Scattering amplitudes for Kerr black holes and higher-spin symmetry

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Based on refs: Chiodaroli, HJ, Pichini [2107.14779]; Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov [2212.06120], [2311.14668], [2312.14913]

Kerr BH Compton scattering

- Eternal BHs = asymptotic states $(m, S, Q = 0)$
- Loops probe finite-size effects (horizon, tidal effects, QNM, etc.)
- Tree-level = superextremal Kerr
- Point-particle approximation valid
- Compton \rightarrow BH dynamics \rightarrow BH EFT
-
- $r_S=2Gm$

$$
Gm \ll S/m
$$

Outline

O Motivation

- **The AHH higher-spin amplitudes**
- **The problem of Compton scattering**
- **A** Higher-spin gauge symmetry and EFTs
- Chiral HS fields and Compton spin-s result

Conclusion

Linearized energy-momentum tensor for Kerr source Vines ('17)

here commutation \mathbf{C} relations \mathbf{C}

imple linear relations Due to the Lie algebra of the gauge symmetry, color factors obey ^s

$$
T^{\mu\nu}(-k) = 2\pi \,\delta(p \cdot k) \, p^{(\mu} \exp(m^{-1}S * ik)^{\nu)}_{\rho} \, p^{\rho}
$$

Non-minimal worldline action for Kerr:
\n
$$
L_{SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1 \mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}
$$
\n
$$
+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1 \mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}
$$
\n(spin-multipole expansion)

Spin operator (QM)

Introduce projective 3-sphere coordinates

$$
z^a = (x_1 + ix_2, x_3 + ix_4) \quad \to \quad 1 = z^a \bar{z}_a = |x|^2
$$

parametrizes SU(2) \iff spin wavefn $z^a \sim (|\uparrow\rangle, |\downarrow\rangle)$

Classical and quantum spin related as:

massive spinor-helicity formalism Properties:

Transversality of spin vector: $p_1 \cdot S = 0$

Equals an expectation value: $S^{\mu} = \langle \hat{S}^{\mu} \rangle \equiv (\bar{z})^{2s} \cdot \hat{S}^{\mu} \cdot (z)^{2s}$ Gives spin operator: $[\hat{S}^{\mu}, \hat{S}^{\nu}] = i\epsilon^{\mu\nu\rho}\hat{S}_{\rho}$ $\hat{S}^{2} = s(s+1)\mathbb{1}$

Massive spinor helicity 2.2 Bookkeeping of little-group indices, polarizations and projectors other identities familiar from the mass spinor-helicity formalism still hold, such and Fierz identity for the mass of the mass <u>viassive sp</u>

 I Following AHH bold massive spinors \leftrightarrow symmetrized little group indices as School (Fierz identities: The Fierz identities) !12"!34" ⁺ !23"!14" ⁺ !31"!24" = 0 , !1|σ^µ|2]!3|σµ|4]=2!13"[42] . (2.7) − 1 _≀old massive sp $\frac{1}{s}$ for the field $\frac{1}{s}$ (\rightarrow symmetrized ⁱ p^ν i d lit
 α aroun indices

$$
| \hspace{.06cm} \mathbf{i} \hspace{.06cm} \rangle \equiv | i^a \rangle z_{i,a} \hspace{.1cm} , \hspace{.1cm} | \hspace{.06cm} \mathbf{i} \hspace{.06cm}] \equiv | i^a] z_{i,a} \hspace{.7cm} \textsf{AHH}
$$

(spinors define maps: $SL(2,\mathbb{C})\rightarrow SU(2)$) $2\pi(2, 0)$ is $2\pi(2, 0)$ t_{minors} define mane: $\mathcal{C}I(2,\mathbb{C})$, $\mathcal{C}II(2)$ the derivative operator introduced here will be computing state sums $\mathcal{L}(\mathcal{A})$ (spinors define maps: $SL(2,\mathbb{C})\rightarrow SU(2)$)

completeness relation for the transverse part of the Lorentz group,

Analytic functions of spinors now possible: |
|-
|-Andryne functions or spinors now possible.

$$
\langle {\bf 12}\rangle^{2s} = \text{degree-}4s
$$
 polynomial in (z_1^a, z_2^a)

 m ussive poturizumens ure non vectors with omedicity, i.e., i.e. m Massive polarizations are null vectors

Chiodaroli, HJ, Pichini

$$
\varepsilon_i^{\mu} = \frac{\langle \mathbf{i} | \sigma^{\mu} | \mathbf{i} \rangle}{\sqrt{2}m_i} = \frac{[\mathbf{i} | \bar{\sigma}^{\mu} | \mathbf{i} \rangle}{\sqrt{2}m_i} = (z_i^1)^2 \varepsilon_{i,-}^{\mu} - \sqrt{2} z_i^1 z_i^2 \varepsilon_{i,L}^{\mu} - (z_i^2)^2 \varepsilon_{i,+}^{\mu}
$$

