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# Black Hole Singularity from Operator Product Expansion (OPE)

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Based on 2404.17286 with Liu, Parnachev, and Valach + ongoing work.

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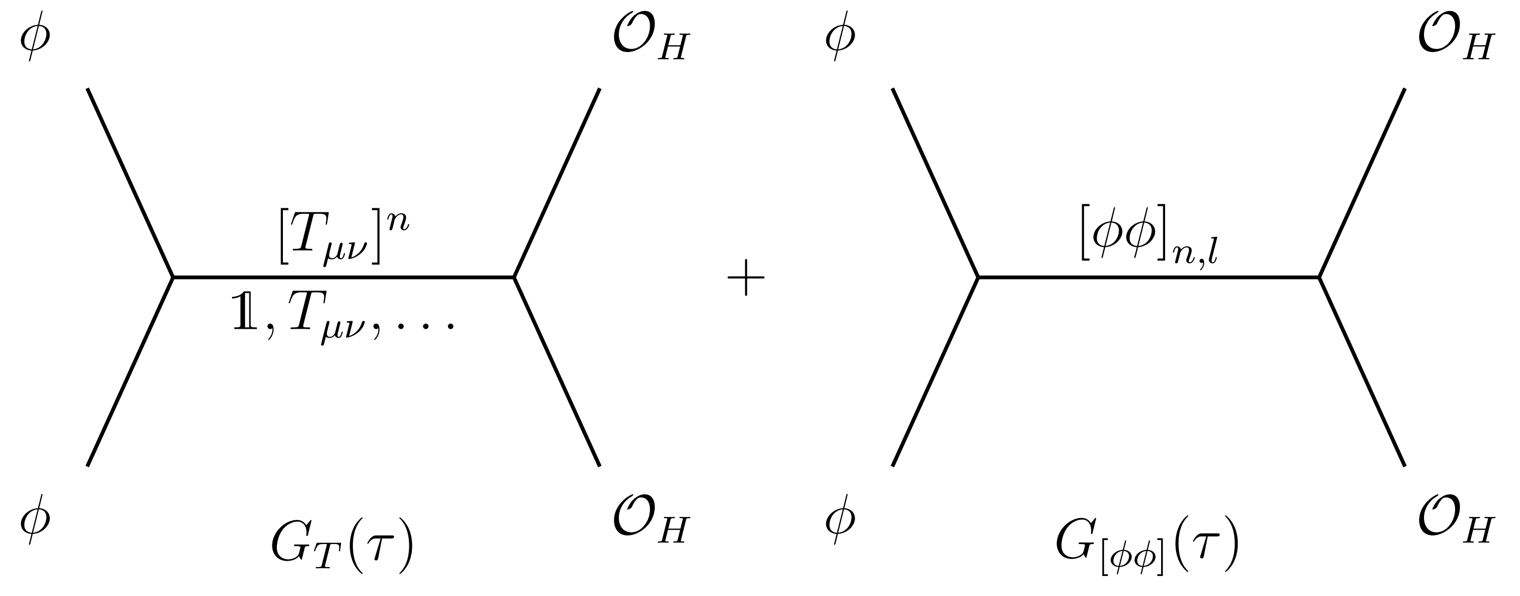
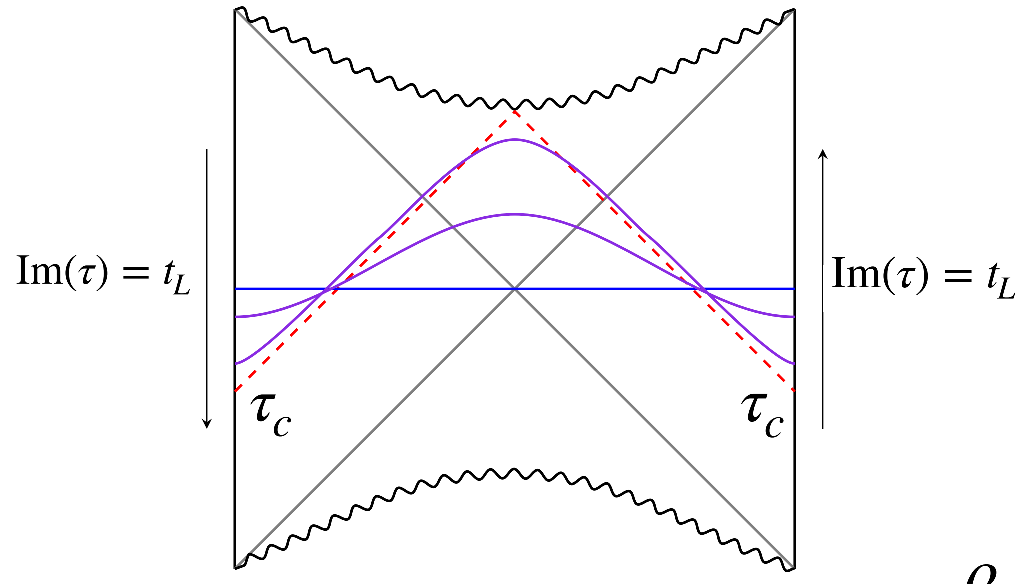
# Singularities of the thermal correlator

