Flat Space Holography

- a review -

Laura Donnay

European Research Council
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SISSA

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes ($\Lambda = 0$)?

→ These spacetimes are relevant from collider physics … to astrophysics (< cosmological scales)

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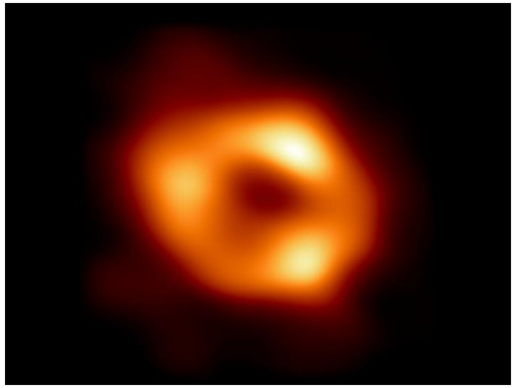
$$\boxed{S_{BH} = \frac{\mathcal{A}c^3}{4G\hbar}}$$

[Bekenstein][Hawking]

$\mathcal{A}$: event horizon area



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[Bekenstein][Hawking]

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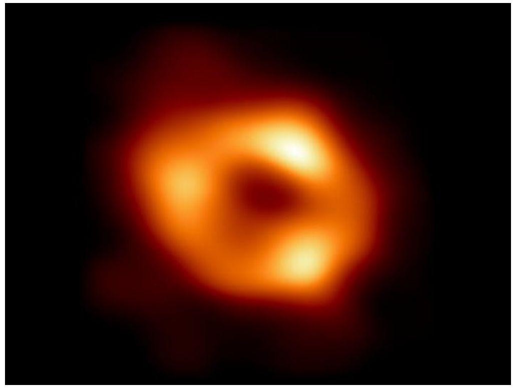
↓

Holography beyond Anti-de Sitter/CFT?

$$\Lambda < 0$$



Event Horizon Telescope

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Early attempts:

[Susskind ‘99][Polchinski ‘99][Giddings ’99]
[de Boer, Solodukhin ‘03][Arcioni, Dappiaggi ’03 ’04]
[Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06]…

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…and even earlier
[Penrose ‘76][Newman ’76]

→ aimed at a reconstruction of the bulk spacetime from quantities defined only at null infinity $\mathscr{I}$

General Relativity and Gravitation, Vol. 7, No. 1 (1976), pp. 107–111

Heaven and Its Properties

EZRA T. NEWMAN

Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213

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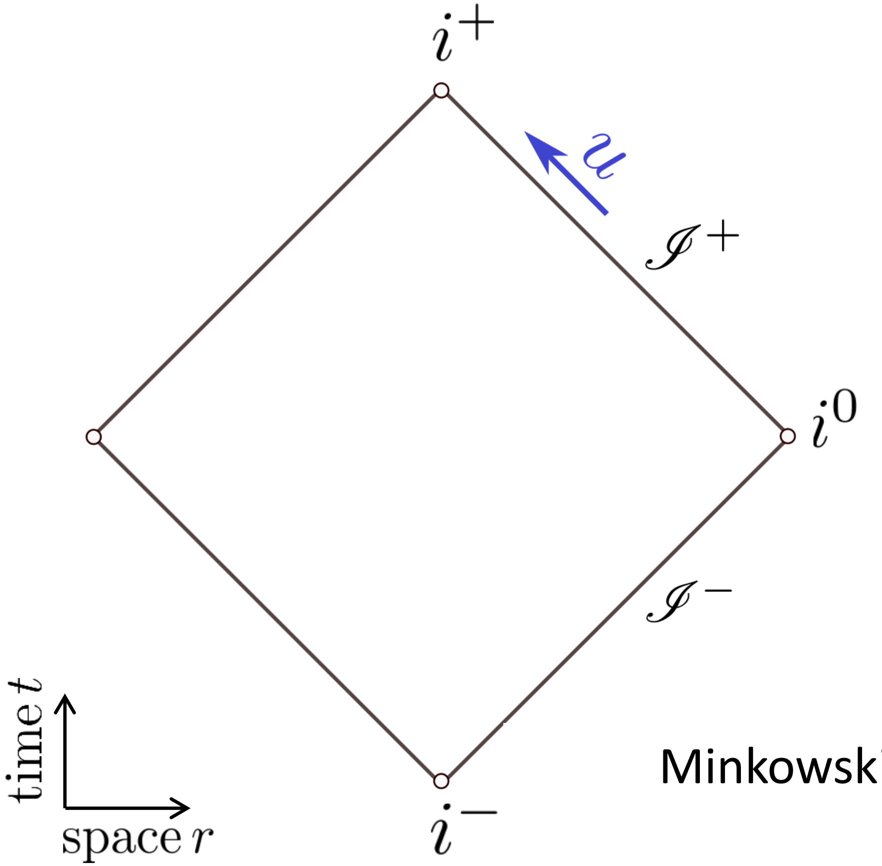
Main obstructions/difficulties:



Minkowski

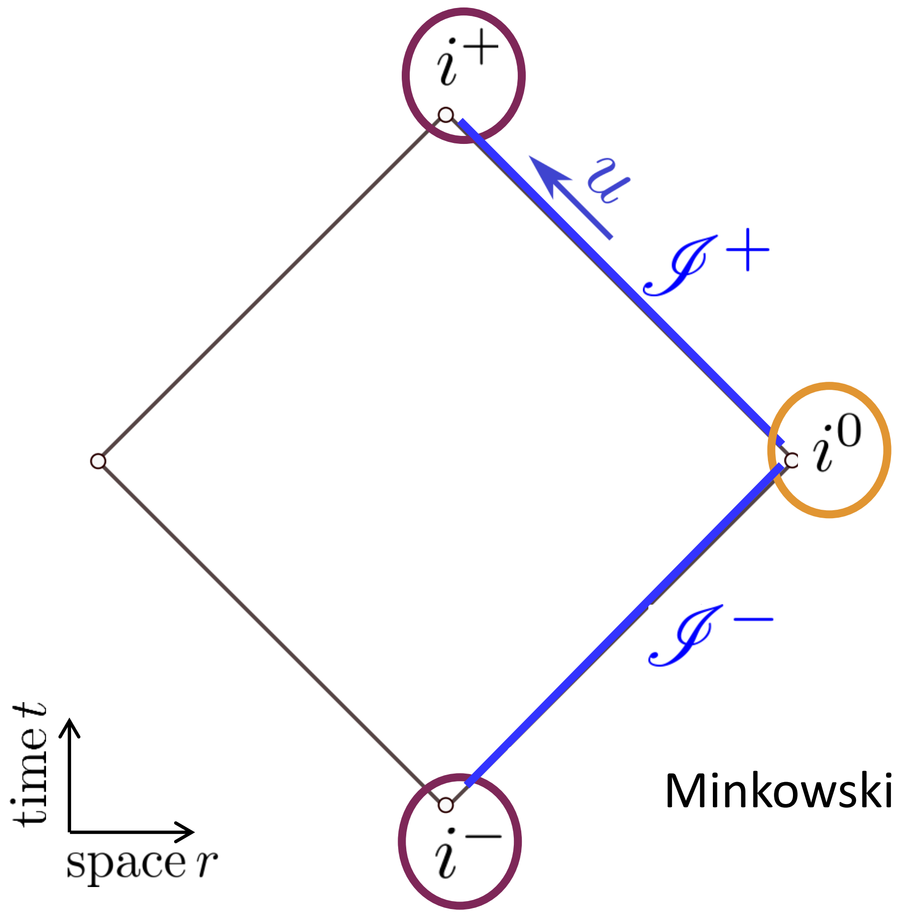
Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Main obstructions/difficulties:

1 The conformal **boundary** includes

future/past timelike infinity
future/past null infinity
spatial infinity



Flat space holography

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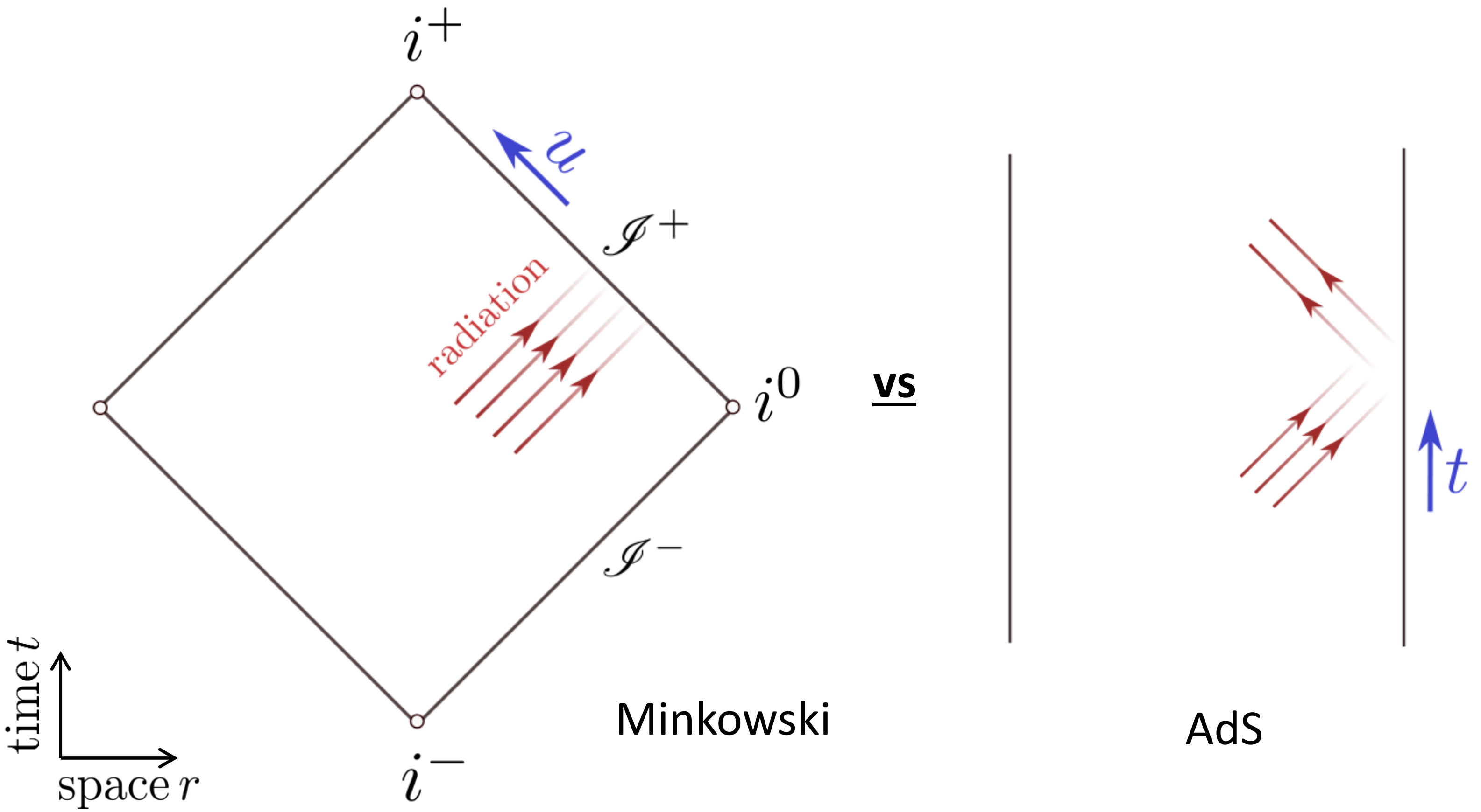

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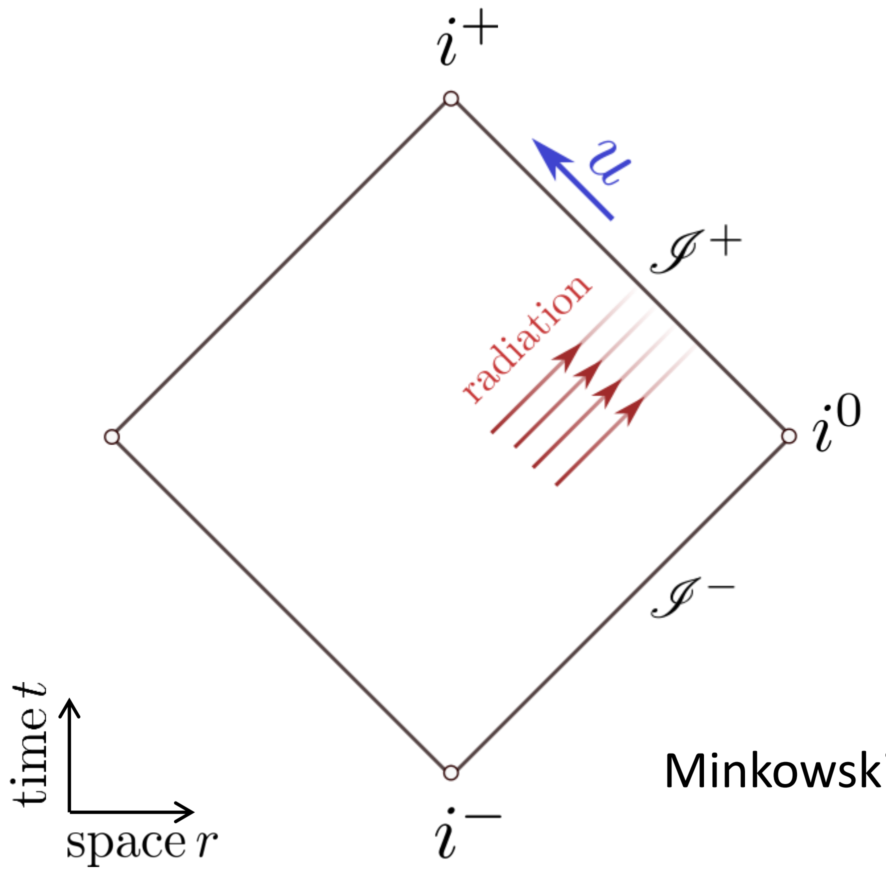

