The Dual of Semi-Classical Gravity

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Based on work with Alex Belin, Diego Liska, Pranjal Nayak, Tarek Anous, Julian Sonner, Daniel Jafferis, Boris Post, Martin Sasieta, Jildou Hollander, Andrew Rolph, Ramesh Chandra, Igal Arav, Shira Chapman ‘20-’24

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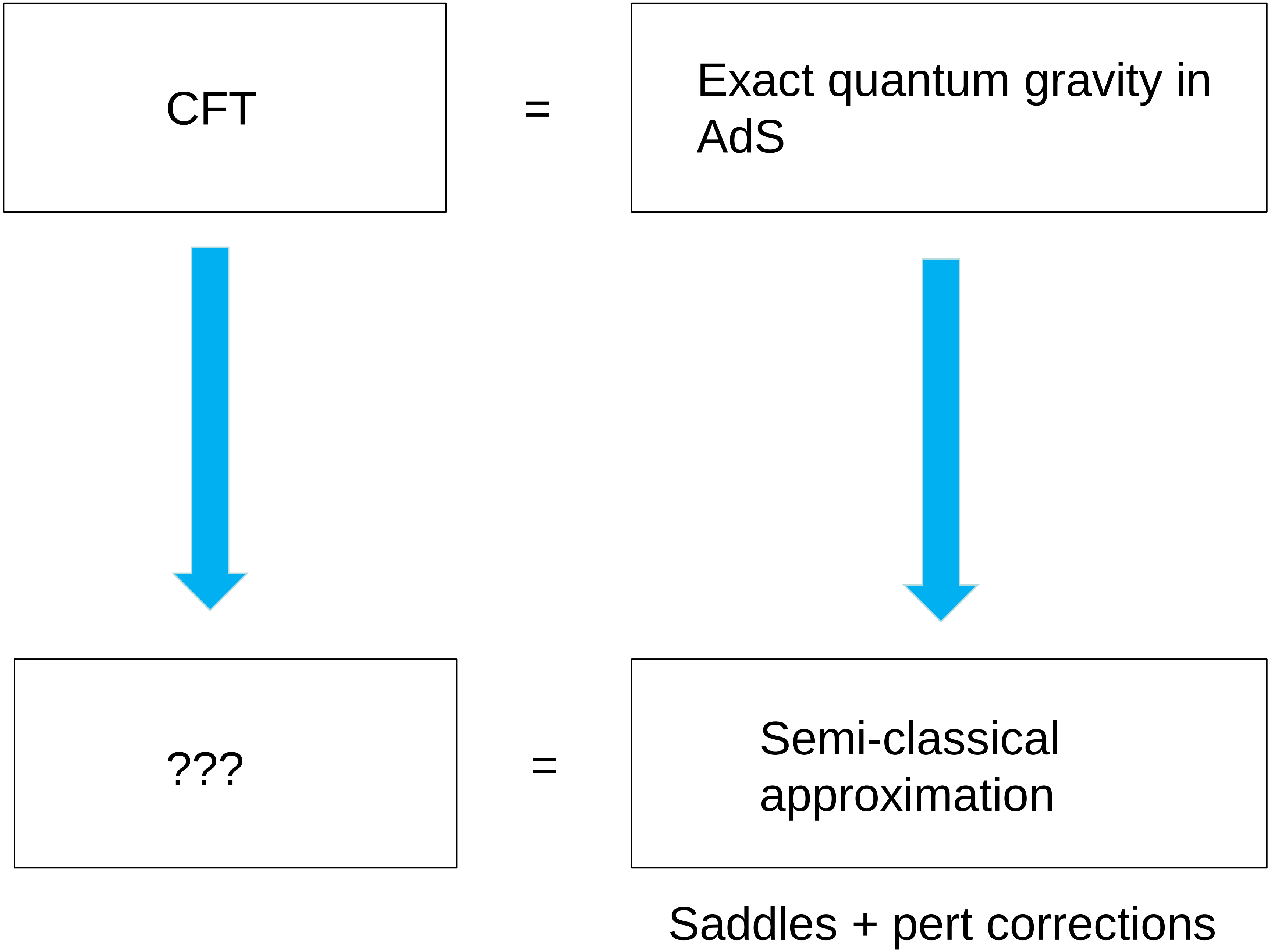

CFT
=
Exact quantum gravity in AdS
???
=
Semi-classical approximation
Saddles + pert corrections

$$Z(\beta) = \sum_i e^{-\beta E_i}$$ = ???

CFT

GRAVITY

??? = $$Z(\beta) \sim \exp(c/\beta^{d-1})$$

Saddles + pert corrections

$$Z(\beta) = \sum_i e^{-\beta E_i}$$

=

???

CFT

GRAVITY

$$Z(\beta) = \int dE\, \rho(E) e^{-\beta E}$$

$$\rho(E) \sim E^{\frac{d}{d-1}}$$

=

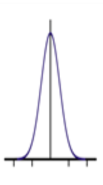
$$Z(\beta) \sim \exp(c/\beta^{d-1})$$

Saddles + pert corrections

In the CFT, the exact spectrum gets replaced by a continuous coarse grained spectral density.

