Hidden Simplicity of Cosmological Correlators

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Based on:

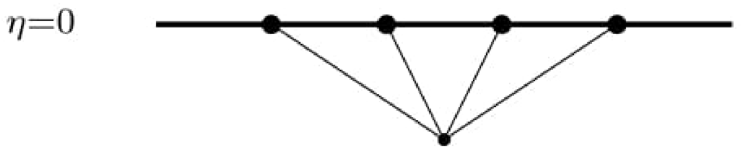
2312.13803 with C. Chowdhury, J. Mei, I. Sachs, P. Vanhove

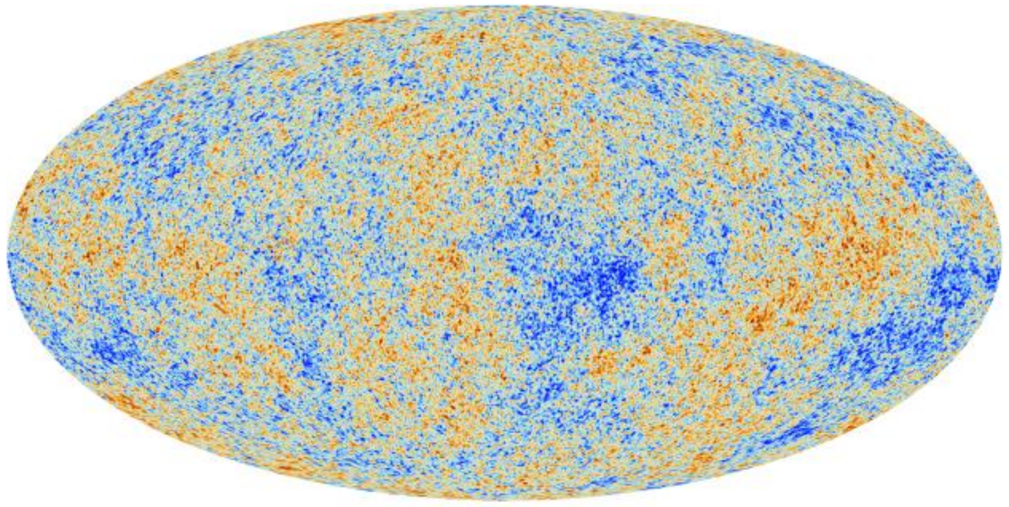
24xx.xxxxx with C. Chowdhury, J. Mei, J. Marshall, I. Sachs

Cosmological Observables

- Inflation: early Universe approximately described by dS_4. CMB comes from correlations on future boundary

$$ds^2 = \frac{-d\eta^2 + (dx^i)^2}{\eta^2}, \quad -\infty < \eta < 0$$





Cosmological Wavefunction

- In-in correlators (Maldacena,Weinberg):

$$\left\langle \phi(\vec{k}_1)...\phi(\vec{k}_n) \right\rangle = \frac{\int \mathcal{D}\phi \, \phi(\vec{k}_1)...\phi(\vec{k}_n) \left|\Psi\left[\phi\right]\right|^2}{\int \mathcal{D}\phi \left|\Psi\left[\phi\right]\right|^2}$$

- Wavefunction:

$$\ln \Psi\left[\phi\right] = -\sum_{n=2}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \frac{\mathrm{d}^d k_i}{(2\pi)^d} \psi_n \left(\vec{k}_1, ...\vec{k}_n\right) \phi(\vec{k}_1)...\phi(\vec{k}_n)$$

- ψ_n can be computed from analytically continued Feynman diagrams in EAdS (Maldacena,Pimentel,McFadden,Skenderis)

AdS Momentum Space

- Poincaré patch of Euclidean AdS_4 with unit radius:

$$ds^2 = \frac{dz^2 + \left(dx^i\right)^2}{z^2}$$

- Wick rotation $z = \pm i\eta$ gives dS_4 metric

- Boundary momenta: $\sum_{i=1}^{n} \vec{k}_i = 0$

- Energy: $k_{12\ldots n} = \sum_{i=1}^{n} k_i, \;\; k_i = \left|\vec{k}_i\right|$

Shadow Formalism

- In-in correlators can be computed from Witten diagrams in EAdS using the following Lagrangian:

$$iS_c = \int_0^\infty \frac{\mathrm{d}z\mathrm{d}^d x}{z^{d+1}} \Bigg[\sin\left(\pi(\Delta_+ - \frac{d}{2})\right) \left((\partial\phi_+)^2 - m^2 {\phi_+}^2\right) + \sin\left(\pi(\Delta_- - \frac{d}{2})\right) \left((\partial\phi_-)^2 - m^2 {\phi_-}^2\right) + e^{i\pi\frac{d-1}{2}} V\left(e^{-i\frac{\pi}{2}\Delta_+}\phi_+ + e^{-i\frac{\pi}{2}\Delta_-}\phi_-\right) + e^{-i\pi\frac{d-1}{2}} V\left(e^{i\frac{\pi}{2}\Delta_+}\phi_+ + e^{i\frac{\pi}{2}\Delta_-}\phi_-\right)\Bigg]$$