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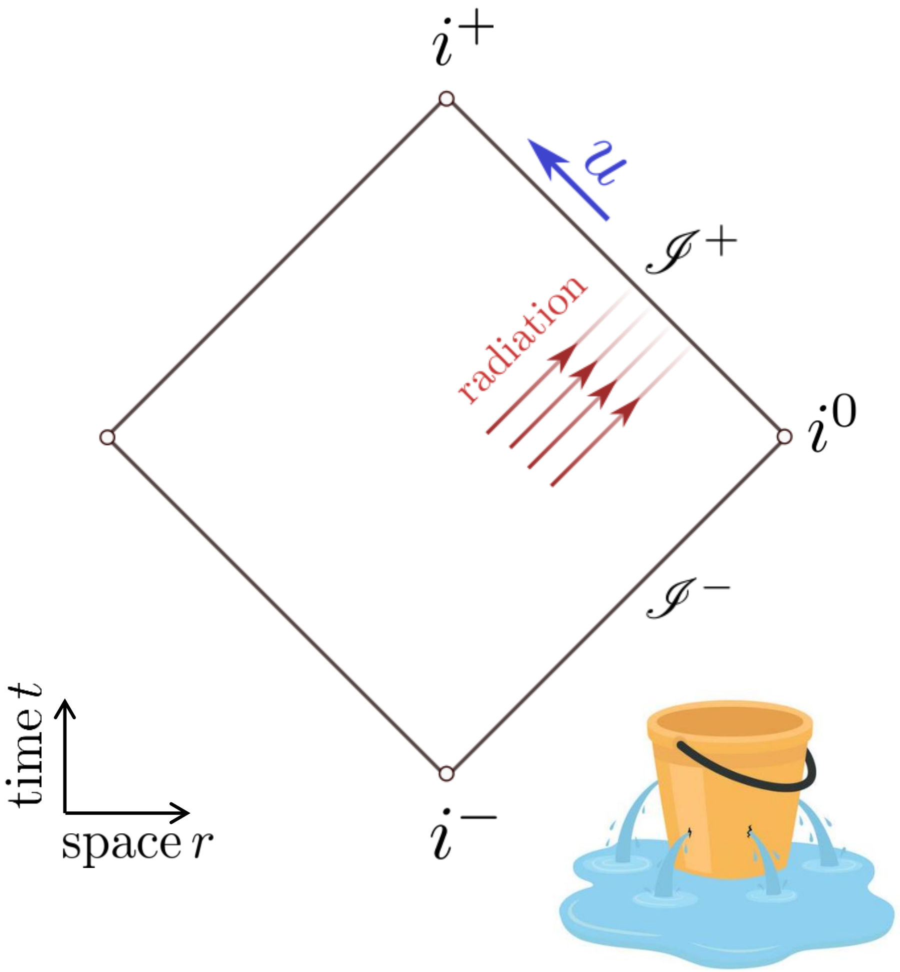
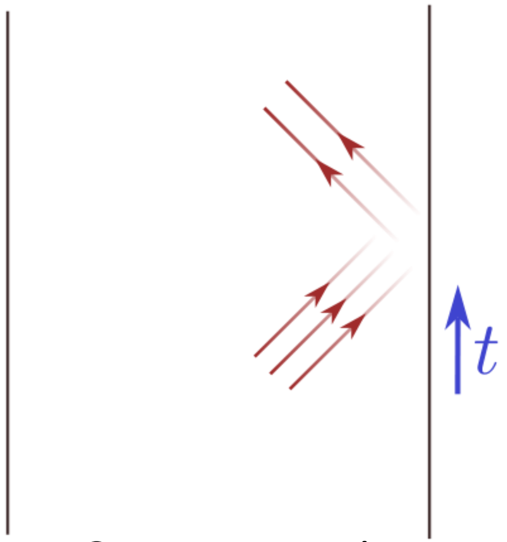


vs

AdS



Quantum gravity 'in a **box**'

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

--> Road map: symmetries

What are the symmetries of asymptotically flat spacetimes?

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

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What are the symmetries of asymptotically flat spacetimes?

what was expected

Poincaré

what was found

Bondi-Metzner-Sachs (BMS) ('62)

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

- - -> Road map: symmetries

What are the symmetries of asymptotically flat spacetimes?

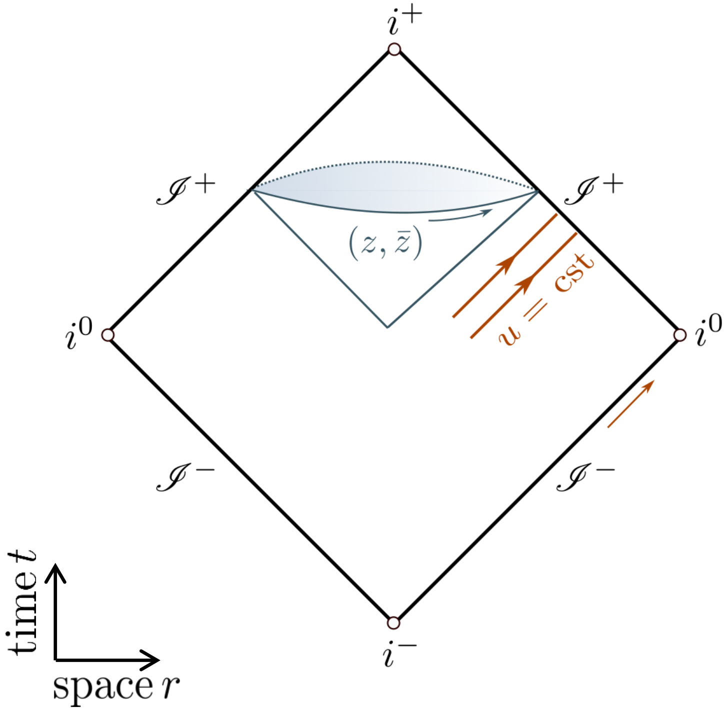
→ infinite-dimensional extension of Poincaré!

4 Poincaré translations

↓ *Symmetry enhancement*

∞ BMS supertranslations

[Bondi, van der Burg, Metzner ’62] [Sachs ’62]



$$\xi = \boxed{\mathcal{T}(z,\bar{z})}\partial_u + \cdots$$

arbitrary function
on the celestial sphere

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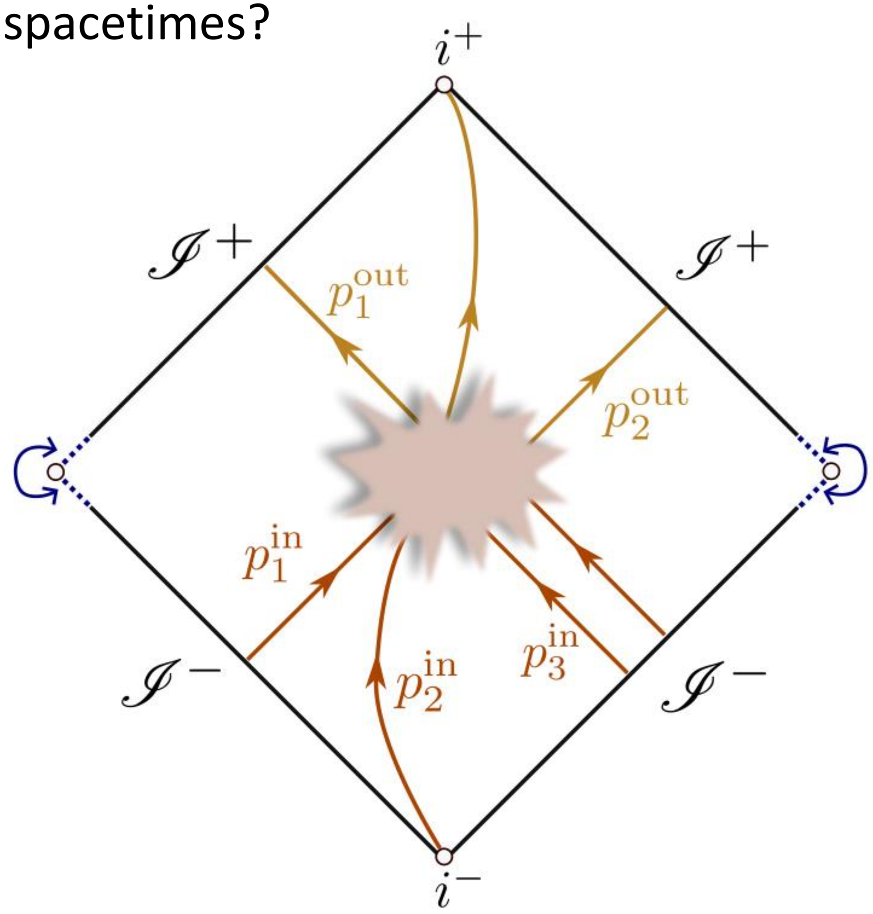

$$\langle \text{out}|S|\text{in}\rangle$$

While BMS symmetries were originally disregarded, it was realized (50 years later, [Strominger '13]) that they

- constrain the gravitational **S-matrix**

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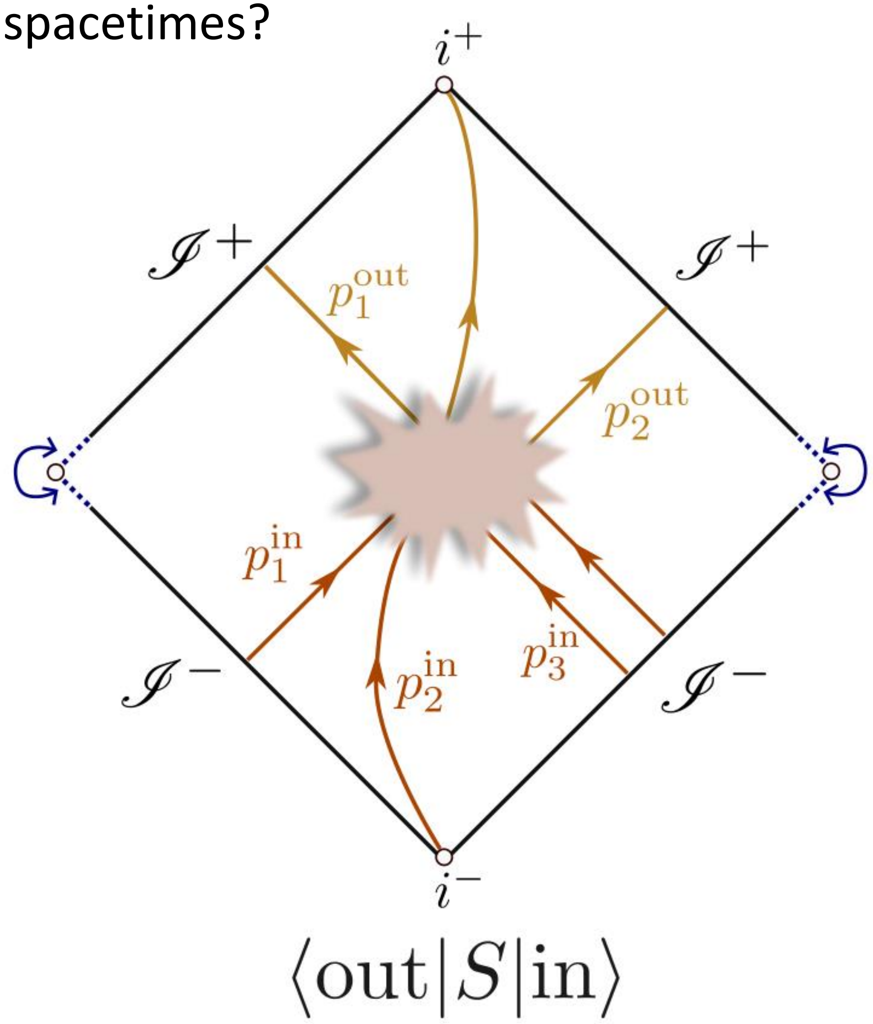

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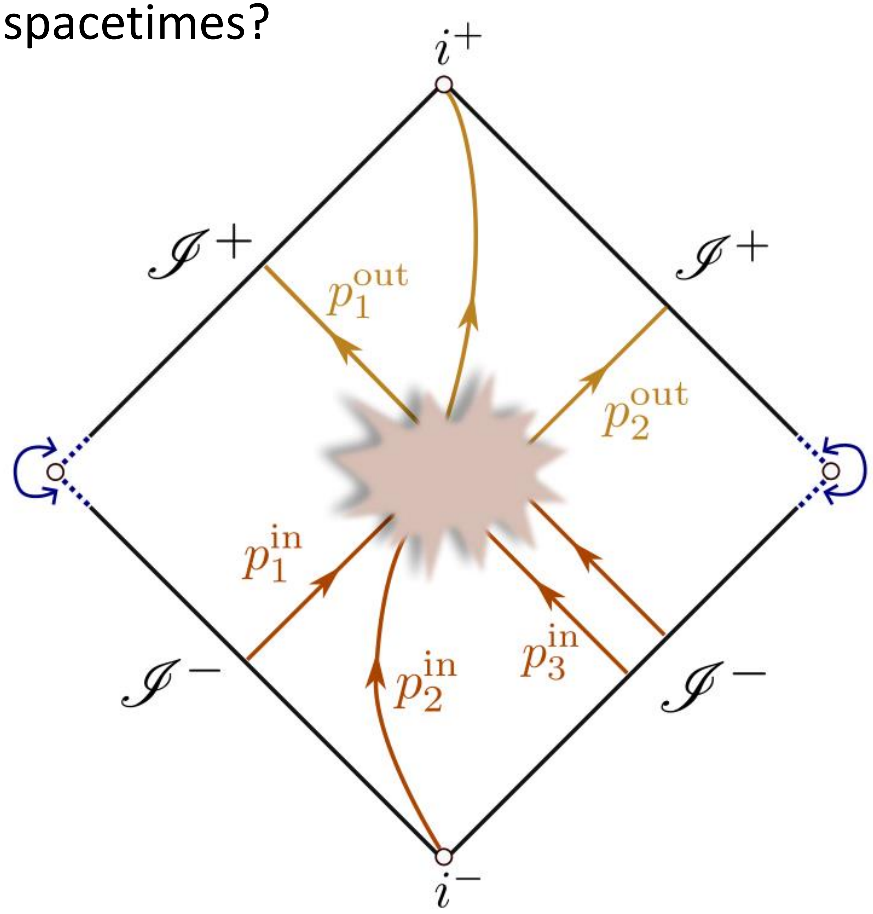
- allow further extensions, including the local **conformal** group



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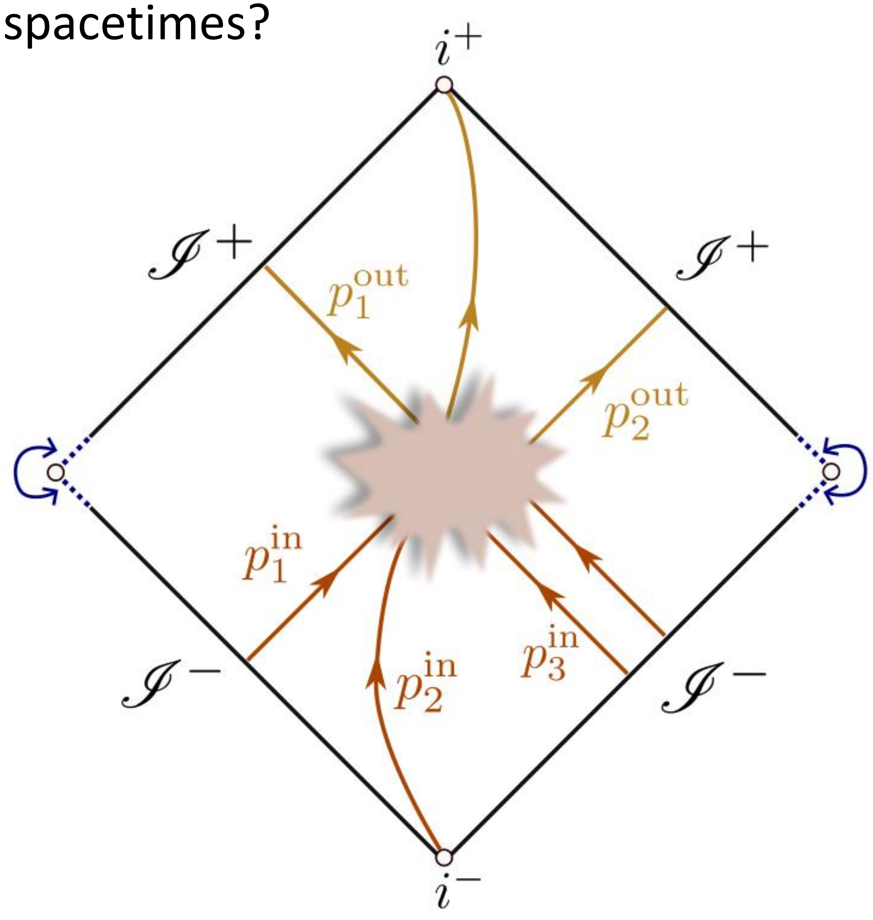
revival of **flat holography**



$\langle \text{out}|S|\text{in}\rangle$

Which boundary?

