WARPED BLACK HOLES AS A DOUBLE COPY FROM MINI-TWISTOR SPACE

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IMPERIAL

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MEETS
FUNDAMENTAL PHYSICS UK

Gauge theories

Gravity

Rer

Inv

- Pl
- G

Is there a better description?

$V^{abc}_{3\,\mu\nu\sigma}($

$\cdot p_2\eta_{\mu\nu}\eta_{\rho\lambda}\eta_{\sigma\tau})$

$\eta_{\rho\nu}\eta_{\sigma\tau})$

$\eta_{\rho\sigma})$

$+ 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho}) - 2P_3(p_1 \cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\Big]\,,$

BCJ DOUBLE COPY & THE COLOR-KINEMATICS DUALITY

$$A^a_\mu : \quad \mathcal{A}_{YM} = \sum_{i \,\epsilon\, \text{trivalent}} \frac{c_i n_i}{d_i}$$

$$h_{\mu\nu},\ \phi,\ B_{\mu\nu} : \quad \mathcal{M}_G = \sum_{i \,\epsilon\, \text{trivalent}} \frac{n_i n_i}{d_i}$$

$$\phi^{a\,a'} : \quad \mathcal{A}_{\phi^3} = \sum_{i \,\epsilon\, \text{trivalent}} \frac{c_i c_i}{d_i}$$

$\mathbf{c} \equiv \mathbf{c}(f^{abc}) \qquad \mathbf{n} \equiv \mathbf{n}(p^\mu, \epsilon^\mu)$

Jacobi relations for color-kinematics duality

$$c_i + c_j + c_k = 0$$

$$n_i + n_j + n_k = 0$$

- Required for gauge/diffeo invariance.

TOPOLOGICALLY MASSIVE THEORIES

TMYM^2 = TMG

Parity breaking

1 d.o.f.

$$\boxed{S_{\mathrm{TMYM}} = -\frac{1}{4g^2}\int \mathrm{Tr}\left(F\wedge *F + m\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)\right)}$$

$$\boxed{\varepsilon^a_\mu + \frac{i}{m}\epsilon_{\mu\nu\rho}p^\nu\varepsilon^a_\rho = 0}$$

$$\boxed{S_{\mathrm{TMG}} = -\frac{1}{2\kappa}\int\left(*R + m\left(\Gamma\wedge d\Gamma + \frac{2}{3}\Gamma\wedge\Gamma\wedge\Gamma\right)\right)}$$

$$\boxed{\varepsilon_{\mu\nu} = \varepsilon_\mu\varepsilon_\nu}$$

TOPOLOGICALLY MASSIVE THEORIES

$$\mathrm{TMYM}^2 = \mathrm{TMG}$$

- Amplitudes DC up to 5 points **MCG**, Momeni, Rumbutis
- Amplitudes DC in eikonal limit at all loop orders
- Classical double copy in coordinate space

- Classical double copy in Minkowski twistor space

MCG, Emond, Moynihan, Rumbutis, White

COTTON DOUBLE COPY

Cotton tensor (3d conformal tensor)

$$C_{\mu\nu} = \epsilon_{\mu\rho\sigma} D^{\rho} \left(R^{\sigma}_{\nu} - \frac{1}{4}\delta^{\sigma}_{\nu} R \right)$$

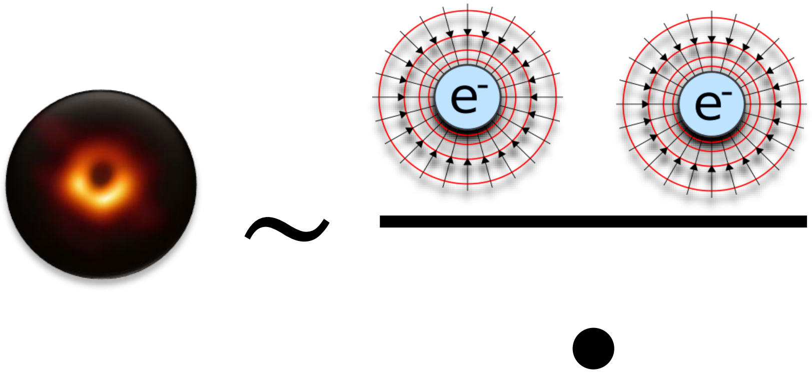
$$\boxed{C_{\alpha\beta\gamma\delta} = C_{\mu\nu}\sigma^{\mu}_{\alpha\beta}\sigma^{\nu}_{\gamma\delta}}$$