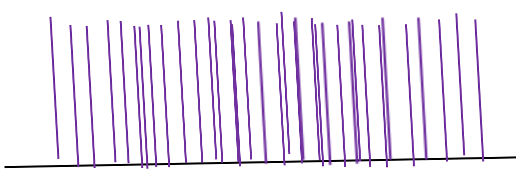
What is coarse graining?

$$\rho_{\mathrm{semi-classical}}(E) = \int dE'\, K(E,E')\, \rho_{\mathrm{exact}}(E')$$



Alternatively, we can write an equation which involves some notion of averaging

$$\rho_{\mathrm{semi-classical}}(E) = \int d\alpha\, \mu[\alpha]\, \rho^{\alpha}_{\mathrm{exact}}(E)$$



where $\rho^{\alpha}_{\mathrm{exact}}$ is semi-classically indistinguishable from ρ_{exact} .

Coarse graining/averaging is an ambiguous procedure.

Statistical physics gives us a preferred method to deal with situations like this (Wigner ‘55 Balian ’68).

Maximize ignorance (=entropy) subject to the constraints imposed by the semi-classical approximation:

$$\int dH \; -\mu[H]\log\mu[H] + \mu[H]\int d\beta\lambda(\beta)\left(\mathrm{Tr}(e^{-\beta H}) - Z(\beta)\right)$$

One finds

$$\mu[H] \sim \exp\left(\int d\beta\lambda(\beta)\mathrm{Tr}(e^{-\beta H})\right) \sim \exp(-\mathrm{Tr}V(H))$$

where V is arbitrary but needs to be fixed to yield the right partition function (or spectral density).

JdB, Liska, Post, Sasieta ‘23

This shows that *in the absence of other information* the best description of the Hamiltonian of a theory with a continuous spectral density is in terms of a matrix model.

For a chaotic theory, it may be difficult to obtain more detailed information about the spectrum and this may be the best one can do.

(Black holes are very chaotic Maldacena, Stanford, Shenker ’15)

This would resonate with the Bohigas–Giannoni–Schmit (BGS) conjecture(1984) which asserts that the spectral statistics of quantum systems whose classical counterparts exhibit chaotic behavior are described by random matrix theory.

One can play a similar game for much more general choices of data. Suppose for example that we know some correlators of an operator A and we want to extract a probability distribution $\mu[A]$ on the space of operators.

The general picture is one where if one e.g. inputs connected ≤k-point correlators, one gets a “matrix model” with up to k-th order interactions in the exponent.

Lagrange multipliers

$$\int dA d\lambda_i \left(-\mu[A] \log \mu[A] + \sum_i \lambda_i \, \mu[A] \left(f_i[A] - c_i \right) \right)$$

Shannon entropy

Input observations

$$\Rightarrow \mu[A] \sim e^{-\sum_i \lambda_i f_i[A]}$$

Consider for example the finite temperature one and two-point functions of some operator A.

$$\langle A(0)A(t)\rangle_\beta = \sum_{i,j} e^{-\beta(E_i+E_j)/2} e^{i(E_i-E_j)t} |\langle i|A|j\rangle|^2$$

These correlation functions (which can be semi-classically computed by a propagator in a black hole background) can be used to produce a statistical model for $\langle i|A|j\rangle$ and the result is a quadratic matrix model.

This quadratic matrix model is a familiar result. It is usually stated as the so-called Eigenstate Thermalization Hypothesis:

$$\langle E_i|O_a|E_j\rangle = \delta_{ij} f_a(\bar{E}) + e^{-S(\bar{E})/2} g_a(\bar{E},\Delta E) R^a_{ij}$$

Deutsch '91
Srednicki '94
Foini, Kurchan '19

$f_a(\bar{E})$: one point functions of simple operators
$g_a(\bar{E},\Delta E)$: two point functions of simple operators
R^a_{ij} : Gaussian random variables

$$\langle R^a_{ij}\rangle = 0, \qquad \langle R^a_{ij} R^b_{kl}\rangle = \delta^{ab}\delta_{il}\delta_{jk}$$

JdB, Liska, Post, Sasieta, '23

As before, this shows that *in the absence of other information* the best description of the matrix elements of a simple operator is in term of ETH.

As before, for a chaotic theory, it may be difficult to obtain more detailed information about these matrix elements and this may be the best one can do.

One can also include thermal higher-point functions and these will give rise to higher order moments for the matrices R^a_{ij} (non-gaussianities).

We now apply this logic to semi-classical gravity in AdS.

This will result in a statistical description which contains all (not necessarily consistent) microscopic theories which are semi-classically indistinguishable.

Semi-classical gravity involves a coarse-graining over high-energy microstates (i.e. black hole microstates). Therefore the ignorance and statistical description will mostly involved the high-energy spectrum. → *typicality*

The equivalence between semi-classical gravity and a statistical description is the coarse grained version of holograpy we were looking for.

To be concrete will mostly focus on AdS3 from now on.

As argued, the spectrum as obtained from semi-classical black hole computations is best modeled by a (double scaled) random matrix model with a suitable choice of potential V(M).

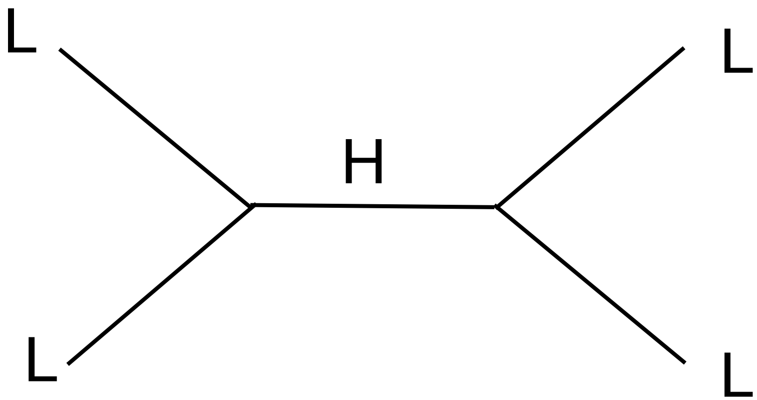
In the presence of additional quantum numbers, we can compute black hole entropy as a function of energy and the charges.

