Computation of Fermion Masses and Mixing in Geometric String Compactifications

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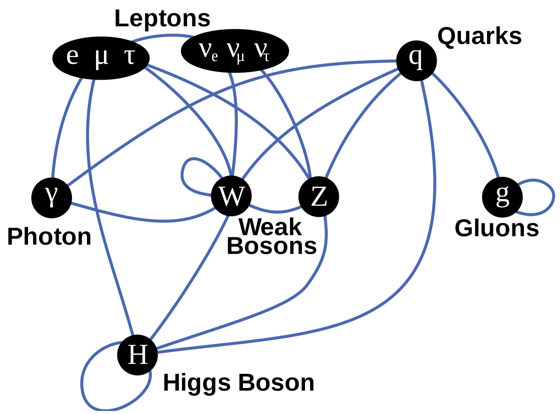
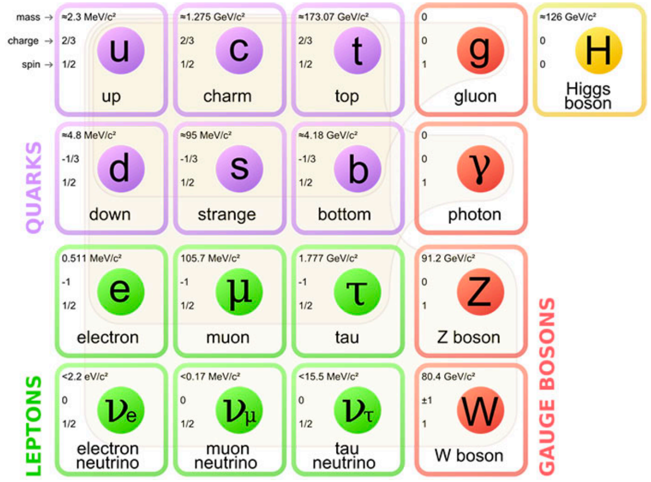
2402.01615, 2401.14463, 2306.03147, 2112.12107

