Bulk quantum corrections for non-spatial holographic entanglement

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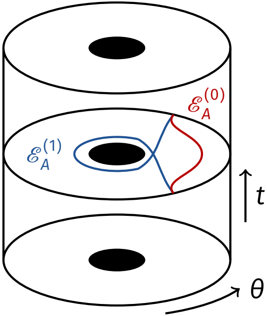
Eurostrings 2024

Joint work with Dongming He to appear soon

Introduction

What is the field theory dual to the area of extremal surfaces $\mathscr{E}_A$ that are topologically distinct from RT surfaces?

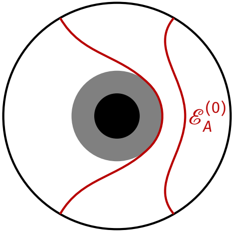


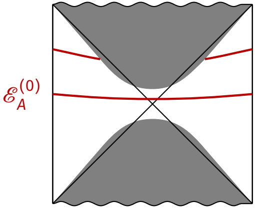
$$\frac{\text{Area}(\mathscr{E}_A^{(w)})}{4G_N} \stackrel{?}{=} S_A^{(w)} = -\text{Tr}(\rho_A^{(w)} \log \rho_A^{(w)})$$

→ Entanglement between different fields as well as between spatial DoF

Motivation

- "entanglement builds geometry" [Swingle (2009), Van Raamsdonk (2010)]: bulk geometry encoded in terms of entanglement
- entanglement shadows [Freivogel, Jefferson, Kabir, Mosk, Yang (2014)]: RT surfaces don't probe all of the spacetime





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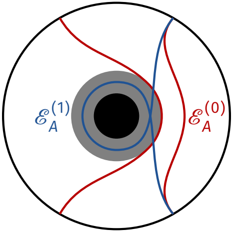
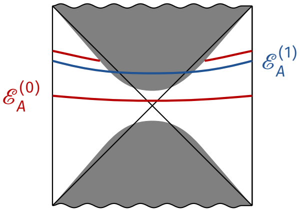




- Non-minimal extremal surfaces can probe closer to singularities/horizons
  - → resolve entanglement shadows
- Expect large quantum corrections close to singularities
  - → breakdown of "entanglement builds geometry"?

Entwinement: definition

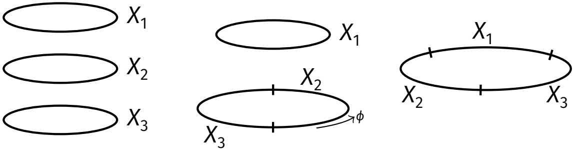
[Balasubramanian, Chowdhury, Czech, de Boer (2014); Balasubramanian, Bernamonti, Craps, de Jonckheere, Galli (2016); MG (2021)]

- bottom-up AdS_3/CFT_2 setup: S_N orbifold CFT of generic seed CFT (e.g. D1/D5 system or $AdS_3 \times X$ with pure NS-NS flux [Eberhardt (2021); Knighton, Sriprachyakul (2024)])

$$S[X] = \sum_{i=1}^{N} S_{\text{seed}}[X_i] + \text{marginal deformations}$$

- field content: N indistinguishable copies of seed fields X_i
- Hilbert space contains states with long strands joining together multiple fields

example: $N = 3$

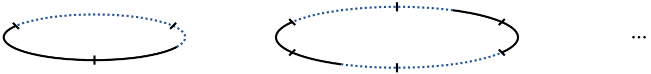


Entwinement: definition

[Balasubramanian, Chowdhury, Czech, de Boer (2014); Balasubramanian, Bernamonti, Craps, de Jonckheere, Galli (2016); MG (2021)]

- ordinary entanglement entropy: consider one or multiple intervals of length L on long strands of arbitrary size

- entwinement: subsystem parametrized by two integers $w < m$ and interval length L
  - → consider one or multiple intervals of length $w + L$ on long strands of length $m\mathbb{Z}$



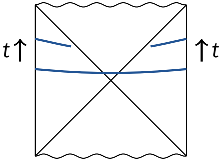
Entwinement at large N: bulk dual

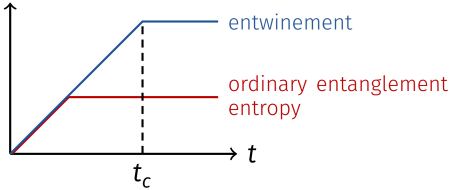
- Explicit computations using large N techniques give

$$S_A^{(w,m)} = \frac{1}{m}\frac{\text{Length}[\mathscr{E}_A^{(w)}]}{4G_N} + O(1)$$

for conical defects [Balasubramanian, Chowdhury, Czech, de Boer (2014)]
and BTZ black holes [MG (2021)]

- Two-sided black hole: phase transition at $t_c \sim w$





- Winding number limit $w < N \sim 1/G_N$: cannot probe arbitrarily close to the horizon/singularity

→ How reliable is the entanglement/geometry connection for finite w and finite N?

