Scattering amplitudes for Kerr black holes and higher-spin symmetry

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Sept 6, 2024

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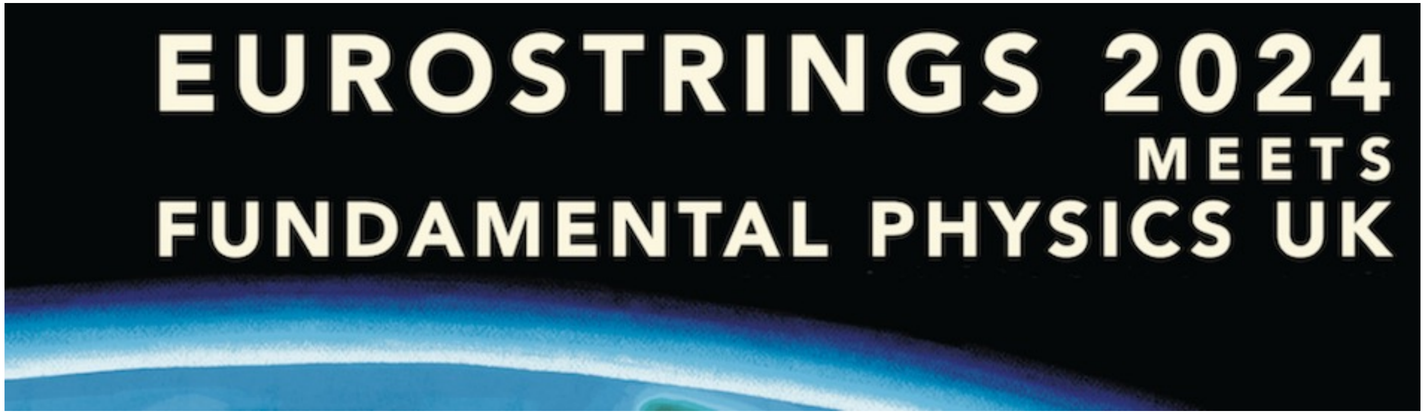


Based on refs: Chiodaroli, HJ, Pichini [2107.14779]; Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov [2212.06120], [2311.14668], [2312.14913]

Classical BH dynamics – PN, PM, spin

Post-Newtonian (PN) expansion:

expand in G and v

Bound systems:

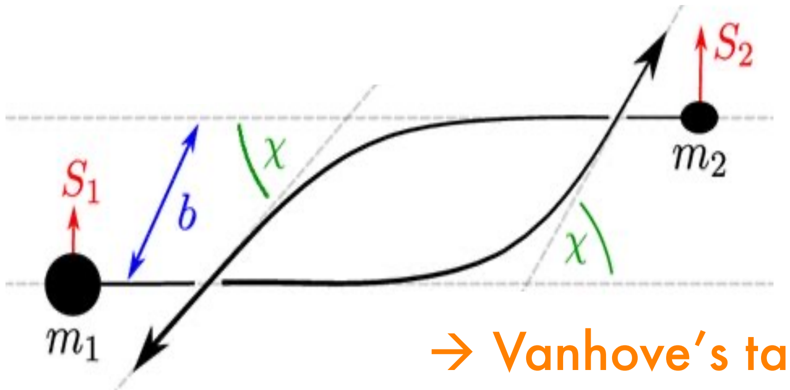
$$v^2 \sim \frac{GM}{r}$$

(virial theorem)

Post-Minkowskian (PM) expansion:

Gravitational scattering:

expand in G → loop expansion

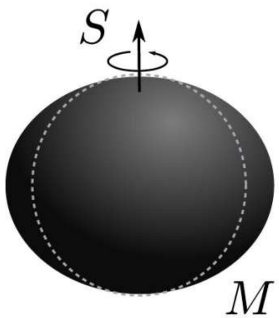


→ Vanhove's talk

Spin-multipole expansion:

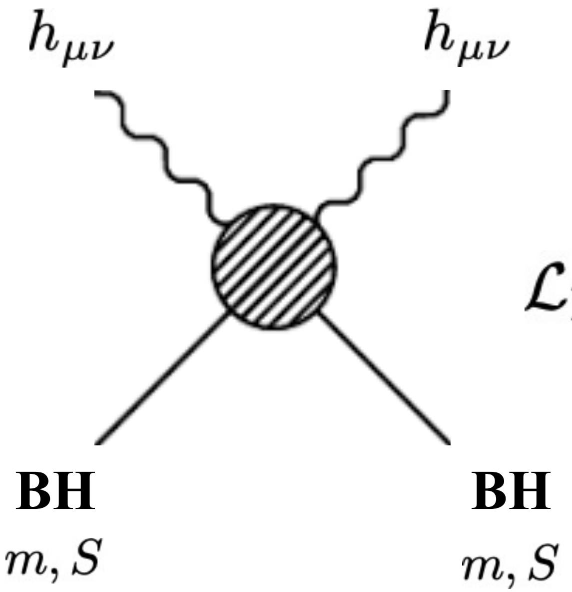
Rotating black holes

expand in S_1 and S_2



Methods
- BH perturbation theory
- Worldline EFTs
- Quantum scattering ampl's
- *higher-spin QFTs*