Charges are quantized and not chaotic. So we get a family of random matrices labeled by the additional quantum numbers. Will mostly ignore this.

Beyond the spectrum, an important part of any CFT are the Operator Product Expansion (OPE) coefficients. How to model those with semi-classical computations?

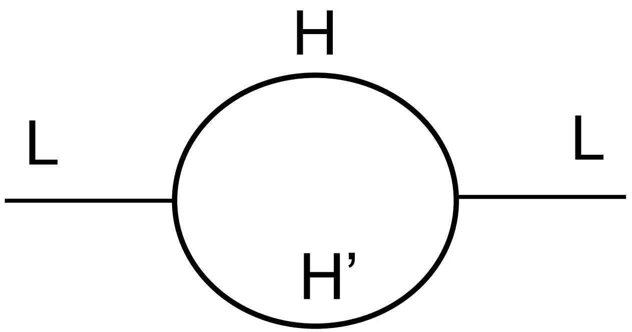
For 2d theories with a holographic AdS3 dual:

- we only have explicit access to low-lying operators (denoted L) and not to very high dimension operators corresponding to black holes (denoted H)
- we can compute correlation functions where the number of operators is $\ll c$
- we can compute partition functions on surfaces with genus $\ll c$
- we can compute correlation functions in Lorentzian signature as long as the center of mass is sub Planckian
- all computations are at best done up to non-perturbative errors of order e^{-c}



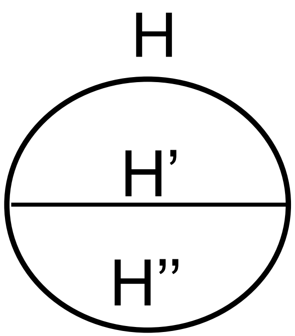
$$\sum_{H} C^2_{LLH}$$

4 point correlator on S^d



$$\sum_{H,H'} C^2_{LHH'}$$

2 point correlator on $S^{d-1} x S^1$



$$\sum_{H,H',H''} C^2_{HH'H''}$$

Connected sum of two times $S^{d-1} x S^1$

Benjamin, Lee, Ooguri, Simmons-Duffin ‘23

Input gives rise to quadratic matrix model for the C’s

Semi-classical gravity = $\overline{CC} = \int dC\, e^{-V[C]}\, CC$

$$Z(\beta) = e^{\frac{\pi^2 c}{3\beta}} = \int dM e^{-\mathrm{Tr} V[M]}\, \mathrm{Tr}\left(e^{-\beta M}\right)$$

This is what gave rise to the OPE randomness hypothesis (Belin, JdB '20):

$$C_{LLH} \sim f(E_H) R_H$$

$$C_{LHH'} \sim f(\bar{E_H}, \Delta E_H) R_{HH'}$$

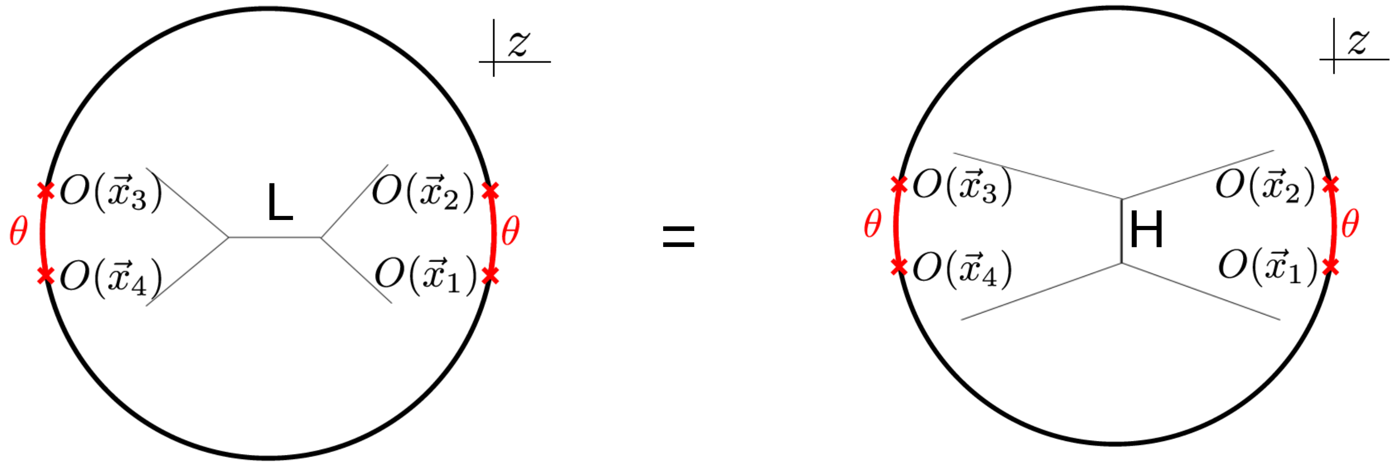
$$C_{HH'H''} \sim f(\bar{E_H}, \Delta E_H) R_{HH'H''}$$

Example of ETH

Slowly varying function of arguments

- Pseudorandom
- Mean=0
- Variance=1
- Can have higher moments which are exponentially suppressed.

