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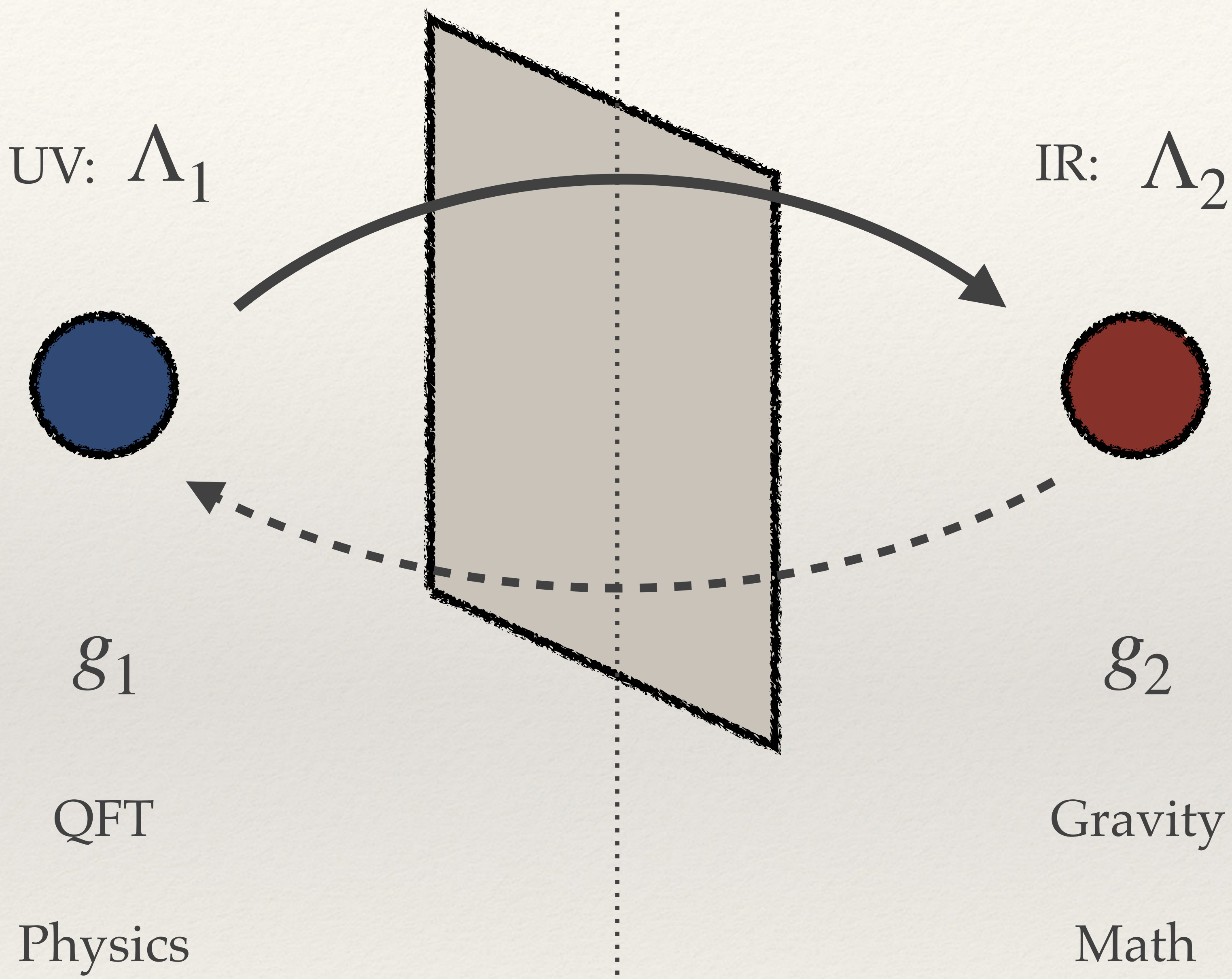
On recent developments in formal SQFT: a duality perspective

5/9/2024
EuroStrings 2024
Southampton

`` Formal QFT ”

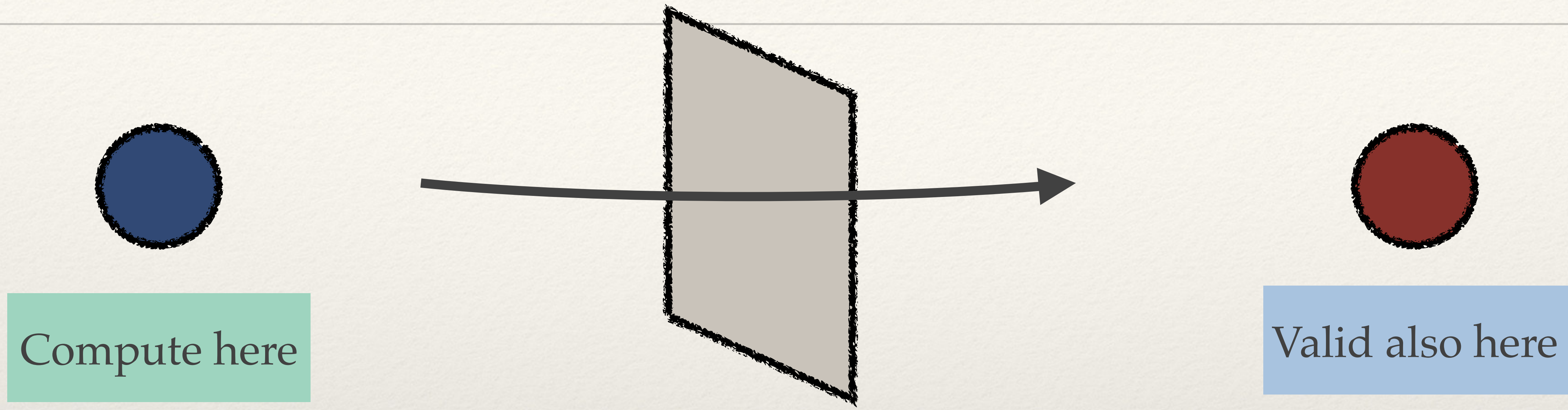
- QFT research driven by theoretical laboratories
- Toy models exhibiting non trivial effects
- Motivations:
- Computability in strong coupling
- Envisioned applications: ``QCD”; ``Gravity”; Mathematical Physics
- Interplay between these

Beyond the strong coupling barrier



- Toy models providing tools to penetrate the barrier of “complicated”

Beyond the strong coupling barrier: Tools



See Shao EuroStrings 2023

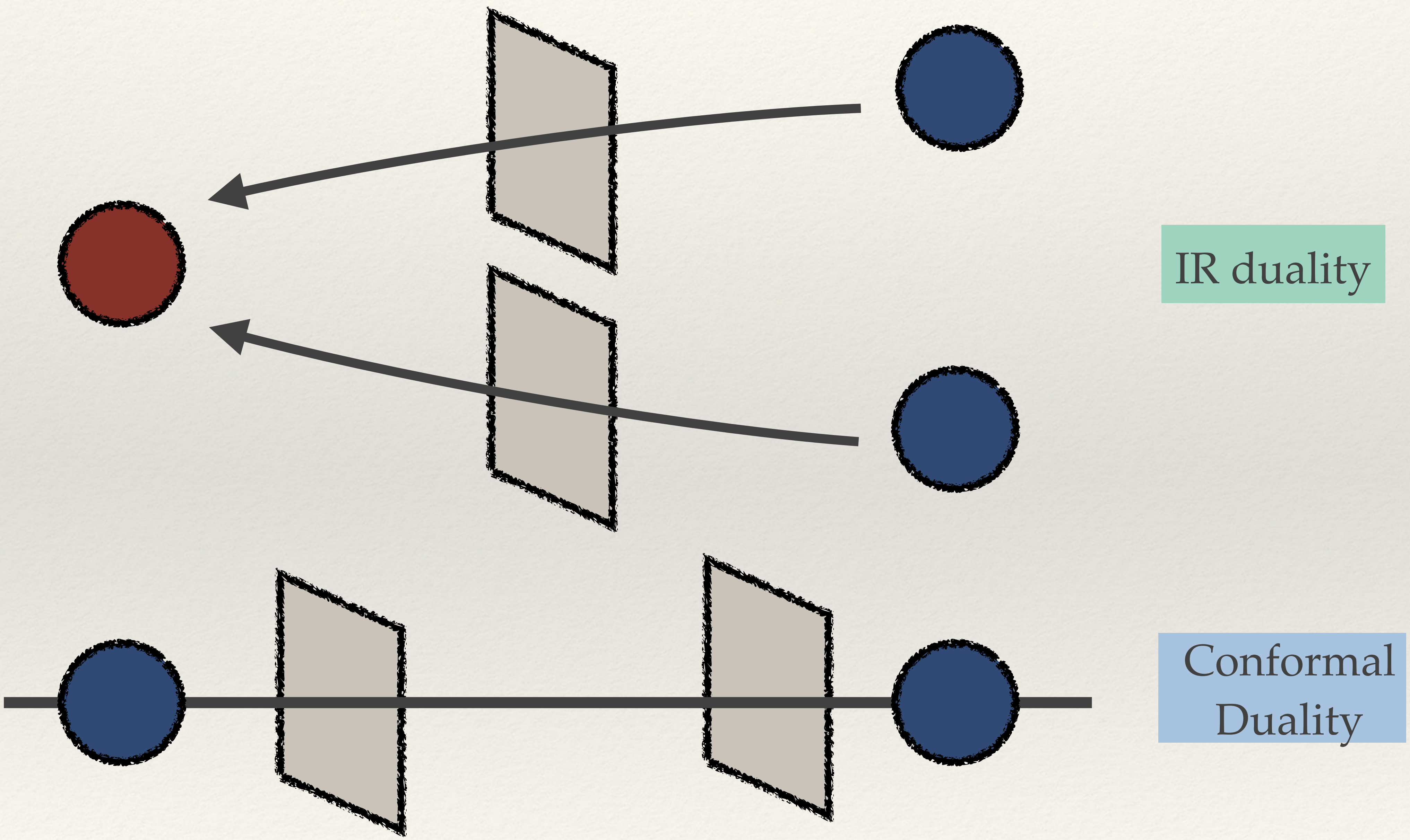
- Symmetries and anomalies (generalized)
- Partition functions (PF) (supersymmetry)
- Counting problems

} Independent of continuous parameter (Λ, λ)

- Dependence on parameters is ``harder'' Eg: Integrability, holography, AGT

Beyond the strong coupling Barrier: Duality

- What can one deduce from these counting exercises?