Higher-spin states automatically symmetric, transverse, traceless

$$
\varepsilon_i^{\mu_1\mu_2\cdots\mu_s} \equiv \varepsilon_i^{\mu_1} \varepsilon_i^{\mu_2} \cdots \varepsilon_i^{\mu_s} = \text{degree-2s polynomial in } z_i^a
$$

AHH amplitudes = Kerr BHs

Relate in/out states by Lorentz transf.

$$
|{\bf 2}\rangle:=|\bar{{\bf 1}}\rangle+p_3\cdot\sigma|\bar{{\bf 1}}]/(2m).
$$

1 2 3

AHH factor \rightarrow exponential of spin operator:

$$
\frac{\langle \boldsymbol{12}\rangle^{2s}}{m^{2s}}=\Big\langle \sum_{n=0}^{2s}\frac{1}{n!}\Big(\frac{p_3\cdot \hat{S}}{m}\Big)^n\Big\rangle=\big\langle e^{p_3\cdot \hat{a}}\big\rangle
$$

Quantum Kerr and root Kerr 3pt \rightarrow Quantum Newman-Janis shift

$$
M_{3,\pm}^{\text{Kerr}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle M_{3,\pm}^{\text{Schwarzchild}}
$$

\n
$$
A_{3,\pm}^{\sqrt{\text{Kerr}}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle A_{3,\pm}^{\text{Coulomb}}
$$

\nwith ring-radius operator: $\hat{a}^{\mu} = \frac{\hat{S}^{\mu}}{m}$

(original argument: Guevara, Ochirov, Vines; see also Chung, Huang, Kim, Lee)

cannot be ruled out that it receives corrections that it receives corrections that it receives a spin. Any spin \mathbf{r}

this suggests that it should not be corrected by corrected by contact terms, although a priori it should be co
It should not be contact that it should be contact that it should be contact that it is not be contact that it

in a unique way has not yet been firmly established. In contrast, we see that the that the sees that the see t
In contrast, we see that the sees that t

 $\frac{1}{[4|p_1|3\rangle^{2s-4}}$

Gauge theory root-Kerr

Not needed for physics purposes, but provide useful toy model!

Again, for
$$
s > 1
$$
 spurious pole $\frac{1}{[4|p_1|3\rangle^{2s-2}} \rightarrow$ need corrections

Which quantum EFTs give Kerr amplitudes ?

EFTs behind root-Kerr two massive higher-spin particles and a gauge boson of the spin particles and a gauge bos showled be the following maximally-chiral objects:

Identify EFTs from covariant formulas:

$$
A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx \frac{\langle \mathbf{12} \rangle^{2s}}{m^{2s}}
$$

$$
\mathsf{spin}\text{-}\mathbf{0} \qquad \qquad A(1\phi^0, 2\bar{\phi}^0, 3A) = \varepsilon_3 \cdot (p_1 - p_2) \equiv A_{\phi\phi A} \qquad \qquad \text{(scalar)}
$$

- spin-1/2: $A(1\phi^{1/2}, 2\bar{\phi}^{1/2}, 3A) = \bar{u}_2 \notin_3 u_1 \equiv A_{\lambda\lambda A}$ (fermion)
- spin-1: $A(1\phi^1, 2\bar{\phi}^1, 3A) = 2(\varepsilon_1 \cdot \varepsilon_2 \varepsilon_3 \cdot p_2 + \varepsilon_2 \cdot \varepsilon_3 \varepsilon_1 \cdot p_3 + \varepsilon_3 \cdot \varepsilon_1 \varepsilon_2 \cdot p_1)$ $\equiv A_{WWA}$ (W-boson) s_n : $\frac{1}{2}$ /2.

$$
A(1\phi^{3/2}, 2\bar{\phi}^{3/2}, 3A) = \bar{u}_2^{\mu} \dot{\phi}_3 u_{1\mu} - \frac{2}{m} \bar{u}_{2\mu} f_3^{\mu\nu} u_{1\nu} - \frac{1}{2m} \bar{u}_2^{\mu} f_3^{\rho\sigma} \gamma_{\rho} \gamma_{\sigma} u_{1\mu} \equiv A_{\psi\psi A}
$$
\n(gravitino)

general spin-s given as a generating function:
\n
$$
\sum_{s=0}^{\infty} A(1\phi^s, 2\bar{\phi}^s, 3A) = A_{\phi\phi A} + \frac{A_{WWA} - (\epsilon_1 \cdot \epsilon_2)^2 A_{\phi\phi A}}{(1 + \epsilon_1 \cdot \epsilon_2)^2 + \frac{2}{m^2} \epsilon_1 \cdot p_2 \epsilon_2 \cdot p_1}
$$
\n**Chiodaroli,** HJ, Pichini

