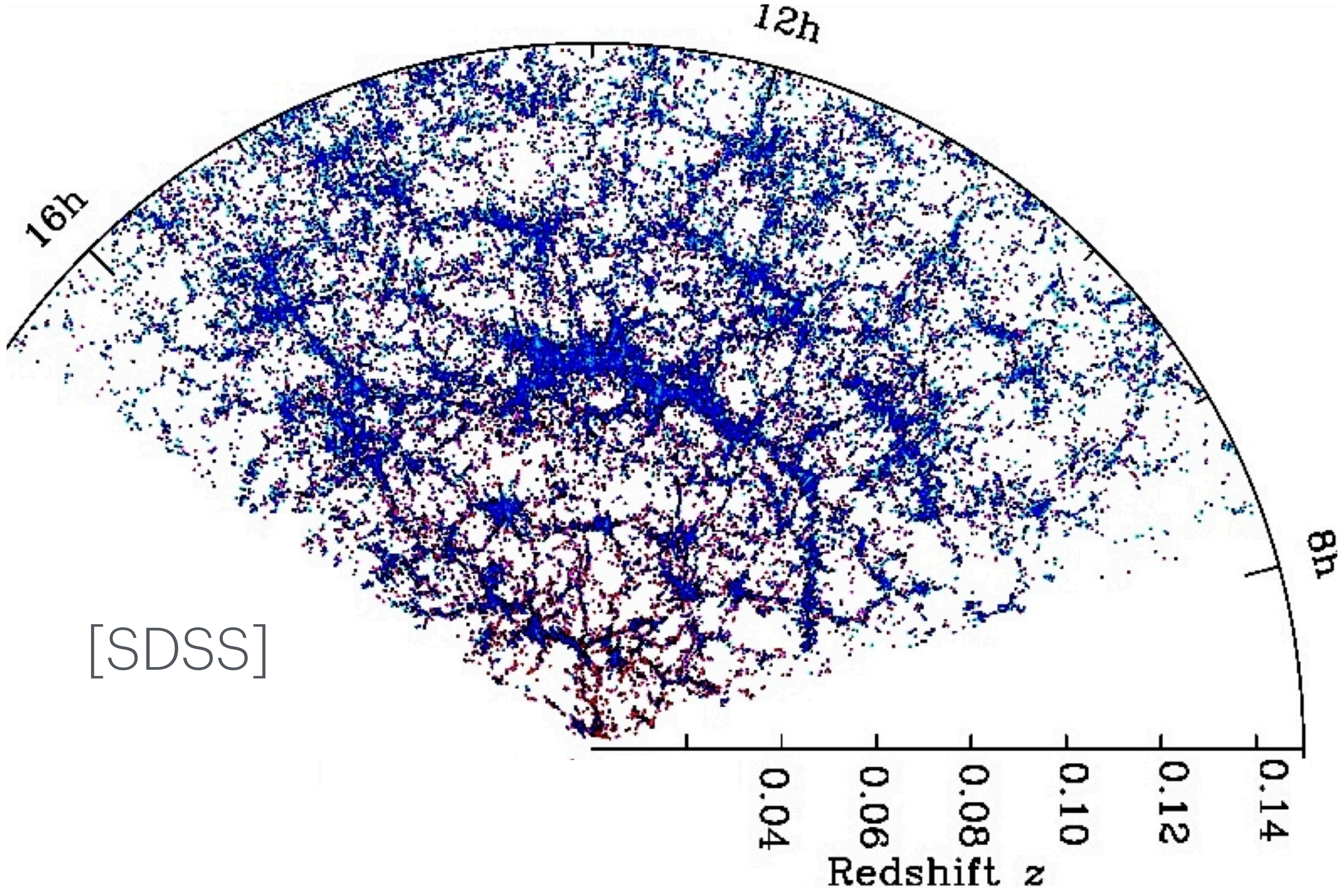
Measuring B-mode polarisation has potential to detect $\langle \gamma\gamma \rangle$, *i.e.*, primordial gravitational waves.

Key signature of inflation. Bound on tensor-to-scalar ratio expected to reduce from $r < 0.06$ to 10^{-3}.

The decade ahead…

- Large-scale structure:

Many new experiments measuring distribution of ~ 1 billion galaxies up to redshifts $z < 5$, and ~ 100 million spectra:



[SDSS]

DESI, Euclid (both operational)

Rubin (LSST) (2025-'27), SPHEREx (launch 2025)

Access to many new Fourier modes since measuring structure over significant fraction of past light cone.

Have to extract primordial non-Gaussianity from that generated by non-linear evolution.

SPHEREx has potential to *rule out* all single-field inflationary models by constraining $\sigma(f_{NL}^{loc}) < 1$.

Outline

- Theoretical background: wavefunction of the universe, Schwinger-Keldysh formalism
- Key themes: special kinematic limits & singularities
  role of symmetry
- Developments: the cosmological collider,
  bootstrapping the collider,
  from dS to AdS, IR divergences,
  kinematic flow
- Other directions; outlook

Inflationary paradigm

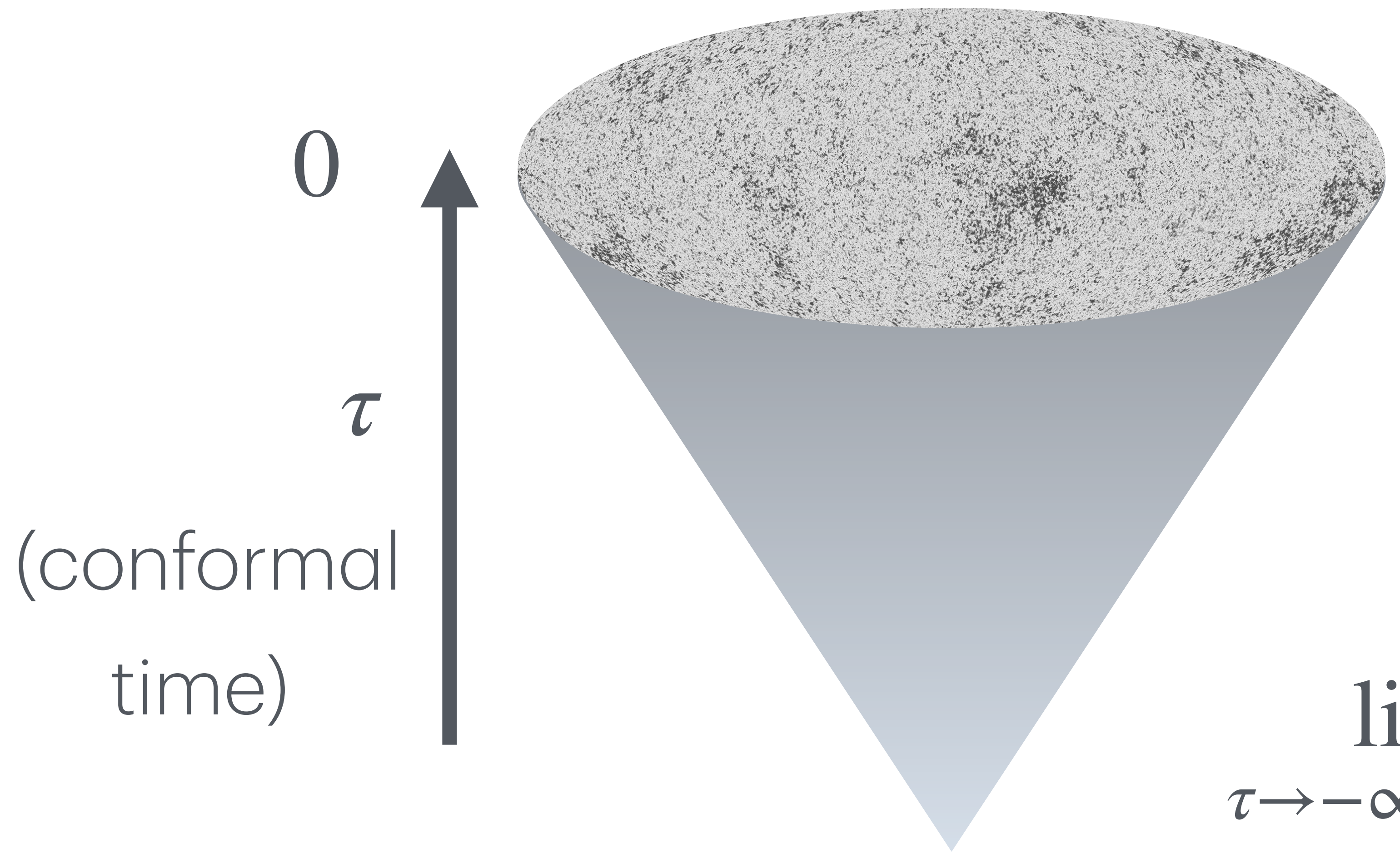
Cosmological correlators can be calculated using either of two (equivalent) formalisms:

- Wavefunction of the universe:

$$\Psi[\varphi_{(0)}] = \langle \varphi_{(0)}(\boldsymbol{x}) \,|\, 0 \rangle = \int_{\text{b.c.}} \mathscr{D}\varphi \, e^{iS}$$