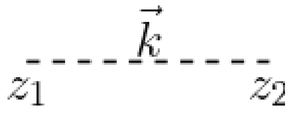
(Sleight,Taronna,Di Pietro,Gorbenko,Komatsu)

- ϕ_+ is a ghost and only appears on internal lines

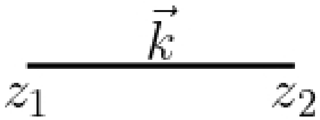
Conformally coupled φ⁴

- Choose d=3 and Δ_-=d- Δ_+=1. $V(\phi_+,\phi_-) = \frac{1}{2}\frac{\lambda}{4!}\left(\phi_+^4 - 6\phi_+^2\phi_-^2 + \phi_-^4\right)$

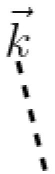
- Feynman rules: (ϕ_- dotted, ϕ_+ solid)



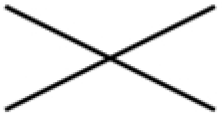
$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dp}{p^2+k^2}\cos(pz_1)\cos(pz_2)$$



$$= -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dp}{p^2+\vec{k}^2}\sin(pz_1)\sin(pz_2)$$



$$= \quad e^{-kz}$$

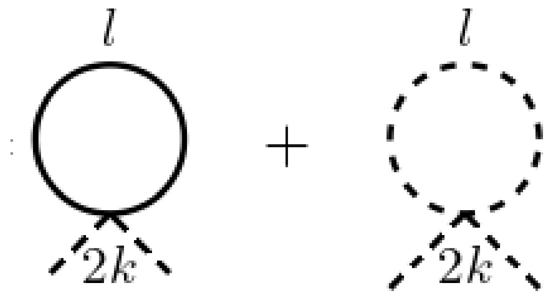


$$= \frac{\lambda}{2},$$

$$= -3\lambda,$$

$$= \frac{\lambda}{2}$$

1-loop 2-point

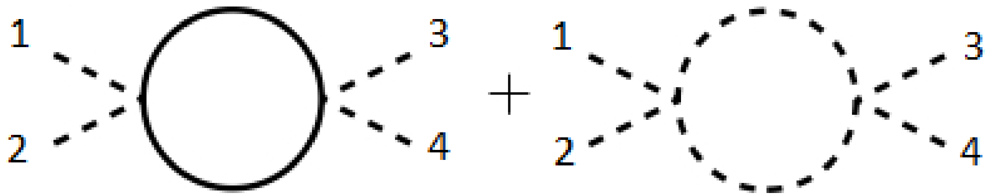


$$= \lambda \int \frac{d^3l dp}{(2\pi)^4} \frac{1}{p^2 + l^2} \int_0^\infty dz e^{-2kz} \left(\sin^2(pz) + \cos^2(pz) \right)$$

$$= \frac{\lambda}{k} \int \frac{d^4L}{(2\pi)^4} \frac{1}{L^2} , \quad L^\mu = \left(p, \vec{l} \right)$$

- 4d Lorentz invariance is restored
- Doesn’t happen for wavefuntion

1-loop 4-point



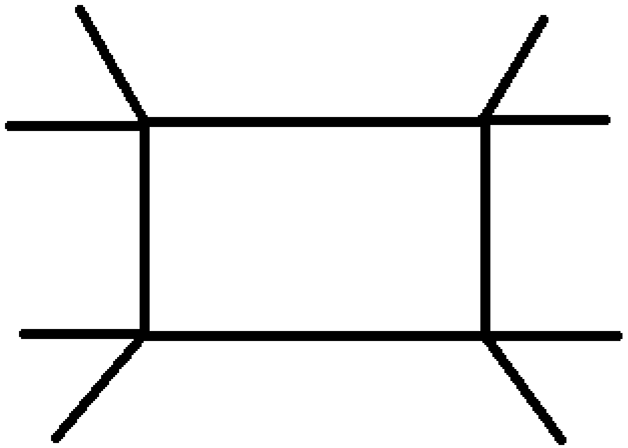
$$= \left(\frac{\lambda}{2}\right)^2 k_{12}k_{34}\int \frac{dp}{(2\pi)^5}\frac{1}{\left(p^2+k_{12}^2\right)\left(p^2+k_{34}^2\right)}\int \frac{d^4L}{L^2(L-P)^2}, \quad P^\mu = \left(p, \vec{k}_{12}\right)$$

$$= \frac{1}{(2\pi)^2}\frac{\lambda^2}{32(k_{12}+k_{34})}\left[\ln\left(\frac{(k_{12}+|\vec{k}_{12}|)(k_{34}+|\vec{k}_{12}|)}{4\Lambda^2}\right)+\frac{k_{12}+k_{34}}{k_{12}-k_{34}}\ln\left(\frac{k_{34}+|\vec{k}_{12}|}{k_{12}+|\vec{k}_{12}|}\right)\right], \quad \vec{k}_{ij} = \vec{k}_i + \vec{k}_j$$

