Pseudospectrum Of Black Holes in AdS

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Spectrum: Quasinormal Modes

- Control the response to (linear) perturbations of the BH spacetime.
- Contain intrinsic, geometric information.
- Applications: astrophysics, mathematical relativity, fundamental gravitational physics, Gauge/Gravity.

Structural stability of spectrum still not fully explored!

$\Updownarrow$

How does the spectrum change under small perturbations of the theory?

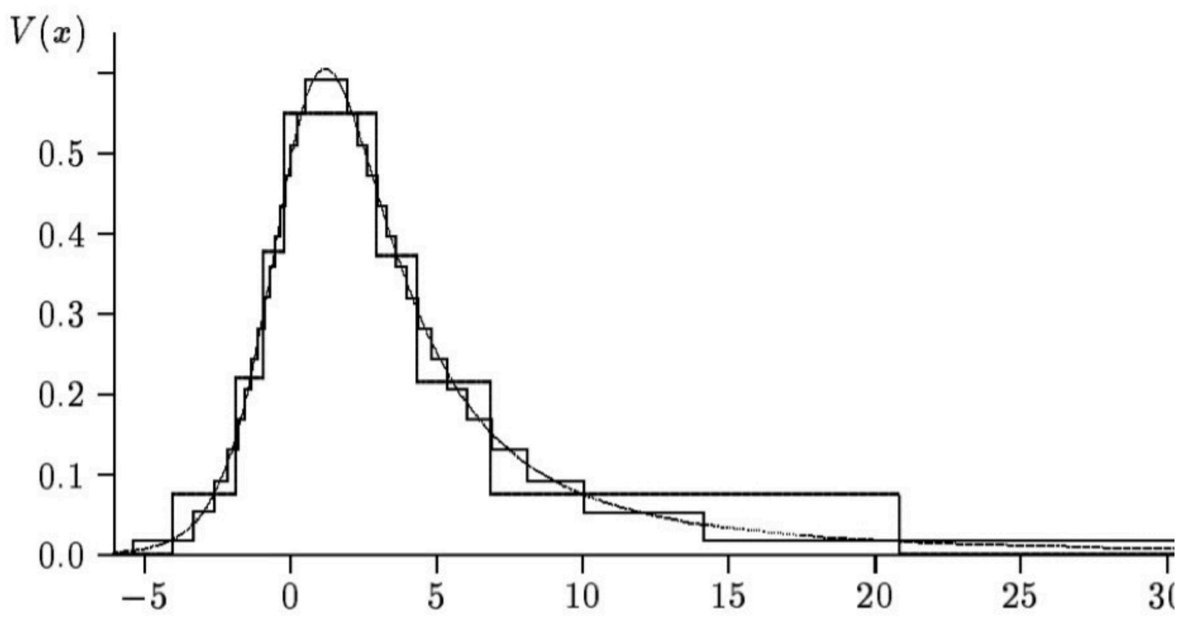
Spectral theorem

★ **Spectral theorem** for **normal** operators:

- Eigenvectors are orthogonal and complete set
- Eigenvalues are stable

★ **No spectral theorem** for **non-normal** operators:

- Neither complete nor orthogonal
- Eigenvalues complex and potentially unstable
- For BHs, first evidence of spectral instability by [Nollert '96, Nollert-Price '99]



★ **Pseudospectrum**

- Not a new idea: hydro stability and turbulence [Trefethen, …]
- In GR: introduced by [Jaramillo et al. '21] for Schw BH flat, RN, ECOs, dS, AdS [Destounis, Panoso, Boyanov, Cardoso,..] [Arean et al. 23][Boyanov et al. '23]

- Topographic map of eigenvalue migration under **generic perturbations** of order ϵ : non-modal, not intrinsic (requires a norm).
- Schematically for black holes:

In this talk

Study the pseudospectrum for D=5 AdS-RN:

- For scalar, electromagnetic, gravitational perturbations.
- Null coordinates: generalised e.v. problem.

