Brane scattering from $\mathcal{N} = 4$ integrated correlators

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EuroStrings 2024

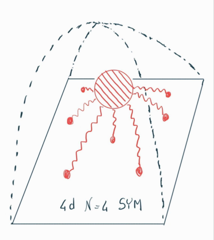
Southampton, 2 September 2024

Based on [2403.17263] with G. Brown, C. Wen
[2308.16575], [2405.10862] with M. Billò, M. Frau, A. Lerda

Motivations: new scattering processes in AdS/CFT

- We present QFT techniques in $\mathcal{N}=4$ SYM with a string theory interpretation

- AdS/CFT dictionary:
  boundary n-pt correlators $\leftrightarrow$ superstring scattering amplitudes



- What we study: integrated correlators in $\mathcal{N}=4$
  [Binder, Chester, Pufu, Wang, Green, Wen, Dorigoni... 2019-21]

see also talks by Dorigoni, Vallarino, Chester, Hansen, Zhang, Alday...

- Technique: supersymmetric localisation [Pestun, 2007]

$$\int [\mathcal{D}\Phi] \xrightarrow{Q} \int da_{\,[N\times N]}$$

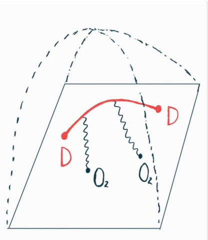
- In this talk: integrated correlators dual to **scattering with extended Branes**

Brane/string scattering from integrated correlators

- Four-point correlator with **determinant operators**

$$\langle O_2(x_1) O_2(x_2) D(x_3) D(x_4) \rangle$$

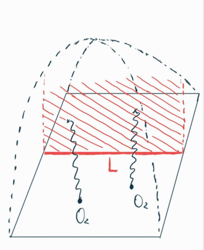
Heavy $D(x)$ operators create a D3 brane in the bulk



- Two-point integrated correlator with a **Line defect**

$$\langle O_2(x_1) O_2(x_2)\, L \rangle$$

Set up dual to graviton scattering off a long string



Brane/string scattering from integrated correlators

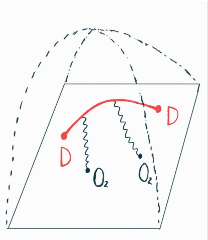
- Four-point correlator with **determinant operators**

$$\langle O_2(x_1) O_2(x_2) D(x_3) D(x_4) \rangle$$

where $O_2(x) = \mathrm{tr}\, \varphi^2$

$$D(x) = \det_N \varphi \qquad \varphi = \phi_I(x) Y^I \, , I = 1, \dots, 6$$

- half-BPS $\Rightarrow$ fixed $\Delta_{\mathcal{D}} = N$
- Baryon-like operator in $\mathcal{N} = 4$ SYM
- Dual to **Giant Graviton** D3-branes along S^5



Integrated correlators via localisation

- The four-point function is fixed by superconformal symmetry as:

$$\langle O_2 O_2 D\, D \rangle = \mathcal{H}_{\text{free}}(x_i, Y_i) + \mathcal{I}_4(x_i, Y_i)\mathcal{H}_{\mathcal{D}}(u, v; \tau, \bar{\tau})$$

where $\tau = \frac{\theta}{2\pi} + \mathrm{i}\frac{4\pi}{g^2_{\text{YM}}}$, $\quad u = \frac{x^2_{12}x^2_{34}}{x^2_{13}x^2_{24}}$, $v = \frac{x^2_{14}x^2_{23}}{x^2_{13}x^2_{24}}$.

- Localisation procedure for integrated correlators: [Binder, Chester,Pufu, Wang 2019]

$$Z^{\mathbb{R}^4}_{\mathcal{N}=4} \xrightarrow{\text{conf. map}} Z^{S^4}_{\mathcal{N}=4} \xrightarrow{\text{deformation}} S_{\mathcal{N}=4} + \lambda^a \int d^4x\, \sqrt{g} O_a(x) \xrightarrow{\text{localisation}} \mathcal{Z}^{m.m.}_{\mathcal{N}=2^*}(\lambda^a)$$

λ^a = deformations preserving at least $\mathcal{N} = 2$ (mass, squashing,...)

- Then the integrated correlator becomes

$$\int d\mu(u, v)\mathcal{H}_{\mathcal{D}}(u, v; \tau, \bar{\tau}) = \frac{\partial_{\mathcal{D}}\partial_{\mathcal{D}}\partial^2_m \log \mathcal{Z}(\tau, \tau'; m)\,|_{\tau',m=0}}{\partial_{\mathcal{D}}\partial_{\mathcal{D}} \log \mathcal{Z}(\tau, \tau'; m)\,|_{\tau',m=0}} = \mathcal{C}_{\mathcal{D}}(\tau, \bar{\tau}; N)$$

- New matrix model techniques for dealing with heavy operators [Brown, FG, Wen]

Topological expansion and $SL(2,\mathbb{Z})$ completion

- Result in 't Hooft limit (large N - fixed $\lambda = g_{\rm YM}^2 N$) in powers of N^{-1}

$$C_{\mathcal{D}}(\lambda; N) = \sum_{g=0}^{\infty} N^{1-g} C_{\mathcal{D}}^{(g)}(\lambda)$$