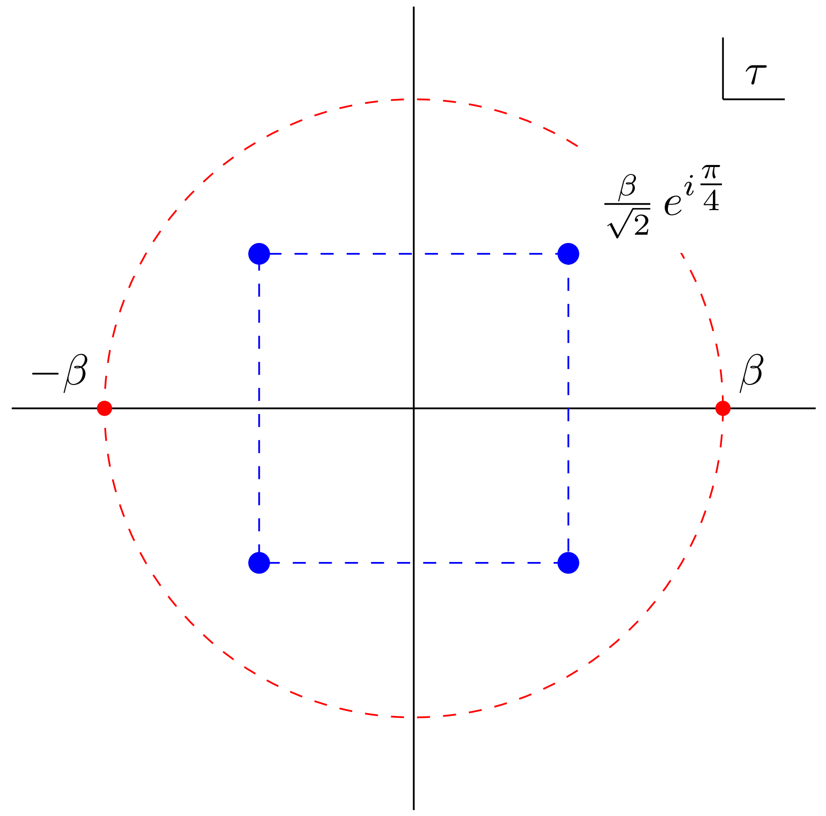
$$G(\tau) = \left\langle \phi_\Delta(\tau)\,\phi_\Delta(0) \right\rangle_\beta \sim \left\langle \mathcal{O}_H\,\phi_\Delta(\tau)\,\phi_\Delta(0)\,\mathcal{O}_H \right\rangle$$




$$G(\tau) = G_T(\tau) + G_{[\phi\phi]}(\tau)$$




$$\lim_{\Delta\to\infty} \hat{G}(\tau) \sim (\tau_c - \tau)^{2\Delta} \qquad \tau_c = \frac{\beta}{\sqrt{2}} e^{i\frac{\pi}{4}}$$




$$G_T(\tau) \sim \left(\tau_c - \tau\right)^{-(2\Delta-2)}$$

$$\tau_c = \frac{\beta}{\sqrt{2}} e^{i\frac{\pi}{4} + i\frac{k\pi}{2}}$$

The Stress-energy tensor sector of holographic correlation functions contains information about the black hole singularity

$\tau = t_E + i\,t_L$ — Complexified time, $\beta = T^{-1}$ —Inverse temperature, $\Delta$ — Conformal dimension



$k \in \mathbb{Z}$

# Setup

- Holographically evaluate

$$G(\tau) = \langle \phi_\Delta(\tau)\,\phi_\Delta(0) \rangle_\beta \qquad\qquad \tau = t_E + i\,t_L, \qquad t_L \in \mathbb{R}, \quad 0 \le t_E < \beta$$