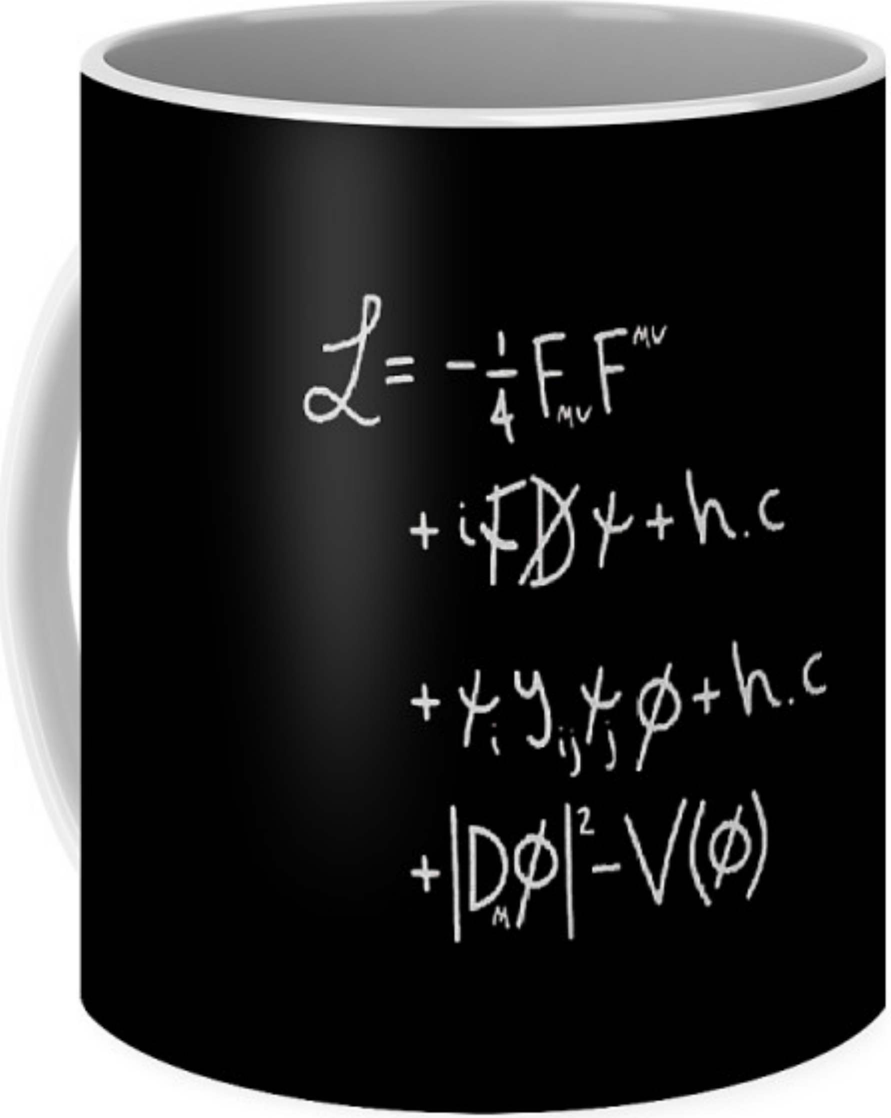


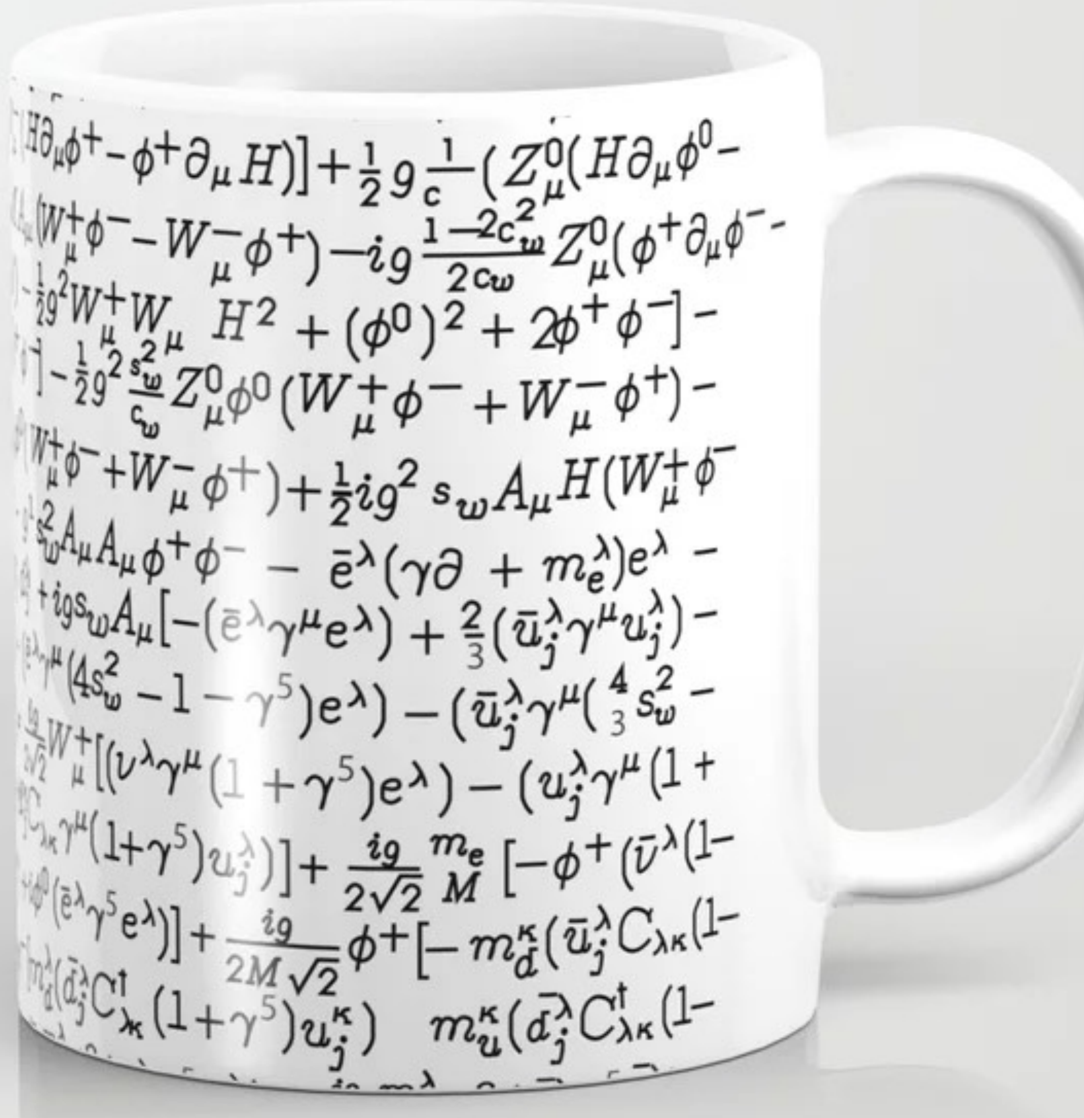


Aim: explain the core structures of the SM in terms of structures present in the fabric of space-time:

- explain the particle content, e.g. why there are three generations of quarks and leptons
- explain the hierarchy of masses

$$m_{\rm top} = 173 \cdot 10^3 \text{ MeV}, \ m_e = 0.511 \text{ MeV}, \ m_\nu < 2.2 \cdot 10^{-6} \text{ MeV}$$


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c$
$+ \psi_i y_{ij} \psi_j \phi + h.c$
$+ |D_\mu \phi|^2 - V(\phi)$



Three steps:

- identify string models that have the correct gauge group and particle content
- compute Yukawa couplings (quark and lepton masses and mixing parameters) as functions of the moduli
- stabilise all moduli

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$E_8 \times E_8$ Heterotic string - from 10d to 4d

- $X_{10} = X_6 \times M_4$
- $E_8 \to G_{\text{bundle}} \times G_{\text{GUT}} \qquad G_{\text{GUT}} \to G_{\text{finite}} \times G_{\text{SM}}$
- matter fields:

$$\mathbf{248} \to (\mathbf{1}, \text{Ad}_{G_{\text{GUT}}}) \oplus \bigoplus_i (R_i, r_i) \qquad n_{r_i} = h^1(X, V_{R_i})$$

keep N=1 SUSY in 4d:

- X_6 Calabi-Yau, $R_{a\bar{b}} = 0$
- V holomorphic and poly-stable,

$$F_{ab} = F_{\bar{a}\bar{b}} = g^{a\bar{b}} F_{a\bar{b}} = 0$$

- matter fields: cohomology, harmonic forms

A heterotic line bundle example

[AC, Fraser-Taliente, Harvey, Lukas, Ovrut ’24]
[Buchbinder, AC, Lukas ’13]

$$X = \begin{matrix} \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \end{matrix} \left[\begin{matrix} 2 \\ 2 \\ 2 \\ 2 \end{matrix} \right]^{4,68}_{-128}$$

$$\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$E_8 \to$$
$$SU(5) \times S(U(1)^5) \to$$
$$G_{SM} \times S(U(1)^5)$$

Basic properties

standard model? True massless U(1): 1 number of 5 $\overline{5}$ pairs: 3 $c_2(V)$ = {24, 8, 20, 12}

$$V:\ (k^i{}_a) = \begin{pmatrix} -1 & -1 & 0 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -2 \end{pmatrix}$$

Cohomology of V:

L_2	=	{−1, −3, 2, 2}	$h[L_2]$	=	{0, 8, 0, 0}	$h[L_2,R]$	= {{0, 0, 0, 0}, {2, 2, 2, 2}, {0, 0, 0, 0}, {0, 0, 0, 0}}
L_5	=	{1, 1, 0, −2}	$h[L_5]$	=	{0, 4, 0, 0}	$h[L_5,R]$	= {{0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_2{\times}L_4$	=	{0, −2, 1, 1}	$h[L_2{\times}L_4]$	=	{0, 4, 0, 0}	$h[L_2{\times}L_4,R]$	= {{0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_2{\times}L_5$	=	{0, −2, 2, 0}	$h[L_2{\times}L_5]$	=	{0, 3, 3, 0}	$h[L_2{\times}L_5,R]$	= {{0, 0, 0, 0}, {0, 1, 1, 1}, {0, 1, 1, 1}, {0, 0, 0, 0}}
$L_4{\times}L_5$	=	{2, 2, −1, −3}	$h[L_4{\times}L_5]$	=	{0, 8, 0, 0}	$h[L_4{\times}L_5,R]$	= {{0, 0, 0, 0}, {2, 2, 2, 2}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_1{\times}L_2^*$	=	{0, 3, −2, −1}	$h[L_1{\times}L_2^*]$	=	{0, 0, 12, 0}	$h[L_1{\times}L_2^*,R]$	= {{0, 0, 0, 0}, {0, 0, 0, 0}, {3, 3, 3, 3}, {0, 0, 0, 0}}
$L_1{\times}L_5^*$	=	{−2, −1, 0, 3}	$h[L_1{\times}L_5^*]$	=	{0, 0, 12, 0}	$h[L_1{\times}L_5^*,R]$	= {{0, 0, 0, 0}, {0, 0, 0, 0}, {3, 3, 3, 3}, {0, 0, 0, 0}}
$L_2{\times}L_3^*$	=	{−1, −4, 3, 2}	$h[L_2{\times}L_3^*]$	=	{0, 20, 0, 0}	$h[L_2{\times}L_3^*,R]$	= {{0, 0, 0, 0}, {5, 5, 5, 5}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_2{\times}L_4^*$	=	{−2, −4, 3, 3}	$h[L_2{\times}L_4^*]$	=	{0, 12, 0, 0}	$h[L_2{\times}L_4^*,R]$	= {{0, 0, 0, 0}, {3, 3, 3, 3}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_3{\times}L_5^*$	=	{−1, 0, −1, 2}	$h[L_3{\times}L_5^*]$	=	{0, 0, 4, 0}	$h[L_3{\times}L_5^*,R]$	= {{0, 0, 0, 0}, {0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}}

Wilson line: {{0, 0}, {0, 1}} Equivariant structure: {{0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}} Higgs pairs: 1

Downstairs spectrum: $\{2\,10_2,\ 10_5,\ \overline{5}_{2,4},\ 2\,\overline{5}_{4,5},\ H_{2,5},\ \overline{H}_{2,5},\ 3\,S_{2,1},\ 3\,S_{5,1},\ 5\,S_{2,3},\ 3\,S_{2,4},\ S_{5,3}\}$ Phys. Higgs: $\{H_{2,5},\ \overline{H}_{2,5}\}$

Transfer format: {{6, 1, 1, 4, 6, 5, 9, 9, 8, 10, 1, 7, 17}, {6, 6, −1, −1, −1, −1}}

$rk(Y^{(u)})$ = {2, 2} $rk(Y^{(d)})$) = {0, 0} dim. 4 operators absent: {True, True} dim. 5 operators absent: {True, True}

A heterotic line bundle example

$$X = \begin{array}{c} \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \\ \mathbb{CP}^1 \end{array} \left[\begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \end{array} \right]^{4,68}_{-128}$$

$$\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$$

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Operators

basic superpotential terms:

$\bar{H}10^p10^q$: $Y^{(u)} = \begin{pmatrix} (0) & (0) & (1) \\ (0) & (0) & (1) \\ (1) & (1) & (0) \end{pmatrix}$

$H\bar{5}^p10^q$: $Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$

$H\bar{H}$: $\mu = \{1\}$

$W_{sing} = \{0\}$

R–parity violating terms in superpotential:

$\bar{H}L^p$: $\rho = \begin{pmatrix} 0 \\ S_{2,4} \\ S_{2,4} \end{pmatrix}$

$10^p\bar{5}^q\bar{5}^r$: λ = {{{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}}

Dimension 5 operators in superpotential:

$\bar{5}^p10^q10^r10^s$: λ' = {{{{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}},
{{{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}},
{{{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}}}

- correct spectrum
- up-Yukawa: rank 2 (perturbatively)
- vanishing lepton and down-Yukawas
- right-handed neutrinos
- no proton decay operators

Heuristic searches for models with the correct particle spectrum

Heterotic line bundle models: searches

Overview:

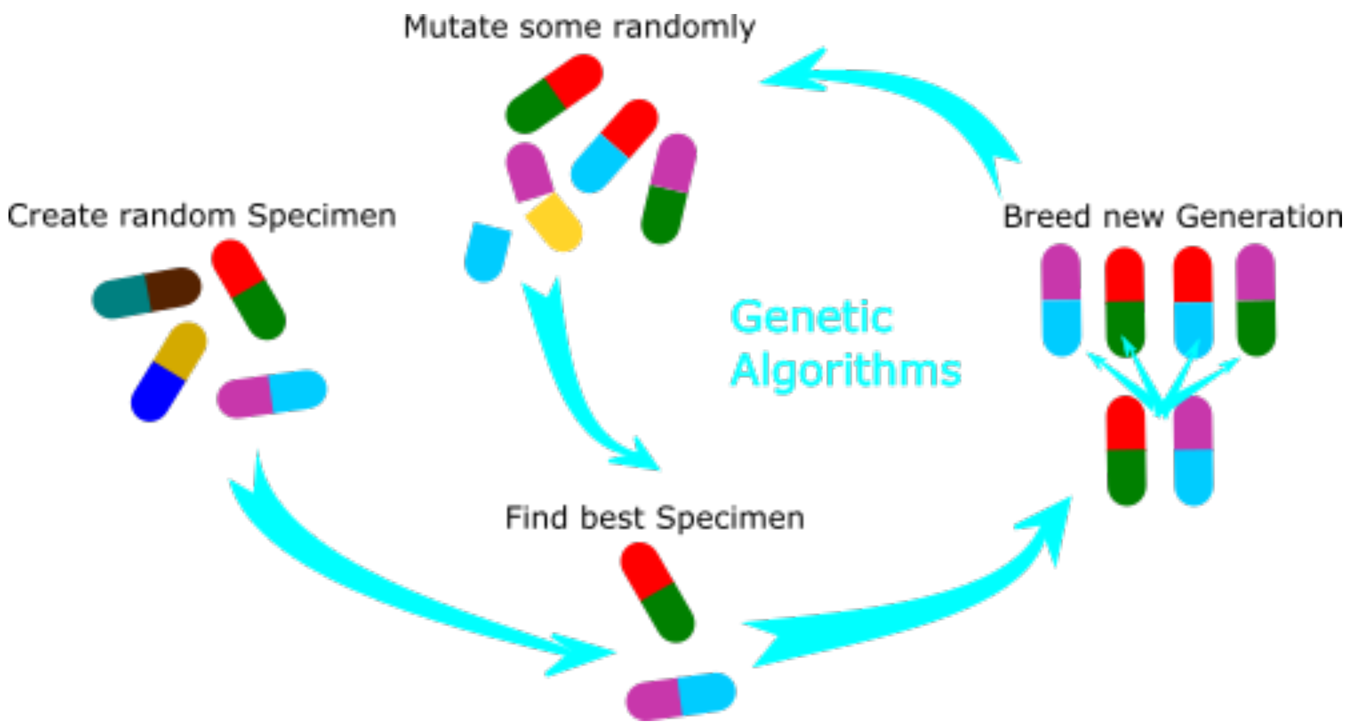
- Situation about 10 years ago: only a handful of models that recovered the correct spectrum were known
- Systematic searches: in 2013 we undertook a massive search, scanning essentially over some 10^{40} (X, V)-pairs; this resulted in several million heterotic line bundle models with the correct particle content