- Exact results in λ:

$$C_{\mathcal{D}}^{(0)}(\lambda) = -\int_0^\infty \frac{8w\,dw}{\sinh(w)^2}\,\frac{1}{v}\left(J_0\left(v\right)-1\right)J_1\left(v\right)\ , \qquad v = \frac{w\sqrt{\lambda}}{\pi},\ \ J_\ell = \text{Bessel}$$

$$C_{\mathcal{D}}^{(1)}(\lambda) = \int_0^\infty \frac{2w\,dw}{\sinh(w)^2}\left[J_1\left(v\right)\left(J_1\left(v\right)-\frac{v}{2}\right)-\left(J_0\left(v\right)-1\right)^2\right]$$

Topological expansion and $SL(2,\mathbb{Z})$ completion

- Result in 't Hooft limit (large N - fixed $\lambda = g^2_{\rm YM} N$) in powers of N^{-1}

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$$C^{(1)}_{\mathcal{D}}(\lambda) = \int_0^\infty \frac{2w\,dw}{\sinh(w)^2}\left[J_1(v)\left(J_1(v) - \frac{v}{2}\right) - (J_0(v) - 1)^2\right]$$

- Result at large N - finite τ in terms of $SL(2,\mathbb{Z})$-invariant functions:

$$C_{\mathcal{D}}(\tau,\bar\tau; N) = (2N-2) - \log(N) - E(1;\tau,\bar\tau) - \frac{E(3/2;\tau,\bar\tau)}{2\,N^{1/2}} + O(N^{-3/2})\,,$$

- Modular functions (non-holomorphic Eisenstein series) consistent with Brane/bulk amplitudes in flat space. [Bachas, Bain, Green, Gutperle, Basu, Lin, Shao, Wang, Yin...]

Brane/string scattering from integrated correlators

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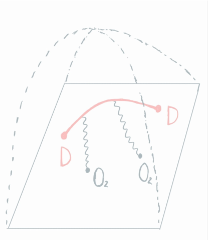

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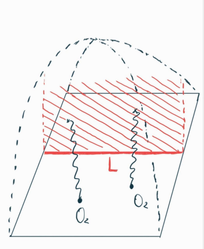
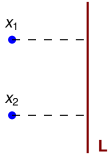


Integrated two-point function with a half-BPS line defect

- superconformal set up [Barrat, Gimenez-Grau, Liendo, Plefka]



$$\xi = \frac{(\vec{x}_1 - \vec{x}_2)^2}{r_1\, r_2} \,, \qquad \eta = \frac{\vec{x}_1 \cdot \vec{x}_2}{r_1\, r_2} \,,$$

- simplest case: **Wilson line** $\quad W = \frac{1}{N} \mathrm{tr}\, \mathcal{P} \exp\left\{ \int d\tau \left[\mathrm{i}\, A_4 + \theta_I\, \phi^I(x) \right] \right\}$

$$\left\langle O_2 O_2 \right\rangle_W = \frac{1}{r_1^2\, r_2^2} \left(\mathcal{I}_0(x_i, Y_i)\, F_0(\xi, \eta) + \text{higher R-symmetry channels} \right)$$

- Localisation: similar as before

$$\langle W \rangle^{\mathbb{R}^4}_{\mathcal{N}=4} \xrightarrow{\text{conf. map}} \ldots \xrightarrow{\text{localisation}} \langle W(a) \rangle^{m.m.}_{\mathcal{N}=2^*} (\lambda^a)$$

- so we define the integrated correlator as

$$\int d\eta d\xi \;\; \widehat{\mu}(\xi, \eta)\, F_0(\xi, \eta) = \partial_m^2 \log \langle W \rangle_{\mathcal{N}=2^*} \Big|_{m=0} = \mathcal{I}_W(\tau, \bar{\tau}, N)$$

Results for line integrated correlator

- Derivation of the **integration measure** from superconformal Ward identities (surprisingly simple!) [Billo, Frau, FG, Lerda], see also [Dempsey, Offertaler, Pufu, Wang]

$$\widehat{\mu}(\xi,\eta) \sim 1$$

- Exact result from matrix model in the 't Hooft limit

$$\mathcal{I}_W = \frac{\lambda}{I_1(\sqrt{\lambda})} \int_0^\infty \frac{dx}{x}\, \chi\Big(\frac{2\pi x}{\sqrt{\lambda}}\Big) \frac{\sqrt{\lambda}\, I_0(\sqrt{\lambda})\, J_1(x)^2 - x\, I_1(\sqrt{\lambda})\, J_0(x)\, J_1(x)}{x^2+\lambda} + \mathcal{O}(N^{-2})\,.$$

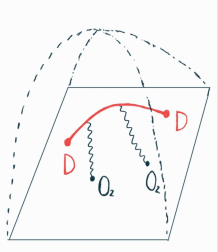
- Defect correlator not modular invariant → e.g. S-transformation now acts as

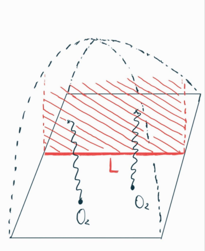
$$\langle O_2 O_2\rangle_W \mapsto \langle O_2 O_2\rangle_T$$

- Interesting modular properties yet to be understood (see also Dorigoni's talk)

Conclusion and outlook

- Two classes of integrated correlators, realising **dual brane scattering processes**





- In the future:
  - Different classes of Heavy operators in four-point functions
  - Different classes of deformation parameters and defects
  - Fully explore $SL(2,\mathbb{Z})$ properties for general correlators