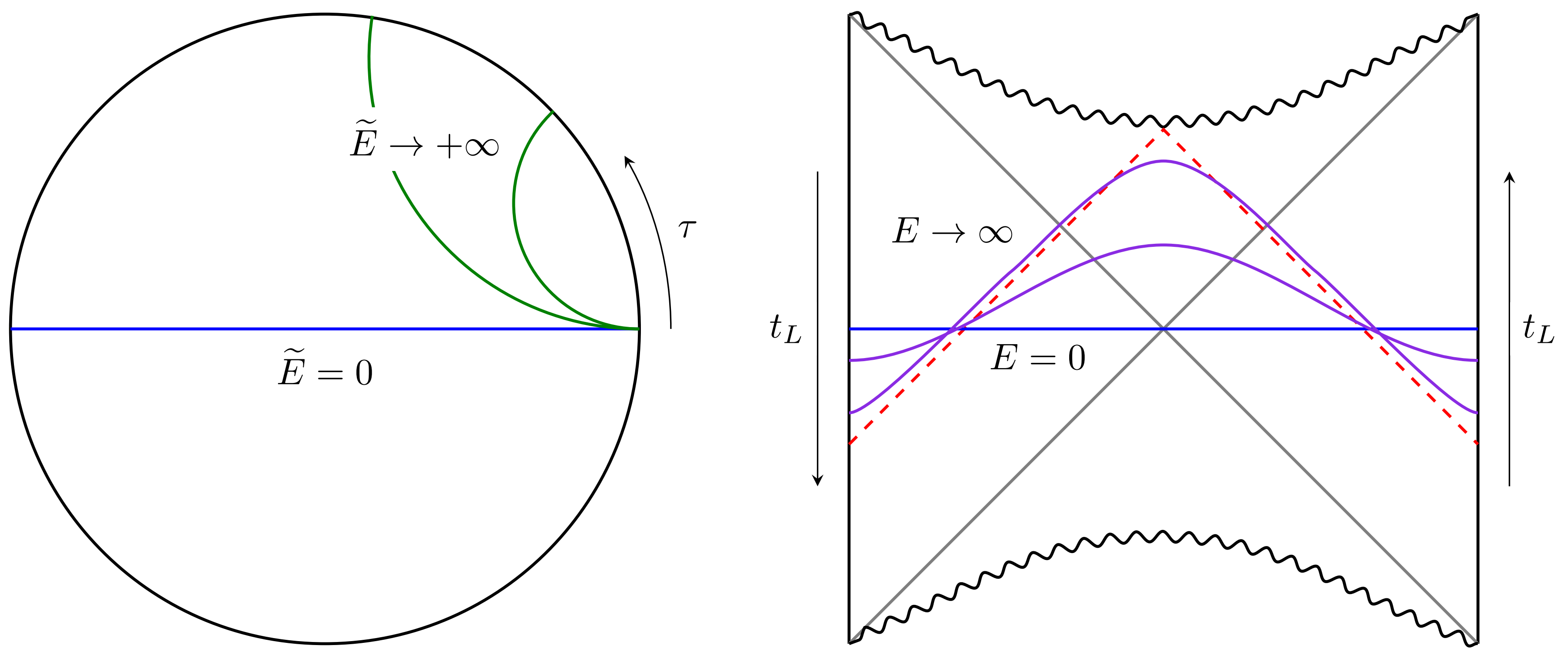
- Solve the Klein-Gordon equation $(\Box - m^2)\phi(\tau, r) = 0$ on planar black hole background

$$ds^2 \;=\; r^2 f(r)\,d\tau^2 \;+\; \frac{dr^2}{r^2 f(r)} \;+\; r^2\,d\vec{x}^2\,, \qquad f(r) = 1 - \frac{\pi^4}{\beta^4\,r^4}$$

- In the $\Delta \to \infty$ limit

$$G(\tau) \sim \sum_{\text{Geodesics}} e^{-\Delta\,L(\tau)}$$

Euclidean Section
$(\tau = t_E)$




Lorentzian section
$(\tau = \beta/2 + i\,t_L)$