Kerr BH Compton scattering



$$\mathcal{L}_{\text{EFT}} \sim \Phi^s(\Box + m^2 + R + R^2 + \ldots)\Phi^s$$

- Eternal BHs = asymptotic states $(m, S, Q = 0)$
- Loops probe finite-size effects (horizon, tidal effects, QNM, etc.) $r_S = 2Gm$
- Tree-level = superextremal Kerr $Gm \ll S/m$
- Point-particle approximation valid
- Compton → BH dynamics → BH EFT

Outline

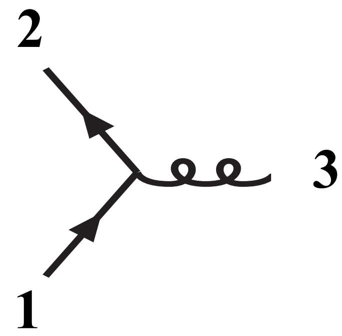
- Motivation
- The AHH higher-spin amplitudes
- The problem of Compton scattering
- Higher-spin gauge symmetry and EFTs
- Chiral HS fields and Compton spin-s result
- Conclusion

Higher-spin 3pt amplitudes & Kerr BH

Natural higher-spin gravitational 3pt amplitudes:

$$M(1\Phi^s, 2\Phi^s, 3h^+) = (\varepsilon_3^+ \cdot p_1)^2 \frac{\langle \mathbf{12} \rangle^{2s}}{m^{2s}}$$

$$M(1\Phi^s, 2\Phi^s, 3h^-) = (\varepsilon_3^- \cdot p_1)^2 \frac{[\mathbf{12}]^{2s}}{m^{2s}}$$



Arkani-Hamed, Huang, Huang ('17)

Linearized energy-momentum tensor for Kerr source

Vines ('17)

$$T^{\mu\nu}(-k) = 2\pi\,\delta(p \cdot k)\, p^{(\mu} \exp(m^{-1} S * ik)^{\nu)}{}_{\rho}\, p^{\rho}$$

Non-minimal worldline action for Kerr:

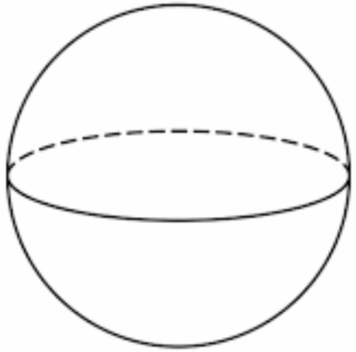
Levi, Steinhoff ('15)

$$L_{\rm SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

(spin-multipole expansion)

Spin operator (QM)

Introduce projective 3-sphere coordinates



$$z^a = (x_1 + ix_2, x_3 + ix_4) \quad \rightarrow \quad 1 = z^a \bar{z}_a = |x|^2$$

parametrizes SU(2) ←→ spin wavefn $\quad z^a \sim (|\uparrow\rangle, |\downarrow\rangle)$

Classical and quantum spin related as:

$$S^\mu = \frac{s}{2m} (\bar{z}^a z_a)^{2s-1} \left(\langle \bar{\mathbf{1}} | \sigma^\mu | \mathbf{1}] + \langle \mathbf{1} | \sigma^\mu | \bar{\mathbf{1}}] \right)$$

massive spinor-helicity formalism

Properties:

Transversality of spin vector: $\quad p_1 \cdot S = 0$

Equals an expectation value: $\quad S^\mu = \langle \hat{S}^\mu \rangle \equiv (\bar{z})^{2s} \cdot \hat{S}^\mu \cdot (z)^{2s}$

Gives spin operator: $\quad [\hat{S}^\mu, \hat{S}^\nu] = i\epsilon^{\mu\nu\rho} \hat{S}_\rho \qquad \hat{S}^2 = s(s+1)\mathbb{1}$

Massive spinor helicity

Following AHH bold massive spinors ←→ symmetrized little group indices

$$|\mathbf{i}\rangle \equiv |i^a\rangle z_{i,a}\,, \qquad |\mathbf{i}] \equiv |i^a] z_{i,a}$$

AHH

(spinors define maps: $SL(2,\mathbb{C}) \to SU(2)$)

Analytic functions of spinors now possible:

$$\langle \mathbf{12}\rangle^{2s} = \text{degree-}4s \text{ polynomial in } (z_1^a, z_2^a)$$

Massive polarizations are null vectors

Chiodaroli, HJ, Pichini

$$\varepsilon_i^\mu = \frac{\langle \mathbf{i}|\sigma^\mu|\mathbf{i}]}{\sqrt{2}m_i} = \frac{[\mathbf{i}|\bar{\sigma}^\mu|\mathbf{i}\rangle}{\sqrt{2}m_i} = (z_i^1)^2 \varepsilon^\mu_{i,-} - \sqrt{2} z_i^1 z_i^2 \varepsilon^\mu_{i,L} - (z_i^2)^2 \varepsilon^\mu_{i,+}$$