Which boundary?

null infinity

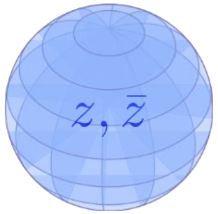
lighlike 3d hypersurface

$$\mathscr{I} = \mathbb{R} \times S^2$$

Looking for a
3d 'BMS field theory'

celestial sphere

Euclidean 2d-sphere



Looking for a
2d 'celestial CFT'

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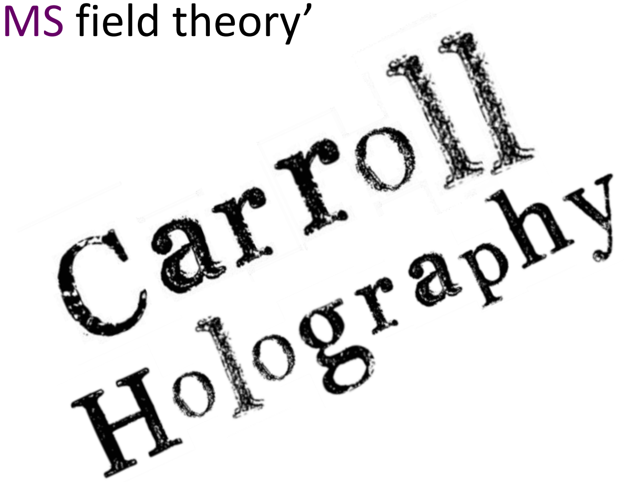

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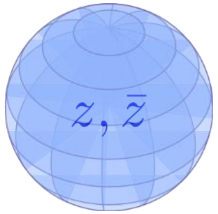

Looking for a
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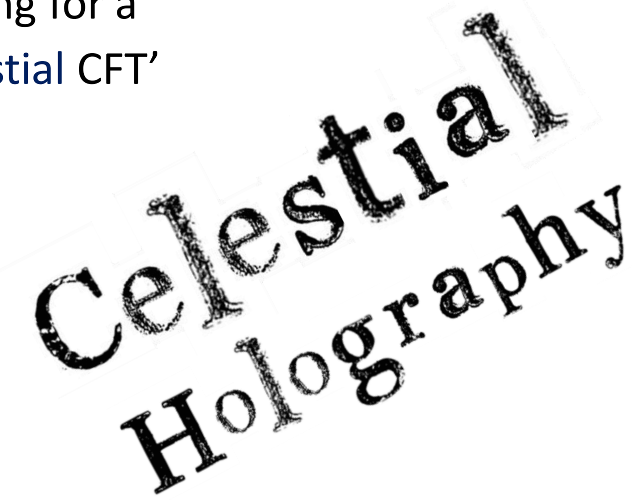
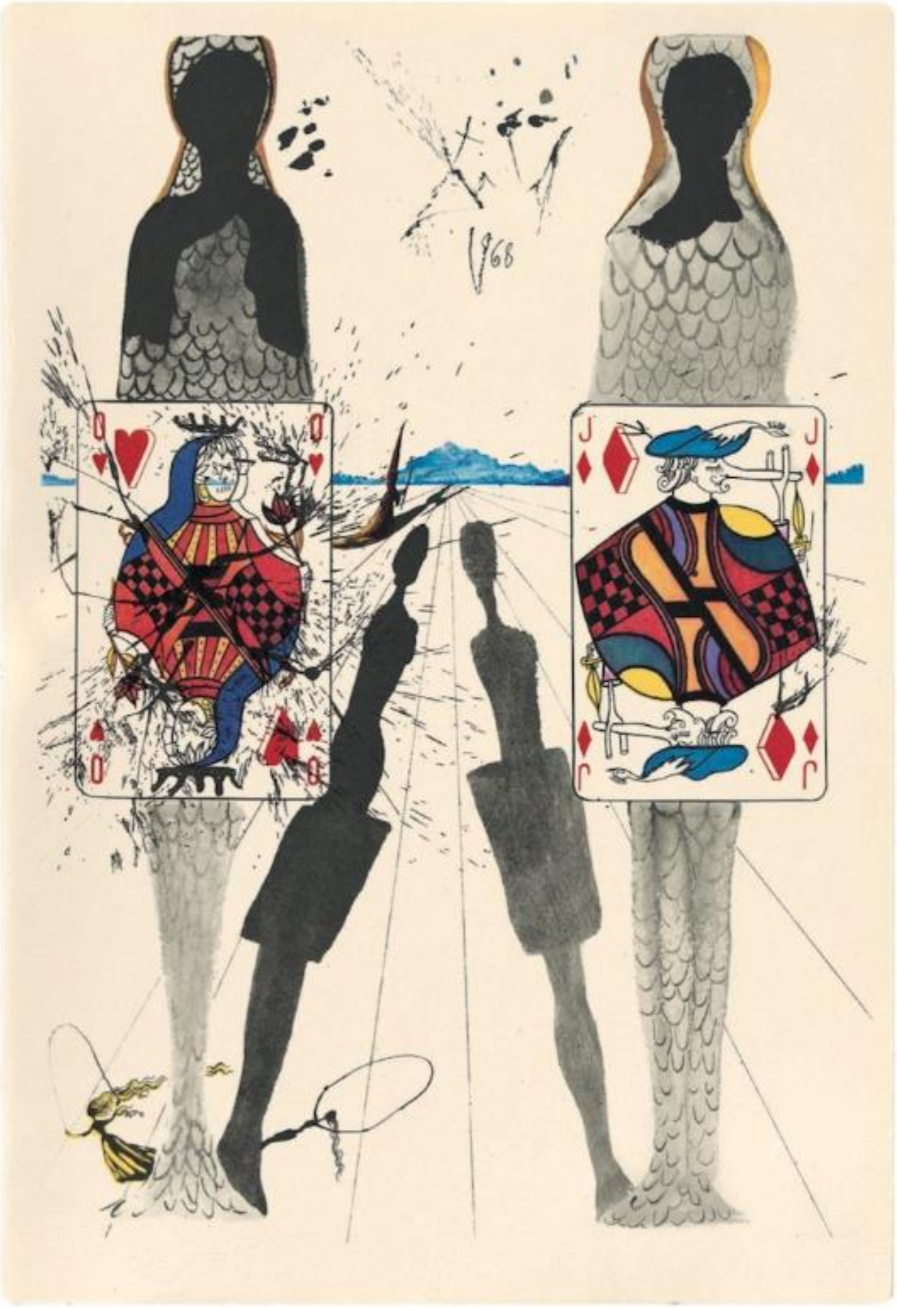


Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



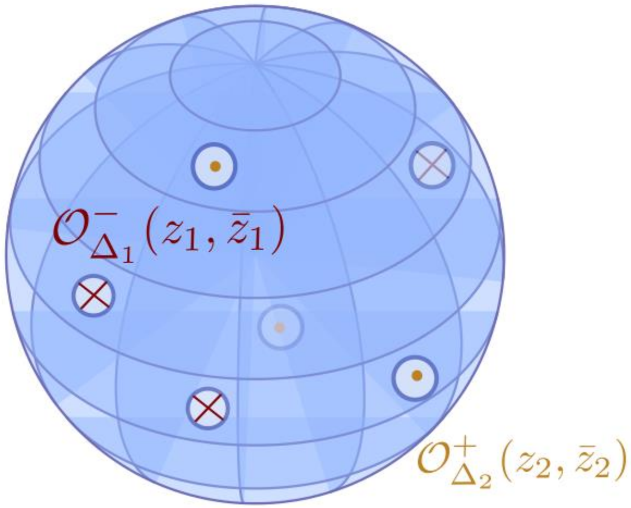
Outline

Celestial holography

Carrollian holography

$\mathscr{L}w_{1+\infty}$ symmetries

Final remarks



$$\langle \mathcal{O}^{\pm}_{\Delta_1}(z_1, \bar{z}_1) \ldots \mathcal{O}^{\pm}_{\Delta_N}(z_N, \bar{z}_N) \rangle$$

Celestial Holography

'If you look up at the sky on a clear cloudless night, you appear to see a hemispherical dome above you, punctuated by myriads of stars.'

R. Penrose, *The road to reality*, 2004

Reviews

Infrared structure of gravity

- A. Strominger, *Lectures on the Infrared Structure of Gravity and Gauge Theory*
Princeton University Press, 3 (2018), 1703.05448

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Celestial holography

- S. Pasterski, M. Pate, A.-M. Raclariu, *Celestial Holography*, in 2022 Snowmass Summer Study 11, 2111.11392
- A.-M. Raclariu, *Lectures on Celestial Holography*, 2107.02075
- S. Pasterski, *Lectures on celestial amplitudes*, Eur. Phys. J. C 81 (2021) no. 12 1062, 2108.04801
- T. McLoughlin, A. Puhm, A.-M. Raclariu, *The SAGEX review on scattering amplitudes chapter 11: soft theorems and celestial amplitudes*, J. Phys. A 55 (3, 2022) 443012, 2203.13022
- L. Donnay, *Celestial holography: an asymptotic symmetry perspective*, Phys. Rept. 1073 (2024), 2310.12922

Celestial holography

The 4d spacetime S-matrix is encoded in a 2d ‘Celestial Conformal Field Theory’

Celestial holography

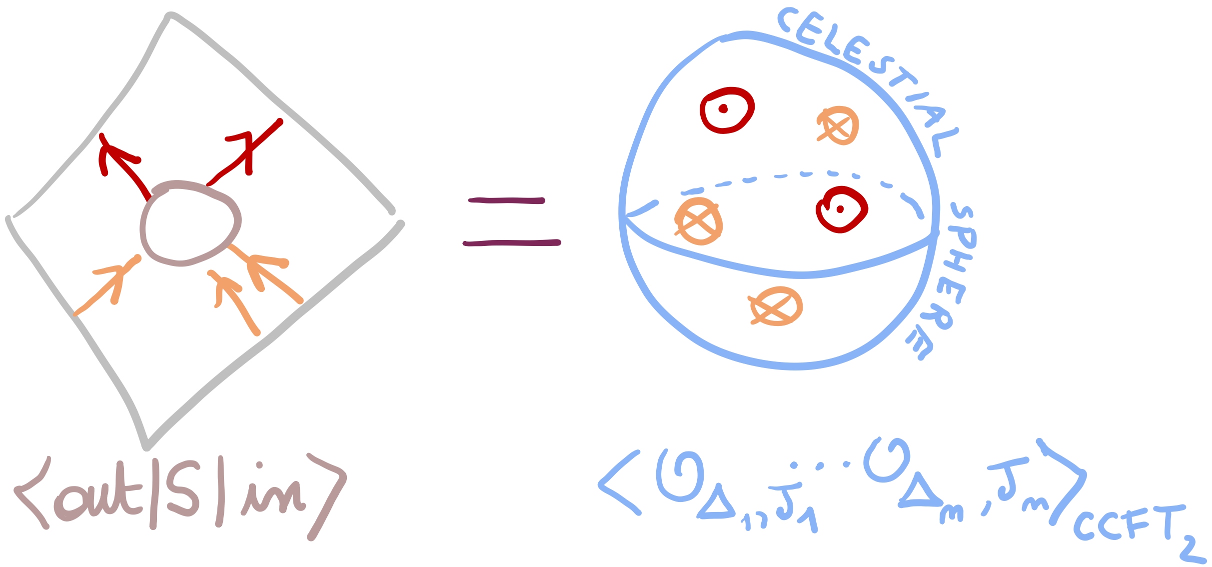
The 4d spacetime S-matrix is encoded in a 2d ‘Celestial Conformal Field Theory’

momentum of a massless particle

$$p^\mu = \omega q^\mu(z, \bar{z})$$

ω: energy

$(z, \bar{z})$: a point on $\mathcal{CS}^2$



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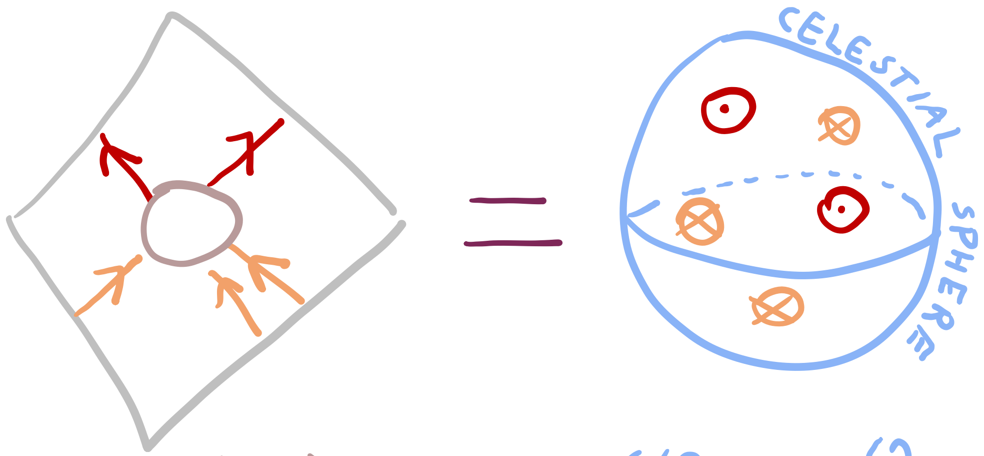