[Anderson, AC, Gray, Lukas, Palti '13]

- Heuristic searches: more recently (in the last two years), we used Genetic Algorithms and Reinforcement Learning to search in even larger regions of the string landscape. Viable models can now be generated on demand (at a rate of hundreds or thousands per day).

Genetic Algorithms

[Abel, AC, Harvey, Lukas, Nutricati ’23]



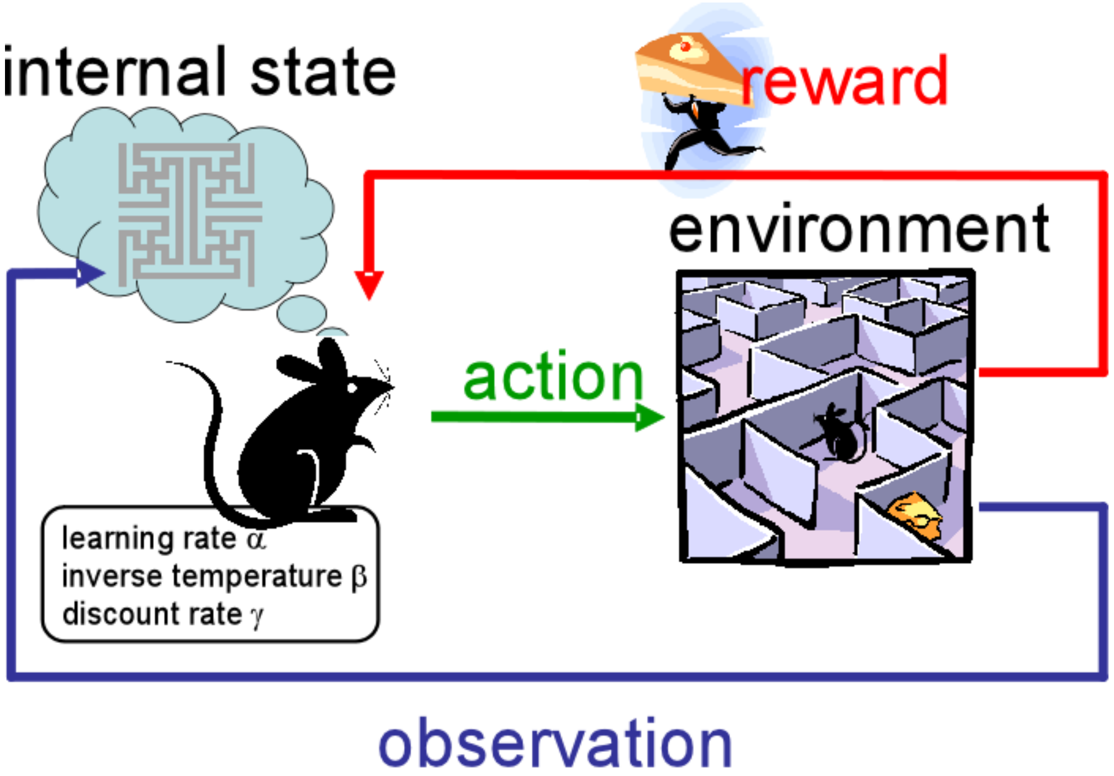
Genotype: for fixed X, encode the line bundle integers into a binary sequence

Phenotype: three generations, no exotics, Higgs field, absence of gauge and gravitational anomalies, supersymmetry, equivariance

Bonus: GAs perform better when enhanced with a Quantum Annealing ‘intrinsic’ mutation

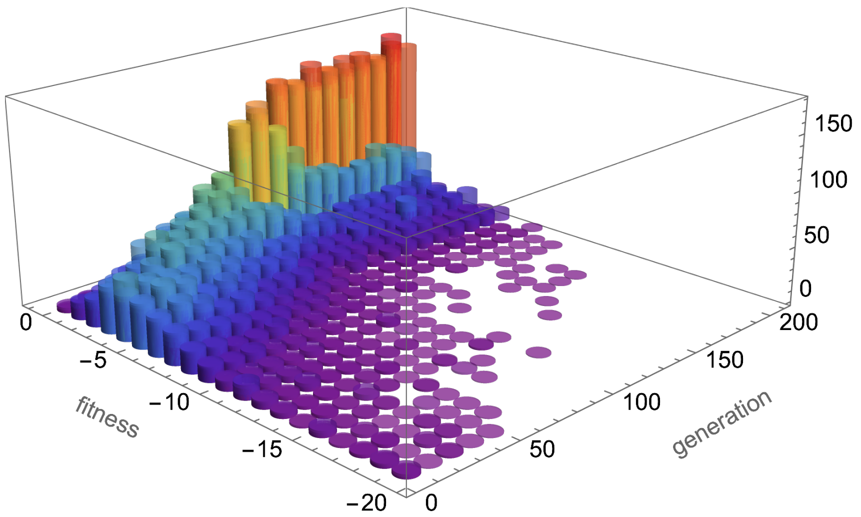
Reinforcement Learning

[Abel, AC, Harvey, Lukas '21]



Mathematical structure: (Stochastic) Markov Decision Processes.
Simplest version: policy-based RL. The policy is controlled by a NN and learnt without any prior knowledge of the environment.