(similarly for g_{ij})

Boundary conditions:



$$\lim_{\tau \to 0^-} (-\tau)^{\Delta - d} \varphi(\tau, \boldsymbol{x}) = \varphi_{(0)}(\boldsymbol{x})$$

$$\lim_{\tau \to -\infty(1-i\varepsilon)} \varphi(\tau, \boldsymbol{x}) = 0$$

Bunch-Davies vacuum

Correlators of late-time fields $\varphi_{(0)}(\boldsymbol{x})$ obtained by a further functional integral:

$$\langle \varphi_{(0)}(\boldsymbol{x}_1) \dots \varphi_{(0)}(\boldsymbol{x}_n) \rangle = \frac{\int \mathscr{D}\varphi_{(0)} \, \varphi_{(0)}(\boldsymbol{x}_1) \dots \varphi_{(0)}(\boldsymbol{x}_n) \, |\Psi[\varphi_{(0)}]|^2}{\int \mathscr{D}\varphi_{(0)} \, |\Psi[\varphi_{(0)}]|^2}$$

Wavefunction formalism

Expanding perturbatively in late-time fields defines the *wavefunction coefficients* ψ_n:

$$\ln \Psi[\varphi_{(0)}] = \sum_{n=2}^{\infty} \frac{(-1)^n}{n!} \int \prod_{i=1}^{n} \frac{\mathrm{d}^d \boldsymbol{k}_i}{(2\pi)^d} (2\pi)^d \delta(\boldsymbol{k}_1 + \ldots + \boldsymbol{k}_n)\, \psi_n(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n)\, \varphi_{(0)}(-\boldsymbol{k}_1) \ldots \varphi_{(0)}(-\boldsymbol{k}_n)$$

Performing the functional integral then relates the ψ_n to the late-time correlators:

e.g., at tree-level,

$$\langle\!\langle \varphi_{(0)}(\boldsymbol{k}) \varphi_{(0)}(-\boldsymbol{k}) \rangle\!\rangle = -\frac{1}{2} \frac{1}{\mathrm{Re}\, \psi_2(k)}, \qquad \langle\!\langle \varphi_{(0)}(\mathbf{k}_1) \varphi_{(0)}(\mathbf{k}_2) \varphi_{(0)}(\mathbf{k}_3) \rangle\!\rangle = \frac{1}{4} \frac{\mathrm{Re}\, \psi_3(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)}{\prod_{i=1}^{3} \mathrm{Re}\, \psi_2(k_i)}$$

etc.

Wavefunction formalism

Wavefunction coefficients can be computed diagrammatically:

e.g., for scalar of mass $m^2 = \Delta(d-\Delta)$ on de Sitter background $\mathrm{d}s^2 = \tau^{-2}(-\mathrm{d}\tau^2 + \mathrm{d}\boldsymbol{x}^2)$

- Bulk-to-boundary propagator: $K(k,\tau) = -\,i\pi \dfrac{(-\tau)^{d/2}k^{\beta}}{2^{\beta}\Gamma(\beta)} H_{\beta}^{(2)}(-k\tau)$ $\qquad \beta = \Delta - d/2$

- Bulk-to-bulk propagator:

$$G(k;\tau,\tau') = \frac{i\pi}{4}(-\tau)^{d/2}(-\tau')^{d/2}\Big[H_{\beta}^{(2)}(-k\tau)\Big(H_{\beta}^{(1)}(-k\tau') + H_{\beta}^{(2)}(-k\tau')\Big)\theta(\tau'-\tau) + (\tau \leftrightarrow \tau')\Big]$$

- Vertices correspond to time integrals, $\quad$ e.g., $\lambda_n \varphi^n \quad \Rightarrow \quad -i\lambda_n \displaystyle\int_{-\infty(1-i\varepsilon)}^{0} \frac{\mathrm{d}\tau}{(-\tau)^{d+1}}$

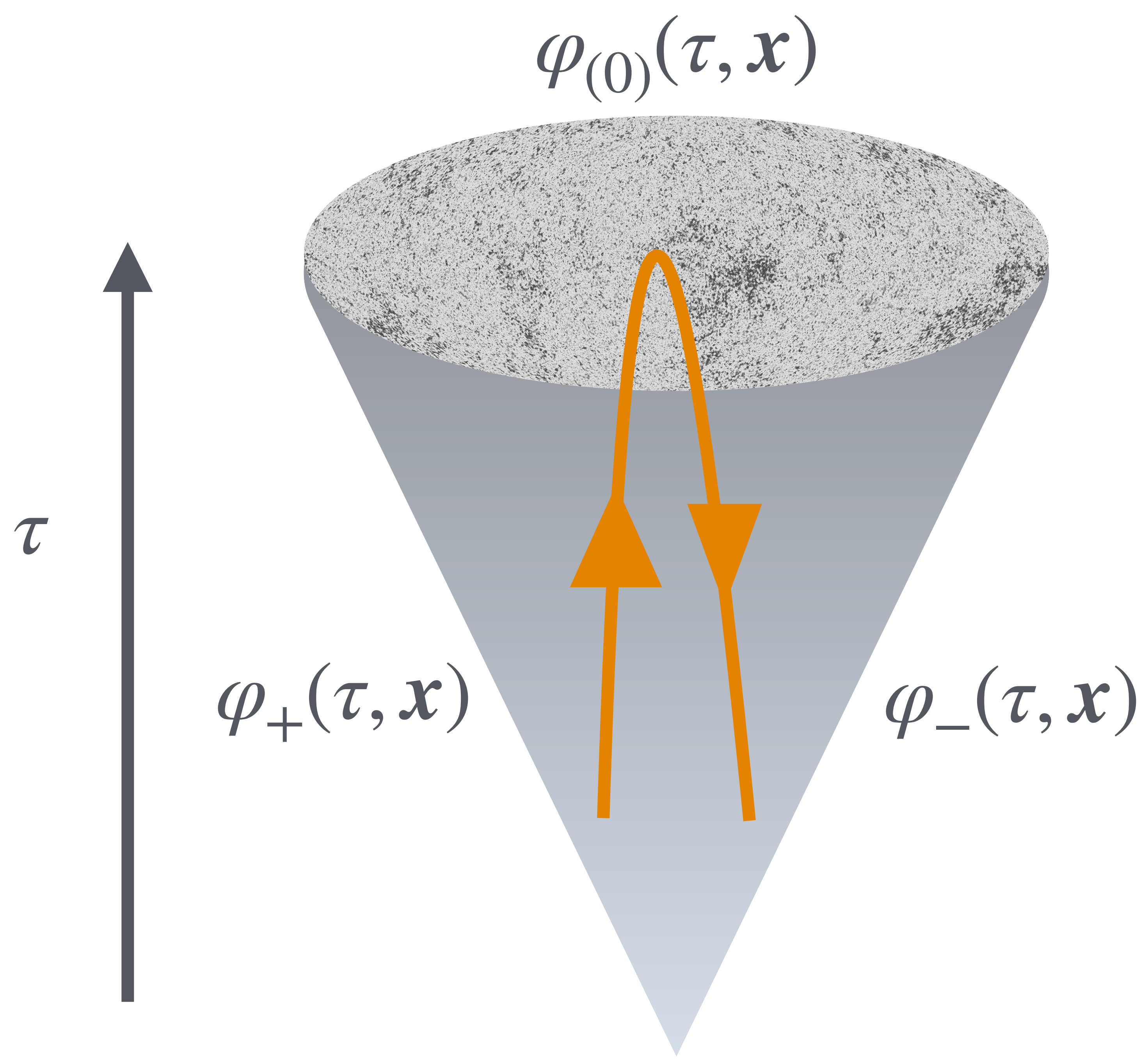
- Momentum conservation is enforced at vertices

In-in formalism

The *in-in*, or *Schwinger-Keldysh* formalism: condenses the wavefunction calculation into a single *closed-time* path integral:

$$\langle \varphi_{(0)}(\boldsymbol{x}_1)\ldots\varphi_{(0)}(\boldsymbol{x}_n)\rangle = \int \mathscr{D}\varphi_+(\tau,\boldsymbol{x})\mathscr{D}\varphi_-(\tau,\boldsymbol{x})\left(\prod_{i=1}^{n}\varphi_{(0)}(\boldsymbol{x}_i)\right)\exp\Big(iS_+[\varphi_+]-iS_-[\varphi_-]\Big)$$

where we rotate $\tau \to \tau(1 \mp i\varepsilon)$ for $S_\pm[\varphi_\pm]$.



$\varphi_+(\tau,\boldsymbol{x})$ lives on the *forward* part of the contour (computing Ψ)

and $\varphi_-(\tau,\boldsymbol{x})$ lives on the *backwards* part (computing Ψ^*).