For $s > 1$ \rightarrow higher-derivative HS effective theories (no massless limit)

It is clear from the spinor-kerr double copy and the spinor-helicity expressions that the gauge-theory and the gauge-theory are related to the gauge-theory and the gauge-theory and the gauge-theory amplitudes. Using the double copy, it is a small step to show that a generating function for $\frac{1}{2}$ generating function for $\frac{1}{2}$

Chiodaroli, Chioda

Chiodaroli, HJ, Pichini

 $M(1\phi^s,2\bar\phi^s,3h^\pm)=iA(1\phi^{s_{\rm L}},2\bar\phi^{s_{\rm L}},3A^\pm)A(1\phi^{s_{\rm R}},2\bar\phi^{s_{\rm R}},3A^\pm)$ generations. However, however, because the spin s can be decomposed into sL \sim sR $M(1\phi^3, 2\phi^3, 3h^+) = \imath A(1\phi^{3L}, 2\phi^{3L}, 3A^+) A(1\phi^{3R}, 2\phi^{3R}, 3A^+)$

The general spin-s 3pt amplitude \rightarrow generating fn The general spin-s 3pt amplitude \rightarrow generating fn

$$
\sum_{2s=0}^{\infty} M(1\phi^s, 2\bar{\phi}^s, 3h) = M_{0\oplus 1/2} + A_{WWA} \Big(A_{0\oplus 1/2} + \frac{A_{1\oplus 3/2} - (\varepsilon_1 \cdot \varepsilon_2)^2 A_{0\oplus 1/2}}{(1 + \varepsilon_1 \cdot \varepsilon_2)^2 + \frac{2}{m^2} \varepsilon_1 \cdot p_2 \varepsilon_2 \cdot p_1} \Big)
$$

gauge-theory amplitudes. I follit double copy, showing, we can find for show that a generating function for $\frac{1}{2}$ From double-copy structure, we can infer: we use the short-integer spin s. Here we use the short-integer spin s

 $\frac{1}{2}$ cova $\frac{1}{2}$ Reil $\frac{1}{2}$ ingher-defivative form of the form $\Gamma_{\text{max}} > 2$ Kem Δ kirker derivative HC EFT, $\ln e$ mereless $\ln 2$ $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ are more as well-known three-point and $\frac{1}{2}$ are more in the $\frac{1}{2}$ For $s > 2$ Kerr \rightarrow higher-derivative HS EFTs (no massless limit)

Higher-spin (HS) theories

What special about the low-spin EFTs ?

Kerr (root-Kerr) EFTs for $s \leq 2$ $(s \leq 1)$

 \rightarrow well-behaved massless limit

Chiodaroli, HJ, Pichini

 \rightarrow exhibits gauge symmetry (SSB)

 $s = 1$ (YM + W-boson) \rightarrow non-abelian gauge symmetry

 $s = 3/2$ (GR + massive gravitino) \rightarrow supersymmetry

 $(GR + \text{massive KK graviton}) \rightarrow \text{General covariance}$ $s=2$

Furthermore: satisfy a current constraint

$$
p_1 \cdot J = \mathcal{O}(m)
$$

1 2 $J = \sum e e 3$

Connected to tree-level unitarity constraint; Porrati et al.

ic relations: $\frac{1}{2}$ the Jacobi Pictorial form of the basic color and kinematic Lie-algebra $\frac{1}{2}$ longitudinal modes suppressed in low-mass (high-energy) limit

Using HS gauge invariance

Consider spin-2 root-Kerr case: Consider spin-2 root-Kerr case:

Cangemi, Chiodaroli, HJ,

$$
\textsf{physical field:} \;\; \Phi_{\mu\nu} \qquad \qquad \textsf{Stückelberg fields:} \;\; \big\{ B_\mu, \varphi \big\}
$$

Imposing a linearized massive higher-spin gauge transformation:

$$
\delta \Phi_{\mu\nu} = \frac{1}{2} \partial_{\mu} \xi_{\nu} + \frac{1}{2} \partial_{\nu} \xi_{\mu} + \frac{m}{\sqrt{2}} \eta_{\mu\nu} \xi,
$$

\n
$$
\delta B_{\mu} = \partial_{\mu} \xi + \frac{m}{\sqrt{2}} \xi_{\mu},
$$

\n
$$
\delta \varphi = \sqrt{3} m \xi,
$$

Makes sure that:

 \rightarrow DOFs are correct,

 \rightarrow small-mass limit better behaved than naively expected

Massive Ward identities

We write down ansatz for off-shell interactions: Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

$$
V_{\Phi\overline{\Phi}A} \sim m (\epsilon_1)^2 (\epsilon_2)^2 \epsilon_3 \left(\frac{p^3}{m^3} + \frac{p}{m}\right),
$$

$$
V_{B\overline{\Phi}A} \sim m (\epsilon_1) (\epsilon_2)^2 \epsilon_3 \left(\frac{p^2}{m^2} + 1\right),
$$

$$
V_{\varphi\overline{\Phi}A} \sim m (\epsilon_2)^2 \epsilon_3 \left(\frac{p}{m}\right),
$$

and constrain them using Ward identities

$$
V_{\xi\overline\Phi A}\big|_{(2,3)}\hspace{-.1cm}=\hspace{-.1cm}V_{\zeta\overline\Phi A}\big|_{(2,3)}\hspace{-.1cm}=\hspace{-.1cm}0
$$

where the vertices corresponding to gauge parameters are:

$$
\begin{split} V_{\xi\overline{\Phi}A}&:=\frac{m}{\sqrt{2}}V_B\overline{\Phi}A-\frac{i}{2}p_1\cdot\frac{\partial}{\partial\epsilon_1}V_{\Phi}\overline{\Phi}A,\\ V_{\zeta\overline{\Phi}A}&:=\sqrt{3}mV_{\varphi\overline{\Phi}A}-ip_1\cdot\frac{\partial}{\partial\epsilon_1}V_{B}\overline{\Phi}A+\frac{m}{2\sqrt{2}}\left(\frac{\partial}{\partial\epsilon_1}\right)^2V_{\Phi}\overline{\Phi}A. \end{split}
$$

 \rightarrow 3pt amplitude: $A(\Phi_1^2 \overline{\Phi}_2^2 A_3^+) = A_0 \frac{\langle 12 \rangle^3}{m^4} (c_1[12] + (1-c_1)\langle 12 \rangle)$ unique after current constraint: $c_1 = 0$

General spin-s EFTs

Consider tower $k = 0, 1, 2, ..., s$ of HS fields and gauge parameters:

$$
\Phi^k := \Phi^{\mu_1 \mu_2 \cdots \mu_k}, \qquad \xi^k := \xi^{\mu_1 \mu_2 \cdots \mu_k} \qquad \text{Zinoviev (2001)}
$$
\n(double-traceless)

Gauge transformation:

$$
\delta\Phi^k=\partial^{(1}\xi^{k-1)}+m\alpha_k\xi^k+m\beta_k\eta^{(2}\xi^{k-2)}
$$

$$
\alpha_k = \frac{1}{k+1} \sqrt{\frac{(s-k)(s+k+1)}{2}}, \quad \beta_k = \frac{1}{2} \frac{k}{k-1} \alpha_{k-1},
$$

Minimal Lagrangian:

Gauge-fixing fn:

Feynman-gauge Lagr:

$$
\mathcal{L}_0 = \mathcal{L}_{\rm F} + \frac{1}{2} \sum_{k=0}^{s-1} (-1)^k (k+1) G^k G^k
$$

$$
G^k = \partial \cdot \Phi^{k+1} - \frac{k}{2} \partial^{(1} \tilde{\Phi}^{k+1}) + m \big(\alpha_k \Phi^k - \gamma_k \tilde{\Phi}^{k+2} - \delta_k \eta^{(2} \tilde{\Phi}^{k)} \big)
$$

$$
\mathcal{L}_{\rm F} = \sum_{k=0}^s \frac{(-1)^k}{2} \bigg[\Phi^k (\Box + m^2) \Phi^k - \frac{k(k-1)}{4} \tilde{\Phi}^k (\Box + m^2) \tilde{\Phi}^k \bigg]
$$

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Non-minimal interactions

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

3pt vertex: $V_{\Phi^k \Phi^s A^{\mathfrak{h}}} = V_{\Phi^k \Phi^s A^{\mathfrak{h}}}^{\min.} + V_{\Phi^k \Phi^s A^{\mathfrak{h}}}^{\min.}$

Ward identities:
$$
V_{\xi^k \Phi^s A^{\mathfrak{h}}} := m \alpha_k V_{\Phi^k \Phi^s A^{\mathfrak{h}}} - \frac{i}{k+1} p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^{k+1} \Phi^s A^{\mathfrak{h}}}
$$

 $+ \frac{m \beta_{k+2}}{(k+2)(k+1)} \frac{\partial}{\partial \epsilon_1} \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^{k+2} \Phi^s A^{\mathfrak{h}}}$

Constraints imposed:

$$
\text{(WI)}\;\;\text{Ward identities}\;V_{\xi^k\Phi^sA^{\mathfrak h}}\big|_{(2,3),\epsilon_1^2\to 0}=0;
$$

(CC) Current constraint
$$
p_1 \cdot \frac{\partial}{\partial \epsilon_1} V_{\Phi^s \Phi^s A^{\mathfrak{h}}}|_{(2,3), \epsilon_1^2 \to 0} = \mathcal{O}(m)
$$
.

