Gapped Phases and Phase Transitions with Categorical Symmetries

Lea Bottini - University of Oxford

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Based on works with Lakshya Bhardwaj (Oxford), Daniel Pajer (Oxford),
Sakura Schäfer-Nameki (Oxford) & Apoorv Tiwari (Copenhagen)

This talk:

following Lakshya’s talk

- Categorical Landau Paradigm via SymTFT

    - Extend standard Landau paradigm to $\mathcal{S}$ = fusion category (finite!)

    - Will strictly be in 2d (space-time)

  [Kong, Lan, Wen, Zhang, Zheng ’20; Chatterjee, Ji, Wen ‘22; Chatterjee, Wen ‘23; Bhardwaj, Schafer-Nameki ’23; Huang, Cheng ‘23; Wen, Potter ’23; Bhardwaj, LB, Pajer, Schafer-Nameki ’23;….]

- Anyon chains

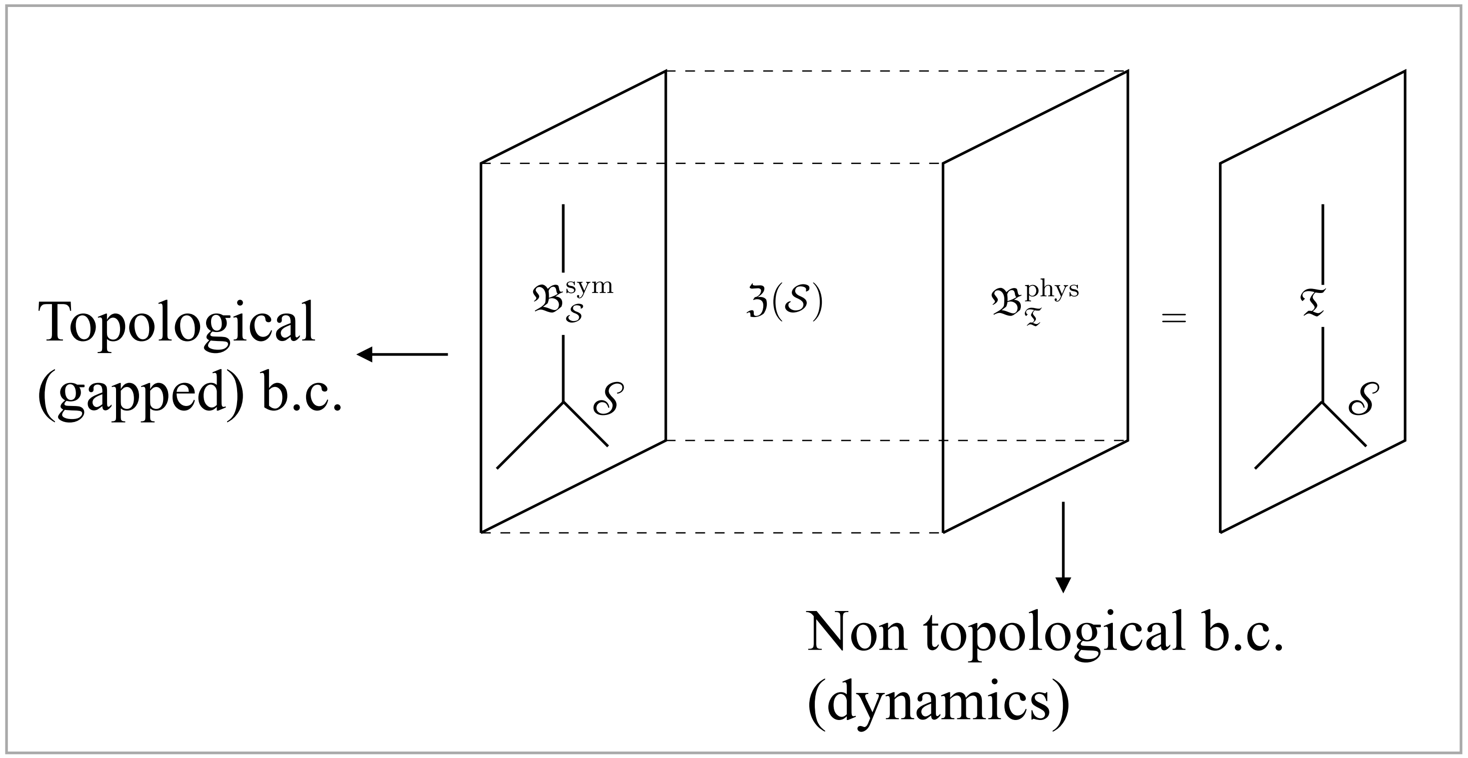
  [Aasen, Mong, Fendley ‘20; Lootens, Delcamp, Ortiz, Verstraete ‘21; Inamura, Ohmori ‘23; Bhardwaj, LB, Schafer-Nameki, Tiwari ’24;….]