$\langle out | S | in \rangle = \langle \mathcal{O}_{\Delta_1, J_1} \cdots \mathcal{O}_{\Delta_m, J_m} \rangle_{CCFT_2}$

Simple idea: make conformal properties manifest

Celestial holography

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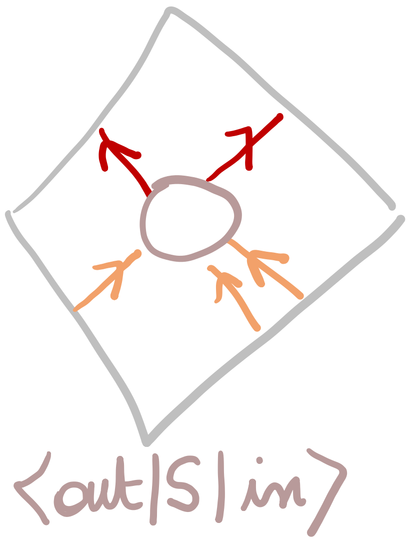

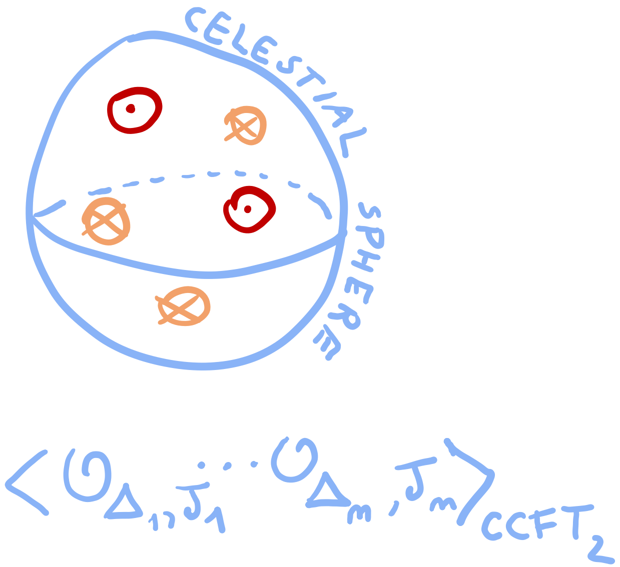

Simple idea: make conformal properties manifest

→ Plane waves are mapped to

$$\Psi^\pm_\Delta(X; z, \bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i p \cdot X}$$

$$\Psi_{h,\bar{h}}(z, \bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \Psi_{h,\bar{h}}(z, \bar{z})$$

Primary field of weight $\Delta = h + \bar{h}$

Celestial amplitudes

Mellin-transformed massless scattering amplitudes = celestial correlators

$$\prod_{k=1}^{n}\int_0^\infty d\omega_k\ \omega_k^{\Delta_k-1}\mathcal{A}(\omega_1,z_1,\bar{z}_1,\dots\omega_n,z_n,\bar{z}_n)=\langle\mathcal{O}_1(\Delta_1,z_1,\bar{z}_1,\dots,\Delta_n,z_n,\bar{z}_n)\rangle_{CCFT_2}$$

loads of these celestial amplitudes have been explicitly computed

Note: a map for **massive** particles also exists

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Some features

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better behavior
- $\rightarrow$ celestial amplitudes in string theory
- $\rightarrow$ turn on a background field [see Giuseppe Bogna's talk]
- $\rightarrow$ work in the eikonal approximation [see Piotr Tourkine's talk]

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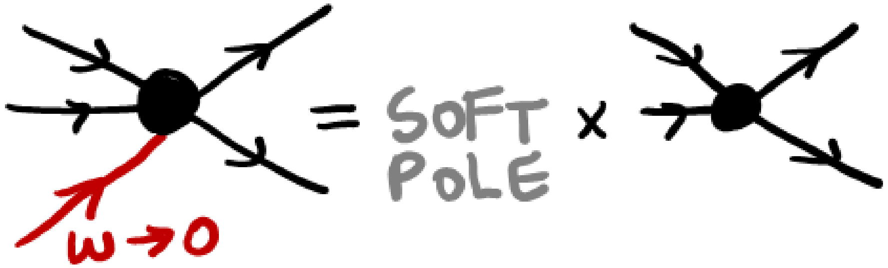
- $\boxed{\textbf{universal behavior} \leftrightarrow \text{symmetry algebra}}$

Celestial currents

The soft sector of celestial CFT is captured by 2d celestial currents.



[Weinberg '65][...]

(here: tree level)

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1} S_n^{(0)} + \omega^{0} S_n^{(1)}\right] \mathcal{A}_n + \mathcal{O}(\omega)$$

leading subleading

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soft theorem

Ward identity

2d current

leading

ω^{-1}

supertranslations

$\delta C_{zz} = \partial_z^2 f$

Celestial currents

The soft sector of celestial CFT is captured by 2d celestial currents.

soft theorem	Ward identity	2d current
leading	supertranslations	$P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$
ω^{-1}	$\delta C_{zz} = \partial_z^2 f$	$\left(\frac{3}{2},\frac{1}{2}\right)$ primary $\quad \Delta = 1$

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subleading ω^{0}	superrotations $\delta C_{zz} = u\partial_z^3 Y^z$	$T(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{h}{(z-w)^2}\mathcal{O}_{h,\bar{h}}(w,\bar{w}) + \frac{\partial\mathcal{O}_{h,\bar{h}}(w,\bar{w})}{z-w}$ T: $(2,0)$ primary $\quad$ Shadow of $\Delta = 0$

[Kapec, Mitra, Raclariu, Strominger '16] [Cheung, de la Fuente, Sundrum '17][LD, Puhm, Strominger '18] [Fotopoulos, Stieberger, Taylor '20] …

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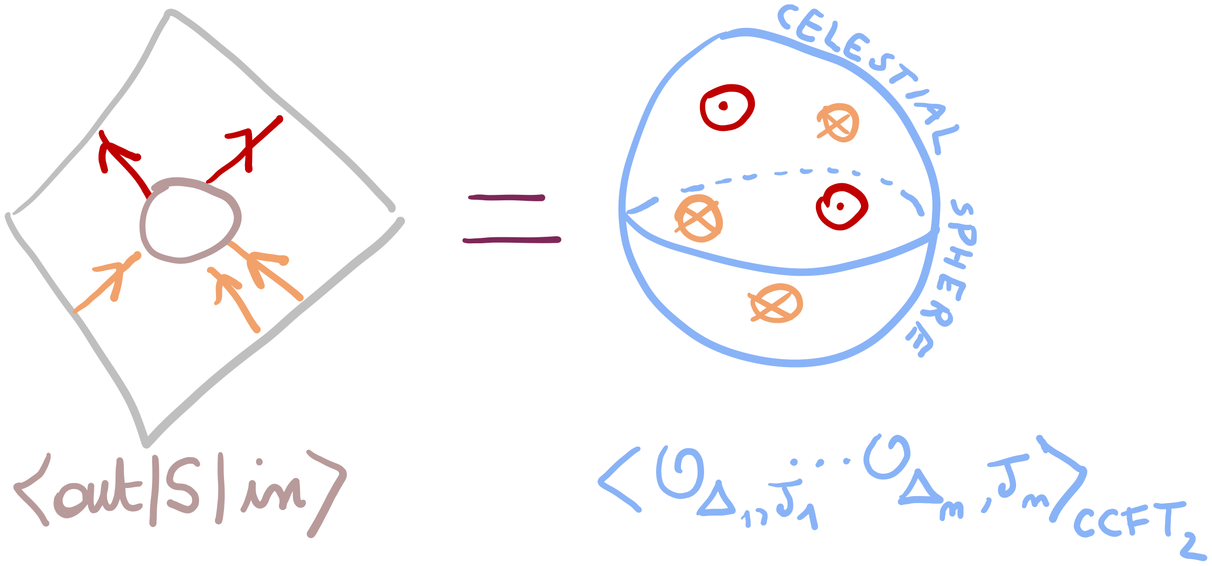
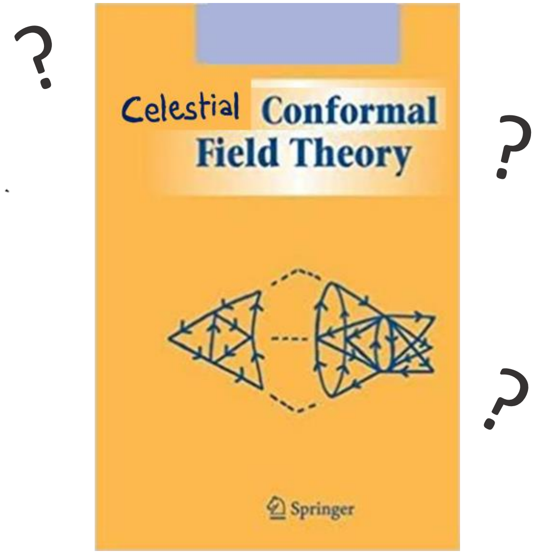
… and more! (see later)

[Kapec, Mitra, Raclariu, Strominger '16] [Cheung, de la Fuente, Sundrum '17][LD, Puhm, Strominger '18] [Fotopoulos, Stieberger, Taylor '20] …

Celestial Holography





collinear limits $p_1^\mu \parallel p_2^\mu$

low point amplitudes

asymptotic symmetries

celestial **OPEs**

kinematic singularities

2*d* currents

spectrum? non-unitary?

…

Carrollian Holography

‘We’re all mad here’

Lewis Carroll, *Alice’s Adventures in Wonderland*

Carrollian physics

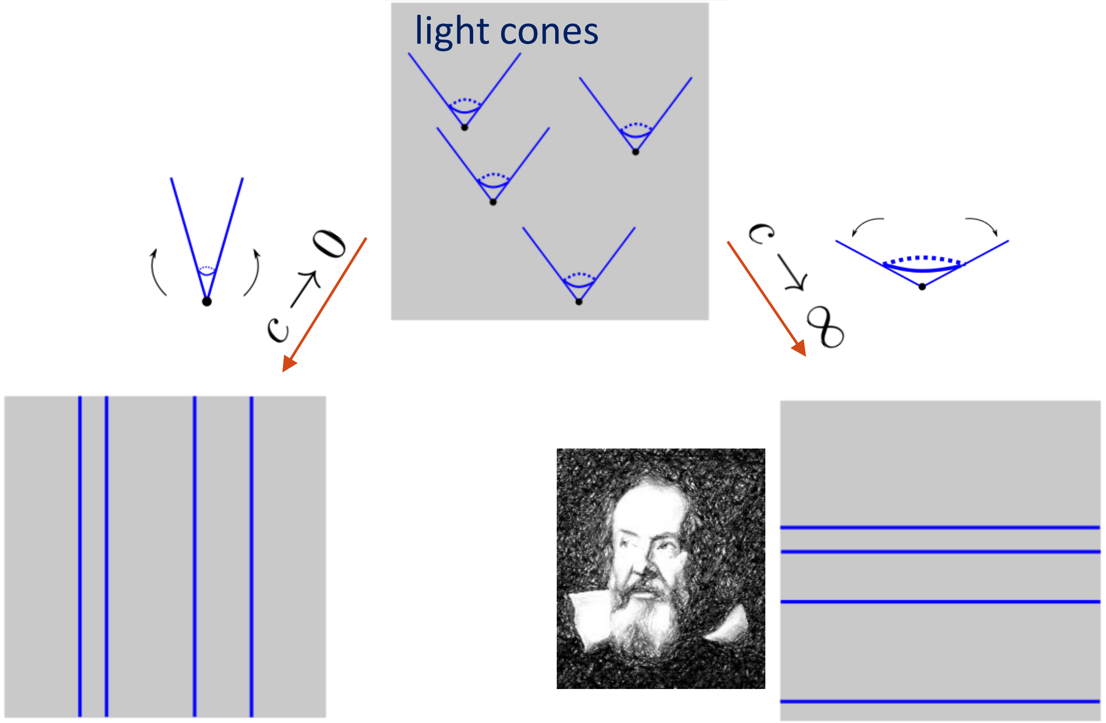
- 1965: A curiosity of Lévy-Leblond (also independently by Sen Gupta 1966)

  The $c \to \infty$ limit of the Poincaré group leads to the Galilean group.