- (PC) Power-counting bound on derivatives in nonminimal vertices: $V^{\text{non-min}}_{\Phi^{s_1} \Phi^{s_2} A^{\mathfrak{h}}} \sim \partial^{s_1+s_2-2\mathfrak{h}} (F_{\mu\nu})^{\mathfrak{h}};$
- (ND) Near-diagonal interactions: if $|s_1-s_2| > \mathfrak{h}$ then $V_{\Phi^{s_1}\Phi^{s_2}A^{\mathfrak{h}}}=0.$

Gives unique Kerr and root-Kerr 3pt amplitudes (matching AHH)

HS perturbation theory

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov Calculations expected to simplify in Feynman gauge:

Feynman-gauge propagator for any field obtained as generating fn:

$$
\Delta(\epsilon,\bar{\epsilon}) = \sum_{s=0}^{\infty} (\epsilon)^s \cdot \Delta^{(s)} \cdot (\bar{\epsilon})^s = \frac{1}{p^2 - m^2 + i0} \frac{1 - \frac{1}{4} \epsilon^2 \bar{\epsilon}^2}{1 + \epsilon \cdot \bar{\epsilon} + \frac{1}{4} \epsilon^2 \bar{\epsilon}^2}
$$

e.g. for root-Kerr Compton amplitude, we obtain

$$
A(\Phi_1^s \Phi_2^s A_3^- A_4^+) = \frac{\langle 3|1|4|^2 (U+V)^{2s}}{m^{4s} t_{13} t_{14}} + \frac{\langle 3|1|4|\langle 13\rangle [24] P^{(2s)}}{m^{4s} t_{13}} + \langle 13\rangle \langle 32\rangle [14][42] \frac{P^{(2s-1)}}{m^{4s}} + C_s,
$$

with a polynomial:
$$
P^{(k)} = \frac{\zeta_1^k - \zeta_2^k}{\zeta_1 - \zeta_2}
$$
and variables
$$
\zeta_1 = \langle 1|1+4|2], \ \zeta_2 = \langle 2|2+3|1]
$$

Chiral fields (2s,0)

Chiral higher-spin approach action (4.1). This corresponds to multiplying the path integral by an integral by a

In the construction \mathcal{L} and \mathcal{L} compton amplitudes \mathcal{L} and \mathcal{L} of \mathcal{L}

*^m*² ^h*[|]*

D|

Ochirov, Skvortsov; Cangemi, et al. ² *^m* $\textsf{Change Lorentz rep.} \quad \textit{(S, S)} \longrightarrow \textit{(2S, O)}$ $\vert \Phi \rangle := \Phi_{\alpha_1...\alpha_{2s}} \qquad \text{SL}(2,\mathbb{C}) \text{ indices}$ ${\cal L}^{(s)}_{\rm min} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle$ *L*(*s*) $\langle \alpha^s, 2^s, 3^+, 4^+, \ldots, n^+ \rangle = \langle \mathbf{12} \rangle^{2s} A_n^{\rm scalar}$ Con Formalism for Con Trade SO(1*,* 3) tensors *µ*1*...µ^s* ! SL(2*,* C) chiral symmetric tensors ↵1*...*↵2*^s* Chiral fields indices $\epsilon_{\min} = (\frac{D_\mu \Psi}{D' \Psi})$ $e^{i\phi}$ a⁺ chiral formulation $e^{i\phi}$ or $e^{i\phi}$ for $e^{i\phi}$ and $e^{i\phi}$ $\lambda_1, \lambda_2, \ldots, n$ $\lambda_n = \langle 12 \rangle - A_n$ Easier way to get correct DOFs: Minimal Lagrangian Gives "correct" all-plus helicity amplitudes: This is convenient when the number of spinorial indices becomes Gaussian integral. The new Lagrangian integral α by parts and rewritten as integrated by parts and rewritten as ² (*Wµ*) p2 *W*_{μ} tr(*u*¹, α where the SL(2^{*, C})* is over the SL(2*, C*) indices. We can now integrate out the original vector of original vector original vector of α </sup> $|\Psi\rangle - \Psi$ $)$ @*µ*↵@*µ*↵ + 2 ↵↵ =: ¹ 2 ^h@*µ|*@*µ*ⁱ ⁺ T the chiral gauge-interacting Lagrangian for massive spin-1 was first introduced intro ref. [81] for the purpose of describing electroweak vector bosons. Adapting it to a general

However, breaks parity badly, and also naive renormalizability… non-abelian gauge group, with *|*i transforming in a matter representation, we find the parity baary, and also haive renormanzability...