$$\sigma : \ \mathrm{SO}(2,1) \to \mathrm{SL}(2,\mathbb{R})$$

$$v_{\alpha\beta} = v_{\mu}\sigma^{\mu}_{\alpha\beta}$$

$$\boxed{\Phi_{\alpha\beta} = \frac{1}{2}\epsilon_{\mu\nu\rho}F^{\nu\rho}\sigma^{\mu}_{\alpha\beta}}$$

$$\boxed{C_{\alpha\beta\gamma\delta} = \frac{\Phi_{(\alpha\beta}\Phi_{\gamma\delta)}}{\varphi}}$$



COTTON DOUBLE COPY FOR GRAVITATIONAL WAVES

MCG, Momeni, Rumbutis

TMG = TMYM^2

$$\nabla_\alpha{}^\epsilon C_{\beta\gamma\delta\epsilon} = m C_{\alpha\beta\gamma\delta}$$

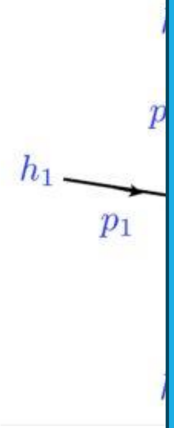
$$C_{\alpha\beta\gamma\delta} = \frac{\Phi_{(\alpha\beta}\Phi_{\gamma\delta)}}{\varphi}$$

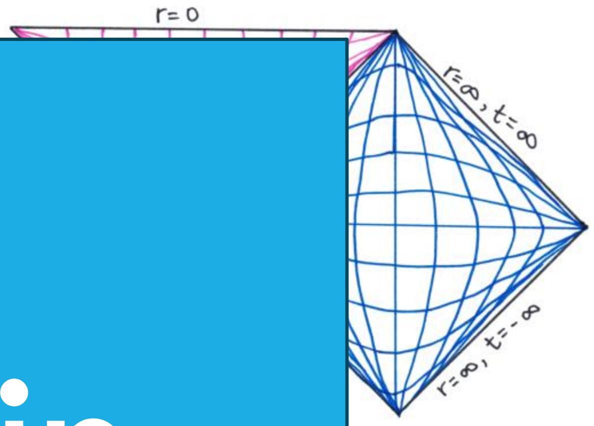
$$\nabla_\alpha{}^\gamma \Phi_{\beta\gamma} = m\Phi_{\alpha\beta}$$

$$\nabla^2\varphi = \left(m^2 + \frac{R}{6}\right)\varphi$$

$$C_{\alpha\beta\gamma\delta} = \Psi_4 o_\alpha o_\beta o_\gamma o_\delta \qquad \Phi_{\alpha\beta} = \Phi_2 o_\alpha o_\beta$$

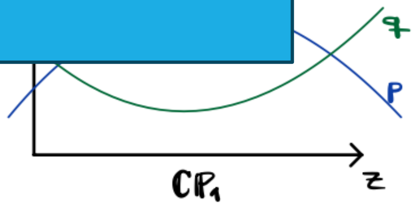
DOUBLE COPY IN DIFFERENT SPACES





What happens in curved spacetimes?

Cohomology classes



ADS3 MINI-TWISTOR SPACE

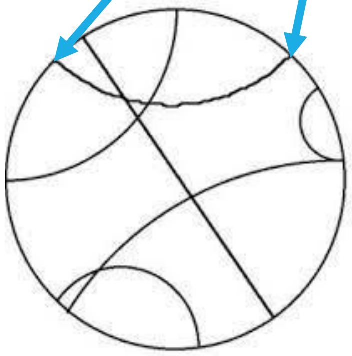
Space of geodesics

Bulk point in embedding space

Boundary points

$$x^{\alpha\dot{\alpha}} \sim \bar{\mu}^{\alpha}\mu^{\dot{\alpha}} + t\lambda^{\alpha}\bar{\lambda}^{\dot{\alpha}}$$

$$(\lambda_\alpha, \mu^{\dot{\alpha}}) \sim (r'\lambda_\alpha, r\mu^{\dot{\alpha}})$$