  But what if we take the $c \to 0$ *limit instead?*

  → '**Carroll** group'

"Alice's Adventures in Wonderland"
Lewis Carroll (1865)



Carrollian spacetime
(space is absolute)

Galilean spacetime
(time is absolute)

Carrollian physics

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  → '**Carroll** group'

- Weird features… but (lately) found to be relevant for

  - Hamiltonian analysis of GR [Henneaux '79]
  - fluid/gravity correspondence
    [Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18]
    [de Boer, Hartong, Obers, Sybesma, Vandoren '22]
  - black hole near-horizon physics [Penna'18][LD, Marteau '18]
  - cosmology [de Boer, Hartong, Obers, Sybesma, Vandoren '22]
  - …flat space holography
    [Bagchi, Mehra, Nandi '19][LD, Fiorucci, Herfray, Ruzziconi '22][Bagchi, Banerjee, Basu, Dutta '22]

BMS = conformal Carrollian symmetries

- BMS symmetries = **conformal** symmetries of a **Carrollian** structure at null infinity
  [Geroch][Penrose][Henneaux][Duval, Gibbons, Horvathy][Hartong][Ciambelli, Leigh, Marteau, Petropoulos][Bekaert, Morand][Herfray]…

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Carrollian geometry

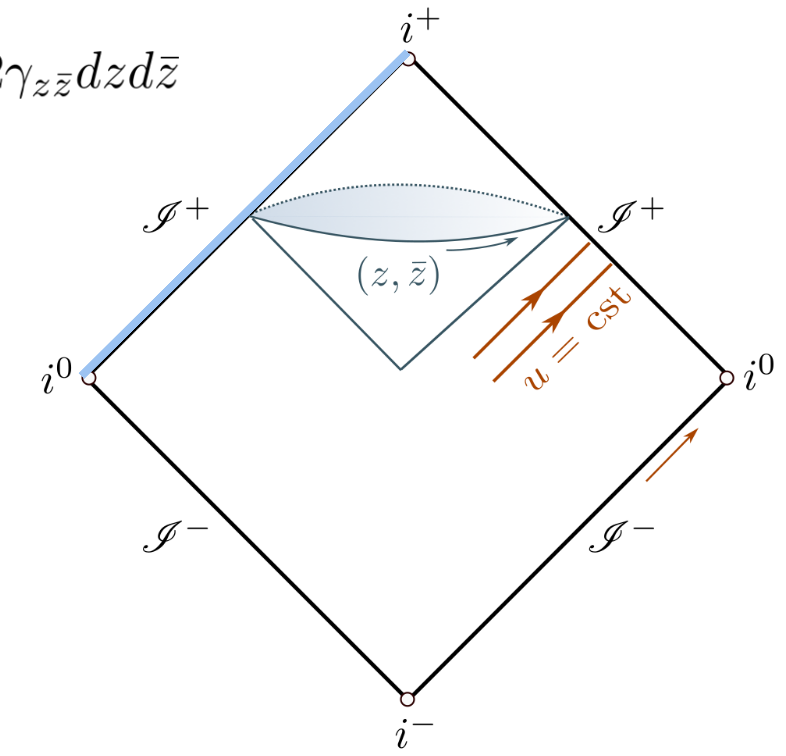
q_{ab} : a *degenerate* metric

a vector field satisfying $q_{ab}n^b = 0$

$\mathscr{I}^+$ $\quad x^a = (u, z, \bar{z})$

$$q_{ab}dx^a dx^b = 0 \times du^2 + 2\gamma_{z\bar{z}}dzd\bar{z}$$

$$n = \partial_u$$



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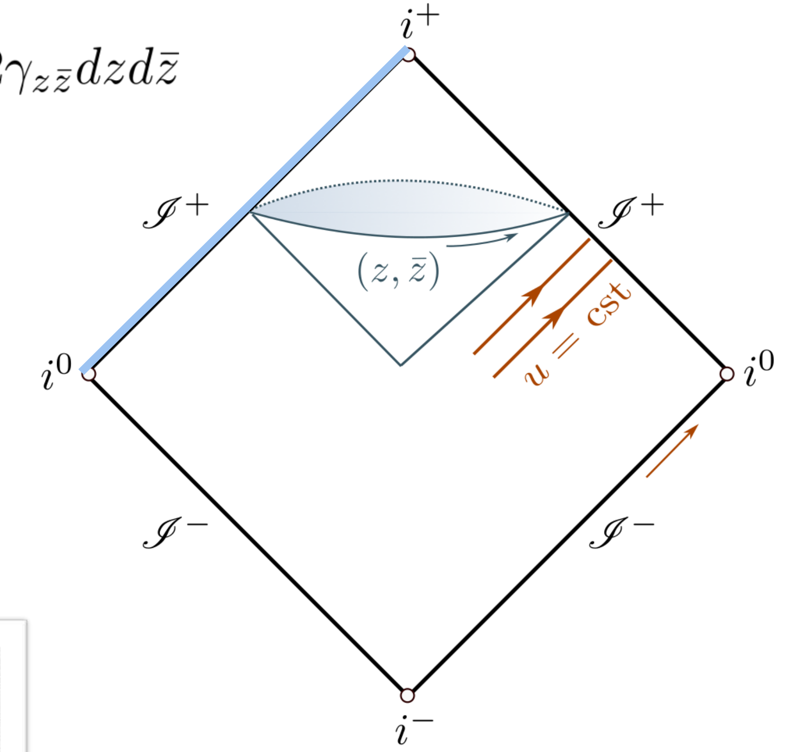

Conformal Carrollian symmetries:

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab} \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$$

$$\alpha := \frac{1}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})$$

$$\bar{\xi} = \left[\mathcal{T} + \frac{u}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

$$\boxed{\mathfrak{CCarr}_d = \mathfrak{bms}_{d+1}}$$



Carrollian 'dictionary'

Observables: S-matrix elements as correlators of a '**Carrollian**' field theory

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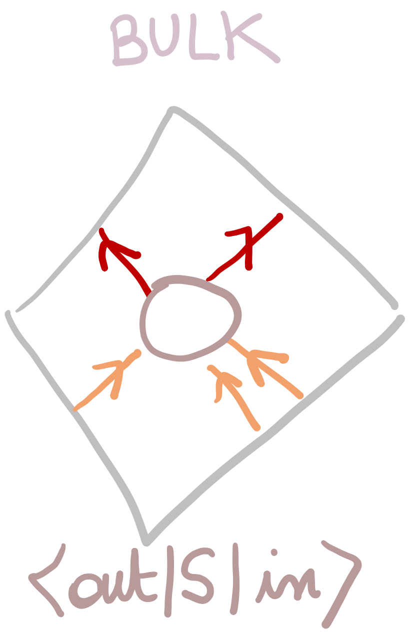

=

$$\left\langle \sigma_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}_1)\ldots\sigma_{k_m,\bar{k}_m}(u_m,z_m,\bar{z}_m)\right\rangle_{\mathrm{Carr}}$$

Field-operator map
(for outgoing massless spin s field)

$$\Phi^{(s)}(X) \overset{\mathscr{I}^+}{\sim} r^{s-1}\boxed{\sigma^{\mathrm{out}}_{k,\bar{k}}(u,z,\bar{z})}$$

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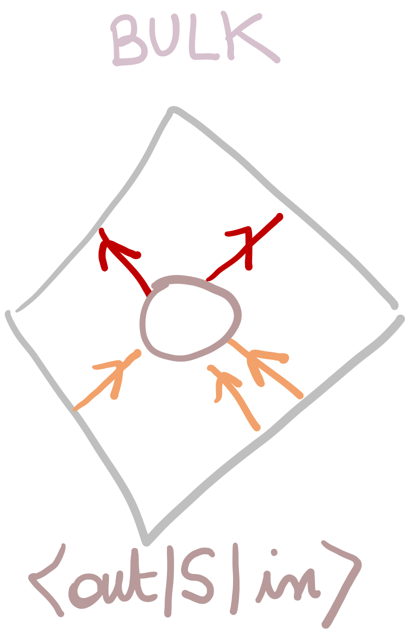

$$=$$

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transform as a 'conformal Carrollian primary' of weights $(k,\bar{k})$

$$\delta_{\bar{\xi}}\sigma_{k,\bar{k}} = \left[\left(\mathcal{T}+\tfrac{u}{2}(\partial\mathcal{Y}+\bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\sigma_{k,\bar{k}}$$

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  - ➔ much richer geometric structure at timelike and spatial infinity
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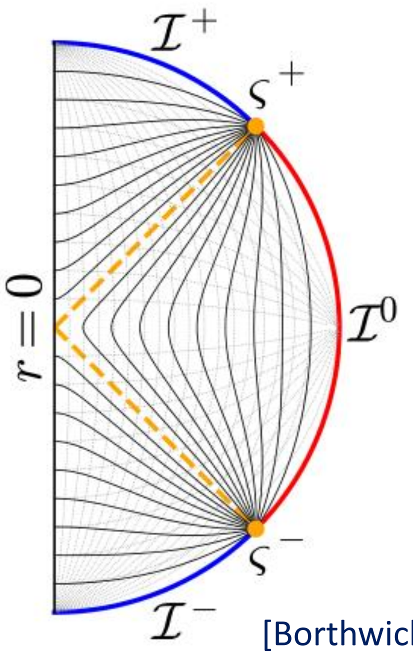
  - ➔ unified framework for generic scattering processes
    [Have, Nguyen, Prohazka, Salzer '24][Borthwick, Chantreau, Herfray '24]



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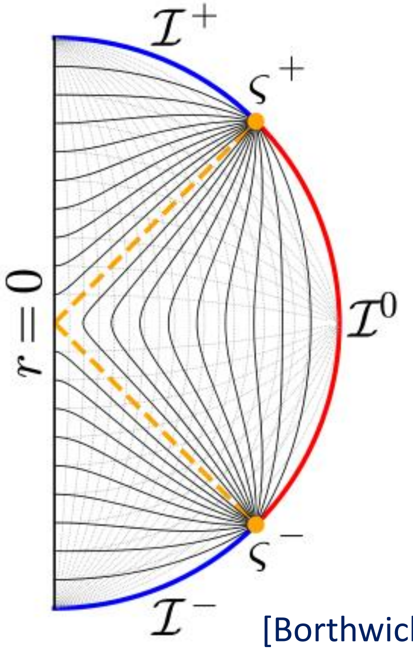

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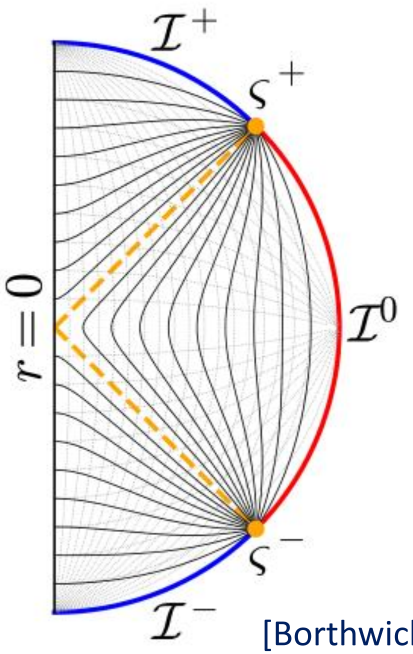

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  → subtle! Infinite degeneracies in the spectrum, non-normalizable ground states,..

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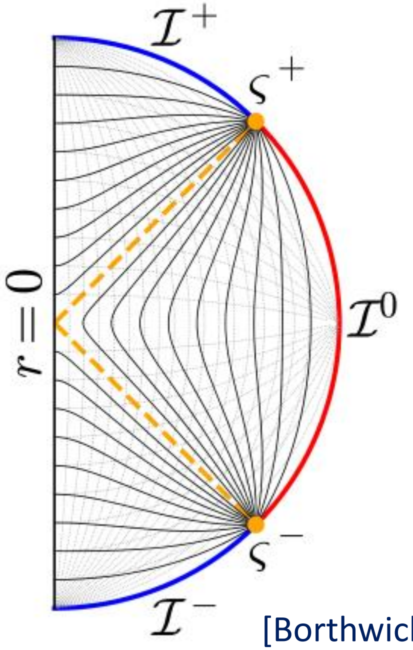

[Borthwick, Herfray '24]

- How to quantize a Carrollian field theory?
  - → subtle! Infinite degeneracies in the spectrum, non-normalizable ground states,..
    [de Boer, Hartong, Obers, Sybesma, Vandoren '23]
  - → strong UV sensitivity, UV/IR mixing (but tractable)
    can be regulated when placed on a lattice [Cotler, Jensen, Prohazka, Raz, Riegler, Salzer '24]

- … and more features! → see talks of Sucheta Majumdar, Kevin Nguyen, Gerben Oling

From **Carrollian** to **celestial** ‘dictionary’

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Observables: S-matrix elements as correlators of a ‘**Carrollian**’ field theory

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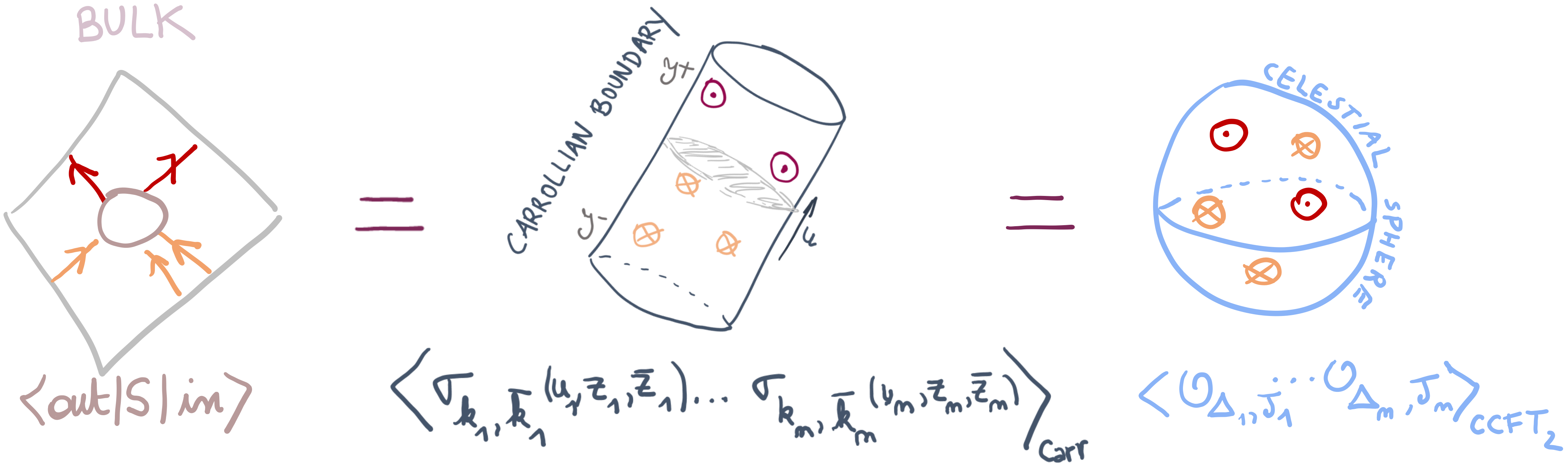