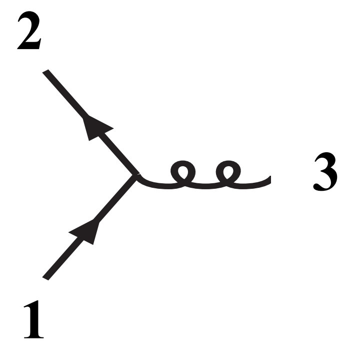
Higher-spin states automatically symmetric, transverse, traceless

$$\varepsilon_i^{\mu_1\mu_2\cdots\mu_s} \equiv \varepsilon_i^{\mu_1}\varepsilon_i^{\mu_2}\cdots\varepsilon_i^{\mu_s} = \text{degree-}2s \text{ polynomial in } z_i^a$$

AHH amplitudes = Kerr BHs

Relate in/out states by Lorentz transf.

$$|\mathbf{2}\rangle := |\bar{\mathbf{1}}\rangle + p_3 \cdot \sigma|\bar{\mathbf{1}}]/(2m).$$



AHH factor → exponential of spin operator:

$$\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}} = \Big\langle \sum_{n=0}^{2s} \frac{1}{n!}\Big(\frac{p_3 \cdot \hat{S}}{m}\Big)^n \Big\rangle = \langle e^{p_3 \cdot \hat{a}} \rangle$$

Quantum Kerr and root Kerr 3pt → Quantum Newman-Janis shift

$$M_{3,\pm}^{\text{Kerr}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle M_{3,\pm}^{\text{Schwarzchild}}$$

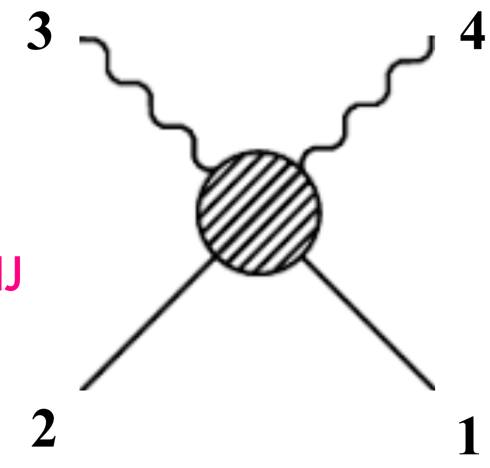
$$A_{3,\pm}^{\sqrt{\text{Kerr}}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle A_{3,\pm}^{\text{Coulomb}}$$

with ring-radius operator: $\hat{a}^\mu = \dfrac{\hat{S}^\mu}{m}$

(original argument: Guevara, Ochirov, Vines; see also Chung, Huang, Kim, Lee)

Kerr Compton amplitudes

Candidate Compton amplitudes via BCFW:



same helicity case:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+, 4h^+) = i\frac{\langle \mathbf{12}\rangle^{2s}[34]^4}{m^{2s-4}s_{12}t_{13}t_{14}}$$

Ochirov, HJ

oppsite helicity case:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-, 4h^+) = i\frac{[4|p_1|3\rangle^{4-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{s_{12}t_{13}t_{14}}$$

AHH

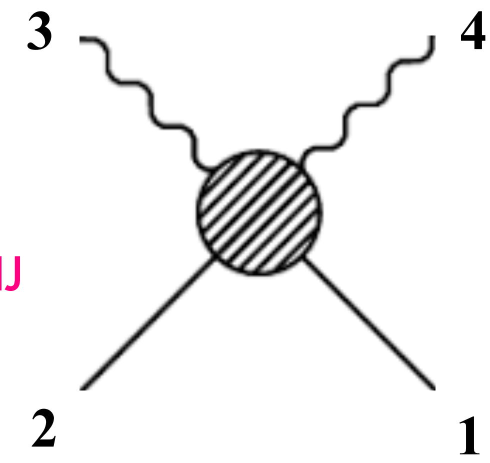
Needed for NLO calculations:

However, for $s > 2$ there is a spurious pole → need corrections

$$\frac{1}{[4|p_1|3\rangle^{2s-4}}$$

Gauge theory root-Kerr

Higher-spin gauge-theory: root-Kerr amplitudes



same helicity case:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+, 4A^+) = i\frac{\langle\mathbf{12}\rangle^{2s}[34]^2}{m^{2s-2}t_{13}t_{14}}$$

Ochirov, HJ

oppsite helicity case:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^-, 4A^+) = -i\frac{[4|p_1|3\rangle^{2-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{t_{13}t_{14}}$$

AHH

Not needed for physics purposes, but provide useful toy model!

Again, for $s > 1$ spurious pole $\frac{1}{[4|p_1|3\rangle^{2s-2}}$ → need corrections

Which quantum EFTs give Kerr amplitudes ?