Both fields are constrained to match at late times:

$$\lim_{\tau\to 0^-}(-\tau)^{\Delta-d}\varphi_\pm(\tau,\boldsymbol{x}) = \varphi_{(0)}(\boldsymbol{x})$$

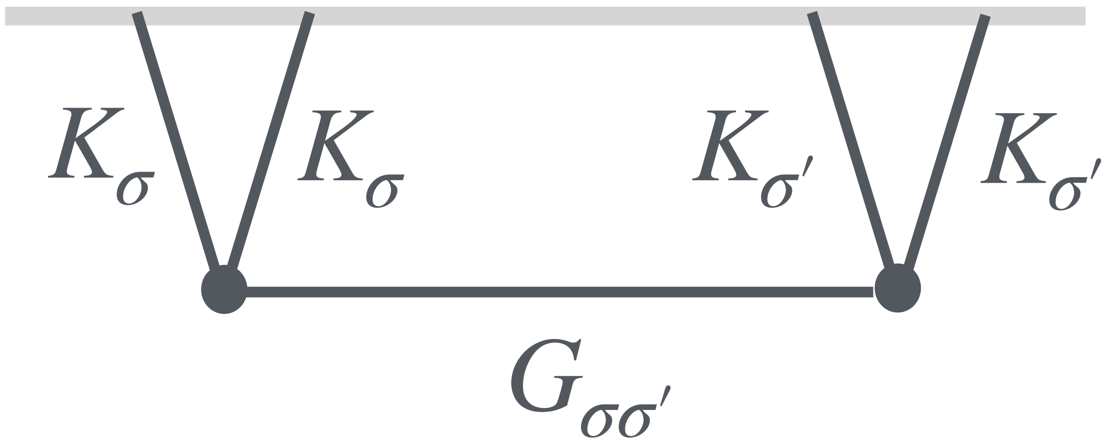
In-in formalism

This leads to a diagrammatic formalism analogous to that for the wavefunction, except that now we have:

Review: [Chen, Wang, Xianyu '17]

- *Two* types of vertices ($\pm$) according to location on forwards/backwards contour.
- *Two* bulk-boundary propagators $K_{\pm}(k, \tau)$
- *Four* bulk-bulk propagators $G_{\sigma,\sigma'}(k; \tau, \tau')$ where $\{\sigma, \sigma'\} = \{++, +-, -+, --\}$

We sum over vertex types: e.g., for exchange diagram $\sum_{\{\sigma,\sigma'\}}$



While more complicated, this has the advantage of computing the correlators directly.

Kinematic limits

An important role is played by a number of special kinematic limits:

1. Collinear limits:

Correlators computed using the adiabatic (Bunch-Davies) vacuum are *non-singular* as momenta become collinear.

In contrast, excited initial states exhibit collinear singularities: recent work includes [Ansari, Banerjee, Dhivakar, Jain, Kundu '24] [Ghosh, Pajer, Ullah '24] [Chopping, Sleight, Taronna '24]

2. Soft limits:

n-point correlators in *slow-roll inflation* can be obtained by taking the soft-limit $k_{n+1} \to 0$ of an external leg in an (n+1)-point *de Sitter* correlator.

[Maldacena '03] [Creminelli '04] [Bzowski, PM, Skenderis '12] [Kundu, Shukla, Trivedi '14, '15] [Arkani-Hamed, Maldacena '15] [Baumann, Duaso Pueyo, Joyce, Lee, Pimentel '19]

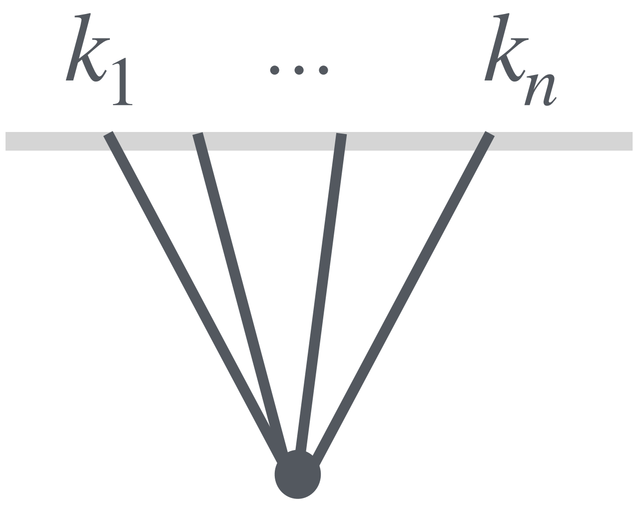
Kinematic limits

3. Flat-space limit:

Wavefunction coefficients *contain* flat-space scattering amplitudes, which appear as the residue of (unphysical) singularities arising when certain sums of 'energies' $k_i = |\boldsymbol{k}_i|$ vanish:

[Maldacena, Pimentel '11] [Raju '12]

- *Total energy* singularity:



$$E = \sum_{i=1}^{n} k_i$$

$$\lim_{E\to 0} \psi_n \sim \frac{A_n}{E^p}$$

A_n amplitude in flat *bulk* spacetime:

function of null momenta $k_i^\mu = (k_i, \boldsymbol{k}_i)$

$$\sum_i k_i^\mu = (E, \boldsymbol{0})$$

Result of *early-time* behaviour:

as $\tau \to -\infty$, propagators $\sim e^{ik_i\tau}$

$$\psi_n \sim A_n \int_{-\infty(1-i\varepsilon)}^{0} \mathrm{d}\tau\, \tau^{p-1} e^{iE\tau}.$$

Unsuppressed when $E \to 0$.

Kinematic limits

Note these singularities can't be reached for *physical* momentum configurations

for which all 'energies' $k_i \geq 0$. Accessible only via analytic continuation.

- *Partial energy* singularities:

Singularities also arise when the 'energy' of an individual vertex vanishes.

See, *e.g.*, [Benincasa '18] [Baumann, Chen, Pueyo, Joyce, Lee, Pimentel '21]

Easiest to see from the *cutting rule* for bulk-bulk propagator of wavefunction:

$$\mathrm{Disc}\, G(k; \tau, \tau') = 2P(k)\, \mathrm{Disc}\, K(k, \tau)\, \mathrm{Disc}\, K(k, \tau')$$

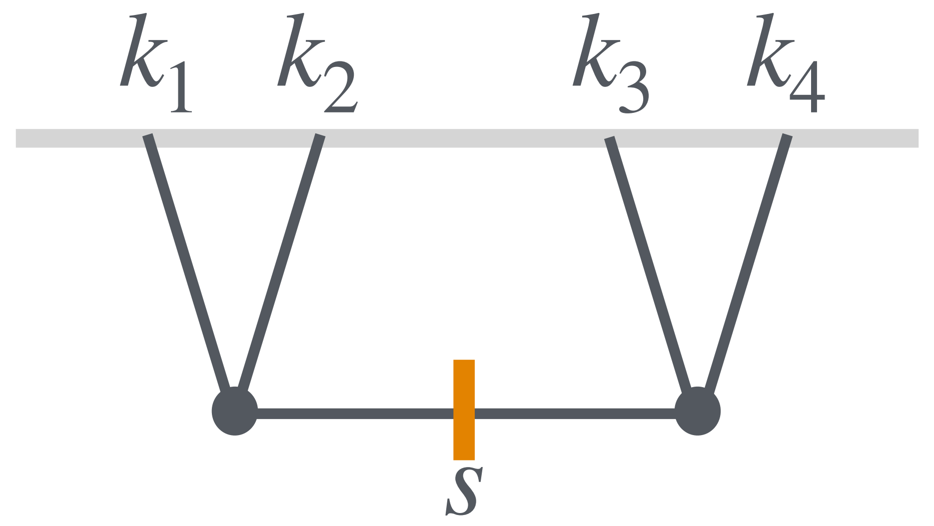
[Meltzer '21]

[Goodhew, Jazayeri, Pajer '20]

where $P(k) = \dfrac{4^{\beta-1}\Gamma^2(\beta)}{\pi\, k^{2\beta}}$ is the power spectrum and $\mathrm{Disc}\, K(k, \tau) = \dfrac{1}{2i}\Big(K(k, \tau) - K(e^{-i\pi}k, \tau)\Big)$

Kinematic limits

For, e.g., an s-channel exchange,



$$\mathrm{Disc}_s\, \psi_4(k_1, k_2, k_3, k_4, s) = 2P(s)\, \mathrm{Disc}_s\, \psi_3(k_1, k_2, s)\, \mathrm{Disc}_s\, \psi_3(k_3, k_4, s)$$

In the limit $E_L = k_1 + k_2 + s \to 0$,

$$\lim_{E_L \to 0} \mathrm{Disc}_s\, \psi_3(k_1, k_2, s) = \lim_{E_L \to 0} \frac{1}{2i}\Big(\psi_3(k_1, k_2, s) - \psi_3(k_1, k_2, -s)\Big) \sim \frac{A_L(k_1, k_2, s)}{E_L^p}$$

gives 3-point total energy singularity

regular since no collinear singularities

(similar argument for $\mathrm{Disc}_s\, \psi_4$)

Partial energy singularity:

$$\lim_{E_L \to 0} \psi_4 \sim \frac{A_L(k_1, k_2, s)}{E_L^p} P(s)\, \mathrm{Disc}_s\, \psi_3(k_3, k_4, s)$$