- Conclusions & future directions

SymTFT

[Ji, Wen ’19; Gaiotto, Kulp ‘20; Apruzzi, Bonetti, Etxebarria, Hosseini, Schafer-Nameki ’21; Freed, Moore, Teleman ‘22; ….]

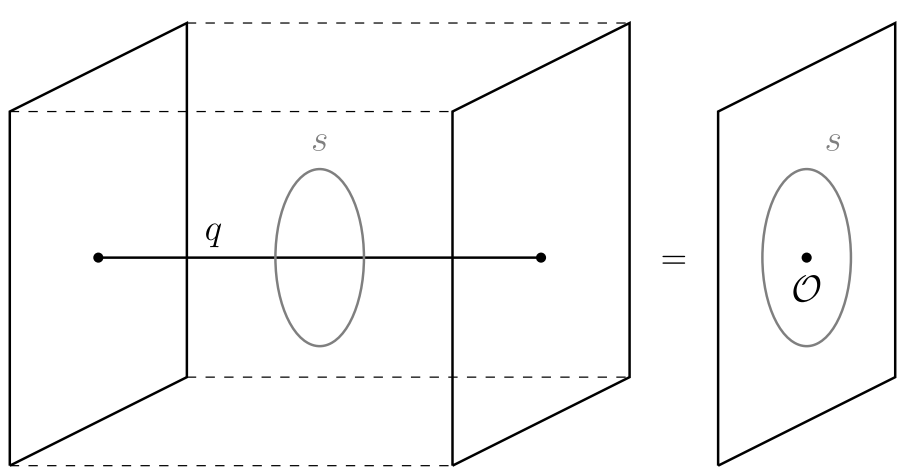
- 2d QFT $\mathfrak{T}$ with symmetry $\mathcal{S}$ $\longrightarrow$ 3d TQFT $\mathcal{Z}(\mathcal{S})$ with topological lines (anyons)