EFTs behind root-Kerr

Identify EFTs from covariant formulas:

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx\frac{\langle \mathbf{12}\rangle^{2s}}{m^{2s}}$$

spin-0: $A(1\phi^0, 2\bar{\phi}^0, 3A) = \varepsilon_3 \cdot (p_1 - p_2) \equiv A_{\phi\phi A}$ (scalar)

spin-1/2: $A(1\phi^{1/2}, 2\bar{\phi}^{1/2}, 3A) = \bar{u}_2 \not{\varepsilon}_3 u_1 \equiv A_{\lambda\lambda A}$ (fermion)

spin-1: $A(1\phi^1, 2\bar{\phi}^1, 3A) = 2(\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2\, \varepsilon_3 \cdot p_2 + \boldsymbol{\varepsilon}_2 \cdot \varepsilon_3\, \boldsymbol{\varepsilon}_1 \cdot p_3 + \varepsilon_3 \cdot \boldsymbol{\varepsilon}_1\, \boldsymbol{\varepsilon}_2 \cdot p_1)$

$\equiv A_{WWA}$ (W-boson)

spin-3/2:

$$A(1\phi^{3/2}, 2\bar{\phi}^{3/2}, 3A) = \bar{u}_2^{\mu} \not{\varepsilon}_3 u_{1\mu} - \frac{2}{m}\bar{u}_{2\mu} f_3^{\mu\nu} u_{1\nu} - \frac{1}{2m}\bar{u}_2^{\mu} f_3^{\rho\sigma}\gamma_\rho\gamma_\sigma u_{1\mu} \equiv A_{\psi\psi A}$$

(gravitino)

general spin-s given as a generating function:

$$\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A) = A_{\phi\phi A} + \frac{A_{WWA} - (\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 A_{\phi\phi A}}{(1 + \boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2)^2 + \frac{2}{m^2}\boldsymbol{\varepsilon}_1 \cdot p_2\, \boldsymbol{\varepsilon}_2 \cdot p_1}$$

Chiodaroli, HJ, Pichini

For $s > 1$ → higher-derivative HS effective theories (no massless limit)

Kerr/root-Kerr double copy

Kerr amplitudes related to gauge th. via double copy

Chiodaroli, HJ, Pichini

$$M(1\phi^s, 2\bar{\phi}^s, 3h^\pm) = iA(1\phi^{s_\mathrm{L}}, 2\bar{\phi}^{s_\mathrm{L}}, 3A^\pm)A(1\phi^{s_\mathrm{R}}, 2\bar{\phi}^{s_\mathrm{R}}, 3A^\pm)$$

The general spin-s 3pt amplitude → generating fn

$$\sum_{2s=0}^{\infty} M(1\phi^s, 2\bar{\phi}^s, 3h) = M_{0\oplus 1/2} + A_{WWA}\Big(A_{0\oplus 1/2} + \frac{A_{1\oplus 3/2} - (\varepsilon_1\cdot\varepsilon_2)^2\, A_{0\oplus 1/2}}{(1+\varepsilon_1\cdot\varepsilon_2)^2 + \frac{2}{m^2}\varepsilon_1\cdot p_2\,\varepsilon_2\cdot p_1}\Big)$$

From double-copy structure, we can infer:

EFTs	$s=1/2$	$s=1$	$s=3/2$	$s=2$	$s=5/2$	$s\geq 3$
Kerr	Major.	Proca	Rar.-Sch.	KK grav.	HS	HS
$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	HS	HS	HS

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

For $s>2$ Kerr → higher-derivative HS EFTs (no massless limit)

Higher-spin (HS) theories

What special about the low-spin EFTs ?

Kerr (root-Kerr) EFTs for $s \leq 2 \ (s \leq 1)$

Chiodaroli, HJ, Pichini

→ well-behaved massless limit

→ exhibits gauge symmetry (SSB)

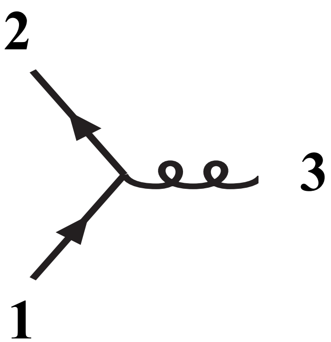
$$s = 1 \qquad \text{(YM + W-boson)} \rightarrow \text{non-abelian gauge symmetry}$$

$$s = 3/2 \quad \text{(GR + massive gravitino)} \rightarrow \text{supersymmetry}$$

$$s = 2 \qquad \text{(GR +massive KK graviton)} \rightarrow \text{General covariance}$$

Furthermore: satisfy a current constraint

$$p_1 \cdot J = \mathcal{O}(m)$$

$J =$ 

Connected to tree-level unitarity constraint;

Porrati et al.