Cutting rules

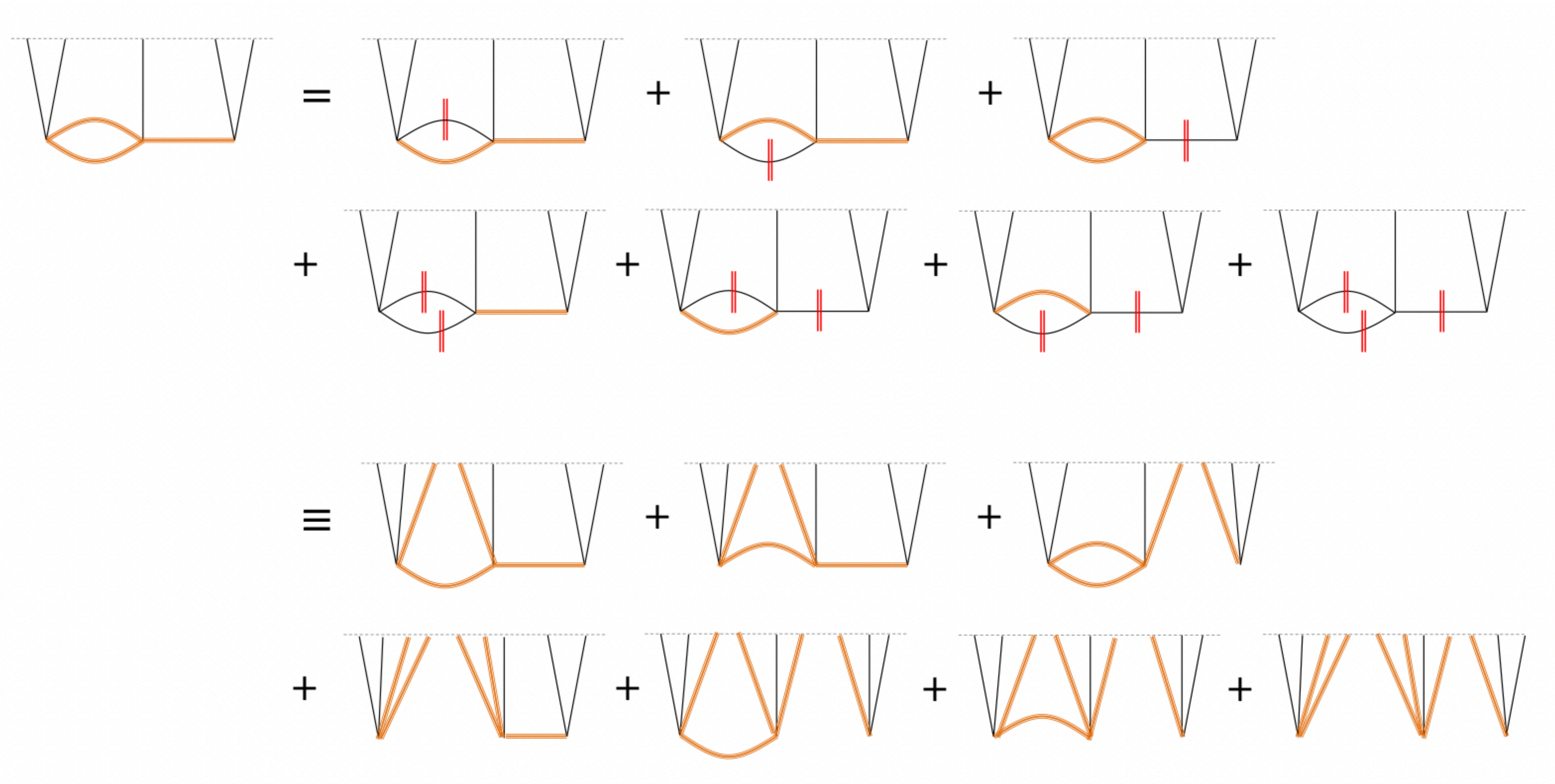
The cutting rule also holds replacing $\mathbf{Disc} \to \mathbf{Im}$ where it encodes constraints of perturbative unitarity:

$$\psi_4(k_i, s) + \psi_4^*(-k_i, s) = P(s)(\psi_3(k_1, k_2, s) + \psi_3^*(-k_1, -k_2, s))(\psi_3(k_3, k_4, s) + \psi_3^*(-k_3, -k_4, s))$$

The cosmological optical theorem [Goodhew, Jazayeri, Pajer '20]

For further applications, see

[Melville, Pajer '21]

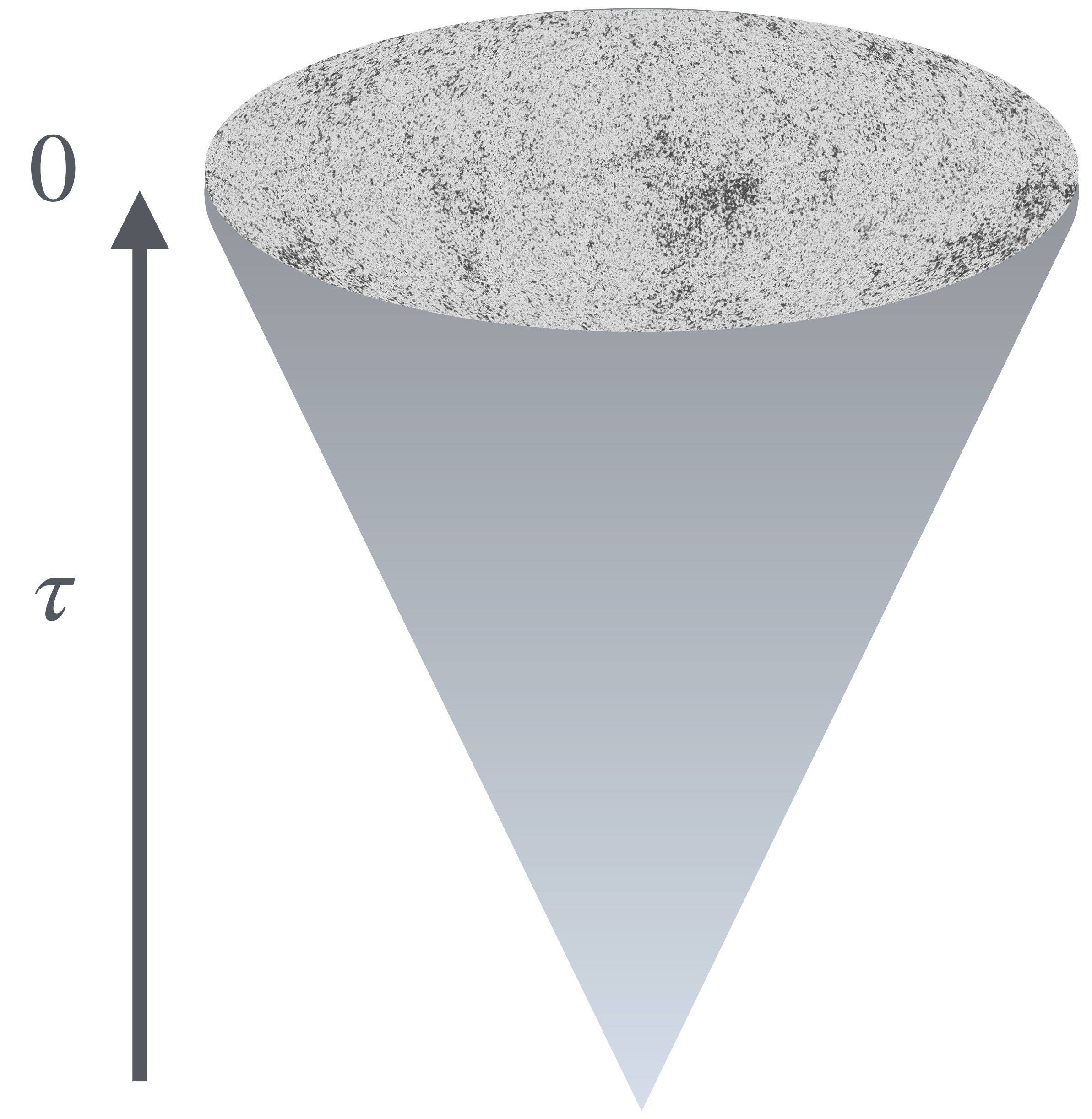


Also: Boostless bootstrap -- often possible to bootstrap correlators using the cosmological optical theorem & other constraints (*e.g.*, 'manifestly local test'), without assuming de Sitter boost symmetry.

see, *e.g.*, [Jazayeri, Pajer, Stefanyszyn '21]

Symmetry constraints

Further constraints on cosmological correlators arise from the action of symmetries:



Famously, bulk de Sitter isometries act on the boundary as conformal tranformations:

[Strominger '01] [Maldacena '03]

$$\xi^\mu_D \partial_\mu = \tau\partial_\tau + \boldsymbol{x}\cdot\partial$$

$$\xi^\mu_{SCT}\partial_\mu = -2(\mathbf{b}\cdot\boldsymbol{x})\tau\partial_\tau + [(-\tau^2 + x^2)\,\mathbf{b} - 2(\mathbf{b}\cdot\boldsymbol{x})\,\boldsymbol{x}]\cdot\partial$$

For fall-off $\varphi(\tau,\boldsymbol{x}) = (-\tau)^{d-\Delta}\varphi_{(0)}(\boldsymbol{x}) + \ldots + (-\tau)^{\Delta}\varphi_{(\Delta)}(\boldsymbol{x}) + \ldots$ as $\tau\to 0$,

the bulk dS Ward identities $0 = \sum_{i=1}^{n}\xi^\mu(\boldsymbol{x}_i)\frac{\partial}{\partial x_i^\mu}\langle\varphi(\tau,\boldsymbol{x}_1)\ldots\varphi(\tau,\boldsymbol{x}_n)\rangle$ reduce to

boundary conformal Ward identities for fields $\varphi_{(0)}(\boldsymbol{x})$ of the *shadow* dimension $\bar{\Delta} = d - \Delta$.