$R_{\text{AdS}} = 1, \quad m^2 = \Delta(\Delta - 4)$

$L(\tau)$ is the proper length of geodesics connecting the insertion points



$\vec{x}$—dependence is suppressed throughout

# Bouncing Singularity

[Fidkowski+Hubeny+Kleban+Shenker '03, Festuccia+Liu '05]

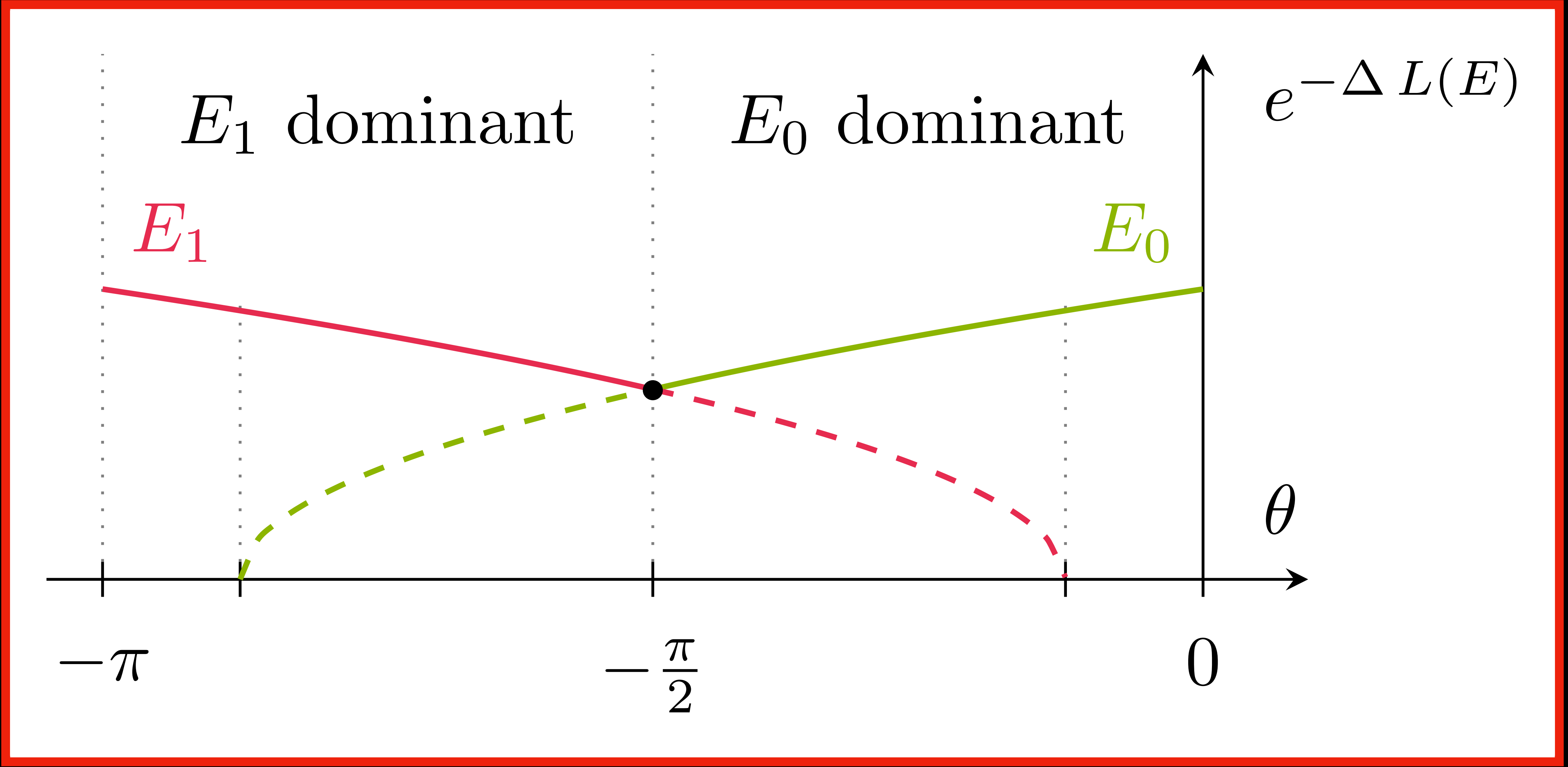
- If the real geodesic contributes, then $G(\tau) \sim (\tau_c - \tau)^{-2\Delta}$, as $\tau \to \tau_c = \frac{\beta}{\sqrt{2}} e^{i\frac{\pi}{4}}$, which is not allowed.