Example:



$$\frac{1}{\theta^{4\Delta_L}} \simeq \sum_H C^2_{LLH} \left(\cos\frac{\theta}{2}\right)^{2\Delta_H}$$

$$\overline{|C_{LLH}|^2} \sim \frac{\Delta_H^{2\Delta_L-1}}{\Gamma(2\Delta_L)\rho(\Delta_H)}$$

Pappadopulo, Rychkov, Espin, Ratazzi '12

Of course, general computations involve both the OPE coefficients and the spectrum of the theory so there will also be cross-correlations.

In 2d, the result of all of this is a mixed matrix/tensor model which encodes statistics in the spectrum and statistics of OPE coefficients.

cf Jafferis, Kolchmeyer, Mukhametzhanov, Sonner '22

In d>2, we do not know what the minimal set of data is to fully describe a CFT, but whatever those are, we get a corresponding statistical model. (Casimir energy on T^{d-1} is not obviously expressible in terms of Δ_i and C_{ijk})

Belin, JdB, Kruthoff, Michel, Shaghoulian, Shyani '16
Belin, JdB, Kruthoff''18

Semi-classical gravity = $\overline{CC} = \int dC\, e^{-V[C]}\, CC$

+

$$Z(\beta) = e^{\frac{\pi^2 c}{3\beta}} = \int dM e^{-\mathrm{Tr} V[M]}\, \mathrm{Tr}\left(e^{-\beta M}\right)$$

+ interactions

Overlaps of states

In the same spirit, suppose we prepare semiclassically a set of states $|\psi_i\rangle$ with some high energy E. If these states form black holes (possibly after some time evolution) they become semi-classically indistinguishable.

Model these states as $|\psi_i\rangle = C_{ia}|\psi_a\rangle$ with $(C_i)_a$ some random unit norm vector acting in a microcanonical energy window and $|\psi_a\rangle$ some fixed orthonormal basis for that window.

Semiclassical computation yields $\langle\psi_i|\psi_j\rangle = \delta_{ij}$

Maximal ignorance principle yields the flat measure on the C's. Indeed

$$\overline{\langle\psi_i|\psi_j\rangle} = \int dC\, \langle\psi_a|C^*_{ia}C_{jb}|\psi_b\rangle = \delta_{ij}$$

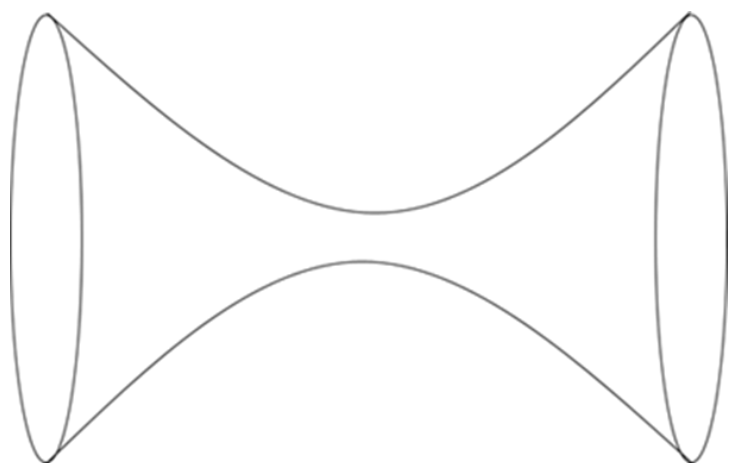
What is this good for???

- It sheds light on the chaotic nature of CFTs
- It explains the factorization problem
- It explains replica wormholes and state overlaps
- It sheds light on how the semi-classical approximation is still compatible with the Page curve
- It is a useful perspective on situations with multiple boundaries or replicas

The factorization problem

Harlow '15
Guica, Harlow '15
Harlow, Jafferis '18
Saad, Shenker, Stanford '19
Marolf, Maxfield, '20
……..

The gravitational path integral includes connected geometries with multiple boundaries



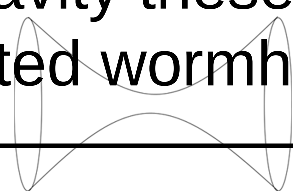
These seem to violate factorization of the CFT on disconnected manifold: $\langle Z(\beta_1)Z(\beta_2)\rangle \neq \langle Z(\beta_1)\rangle\langle Z(\beta_2)\rangle$

This *apparent* lack of factorization arises as follows: above we built a statistical model using gravitational computations with a single boundary.
This produced a “single trace” matrix/tensor model.

Statistical models predict correlations between multiple copies of the theory.

$$\int dH\,\mu[H]\mathrm{Tr}(e^{-\beta_1 H})\mathrm{Tr}(e^{-\beta_2 H}) - \int dH\,\mu[H]\mathrm{Tr}(e^{-\beta_1 H}) \int dH\,\mu[H]\mathrm{Tr}(e^{-\beta_2 H}) \neq 0$$

We propose that in gravity these correlations precisely correspond to connected wormhole geometries.



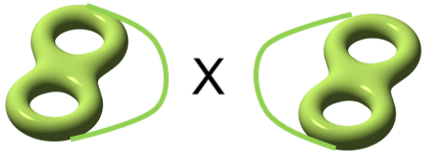
Conjecture: wormholes compute the correlations of the one-sided statistical model. They contain no new information.