- Conjectural statements based on counting

Plan

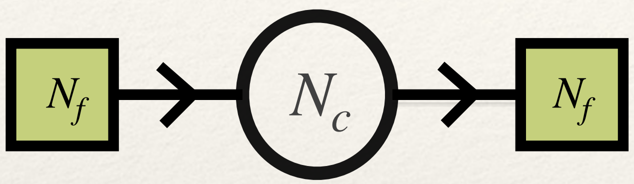
- “Dualities” turns out to be a rich theoretical laboratory
- Many of the developments in formal QFT in recent years are directly or indirectly related to dualities
- We will organize thus our discussion around the notion of duality
    - Understanding better QFT constructions and RG flows
    - Mathematical physics following from dualities
    - Dualities following from mathematical physics

See SSR Strings 2022

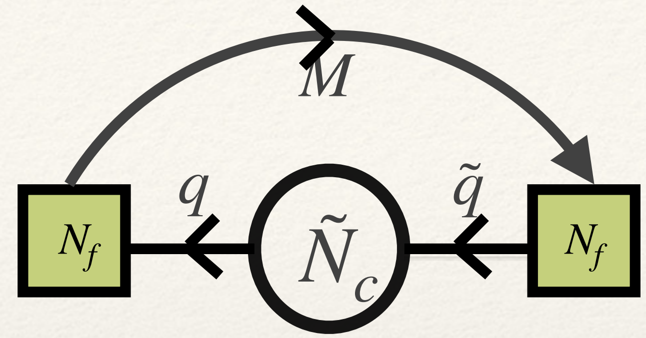
Simplest IR duality examples, 4d and 3d

$4d$

SU global



$SU(N_c)$ *gauging*



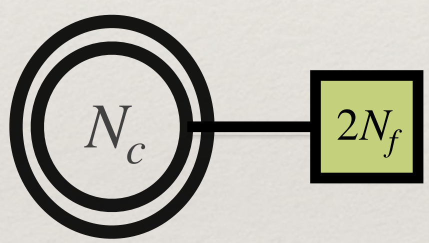
$\tilde{N}_c = N_f - N_c$

$W = q\tilde{q}\,M$

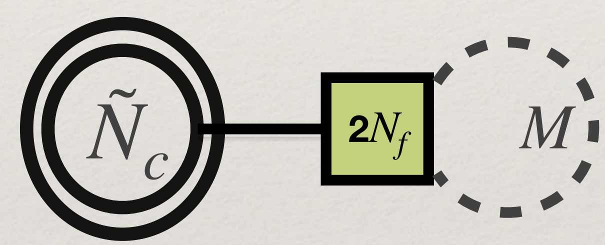
Seiberg

(Se)

$4d$



$USp(2N_c)$ *gauging*



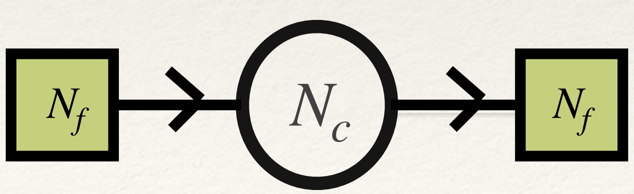
$\tilde{N}_c = N_f - N_c - 2$

$W = q\,q\,M$

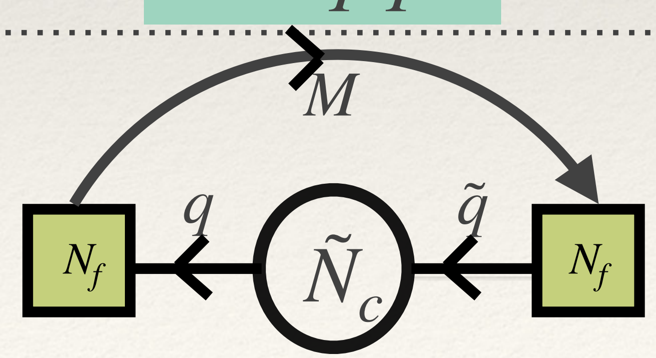
Intriligator-Pouliot

(IP)

$3d$



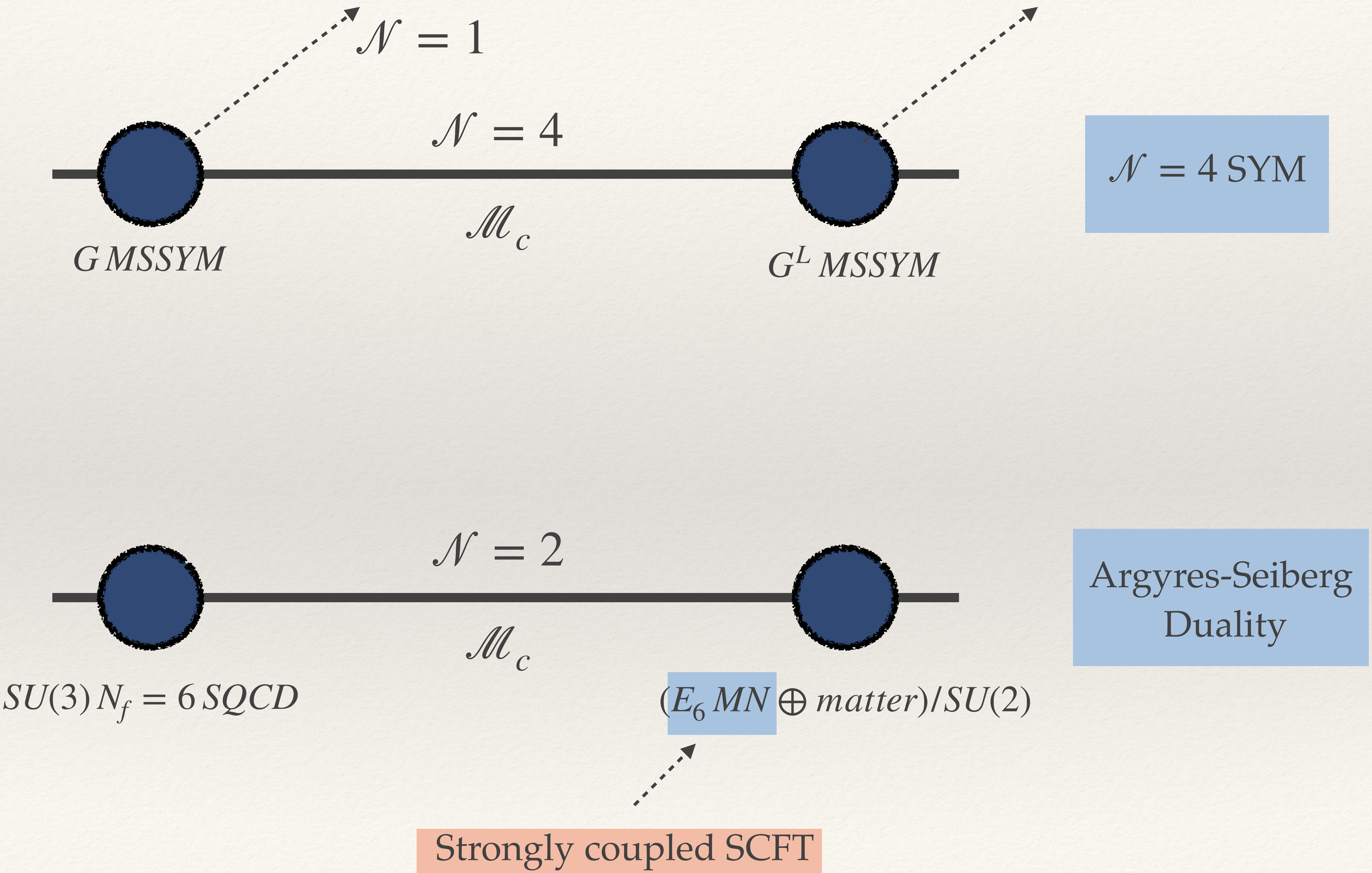
$U(N_c)$ *gauging*



$\tilde{N}_c = N_f - N_c$ $\quad W = q\tilde{q}\,M + \textit{Monopoles}$

Aharony

Conformal duality examples, 4d



Strongly coupled SCFTs

CFTs in 6d

See Heckman, Rudelius 2018 (Review)

CFTs in 5d

Seiberg CFTs in 4d

Argyres-Douglas theories

Minahan-Nemeschansky theories

3d Gauge theories

- *Non Lagrangian theories: theories for which a UV weakly coupled description in terms of free fields RG flowing to them is not known at the moment*

The SCFT universe

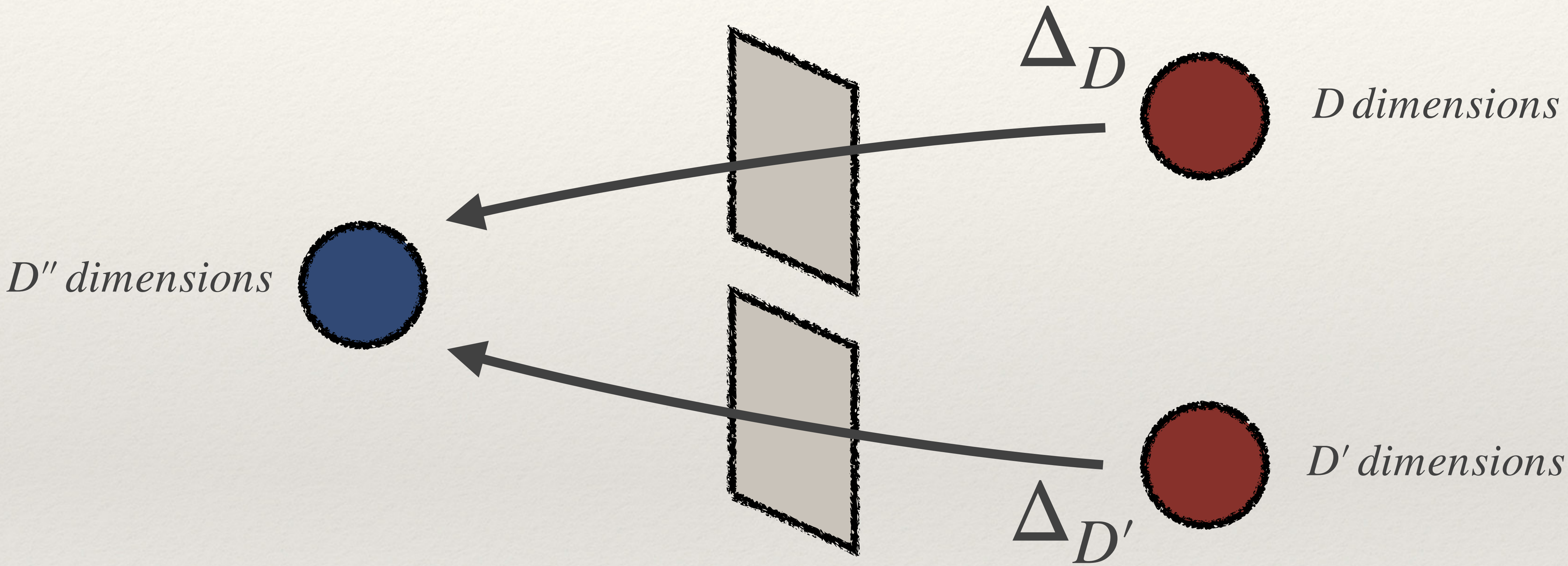
(See talk by Ben Gripaios)

- Many scattered instances of CFTs and dualities relating them
- Evidence for existence of new CFTs
- Are there schemes to organize this data?
- Is there a structure to the space of all (S)CFTs and RG flows?
  - A geometric scheme : geometrize the problem
  - A reductionism scheme : basic sets of facts from which all follows
  - "Big data" scheme? : look for patterns
- *Comments on relations to math and more*