- Euclidean geodesic ($\tau < \beta/2$) is unique, but at $\tau = \beta/2$ we encounter a branch point

$$\left(\frac{\beta}{2} - \tau\right) \sim E^3, \qquad L \sim E^4, \qquad L \sim \left(\frac{\beta}{2} - \tau\right)^{\frac{4}{3}}$$

There are three branches, two complex and one real at $\tau = \beta/2 + i\, t_L$: $\left(\frac{\beta}{2} - \tau\right) \sim e^{i\theta}$

$$E_0 \sim e^{\frac{i\theta}{3}}$$
$$E_1 \sim E_0\, e^{-\frac{2\pi i}{3}}$$
$$E_2 \sim E_0\, e^{-\frac{4\pi i}{3}}$$




$E_2$ branch, which contains the bouncing singularity, never contributes to the correlation function.
That information is encoded in the analytic properties of the correlator through analytic continuation.

$E$ is the conserved charge along the geodesic associated with time translation invariance

# OPE decomposition of the thermal correlator

- Separate the correlator into the Stress-tensor and Double-trace contributions:

$$G(\tau) = G_T(\tau) + G_{[\phi\phi]}(\tau)\,, \qquad G_T(\tau) = \frac{1}{\tau^{2\Delta}} \sum_{n=0}^{\infty} \Lambda_n \left(\frac{\tau}{\beta}\right)^{d\,n}, \qquad G_{[\phi\phi]}(\tau) = \frac{1}{\tau^{2\Delta}} \sum_{k=0}^{\infty} D_k \left(\frac{\tau}{\beta}\right)^{2\,k+2\Delta}$$

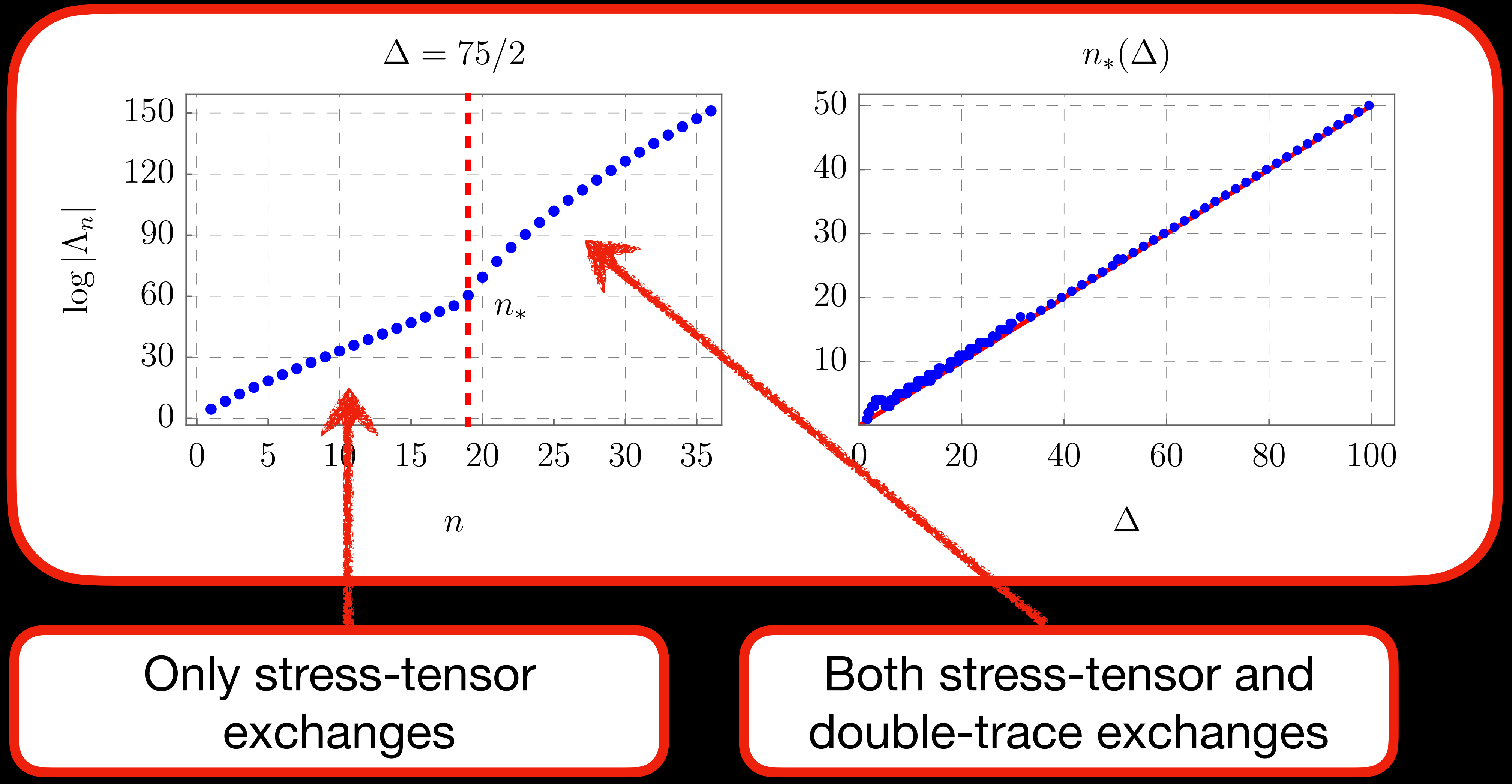
- The stress-tensor sector ($\Lambda_n$) can be obtained from a near-boundary analysis order by order in $n$. For $d = 4$