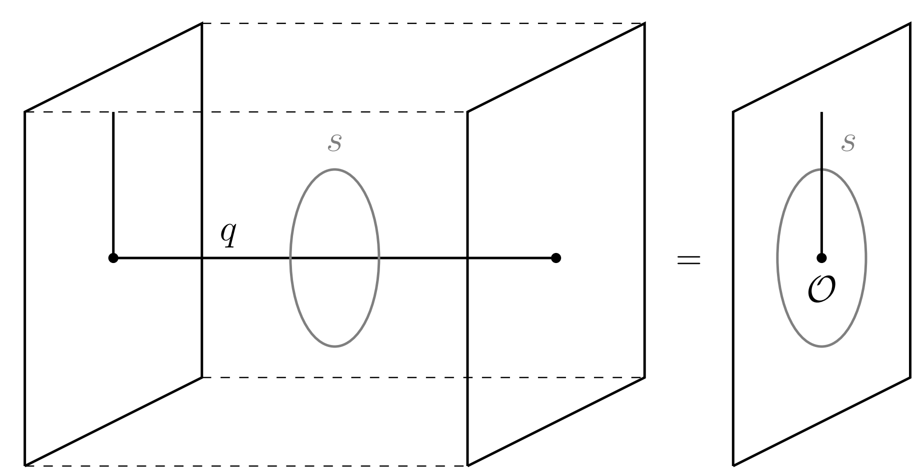
<u>Sandwich construction</u>

Separates symmetry aspects of the theory from the dynamics

- Gives convenient characterisation of charged operators



Local (untwisted) operators



Twisted sector operators

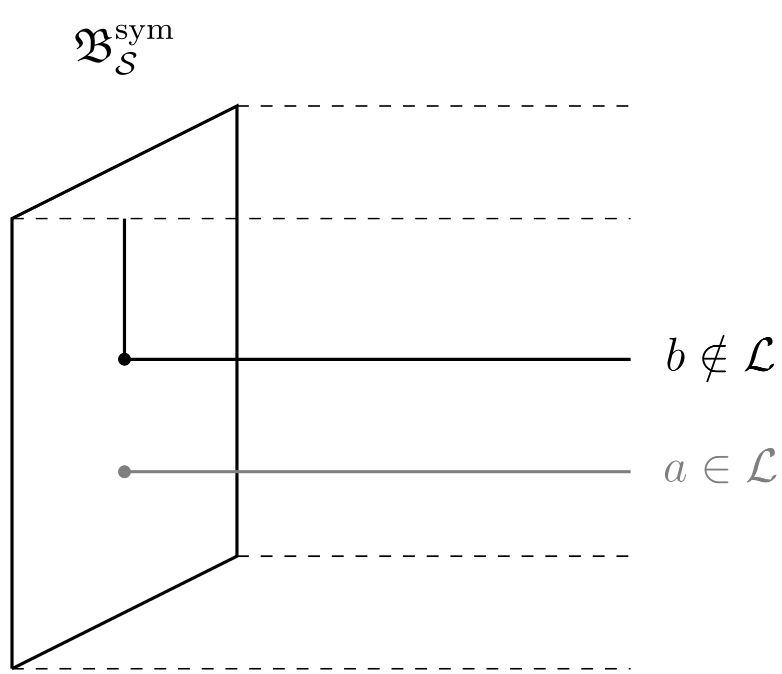
SymTFT

- Gapped boundary specified by maximal set of anyons that can end on it

$$\updownarrow$$

Lagrangian algebras in $\mathscr{Z}(\mathscr{S})$

$$\mathscr{L} = \bigoplus_{a \in \mathscr{Z}(\mathscr{S})} n_a a$$



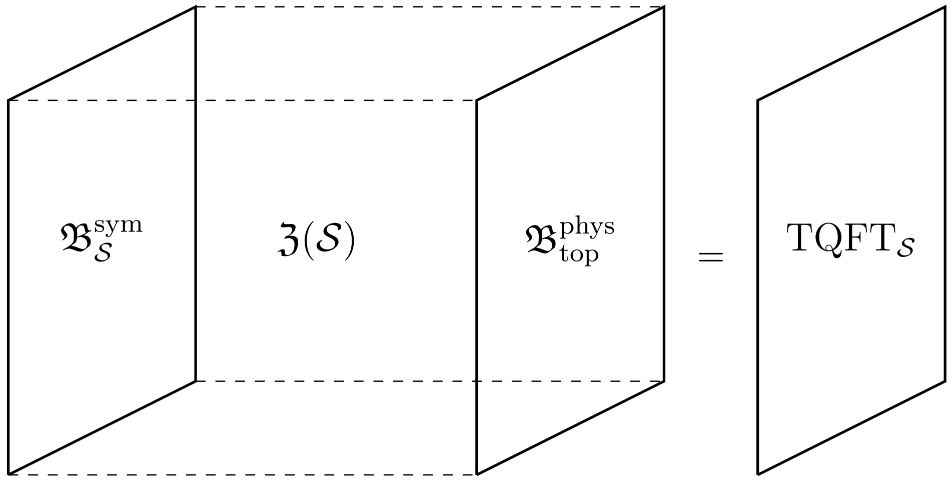
- Anyons $b \notin \mathscr{L}$ survive on $\mathscr{B}^{\text{sym}}$ and generate $\mathscr{S}$ (Neumann b.c.)