Some results



[Abel, AC, Harvey, Lukas '21]

Manifold	h	$\|\Gamma\|$	Range	GA	Scan	Found	Explored
7862	4	2	[-7,8]	5	5	100%	10^{-10}
7862	4	4	[-7,8]	30	31	97%	10^{-10}
7447	5	2	[-7,8]	38	38	100%	10^{-14}
7447	5	4	[-7,8]	139	154	90%	10^{-14}
5302	6	2	[-7,8]	403	442	93%	10^{-19}
5302	6	4	[-7,8]	722	897	80%	10^{-19}
4071	7	2	[-3,4]	11,937	N/A	N/A	10^{-14}

[Abel, AC, Harvey, Lukas, Nutricati '23]

Comparison with systematic scans: virtually the same results while scanning only a fraction of $\sim 10^{-20}$

Comparison between GA and RL: very different philosophies, similar results

Particle spectra and cohomology computations

Particle content and cohomology: recap

Compactification data for the $E_8 \times E_8$ heterotic string: (X, V)

Want manifolds and bundles that can be given very explicit presentations.

Best choice: X CICY in product of projective spaces and $V = L_1 \oplus L_2 \oplus L_3 \oplus L_4 \oplus L_5,\ c_1(V) = 0$

This leads to $SU(5) \times S(U(1)^5)$ GUTs. Further breaking to the SM gauge group using discrete Wilson lines.

repr.	cohomology	total number	required for MSSM
$\mathbf{1}_{a,b}$	$H^1(X, L_a \otimes L_b^*)$	$\sum_{a,b} h^1(X, L_a \otimes L_b^*) = h^1(X, V \otimes V^*)$	-
$\mathbf{5}_{a,b}$	$H^1(X, L_a^* \otimes L_b^*)$	$\sum_{a<b} h^1(X, L_a^* \otimes L_b^*) = h^1(X, \wedge^2 V^*)$	n_h
$\overline{\mathbf{5}}_{a,b}$	$H^1(X, L_a \otimes L_b)$	$\sum_{a<b} h^1(X, L_a \otimes L_b) = h^1(X, \wedge^2 V)$	$3\lvert\Gamma\rvert + n_h$
$\mathbf{10}_a$	$H^1(X, L_a)$	$\sum_a h^1(X, L_a) = h^1(X, V)$	$3\lvert\Gamma\rvert$
$\overline{\mathbf{10}}_a$	$H^1(X, L_a^*)$	$\sum_a h^1(X, L_a^*) = h^1(X, V^*)$	0

A good starting point

Line bundles on $\mathbb{P}^n$. Cohomology dimensions given by the Bott formula:

$$h^0(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(k)) = \binom{k+n}{n} = \frac{1}{n!}(1+k)\dots(n+k)\ , \ \text{if } k \geq 0, \ \text{and 0 otherwise.}$$

$$h^i(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(k)) = 0\ , \ \text{if } 0 < i < n\ .$$

$$h^n(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(k)) = \binom{-k-1}{-n-k-1} = \frac{1}{n!}(-n-k)\dots(-1-k)\ , \ \text{if } k \leq -n-1,$$

and 0 otherwise.

A good starting point

Writing

$$\mathbb{P}^n = \frac{U(n+1)}{U(1) \times U(n)} ,$$

Bott's formula can be regarded as a special case of the Borel-Weil-Bott theorem which deals with flag varieties. Using this, it is possible to represent the cohomology groups of line bundles over products of projective spaces as irreducible representations of unitary groups. This technique provides a simple and computationally useful representation for the cohomology groups.

On toric varieties there is an algorithm due to Blumenhagen, Jurke, Rahn, Thorsten, Roschy which allows the computation of line bundle cohomology.

$X \subset \mathcal{A} = \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \ldots \times \mathbb{P}^{n_m}$

Let $L \to X$ be a line bundle over X and $\mathcal{L}_{\mathcal{A}}$ the corresponding line bundle.

Write the Koszul complex associated with L:

$$0 \;\to\; \mathcal{L}_{\mathcal{A}} \otimes \wedge^K \mathcal{N}^* \;\to\; \mathcal{L}_{\mathcal{A}} \otimes \wedge^{K-1} \mathcal{N}^* \;\to\; \ldots \;\to\; \mathcal{L}_{\mathcal{A}} \;\to\; L \;\to\; 0$$

We automatised the Leray spectral sequence machinery.

[CIPro package, Anderson, AC, Gray, He, Lee, Lukas - to become publicly available later in ’24]

[pyCICY by Larfors & Schneider ‘19]

Computational cost of line bundle cohomology (using spectral sequences):

$$\sim O\left(\left(\rho(X)^{\dim(X)}\deg(L)^{\dim(X)}\right)^3\right)$$

Example: for a line bundle of (multi)-degree 10 on a Calabi-Yau threefold with $h^{1,1}(X) = \rho(X) = 4$ Kähler parameters, the estimate is

$\sim 10^{14}$ elementary operations

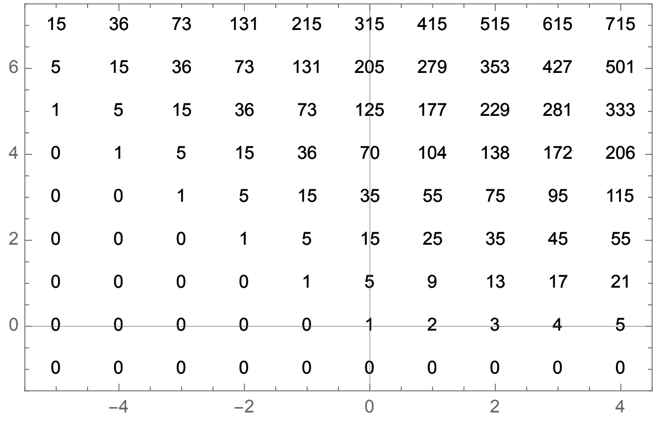
which reaches the limits of a standard machine