A geometric scheme

A geometric scheme

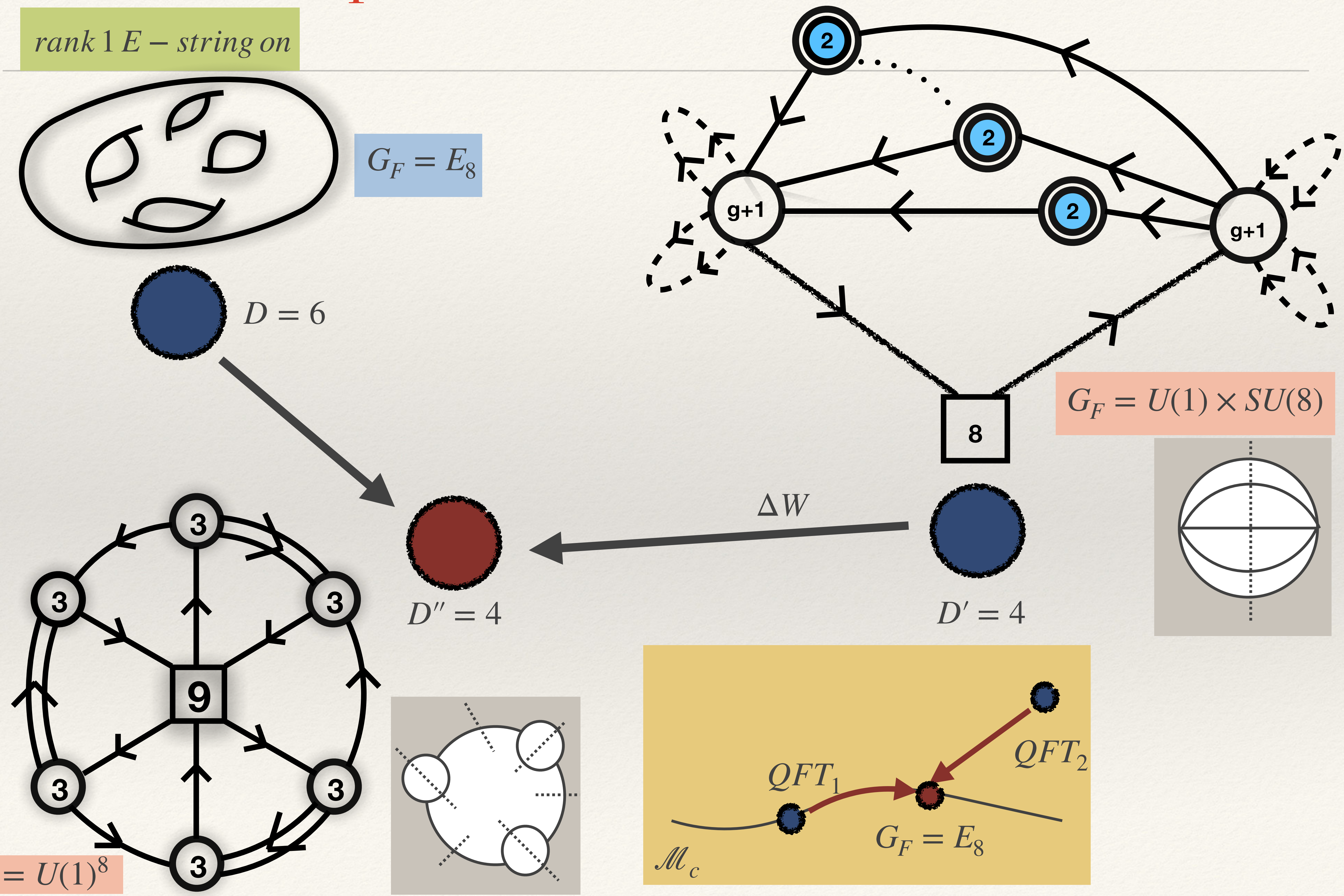
- IR Dualities across dimensions



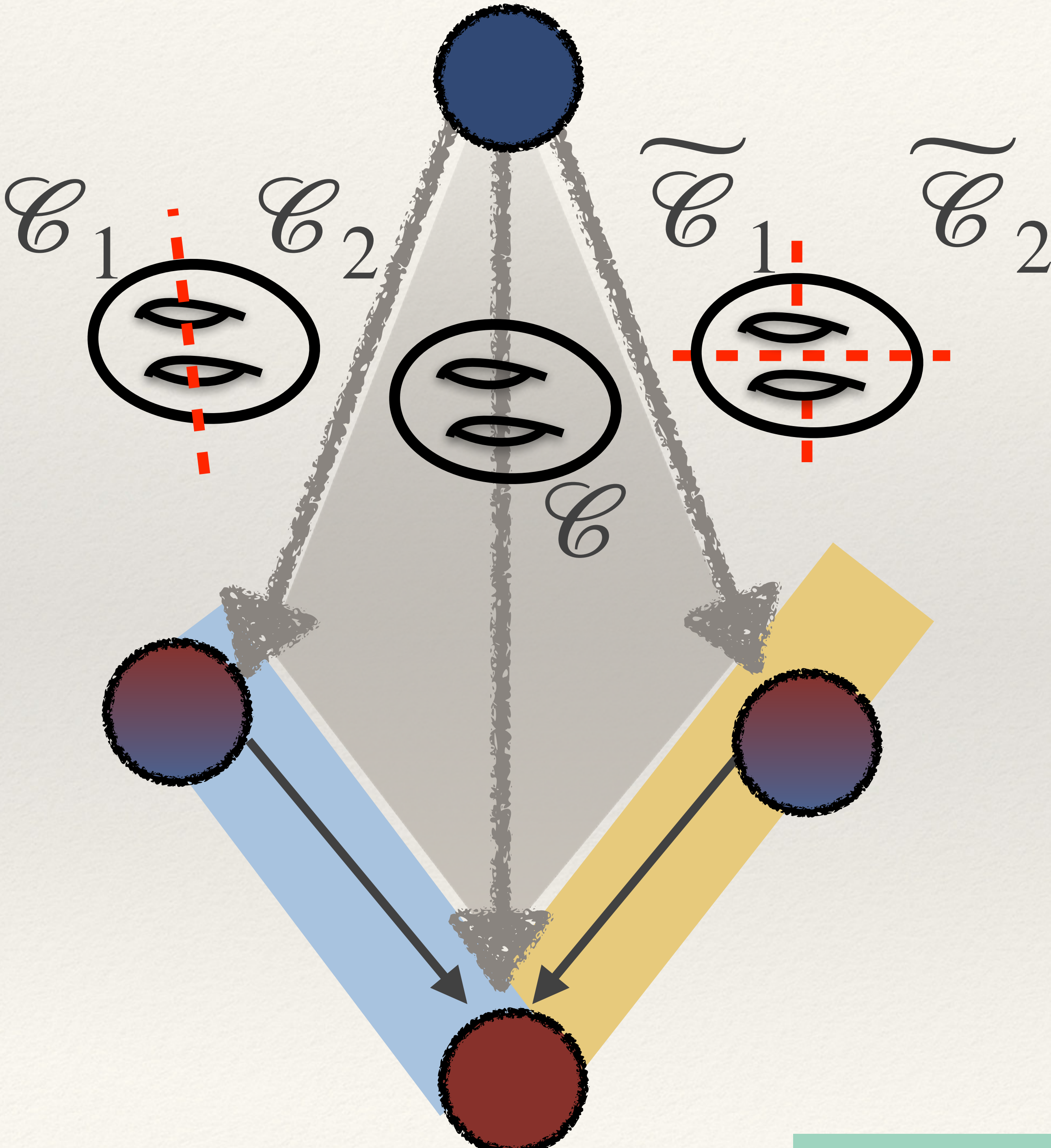
- Implicitly assumed till now that all flows happen in given dimensionality
- However, this can be generalized to UV starting points and IR end points being in different dimensions
- The deformations then can be geometric

See SSR, Sabag, Sela, Zafrir 2022 (Review)

Example: $D = 6, D' = 4, D'' = 4$



Geometric derivation of dualities



Geometric operation

$$\mathscr{C} = \oplus_i \mathscr{C}_i = \oplus_j \widetilde{\mathscr{C}}_j$$

$$T_{D''}[T_D; \mathscr{C}_{D'}] =$$

$$= \otimes_i T_{D''}[T_D; \mathscr{C}^i_{D'}]$$

QFT operation (gauging, superpotential)

$$= \otimes_j T_{D''}[T_D; \widetilde{\mathscr{C}}^i_{D'}]$$

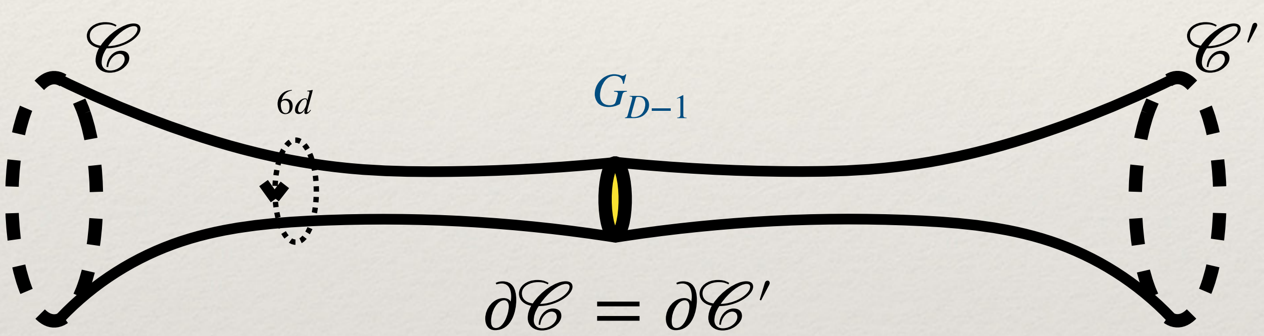
Duality in $\mathscr{D}''$

In $\mathscr{D}$ dualities follow from consistency of across $\mathscr{D}$ dualities

Starting with Gaiotto 2009

Cutting surfaces: Punctures

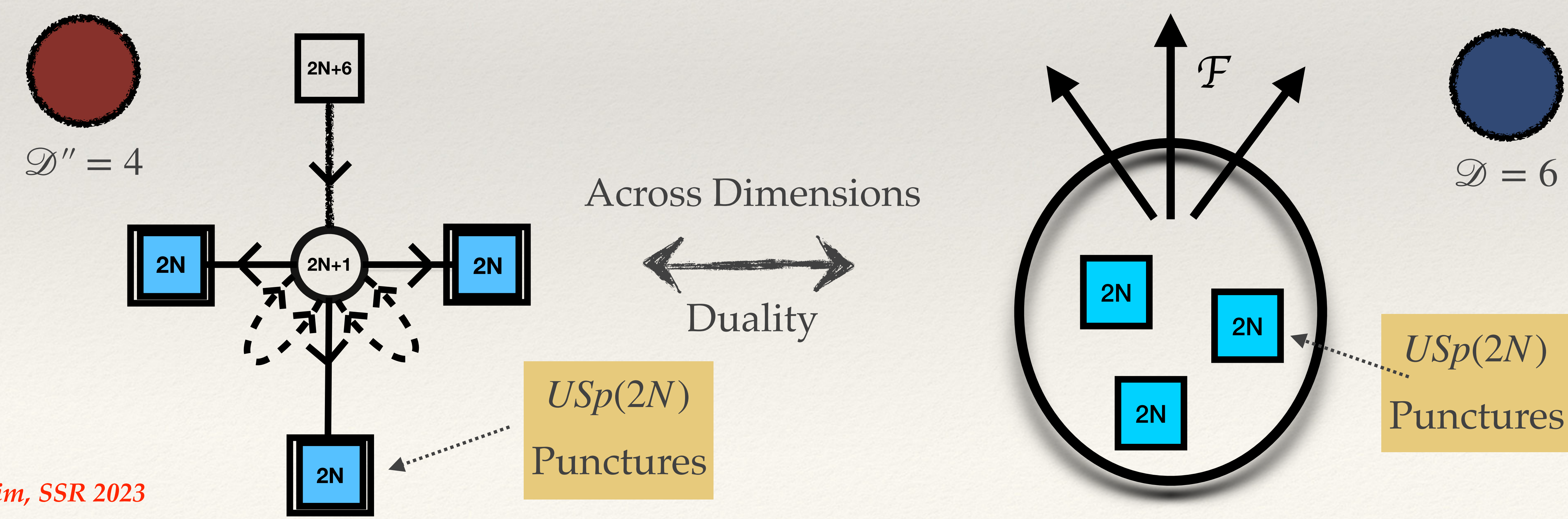
- Cutting surface and gluing them is important to understand dualities
- Cutting leads to surfaces with boundaries (punctures when $\mathscr{D}' = 2$)



- Input: compactifications on $\mathbb{S}^1$
- Input: boundary conditions in $\mathscr{D} - 1$

Example of punctures

- In many cases studied theory in $\mathscr{D}-1$ is a gauge theory
- Eg: $\mathscr{D}=6\, ADE\, (2{,}0) \rightarrow \mathscr{D}=5\, MSSYM$
- Eg: $\mathscr{D}=6\,(1{,}0)\,(D_{N+3}, D_{N+3})\, min\,.\, conf\,.\, matter \rightarrow$
  $\mathscr{D}=5\; USp(2N)\; N_f = 2N+6$
- Punctures are choices for bc for these fields / gluing is undoing the bc



Kim, SSR 2023

Generalized punctures – $\mathscr{D} = 6$ (2,0)

- Classification of punctures is important to understand all the geometric constructions
- Eg: Compactifications of *ADE* (2,0)
- $\mathcal{N} = 2$ preserving punctures
- A: Regular B: Irregular (leading to Argyres-Douglas theories)
- $\mathcal{N} = 1$ preserving punctures
- A: More general boundaries B: Spindles ($\mathscr{D}'' = 4$ QFT duals)??