$$G_T(\tau) \approx \frac{1}{\tau^{2\Delta}}\left[1 + \frac{\pi^4\,\Delta}{40}\left(\frac{\tau}{\beta}\right)^4 + \frac{\pi^8\,\Delta\left(63\Delta^4 - 413\Delta^3 + 672\Delta^2 - 88\Delta + 144\right)}{201600(\Delta-4)(\Delta-3)(\Delta-2)}\left(\frac{\tau}{\beta}\right)^8 + \ldots\right], \qquad \Lambda_n \sim \frac{h_n(\Delta)}{\prod_{k=2}^{2n}(\Delta - k)}$$

- A crossover at

$$n_* = \frac{2\,\Delta}{d} \iff \Delta^*_{T^n} = n_*\,d = 2\Delta$$

Where double-traces start to contribute ($\Delta_{[\phi\phi]_k} = 2\Delta + 2k$)




$d = 4$ corresponds to $AdS_5$. All explicit results are for $AdS_5$.
$h_n(\Delta)$ is some polynomial in $\Delta$.

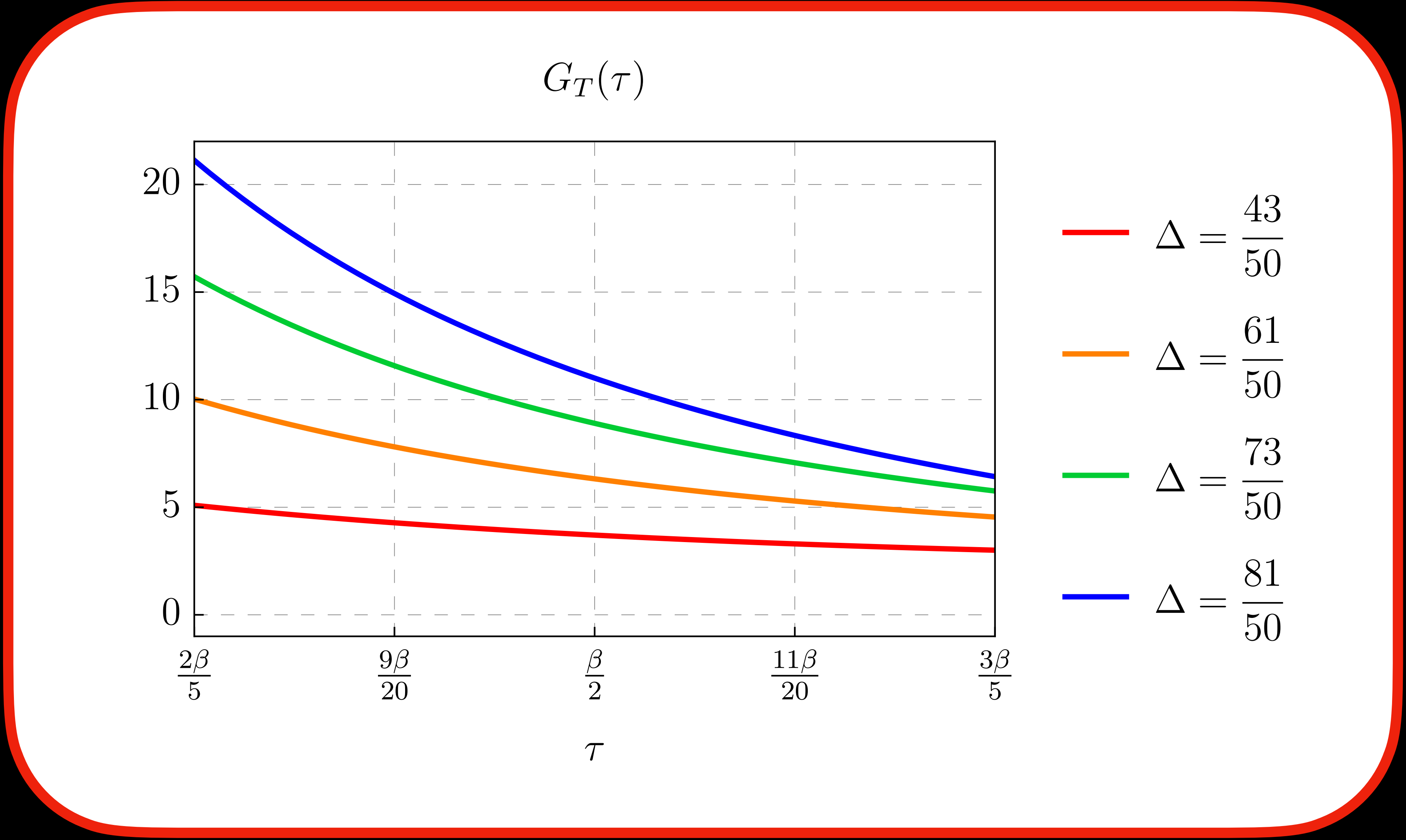
# Role of double trace operators

- Double-trace operators are necessary to ensure KMS condition

$$G(\tau) = G(\beta - \tau)$$

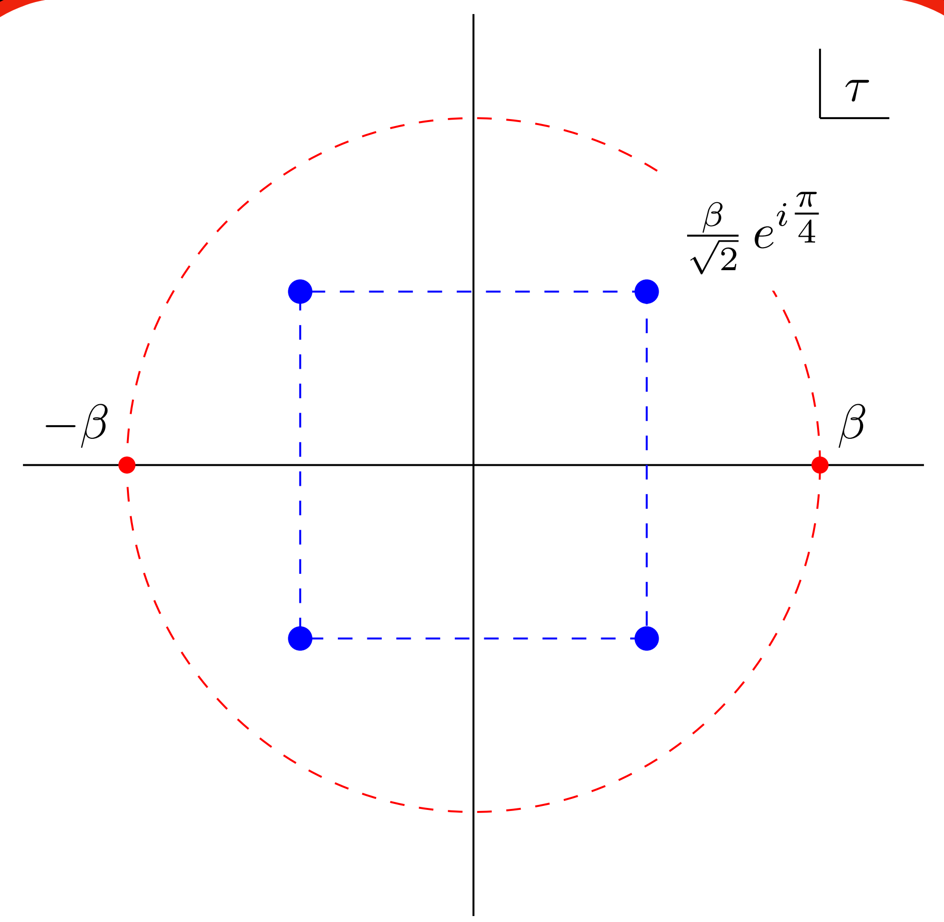
  Full thermal correlator needs to be symmetric around $\tau = \beta/2$.

- $G_T(\tau)$ does not satisfy the KMS condition by itself:




- $G_{[\phi\phi]}(\tau)$ needed even at finite $\Delta$.