longitudinal modes suppressed in low-mass (high-energy) limit

Using HS gauge invariance

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

Consider spin-2 root-Kerr case:

physical field: $\Phi_{\mu\nu}$

Stückelberg fields: $\{B_\mu, \varphi\}$

Imposing a linearized massive higher-spin gauge transformation:

$$\delta\Phi_{\mu\nu} = \frac{1}{2}\partial_\mu\xi_\nu + \frac{1}{2}\partial_\nu\xi_\mu + \frac{m}{\sqrt{2}}\eta_{\mu\nu}\xi,$$
$$\delta B_\mu = \partial_\mu\xi + \frac{m}{\sqrt{2}}\xi_\mu,$$
$$\delta\varphi = \sqrt{3}m\xi,$$

← gauge parameter

Makes sure that:

→ DOFs are correct,

→ small-mass limit better behaved than naively expected

Massive Ward identities

We write down ansatz for off-shell interactions:

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$V_{\Phi\overline{\Phi}A} \sim m\,(\epsilon_1)^2\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p^3}{m^3}+\frac{p}{m}\right),$$

$$V_{B\overline{\Phi}A} \sim m\,(\epsilon_1)\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p^2}{m^2}+1\right),$$

$$V_{\varphi\overline{\Phi}A} \sim m\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p}{m}\right),$$

and constrain them using Ward identities

$$V_{\xi\overline{\Phi}A}\big|_{(2,3)} = V_{\zeta\overline{\Phi}A}\big|_{(2,3)} = 0$$

where the vertices corresponding to gauge parameters are:

$$V_{\xi\overline{\Phi}A} := \frac{m}{\sqrt{2}}V_{B\overline{\Phi}A} - \frac{i}{2}p_1\cdot\frac{\partial}{\partial\epsilon_1}V_{\Phi\overline{\Phi}A},$$

$$V_{\zeta\overline{\Phi}A} := \sqrt{3}mV_{\varphi\overline{\Phi}A} - ip_1\cdot\frac{\partial}{\partial\epsilon_1}V_{B\overline{\Phi}A} + \frac{m}{2\sqrt{2}}\left(\frac{\partial}{\partial\epsilon_1}\right)^2 V_{\Phi\overline{\Phi}A}.$$

→ 3pt amplitude: $A(\Phi_1^2\,\overline{\Phi}_2^2\,A_3^+) = A_0\frac{\langle\mathbf{12}\rangle^3}{m^4}\left(c_1[\mathbf{12}] + (1-c_1)\langle\mathbf{12}\rangle\right)$

unique after current constraint: $c_1 = 0$

General spin-s EFTs

Consider tower $k = 0, 1, 2, \ldots, s$ of HS fields and gauge parameters:

$$\Phi^k := \Phi^{\mu_1 \mu_2 \cdots \mu_k}, \qquad \xi^k := \xi^{\mu_1 \mu_2 \cdots \mu_k}$$

(double-traceless) (traceless)

Zinoviev (2001)

Gauge transformation:

$$\delta \Phi^k = \partial^{(1} \xi^{k-1)} + m \alpha_k \xi^k + m \beta_k \eta^{(2} \xi^{k-2)}$$

$$\alpha_k = \frac{1}{k+1} \sqrt{\frac{(s-k)(s+k+1)}{2}}, \quad \beta_k = \frac{1}{2} \frac{k}{k-1} \alpha_{k-1}$$

Minimal Lagrangian:

$$\mathcal{L}_0 = \mathcal{L}_{\rm F} + \frac{1}{2} \sum_{k=0}^{s-1} (-1)^k (k+1) G^k G^k$$

Gauge-fixing fn:

$$G^k = \partial \cdot \Phi^{k+1} - \frac{k}{2} \partial^{(1} \tilde{\Phi}^{k+1)} + m \big(\alpha_k \Phi^k - \gamma_k \tilde{\Phi}^{k+2} - \delta_k \eta^{(2} \tilde{\Phi}^{k)} \big)$$

Feynman-gauge Lagr:

$$\mathcal{L}_{\rm F} = \sum_{k=0}^{s} \frac{(-1)^k}{2} \left[\Phi^k (\Box + m^2) \Phi^k - \frac{k(k-1)}{4} \tilde{\Phi}^k (\Box + m^2) \tilde{\Phi}^k \right]$$

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

Non-minimal interactions

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

3pt vertex: $V_{\Phi^k \Phi^s A^\mathfrak{h}} = V^{\text{min.}}_{\Phi^k \Phi^s A^\mathfrak{h}} + V^{\text{non-min.}}_{\Phi^k \Phi^s A^\mathfrak{h}}$

Ward identities:
$$V_{\xi^k \Phi^s A^\mathfrak{h}} := m\alpha_k V_{\Phi^k \Phi^s A^\mathfrak{h}} - \frac{i}{k+1} p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^{k+1} \Phi^s A^\mathfrak{h}} + \frac{m\beta_{k+2}}{(k+2)(k+1)} \frac{\partial}{\partial \epsilon_1} \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^{k+2} \Phi^s A^\mathfrak{h}}$$

Constraints imposed:

(WI) Ward identities $V_{\xi^k \Phi^s A^\mathfrak{h}}\big|_{(2,3),\epsilon_1^2 \to 0} = 0;$

(CC) Current constraint $p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^s \Phi^s A^\mathfrak{h}}\big|_{(2,3),\epsilon_1^2 \to 0} = \mathcal{O}(m)$.