Intuition for the conjecture: one-sided computations allow one to reconstruct the bulk Lagrangian. Crossing symmetry is closely related to bulk locality. So all information which is needed to compute wormholes semi-classically is *in principle* available

This conjecture has been tested fairly extensively (Alex Belin, JdB ’20; Chandra, Collier, Hartman, Maloney ’22) for computations involving OPE coefficients in AdS3. More general understanding for pure 3d gravity follows from the Virasoro TQFT (Collier, Eberhardt, Zhang ‘23).

A simple example

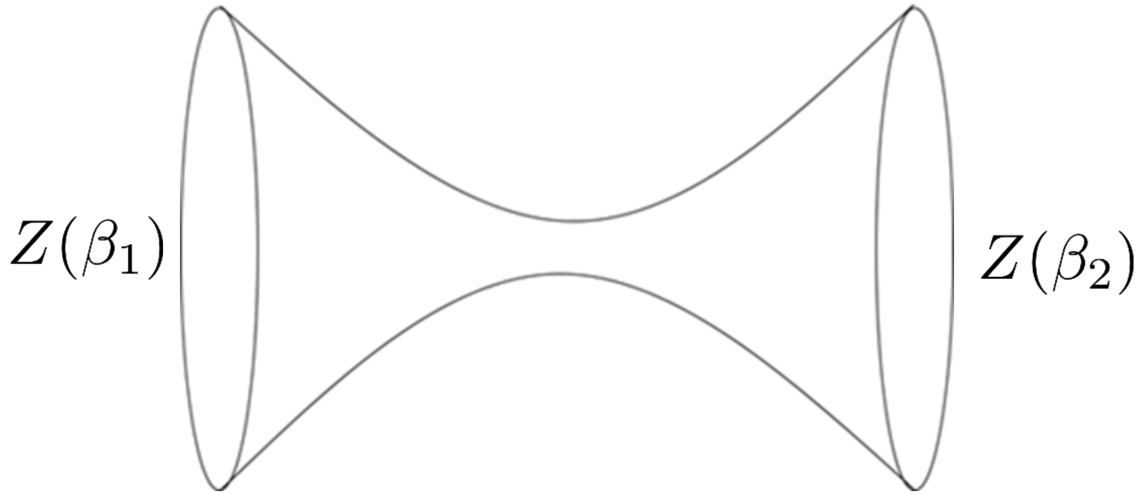
$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta} \right) \left(\sum_{lmn} C_{llm} C^*_{mmn} e^{-3\beta\Delta} \right) \right\rangle$$



$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij} C^*_{jkk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{llm} C^*_{mnn} e^{-3\beta\Delta} \right) \right\rangle$$

Belin, JdB, '20

Test of the conjecture for the spectral part of the theory



Cotler, Jensen ’21 – see also Di Ubaldo, Perlmutter ’23 and Haehl, Reeves, Rozali ‘23

The off-shell gravity computation agrees to leading order with the universal random matrix theory result

$$\langle Z(\beta_1)Z(\beta_2)\rangle = Z(\beta_1)Z(\beta_2) + \frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2} + \ldots$$

Ambjørn, Jurkiewicz, Makeenko ’90
Saad, Shenker, Sanford ’19

Overlaps of states and replica wormholes

Recall that semiclassically prepared states were modeled as $|\psi_i\rangle = C_{ia}|\psi_a\rangle$ with the flat measure on C. Then

$$\overline{\langle\psi_i|\psi_j\rangle} = \int dC\,\langle\psi_a|C^*_{ia}C_{jb}|\psi_b\rangle = \delta_{ij}$$

$$\overline{\langle\psi_i|\psi_j\rangle\langle\psi_k|\psi_l\rangle} = \int dC\,\langle\psi_a|C^*_{ia}C_{jb}|\psi_b\rangle\langle\psi_c|C^*_{kc}C_{ld}|\psi_d\rangle$$

$$= \delta_{ij}\delta_{kl} + e^{-S}(\delta_{il}\delta_{jk} - \delta_{ij}\delta_{jk}\delta_{kl})$$

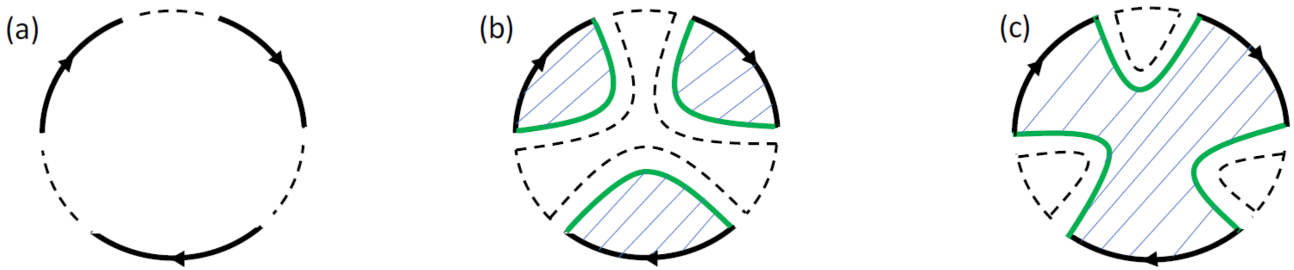
In particular

$$\overline{|\langle\psi_i|\psi_j\rangle|^2} = \delta_{ij} + e^{-S}(1-\delta_{ij})$$

This is sometimes written as (R is unit random matrix)