Xie 2013
Heckman, Jefferson, Rudelius, Vafa 2016

See several talks here *Bomans, Couzens 2024*

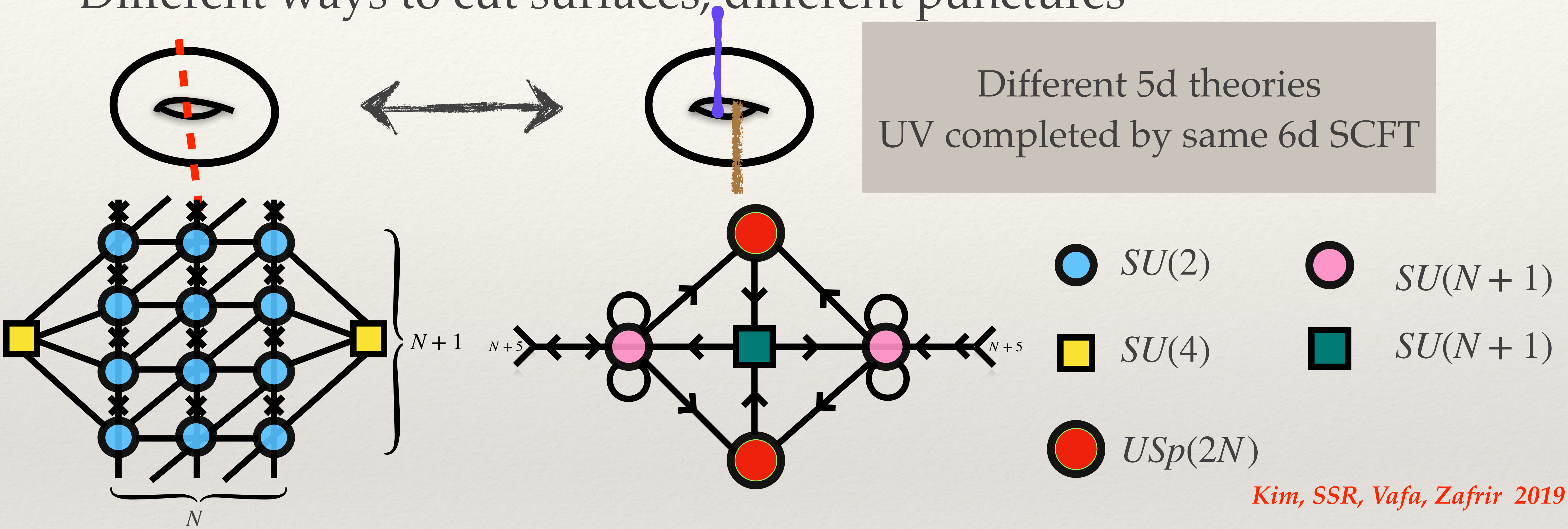
** *Holographic understanding of punctures:* *Eg: Gaiotto,Maldacena 2009, Bah, Bonetti, Nardoni, Waddleton 2022 Bah, Bonetti, Minasian, Nardoni 2021, Couzens, Kim, Kim Lee 2022*

Generalized punctures: $\mathscr{D} = 5$ SCFTs

- Gauge theories relevant for punctures in $\mathscr{D} = 5$ are UV completed by $\mathscr{D} = 6$ SCFTs
- One can consider also deformed $\mathscr{D} = 5$ SCFTs which are UV completed by $\mathscr{D} = 6$ SCFTs
- Such $\mathscr{D} = 5$ can be relevant for the geometric scheme
- Eg: $\mathscr{D} = 6$ SCFT is 2 M5 branes probing $\mathbb{Z}_k$ singularity *SSR, Sabag 2019*
- $\mathscr{D} = 5$ can be described as $SU(2)^k$ gauge theory
- Or as $SU(2)$ gauging of a strongly coupled $\mathscr{D} = 5$ SCFT *Ohmori, Shimizu, Tachikawa, Yonekura 2015*
- *(This SCFT has a deformation such that it flows to $SU(k)$ gauge theory with instanton $U(1)$ enhancing to $SU(2)$ in UV)*
- *This description can be used to cut and glue surfaces: $SU(k) \to SU(2)$ gauging*

More dualities from geometry

- Different ways to cut surfaces, different punctures



Kim, SSR, Vafa, Zafrir 2019

- We can start with $D = D' = 6$ and flow to $D'' = 4$
- Ex : $D = 6$: compactifications of (2,0) on general surface
- $D' = 6$: compactifications of (1,0) on tori

Ohmori, Tachikawa, Zafrir 2018

Distler, Elliot, Kang, Lawrie 2022

Heckman, Lawrie, Lin, Zhang, Zoccarato 2022

- More generally: Different (1,0) theories on different surfaces

SSR, Sabag, Sela, Zafrir 2022 (Review)

Obscure $\mathcal{N} = 2$ SCFTs from Geometry

- Can all $\mathscr{D} = 4$ SCFT be engineered in $\mathscr{D} = 6$?

- Ex 1:

$\mathcal{N} = 2 \quad USp(4)$
Gauge theory with
half hyper in **16**

Across Dimensions
⟷
Duality

(1,0) SCFT
On torus
with SW class
(+deformation)

Giacomelli, Savelli, Zoccarato 2024

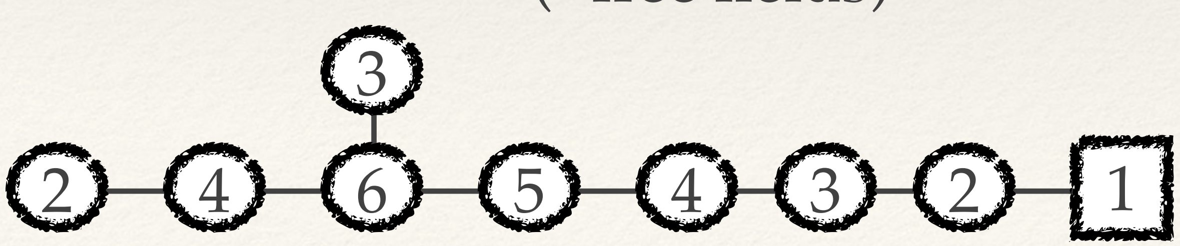
- Ex 2:

$\mathcal{N} = 2$ Quiver
Shaped as
E_8 Dynkin diagram
(+free fields)

Across Dimensions
⟷
Duality

$(1,0)\ (E_8, E_8)$
Min. Conf. SCFT
On torus with flux

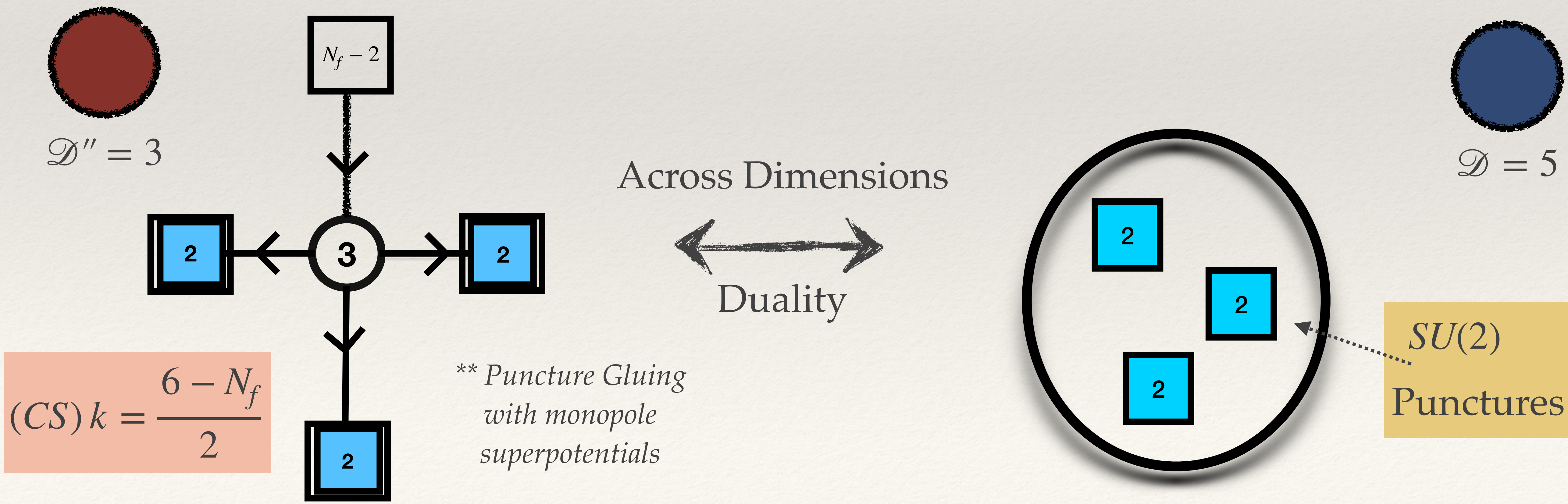
Kim, SSR, Vafa, Zafrir 2018



Example: $D = 5, D' = 3, D'' = 3$

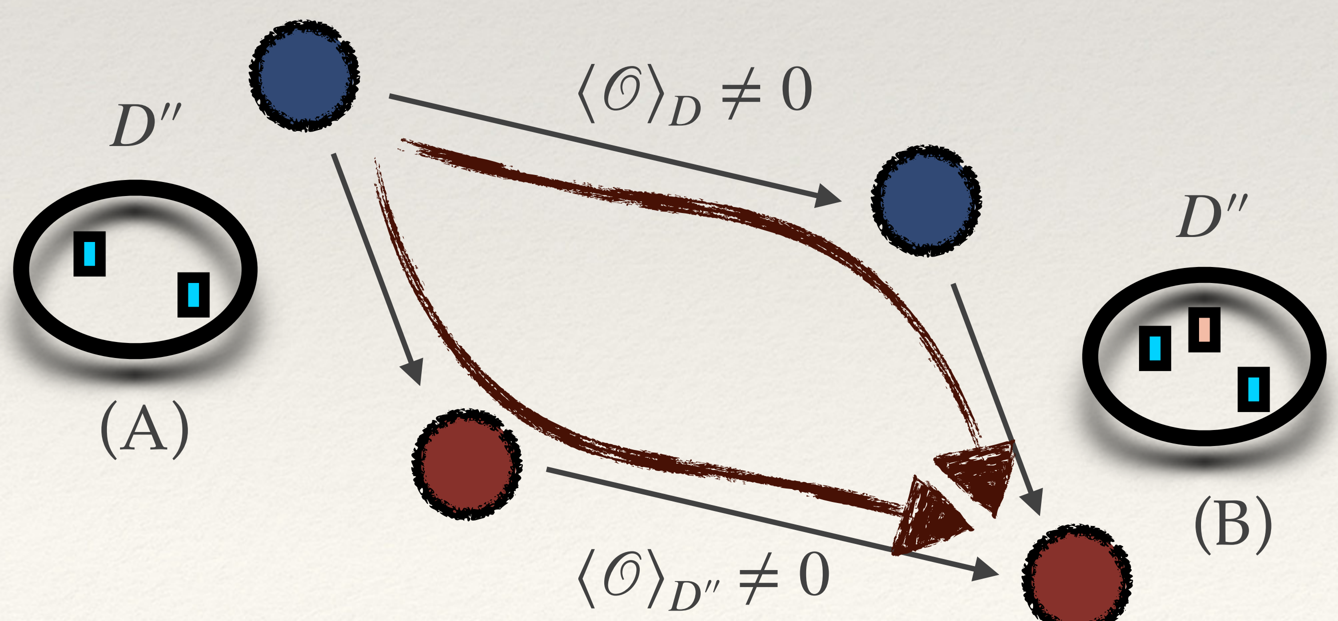
- Can discuss similar constructions in odd dimensions
- No anomalies for continuous symmetries
- However, can utilize discrete symmetry and symmetry enhancement
- Eg: Compactifications of E_{N_f+1} Seiberg CFTs (UV completion of $SU(2)\, N_f\, SQCD$)