Isometries of geodesics

$$\mu^{\dot{\alpha}} = x^{\alpha\dot{\alpha}}\lambda_\alpha$$

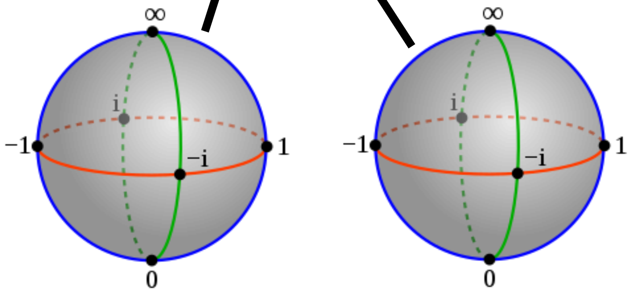
Incidence relation

Isometries of geodesics

$$\mathbb{MT}$$

$$\cong (\mathrm{SL}(2,\mathbb{C})) \times \mathrm{SL}(2,\mathbb{C}))/(Q \times Q^{\intercal})$$

$$(\lambda_\alpha, \mu^{\dot{\alpha}}) \in \mathbb{CP}^1 \times \mathbb{CP}^1$$

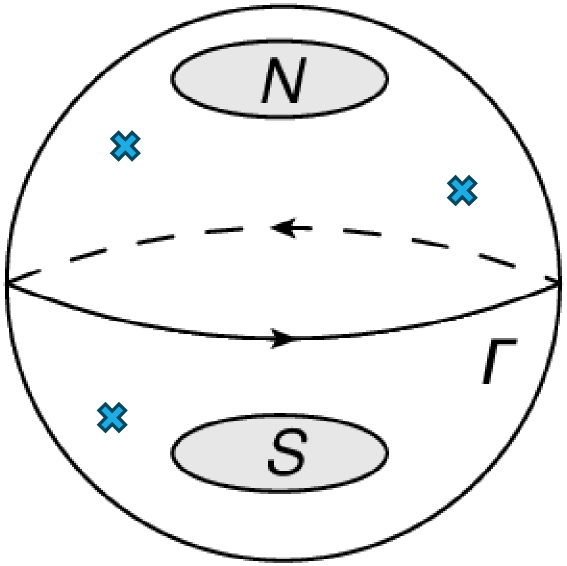


PENROSE TRANSFORM

Isometries of geodesics

$$f(\alpha\lambda, \alpha'\mu) = \alpha^{\tilde{m}-s-1}\alpha'^{-\tilde{m}-s-1}f(\lambda, \mu)$$

$$\phi_{\alpha_1 \ldots A\alpha_{2s}} = \oint_\Gamma \langle \lambda d\lambda \rangle \lambda_{\alpha_1} \ldots \lambda_{\alpha_{2s}} \check{f}(\lambda, \mu)\,|_x$$



$$\nabla_\alpha{}^\beta \Psi_{\beta\alpha_2 \ldots \alpha_{2s}} = m\Psi_{\alpha_1 \ldots \alpha_{2s}}$$

$$\nabla^2\psi = (m^2 - 1)\psi$$

AdS3 mini-twistor space double copy

MCG, Keseman, Jaitly

In flat space

$$\check{f}_{s=2,m} = \frac{\check{f}_{s=1,m}\check{f}_{s=1,m}}{\check{f}_{s=0,m}}$$

Bulk to boundary propagator

$$\frac{A_{\alpha_1} \ldots A_{\alpha_{2s}}}{(v \cdot x)^{s+\Delta_+}}$$

Boundary point

Bulk point

In AdS_3 $\quad so(2,2) \simeq sl(2,\mathbb{R}) \oplus \overline{sl(2,\mathbb{R})}$

$$\Delta_\pm = h + \bar{h} = 1 \pm m$$

$$\boxed{\check{f}_{s=2,\Delta_\phi-2} = \frac{\check{f}_{s=1,\Delta_\phi-1}\check{f}_{s=1,\Delta_\phi-1}}{\check{f}_{s=0,\Delta_\phi}}}$$