Symmetry constraints

Thus, de Sitter correlators obey the same *kinematic constraints* as CFT correlators.

2- and 3-point functions are fixed up to constants

→ for cosmology, want CFT correlators in *momentum space.*

[Antoniadis, Mazur, Mottola '11]
[Coriano, delle Rose, Mottola, Serino '13]
[Bzowski, PM, Skenderis '13] ...

In fact, *slow-roll* 2- and 3-point correlators can also be found via deformations of CFT:

$\langle\gamma\gamma\gamma\rangle$ (insensitive to deviation from dS) [Maldacena, Pimentel '11]

$\langle\zeta\zeta\zeta\rangle, \langle\zeta\zeta\gamma\rangle, \langle\zeta\gamma\gamma\rangle$ slow-roll η corrections [Bzowski, PM, Skenderis '12]

$\langle\zeta\zeta\gamma\rangle$ [Mata, Raju, Trivedi '12]

$\langle\zeta\zeta\rangle$ order η^2 corrections [PM '13]

$\langle\zeta\zeta\zeta\rangle$ slow-roll ϵ corrections [Ghosh, Kundu, Raju, Trivedi '14]

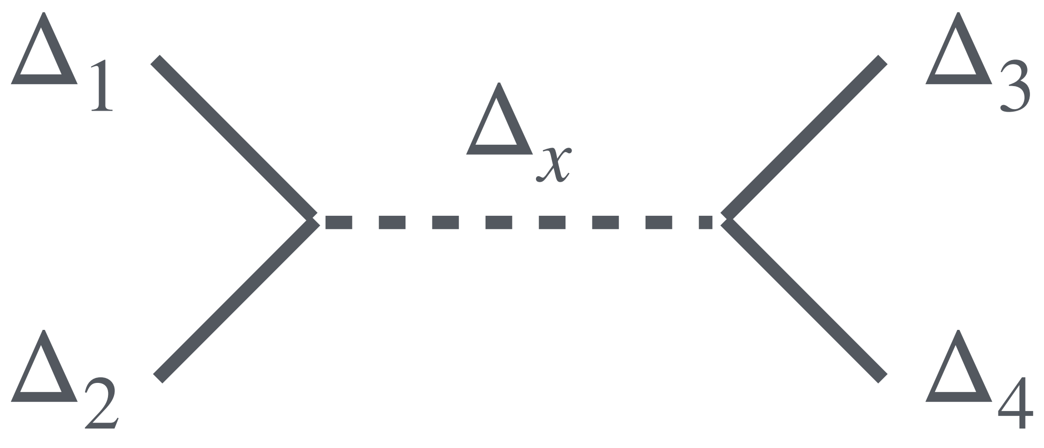
For recent work, see:

[Baumann, Duaso Pueyo, Joyce, Lee, Pimentel '19, '20]

Cosmological colliders

At 4-points and higher, can anticipate signatures of exchanged particles using the OPE:

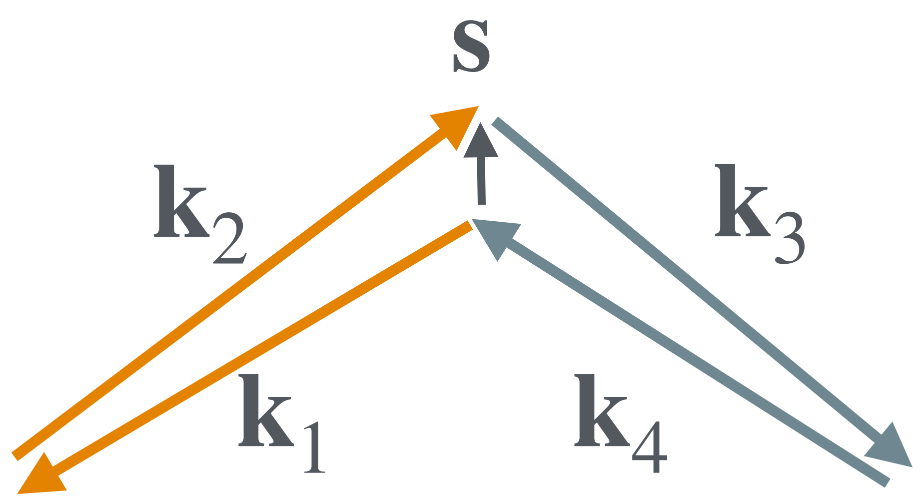
[Arkani-Hamed, Maldacena '15]



$$\mathcal{O}_{\Delta_i}(x_i)\mathcal{O}_{\Delta_j}(x_j) \sim \frac{C_{ijx}}{x_{ij}^{\Delta_i+\Delta_j-\Delta_x}}\mathcal{O}_{\Delta_x}(x_i)$$

as $x_{12}^2 \to 0,\ x_{34}^2 \to 0,$ 4-pt fn $\sim x_{12}^{\Delta_x-\Delta_1-\Delta_2} x_{13}^{-2\Delta_x} x_{34}^{\Delta_x-\Delta_3-\Delta_4}$

In momentum space, this is $s \to 0$ with $k_1 \approx k_2$ and $k_3 \approx k_4$ giving $\sim k_1^{\Delta_1+\Delta_2-\Delta_x-d} k_3^{\Delta_3+\Delta_4-\Delta_x-d} s^{2\Delta_x-d}$



OPE limit

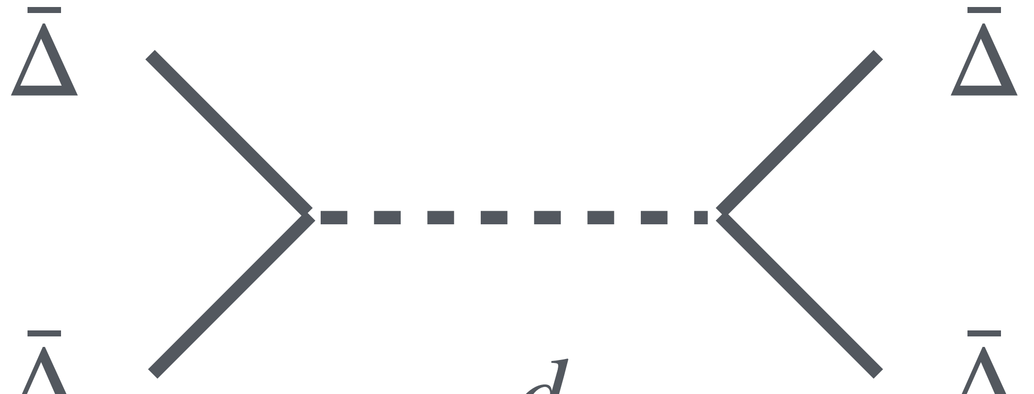
For, *e.g.*, a *heavy* scalar in dS we have $\Delta_\pm = \frac{d}{2} \pm i\mu,$

where $\mu = \sqrt{\frac{m^2}{H^2} - \frac{d^2}{4}}$

Cosmological colliders

[Arkani-Hamed, Maldacena '15]