- Anyons $a \in \mathscr{L}$ are annihilated on $\mathscr{B}^{\text{sym}}$ (Dirichlet b.c.)

Classifying $\mathcal{S}$-symmetric gapped phases in (1+1)d

[Bhardwaj, LB, Pajer, Schafer-Nameki '23]

1. Start with $\mathcal{S}$ and construct $\mathcal{Z}(\mathcal{S})$
2. Classify all gapped b.c.'s $\longleftrightarrow$ all $\mathcal{L}$'s in $\mathcal{Z}(\mathcal{S})$
3. To study $\mathcal{S}$ symmetric phases, fix $\mathcal{B}^{\text{sym}} = \mathcal{L}_{\mathcal{S}}$
4. To obtain a TQFT, also $\mathcal{B}^{\text{phys}}$ is gapped: $\mathcal{B}^{\text{phys}} = \mathcal{L}_{\text{phys}}$



Fixing $\mathcal{L}_{\mathcal{S}}$ and changing $\mathcal{L}_{\text{phys}}$ we span all $\mathcal{S}$-symmetric TQFTs

5. Generalised order parameters $\longleftrightarrow$ anyons in $\mathcal{L}_{\text{phys}}$
6. Number of vacua $\longleftrightarrow$ anyons completely ending on both boundaries
7. Action of symmetry $\mathcal{S}$ on the vacua
8. Framework can be extended to incorporate phase transitions

Example: Rep(S_3)

$$\text{Rep}(S_3) = \{1, P, E\}$$

$$S_3 = \left\{a, b \,|\, a^3 = 1\,, b^2 = 1\,, bab = a^2\right\}$$

$P \otimes E = E$ $\qquad$ $E \otimes E = 1 \oplus P \oplus E$ $\qquad$ $P \otimes P = 1$] $\mathbb{Z}_2$ subsymmetry

- SymTFT $\mathscr{Z}(\text{Rep}(S_3))$ anyons g_ρ labelled by
  - conjugacy class $[g]$
  - irreducible representation ρ of centraliser H_g

- $[1]$ has $H_1 = S_3 \longrightarrow 1\,, P\,, E$
- $[a] = \{a, a^2\}$ has $H_a = \mathbb{Z}_3 \longrightarrow a_1\,, a_\omega\,, a_{\omega^2}$
- $[b] = \{b, ab, a^2b\}$ has $H_b = \mathbb{Z}_2 \longrightarrow b_+\,, b_-$