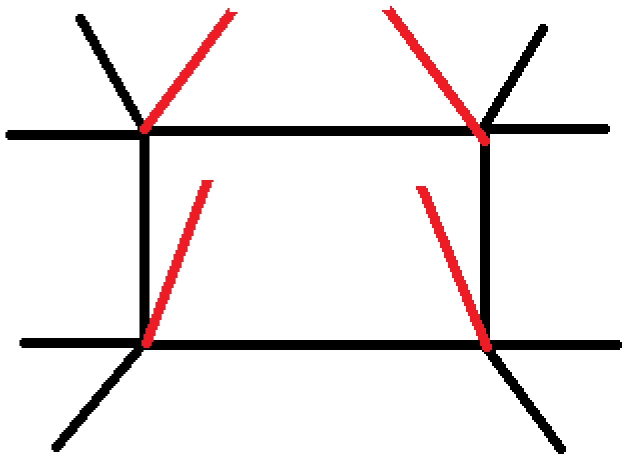
- Same analytic structure as scattering amplitude
- In contrast, wavefunction is a dilog (Albayrak,Chowdhury,Kharel)

Question: How do we make the simplicity of in-in correlators manifest?

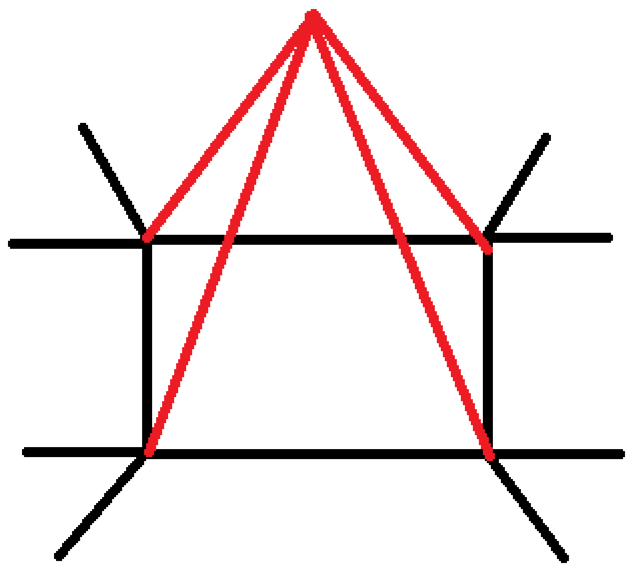
- Start with a flat space Feynman diagram



- Start with a flat space Feynman diagram
- Add an auxiliary propagator at each vertex

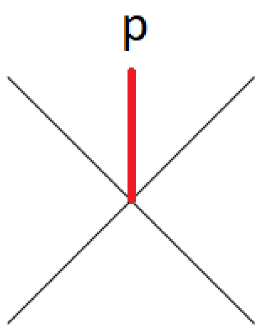


- Start with a flat space Feynman diagram
- Add auxiliary propagator at each vertex
- Connect auxiliary propagators to a common point
- Impose internal energy conservation at each vertex
- Integrate over auxiliary energies



Dressing Rules

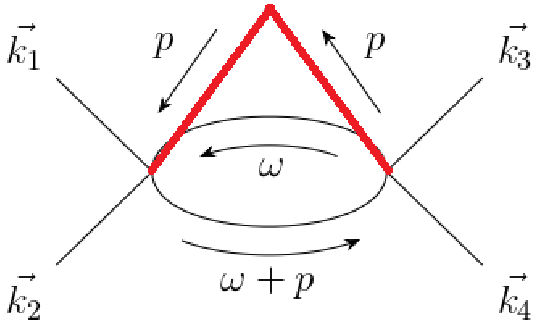
- For conformally coupled ϕ^4, the auxiliary propagator is:



$$\frac{k_{ext}}{p^2 + k_{ext}^2}$$

- k_{ext} = total energy of external legs attached to vertex
- p = total energy of internal legs attached to vertex
- Similar rules can be derived for conformally coupled ϕ^3, massless ϕ^4, and massless ϕ^3

Example:



$$= \left(\frac{\lambda}{2}\right)^2 \int \frac{1}{(2\pi)^5} dp\, d\omega\, d^3 l \left(\frac{k_{12}}{p^2 + k_{12}^2}\right) \left(\frac{k_{34}}{p^2 + k_{34}^2}\right) \left(\frac{1}{\omega^2 + \vec{l}^2}\right) \left(\frac{1}{(\omega + p)^2 + \left(\vec{l} + \vec{k}_{12}\right)^2}\right)$$

$$= \left(\frac{\lambda}{2}\right)^2 k_{12} k_{34} \int \frac{dp}{(2\pi)^5} \frac{1}{\left(p^2 + k_{12}^2\right)\left(p^2 + k_{34}^2\right)} \int \frac{d^4 L}{L^2 (L - P)^2},$$

Conclusion and Future

- In-in correlators are simpler than wavefunctions
- Can be obtained by dressing flat space Feynman diagrams
- Discontinuities? (work in progress with Chowdhury,Jazayeri,Mei,Marshall,Sachs)
- Renormalisation? (see recent work by Bzowski,McFadden,Skenderis)
- General backgrounds?