W-bosons in SM:
$$
\mathcal{L}^{(1)} = \langle \Phi | \left\{ |\overrightarrow{D}|\overrightarrow{D}| \otimes \frac{1}{1 - \frac{ig}{m^2}|F_-|} \right\} | \Phi \rangle - m^2 \langle \Phi | \Phi \rangle + \mathcal{O}(\Phi^4)
$$

Chalmers, Siegel

terms and minimal chiral interactions of ref. [79], and then add appropriate cubic nonminimal interactions to restore particles to restore particles to result and the resulting α

straightforward to generalize it to the spin-*s* case. In particular, we combine the kinetic

Restore parity at $3\text{pts} \rightarrow \text{AHH}$ 3pt amplitudes: Ochirov, Pichini, Skvortsov cangemi, Chiodaroli, HJ,
Deste result with a striple of the NAHH 2nd area relitived assets. Ochirov Pichini, Shvortov Restore parity at 3pts \rightarrow AHH 3pt amplitudes: \qquad Ochirov, Pichini, Skvortso

Cangemi, Chiodaroli, HJ,

\mathbf{r} and be written compactly as \mathbf{r} Root-Kerr non-minimal interactions:

$$
\mathcal{L}^{(s)} = \langle D_{\mu} \Phi | D^{\mu} \Phi \rangle - m^2 \langle \Phi | \Phi \rangle + \sum_{k=0}^{2s-1} \frac{ig}{m^{2k}} \langle \Phi | \left\{ |\stackrel{\leftarrow}{D} |\stackrel{\rightarrow}{D}|^{\odot k} \otimes |F_{-}| \right\} | \Phi \rangle + \mathcal{O}(F^2)
$$

Werr non-minimal interactions: we additionally resort product and the symbol of the west of the west of the symbol of the sy

$$
\mathcal{L}_{\text{Kerr}} = \sqrt{-g} \left\{ \frac{1}{2} \langle \nabla_{\mu} \Phi | \nabla^{\mu} \Phi \rangle - \frac{m^2}{2} \langle \Phi | \Phi \rangle - \frac{1}{4} \sum_{k=0}^{2s-2} \frac{2s-k-1}{m^{2k}} \langle \Phi | \left\{ \left(| \nabla | \vec{\nabla} | \right)^{\odot k} \odot |R_-| \right\} | \Phi \rangle \right\} + \mathcal{O}(R^2)
$$

SU(*N*c) or SO(*N*c) actually have imaginary-valued generators. Interactions behave as geometic series

$$
\frac{1}{1-|D|D|}\odot|F_-|
$$

Omnipresent polynomials

In general:

 $(31 - 52)(51 - 53)$
U_{*L*} *U Complete homogenous symmetric polynomials:*

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$
P_n^{(k)} = \frac{\varsigma_1^k}{(\varsigma_1 - \varsigma_2)(\varsigma_1 - \varsigma_3)\dots(\varsigma_1 - \varsigma_n)} + \text{perm}(\varsigma_1, \varsigma_2, \dots, \varsigma_n)
$$

-*n* = 2 example: Compton spin variables:

$$
\zeta_1 = \langle 1|1+4|2], \ \ \zeta_2 = \langle 2|2+3|1], \ \ \zeta_3 = m\langle 21 \rangle, \ \ \zeta_4 = m[21]
$$

Example 2 Constraints for fixing R² contact term *C*(*s*) = *C*(*s*) [*P*(*k*) *ⁿ*] *.* (5.7) *C*(*s*) = *C*(*s*) [*P*(*k*) *ⁿ*] *.* (5.7) Furthermore, we impose the following heuristic constraints on the complete amplitude:

Constraints. Therefore, to determine *C*(*s*) we assume that it is a linear combination of

 $\mathbf{1}$, so every term in the Compton amplitude is multiplied by some polynomial $\mathbf{1}$ <u>Assumptions</u>: contact terms depend only on $\;\;C^{(s)}=C^{(s)}[P_n^{(k)}]$ $\binom{n}{n}$]

for constraining the contact term *C*(*s*) $\bullet\,$ well-behaved classical limit $s\to\infty;$

i+*j*+*k*+*l*=2*s*4

- $\bullet\,$ compatible with massive higher-spin gauge invariance; *Kerr theory should be a finite*
- \bullet $\,$ s-independent numerical coefficients; • *s*-independent numerical coefficients;
- parity invariance
	- all contact terms have spinor-helicity structure $\sim (\langle 13 \rangle \langle 32 \rangle [14][42])^2$ $\mathbf{e} \cdot \mathbf{e}$
- classical spin hexadecapole $S⁴$ is fixed by $s=2$ amplitude • *s*-independent numerical coecients; • classical spin hexadecapole $S⁴$ is fixed by $s = 2$ ampli
	- improved behavior in $m \to 0$ limit: $M(1^s, 2^s, 3^-, 4^+) \sim$ \bullet is particular invariant $M(18.08.9 - 4+)$; $m = 4s + 4$ • improved behavior in $m \to 0$ limit: $M(1^s, 2^s, 3^-, 4^+) \sim m^{-4s+4}$