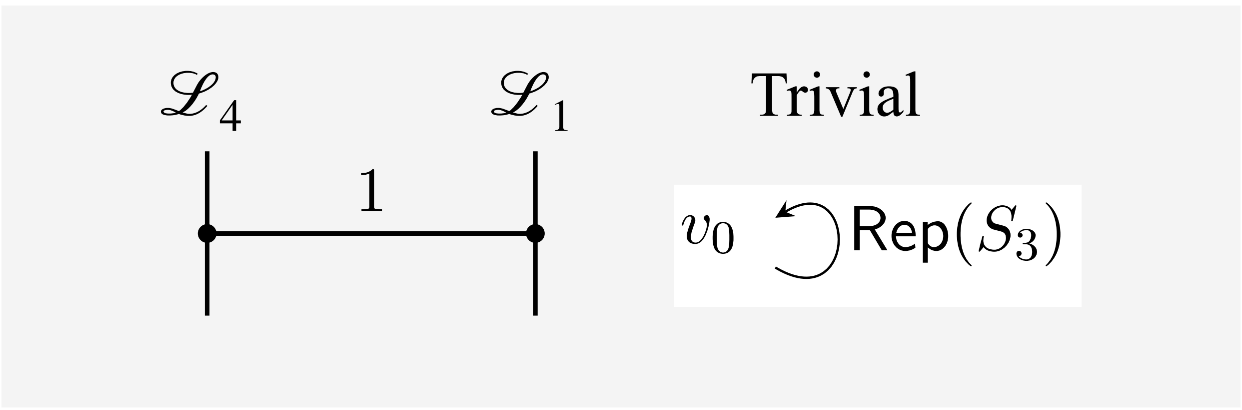
- Lagrangian algebras

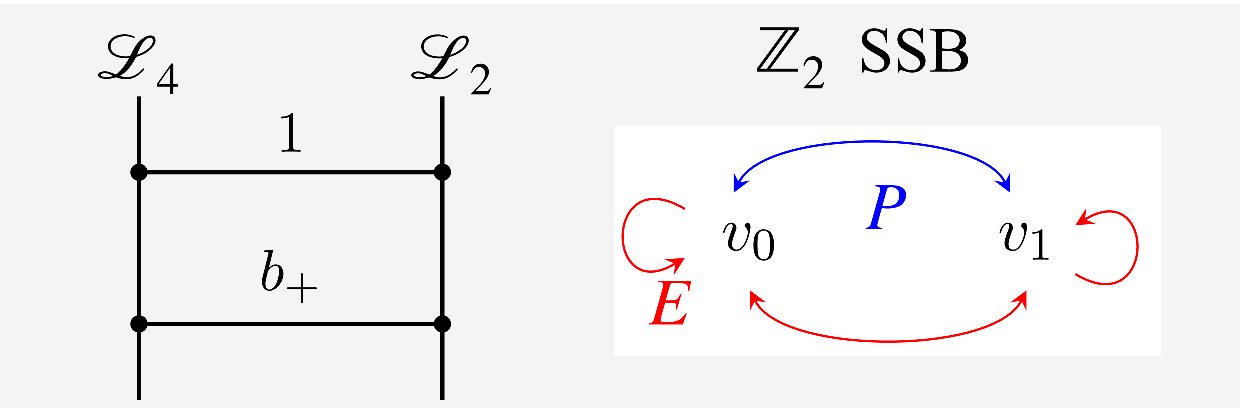
1. $\mathscr{L}_1 = 1 + P + 2E$
2. $\mathscr{L}_2 = 1 + E + b_+$
3. $\mathscr{L}_3 = 1 + P + 2a_1$
4. $\mathscr{L}_4 = 1 + a_1 + b_+$

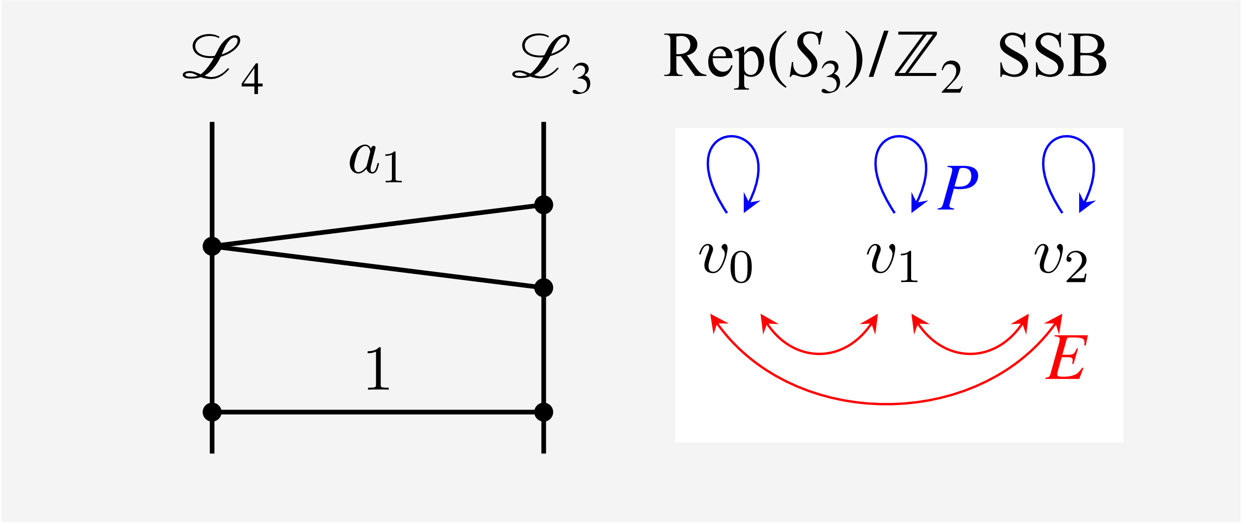
$$\mathscr{B}^{\text{sym}} = \mathscr{L}_4$$