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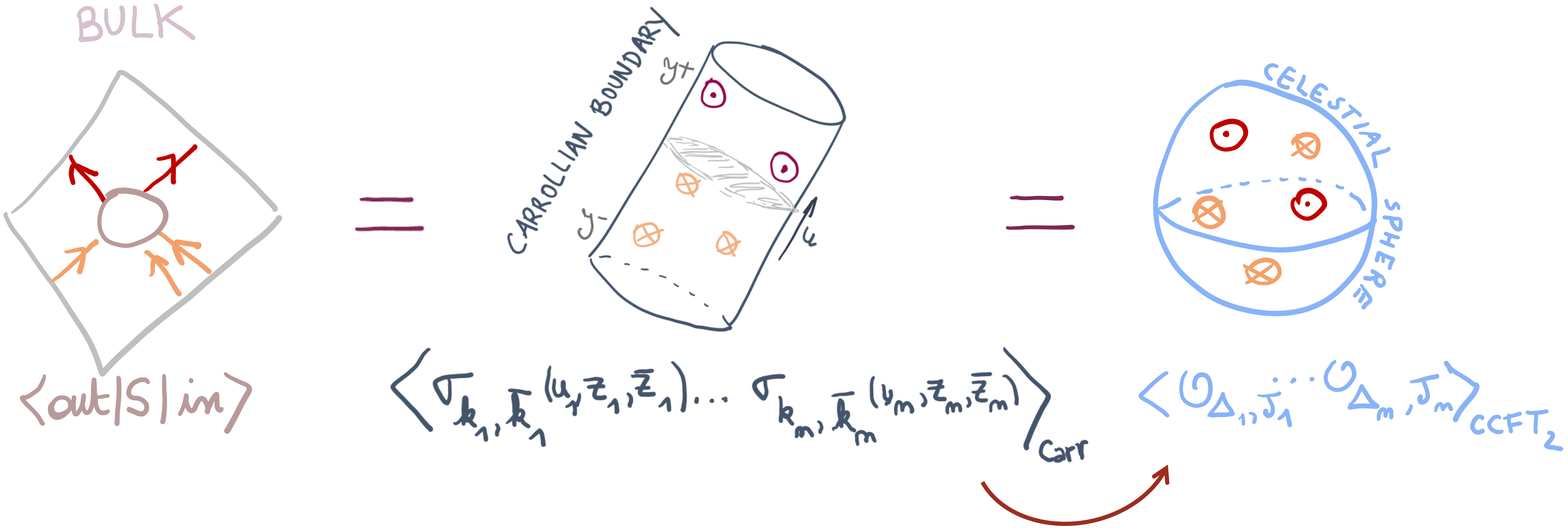


$$\mathcal{O}^{\text{out}}_{(\Delta,J)}(z,\bar{z}) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{du}{(u+i\epsilon)^{\Delta}}\,\sigma^{\text{out}}_{(k,\bar{k})}(u,z,\bar{z})$$

Carrollian – **celestial** operator map

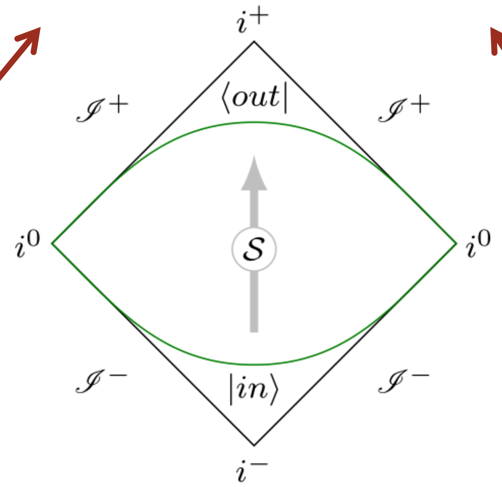
In summary

Momentum basis

$$\mathcal{A}_N = \langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle_{\mathrm{momentum}}$$

Mellin

$$\int_0^{+\infty} d\omega\, \omega^{\Delta-1}$$



Fourier

$$\int_0^{+\infty} d\omega\, e^{-i\omega u}$$

Celestial basis

$$\mathcal{M}_N = \langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle_{\mathrm{boost}}$$

Carrollian basis

$$\mathcal{C}_N = \langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle_{\mathrm{position}}$$

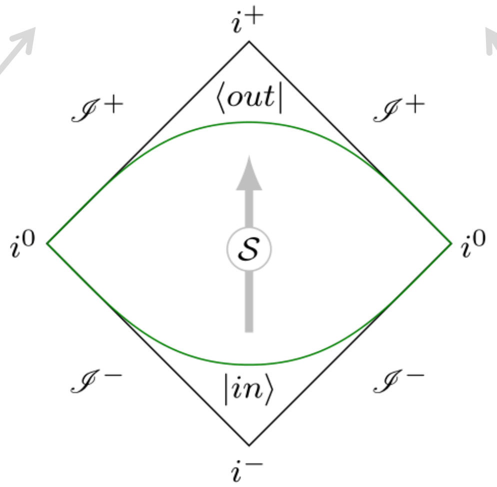
In summary

Momentum basis

$$\mathcal{A}_N = \langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle_{\mathrm{momentum}}$$

$$\int_0^{+\infty} d\omega\, \omega^{\Delta-1}$$

Mellin



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Celestial basis

$$\mathcal{M}_N = \langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle_{\mathrm{boost}}$$

Mellin + Fourier

$$\int_{-\infty}^{+\infty} du\, u^{-\Delta}$$

Carrollian basis

$$\mathcal{C}_N = \langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle_{\mathrm{position}}$$

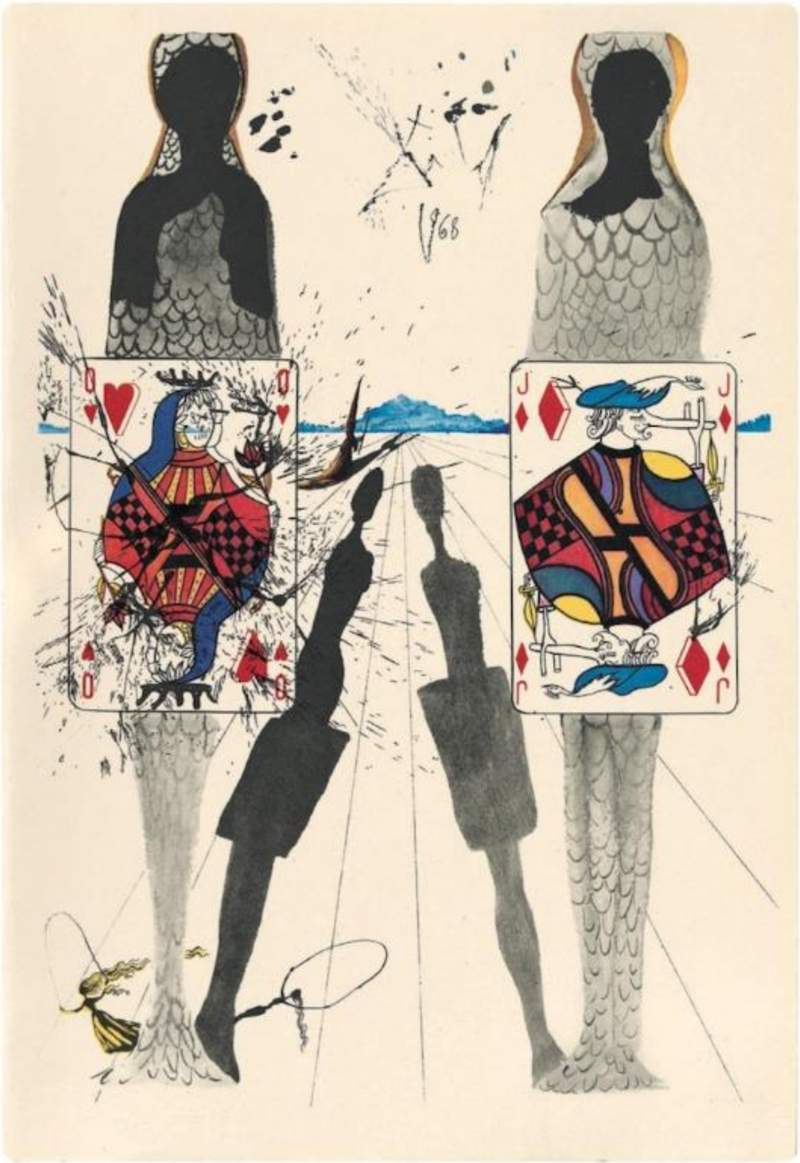
Just a change of basis?

Is this really holography?

Is this useful?

Can we learn something we did not know already?

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



$\mathscr{L}w_{1+\infty}$ symmetries

$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

- Celestial operators of integer conformal dimension give rise to 2d currents

$$H^k(z,\bar{z}) := \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\varepsilon,+2} \qquad k = 2, 1, 0, -1, \dots$$

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Celestial graviton OPE

- $$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\kappa}{2}\frac{1}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{(\bar{z}_{12})^{n+1}}{n!}\bar{\partial}^n\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

[Guevara, Himwich, Pate, Strominger ‘21]

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[Guevara, Himwich, Pate, Strominger '21]

$$H^k(z,\bar z) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar z^{n+\frac{k-2}{2}}}$$

the holomorphic modes close the algebra

$$\left[H^k_m, H^l_n\right] = -\frac{\kappa}{2}\left[n(2-k)-m(2-l)\right]\frac{(\frac{2-k}{2}-m+\frac{2-l}{2}-n-1)!}{(\frac{2-k}{2}-m)!(\frac{2-l}{2}-n)!}\frac{(\frac{2-k}{2}+m+\frac{2-l}{2}+n-1)!}{(\frac{2-k}{2}+m)!(\frac{2-l}{2}+n)!}H^{k+l}_{m+n},$$

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redefining

$$w^p_n = \frac{1}{\kappa}(p-n-1)!(p+n-1)!\, H^{-2p+4}_n$$

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$$\boxed{[w^p_m, w^q_n] = [m(q-1)-n(p-1)]\, w^{p+q-2}_{m+n}}$$

$$p = 1, \frac{3}{2}, 2, \frac{5}{2}, \dots$$

(super)translations

Virasoro

$$1-p \le m \le p-1$$

`wedge'

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(super)translations

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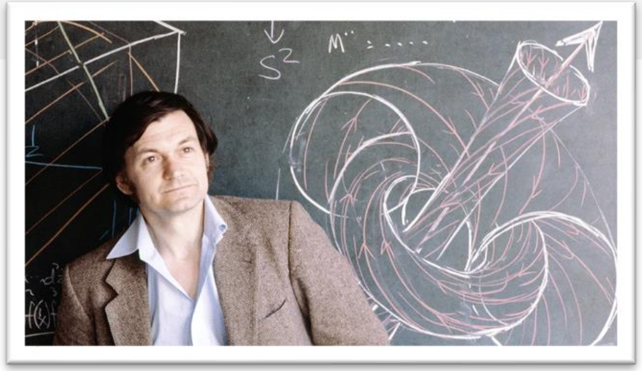
`wedge'

The infinite tower of celestial currents organizes into a single $\mathscr{L}w_{1+\infty}$ algebra ! [Strominger '21]

$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- … but how do these symmetries act on the Carrollian fields ?

$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity



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→ Go to twistor space !

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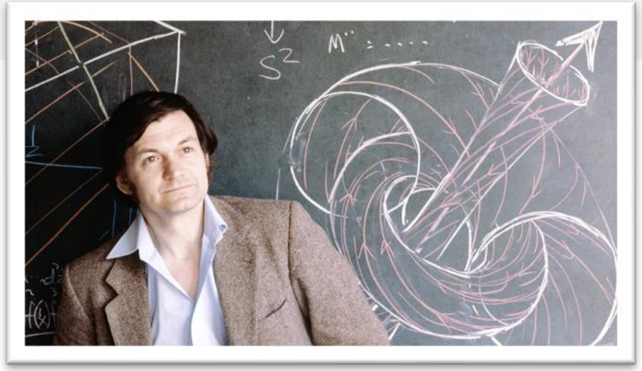

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- The $\mathscr{L}w_{1+\infty}$ algebra has a very natural implementation in twistor space [Penrose '76] [Boyer, Plebanski '85][Adamo, Mason, Sharma '22]

$$[Z^A] = (\mu^{\dot\alpha}, \lambda_\alpha(z)) \quad \in \mathbb{CP}^3$$

$$g = g_0(z) + g_{\dot\alpha}(z)\mu^{\dot\alpha} + g_{\dot\alpha\dot\beta}(z)\mu^{\dot\alpha}\mu^{\dot\beta} + \dots$$

$$\{g_1, g_2\} = \epsilon^{\dot\alpha\dot\beta}\frac{\partial g_1}{\partial\mu^{\dot\alpha}}\frac{\partial g_2}{\partial\mu^{\dot\beta}}$$

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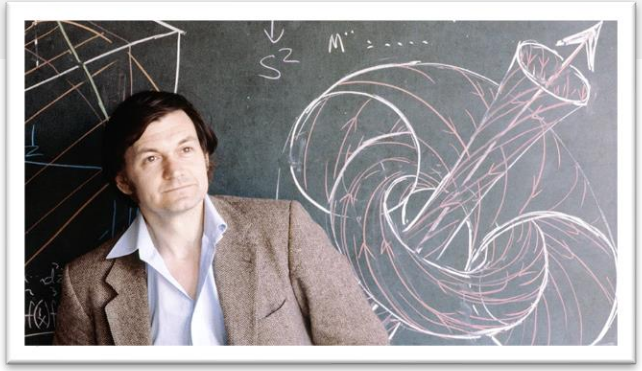

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$\mathscr{I}$

BOUNDARY

$\sigma(u, z, \bar{z})$

Carrollian field of weights

$(k, \bar{k}) = (\frac{1-s}{2}, \frac{1+s}{2})$

Large r expansion /
Kirchoff-d'Adhémar formula

BULK

$\Phi^{(s)}(X)$

zero-rest mass field of helicity s

$\mathbb{M}$

[Eastwood, Tod '82]

[LD, Herfray, Freidel '24]

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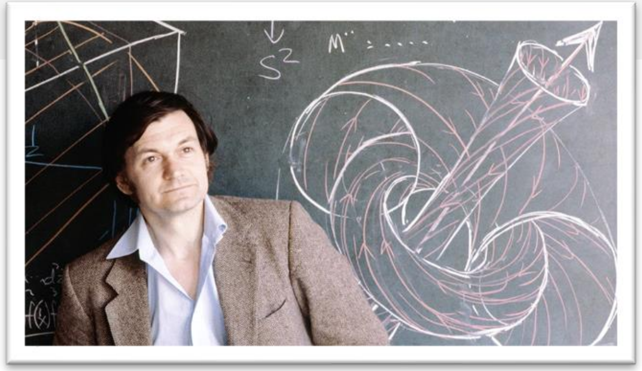

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Carrollian field of weights

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Twistor lift

TWISTOR SPACE

twistor representative

$$H^{0,1}(\mathbb{PT}, \mathcal{O}(2s-2))$$

$\mathbb{PT}$

Large r expansion /
Kirchoff-d'Adhémar formula

BULK

$$\Phi^{(s)}(X)$$

zero-rest mass field of helicity s

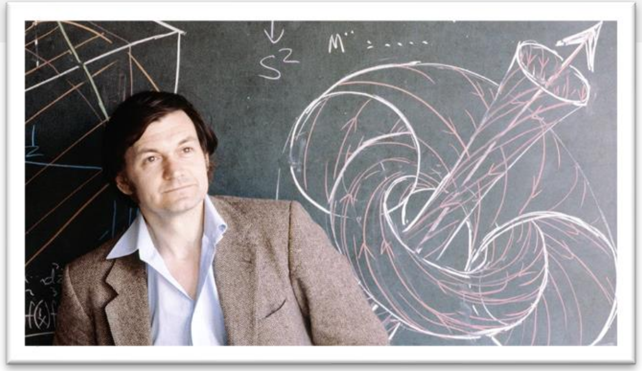
$\mathbb{M}$

Penrose transform
[Penrose '69]

[Eastwood, Tod '82]

[LD, Herfray, Freidel '24]

$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity



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$\mathscr{I}$

$$\bar{\sigma}(u,\lambda)$$

$$\Big\downarrow {\scriptstyle Lw_{1+\infty}}$$

GOAL: $\delta\bar{\sigma}(u,\lambda)$?