Sacchi, Sela, Zafrir 2021-2023 *See Zafrir Eurostrings 2023*



Algorithm to derive across dimension dualities

- How do one derives dualities across dimensions?
- (1) Conjecture: matching symmetries, anomalies, and flows
- (2) Understand building blocks and then play lego
- Building blocks are two and three punctured spheres
- Two punctured spheres can be derived by reducing on a circle and studying domain walls and boundary conditions
- Three punctured spheres, in some cases, can be derived by studying flows between different theories in D dimensions



$\#(punct)_B > \#(punct)_A$ *if*

$\mathcal{F}[\mathcal{O}] \neq 0$

Sabag, SSR, Zafrir 2019

Geometric scheme summary

- Geometric scheme partially systematizes understandings of many scattered SCFT results such as dualities and emergence of symmetry
- Systematically constructs new examples of such phenomena
- To understand across dimension dualities need to integrate many different understandings and techniques
- Classification of 6d SCFTs; 5d SCFTs; relations between the two; duality domain walls in different dimensions; classifications of manifolds and boundaries
- Generalized symmetries, geometry, and compactifications

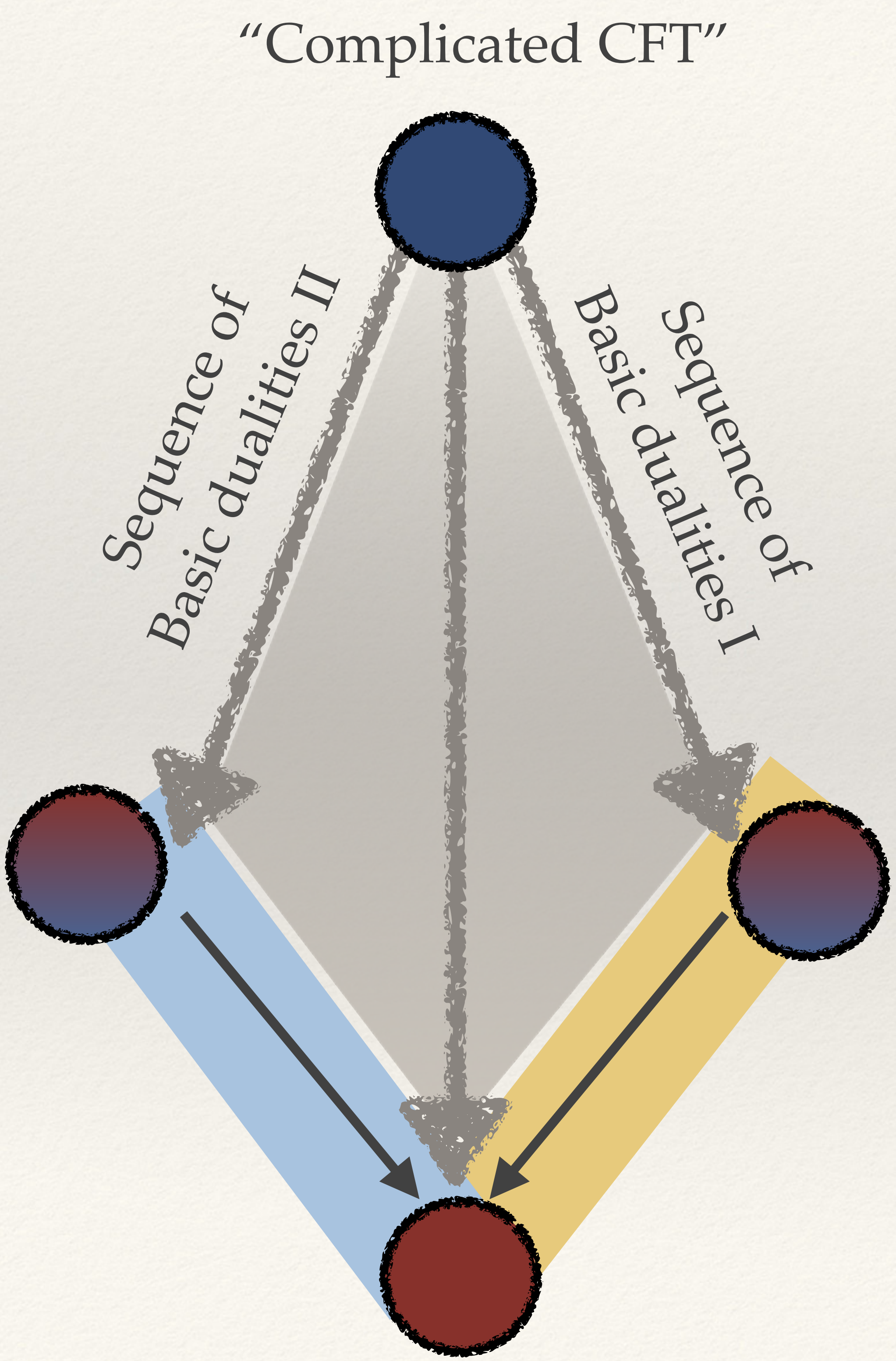
Lee, Ohmori, Tachikawa 2021 *Nardoni, Sacchi, Zafrir, Zheng 2024*
Kaidi, Zafrir, Zheng 2022 *Bashmakov, del Zotto, Hassan, Kaidi 2022*

Bhardwaj, Schafer-Nameki and Co

- Can we understand ALL SCFTs as geometries?

Reductionism scheme

A reductionism scheme

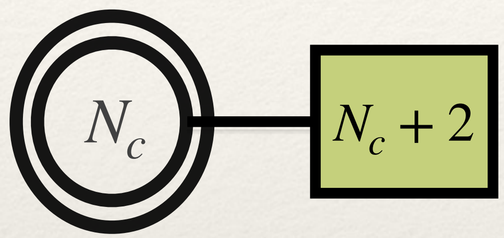


- Start with a complicated QFT
- Use a sequence of basic dualities in different ways
- Arrive at a simple duality

- Eg: Complicated QFT — Gauge theory with a non-simple gauge group
- Basic dualities — Seiberg, Aharony, IP
- Dualities lead to S-confinement and relatively simple theories

S-confinement

- One way to complicate things is to turn free fields into gauge theories

(IP) : 

(S-confinement) $\longrightarrow$ M — Mesons in two index antisymmetric irrep of $SU(N_c + 2)$

Berkooz 1995

Luty, Schmaltz, Terning 1996

Gauge theory $\longleftarrow$ *(Berkooz Deconfinement)* WZ model

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- Use (IP) and (Se) to derive many (all?) S-confinement dualities

Bottini, Hwang, Pasquetti, Sacchi 2022

Bajeot, Benvenuti 2022

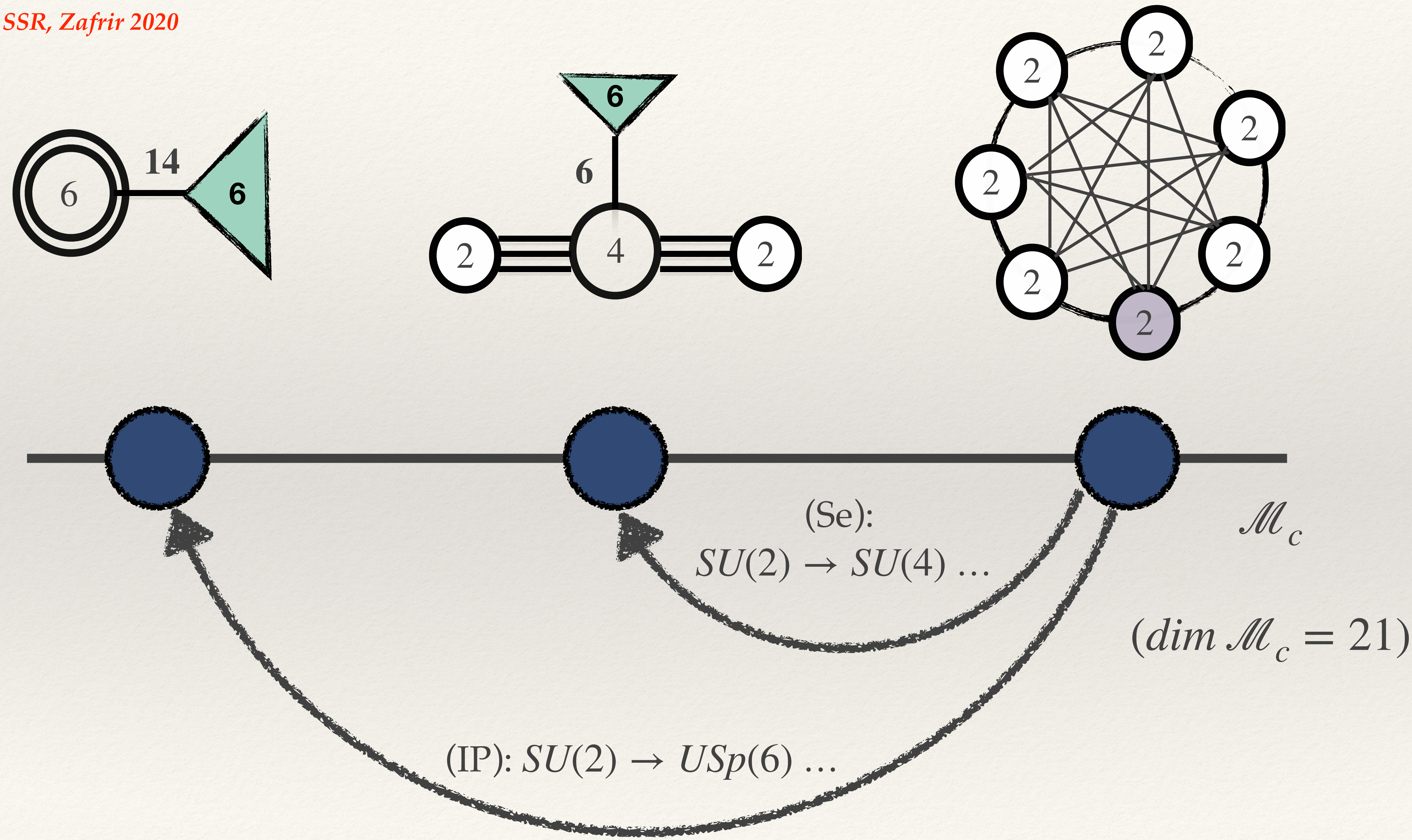
Csaki, Schmaltz, Skiba 1996

Eg: $SU(N_c)$ SQCD with $\square\!\square \oplus N\, \bar{\square} \oplus 4\, \square$ $\longrightarrow$ Gauge singlets

- Is there a set of basic QFT dualities from which all other dualities can be derived?