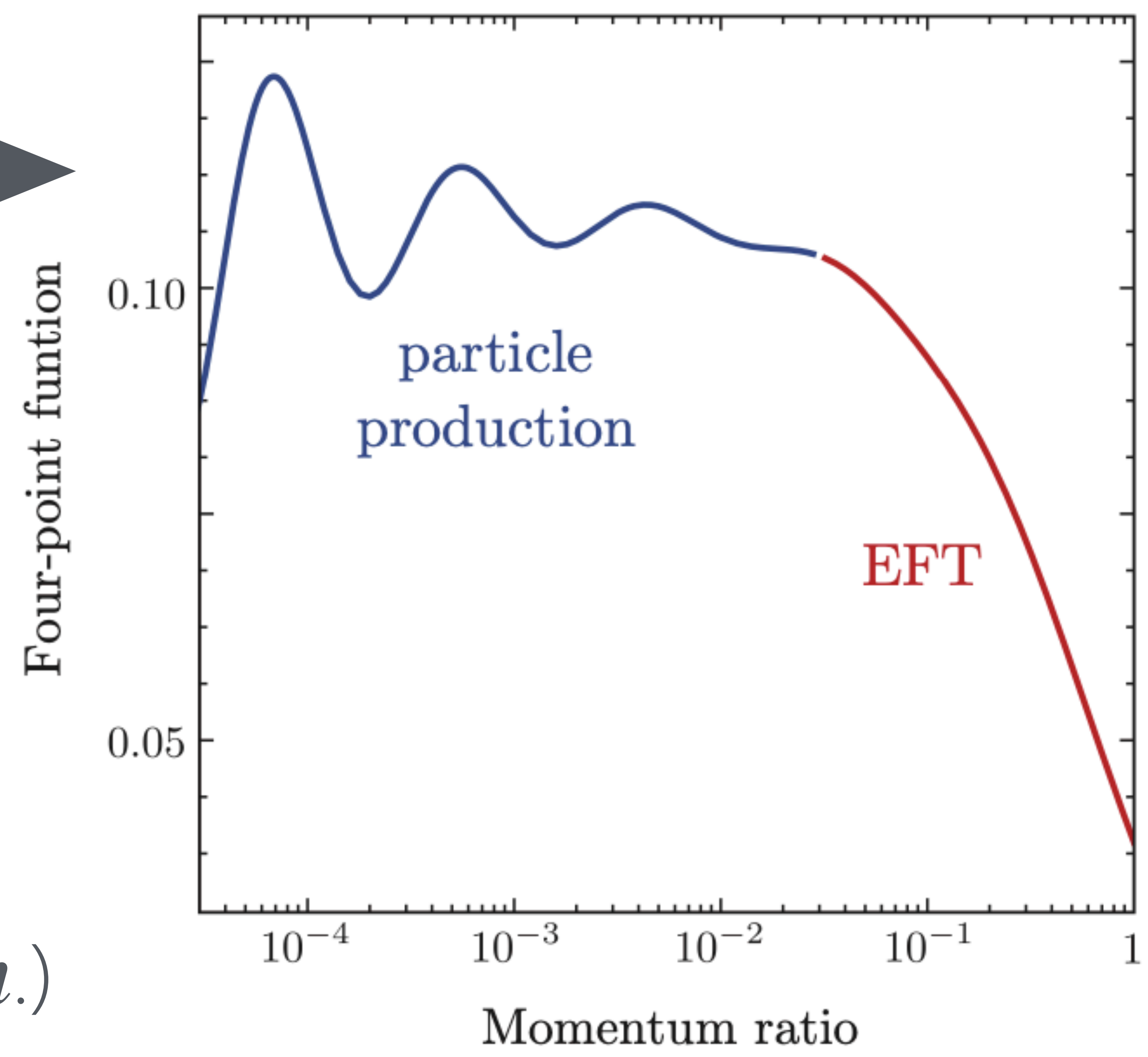
Using the shadow dimension $\bar{\Delta}$ for all external fields and both $\Delta_\pm$ for the exchanged field:



$$\Delta_\pm = \frac{d}{2} \pm i\mu$$

$$\lim_{s\to 0} \text{ dS 4-pt fn } \sim (k_1 k_3)^{2\bar{\Delta}-\frac{3d}{2}} \left[a_{\bar{\Delta}}(\mu) \left(\frac{s^2}{k_1 k_3} \right)^{i\mu} + \text{c.c.} \right]$$

Oscillating signal: $e^{i\mu \ln\left(\frac{s^2}{k_1 k_3}\right)}$



Additional angular dependence for spinning exchanges.

Identifying coefficient $a_{\bar{\Delta}}(\mu)$ requires detailed calculation.

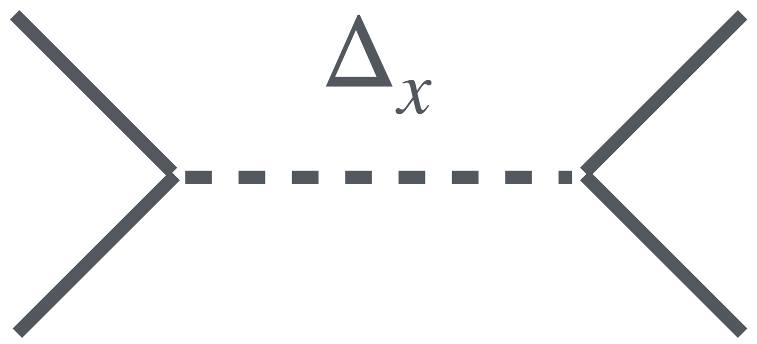
Exponentially suppressed for large masses $a_{\bar{\Delta}}(\mu) \sim e^{-\pi m/H}$

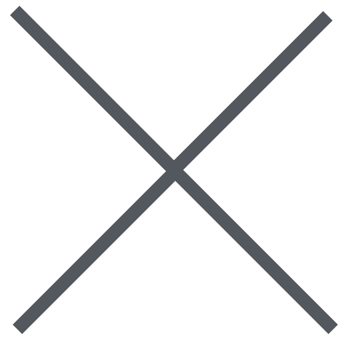
(Effect not captured by EFT which is expansion in powers of H/m.)

Plot: [Arkani-Hamed, Baumann, Lee, Pimentel '18]

Cosmological bootstrap

For external conformal scalars, a neat alternative to the full in-in calculation is to bootstrap the Casimir equation: [Arkani-Hamed, Baumann, Lee, Pimentel '18]

$(C_{12} + m_x^2)$ 

$=$ 

For $\bar{\Delta} = (d-1)/2$, the bulk-boundary propagators $K(k_i, \tau) \sim e^{ik_i\tau}$ meaning s-channel exchange diagram is a function of only *two* variables: can choose, *e.g.*, $u = \dfrac{s}{k_1 + k_2}$ and $v = \dfrac{s}{k_3 + k_4}$

Casimir equation reduces from a *PDE* to an *ODE* in u with inhomogeneous source term

→ *Solvable*!

Cosmological bootstrap

Boundary conditions:

- regular as $u \to 1$ (*i.e.*, absence of collinear singularity for $s = k_1 + k_2$)
- correct partial energy singularity as $u \to -1$ (*i.e.*, as $E_L = k_1 + k_2 + s \to 0$)

Result can then be generalised to massless external scalars (and other integer $\bar{\Delta}$) as well as spinning fields using weight-shifting/spin-raising operators, as well the transverse Ward-Takahashi identity.

[Baumann, Duaso Pueyo, Joyce, Lee, Pimentel '19, '20]

Cosmological collider signals can also be obtained by directly computing dS exchanges, for general kinematics, exchanged masses and spins, using their *Mellin representation* .

[Sleight '19] [Sleight, Taronna '19, '20]

Searching for 'collider' signals

Taking the soft limit of the above calculations for the dS 4-pt function yields the corresponding 3-pt function in a slow-roll background. [Arkani-Hamed, Maldacena '15]

Result suppressed by a slow-roll parameter hence likely small (unless couplings very large).

No sign in present data:

- BOSS constraints on massive particles during inflation: the cosmological collider in action

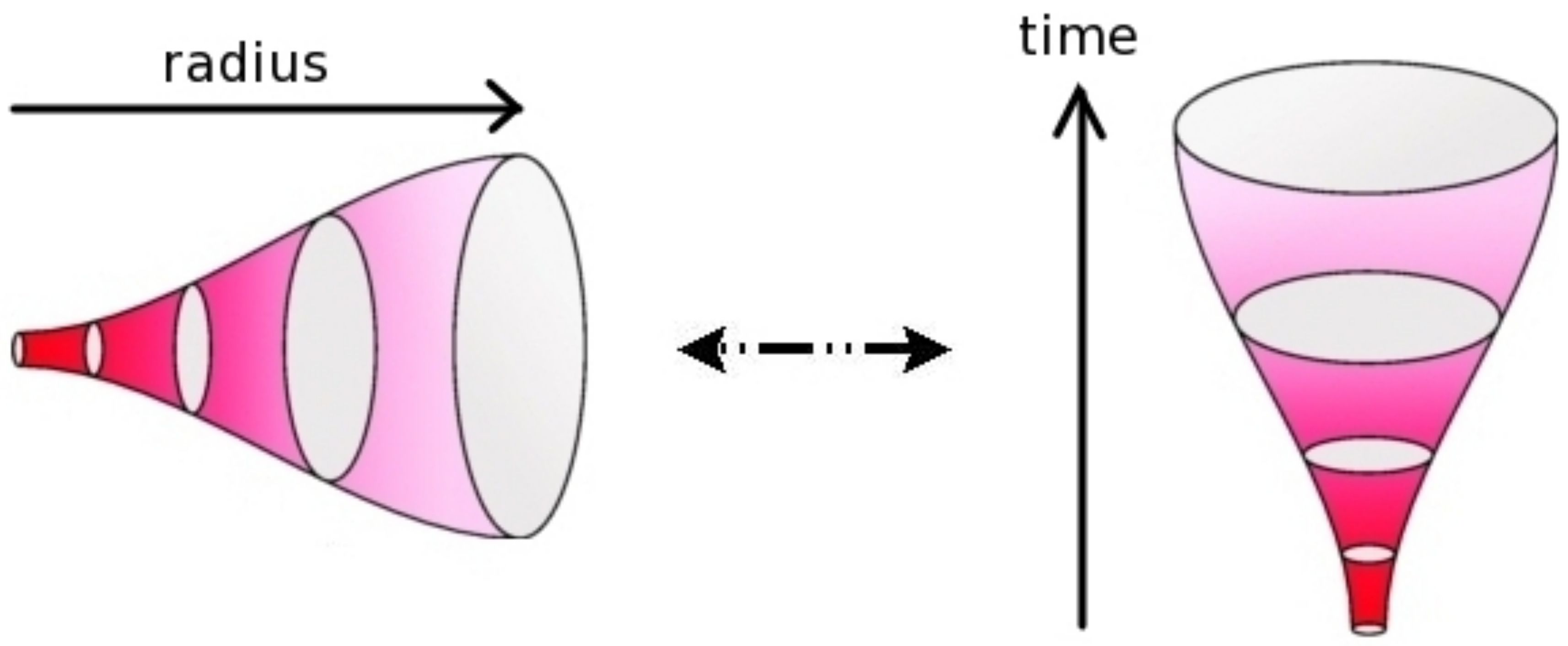
  [2404.01894] Cabass, Oliver, Philcox, Ivanov, Akitsu, Chen, Simonović, Zaldarriaga