(PC) Power-counting bound on derivatives in non-minimal vertices: $V^{\text{non-min.}}_{\Phi^{s_1} \Phi^{s_2} A^\mathfrak{h}} \sim \partial^{s_1+s_2-2\mathfrak{h}} (F_{\mu\nu})^\mathfrak{h};$

(ND) Near-diagonal interactions: if $|s_1 - s_2| > \mathfrak{h}$ then $V_{\Phi^{s_1} \Phi^{s_2} A^\mathfrak{h}} = 0.$

Gives unique Kerr and root-Kerr 3pt amplitudes (matching AHH)

HS perturbation theory

Calculations expected to simplify in Feynman gauge:

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Feynman-gauge propagator for any field obtained as generating fn:

$$\Delta(\epsilon,\bar{\epsilon}) = \sum_{s=0}^{\infty} (\epsilon)^s \cdot \Delta^{(s)} \cdot (\bar{\epsilon})^s = \frac{1}{p^2 - m^2 + i0} \frac{1 - \frac{1}{4}\epsilon^2\bar{\epsilon}^2}{1 + \epsilon\cdot\bar{\epsilon} + \frac{1}{4}\epsilon^2\bar{\epsilon}^2}$$

e.g. for root-Kerr Compton amplitude, we obtain

$$A(\Phi_1^s \Phi_2^s A_3^- A_4^+) = \frac{\langle 3|1|4]^2 (U+V)^{2s}}{m^{4s} t_{13} t_{14}} + \frac{\langle 3|1|4] \langle \mathbf{1}3\rangle [\mathbf{2}4] P^{(2s)}}{m^{4s} t_{13}} + \langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle [\mathbf{1}4][4\mathbf{2}] \frac{P^{(2s-1)}}{m^{4s}} + C_s,$$

contact term $C_{s<2} = 0$

with a polynomial: $P^{(k)} = \dfrac{\varsigma_1^k - \varsigma_2^k}{\varsigma_1 - \varsigma_2}$

and variables $\varsigma_1 = \langle \mathbf{1}|1+4|\mathbf{2}]\,, \quad \varsigma_2 = \langle \mathbf{2}|2+3|\mathbf{1}]$

Chiral fields (2s,0)

Chiral higher-spin approach

Ochirov, Skvortsov;
Cangemi, et al.

Easier way to get correct DOFs:

Change Lorentz rep. $$(s, s) \longrightarrow (2s, 0)$$

Chiral fields $$|\Phi\rangle := \Phi_{\alpha_1 \ldots \alpha_{2s}}$$ $\mathrm{SL}(2, \mathbb{C})$ indices

Minimal Lagrangian $$\mathcal{L}^{(s)}_{\mathrm{min.}} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle$$

Gives ''correct'' all-plus helicity amplitudes:

$$A_n(1^s, 2^s, 3^+, 4^+, \ldots, n^+) = \langle \mathbf{12} \rangle^{2s} A_n^{\mathrm{scalar}}$$

However, breaks parity badly, and also naive renormalizability...

W-bosons in SM: $$\mathcal{L}^{(1)} = \langle \Phi | \left\{ |\overleftarrow{D}|\overrightarrow{D}| \otimes \frac{1}{1 - \frac{ig}{m^2}|F_-|} \right\} |\Phi\rangle - m^2 \langle \Phi | \Phi \rangle + \mathcal{O}(\Phi^4)$$

Chalmers, Siegel

Non-minimal chiral interactions

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Restore parity at 3pts → AHH 3pt amplitudes:

Root-Kerr non-minimal interactions:

$$\mathcal{L}^{(s)} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle + \sum_{k=0}^{2s-1} \frac{ig}{m^{2k}} \langle \Phi | \Big\{ |\overleftarrow{D}|\overrightarrow{D}|^{\odot k} \otimes |F_-| \Big\} | \Phi \rangle + \mathcal{O}(F^2)$$

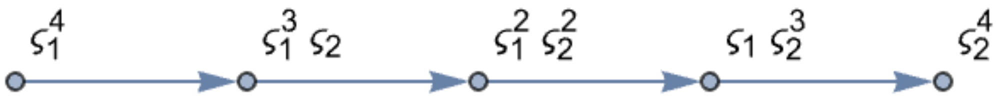
Kerr non-minimal interactions:

$$\mathcal{L}_{\text{Kerr}} = \sqrt{-g} \Big\{ \frac{1}{2} \langle \nabla_\mu \Phi | \nabla^\mu \Phi \rangle - \frac{m^2}{2} \langle \Phi | \Phi \rangle - \frac{1}{4} \sum_{k=0}^{2s-2} \frac{2s-k-1}{m^{2k}} \langle \Phi | \Big\{ \big(|\overleftarrow{\nabla}|\overrightarrow{\nabla}| \big)^{\odot k} \odot |R_-| \Big\} | \Phi \rangle \Big\}$$
$$+ \mathcal{O}(R^2)$$