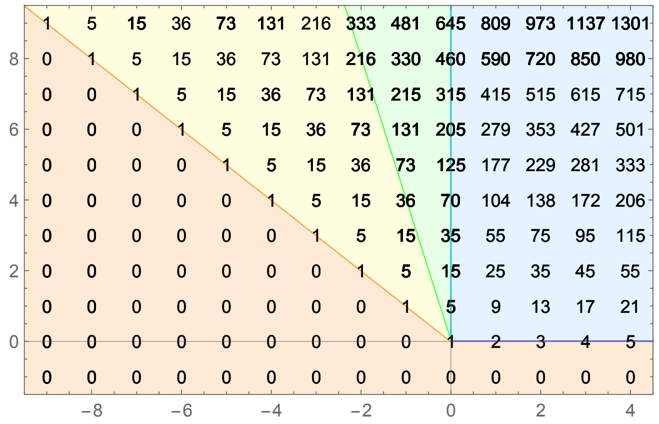
An exercise in pattern recognition

$$X = \begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^4 \end{matrix} \left[\begin{matrix} 1 & 1 \\ 4 & 1 \end{matrix} \right]^{2,86}$$

look at patterns in the data for

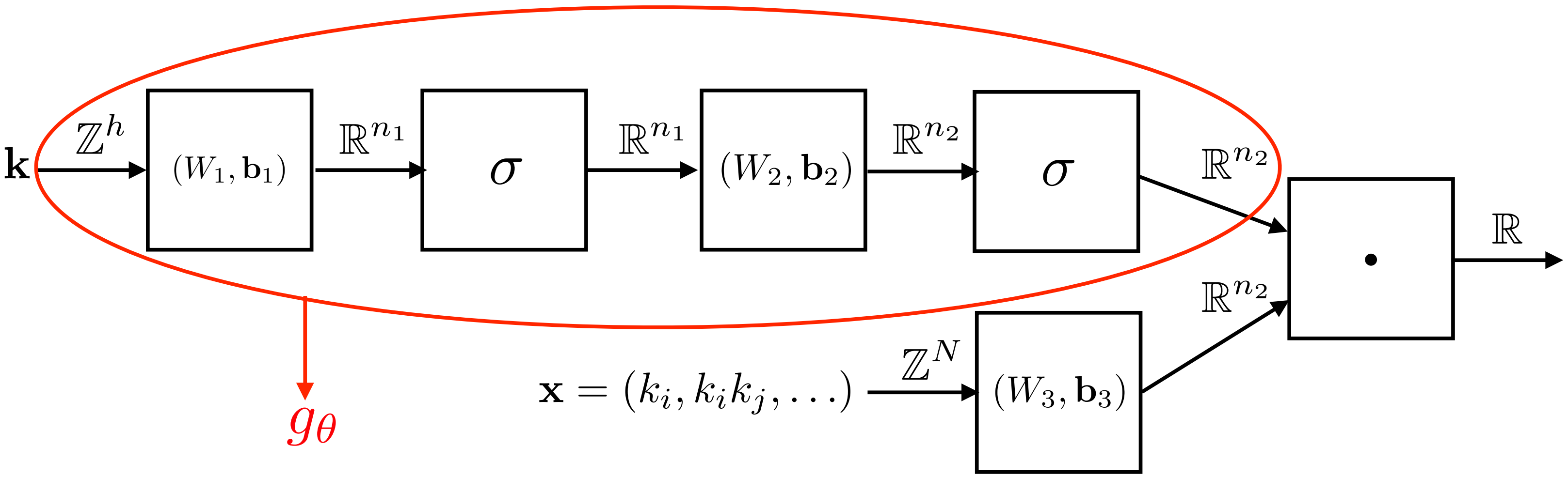
$h^0(X, L)$, $L \in \text{Pic}(X)$





region in eff. cone	$h^0(X, L = \mathcal{O}_X(D = k_1 D_1 + k_2 D_2))$
blue	$2k_1(1 + k_2^2) + \frac{5}{6} k_2(5 + k_2^2)$
green	$2k_1(1 + k_2^2) + \frac{5}{6} k_2(5 + k_2^2) + \frac{8}{3} k_1(1 - k_1^2)$
yellow	$2k_1(1 + k_2^2) + \frac{5}{6} k_2(5 + k_2^2) + \frac{8}{3} k_1(1 - k_1^2) +$ $+ \frac{1}{2}(1 - (4k_1 + k_2)^2) \left\lceil \frac{4k_1 + k_2}{-3} \right\rceil$
$k_1 > 0, \; k_2 = 0$	$k_1 + 1$
$-k_1 = k_2 \geq 0$	1

It is possible to train a neural network (supervised learning) to identify the different regions and the formulae that hold within each.



[Brodie, AC, Deen, Lukas, 1906.08730]

see also: [Klaewer, Schlechter, 1809.02547]

The training data consists of pairs $(\mathbf{k}, h^i(X, \mathcal{O}_X(\mathbf{k})))$.

Drawback: the amount of training data is limited by the slow algorithmic computation. For larger Picard number manifolds it is not feasible to generate enough training data. Nevertheless, this ML exercise was useful to generate conjectures.

Conjecture 5. *Let X be a general complete intersection of two hypersurfaces of bi-degrees $(1,1)$ and $(1,4)$ in $\mathbb{P}^1 \times \mathbb{P}^4$, belonging to the deformation family with configuration matrix*

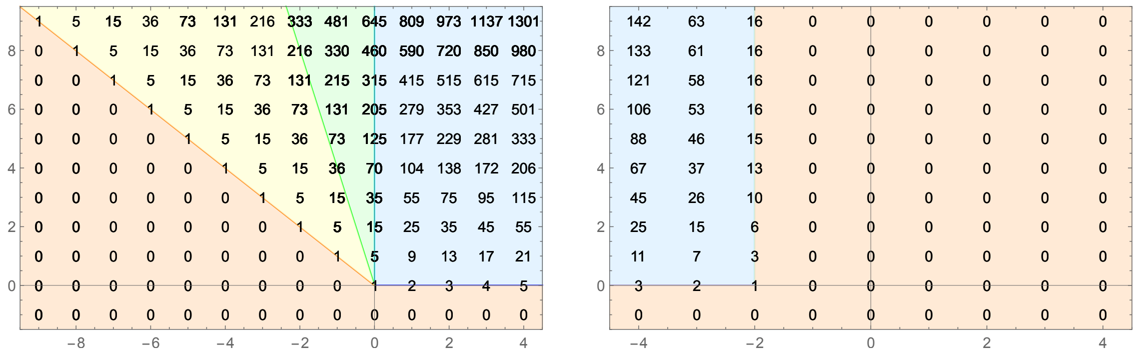
$$\begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^4 \end{matrix} \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} . \tag{1.16}$$