WARPED BLACK HOLES

Warped AdS$_3$ $\quad SL(2,\mathbb{R}) \times U(1)$

$$ds^2 = (L_{\mathrm{AdS}_2})^2 \left[\frac{-dt^2 + dx^2}{x^2} + \lambda^2 \left(dz + \frac{dt}{x}\right)^2\right]$$

Squashing parameter

Killing vector

$$m^\mu \partial_\mu = \frac{1}{\lambda L_{\mathrm{AdS}_2}} \partial_z$$

AdS$_3$ $\quad \lambda = 1 \to m = \pm 3$

Quotient by isometry Γ

Warped Black Holes

Kerr-Schild form

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + k_\mu k_\nu \phi$$

$$m\bar{L} = \pm 3$$

$$k_\mu dx^\mu = dt + dx$$

$$\phi = \frac{L_{\mathrm{AdS}_2}}{x^2}(\lambda - 1)$$

MCG, Beetar

Mini-twistor space

$$\boxed{\check{f}_{s=2,\Delta=1+3} = \frac{1}{\langle\lambda A\rangle^3 \langle\lambda B\rangle^3}}$$

Coordinate space

$$\boxed{C_{\alpha\beta\gamma\delta}(x) = \langle AB\rangle^{-3} o_{(\alpha} o_\beta r_\gamma r_{\delta)}} \quad \langle AB\rangle^{-3} = \lambda \frac{(1-\lambda^2)}{L^3_{\mathrm{AdS}_2}}$$

WARPED BLACK HOLES SINGLE COPY

Mini-twistor space

$$\boxed{\check{f}_{s=1,\Delta=1+2} = \frac{1}{\langle\lambda A\rangle^2\langle\lambda B\rangle^2}}$$

Coordinate space

$$\boxed{\Phi_{\alpha\beta}(x) = \langle AB\rangle^{-2} o_{(\alpha} r_{\beta)}}$$

Electric Field

$$^*F^\mu \propto \left(\lambda(1-\lambda^2)\right)^{-2/3} m^\mu$$

Killing vector of WAdS

$$m^\mu\partial_\mu = \frac{1}{\lambda L_{\mathrm{AdS}_2}}\partial_z \qquad \mathcal{L}_m F = 0$$

TMG solution can be reproduce from Killing vector

Gürses; 2010

Double Copy

Charged BH?

Zeroth copy

$$\check{f}_{s=1,\Delta=1+1} = \frac{1}{\langle\lambda A\rangle\langle\lambda B\rangle}$$

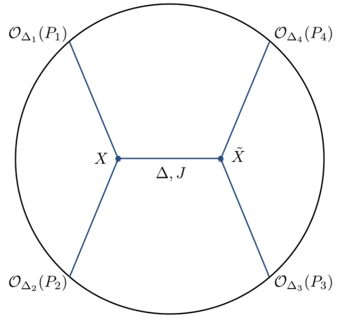
$$\varphi = \langle AB\rangle^{-1}$$

✓ Mini-twistor space

$$\boxed{\check{f}_{s=2,M=3} = \frac{\check{f}_{s=1,M=2}\check{f}_{s=1,M=2}}{\check{f}_{s=0,M=0}}}$$

✓ Coordinates space

$$\boxed{C_{\alpha\beta\gamma\delta} = \frac{\Phi_{(\alpha\beta}\Phi_{\gamma\delta)}}{\varphi}}$$



Perturbative

Momentum space ?

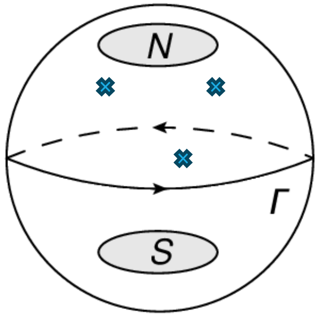
AdS3 double copy

Coordinate space

Mini-twistor space

On-shell solutions

Non- perturbative in some cases



Special solutions