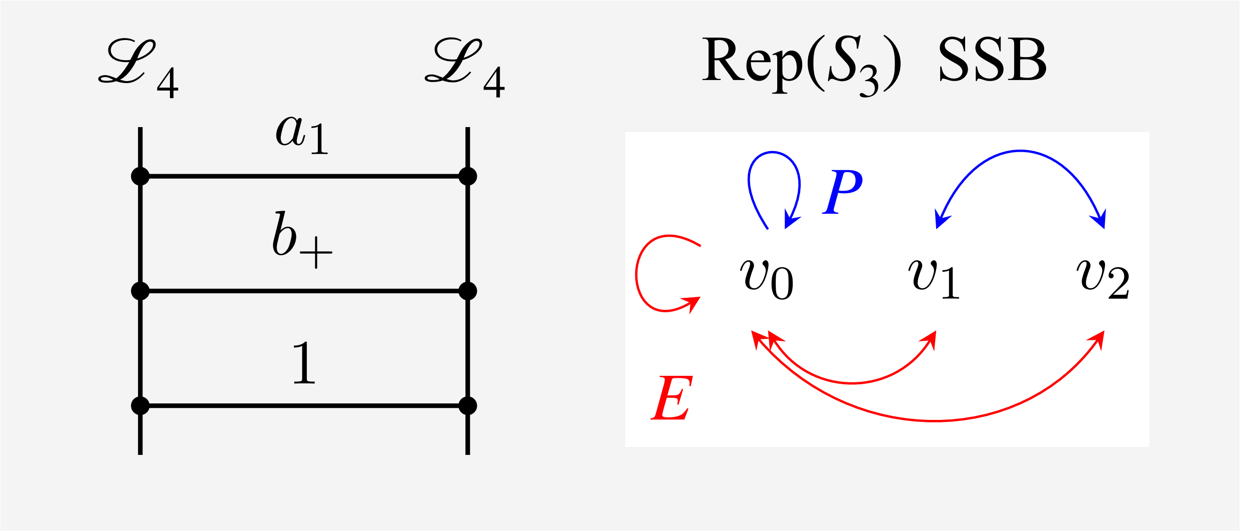
Example: Rep(S_3)