# Asymptotic form of $\Lambda_n$ at finite $\Delta$




- Fix $\Delta < \infty$ and analyse $\Lambda_n$ at $n \gg n_*$

$$\Lambda_n \sim \frac{n^{2\Delta-3}}{\left(\frac{1}{\sqrt{2}}\right)^{4n} e^{i\pi n}}$$

- Allows us to resum the stress-tensor contribution

$$G_T(\tau) \sim \frac{1}{\tau^{2\Delta}} \left[ -\log\left( \frac{\tau^4}{\left(\frac{\beta}{\sqrt{2}}\right)^4 e^{i\pi}} \right) \right]^{-(2\Delta-2)} \longrightarrow \quad G_T(\tau \to \tau_c) \sim (\tau_c - \tau)^{-(2\Delta-2)}, \qquad \tau_c = \frac{\beta}{\sqrt{2}} e^{i\frac{\pi}{4} + i\frac{k\pi}{2}}$$

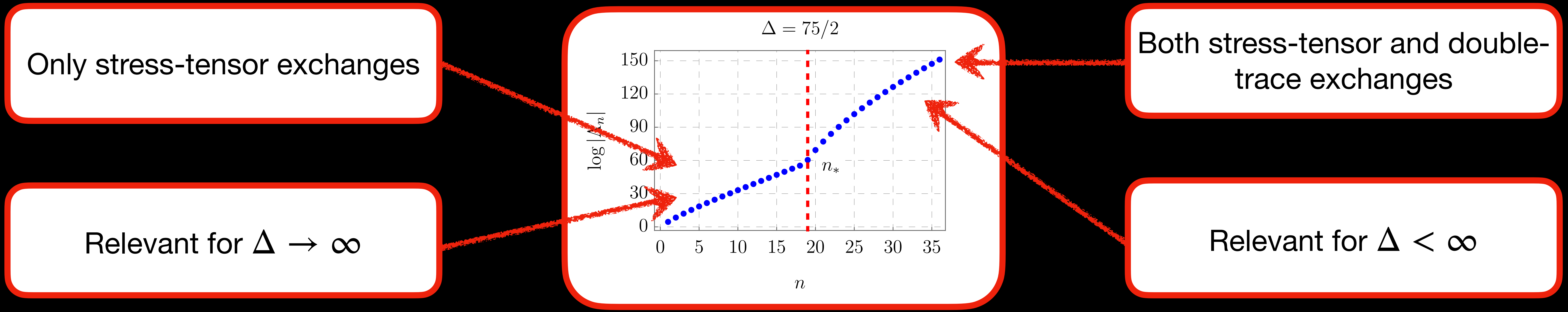
- At finite $\Delta$, the stress-tensor sector has a divergence at the same location as the bouncing singularity

$G_T(\tau)$ contains information about the black hole singularity

$k \in \mathbb{Z}$

# Asymptotic form of $\Lambda_n$ as $\Delta \to \infty$

- Taking $\Delta \to \infty$ at $G_T(\tau \to \tau_c) \sim (\tau_c - \tau)^{-(2\Delta-2)}$ reproduces the bouncing geodesic result, in contradiction with CFT.
- Order of limits is important: $\Delta \to \infty$ needs to be taken before analysing $\Lambda_n$, as different data becomes important




Cross-over at $n_* \sim \Delta$ where double-trace operators start to contribute: When $\Delta \to \infty$, only stress-tensor sector

- At $\Delta \to \infty$, analyse the analogue of the proper length

$$\mathscr{L}_T(\tau) = -\lim_{\Delta\to\infty} \frac{1}{\Delta} \log G_T(\tau) = 2\log\tau - \frac{\pi^4}{40}\left(\frac{\tau}{\beta}\right)^4 - \frac{11\,\pi^8}{14400}\left(\frac{\tau}{\beta}\right)^8 + \ldots$$

**The stress-tensor sector reproduces the proper length of the one sided (Euclidean) geodesic: $L(\tau) = \mathscr{L}_T(\tau)$, for $\tau < \beta/2$**

- Asymptotic analysis of expansion reproduces the geodesic branch point at $\beta/2 < |\tau_c|$: Bouncing singularity is not seen

$$\mathscr{L}_T(\tau) \sim \left(\frac{\beta}{2} - \tau\right)^{\frac{4}{3}}$$

# Role of double trace operators

- Decomposition of the correlator into the stress-tensor and double-trace sector

$$G(\tau) = G_T(\tau) + G_{[\phi\phi]}(\tau)\,, \qquad G_T(\tau) = \frac{1}{\tau^{2\Delta}} \sum_{n=0}^{\infty} \Lambda_n \left(\frac{\tau}{\beta}\right)^{dn}, \qquad G_{[\phi\phi]}(\tau) = \frac{1}{\tau^{2\Delta}} \sum_{k=0}^{\infty} D_k \left(\frac{\tau}{\beta}\right)^{2k+2\Delta}$$

$$\frac{G_{[\phi\phi]}(\tau)}{G_T(\tau)} \sim \left(\frac{\tau}{\beta}\right)^{2\Delta} \ll 1 \qquad \text{for} \quad \tau < \beta \ \text{ as } \ \Delta \to \infty$$