$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- … but how do these symmetries act on the Carrollian fields ?

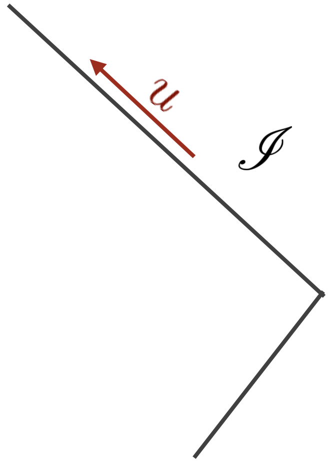
$\mathscr{I}$ $\qquad\qquad\qquad\qquad$ $\mathbb{PT}$

$$
\begin{tikzcd}
\bar\sigma(u,\lambda) \arrow[r] \arrow[d, "Lw_{1+\infty}"'] & h(u,\lambda) = \partial_u^{-1}\,\bar\sigma \arrow[r, "(i)", "\text{twistor lift}"'] & \mathbf{h} = h(u=\mu\lambda,\lambda)\, D\bar\lambda \arrow[d, "Lw_{1+\infty}"] \\
\delta\bar\sigma(u,\lambda) & \delta\Phi(x) = \displaystyle\int_{\mathbb{CP}^1} \frac{\partial^2 \delta\mathbf{h}}{\partial\mu\partial\mu} \arrow[l, "\text{large } r"', "(iii)"] & \delta\mathbf{h} = \{g, \mathbf{h}\} \arrow[l, "\text{Penrose transform}"', "(ii)"]
\end{tikzcd}
$$

$Z^A = (\mu^{\dot\alpha}, \lambda_\alpha) \in \mathbb{C}^4$

[LD, Herfray, Freidel '24]

- … but how do these symmetries act on the Carrollian fields ?



Main result:

$$\delta_n \bar{\sigma} = \sum_{\ell=0}^{n} \bar{\eth}^{n-\ell}\left(g_{\dot{\alpha}(n)}\bar{\lambda}^{\dot{\alpha}(n)}\right)\frac{\ell}{(n-\ell)!}\,\partial_u^3\left(u^{n-\ell}\,\partial_u^{-1-\ell}\,\bar{\eth}^{\ell-1}\bar{\sigma}\right) \qquad s=+2$$

$$\delta_n \sigma = \sum_{\ell=0}^{n} \bar{\eth}^{n-\ell}\left(g_{\dot{\alpha}(n)}\bar{\lambda}^{\dot{\alpha}(n)}\right)\frac{\ell}{(n-\ell)!}\,\partial_u^{-1}\left(u^{n-\ell}\,\partial_u^{3-\ell}\,\bar{\eth}^{\ell-1}\sigma\right) \qquad s=-2$$

$\mathscr{L}w_{1+\infty}$ generators

$n = 0, 1, 2, \ldots$

non-local action
(vs local in twistor space)

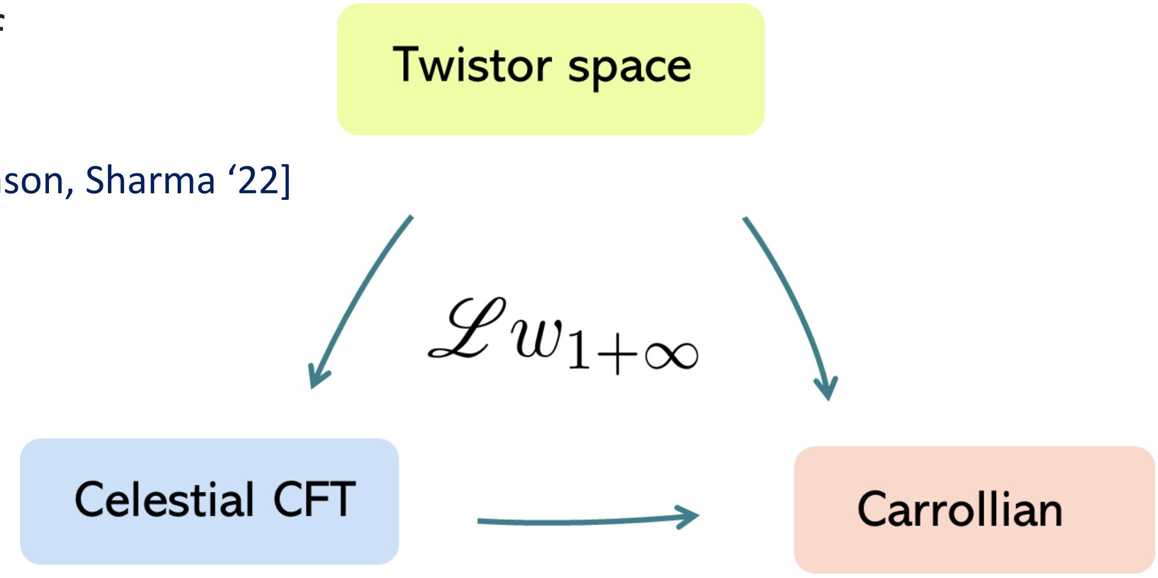
[LD, Herfray, Freidel '24]

✓ explicit match with [Freidel, Raclariu, Pranzetti '21]

In summary

- $\mathscr{L}w_{1+\infty}$ symmetries organize an infinite tower of celestial currents at tree level

  [Guevara, Himwich, Pate, Strominger '21][Strominger '21][Adamo, Mason, Sharma '22]



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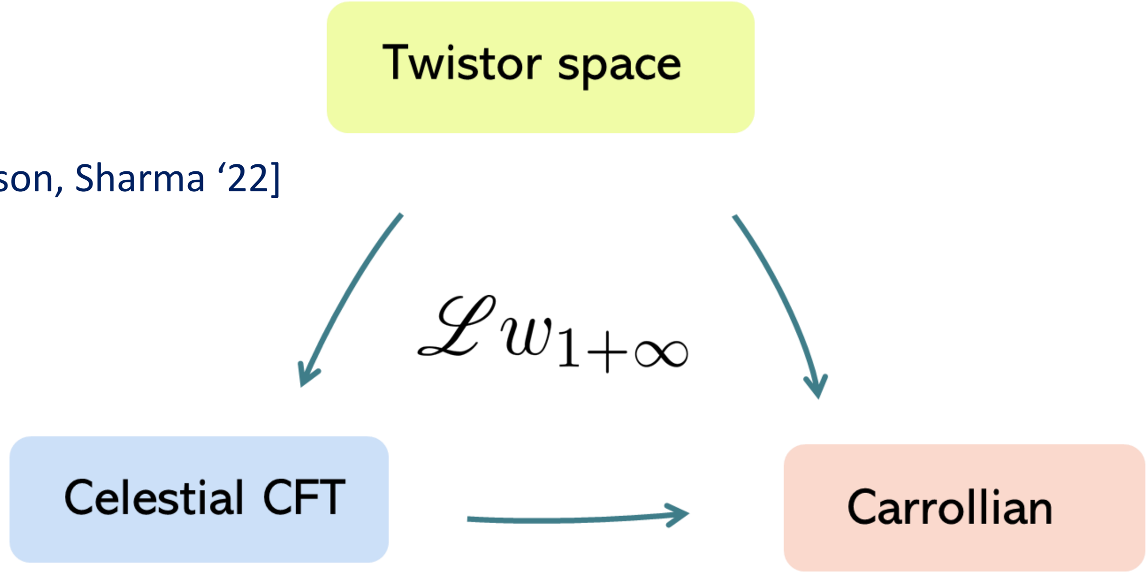
- There is an explicit realization of these symmetries for Carrollian fields at null infinity.

  [Freidel, Raclariu, Pranzetti '21][Geiller '24]
  [LD, Herfray, Freidel '24][Kmec, Mason, Ruzziconi, Srikant '24]

  The action of these symmetries is local in twistor space but non-local in spacetime.



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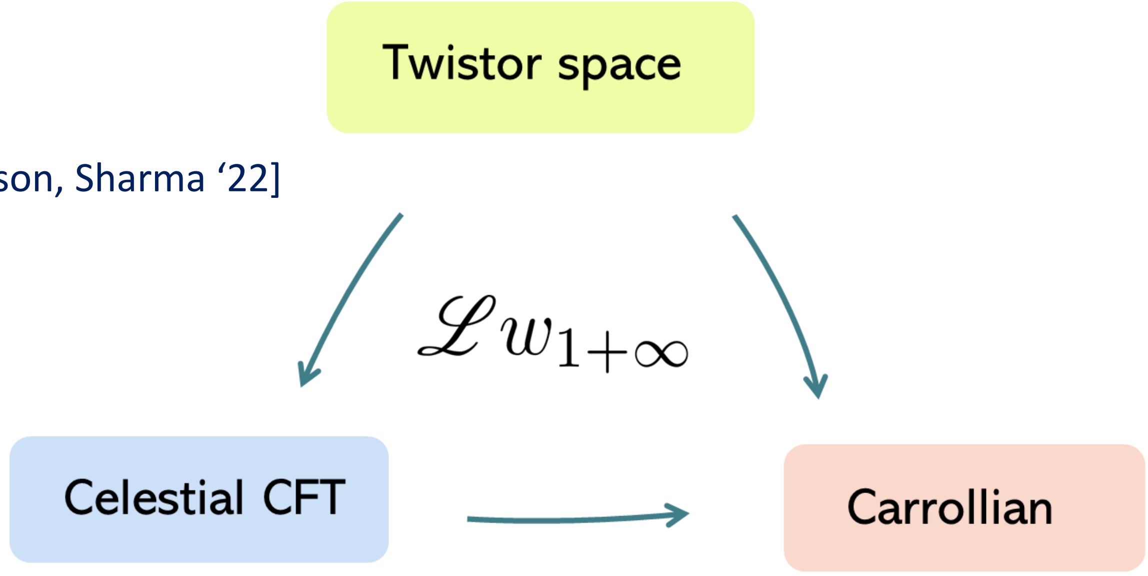

- Note: $\mathscr{L}w_{1+\infty}$ algebra deformation for $\Lambda \neq 0$ $\longrightarrow$ see Bin Zhu's talk

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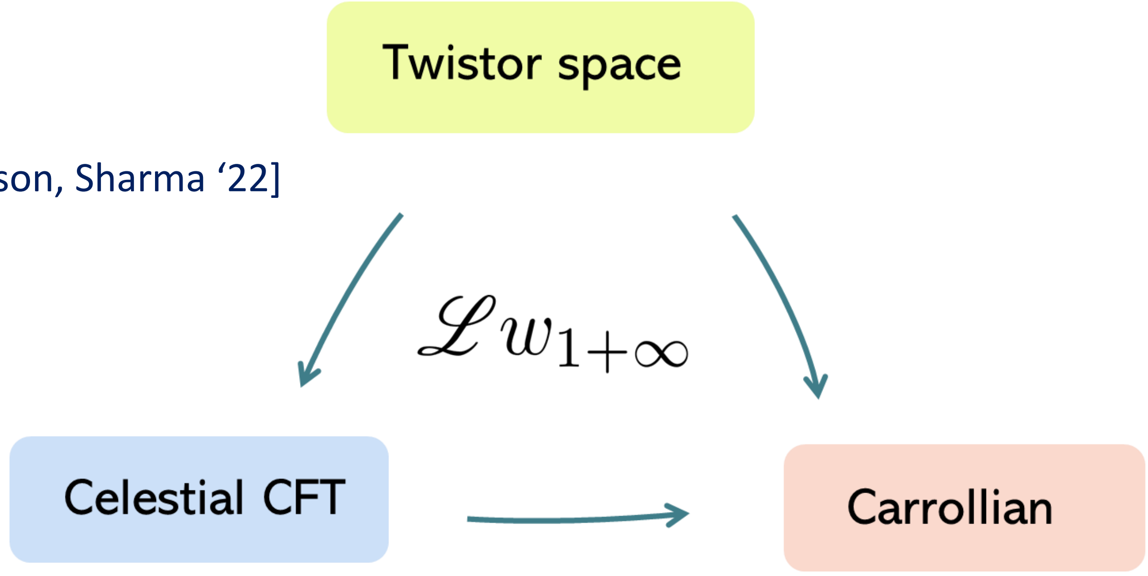


- Note: $\mathscr{L}w_{1+\infty}$ algebra deformation for $\Lambda \neq 0$ $\longrightarrow$ see Bin Zhu's talk

$\longrightarrow$ What is the faith of these symmetries beyond tree level ?