- 4 gapped boundaries $\Longrightarrow$ 4 gapped phases









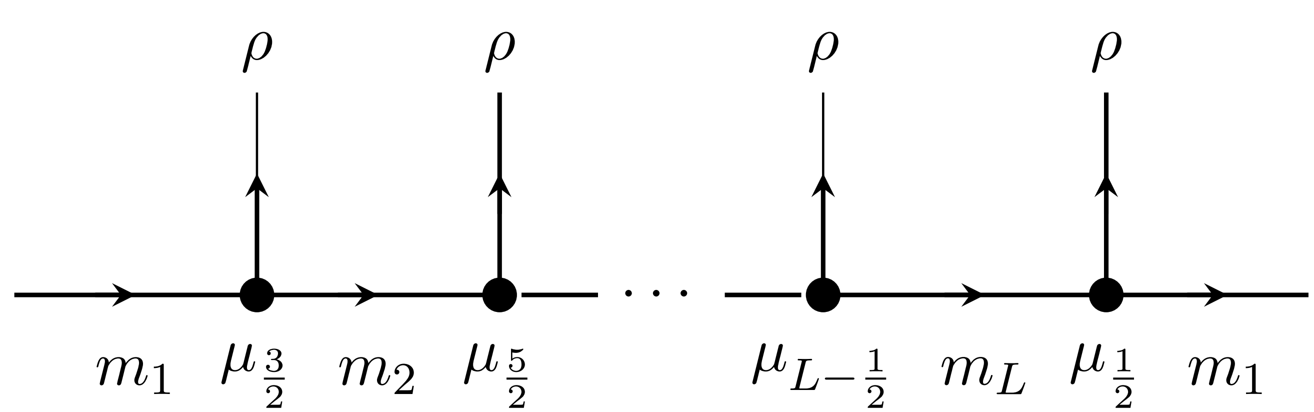
Lattice models

[Bhardwaj, LB, Schafer-Nameki, Tiwari ’24]

- Goal: construct (1+1)d lattice models flowing to gapped and gapless phases with symmetry $\mathscr{S}$
- Use anyon chains: naturally defined with fusion category symmetry $\mathscr{S}$
- Data defining the model:

1. Input fusion category $\mathscr{C}$
2. $\mathscr{C}$-module category $\mathscr{M}$
   (Something with a left $\mathscr{C}$ action: $\mathscr{C} \times \mathscr{M} \to \mathscr{M}$)
3. Some object $\rho \in \mathscr{C}$

- State space of the model spanned by fusion diagrams of the form



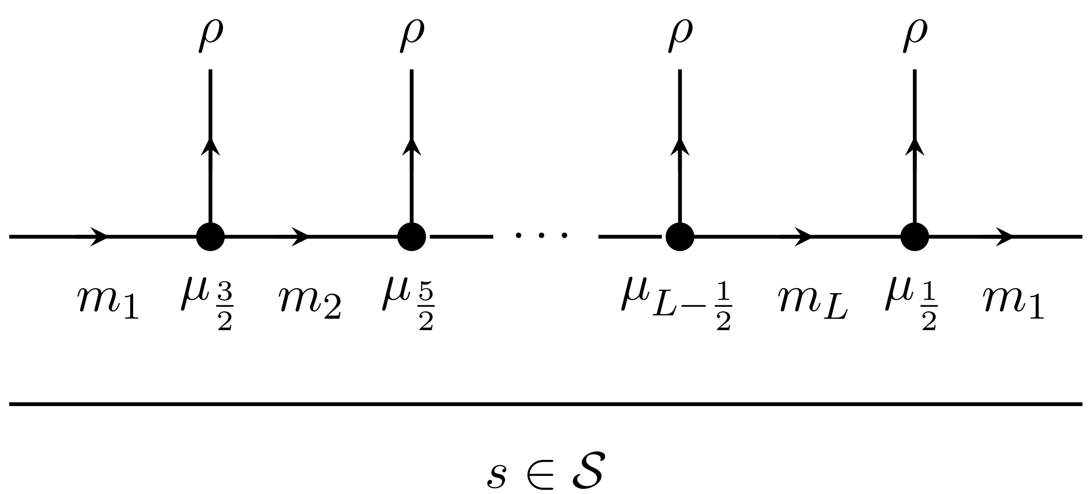
Lattice models

[Bhardwaj, LB, Schafer-Nameki, Tiwari '24]