Kerr amplitude from chiral fields + contact

Cangemi, Chiodaroli, HJ,

Ochirov, Pichini, Skvortsov Final Kerr Compton amplitude (quantum spin):

$$
M(\mathbf{1}^{s}, \mathbf{2}^{s}, 3^{-}, 4^{+}) = \frac{\langle 3|1|4]^{4} P_{1}^{(2s)}}{m^{4s} s_{12} t_{13} t_{14}} - \frac{\langle 13 \rangle [42] \langle 3|1|4]^{3}}{m^{4s} s_{12} t_{13}} P_{2}^{(2s)} + \frac{\langle 13 \rangle \langle 32 \rangle [14][42]}{m^{4s} s_{12}} \left(\langle 3|1|4]^{2} P_{2}^{(2s-1)} + m^{4} \langle 3|\rho|4]^{2} P_{4}^{(2s-1)} \right) + \frac{\langle 13 \rangle \langle 32 \rangle [14][42]}{m^{4s-2} s_{12}} \langle 3|1|4] \langle 3|\rho|4] \left(P_{2}^{(2s-2)} - m^{2} \langle 12 \rangle [12] P_{4}^{(2s-2)} \right) + \frac{\langle 13 \rangle^{2} \langle 32 \rangle^{2} [14]^{2} [42]^{2}}{2m^{4s-4}} \langle 12 \rangle [12] \left[(1+\eta) P_{5|\varsigma_{1}}^{(2s-2)} + (1-\eta) P_{5|\varsigma_{2}}^{(2s-2)} \right] + \alpha C_{\alpha}^{(s)}.
$$

Includes some dissipative effects after matching to BHPT Bautista, Guevara, Kavanagh, Vines, et al

Does not incude: near-zone contributions or loop corrections

(similar expression for root-Kerr gauge theory)

Root-Kerr Lagrangian and classical amplitude IUULTIGII LAGIANGIAN ANU VIASSIVAI ANIPIILUUG the final spin-*^s* Compton amplitudes, given later in this section. That is, the ^p In order to set the stage, we quote a chiral Lagrangian that is fully compatible with the final spin-bot-kerring spin-term and spin-term in the post-Kerrin Lagrangian and classical amplitude functions *^E*˜*, ^E*˜⁰ *, ^E*˜⁰⁰ can be read o↵ from the amplitude and correspond to the functions **Root-Kerr Lagrang** As mentioned at the end of Section 5.1, the choice of quantum contact term *C*(*s*) is

Chiral spin-s Lagrangian (gauge theory) 2 *s*1 Chiral spin-s Lagrangian (gauge theory) Cangemi, Chioda term which is fully consistent in \mathbf{v}

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$
\mathcal{L} = \langle D_{\mu} \Phi | D^{\mu} \Phi \rangle - m^{2} \langle \Phi | \Phi \rangle + \sum_{k=0}^{2s-1} \frac{ig}{m^{2k}} \langle \Phi | \left\{ |D^{\mu} \overrightarrow{D} |^{\odot k} \odot |F_{-}| \right\} | \Phi \rangle + \mathcal{O}(|F_{-}|^{2})
$$

+
$$
\sum_{k \leq l=0}^{2s-4} \sum_{j=0}^{2s-3-l} \frac{g^{2}}{m^{2(j+l)+6}} \langle \Phi | \left\{ (|D^{\mu} \overrightarrow{D}| - m^{2}) \odot |D^{\mu} \overrightarrow{D}|^{\odot j} \odot |D^{\mu} \overrightarrow{D} + |^{\odot k} \odot |D^{\mu} \overrightarrow{D}|^{\odot (l-k)} \odot \mathfrak{F}_{6} \right\} | \Phi \rangle
$$

 $\mathbf{Field}\text{-}\mathbf{strength}\text{ dependent:}\quad \mathfrak{F}_6 = \frac{1}{4}\{T^c,\$ $T^{c'}\}$ |*l* $\overline{\Gamma}C$ $\mathbf{B} = \frac{1}{4} \{T^c, T^{c'}\} |F^c_-| \odot |D| F^c_+ |D|$ $\overleftarrow{D}|F_{+}^{c'}|$ $\overrightarrow{\bf n}$ D [|] Field-strength dependence: $\begin{array}{ccc} \mathbf{r} \cdot \mathbf{1} & \math$

Classical root-Kerr amplitude:
$$
\lim_{s \to \infty, \hbar \to 0} \mathcal{A}(1, 2, 3^-, 4^+)
$$