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + e^{-S/2}R_{ij}$$

Correction is due to “replica wormholes



picture from Liu, Vardhan ‘20

Versions of this statistical picture appears in many papers, for example

Goel, Lam, Turiaci, Verlinde ‘18
Penington, Shenker, Stanford, Yang ’20
Pollack, Rozali, Sully, Wakeham ‘20
Liu, Vardhan ’20
Freivogel, Nikolakopoulou, Rotundo ’21
Chadra, Hartman ’22
Bah, Chen, Maldacena ’22
Balasubramanian, Lawrence, Magan, Sasieta ’22
JdB, Liska, Post, Sasieta ‘23
Climent, Emparan, Magan, Sasieta, Vilar Lopez ’24
Iliesiu, Levine, Lin, Maxfield, Mezei ‘24

Information recovery in the semiclassical approximation

JdB, Hollander, Rolph '23

Time evolution of an initial state

$$\rho_0 \Rightarrow \overline{\rho(t)} = \int dH\, \mu[H]\, e^{-iHt} \rho e^{iHt}$$

produces a *classical statistical* mixture of states.

In general $S(\overline{\rho(t)})$ will increase: information loss. But since

$$\mathrm{Tr}(\overline{\rho(t)^n}) = \mathrm{Tr}(\rho_0^n) \Rightarrow \overline{S(\rho(t))} = S(\rho_0)$$

a suitable semi-classical replica computation knows that information is actually not lost.

Penington '19
Almheiri, Engelhardt, Marolf, Maxfield '19
Penington, Shenker, Stanford, Yang '19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '19

Can be illustrated with a simple model which is a cartoon of the microcanonical Hilbert space of a black hole coupled to a bath

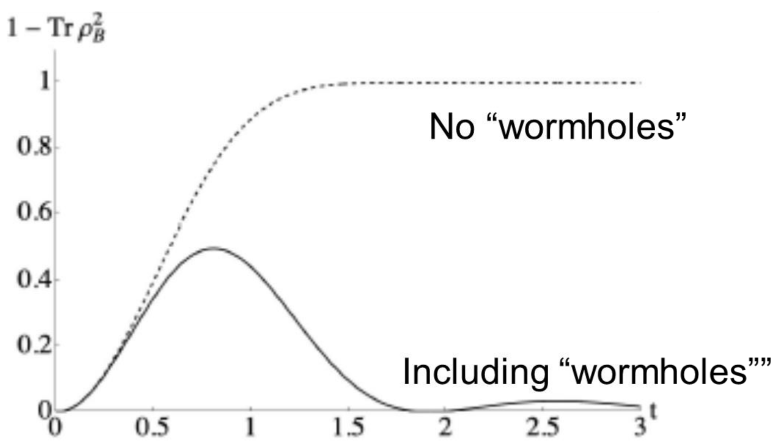
$$\mathcal{H} = (|1\rangle \otimes \mathcal{H}_1) \oplus (|0\rangle \otimes \mathcal{H}_0)$$

Black hole

Vacuum

with $\dim \mathcal{H}_1 \ll \dim \mathcal{H}_0$

Full random unitary dynamics:



Features:

$$g(t) = \mathrm{Tr}(e^{iHt})\mathrm{Tr}(e^{-iHt})$$

- Timescale is set by spectral form factor of matrix model

- “Wormholes” are related to types of contractions of random unitaries.

- Model has dynamics as opposed to many discrete qubit setups people have considered

- Mechanism is very simple and robust

- The final result is a classical statistical ensemble of pure states, not a mixed state. A replica computation like $\overline{\mathrm{Tr}(\rho^2)}$ can distinguish the two. A non-replica computation can not.

- No prediction for final state, just for unitarity.

A few issues

Restoring factorization?

It is an interesting question what the minimum number of ingredients are that we need to add to semiclassical gravity in order to uncover more detailed features of the UV and restore factorization.

Several suggestions exist in the literature, like half-wormholes, various branes, non-local interactions, ….

A simple universal explanation could be that wormholes are unstable due to brane creation by an analogue of Schwinger pair production. The Swampland cobordism conjecture suggests that such branes always exist.

But cf Marolf Santos '21

Or overcounting? Eberhardt '20'21

See e.g.
Gao, Jafferis, Kolckmeyer '21
Saad, Shenker, Stanford, Yao '21
Blommaert, Kruthoff '21
Mukhametzhanov '21
Blommaert, Iliesiu, Kruthoff '21

Alternative: gauging higher-form symmetries.

Benini, Copetti, Di Pietro '22

Are off-shell configurations needed to make this work?

Spectral correlations of the matrix model are obtained from geometries with two $S^{d-1} \times S^1$ boundaries. There are no on-shell geometries except the on-shell double cone (identify $t \sim t + T$ in a two-sided black hole geometry)

To reproduce the matrix model result we need to integrate over some off-shell configurations with the “constrained instanton” method (Cotler, Jensen ‘21)

Due to their topological nature, can do something more precise in JT gravity (Saad, Shenker, Stanford ‘19) and in 3d gravity (Cotler, Jensen ‘20)

But in general rules of the game are unclear.

Perhaps one can reverse engineer a suitable gravitational prescription which agrees with the matrix model.