- Symmetry Action

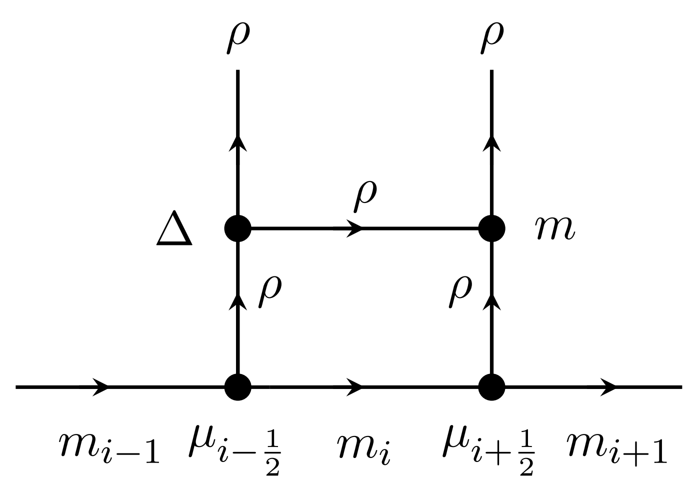
$\mathcal{S} = \mathcal{C}^*_{\mathcal{M}}$

(Dual category obtained from $\mathcal{C}$ via generalised gauging)



- Gapped Hamiltonians

Specialise ρ to be an algebra object in $\mathcal{C}$



- Symmetry commutes with hamiltonian as H acts from top and $\mathcal{S}$ acts from below

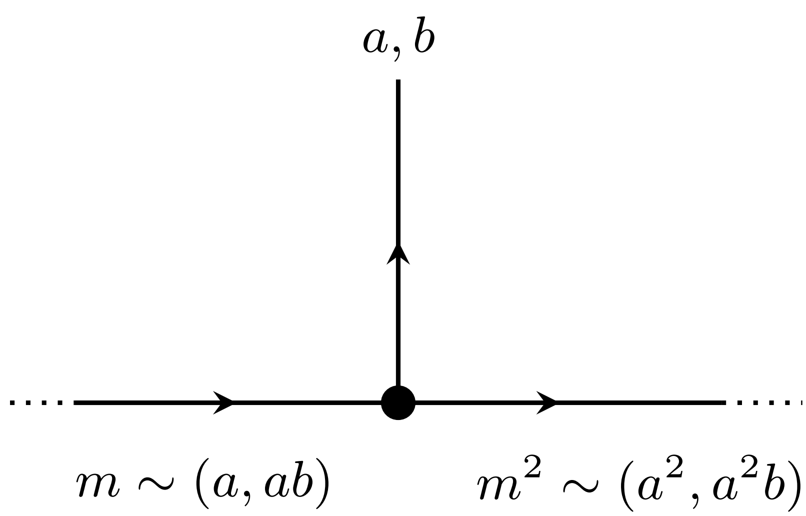
- Gapped boundaries of SymTFT $\mathcal{Z}(\mathcal{S})$ ↔ Algebra objects in $\mathcal{C}$ ↔ $\mathcal{S}$-symmetric TQFTs

Example: Rep(S_3)

- Defining data: $\mathscr{C} = S_3$ | $\mathscr{M} = \mathscr{M}(\mathbb{Z}_2^b) = \{1 \sim (1,b)\,, m \sim (a, ab)\,, m^2 \sim (a^2, a^2 b)\}$ | $\rho = \bigoplus_{g \in S_3} g$

$$\mathscr{S} = \mathscr{C}^*_{\mathscr{M}} = \mathrm{Rep}(S_3)$$

- State space:



Total space decomposes into tensor product of $\mathbb{C}^3$ at integer sites and $\mathbb{C}^2$ at half-integer sites

- Anyon chain admits tensor product Hilbert space: can write this as standard spin chain Hamiltonian

Conclusions & Future Directions

- SymTFT gives powerful machinery to study gapped and gapless phases in (1+1)d
- Applicable to any fusion category $\mathcal{S}$
- Lattice models provide concrete UV realisation of this paradigm
- Extension to higher dimensions
- Experimental realisations

Thank you!