EOM: Spectrum:

$$A\psi = \omega B\psi \Rightarrow \sigma(A,B) = \{\omega \in \mathbf{C} : det(\omega B - A) = 0\}$$

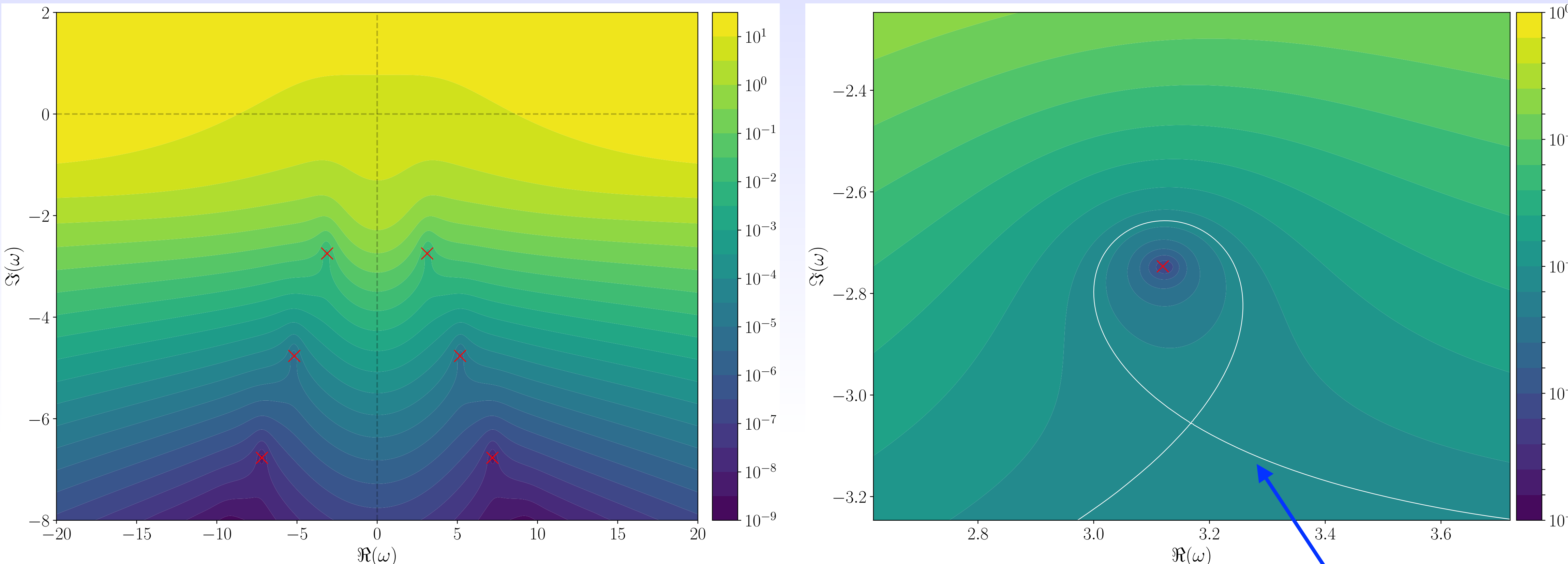
Pseudospectrum: physics only in A, so only perturb A

$$\begin{aligned}\sigma^{\epsilon}(A,B) &= \{\omega \in \mathbf{C} : \exists\, \delta A \text{ with } ||\delta A|| < \epsilon : \omega \in \sigma(A+\delta A, B)\} \\ &= \{\omega \in \mathbf{C} : ||R_{A,B}(\omega)|| \equiv ||(\omega B - A)^{-1}|| > 1/\epsilon\}\end{aligned}$$

↑ Resolvent

- Energy norm: integrate stress tensor on a null hypersurface.