- Searching for cosmological collider in the Planck CMB data

  [2404.07203], Sohn, D.G. Wang, Fergusson, Shellard

From dS to AdS

Another perspective on cosmological correlators follows from mapping to AdS/CFT:



$$ds^2 = \frac{\ell_{AdS}^2}{z^2}(dz^2 + d\boldsymbol{x}^2) \qquad ds^2 = \frac{\ell_{dS}^2}{\tau^2}(-d\tau^2 + d\boldsymbol{x}^2)$$

$$\ell_{AdS} = i\ell_{dS} \qquad z = -i\tau$$

$$\Psi_{dS} = Z_{CFT}\Big|_{\ell_{AdS}\to i\ell_{dS}}$$ [Maldacena '03]

Wavefunction propagators on dS continue to standard AdS/CFT propagators, relating ψ_n to CFT of *canonical* operator dimensions:

$$\psi_n(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) = e^{-\frac{i\pi}{2}n(d-\Delta)}\langle\langle \mathcal{O}_{\Delta_1}(\boldsymbol{k}_1)\ldots\mathcal{O}_{\Delta_n}(\boldsymbol{k}_n)\rangle\rangle\Big|_{\ell_{AdS}\to i\ell_{dS}}$$

For recent discussion: [Bzowski, PM, Skenderis '23]

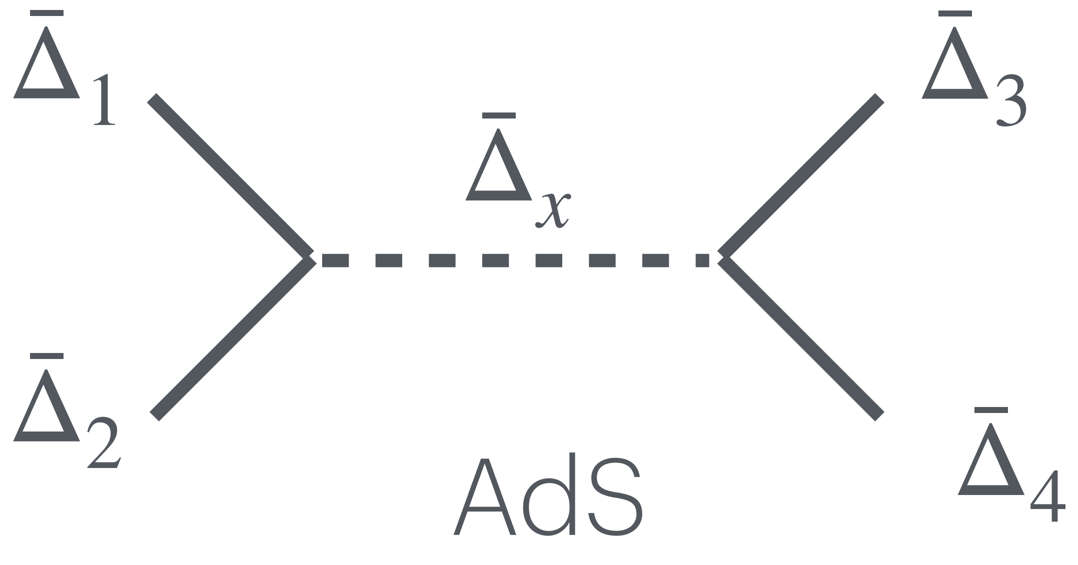
Can also map dS *correlators* to AdS by appropriate continuation of ± contours $z = (-\tau)e^{\pm i\pi/2}$ (consistent with $i\epsilon$ prescription) in Schwinger-Keldysh formalism. [Sleight, Taronna '19, '20]

From dS to AdS

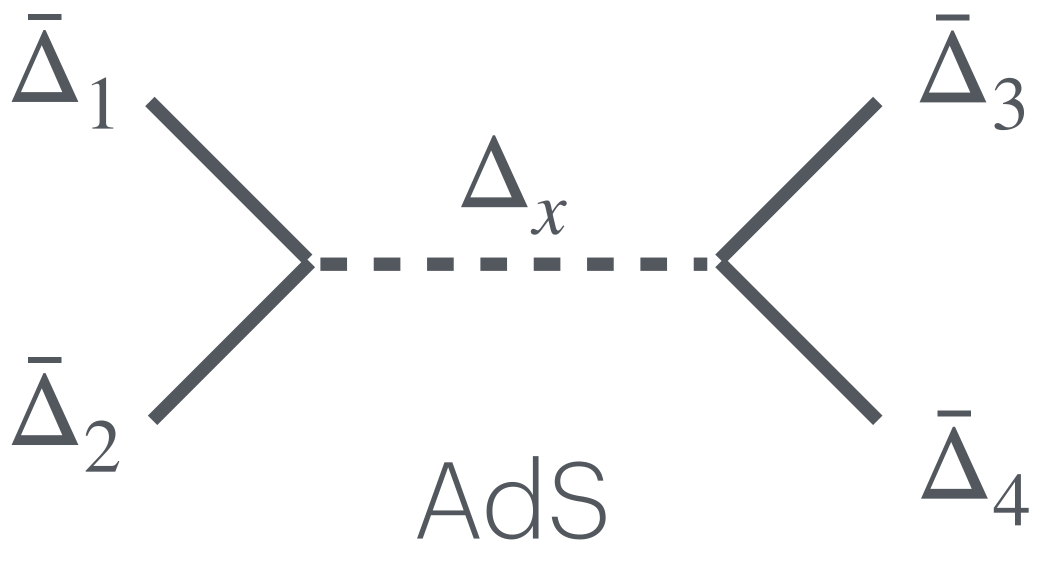
Leads to formulae for dS correlators in terms of AdS correlators of the *shadow* dimension, as expect from dS Ward identities. [Sleight, Taronna '19, '20]

For dS exchanges, shadow symmetry $\Delta_x \leftrightarrow \bar{\Delta}_x = d - \Delta_x$ requires combination:

dS exchange $= C(\bar{\Delta}_x, \bar{\Delta}_i)$



$+ C(\Delta_x, \bar{\Delta}_i)$



[See talk in parallel session]

Such formulae can even be recovered from an effective AdS action. [di Pietro, Gorbenko, Komatsu '21]

However, these shadow CFT formulae hold only for generic dimensions: they break down for cases involving *IR divergences* and cannot be renormalised, unlike those for the wavefunction.

[Bzowski, PM, Skenderis '23]

IR divergences

IR divergences arise in many dS correlators, including those of massless and conformal scalars at tree level, both with and without derivative vertices.

They can be renormalised via addition of boundary counterterms. [Bzowski, PM, Skenderis '23]

The structure of bulk IR divergences in the dS wavefunction is consistent with that of UV divergences in a local boundary CFT of the canonical dimensions. $\Psi_{dS} = Z_{CFT}\Big|_{\ell_{AdS}\to i\ell_{dS}}$

This *isn't* the case for dS correlators and a boundary CFT of the shadow dimensions:

e.g. dS correlator of one massless & two conformal scalars requires renormalisation,

$\mathrm{dS}^{ren}_{[322]} = \dfrac{1}{k_1^3 k_2 k_3}\Big(-k_1 + (k_2+k_3)\Big(\ln\dfrac{k_t}{\mu} + \tilde{a}\Big)\Big)$ while shadow CFT correlator *finite*: $CFT_{[011]} = C\dfrac{(k_2+k_3)}{k_1^3 k_2 k_3}$

(Captures only scheme-dept. terms.)

IR divergences

Other consequences:

Where IR divergences occur, the renormalised dS correlators obey modified inhomogeneous (or *anomalous*) conformal Ward identities.