Conformal triality example

SSR, Zafrir 2020



Amariti, Rota 2022

Ode to Physics and Math

- Seiberg duality (1995) leads to highly non trivial identities of special functions while computing partition functions
- Eg: The superconformal index:

$$I_N^{N_f}(\mathbf{u},\mathbf{v};q,p) = \frac{\kappa^{N-1}}{N!}\oint \prod_{i=1}^{N-1}\frac{dz_i}{2\pi i z_i}\frac{\prod_{a=1}^{N_f}\prod_{j=1}^{N}\Gamma_e^{q,p}(u_a z_i)\Gamma_e^{q,p}(v_a z_i^{-1})}{\prod_{i\neq j}\Gamma_e^{q,p}(z_i/z_j)}$$

$$I_N^{N_f}(\mathbf{u},\mathbf{v};q,p) = I_{N_f-N}^{N_f}(\tilde{\mathbf{u}},\tilde{\mathbf{v}};q,p)\prod_{a\neq b}\Gamma_e^{q,p}(u_a v_b)$$

- This identity was derived and proven independently in the math literature, before the superconformal index was even defined in physics

Spiridonov 2001

Eric Rains

"Transformations of Elliptic Hypergeometric integrals"

0309252

Dolan, Osborn 2011

Romelsberger 0510060

Kinney,Maldacena,Minwalla,Raju

"An index for 4d superconformal theories"

0510251

* Generalization to lens index *Kels, Yamazaki 2017*

* Generalization to reductions of dualities: *Van De Bult thesis 2007* ⟶ *Benini, Closset, Cremonesi 2011*

Ode to Physics from Math

Eric Rains

"Multivariate Quadratic Transformations and the Interpolation Kernel"

1408.0305

Hwang, Pasquetti, Sacchi

"4d Mirror like dualities"

2002.12897

Geometric Scheme

Pasquetti, SSR, Sacchi, Zafrir

"Rank Q E-string on a torus with flux"

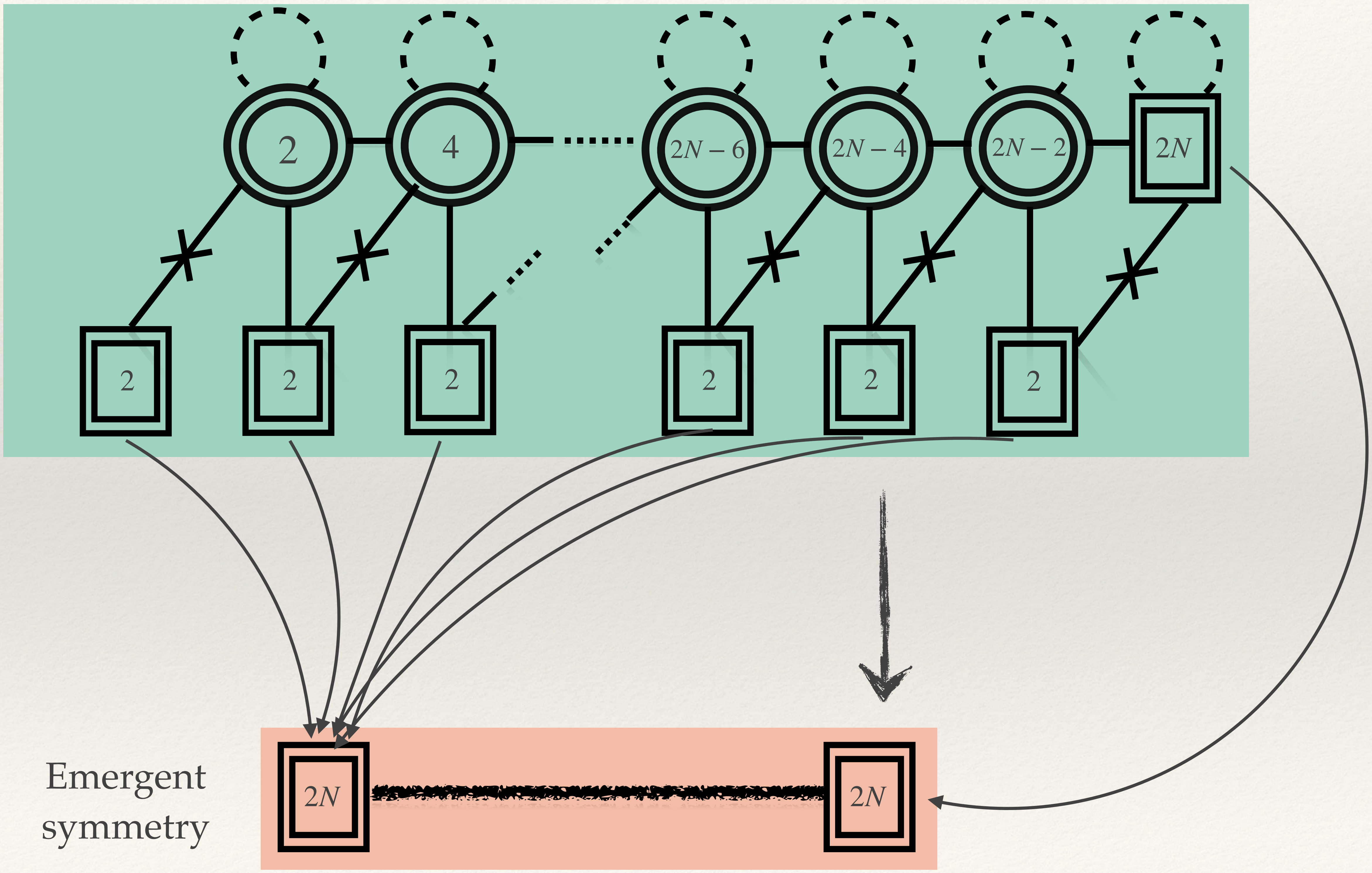
1908.03278

3d mirror symmetry

4d IR dualities

(IP) and (Se)

The master theory of Rains



Emergent symmetry

3d Mirror symmetry from 4d IP

- The master theory is self dual and has emergent symmetry
- *(Without the work of Rains it would be hard to come up with such a model)*
- Upon dimensional reduction to 3d and deformations leads to many known mirror dualities
- Thus it can be viewed as a 4d avatar of mirror symmetry
- Various properties of the master theory can be proven by utilizing (IP) dualities *Hwang, Pasquetti, Sacchi 2020* *Benvenuti, Comi, Pasquetti 2023*

$\mathcal{N} = 4$ unitary groups in 3d

$T[SU(N)]$

Gaiotto, Witten 2008
Benini, Tachikawa, Xie 2010

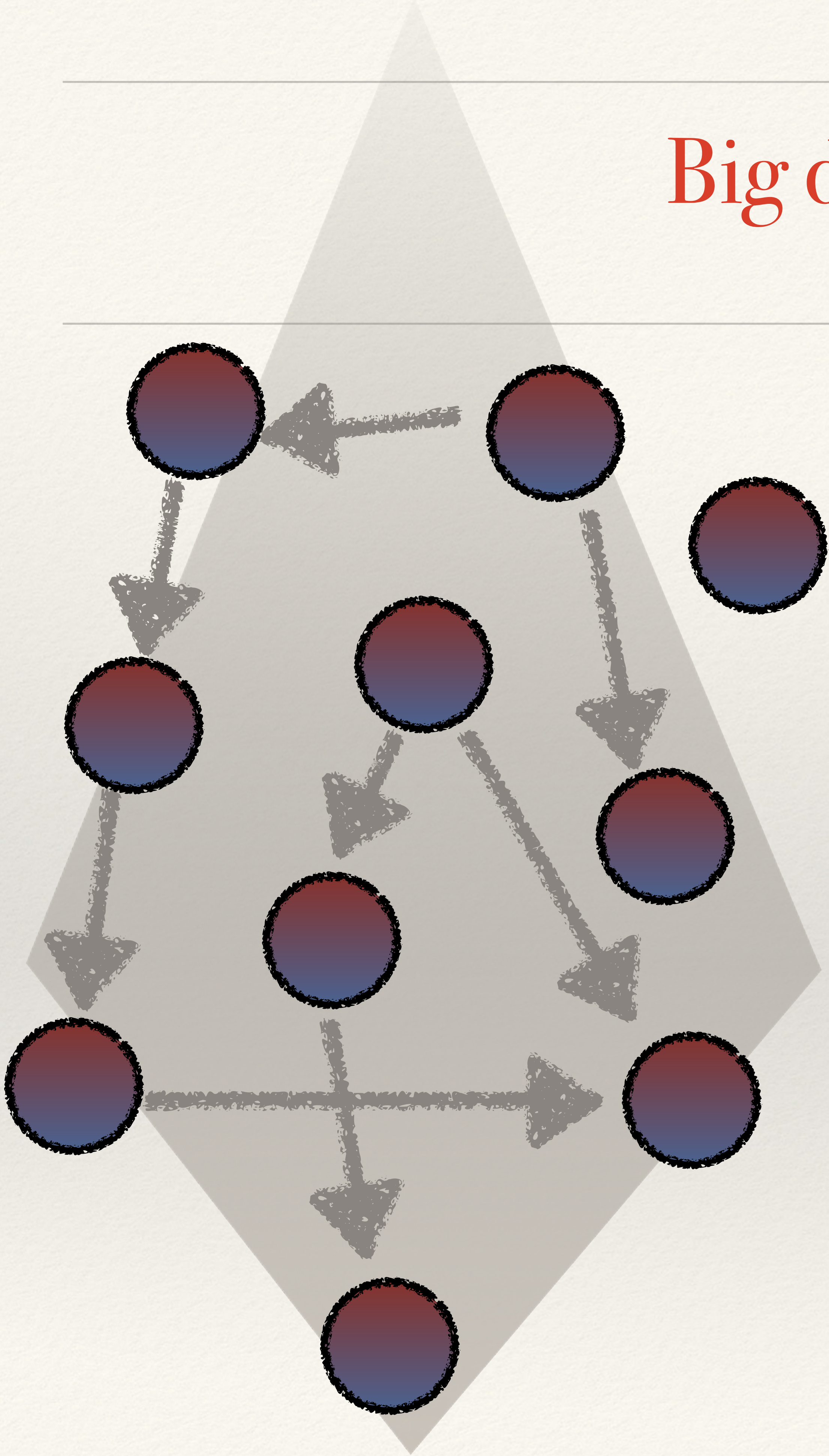
Reductionism scheme summary

- A lot of simple looking dualities in a given number of dimensions can be reduced to "basic moves"
- *(Another example is $USp(2N)$ Kutasov-Schwimmer/Intriligator duality)*
  Benvenuti, Comi, Pasquetti, Sacchi 2024
- Reducing Lagrangian 4d theories on a circle one can produce huge amount of 3d and lower dimensional dualities, some known/some new
  Niarchos 2012 *Aharony, SSR, Seiberg, Willett 2013*
- Reducing further to two dimensions leads to derivations of known dualities but also to interesting puzzles
  Aharony, SSR, Willett 2017; Gadde, SSR, Willett 2015; Nardoni, Sacchi, Zafrir, Zheng 2024 *Sacchi 2020* *Dedushenko, Gukov 2017*

- Understand ALL Lagrangian dualities in terms of a set of basic moves?