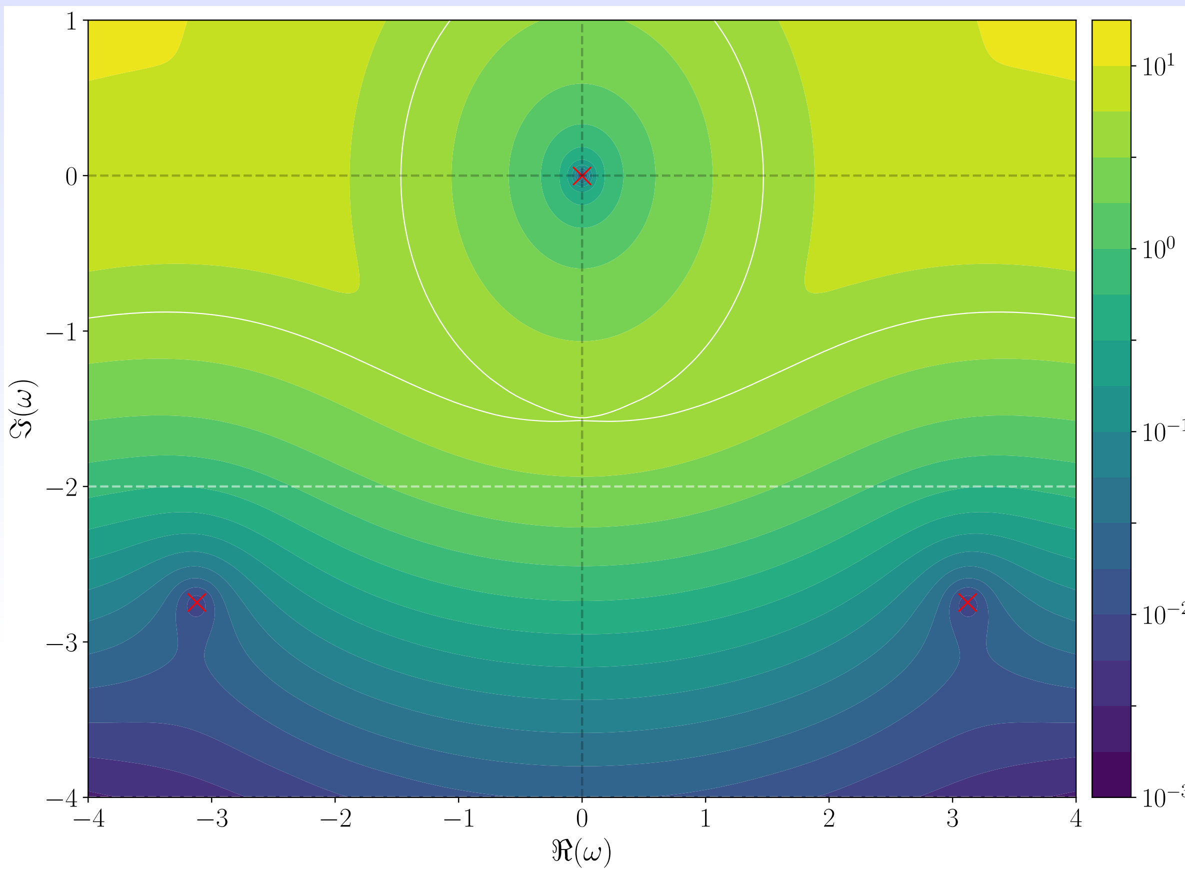
Results: scalar perturbations $k, Q = \text{finite}$