The effective, movable and nef cones of X are given by

$$\begin{gathered} \mathrm{Eff}(X) = \mathbb{R}_{\geq 0} H_1 + \mathbb{R}_{\geq 0}(H_2 - H_1), \; \mathrm{Mov}(X) = \mathbb{R}_{\geq 0} H_1 + \mathbb{R}_{\geq 0}(4H_2 - H_1) \\ \mathrm{Nef}(X) = \mathbb{R}_{\geq 0} H_1 + \mathbb{R}_{\geq 0} H_2 \; , \end{gathered} \tag{1.17}$$

where $H_1 = \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(1,0)|_X$ and $H_2 = \mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^4}(0,1)|_X$. We propose the following generating functions for all line bundle cohomology dimensions in the entire Picard group of X:

$$\begin{aligned} CS^0(X, \mathcal{O}_X) &= \left(\frac{(1-t_2)^2 \left(1-t_2^4\right)^2}{(1-t_1)^2 (1-t_2)^5 \left(1-t_1^{-1} t_2\right) \left(1-t_1^{-1} t_2^4\right)} , \begin{matrix} t_2 & t_1 \\ 0 & 0 \end{matrix} \right) \\ CS^1(X, \mathcal{O}_X) &= \left(\frac{(1-t_2)^2 \left(1-t_2^4\right)^2}{(1-t_1)^2 (1-t_2)^5 \left(1-t_1^{-1} t_2\right) \left(1-t_1^{-1} t_2^4\right)} , \begin{matrix} t_2 & t_1 \\ \infty & 0 \end{matrix} \right) \\ CS^2(X, \mathcal{O}_X) &= \left(\frac{(1-t_2)^2 \left(1-t_2^4\right)^2}{(1-t_1)^2 (1-t_2)^5 \left(1-t_1^{-1} t_2\right) \left(1-t_1^{-1} t_2^4\right)} , \begin{matrix} t_2 & t_1 \\ 0 & \infty \end{matrix} \right) \\ CS^3(X, \mathcal{O}_X) &= \left(\frac{(1-t_2)^2 \left(1-t_2^4\right)^2}{(1-t_1)^2 (1-t_2)^5 \left(1-t_1^{-1} t_2\right) \left(1-t_1^{-1} t_2^4\right)} , \begin{matrix} t_2 & t_1 \\ \infty & \infty \end{matrix} \right) \end{aligned} \tag{1.18}$$



[AC ’24]

Significance of cohomology formulae/generating functions

The existence of line bundle cohomology formulae / generating functions greatly simplifies the analysis of heterotic line bundle models. Calculations that would otherwise take minutes or hours, are now virtually instantaneous.

Moreover, these expressions are of mathematical interest in themselves. I have examples in arbitrary dimension ≥ 2 including varieties of Fano, semi-Fano, CY and general type, including non-Mori dream spaces and complex structure dependence. Aim: convert geometry into algebraic data.

Two surprises:

1. evidence that such generating functions exist
2. the same generating function, expanded around different points, encodes the zeroth and higher cohomology of all line bundles.

Generating functions carry a lot of numerical information about the variety. Do they uniquely determine the variety?

A similar question has been asked for the regularised quantum period of Fano varieties, which is a generating function for certain Gromov-Witten invariants. [Coates, Kasprzyk, Pitton, Tveiten '21]

Computation of Yukawa couplings

Low-energy Lagrangian with chiral matter multiplets $C^I = (c^I, \chi^I)$, corresponding to harmonic forms ν_I

$$\mathscr{L} = -K_{I\bar{J}}\partial_\mu c^I \partial^\mu \bar{c}^{\bar{J}} - iK_{I\bar{J}}\bar{\chi}^{\bar{J}}\bar{\sigma}^\mu \partial_\mu \chi^I + e^{K/2}(\lambda_{IJK}c^I \chi^J \chi^K + c.c.) + \dots$$

The holomorphic Yukawa couplings and the matter field Kähler metric can be computed from the geometry:

$$\lambda_{IJK} \sim \int_X \nu_I \wedge \nu_J \wedge \nu_K \wedge \Omega \qquad K_{I\bar{J}} \sim \int_X \nu_I \wedge \star\,(H_V \bar{\nu}_{\bar{J}})$$

λ_{IJK} is **quasi-topological** - can be calculated without the CY metric, bundle metric and harmonic forms

$K_{I\bar{J}}$ calculation - requires full knowledge of the geometry

Computation of CY metric

Idea: use neural networks as universal approximators to solve PDEs on curved spaces.

Advantage: the solutions are known to exist and are smooth

By using NNs, one can avoid discretisation problem on the manifold

Naively, one would like to solve

$$R_{i\bar{j}} = -\partial_i \bar{\partial}_{\bar{j}} \log(\det(g)) = 0 \qquad\qquad g_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} K$$

This is a terrible 4-th order non-linear PDE (with particularly unpleasant non-linearities in the highest derivatives) in six dimensions. That's not how Yau proved the theorem.

Computation of CY metric

Yau's theorem (1978): A compact, $2n$-dimensional Kähler manifold with vanishing first Chern class admits a unique Ricci-flat Kähler metric in each Kähler class.

Write $g^{(\text{flat})}_{a\bar{b}} = g^{(\text{ref})}_{a\bar{b}} + \partial_a \bar{\partial}_{\bar{b}} \phi$, where ϕ is a global function

For instance, $g^{(\text{ref})}$ can be taken to be the metric induced from the Fubini-Study metric on $\mathbb{P}^m$,

$$g^{(\text{ref})}_{a\bar{b}} = \sum_{i=1}^{4} \frac{t^i}{2\pi} \partial_a \bar{\partial}_{\bar{b}} \ln(\kappa_i) \Big|_X \ , \qquad \kappa_i = 1 + |z_i|^2 .$$

Yau showed instead that the Monge-Ampère equation

$$J^{(\text{flat})} \wedge J^{(\text{flat})} \wedge J^{(\text{flat})} = \kappa\, \Omega \wedge \bar{\Omega} \,, \quad \text{with} \quad J^{(\text{flat})} = J^{(\text{ref})} + \partial\bar{\partial}\phi$$

can be solved.