Big Data scheme

Big data scheme



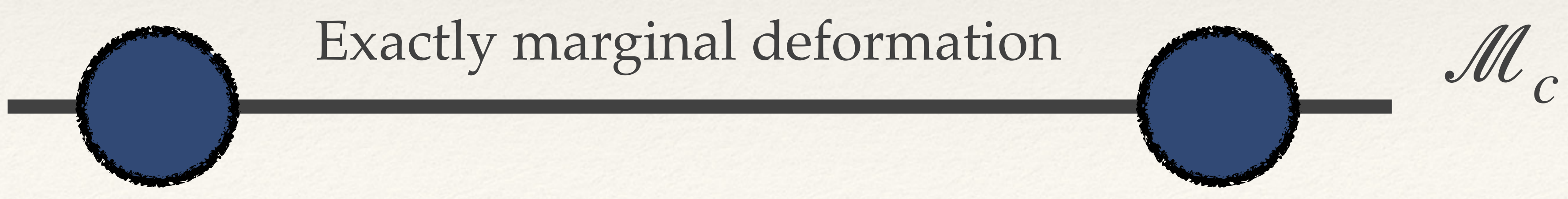
- Can organize the space of theories in various ways systematically and algorithmically
- Can study relations between different theories algorithmically and "experimentally"
- Look for patterns to discover new physics

Ex 1: Conformal Lagrangians

- Classify all interacting conformal gauge theories
- Determined by a choice of gauge group and matter such that exactly marginal deformations of free point exist

Leigh, Strassler 1995
Green, Komargodski, Seiberg, Tachikawa, Wecht 2010

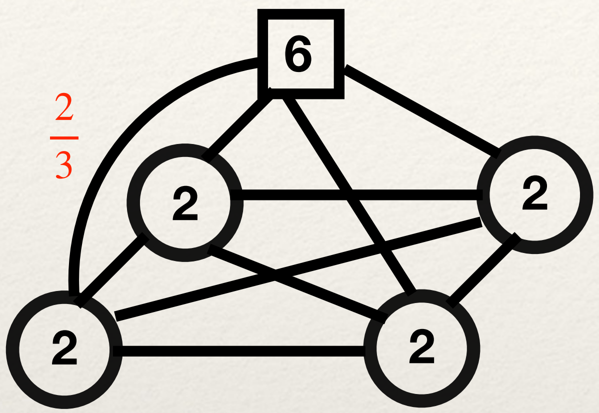
- Study interrelations between the conformal Lagrangians
- Search for conformal theories with identical protected data
- Look for conformal gauge theories matching strongly coupled SCFTs



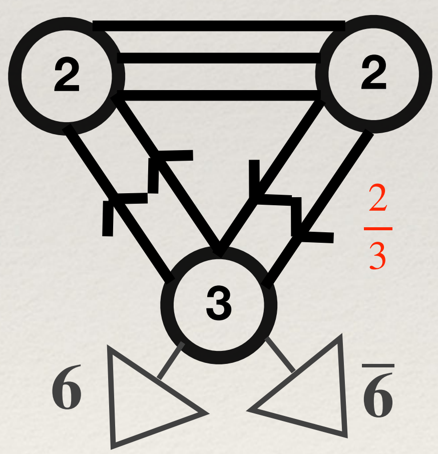
Scattered dualities

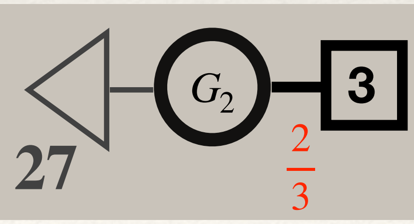
- Examples of dualities derived this way (completely algorithmic)

SSR, Zafrir 2019

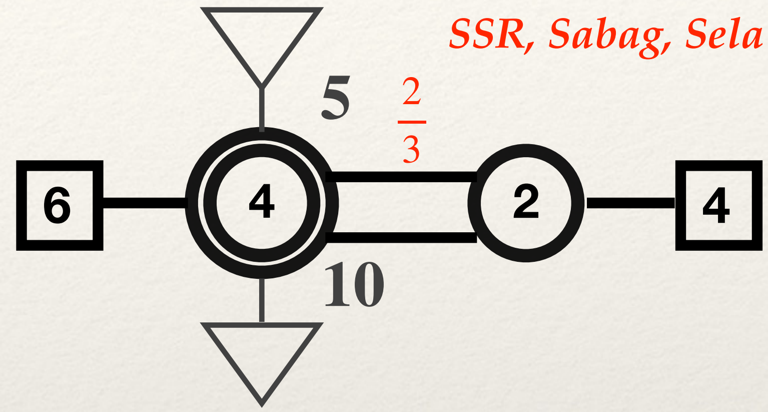


$R_{0,4}$ $\mathcal{N} = 2$ SCFT



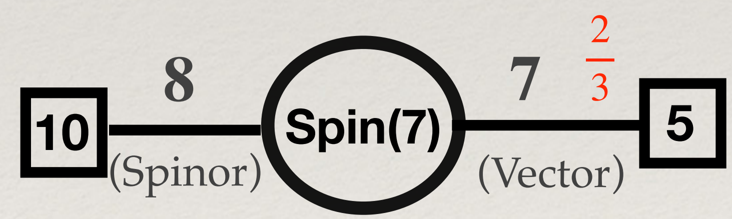


SSR, Sabag, Sela Zafrir 2022



A_2 Class S SCFT

SSR, Zafrir 2020



E_8 MN SCFT $\oplus$ $6\,\square$ / $USp(4)$

- Although derivation is algorithmic the results lack structure

Ex 2: RG fixed points

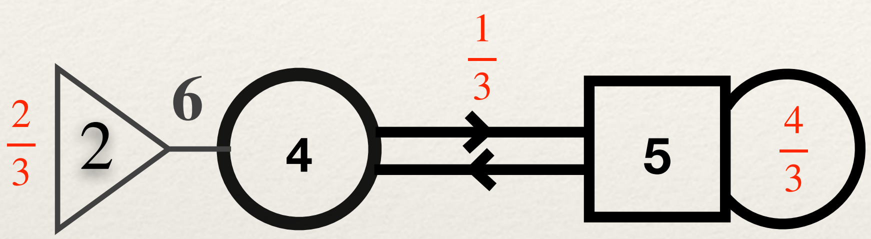
- Classify all RG fixed points
- Organize the classification starting from a set of theories and studying all the relevant deformations
- Organize the classification restricting the values of allowed conformal dimensions (constrain R-charges)

- Some IR fixed points are strongly coupled
- Some can fit interesting SCFTs, existence of which is predicted from elsewhere

Scattered results

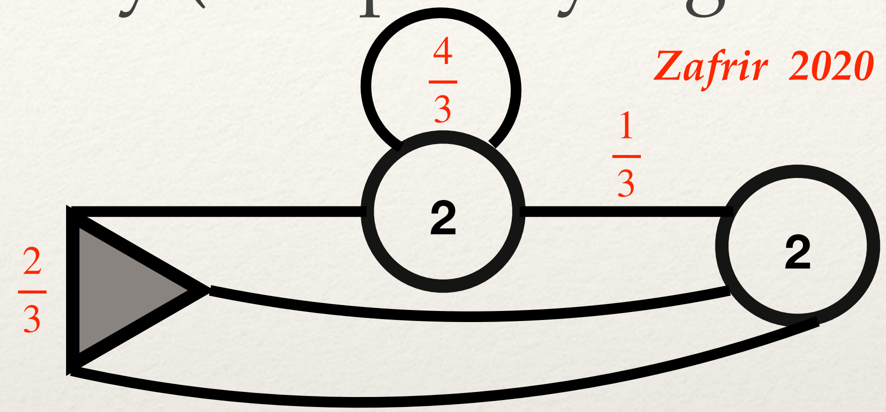
- Examples of results derived this way (completely algorithmic)

Zafrir 2019



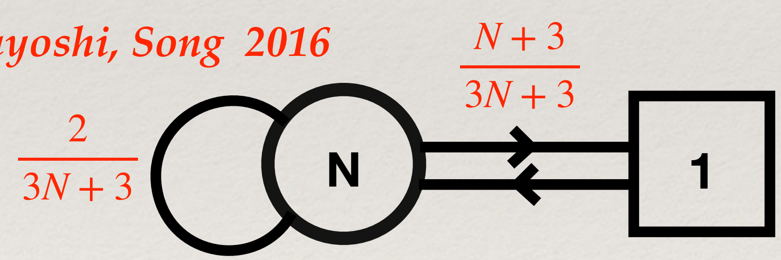
E_6 Minahan-Nemeschansky
$\mathcal{N}=2$ SCFT

Zafrir 2020



$\mathcal{N}=3$ SCFT
With $a=c=5/4$

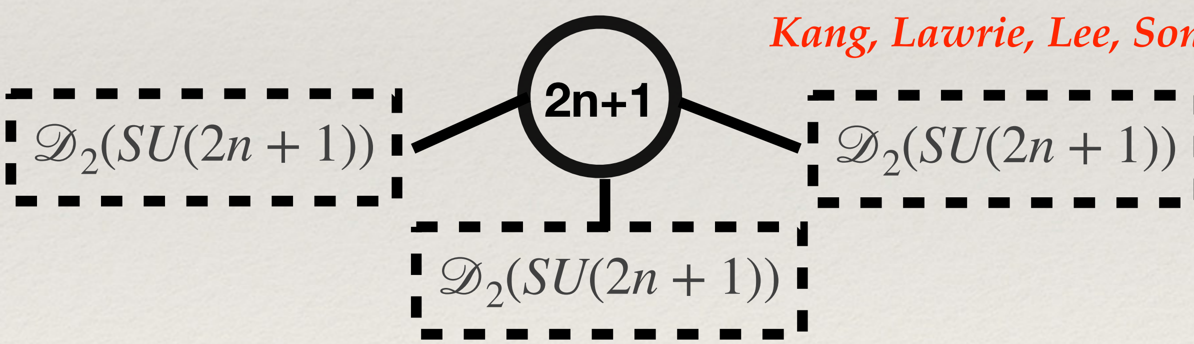
Maruyoshi, Song 2016



+ Singlets

(A_1, A_{2N-1}) AD
$\mathcal{N}=2$ SCFT

Kang, Lawrie, Lee, Song 2023



IR dual of SU(2n+1)
$\mathcal{N}=4$ SYM

- Although derivation is algorithmic the results lack structure

Bounds on central charges

- One can rigorously prove that the conformal anomalies in 4d $\mathcal{N} = 1$ SCFTs have to satisfy the Hofman-Maldacena bounds

$$\frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2}$$

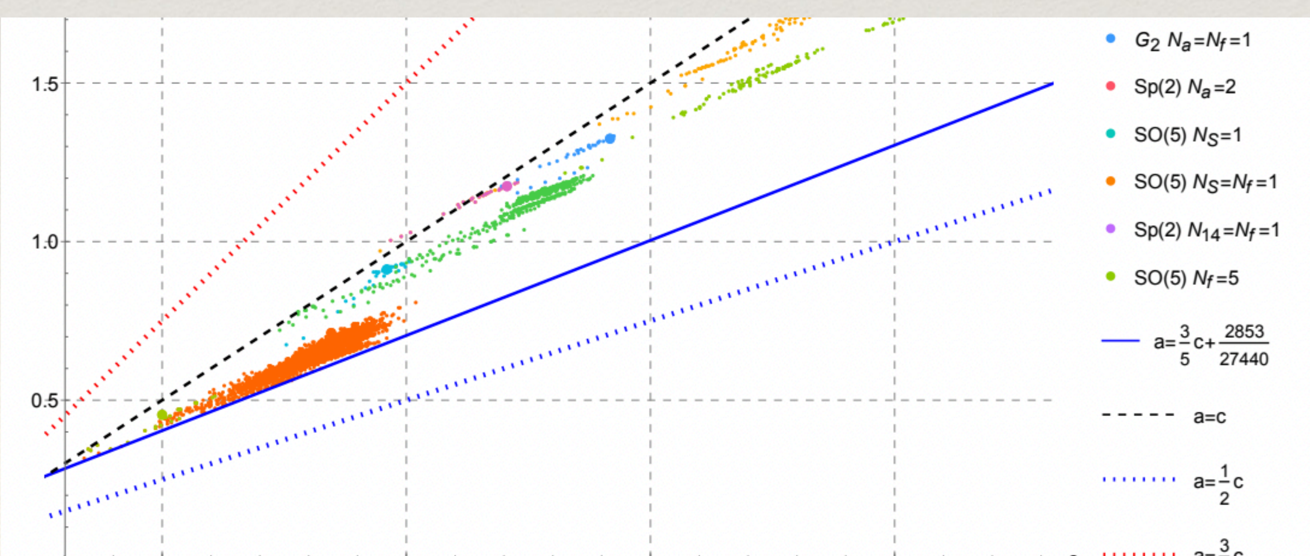
Hofman, Maldacena 2008

- The lower bound is saturated by free chiral superfields and the upper bound by free vector superfields
- However, scanning through RG flows it appears there is a gap: interacting SCFTS have $\frac{3}{5} \leq \frac{a}{c}$

Bobev, SSR unpublished

Benini, Bobev, Crichigno 2015; Bobev, Crichigno 2017

$$Tr\, R^3 \propto 5a - 3c \overset{?}{>} 0$$ *For interacting SCFTS*



Can this stronger bound be proven/debunked?