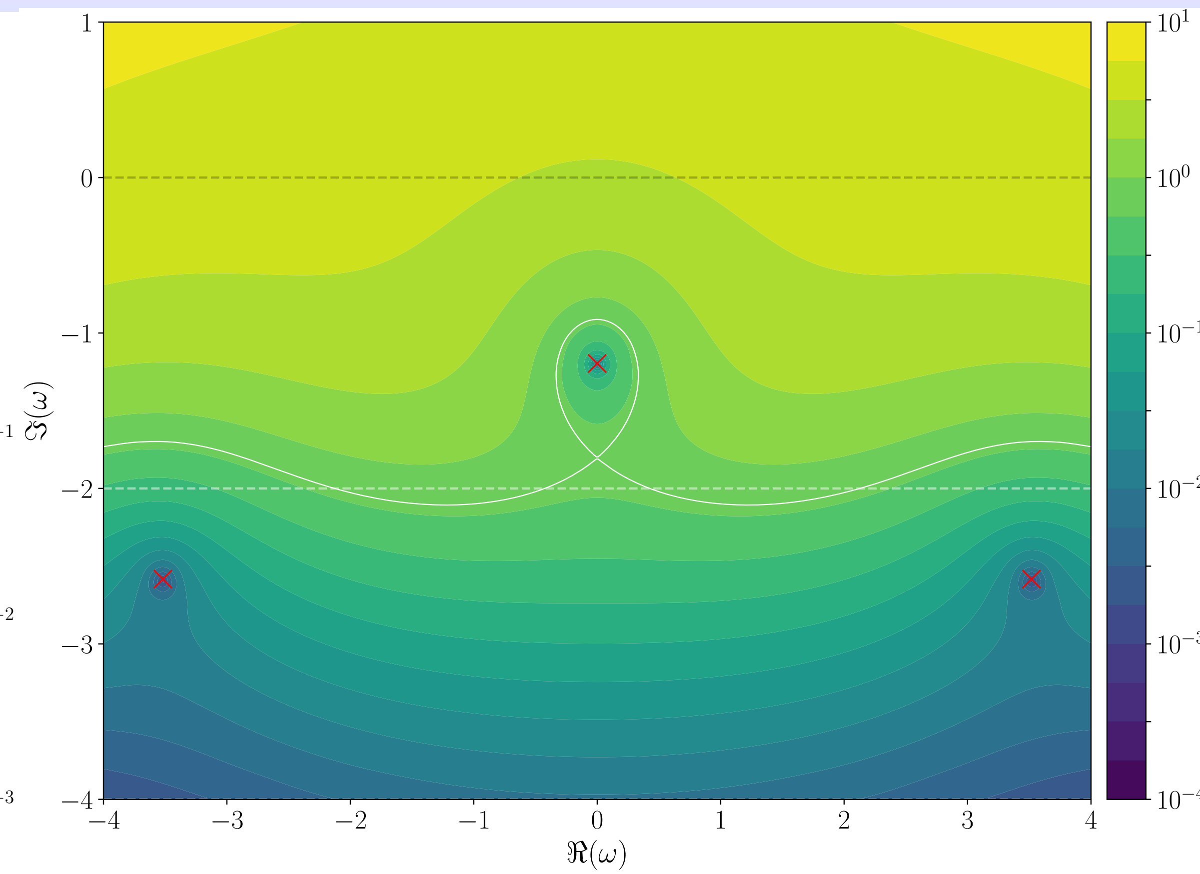
$\epsilon \sim 0.01$

- Instability for large-enough perturbations.
- More unstable as $|Im(\omega)|$: controls k, Q, n behaviour.
- Origin: high-frequency perturbations.
- Contour lines cross to unstable half place: pseudo-resonances and potential transients.

Results: e.g. shear channel



$k = 0, Q = 0$ $\epsilon \sim 1$



$k = 4, Q = 0$ $\epsilon \sim 0.6$

- Like before: dominating mode is most stable —hydro mode for small k, non-hydro modes for larger k.
- Pseudo-resonances and potential transients, even for hydro mode.

Gauge/Gravity interpretation

Spectra:

- Unstable excitation spectrum: short-lived excitations more unstable.
- Quantities extracted from QNM dispersions (e.g. transport coeffs) can be sensitive to perturbations.
- Impact on hydro radius of convergence?

Transients:

- Small error in theory may lead in a completely different equilibrium state.
- For hydro modes: potential transient instability of flow patterns (c.f. turbulence in fluids).

Summary

We have formulated the computation of the pseudospectrum in **null coordinates as Generalised EV problem**:

- Spectral instability for large enough perturbations.
- Potential transient instabilities and pseudoresonances.
- Improved convergence properties!!!

Vs hyperb. slicing

Future directions

- Significance of results in physical situations [Cardoso et al.]
- Physical importance of potential transients [Carballo, Withers]
- Coordinate dependence

Thank you for listening!

Extra

Energy norm

discretisation ↓ Gram Matrix ↓

$$E = \int_\Sigma T_{ab}\,\xi^a \kappa^b\, d\Sigma = \langle \psi, \psi \rangle_E = \psi \cdot G \cdot \psi$$

Pseudospectrum definition:

$$\sigma(A, B) = \{\lambda \in \mathrm{C} : \det(\lambda\, B - A) = 0\}$$

$$\sigma^\epsilon(A, B) = \{\lambda \in \mathrm{C} : \exists\, \delta A \,\mathrm{with} \|\delta A\| < \epsilon : \lambda \in \sigma(A + \delta A, B)\}$$