The topological recursion of matrix theory and its geometric interpretation (Eynard, Orantin ‘07) suggests to look for an effective 2d theory…

JT gravity and 3d gravity

$$\begin{tikzcd} \boxed{???} \arrow[r, phantom, "="] \arrow[d, "\text{CFT}"'] & \boxed{???} \arrow[d, "\text{GRAVITY}"] \\ \boxed{\begin{array}{l}\text{SSS Matrix Model} \\ \\ \text{unknown}\end{array}} \arrow[r, phantom, "="] & \boxed{\begin{array}{l}\text{JT Gravity} \\ \\ \text{Pure 3d gravity}\end{array}} \end{tikzcd}$$

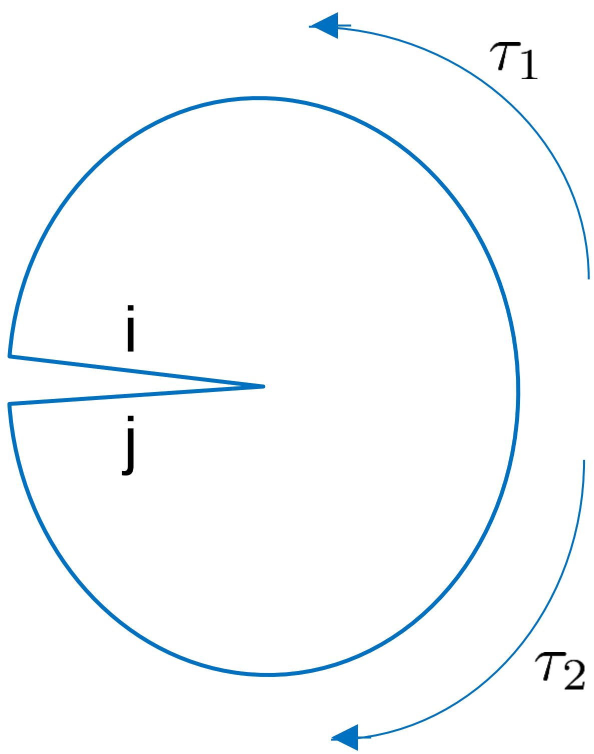
Two alternatives

Alternative 1: State Averaging

So far we looked at semi-classical gravitational computations with a closed boundary.

However, we can also use gravitational path integrals with boundaries to semi-classically produce states.

Freivogel, Nikolapoulou, Rotundo ‘21
Chadra, Hartman ‘22
Penington, Shenker, Stanford, Yang ’19
Bah, Chen, Maldacena ’22
Goel, Lam, Turiaci, Verlinde ‘18
Balasubramanian, Lawrence, Magan, Sasieta ’22
JdB, Liska, Post, Sasieta ‘23



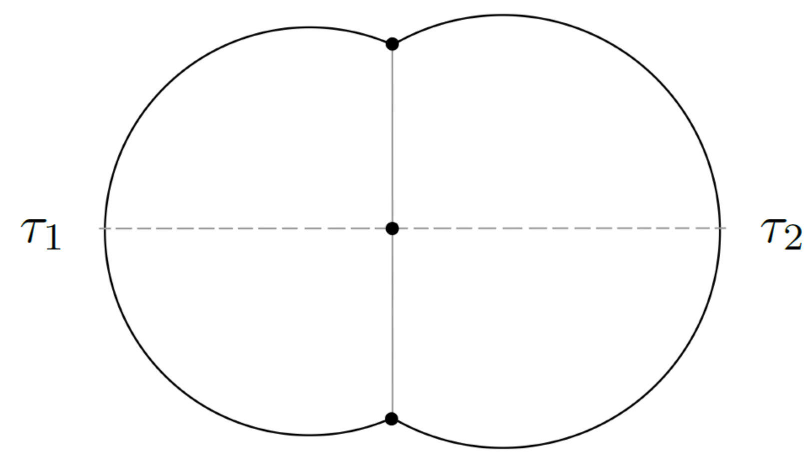
One can derive a suitable state-averaging ansatz for an open path integral.

More precisely, we consider purification of density matrices

$$|\Psi_{\mathrm{sc}}\rangle = \sum_{i,\alpha} A_{i\alpha}|E_i\rangle|E_\alpha\rangle \longrightarrow \rho = \sum_{i,j,\alpha} A_{i\alpha}A^\dagger_\alpha|E_i\rangle\langle E_j|$$

and assume such states can be prepared semiclassically (e.g. TFD state or PETS states). We then compute semiclassical overlaps of the form

$$\mathbf{Z}(\tau_1,\tau_2) := \langle\Psi_{\mathrm{sc}}|\,\mathrm{e}^{-\tau_1 H_L}\mathrm{e}^{-\tau_2 H_R}\,|\Psi_{\mathrm{sc}}\rangle =$$



and apply the maximal ignorance philosophy to obtain a quadratic matrix model for A.

Result:

$$\langle E_i|\rho|E_j\rangle = \delta_{ij}\,\bar{\rho}(E_i) + \frac{e^{-\beta \bar{E}_{ij}}}{Z(\beta)} e^{-S(\bar{E}_{ij})/2} j(\bar{E}_{ij},\omega_{ij})^{1/2} R_{ij}$$

Provides an alternative picture to OPE/spectral statistics. It more directly describes a coarse graining at the level of states. It is in particular useful for cutting/gluing constructions of correlation functions.

It reproduces many results of the OPE/spectral statistics picture.