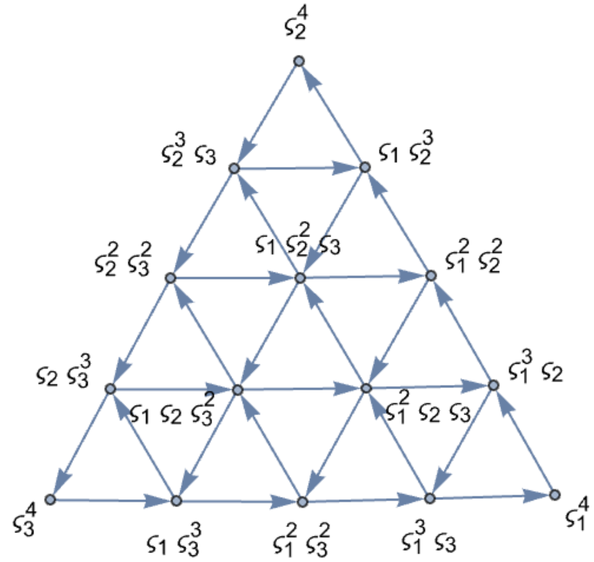
Interactions behave as geometic series $\sim \dfrac{1}{1 - |\overleftarrow{D}|\overrightarrow{D}|} \odot |F_-|$

Omnipresent polynomials

Consider geometric sums:



$$= \frac{\varsigma_1^5 - \varsigma_2^5}{\varsigma_1 - \varsigma_2}$$



$$= \frac{\varsigma_1^6}{(\varsigma_1 - \varsigma_2)(\varsigma_1 - \varsigma_3)} + \text{perm}(\varsigma_1, \varsigma_2, \varsigma_3)$$

In general:

Complete homogenous symmetric polynomials:

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$P_n^{(k)} = \frac{\varsigma_1^k}{(\varsigma_1 - \varsigma_2)(\varsigma_1 - \varsigma_3)\ldots(\varsigma_1 - \varsigma_n)} + \text{perm}(\varsigma_1, \varsigma_2, \ldots \varsigma_n)$$

Compton spin variables:

$$\varsigma_1 = \langle \mathbf{1}|1{+}4|\mathbf{2}]\,,\quad \varsigma_2 = \langle \mathbf{2}|2{+}3|\mathbf{1}]\,,\quad \varsigma_3 = m\langle \mathbf{21}\rangle\,,\quad \varsigma_4 = m[\mathbf{21}]$$

Constraints for fixing R^2 contact term

<u>Assumptions</u>: contact terms depend only on $C^{(s)} = C^{(s)}[P_n^{(k)}]$

- well-behaved classical limit $s \to \infty$;
- compatible with massive higher-spin gauge invariance;
- s-independent numerical coefficients;
- parity invariance
- all contact terms have spinor-helicity structure $\sim \big(\langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]\big)^2$
- classical spin hexadecapole S^4 is fixed by $s=2$ amplitude
- improved behavior in $m \to 0$ limit: $M(1^s, 2^s, 3^-, 4^+) \sim m^{-4s+4}$

Kerr amplitude from chiral fields + contact

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Final Kerr Compton amplitude (quantum spin):

$$M(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^4 P_1^{(2s)}}{m^{4s}s_{12}t_{13}t_{14}} - \frac{\langle \mathbf{13}\rangle[4\mathbf{2}]\langle 3|1|4]^3}{m^{4s}s_{12}t_{13}} P_2^{(2s)} + \frac{\langle \mathbf{13}\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s}s_{12}}\left(\langle 3|1|4]^2 P_2^{(2s-1)} + m^4\langle 3|\rho|4]^2 P_4^{(2s-1)}\right)$$
$$+ \frac{\langle \mathbf{13}\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s-2}s_{12}}\langle 3|1|4]\langle 3|\rho|4]\left(P_2^{(2s-2)} - m^2\langle \mathbf{12}\rangle[\mathbf{12}]P_4^{(2s-2)}\right)$$
$$+ \frac{\langle \mathbf{13}\rangle^2\langle 3\mathbf{2}\rangle^2[\mathbf{1}4]^2[4\mathbf{2}]^2}{2m^{4s-4}}\langle \mathbf{12}\rangle[\mathbf{12}]\left[(1+\eta)P_{5|\varsigma_1}^{(2s-2)} + (1-\eta)P_{5|\varsigma_2}^{(2s-2)}\right] + \alpha\, C_\alpha^{(s)}.$$

Includes some dissipative effects after matching to BHPT

Bautista, Guevara, Kavanagh, Vines, et al

Does not incude: near-zone contributions or loop corrections

(similar expression for root-Kerr gauge theory)

Root-Kerr Lagrangian and classical amplitude

Chiral spin-s Lagrangian (gauge theory)