= $-2g^2(p \cdot \chi)^2 \left\{ \left(\frac{[T^{c_3}, T^{c_4}]}{q^2(p \cdot q_\perp)} + \frac{1}{2} \frac{\{T^{c_3}, T^{c_4}\}}{(p \cdot q_\perp)^2} \right) \left(e^x \cosh z - w e^x \sinh z + \frac{w^2 - z^2}{2} E(x, y, z) \right) - \frac{[T^{c_3}, T^{c_4}]}{q^2(p \cdot q_\perp)} \left(x(w^2 + z^2) - w(x^2 - y^2 + z^2) \right) \tilde{E}(x, y, z) \right\}.$

$$
E(x, y, z) = \frac{e^y - e^x \cosh z + (x - y)e^x \sinh z}{(x - y)^2 - z^2} + (y \to -y) \qquad \begin{array}{c} x = a \cdot q_{\perp}, & y = a \cdot q, \\ z = |a| \frac{p \cdot q_{\perp}}{m}, & w = \frac{a \cdot \chi \ p \cdot q_{\perp}}{p \cdot \chi} \end{array}
$$

Final classical results – Kerr BH construction and discuss how it introduces free parameters in the classical amplitude. Color-dressed amplitude. We can now assemble the full, color-dressed classical am-*^M*(1*,* ²*,* ³*,* ⁴⁺) = (*^p ·*)⁴ l assi *^e^x* cosh *^z w e^x*sinhc *^z* ⁺ 2 *w*² *z*² *E*(*x, y, z*) ⁺ (*^p ·*)³ *w*² *z*²

$$
\fbox{Classical root-Kerr amplitude:} \quad \lim_{s\to\infty, \hbar\to 0} \mathcal{A}(1,2,3^-,4^+)
$$

⇣

@

$$
= -2g^{2}(p \cdot \chi)^{2} \left\{ \left(\frac{[T^{c_{3}}, T^{c_{4}}]}{q^{2}(p \cdot q_{\perp})} + \frac{1}{2} \frac{\{T^{c_{3}}, T^{c_{4}}\}}{(p \cdot q_{\perp})^{2}} \right) \left(e^{x} \cosh z - w e^{x} \sinh z + \frac{w^{2} - z^{2}}{2} E(x, y, z)\right) - \frac{[T^{c_{3}}, T^{c_{4}}]}{q^{2}(p \cdot q_{\perp})} \left(x(w^{2} + z^{2}) - w(x^{2} - y^{2} + z^{2})\right) \tilde{E}(x, y, z)\right\}.
$$

 \mathbb{R}^n before considering the spin multipole expansion of the full amplitude, we first note that \mathbb{R}^n

Classical Kerr BH amplitude: Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov
\n
$$
\mathcal{M}(1,2,3^{-},4^{+}) = \frac{(p \cdot \chi)^{4}}{q^{2}(p \cdot q_{\perp})^{2}} \left(e^{x} \cosh z - w e^{x} \sinh z + \frac{w^{2} - z^{2}}{2} E(x,y,z) \right)
$$
\n
$$
+ \frac{(p \cdot \chi)^{4}}{q^{2}(p \cdot q_{\perp})^{2}} \frac{w^{2} - z^{2}}{2} (w - x) \tilde{E}(x,y,z)
$$
\n
$$
- \frac{(p \cdot \chi)^{4}}{(p \cdot q_{\perp})^{4}} \frac{(w^{2} - z^{2})^{2}}{2} \left(\frac{\partial \tilde{E}}{\partial x} + \eta \frac{\partial \tilde{E}}{\partial z} \right) + \alpha z \text{ (polygamma terms)}
$$

@3*E*˜(*x, y, z*) =1 ⁼ ¹ 2 where ⌘ = *±*1 controls the dissipative terms, and ↵ = 1 the polygamma terms. Note $T = \frac{1}{2}$ atthes explicit bit performancie inevity from OK of techology equities of the some complete behavior of the th
Complete the solution of the south of the sout $\mathbf s$ to spin- S^7 (ignoring p The entire function \mathbf{F} Matches explicit BH perturbation theory from GR \rightarrow Teukolsky eqn. up to spin $\,S^7\,$ (ignoring polygamma terms) Bautista, Guevara, Kavanagh, Vines

 Ξ

E(*x, y, z*)*.* (6.62)

i✏(*, p, q, a*)*E*˜(*x, y, z*) + contact terms (6)

⌘

Conclusion: Kerr dynamics from HS

Kerr dynamics is non-trivially constrained by

- massive higher-spin gauge symmetry
- -- power counting, current constraint, …
- Checks: \rightarrow uniquely predicts previously known Kerr 3-4pt amplitudes \rightarrow constrains $s \geq 2$ 4pt contact terms, but not unique...

Additional constraints imposed:

- chiral Lagrangian,
- -- symmetric homogeneous polynomials,…
- classical limit consistency,
- matching to Teukolsky BHPT (mod. polygamma terms)
- Outlook: \rightarrow classical loop corrections to Compton \rightarrow implications for quantum BHs, \rightarrow including absorption and emission effects