Final remarks

Loop corrections to soft theorems

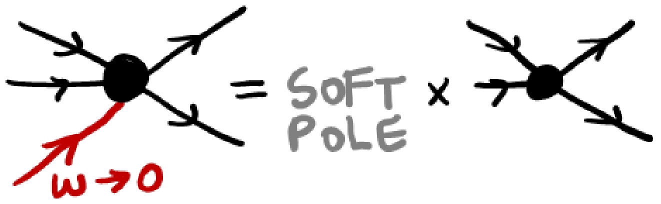
- Tree-level soft graviton theorem
  (power series expansion in the soft momentum $q = \omega\hat{q}$)

[Weinberg '65]
[Cachazo, Strominger '14]

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} + \omega^0\, S_n^{(1)}\right] \mathcal{A}_n + \mathcal{O}(\omega)$$

leading subleading

$$S_n^{(0)} = \frac{\kappa}{2}\sum_{i=1}^{n} \frac{p_i^\mu p_i^\nu \varepsilon_{\mu\nu}(\hat{q})}{p_i \cdot \hat{q}}$$



$$S_n^{(1)} = -\frac{i\kappa}{2}\sum_{i=1}^{n} \frac{p_i^\mu\, \varepsilon_{\mu\nu}(\hat{q})\, q_\lambda}{p_i \cdot q}\, \left(J_i^{\lambda\nu} + S_i^{\lambda\nu}\right)$$

$$\kappa = \sqrt{32\pi G}$$

Logarithmic soft theorems

- One-loop corrections generate logarithmic corrections!

$$\mathcal{A}_{n+1} \stackrel{\omega \to 0}{=} \left[\omega^{-1} S_n^{(0)} - \frac{\kappa^2}{4} \ln \omega \, S_n^{(\ln)} \right] \mathcal{A}_n + \mathcal{O}(\omega^0)$$

dominate over the subleading term

[Laddha, Sen ’18 ‘19] [Sahoo, Sen ‘18] [Saha, Sahoo, Sen ‘19][Krishna, Sahoo ‘23]
[Ciafaloni, Colferai, Veneziano ‘18] [Addazi, Bianchi, Veneziano ’19]
[di Vecchia, Heissenberg, Russo, Veneziano ‘23][Alessio, di Vecchia ‘24]

Logarithmic soft theorems

- One-loop corrections generate logarithmic corrections! [Sahoo, Sen '18][…]

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4} \ln\omega\, S_n^{(\ln)}\right] \mathcal{A}_n + \mathcal{O}(\omega^0)$$

$$\begin{aligned} S_n^{(\ln)} = {} & \frac{i\kappa}{8\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot q} \sum_j \delta_{\eta,\eta_j}\, q\cdot p_j \\ & + \frac{i\kappa}{16\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{p_i \cdot q} \sum_j \delta_{\eta_i,\eta_j} (p_i\cdot p_j)\,(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho) \frac{2(p_i\cdot p_j)^2 - 3p_i^2 p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}} \\ & - \frac{\kappa}{8\pi^2} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu p_i^\nu}{p_i\cdot q} \sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j| \\ & - \frac{\kappa}{32\pi^2} \sum_i \frac{p_i^\mu \varepsilon_{\mu\nu} q_\lambda}{p_i\cdot q} \left(p_i^\lambda \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\lambda}}\right) \sum_j \frac{2(p_i\cdot p_j)^2 - p_i^2 p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2 p_j^2\right]^{1/2}} \ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}\right) \end{aligned}$$

Logarithmic soft theorems

- One-loop corrections generate logarithmic corrections!

[Sahoo, Sen '18][…]

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4}\ln\omega\, S_n^{(\ln)}\right]\mathcal{A}_n + \mathcal{O}(\omega^0)$$

[Agrawal, LD, Nguyen, Ruzziconi '23]

$$S_n^{(\ln)} = \frac{i\kappa}{8\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{p_i\cdot q}\sum_j \delta_{\eta,\eta_j}\, q\cdot p_j$$
$$+\frac{i\kappa}{16\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu q_\rho}{p_i\cdot q}\sum_j \delta_{\eta_i,\eta_j}(p_i\cdot p_j)\left(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right)\frac{2(p_i\cdot p_j)^2 - 3p_i^2p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2p_j^2\right]^{3/2}}$$
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$$-\frac{\kappa}{32\pi^2}\sum_i \frac{p_i^\mu\varepsilon_{\mu\nu}q_\lambda}{p_i\cdot q}\left(p_i^\lambda\frac{\partial}{\partial p_{i\nu}} - p_i^\nu\frac{\partial}{\partial p_{i\lambda}}\right)\sum_j \frac{2(p_i\cdot p_j)^2 - p_i^2p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2p_j^2\right]^{1/2}}\ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2p_j^2}}\right)$$

Can be exactly **derived** from superrotation **symmetry** conservation!

Key ingredient: **Goldstone modes** and **dressing** at timelike infinity

⟶ see talks of Shreyansh Agrawal and Sangmin Choi

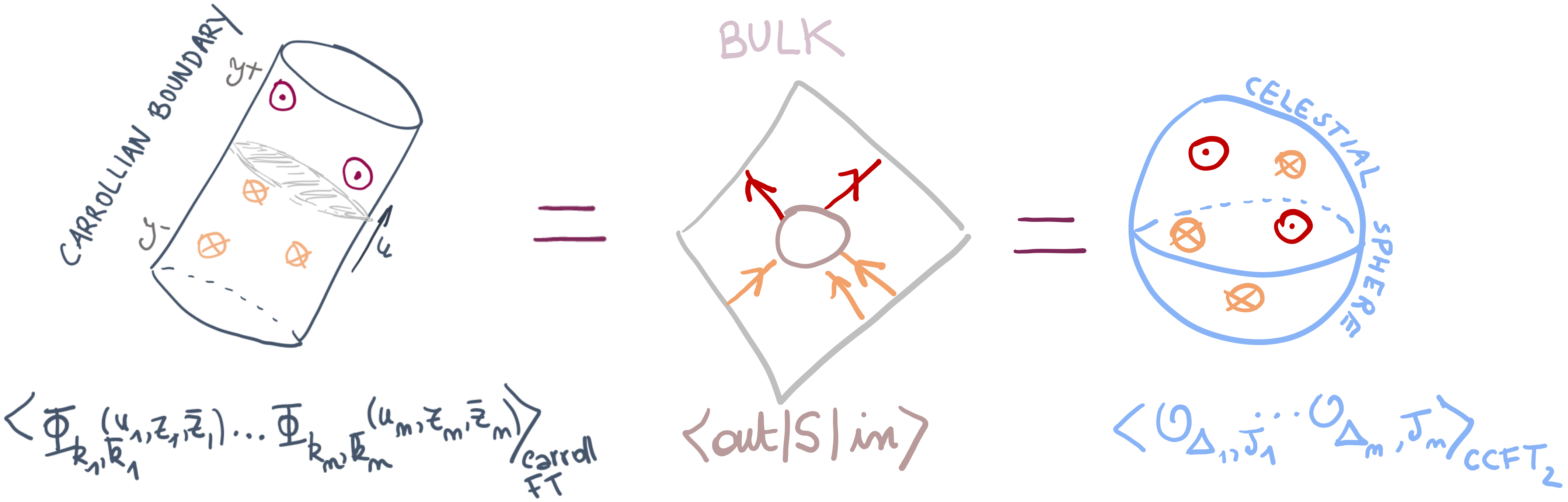
Summary and outlook

Summary and outlook

Celestial **CFT** living on the celestial sphere

$\updownarrow$ $\qquad\longleftrightarrow\qquad$ quantum gravity in flat spacetime

Conformal Carrollian field theory living at null infinity



Summary and outlook

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↕

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What is a **CCFT**?

→ Beyond kinematics? Top-down constructions?

Summary and outlook

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↕ ⟷ quantum gravity in flat spacetime

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What is a **CCFT**?
→ Beyond kinematics? Top-down constructions?

full tower of currents
link with AdS/CFT, dS/CFT → see talks of Karan Fernandes, Raffaele Marotta, Yuyu Mo, Romain Ruzziconi
building representations
loop corrections & log CFT [Bhardwaj, Lippstreu, Ren, Spradlin, Srikant, Volovich '22] [LD, Bissi, Valsesia '24]
bootstrapping CCFT
higher dimensions → see Tim Adamo's talk
adding black holes
… → see Ronnie Rodger's talk

BMS symmetries in the sky

LD, Boris Goncharov, Jan Harms, *Phys. Rev. Lett.* 2024

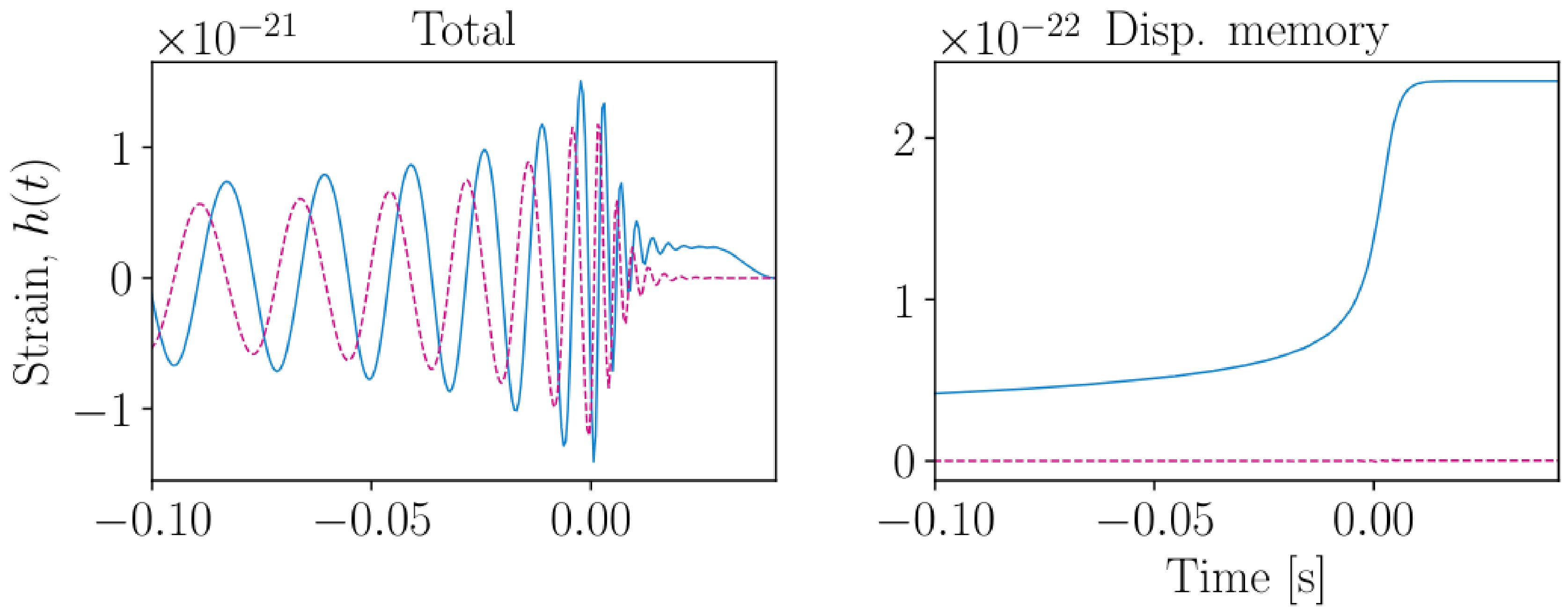
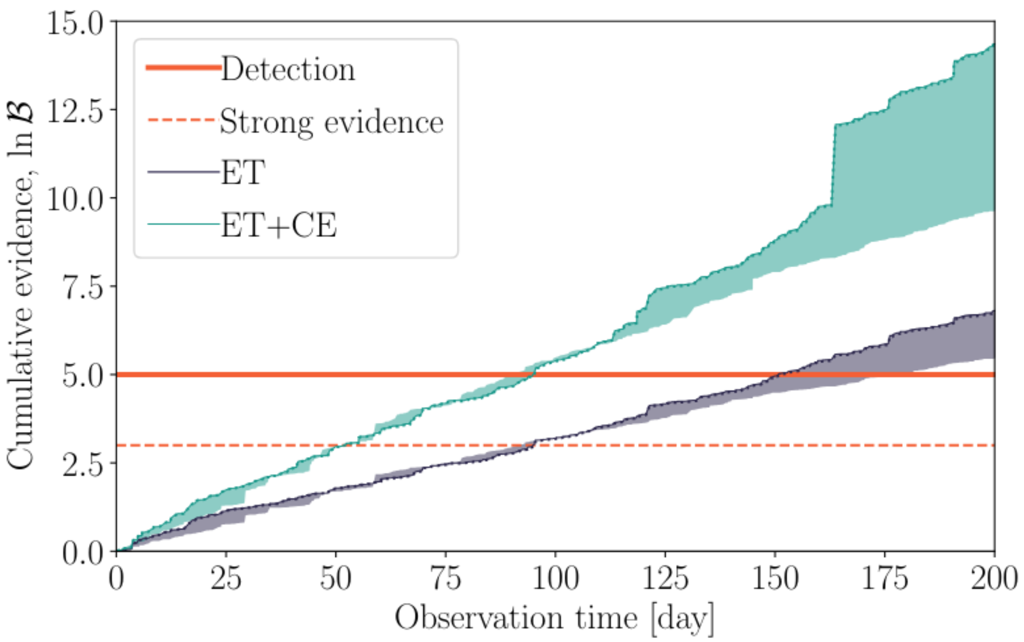


FIG. 4: Demonstration of the GW memory contribution to strain from a merger of two non-spinning BBHs in the extended BMS scenario, $(m_1, m_2, \theta_{jn}, z) = (30\ M_\odot, 30\ M_\odot, \pi/3, 0.06)$. Solid lines show h_+, dashed lines show $h_\times$.

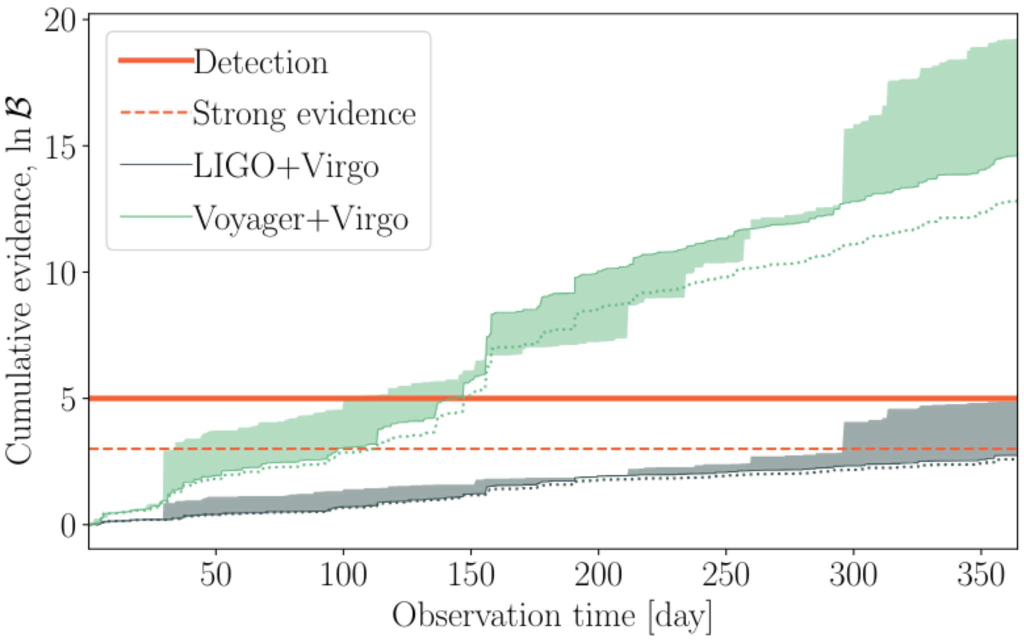
BMS symmetries in the sky

LD, Boris Goncharov, Jan Harms, *Phys. Rev. Lett.* 2024

Model selection between standard and extended (superrotation) BMS symmetries.



Einstein Telescope (ET) and Cosmic Explorer (CE)



LIGO and VIRGO

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gravitational waves observation

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Thank you!