Computation of CY metric

Train on the loss

$$\mathscr{L} = \alpha_1 \mathscr{L}_{\text{MA}} + \alpha_2 \mathscr{L}_{\text{tr.}} + \alpha_3 \mathscr{L}_{\text{Kähler}}$$

$$\mathscr{L}_{\text{MA}}[\phi] = \left|\left|1 - \frac{1}{\kappa} \frac{J(\phi) \wedge J(\phi) \wedge J(\phi)}{\hat{\Omega} \wedge \bar{\hat{\Omega}}}\right|\right|_1$$

$$\mathscr{L}_{\text{tr.}}[\phi] = \sum_{s \neq t} \left|\left|\phi_s - \phi_t\right|\right|_1 .$$

[Larfors, Lukas, Ruehle, Schneider, '22]

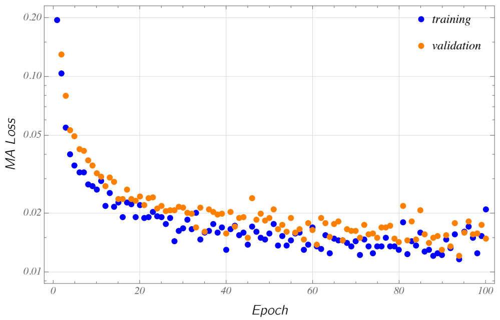
We used the "cymetric" package to realise the ϕ-model.

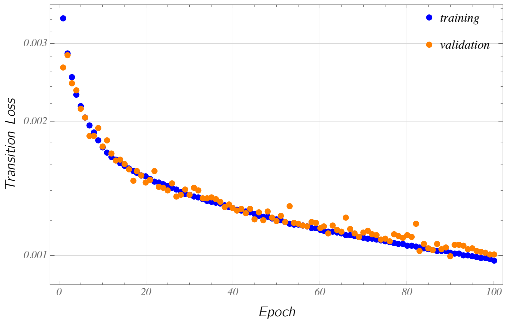
Details of the implementation: a sample of 300,000 points on X, used both for training and Monte-Carlo integration.

The point sample is split into training and validation sets at a ratio of 9:1.

The neural network is fully connected with GeLU activation, four layers and a width of 128.

Training is carried out for 100 epochs, with batch size 64 and learning rate 0.001.





Computation of HYM connection

For the bundle $L = \mathcal{O}_X(\vec{k})$, with reference bundle metric $H^{(\mathrm{ref})}$, reference connection $A^{(\mathrm{ref})} = \partial \ln H^{(\mathrm{ref})}$ and field strength $F^{(\mathrm{ref})} = \bar{\partial}\partial \ln H^{(\mathrm{ref})}$, write

$$H = e^{\beta} H^{(\mathrm{ref})}$$

The HYM equation implies that β must satisfy the following Poisson equation

$$\Delta\beta = \rho_\beta = -g^{a\bar{b}}\partial_a \bar{\partial}_{\bar{b}} \ln\left(\bar{H}^{(\mathrm{ref})}\right).$$

Train on the loss:

$$\begin{aligned}
\mathscr{L} &= \alpha_1 \mathscr{L}_{\mathrm{HYM}} + \alpha_2 \mathscr{L}_{\mathrm{tr.}} \\
\mathscr{L}_{\mathrm{HYM}}[\beta] &= \left|\left|\Delta\beta - \rho_\beta\right|\right|_1 \\
\mathscr{L}_{\mathrm{tr.}}[\beta] &= \sum_{s\neq t} \left|\left|\beta_s - \beta_t\right|\right|_1 ,
\end{aligned}$$

with a similar architecture as before.

[AC, Fraser-Taliente, Harvey, Lukas, Ovrut '24]

Computation of harmonic bundle-valued forms

For the harmonic forms, use reference quantities that can be written as restrictions of forms from the ambient product of projective spaces and

$$\nu = \nu^{(\mathrm{ref})} + \bar{\partial}_{\mathcal{L}}\sigma \ .$$

The Here, σ is a global section of L determined by the Poisson equation

$$\Delta_{\mathcal{L}}\sigma = \rho_\sigma = -g^{a\bar{b}}\partial_a\left(H\nu_{\bar{b}}^{(\mathrm{ref})}\right)$$

Train on the following loss, with a similar architecture:

$$\mathscr{L} = \tilde{\alpha}_1\mathscr{L}_\Delta + \tilde{\alpha}_2\mathscr{L}_{\mathrm{tr.}}$$
$$\mathscr{L}_\Delta[\sigma] = \left|\left|\Delta_{\mathcal{L}}\sigma - \rho_\sigma\right|\right|_1$$
$$\mathscr{L}_{\mathrm{tr.}}[\sigma] = \sum_{s\neq t}\left|\left|\sigma_s - T_{(s,t)}\sigma_t\right|\right|_1$$

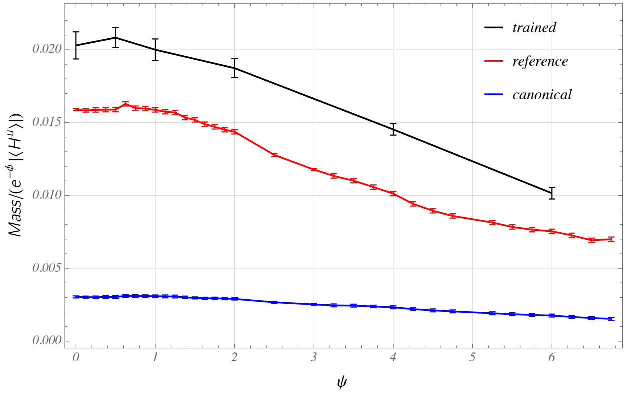
[AC, Fraser-Taliente, Harvey, Lukas, Ovrut '24]

A similar approach for standard embedding ($V = TX$) compactifications was carried out, which matches spectacularly well the analytic results that can be performed in this setting.

[Butbaia, Pena, Tan, Berglund, Hubsch, Jejjala, Mishra, '24]

Application to the model we started with

Plot for the top-quark mass as a function of (one) complex modulus:



Preliminary exploration of the moduli space: a hierarchy factor of 20 (possibly more) between top and charm can be achieved. This is somewhat too small (the measured factor is approx 137).

However, we have a database of millions of line bundle models with the correct spectrum to which this method can now be applied. A full-fledged embedding of the SM in string theory is achievable.

[AC, Fraser-Taliente, Harvey, Lukas, Ovrut '24]

Major piece of work left: understand moduli stabilisation.

Summary

Connecting String Theory and particle Physics: a hard, but worthwhile problem.
AI tools likely to bring the solution within reach.

The size of the string landscape: the spectacular success of heuristic search methods seems to indicate that this is no longer a problem.

Fast line bundle cohomology computations: an essential tool for model building.

Computation of physical parameters (quark and lepton masses): now feasible in realistic string models.