➡ Need to solve these *anomalous* Ward identities to bootstrap IR divergent correlators.

See also: [D.G. Wang, Pimentel, Achúcarro '22]

The naive use of weight-shifting operators will also fail as these map between solutions of the *homogeneous* conformal Ward identities.

For other recent perspectives on IR divergences, see e.g., [Gorbenko, Senatore '19]

[Céspedes, Davis, D.G. Wang '23] [Benincasa, Vazão '24]

Kinematic flow

Beyond the conformal Ward identities for de Sitter backgrounds, what other classes of differential equations might cosmological correlators obey?

For *conformal scalars* on power-law FRW backgrounds $\mathrm{d}s^2 = (-\tau)^{-2(1+\epsilon)}(-\mathrm{d}\tau^2 + \mathrm{d}\boldsymbol{x}^2)$ with polynomial interactions, can express wavefunction coefficients as integrals of form:

$$\psi_{FRW}(E_v, E_I) = \int_0^\infty \left(\prod_v \mathrm{d}\omega_v\, \omega_v^\epsilon\right) \psi_{flat}(E_v + \omega_v, E_I)$$

Shift of vertex energy

Trick is to transform to flat space with time-dependent interactions, and hide these by writing

$\tau^{-1-\epsilon} \sim \int_0^\infty \mathrm{d}\omega\, \omega^\epsilon e^{i\omega\tau}$. The $e^{i\omega\tau}$ combines with $e^{iE_v\tau}$ from bulk-bdy propagators giving $\psi_{flat}(E_v + \omega_v, E_I)$.

Cosmological polytopes: [Arkani-Hamed, Benincasa, Postnikov '17]

Kinematic flow

Crucially, $\psi_{flat}(E_v, E_I)$ is a product of *powers* of factors that are *linear* in the energies E_v, E_I.

Taking derivatives wrt the energies produces integrals with shifted powers. Using integration by parts, we can reduce back to a finite set of master integrals $\vec{I}$.

Gives *1st order* PDEs of a form familiar from study of multi-loop Feynman integrals: cf. [Henn '13]

$$\mathrm{d}\vec{I} = \epsilon A \vec{I}$$

where $\mathrm{d} = \sum_i \frac{\partial}{\partial Z_i} \mathrm{d}Z_i$ with $Z = (E_v, E_I)$ and $A = \sum_j \alpha_j \, \mathrm{d} \log \Phi_j(Z)$ is a matrix-valued 1-form.

'Letters' encode singularities.

For $\epsilon \ll 1$ (power-law inflation) can solve as Chen iterated integral (truncated Dyson series).

The matrix A can be constructed independently via a set of *graphical rules* involving 'tubings'.

[Arkani-Hamed, Baumann, Hillman, Joyce, Lee, Pimentel '23]

Kinematic flow

This gives a concrete toy model in which the differential equations for any tree diagram contributing to the wavefunction can be computed, purely from the boundary…

$\Rightarrow$ Autonomous boundary encoding of bulk time evolution.

… ultimately would like to move beyond individual diagrams to the full wavefunction.

Other directions…

Since cosmological correlators *contain* scattering amplitudes, many developments applying amplitudes ideas to cosmology including:

- Cosmological polytopes [Arkani-Hamed, Benincasa, Postnikov '17] [Benincasa '18] …

  For introduction: see Benincasa [2203.15330]

- Double copy structure: via weight-shifting operators [Lee, X. Wang '22];

  for 4-graviton amplitude [Armstrong, Goodhew, Lipstein, Mei '23]

  following direct calculation [Bonifacio, Goodhew, Joyce, Pajer, Stefanyszyn '22]

- Cosmological scattering equations [Gomez, Jusinskas, Lipstein, 21] …
- Recursion relations: [Raju '11, '12] … [Jazayeri, Pajer, Stefanyszyn '21] … [Albayrak, Kharel '23]
- Defining a de Sitter S-matrix [Melville, Pimentel '23, '24]

Other directions...

Also much progress on loop corrections:

e.g.,

[Benincasa, Brunello, Mandal, Mastrolia, Vazão '24]

[Cacciatori, Epstein, Moschella '24] ← see talks in parallel session

[Chowdhury, Chowdhury, Moga, Singh '24]

[Chowdhury, Lipstein, Mei, Sachs, Vanhove '23] ←

[Beneke, Hagen, Sanfilippo '23]

[Xianyu, Qin '22, '23]

[Bañados, Bianchi, Muñoz, Skenderis '22]

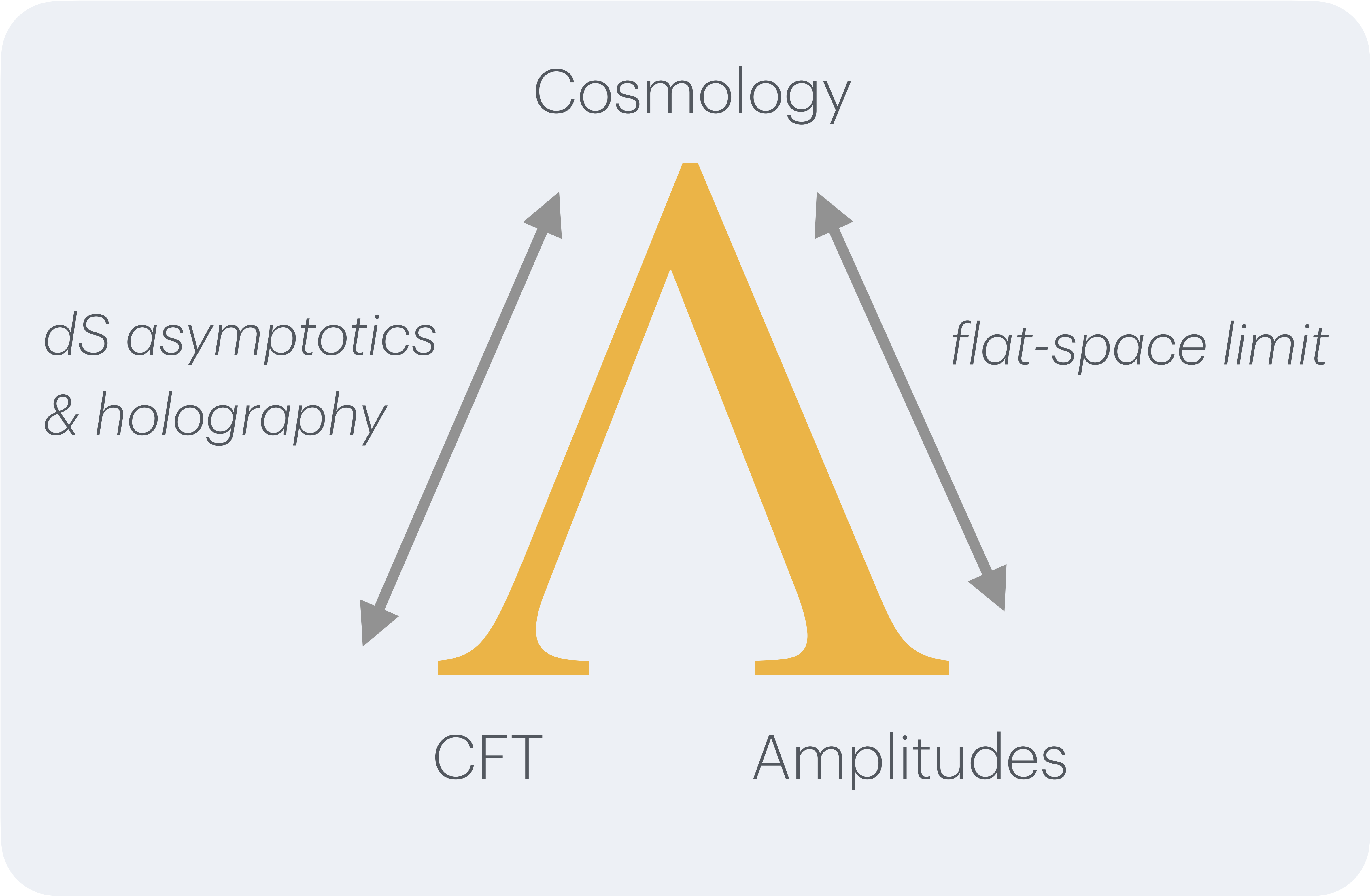
and many other authors...

And programme to apply non-perturbative CFT bootstrap methods to QFT on dS:

[Hogervorst, Penedones, Vaziri, '21] [Penedones, Vaziri, Sun '23] [Loparco, Penedones, Varziri, Sun '23]

Outlook

Many new connections to be made!



Amplitudes-inspired techniques:

- Can we move beyond conformal scalars?
- Beyond analysis of individual diagrams?

CFT & holography:

- Can we bootstrap using general solution of CWI?
- How is bulk unitarity encoded on the boundary?

Connecting with observations:

- Better understanding of future prospects & strategies for detecting 'collider'-type signals

e.g., 21cm tomography: probes distribution of neutral hydrogen between $2 < z < 6$