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

$$\mathcal{L} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle + \sum_{k=0}^{2s-1} \frac{ig}{m^{2k}} \langle \Phi | \left\{ |\overleftarrow{D}|\overrightarrow{D}|^{\odot k} \odot |F_-| \right\} |\Phi\rangle + \mathcal{O}(|F_-|^2)$$

$$+ \sum_{k\leq l=0}^{2s-4} \sum_{j=0}^{2s-3-l} \frac{g^2}{m^{2(j+l)+6}} \langle \Phi | \left\{ (|\overleftarrow{D}|\overrightarrow{D}| + m^2) \odot |\overleftarrow{D}|\overrightarrow{D}|^{\odot j} \odot |\overleftarrow{D}|\overrightarrow{D}_+|^{\odot k} \odot |\overleftarrow{D}_+|\overrightarrow{D}|^{\odot (l-k)} \odot \mathfrak{F}_6 \right\} |\Phi\rangle$$

Field-strength dependence: $\mathfrak{F}_6 = \frac{1}{4} \{T^c, T^{c'}\} |F_-^c| \odot |\overleftarrow{D}| F_+^{c'} |\overrightarrow{D}|$

Classical root-Kerr amplitude: $\lim_{s\to\infty, \hbar\to 0} \mathcal{A}(\mathbf{1}, \mathbf{2}, 3^-, 4^+)$

$$= -2g^2 (p\cdot\chi)^2 \left\{ \left(\frac{[T^{c_3}, T^{c_4}]}{q^2 (p\cdot q_\perp)} + \frac{1}{2} \frac{\{T^{c_3}, T^{c_4}\}}{(p\cdot q_\perp)^2} \right) \left(e^x \cosh z - w\, e^x \operatorname{sinhc} z + \frac{w^2 - z^2}{2} E(x,y,z) \right) - \frac{[T^{c_3}, T^{c_4}]}{q^2 (p\cdot q_\perp)} \Big(x(w^2 + z^2) - w(x^2 - y^2 + z^2) \Big) \tilde{E}(x,y,z) \right\}.$$

$$E(x,y,z) = \frac{e^y - e^x \cosh z + (x-y) e^x \operatorname{sinhc} z}{(x-y)^2 - z^2} + (y \to -y)$$

$$x = a\cdot q_\perp, \quad y = a\cdot q, \quad z = |a| \frac{p\cdot q_\perp}{m}, \quad w = \frac{a\cdot\chi\, p\cdot q_\perp}{p\cdot\chi}$$

Final classical results – Kerr BH

Classical root-Kerr amplitude: $\lim_{s\to\infty,\hbar\to 0} \mathcal{A}(\mathbf{1},\mathbf{2},3^-,4^+)$

$$= -2g^2(p\cdot\chi)^2\Big\{\Big(\frac{[T^{c_3},T^{c_4}]}{q^2(p\cdot q_\perp)}+\frac{1}{2}\frac{\{T^{c_3},T^{c_4}\}}{(p\cdot q_\perp)^2}\Big)\Big(e^x\cosh z - w\,e^x\mathrm{sinhc}\,z+\frac{w^2-z^2}{2}E(x,y,z)\Big)$$
$$-\frac{[T^{c_3},T^{c_4}]}{q^2(p\cdot q_\perp)}\Big(x(w^2+z^2)-w(x^2-y^2+z^2)\Big)\tilde{E}(x,y,z)\Big\}.$$

Classical Kerr BH amplitude: Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$\mathcal{M}(\mathbf{1},\mathbf{2},3^-,4^+)=\frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2}\Big(e^x\cosh z - w\,e^x\mathrm{sinhc}\,z+\frac{w^2-z^2}{2}E(x,y,z)\Big)$$
$$+\frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2}\frac{w^2-z^2}{2}(w-x)\tilde{E}(x,y,z)$$
$$-\frac{(p\cdot\chi)^4}{(p\cdot q_\perp)^4}\frac{(w^2-z^2)^2}{2}\Big(\frac{\partial\tilde{E}}{\partial x}+\eta\frac{\partial\tilde{E}}{\partial z}\Big) \qquad +\alpha z\,(\text{polygamma terms})$$

Matches explicit BH perturbation theory from GR → Teukolsky eqn.

up to spin S^7 (ignoring polygamma terms) Bautista, Guevara, Kavanagh, Vines

Conclusion: Kerr dynamics from HS

Kerr dynamics is non-trivially constrained by

– massive higher-spin gauge symmetry
– power counting, current constraint, ...

Checks: → uniquely predicts previously known Kerr 3-4pt amplitudes
→ constrains $s \geq 2$ 4pt contact terms, but not unique...

Additional constraints imposed:

– chiral Lagrangian,
– symmetric homogeneous polynomials,...
– classical limit consistency,
– matching to Teukolsky BHPT (mod. polygamma terms)

Outlook: → classical loop corrections to Compton
→ implications for quantum BHs,
→ including absorption and emission effects