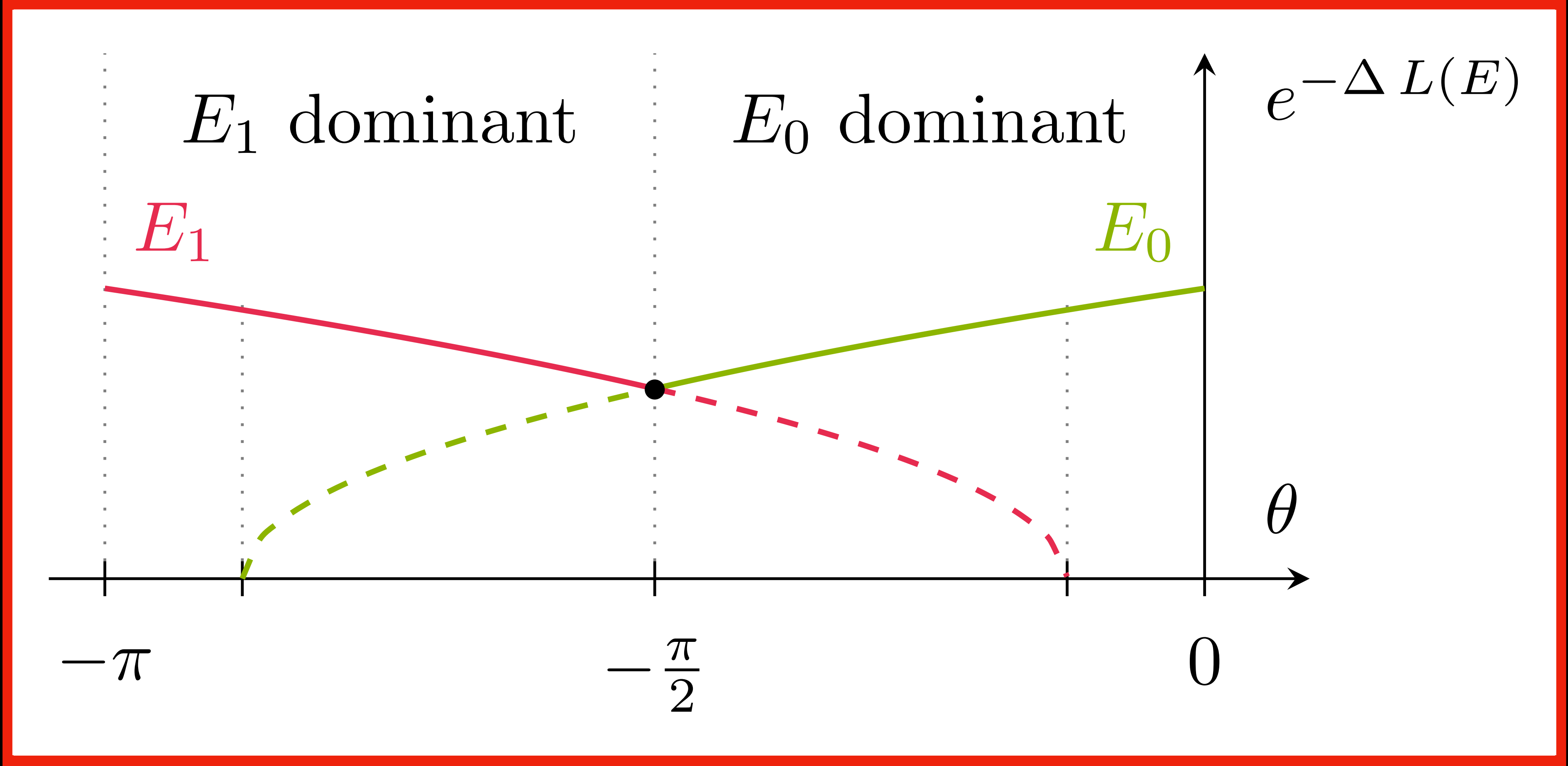
- Assumes that $\Delta \to \infty$ limit can be taken inside the OPE
- At $\Delta \to \infty$, $G_{[\phi\phi]}(\tau)$ becomes important already at $\tau = \beta/2$.
- OPE not uniformly convergent for all $\Delta$ for $\tau > \beta/2$:
  Needs to be resumed first, only then can the $\Delta \to \infty$ limit be taken.

# Identifying geodesic branches

$$E_0 \sim e^{\frac{i\theta}{3}}$$

$$E_1 \sim E_0\, e^{-\frac{2\pi i}{3}}$$

$$E_2 \sim E_0\, e^{-\frac{4\pi i}{3}}$$




$$\lim_{\Delta\to\infty} G(\tau) = \lim_{\Delta\to\infty} \begin{cases} G_T(\tau) & \tau < \frac{\beta}{2} : \quad E_0 \ \text{branch} \\ G_{[\phi\phi]}(\tau) & \tau > \frac{\beta}{2} : \quad E_1 \ \text{branch} \\ G_T(\tau) + G_{[\phi\phi]}(\tau) & \tau = \frac{\beta}{2} + it_L,\ t_L \in \mathbb{R} : \quad E_0 + E_1 \ \text{branches} \end{cases}$$

# Summary and outlook

- The stress-tensor sector contains information about the black-hole singularity.
- The double-trace sector becomes especially important at $\tau > \beta/2$:
  - To ensure KMS condition,
  - Cancel the bouncing singularity of the stress-tensor sector in the full correlator.
- At $\Delta \to \infty$, $G_T(\tau)$ and $G_{[\phi\phi]}(\tau)$ are identified with the two complex geodesics that contribute to the holographic correlator at $\tau = \beta/2 + i\,t_L$.
- New approach to analyse bulk curvature singularities from boundary CFT point of view:
  - $\alpha', G_N$ corrections,
  - Addition of matter fields,
  - Other boundary manifolds.
- Direct analysis from the CFT.