Interesting feature: $\overline{S(\rho|\rho_\beta)} \sim \mathcal{O}(1)$

JdB, Liska, Post, Sasieta, '23

Interestingly, the matrix model for A also gives rise to non-gaussianities for ρ

$$\overline{\delta\rho_{ij}\delta\rho_{kl}\delta\rho_{mn}}^{\,\text{conn.}} = \sum_{\alpha,\beta,\gamma} A_{i\alpha}A^*_{j\alpha}A_{k\beta}A^*_{l\beta}A_{m\gamma}A^*_{n\gamma}$$

This makes the model subtly different from operator/spectral averages models, for example

$$\overline{\mathrm{Tr}(\rho\mathcal{O})\,\mathrm{Tr}(\rho\mathcal{O})}^{\,\text{conn.}} = \sum_{ij} \overline{|\delta\rho_{ij}|^2}\,\mathcal{O}_{ij}\mathcal{O}_{ji} \qquad \text{(state averaging)}$$

$$\langle\!\langle \mathrm{Tr}(\rho_\beta\mathcal{O})\,\mathrm{Tr}(\rho_\beta\mathcal{O})\rangle\!\rangle^{\text{conn.}} = \sum (\rho_\beta)^2_{ii}\langle\!\langle \mathcal{O}_{ii}\mathcal{O}_{ii}\rangle\!\rangle^{\text{conn.}} \qquad \text{(operator averaging).}$$

Alternative 2: Matrix/Tensor model for 3d gravity

Belin, JdB, Jafferis, Nayak, Sonner ‘23

Recall that for 2d theories with a holographic AdS3 dual:

- we only have explicit access to low-lying operators (denoted L) and not to very high dimension operators corresponding to black holes (denoted H)
- we can compute correlation functions where the number of operators is $\ll c$
- we can compute partition functions on surfaces with genus $\ll c$
- we can compute correlation functions in Lorentzian signature as long as the center of mass is sub Planckian
- all computations are at best done up to non-perturbative errors of order e^{-c}

In arXiv:2308.03829 we called a set of conformal dimensions and OPE coefficients for which these computations approximately obey the CFT axioms (crossing and modular invariance of 1-pt functions) an *approximate CFT*

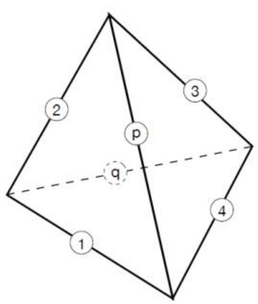
Note: by changing multiple conformal dimensions of heavy operators in a coordinated way, can prove every 2d CFT sits in an island of approximate CFTs

Note: the opposite is not obviously true. An approximate CFT may not be close to an actual CFT. Possible example: approximate 2d CFTs defined by pure 3d AdS gravity.

The idea is now to average over all CFT2 data with a spectrum which is very close to that of 3d gravity, and with a weight schematically of the form

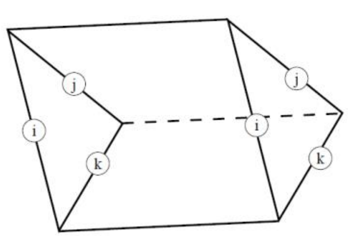
$$P(\Delta_i, C_{ijk}) \sim \exp\left(-a\sum(\text{axioms})^2\right)$$

Result is a quartic tensor model with Feynman rules



$$= \quad \begin{Bmatrix} \mathcal{O}_q & \mathcal{O}_2 & \mathcal{O}_1 \\ \mathcal{O}_p & \mathcal{O}_4 & \mathcal{O}_3 \end{Bmatrix}$$

Virasoro 6j symbol



$$= \quad C_0\left(P_i, P_j, P_k\right) C_0\left(\bar{P}_i, \bar{P}_j, \bar{P}_k\right)$$

Crossing:

$$\sum_H C_{LLH} C_{LLH} \simeq C_{LL\mathbb{I}} C_{LL\mathbb{I}}$$

Square of crossing

$$\left(\sum_H C_{LLH} C_{LLH} - C_{LL\mathbb{I}} C_{LL\mathbb{I}} \right) \left(\sum_H C_{LLH} C_{LLH} - C_{LL\mathbb{I}} C_{LL\mathbb{I}} \right) \simeq 0$$

This is reminiscent of various other discrete descriptions of 3d gravity.

e.g. Regge ’61; Boulatov ‘92; Turaev Viro ’92 + many more

It is also connected to the so-called Teichmüller TQFT (Andersen, Kishaev ‘11 ’13) which was recently connected to 3d gravity (Collier, Eberhardt, Zhang ’23 ’24) (Jafferis, Rosenberg, Wong ‘24)

To be continued…. Important challenge is to count three-manifolds once while the same manifold can combinatorically be obtained in many different ways.

Note: model has same generating function of correlators but is smarter then construction described earlier because it knows about CFT axioms. Model is not weakly coupled.

COMMENT

None of the above implies that AdS/CFT fundamentally requires averaging. Averaging is purely a consequence of the semi-classical approximation. As one improves the description the averaging should become over increasingly smaller sets of data and ultimately disappear.

If the set of data would not become smaller as one would increase accuracy then the dual description would indeed be a proper average. This is what e.g. happens in topological theories like JT gravity.

But there currently is no evidence that anything like this is happening in standard examples of AdS/CFT.

OUTLOOK

- Are different types of coarse graining (operator versus state) somehow equivalent?
- The statistical description of the spectral part of the theory requires further clarification.
- What is the best definition of pure 3d gravity?
- Relation to α-vacua?
- It is not yet clear whether there is a simple universal mechanism which restores factorization in the UV.
- The precise rules for off-shell computations should be understood.
- Many generalizations exist: higher ETH, etc
- There are many different types of wormholes (e.g. axionic) whose interpretation is still somewhat confusing.
- It would be interesting to apply this logic to (observer-centric approaches to) quantum gravity in flat space and de Sitter (bra-ket wormholes?)
- Semi-classical gravity is averaging agnostic.
- Should we stop pretending we are meta-observers who can solve everything? Especially when we are part of a chaotic system ourselves?