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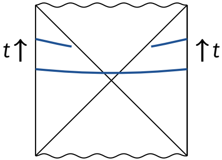

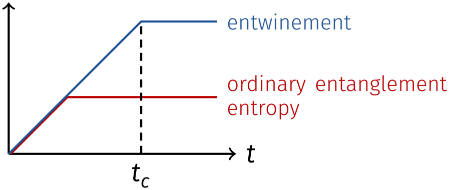

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$1/N$ corrections: formal procedure

- Entwinement ≈ ordinary entanglement entropy in "covering theory" on m-times larger torus

→ Bulk quantum corrections for entwinement follow from [Faulkner, Lewkowycz, Maldacena (2013)] applied to the covering theory

- DoF for entwinement are located on a subset of all possible collections of long strands (subset $\mathscr{H}_m$ of the Hilbert space)
- QFT with spectrum $\mathscr{H}_m$: $S_{N/m}$ orbifold of the same seed CFT where only spins $m\mathbb{Z}$ are allowed,

$$Z(\tau) = \mathrm{Tr}_{\mathscr{H}_m}[q^{L_0}\bar{q}^{\bar{L}_0}] = \sum_{h-\bar{h}\in m\mathbb{Z}} q^{h-\frac{c}{24m}}\bar{q}^{\bar{h}-\frac{c}{24m}}$$

→ can be rewritten as partition function of covering theory,

$$Z(\tau) = \sum a_{h,\bar{h}}\, \chi_h^{(c/m)}(-m/\tau)\chi_{\bar{h}}^{(c/m)}(-m/\bar{\tau})$$

- Entwinement = ordinary entanglement entropy for interval of length $(w+L)/m$ and state $\rho \propto \sum_{h-\bar{h}\in m\mathbb{Z}} q^{h-\frac{c}{24m}}\bar{q}^{\bar{h}-\frac{c}{24m}}\,|h,\bar{h}\rangle\langle h,\bar{h}|$

$1/N$ corrections: explicit computation

- Direct computation of $1/N$ correction to entwinement at finite temperature gives (using techniques from [Barrella, Dong, Hartnoll, Martin (2013)] valid for $m, w = O(1)$)

$$S_A^{(w,m)} = \frac{N c_{\text{seed}}}{3m} \log\Big[\frac{\beta}{2\pi^2 \epsilon_{\text{UV}}} \sinh\Big(\frac{2\pi^2 (L+w)}{\beta}\Big)\Big] + e^{-2m\frac{2\pi}{\beta}} \left[8 - \frac{16\pi^2 (L+w)}{\beta} \coth\left(\frac{2\pi^2 (L+w)}{\beta}\right)\right] + O(e^{-4m\frac{2\pi}{\beta}}) + O(1/N).$$

- Corrections increase ~ lineary with w but decrease exponentially with m

$\Rightarrow$ For small $O(1)$ winding numbers: quantum corrections are tiny, entanglement builds geometry is robust

- large $O(N)$ winding numbers: no universal result for $1/N$ corrections, depends on seed theory in question

$1/N$ corrections: comments

What about the exactly marginal deformations of the S_N orbifold?

- long strands exist as before but spectrum and OPE coefficients changes

→ generalization of FLM: covering theory depends on coupling constant

- explicit computations for small winding numbers: result shown is the universal piece from $1/N$ corrections to the dominant conformal block

→ further non-universal $1/N$ corrections may appear depending on the OPE coefficients and spectrum

Summary

- Entanglement between internal DoF (entwinement) is essential for understanding the entanglement/geometry connection in AdS/CFT
- FLM formula applies to entwinement, given the right choice of covering theory and state
- Obstructions to bulk geometry reconstruction from entwinement due to bulk quantum corrections can generically appear for large $O(N) = O(1/G_N)$ winding numbers

Open questions:

- Entwinement for large winding numbers?
- Entanglement between internal DoF in higher dimensions?
- String theoretic AdS/CFT constructions based on $\mathrm{AdS}_{d+1}\times$ internal space. Probe geometry of the internal space using entanglement? [Mollabashi, Shiba, Takayanagi (2014); Karch, Uhlemann (2015); Taylor (2015); Das, Kaushal, Mandal, Nanda, Radwan, Trivedi (2022)]