Eg: Cho, Maruyoshi, Nardoni, Song 2024

Ex 3: Moduli spaces of $\mathcal{N} = 2$: Coulomb

- $\mathcal{N} = 2$ SCFTs in 4d have a moduli space of vacua associated to them
- This involves in particular the Coulomb branch and the Higgs branch

Argyres, Martone and co 2015-2022

- The coulomb branch can be classified by its rank
- Can systematically classify Coulomb branch geometries rank by rank

- For $\mathcal{N} > 2$ the classification of moduli spaces seems to be related to complex crystallographic reflection groups

Eg Argyres, Bourget, Martone 2019; Tachikawa, Zafrir 2019; Kaidi, Martone, Zafrir 2022

(Deb, Zafrir 2024: 3d $\mathcal{N} = 5$ and quaternionic reflection groups)

- Some of these geometries known to be realized in the geometric scheme
- Many rank one theories have a known Lagrangian flowing to them (eg: E_6 MN, AD theories)

- Do all have Lagrangians?

Ex 4: Higgs : Magnetic quivers

- $\mathcal{N} = 2$ 4d Higgs branches are richer and harder to classify
- Many Higgs branches can be realized as Coulomb branches of three dimensional $\mathcal{N} = 4$ theories: in 3d this is statement of mirror symmetry
- *(Mirror symmetry is an IR duality in 3d exchanging the two branches)*
- Instead of studying the higher dimensional Higgs branches one can then study the three dimensional Coulomb branches
- *The relevant 3d theories are often called Magnetic Quivers*

Cabrera, Hanany, Yagi 2018 *Bourget, Cabrera, Grimminger, Hanany, Sperling, Zayac, Zhong 2019* *Hanany and Co 2018 — 2024 See Zhong Parallel session talk*

- *Tightly related to the Higgs branch is a chiral algebra one can associate to any $\mathcal{N} = 2$ SCFT* *Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees 2013*

Cordova, Gaiotto, Shao 2016 (Relation between Coulomb and Higgs)

Spiridonov, Vartanov 2014

- *$\mathcal{N} = 1$ can be also intricate: eg quantum corrections (PF vanish); singularities away from origin (PF diverges): "Bad" theories. Recent progress understanding PF as distributions*

Gaiotto, Witten 2008 *Yaakov 2013* *Assel, Cremonesi 2017* *Giacomelli, Hwang, Marino, Pasquetti, Sacchi 2023/2024*

Big data scheme summary

- Many of the questions discussed here are answered algorithmically
- One can produce a lot of results
- However, what is the physical significance / reason for the results?
- What is the pattern of the results?
- It seems these questions can be formulated for machine learning algorithms …

Beyond simple Lagrangians

Physics beyond Lagrangians

- We have discussed RG flows and dualities: using Lagrangians
- On the other hand we have considered looking for a structure on the space of theories
- Most theories we have discussed are strongly-coupled and thus direct computations are hard
- However in some cases one can perform computations exploiting a non-Lagrangian definition of the model
- Eg: Geometry defining a model in the geometric scheme: $T_{D''}[T_D, \mathscr{C}_{D'}]$
- Eg: AGT correspondence, the superconformal index, VOAs and $\mathcal{N} = 2$

Alday, Gaiotto, Tachikawa 2009 *Gadde, Pomoni, Rastelli, SSR 2009* *Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees 2013* *Beem, Rastelli 2017*

Integrability and the geometric scheme

- Eg: Given 6d SCFT T_{6d}; 5d gauge theory description with group $\mathcal{G}$;
- and compactification to 4d on genus g surface with s punctures;
- the superconformal index of the 4d theory is given by,

$$\mathcal{I}[T_{4d}] \propto \sum_{\Lambda} (C_\Lambda)^{2g-2+s} \prod_{i=1}^{s} \psi_\Lambda(\mathbf{a}_i^{(\mathcal{G})})$$

See talk by Nedelin

- ψ_Λ are eigenfunctions of a QM integrable system determined by $\{T_{6d}, \mathcal{G}\}$

- For many pairs $\{T_{6d}, \mathcal{G}\}$ the IM is known: Eg ADE (2,0), E-string

ADE Ruijsenaars-Schneider — BC_N van Diejen

- The index can be computed whether Lagrangians are known or not
- Classification of $\{T_{6d}, \mathcal{G}\}$ related to classification of IM

Ode to Math from Physics (and back)

Dyson and statistics
Of energy levels in
Complex Nuclei

1962

Cute evaluation
Identity of an integral

Proven by Gunson and Wilson

Andrews q-deformes
the identity
(Much harder to prove)

1975

Macdonald further generalizes
Andrews' conjectures
(Root systems)

1982

Macdonald polynomials
(And their relatives)
Appear in numerous
Supersymmetric QFT contexts

1990s and on

Modularity of partition functions

- The integrability connection is well understood, however there are other surprising ways to present the index
- Eg: A_1 (2,0) on genus g surface with $2s$ punctures, the *Schur* index is:

Cf Talk by Dorigoni

$$\mathcal{I}_{g,0}(q) \propto \sum_{i=0}^{g-1} \mathfrak{a}_i^{(g,0)} \mathbb{E}_{2i}(\tau)$$

Pan, Peelaers 2021 *Beem, SSR, Singh 2021* *Gun, Li, Pan, Wang 2024*

- It is not clear why the *Schur* index has such a simple expression
- The index is quasi-modular $(\mathbb{E}_2(\tau))$ and in more generality can be found to be Mock modular (non-conformal SQCD)

Dabholkar, Murthy, Zagier 2012 *Cordova, Gaiotto, Shao 2016* *Beem, Rastelli 2017*

- What is the shadow of the Schur index?

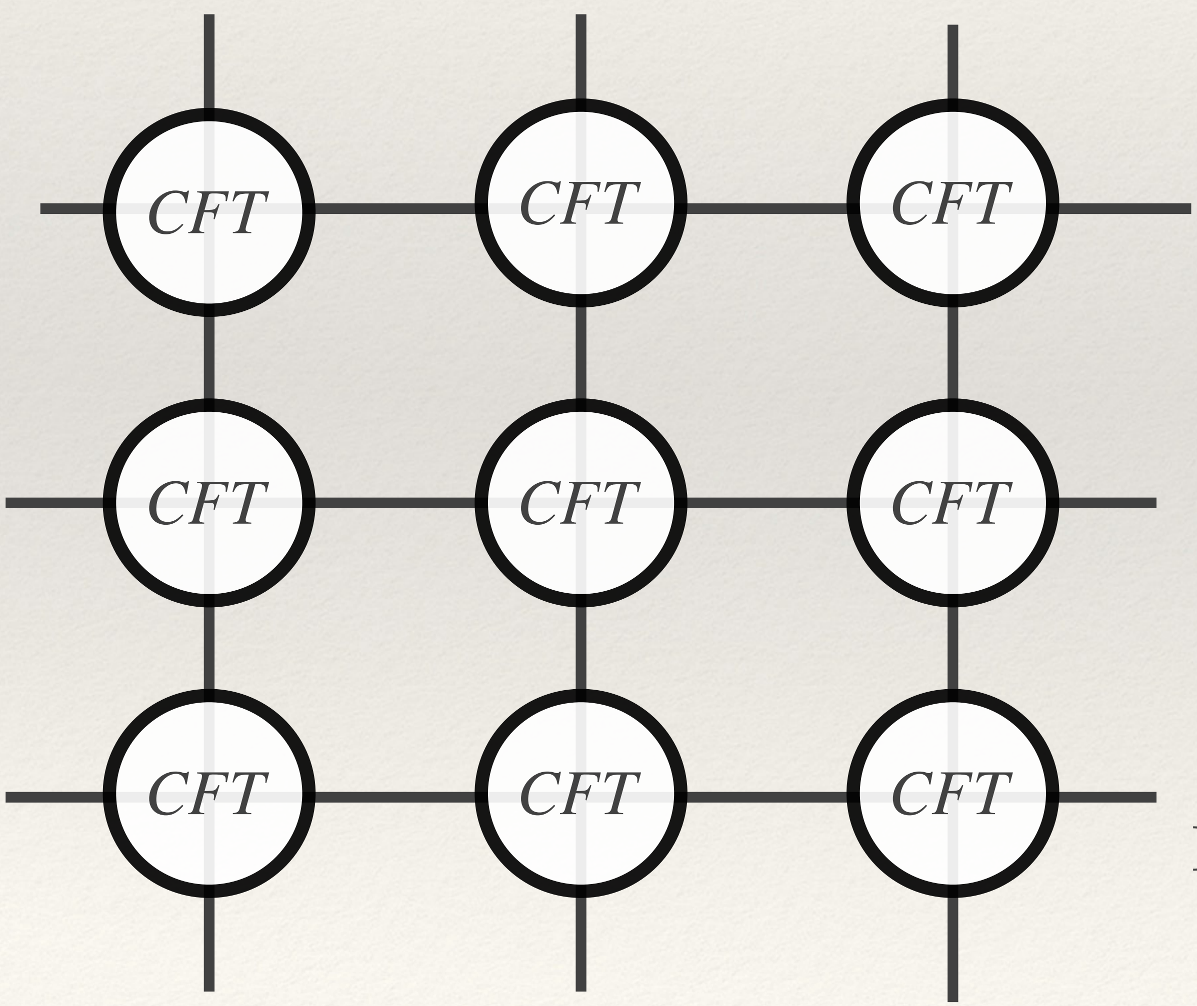
- *** Indices can be expanded in different ways and encode interesting physics (black hole micro states, giant gravitons, etc)*

Kim, Kim, Song 2019

Eg Gaiotto, Lee 2021 *Cassani, Komargodski 2021* *Murthy, Arabi Ardehali 2021* *Benini, Milan 2018* *Bourdier, Drukker, Felix 2015*

"Wire constructions" and Quivers

- We discussed how to get simple physics by complicating things
- We can also discuss complicated physics built from simple things



Lattices of coupled CFTs

Coupling can be through gauging/potentials

Quiver theories are an example in hep-th

In cond. matt.: ``Wire constructions''

Interesting physics in the limit of large lattice?

P. W. Anderson: ``More is different''

Lattices and emergence

- Which lattice limits lead to interesting physics?
- Eg: large number of sites and limits in moduli space in certain 1D lattice of 4D SCFTs leads to 6D SCFT / little string theory

Arkani-Hamed, Cohen, Kaplan, Karch, Motl 03 Hayling, Papageorgakis, Pomoni, Rodriguez-Gomez 17

- Space-time symmetry is believed to emerge in the limit **
- We do not have a QFT definition of the higher D SCFTs beyond attempts which break some space-time symmetry See eg work of Lambert
- Phrasing it differently: Can we reconstruct the higher D SCFTs understanding all of their lower D compactifications?

** *In many examples we have discussed global symmetry/supersymmetry was emergent*

Summary

- In recent years accumulated a lot of data about the space of all SCFTs
- There are many structures and patterns
- We understand the space of SCFTs much better than 20 years ago
- With great knowledge comes great sorrow: one might have the feeling that we are missing the big picture
- … But there is also hope that out of all the data will come a breakthrough/different, more fundamental, way to view QFTs

Thank You!!